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THE FORCES AND MOMENTS ON AIRPLANE ENGINE MOUNTS

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SUMMARY

A résumé of the equations and formulas for the forces and moments on an aircraft-engine mount is presented. In addition, available experimental data have been included to permit the computation of these forces and moments. A sample calculation is made and compared with present design conditions for engine mounts.

INTRODUCTION

In the ordinary design procedure for aircraft-engine mounts, certain dynamic forces and moments have been neglected owing to the lack of information regarding the velocities and accelerations imposed in flight. The purpose of the present paper is to present a summary of the equations and formulas for these forces and moments and to include available information to enable them to be computed.

The problem of finding the forces and moments on an engine mount may be divided into three parts: first, the determination of the qualitative expression for the forces and moments; second, the obtaining of data so that a quantitative solution may be made; and third, the determination of the conditions of critical forces and moments.

The development of the qualitative expressions involves a consideration of the types of forces and moments involved and these, of course, depend on the motions of the airplane, or the flight conditions, as well as on the operating conditions of the engine-propeller units. The forces and moments on the engine mount may generally be classified as direct (thrust and torque) and induced (inertia). Although all the induced forces are of the same fundamental character, they arise from such a variety of causes that it is convenient further to classify them according to source. The following classification and terms
have therefore been adopted for the present paper to facilitate the analysis and discussion:

**Direct forces and moments:**

- A. Thrust
- B. Torque
- C. Air forces on engine or cowlings

**Induced forces and moments:**

A. Those depending primarily on airplane motion
   1. Due to linear accelerations
   2. Due to angular accelerations
   3. Due to angular velocities

B. Those depending primarily on engine and propeller (vibration)

C. Those depending on engine-propeller motion acting with airplane angular velocities
   1. Due to rotation of parts (gyroscopic)
   2. Due to translation of parts

**QUALITATIVE RELATIONSHIPS**

Most of the qualitative relationships or formulas for the several forces and moments involved are well known and need not be discussed in detail. The following remarks,
however, are offered in order to clarify the final formulas, which give the forces and moments along and about the three principal axes of the engine-propeller combination.

**Thrust and torque.** Thrust and torque should, of course, be considered in the engine-mount analysis. Their values depend on the engine and propeller characteristics, as well as on the flight conditions, and may be found by standard methods. The thrust is assumed to be along the X axis.

**Lift and drag.** In radial-engine installations the lift and "antidrag" or the X component of air force on the cowling may be of sufficient magnitude to include in the engine-mount analysis. For the present these forces may be estimated from the data of reference 1 or 2 or from wind-tunnel tests.

**The a forces.** The a forces arise from the acceleration of the airplane as a whole caused by the resultant air force,

\[ F_a = \frac{W}{g} a \]

They may be resolved into components along the principal engine axes.

**The a forces and moments.** The a forces and moments arise from the angular acceleration of the airplane about its center of gravity. The a force is given by

\[ F_\alpha = \frac{W}{g} l \alpha \]

where \( l \) is the distance from the center of gravity of the airplane to the center of gravity of the engine-propeller combination and the moment by

\[ M_\alpha = I \alpha \]

**Centrifugal force.** The centrifugal force arises from the angular velocity, \( \omega \), of the airplane about its center of gravity and is given by

\[ F_c = \frac{W}{g} \omega^2 l \]
Vibrations. — Vibratory forces arising from unbalanced engine parts, etc., are not included in the formulas given herein. The question of vibration is considered to be outside the scope of this paper, inasmuch as it is not normally dealt with in terms of forces that can be included in the usual static-stress calculations. Certain periodic moments due to the gyroscopic action of the propeller are, however, included with the general consideration of the gyroscopic phenomenon.

The g forces and moments. — Gyroscopic couples arise from the rotation of the propeller and certain parts of the engine, both of which act in conjunction with an angular velocity of the airplane. The equations for these couples may be derived from mechanical principles as given in any good textbook on mechanics. (See reference 3.) Propellers having three or more blades give rise to nonoscillatory couples for all practical cases, as given by the general expression

\[ M_g = I \omega_p \omega \]

in which the subscript \( p \) differentiates the angular velocity of the propeller from that of the airplane.

