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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

No. 575

ESTIMATION OF MOMENTS OF INERTIA OF AIRPLANES FROM DESIGN DATA

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SUMMARY

A method of determining the moments of inertia of an airplane from design data pertaining to the weights and locations of the component parts is described. The computations required to ascertain the center-of-gravity position are incorporated with the calculations of moments of inertia. A complete set of data and calculations for a modern airplane is given to illustrate the procedure. From a comparison between calculated values and measured values it is believed that the moments of inertia can be estimated within 10 percent by the use of this method.

INTRODUCTION

A study of the forces and couples involved in the rotational motion of an airplane involves a knowledge of the moments of inertia. A method for determining the moments of inertia of airplanes experimentally is described in reference 1. In some cases, however, the moments of inertia of an airplane may be needed before its construction is actually completed; furthermore, it is not always convenient or possible to measure the moments of inertia owing to lack of equipment even if the airplane is available. There may be frequent cases, therefore, wherein it will be desirable to calculate the approximate moments of inertia from design data. It is believed that a description of a method for making such calculations will prove useful.

The method described makes use of the design data on the weights and the locations of the component parts of the airplane. The calculations of moments of inertia are combined with the calculations necessary to determine the center of gravity of the airplane. A complete set of calculations for an actual airplane is given to illustrate the procedure. This example shows how the calculations can be combined with those required to determine the center of gravity and also how the principal moments of inertia and the angles between the principal axes and the airplane axes can be determined.

METHOD

The first step in the calculations is to set up three mutually perpendicular reference planes such as would be required for calculations of the center-of-gravity position (fig. 1). It is known that the center of gravity will lie in the plane of symmetry so that the plane of symmetry is used as the $X'Z'$ reference plane. For convenience, the $Y'Z'$ reference plane is placed ahead of the forward end of the airplane and the $X'Y'$ reference plane below the lowest part and parallel to the thrust line. The X', Y', and Z' reference axes are the intersections of these planes. For convenience in making the calculations the units used are inches and pounds until the final stage is reached, when the moments of inertia are converted to slug-feet squared.

The method of making the calculations is best illustrated by carrying through a complete set of calculations for an actual airplane. Such calculations are shown in table I. Columns 1, 2, 3, 5, 6, and 7 list items normally required for the computation of the center of gravity. Column 1 shows the element considered and column 2 is its weight. Column 3 shows the distances aft of the $Y'Z'$ plane and column 5 shows the distances above the $X'Y'$ plane. Column 4, the distance of the element from the plane of symmetry, is not required for the center-of-gravity determination, since the center of gravity must lie in this plane, but it is needed for the calculations of the moments of inertia. Columns 6 and 7 give the moments of the elements relative to the reference planes. The summation of column 2 gives the total weight, and the summation of 6 and 7 gives the total moments relative to the $Y'Z'$ and $X'Y'$ reference planes. The distances of the center of gravity aft and above the reference planes are determined by dividing the total moments by the total weight. These distances are designated as $x_{c.g.}$ and $z_{c.g.}$, respectively.

All subsequent calculations are required only for the determination of moments of inertia. Before further reference is made to the table, however, some additional explanation is required. The moment of inertia of a concentrated weight w about the X' reference axis is

$$w \times k_{X'}{}^2$$

where $k_{X'}$ is the radius of gyration

but $$k_{X'}{}^2 = y^2 + z^2$$

where y and z are the distances to the $X'Z'$ and $X'Y'$ reference planes, respectively.

Hence $$wk_{X'}{}^2 = wy^2 + wz^2$$

Similarly $$wk_{Y'}{}^2 = wx^2 + wz^2$$

and

$$wk_{Z'}{}^2 = wy^2 + wx^2$$

Values for wx^2, wy^2, and wz^2 are given in columns 8, 9, and 10 of table I.

Many elements of the airplane are of such size that their masses may not be assumed to be concentrated at their centers of gravity. For such elements it is necessary to add to the moment of inertia of the element, considered as a concentrated mass, its moment of inertia about an axis passing through its own center of gravity. This latter moment of inertia must be estimated. In general, the precision required in the estimation is not very great provided that reasonably small elements are taken, and for many elements this item can actually be neglected. A further discussion of this point is given later. The estimated moments of inertia ΔI of the larger items about axes passing through their own centers of gravity are shown in columns 11, 12, and 13. The symbol ΔI_X indicates a moment of inertia about an axis parallel to the airplane X' reference axis, etc.

The total moments of inertia of the airplane relative to axes passing through its center of gravity are found as follows:

The total moments of inertia of the airplane about the three reference axes are

$$I_{X'} = \Sigma wy^2 + \Sigma wz^2 + \Sigma\Delta I_X$$

$$I_{Y'} = \Sigma wx^2 + \Sigma wz^2 + \Sigma\Delta I_Y$$

$$I_{Z'} = \Sigma wx^2 + \Sigma wy^2 + \Sigma\Delta I_Z$$

The center of gravity lies in the X'Z' plane but is displaced from the Y'Z' and X'Y' planes by distances designated as $x_{c.g.}$ and $z_{c.g.}$, respectively. The total moment of inertia of the airplane about the Y axis passing through the center of gravity is

$$I_Y = I_{Y'} - W(x_{c.g.}{}^2 + z_{c.g.}{}^2)$$

where W is the total weight.

By substitution for $I_{Y'}$ this equation can be reduced to

$$I_Y = \left[\Sigma wx^2 - Wx_{c.g.}{}^2\right] + \left[\Sigma wz^2 - Wz_{c.g.}{}^2\right] + \Sigma\Delta I_Y$$

Since there is no term $y_{c.g.}$ the equations for the other two axes reduce to

$$I_X = \Sigma wy^2 + \left[\Sigma wz^2 - Wz_{c.g.}{}^2\right] + \Sigma\Delta I_X$$

and

$$I_Z = \left[\Sigma wx^2 - Wx_{c.g.}{}^2\right] + \Sigma wy^2 + \Sigma\Delta I_Z$$

These moments of inertia are in units of pound-inches squared, which are then converted to slug-feet squared by multiplying by the appropriate factors, namely,

$$\frac{1}{32.17} \times \frac{1}{144}.$$

In order to determine the locations of the principal axes, it is necessary to find the product of inertia wxz for each item as shown in column 14. In these computations it is permissible to neglect the products of inertia of the larger elements about the axes passing through their own center of gravity. The summation of column 14 represents the products of inertia with respect to the reference planes so that it is necessary to subtract $Wx_{c.g.}z_{c.g.}$ to obtain the product of inertia with respect to the center of gravity. This quantity, like the moments of inertia, is in units of pound-inches squared and is converted to slug-feet squared in the same manner.

If the total product of inertia with respect to the center of gravity is designated by H, the angle η between the principal axes and reference axes is given by

$$\tan 2\eta = \frac{2H}{I_Z - I_X} \qquad \tan 2\eta = \frac{2 \times 181}{9{,}096 - 3{,}061} = 0.05998$$

$$\eta = 1^\circ\ 43'$$

The principal moments of inertia are given by

$$I_{X_{prin}} = I_X \cos^2 \eta + I_Z \sin^2 \eta - H \sin 2\eta$$

$$I_{Y_{prin}} = I_Y$$

$$I_{Z_{prin}} = I_X \sin^2 \eta + I_Z \cos^2 \eta + H \sin 2\eta$$

then since

$$\eta = 1^\circ\ 43'$$

$$\sin \eta = 0.0300$$

$$\cos \eta = 0.9996$$

$$\sin 2\eta = 0.0599$$

and, since the other quantities are as previously determined, it follows that

$$I_{X_{prin}} = 3{,}061 \times (0.9996)^2 + 9{,}096 \times (0.0300)^2 - 181 \times 0.0599 = 3{,}056$$

$$I_{Y_{prin}} = 6{,}650$$

$$I_{Z_{prin}} = 3{,}061 \times (0.0300)^2 + 9{,}096 \times (0.9996)^2 + 181 \times 0.0599 = 9{,}102$$

DISCUSSION

The accuracy with which the moments of inertia of the airplane are determined by this method depends primarily upon the accuracy with which the weight and disposition of the various elements is known. Another factor of some importance is the accuracy with which the moments of inertia of elements about their own centers of gravity are known. In general, the moments of inertia of the various elements about their own centers of gravity are small relative to the total moments of inertia of the airplane so that the accuracy of these items does not need to be very great. For many cases it is permissible to neglect these items altogether but it should be appreciated that the error due to neglecting such items is cumulative, whereas errors due to erroneous estimates are probably random and tend to nullify one another.

Judgment is required in determining how much care must be observed in subdividing the airplane and estimating the moments of inertia of elements in order to keep the errors due to incorrect estimates of the moments of inertia of elements about their own centers of gravity to a minimum. One large item that cannot conveniently be subdivided into desirably small parts is the engine. In the example the estimated moment of inertia of the engine about its X axis is about 2 percent of the total moment of inertia of the airplane about its X axis. A 50-percent error in the estimation of the moment of inertia of the engine about its X axis would therefore result in an error of about 1 percent in the final result. Thus, for an element as large as the engine, it is apparent that reasonable care should be taken in making the estimate.