In the case of 2-blade propellers the couples are oscillatory and, unlike 3-blade propellers, must always be considered with respect to two axes even when the angular velocity of the airplane occurs about only one axis. If \( \psi \) is the angle of the propeller blade from the XZ plane, the instantaneous moments are

\[ M_g = -2 I \omega_p \omega \sin^2 \psi \]

\[ M_{g'} = 2 I \omega_p \omega \sin \psi \cos \psi \]

in which \( M_g \) is the precessional moment of which the frequency is equal to twice the propeller speed, and \( M_{g'} \) is the moment about the axis of rotation of the airplane of which the frequency is also twice the propeller speed. The average value of \( M_{g'} \) is zero; the average value of \( M_g \) is the same as the value of \( M_g \) for a 3-blade propeller of the same polar moment of inertia.
The $g_t$ forces and moments. The $g_t$ forces and moments arise from the reciprocating motion of certain engine parts acting in conjunction with angular velocity of the airplane. They are oscillatory in character and small in magnitude. In the case of V-type and radial engines, analytical expressions for the $g_t$ forces and moments generally cannot be determined and the solutions, if made, must be made graphically. In some special cases expressions can be given as follows:

(1) Four-cylinder in-line engine with cranks 180° apart:

$$M_{g_t} = -2mwV_c \left( 1 - \frac{R}{L} - \frac{R^2}{L^2} \frac{\sin^2 \theta}{2} \right) (\sin 2 \theta)$$

$$F_{g_t} = 2mV_c \omega \frac{R}{L} \sin 2 \theta$$

(2) Six-cylinder in-line engine with cranks 120° apart:

$$M_{g_t} = \frac{3}{2} V_c \omega m R \left[ (\sin \theta - \cos \theta) \left( 1 + 2 \sin^2 \theta \right) \right]$$

$$F_{g_t} = 0$$

In these equations the symbols used are defined as follows:

- $R$, length of crank throw.
- $L$, length of connecting rod.
- $V_c$, crank speed in feet per second.
- $\theta$, crank angle from top dead center.
- $m$, reciprocating mass per cylinder.

For an engine of conventional design of approximately 150 horsepower the maximum value of the $g_t$ moment would be of the order of 20 and 30 foot-pounds for the 4- and 6-cylinder engines, respectively. Likewise in these two cases the unbalanced forces would be of the order of 4 and 0 pounds, respectively.
In view of the small magnitude and complex periodic nature of the \( \varepsilon_t \) forces and moments, they may properly be neglected in static-stress calculations of the engine mount.

**Total forces and moments.** Summation of the important forces and moments gives the total values along and about the three principal axes originating at the center of gravity of the engine. By the use of the symbols defined in table I, and following the N.A.C.A. convention of signs, these forces and moments are:

\[
X = -n_x W_e + \frac{\varepsilon_x x_2}{g} (r^2 + q^2) + \frac{\varepsilon_e}{g} (-\lambda z_3 + \varphi y_3) + T
\]

\[
Y = -n_y W_e + \frac{\varepsilon_y y_2}{g} (p^2 + r^2) + \frac{\varepsilon_e}{g} (\alpha z_3 - \varphi x_3)
\]

\[
Z = -n_z W_e + \frac{\varepsilon_z z_2}{g} (p^2 + q^2) + \frac{\varepsilon_e}{g} (-\alpha y_3 + \lambda x_3)
\]

\[
L = -Q - I_x e \alpha
\]

\[
M = -I_y \lambda - I_p \omega r
\]

\[
N = -I_z \varphi + I_p \omega q
\]

In addition to the foregoing expressions for the principal forces and moments on the engine, the gyroscopic formulas may now be somewhat extended for the case of the 2-blade propeller to include rotation of the airplane about both the pitching and yawing axes. These formulas are:

\[
M = -2I_p \omega (q \sin \psi \cos \psi + r \sin^2 \psi)
\]

\[
N = 2I_p \omega (r \sin \psi \cos \psi + q \cos^2 \psi)
\]

**QUANTITATIVE SOLUTION**

**Critical flight conditions.** The practical quantitative solution of the engine-mount forces involves, first,
a selection of the critical, or most severe, conditions. It is apparent that, since the normal component of the resultant acceleration of the airplane gives rise to the most important single force on the engine mount, the selection of the critical conditions requires primarily an appraisal of the magnitudes of the secondary forces in cases where the normal acceleration is a maximum or nearly so. These cases are:

(1) For acrobatic types:
   a. Abrupt pull-up, power off.
   b. Abrupt pull-up, power on.
   c. Pull-out from fast dive, power off.
   d. Pull-out from fast dive, power on.
   e. Inverted maneuvers.

(2) Nonacrobatic types:
   a. Low-speed gust condition.
   b. High-speed gust condition.

In addition to these cases, there may be some others in which the normal acceleration may be comparatively low and the otherwise secondary forces may become of paramount importance. The most important condition of this character is the inadvertent spin of multiengine airplanes, in which the centrifugal and gyroscopic forces might be large.

Of the foregoing cases, the power-off pull-up is less severe than the power-on pull-up and may therefore be eliminated. The power-on pull-up, while commonly considered to be a maneuver only in the vertical plane, often involves in practice an inadvertent angular velocity in roll at peak acceleration that may be equal in magnitude to the angular velocity in pitch or to that experienced in a deliberate snap roll. It is therefore more severe than the snap roll, since the normal accelerations are greater, and the snap roll need not be considered.

The dive pull-outs, although they do not necessarily involve greater normal accelerations than the abrupt pull-ups, have different conditions of engine speed and should therefore be examined.
Inverted maneuvers are essentially of the same character as normal maneuvers and may be treated as such, bearing in mind the reverse directions of some of the pertinent quantities.

In the gust conditions for the nonacrobatic types, the minimum flying weight is of principal concern.

Quantitative data.—In order to obtain a quantitative solution for the general equations in any given case, certain data concerning moments of inertia and velocities and accelerations in the critical conditions must be available. In the following data and equations, all angular velocities and accelerations apply only to maneuvers, not to the gust conditions for which angular quantities are not known but may be important. In such cases, therefore, the designer will have to make conservative assumptions.

(1) Linear velocity:

The only linear velocity used is the velocity along the X axis that corresponds to the flight condition for which the analysis is made.

(2) Linear accelerations:

The force factor, or acceleration in g units, along the X axis may be calculated from the formula

$$n_X = \frac{T - D \cos \alpha + L \sin \beta + \cos (\gamma + \alpha)}{W}$$

where \(\gamma\) is the flight-path angle measured from the vertical, below, and positive for a dive pull-out.

The acceleration along the Y axis may be calculated in the same manner but, since the data and conditions are vague, it is better to assume a value for \(n_Y\). The maximum recorded value was approximately 2g on an F6C-3 airplane (reference 4).

The force or load factor along the Z axis is calculated in the usual manner from the air speed, lift coefficient, and other pertinent data.
(3) Angular velocity:

The total angular velocity \( q \) in pitch consists of the angular velocity of the airplane about its instantaneous flight-path center plus the rate of change of angle of attack. Conservatively, neglecting the effect of gravity

\[
q = \frac{g n Z}{V} + \frac{d \alpha}{d t}
\]

It is apparent that in maneuvers involving low rates of change of angle of attack at peak normal-load factor, such as pull-outs from fast dives, the second term in the expression is small and may even be negative; on the other hand, in abrupt pull-ups at relatively low speeds, the second term becomes of appreciable importance. In any given case, however, the relative values of the first and second terms will depend on the manner in which the maneuver is performed, which, of course, is affected to some degree by the maneuverability of the airplane. Thus, in figure 1, a considerable scattering of points is noted at the transition from low to high values of \( n Z / V \), with a limited scattering in the mild fast pull-outs and in the abrupt, lower-speed pull-ups, where, respectively, the first and second terms of the expression for \( q \) predominate. For design purposes the upper envelope of the points is recommended.