The accuracy of this method of calculating the moments of inertia was investigated by making the calculations for an airplane for which the moments of inertia were also determined by experiment. The airplane was available in disassembled form so that the weights of various major elements could be accurately determined and checked against weight estimates of small elements. It was found that the calculated values were lower than the experimental values by 6.5, 4, and 1 percent for the X, Y, and Z axes, respectively.

The error in the experimental moments of inertia is greatest for the X axis for which it might amount to as much as 2.5 percent (reference 1). Thus the disagreement between calculated and experimental values in this case could be accounted for by a maximum error of 4 percent in the calculated values. For the more general case in which the weights of elements or assemblies are obtained solely from estimates, some additional error might be incurred but it seems reasonable to expect that the error can be kept within 10 percent without difficulty.

This method of calculating moments of inertia does not take into consideration the entrapped air. The moment of inertia of the entrapped air, however, would increase the calculated values only by a small amount that can safely be neglected.

Langley Memorial Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., July 17, 1936.

REFERENCE

1. Soulé, Hartley A., and Miller, Marvel P.: The Experimental Determination of the Moments of Inertia of Airplanes. T.R. No. 467, N.A.C.A., 1933.

TABLE I

COMPUTATIONS OF MOMENTS OF INERTIA

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.
Item	Weight	x	y	z	wx	wz	wx^2	wy^2	wz^2	ΔI_X	ΔI_Y	ΔI_Z	wxz
Center section nose assembly	108.8	102	-	57	11,098	6,202	1,131,955	-	353,491	261,229	-	261,229	632,563
Center section beam, etc.	204.6	121	-	57	24,757	11,662	2,995,549	-	664,745	491,245	-	491,245	1,411,126
Center section ribs, etc.	84.2	148	-	55	12,462	4,631	1,844,317	-	254,705	202,164	33,680	235,844	685,388
Flap	22.0	180	-	53	3,960	1,166	712,800	-	61,798	48,598	-	48,598	209,880
Outer panel nose	104.6	105	156	65	10,983	6,799	1,153,215	2,545,546	441,935	184,514	-	184,514	713,895
Outer panel beam	155.6	120	156	65	18,672	10,114	2,240,640	3,786,682	657,410	274,478	-	274,478	1,213,680
Outer panel ribs	89.8	139	156	64	12,482	5,747	1,735,026	2,185,373	367,821	158,407	17,601	176,008	798,861
Ailerons	31.4	172	156	62	5,401	1,947	928,938	764,150	120,702	55,390	-	55,390	334,850
Horizontal tail	87.1	367	-	96.7	31,966	8,423	11,731,412	-	814,463	176,378	-	176,378	3,091,083
Vertical tail	31.4	352	-	125	11,053	3,925	3,890,586	-	490,625	110,200	10,174	31	1,381,600
Fuselage skeleton	314.0	176	-	81	55,264	25,434	9,726,464	-	2,060,154	69,394	1,576,594	1,570,000	4,476,384
Engine mount	40.5	60	-	80	2,430	3,240	145,800	-	259,200	5,184	5,184	5,184	194,400
Turtleback (fairing)	48.5	254	-	80	12,319	3,880	3,129,026	-	310,400	15,181	57,861	56,648	985,520
Firewall	11.0	70	-	80	770	880	53,900	-	70,400	2,200	1,100	1,100	61,600
Steps	2.0	170	20	70	340	140	57,800	800	9,800	-	-	-	23,800
N.A.C.A. cowling	70.0	50	-	80	3,500	5,600	175,000	-	448,000	16,940	15,470	15,470	280,000
Cabin and windshield	66.5	146	-	108	9,709	7,182	1,417,514	-	775,656	2,394	106,400	108,794	1,048,572
Foot troughs	2.0	77	5	68	154	136	11,858	50	9,248	-	-	-	10,472
Floor, rear	9.5	210	-	66	1,995	627	418,950	-	41,382	608	1,368	1,976	131,670
Wing fillets	18.5	142	20	58	2,627	1,073	373,034	7,400	62,234	-	18,944	18,944	152,366
Bottom cowling and side frames	27.0	140	11	75	3,780	2,025	529,200	3,267	151,875	-	24,300	24,300	283,500
Arresting door	1.3	284	-	63	369	82	104,853	-	5,160	5	-	5	23,260
Tail-wheel pan, etc.	4.0	365	-	84	1,460	336	532,900	-	28,224	100	-	100	122,640
Side doors	17.0	143	18	82	2,431	1,394	347,633	5,508	114,308	1,088	28,288	27,200	199,342
Baggage door	1.8	165	-	63	297	113	49,005	-	7,144	720	-	720	18,711
Fabric and dope	13.0	254	-	80	3,302	1,040	838,708	-	83,200	4,394	15,509	15,509	264,160
Tail cone	7.5	385	-	91	2,888	683	1,111,688	-	62,108	120	-	120	262,763
Cowling, stations 1-2	12.0	110	-	95	1,320	1,140	145,200	-	108,300	1,200	-	1,200	125,400
Chassis (retracted)	232.4	115	54	51	26,726	11,852	3,073,490	677,678	604,472	-	23,240	23,240	1,363,026
Retracting mechanism	28.6	110	25	67	3,146	1,916	346,060	17,875	128,385	-	-	-	210,782
Wheels, etc.	91.0	141	54	56	12,831	5,096	1,809,171	265,356	285,376	-	-	-	718,536
Tail wheel	26.0	360	-	74	9,360	1,924	3,369,600	-	142,376	26	-	26	692,640
Engine	1049.0	33	-	80	34,617	83,920	1,142,361	-	6,713,600	253,858	253,858	253,858	2,769,360
Engine accessories	90.6	52	-	83	4,711	7,520	244,982	-	624,143	9,060	-	9,060	391,030
Engine controls	11.0	103	10	76	1,133	836	116,699	1,100	63,536	-	-	-	86,108
Propeller	222.0	9.3	-	80	2,065	17,760	19,201	-	1,420,300	177,600	89,300	89,300	165,168
Starting system	37.0	56	-	85	2,072	3,145	116,032	-	267,325	148	333	481	176,120
Lubricating system	26.0	69	-	82	1,794	2,132	123,786	-	174,824	1,274	-	1,274	147,108
Fuel system	82.0	128	-	80	10,496	6,560	1,343,488	-	524,300	22,058	20,008	25,866	839,680
Instruments	38.0	102	-	92	3,876	3,496	395,352	-	321,632	2,432	60,800	63,232	356,592
Surface controls	81.5	160	-	71	13,040	5,787	2,086,400	-	410,842	123,962	130,400	254,362	925,840
Furnishings	160.0	156	-	80	24,960	12,800	3,893,760	-	1,024,000	108,160	400,000	508,160	1,996,800
Electrical equipment	100.7	128	15	81	12,890	8,157	1,649,869	22,658	660,693	-	-	-	1,044,058
Hoist sling	6.0	115	-	102	690	612	79,350	-	62,424	864	-	864	70,380
AIRPLANE EMPTY	3867.4	106.6	0	74.8	412,196	289,138	67,342,573	10,283,443	22,263,736	2,681,572	2,890,404	4,980,508	31,090,714
Pilot	200.0	105	-	90	21,000	18,000	2,205,000	-	1,620,000	33,800	28,800	5,000	1,890,000
Observer	200.0	205	-	89	41,000	17,800	8,405,000	-	1,584,200	33,800	28,800	5,000	3,649,000
Fuel	780.0	132	-	85.5	102,960	66,690	13,590,720	-	5,701,995	141,180	172,380	157,560	8,803,080
Oil	75.0	71	-	85	5,325	6,375	378,075	-	541,875	3,675	-	3,675	452,625
Very Pistol	3.9	195	14	75	761	293	148,298	764	21,938	-	-	-	57,038
Smoke candles	4.0	184	14	96	736	384	135,424	784	36,864	-	-	-	70,656
Float lights	9.0	190	14	72	1,710	648	324,900	1,764	46,656	-	-	-	123,120
Radio	142.7	178	-	85	25,401	12,130	4,521,307	-	1,031,008	5,137	-	5,137	2,159,051
Chart board, etc	3.7	80	3	94	296	348	23,680	33	32,693	-	-	-	27,824
Drift sight	1.6	222	14	84	355	134	78,854	314	11,290	-	-	-	29,837
First aid	4.0	165	10	57	660	228	108,900	400	12,996	-	-	-	37,620
Life raft	34.0	136	-	101	4,624	3,434	628,864	-	346,834	306	-	306	467,024
USEFUL LOAD	1457.9	140.5	0	86.7	204,828	126,464	30,549,022	4,079	10,988,299	217,898	229,980	176,678	17,766,875
TOTALS	5325.3				617,024	414,848	97,891,595	10,287,522	33,252,035	2,899,470	3,120,384	5,157,186	48,857,589
CORRECTION	5325.3	115.9	0	77.8			71,200,000	0	32,260,000				48,018,336
							26,691,595	10,287,522	992,035				839,253
										10,287,522	26,691,595	26,691,595	
										992,035	992,035	10,287,522	
I =										14,179,027	30,804,014	42,136,303	
k^2 =										2,662	5,784	7,912	
k =										51.6	76.1	89	
		$x_{c.g.}$		$z_{c.g.}$			$Wx_{c.g.}^2$		$Wz_{c.g.}^2$	I_X	I_Y	I_Z	H $Wx_{c.g.}z_{c.g.}$

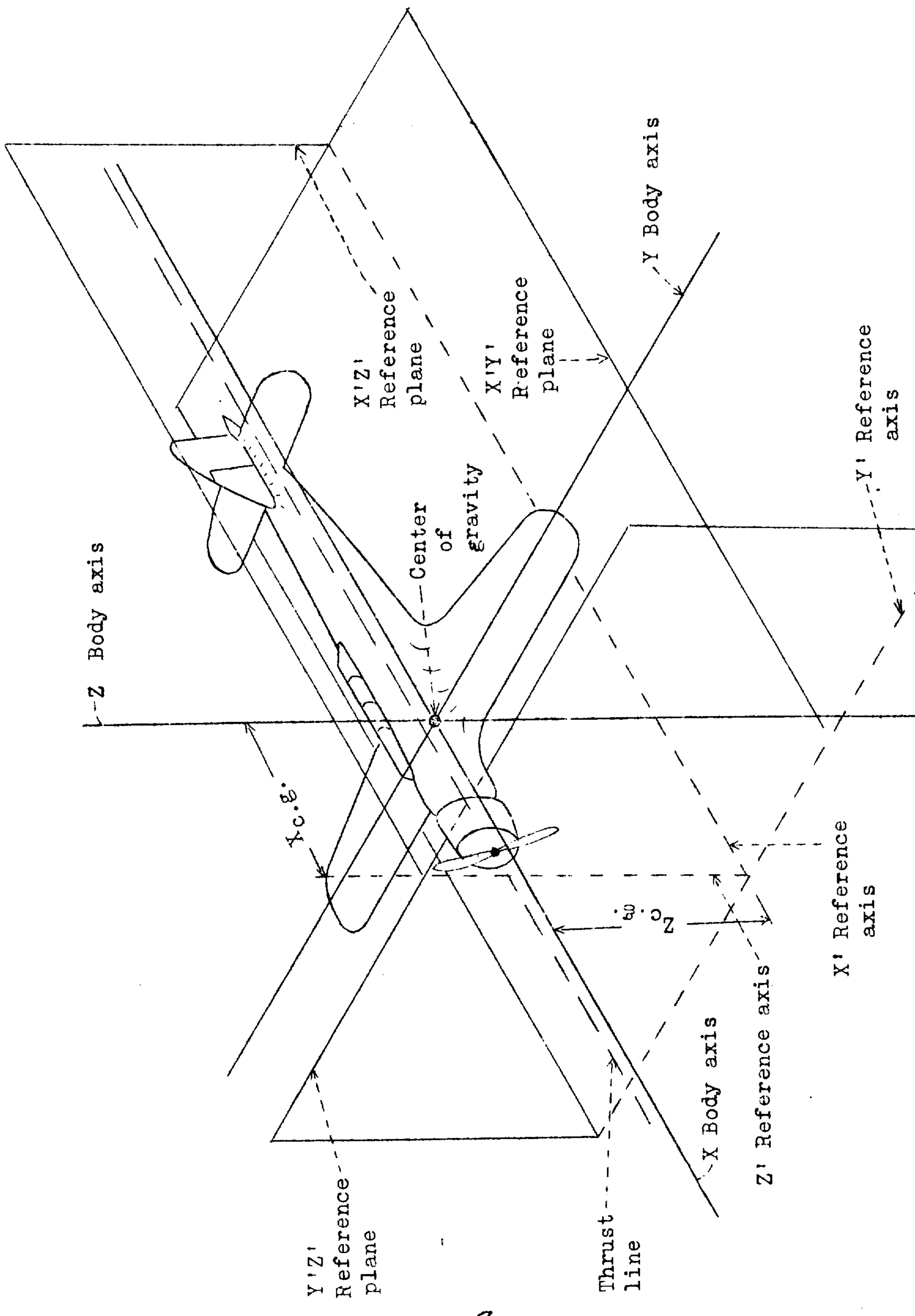
Y Body axis
X'Z' Reference plane
X'Y' Reference plane
Y' Reference axis
Center of gravity
Z Body axis
$x_{c.g.}$
$z_{c.g.}$
X' Reference axis
Z' Reference axis
X Body axis
Y'Z' Reference plane
Thrust line

Figure 1

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