Experimental data on the angular velocities in roll and yaw are scarce; in view of this fact and considering the relatively complex nature of the qualitative relationships for \( p \) and \( r \), the following values based on available experimental data are suggested for design purposes:

<table>
<thead>
<tr>
<th>Maneuver</th>
<th>( p )</th>
<th>( r )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abrupt pull-up</td>
<td>( q_{\text{max}} )</td>
<td>( q_{\text{max}} )</td>
</tr>
<tr>
<td>Snap roll</td>
<td>( q_{\text{max}} )</td>
<td>( q_{\text{max}} )</td>
</tr>
<tr>
<td>Spin</td>
<td>( q_{\text{max}} )</td>
<td>( 2q_{\text{max}} )</td>
</tr>
<tr>
<td>Dive pull-out</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
(4) Angular accelerations:

The angular acceleration in pitch can be calculated fairly accurately for the start of a maneuver such as a pull-up but at the point of interest, namely, at the peak acceleration, the calculation is extremely inaccurate owing to the introduction of damping. The only theoretically correct way of determining the desired acceleration is to solve the complete equations of motion.

Since such a procedure would be not only laborious but also inaccurate, as previously indicated, the following method is suggested for design purposes:

$$\lambda = 8 \frac{1875 \text{ } \text{lt}}{I_{ya} 15}$$

This formula represents the maximum value of \( \lambda \) obtained on the PW-9 airplane in an abrupt pull-up from level flight and assumes for any other airplane that the angular acceleration in pitch is inversely proportional to the moment of inertia and varies directly with the tail length.

Attempts to elaborate on the foregoing formula have been unsuccessful to date, owing to the scarcity of data that could be considered comparable with those obtained on the PW-9 airplane. It is felt that the formula should give conservative results for the abrupt pull-up condition. Inspection of the PW-9 data leads to the conclusion that the angular acceleration is negligible for the dive pull-out condition.

The quantitative solution of angular accelerations in roll and yaw is even more difficult than the solution for \( \lambda \) and, in general, cannot be performed with suitable precision. Perhaps a reasonable, conservative assumption is that the critical instantaneous values of external moments in roll and yaw are equal to those in pitch; then the angular accelerations are inversely proportional to the moments of inertia about the axes in question.

Moments of inertia.—The moments of inertia of the airplane, engine, and propeller may be determined as set forth in the appendix.
EXAMPLE

The following example is given to illustrate the application of the equations and data for the forces and moments on engine mounts. A fighter airplane is used and the analysis is given for assumed applied conditions for both an abrupt pull-up with power on and pull-out from a fast dive with power off.

Airplane ............... F6C-4 (reference 5):
Weight ................. 2,600 pounds
\( \rho X_a \) ............... 12.5 feet\(^2\)
\( \rho Y_a \) ............... 20.0 feet\(^2\)
\( \rho Z_a \) ............... 26.0 feet\(^2\)
\( I X_a \) ............... 1,010 slug-feet\(^2\)
\( I Y_a \) ............... 1,614 slug-feet\(^2\)
\( I Z_a \) ............... 2,100 slug-feet\(^2\)

Engine ................. Pratt & Whitney 1300
425 hp. at 1,900 r.p.m.
Weight ................. 800 pounds
\( I X_e \) ............... 50 slug-feet\(^2\)
\( I Y_e \) ............... 25 slug-feet\(^2\)
\( I Z_e \) ............... 25 slug-feet\(^2\)

Propeller diameter ........ 9 feet
\( I p \) ............... 4.7 slug-feet\(^2\)
Weight ................. 70 pounds
Weight of propeller
plus engine ............... 870 pounds
\[ x = -5.5 \text{ feet} \quad y = 0 \quad z = 0 \]
\[ x_1 = -4.2 \text{ feet} \quad y_1 = 0 \quad z_1 = 0 \]
\[ x_2 = -4.3 \text{ feet} \quad y_2 = 0 \quad z_2 = 0 \]

Assumptions:

\[ n_x = -1 \]
\[ n_y = 0 \]
\[ n_z = -8 \]

Terminal velocity, \( V_t \) \hspace{1cm} 260 \text{ miles per hour}

Propeller speed
\[
\begin{align*}
\frac{n_t}{\omega_t} &= 46.8 \text{ revolutions per second} \\
\omega_t &= 283 \text{ radians per second}
\end{align*}
\]

High-speed level flight, \( V_L \) \hspace{1cm} 162 \text{ miles per hour}

Propeller speed at high speed
\[
\begin{align*}
\frac{n_L}{\omega_L} &= 31.7 \text{ revolutions per second} \\
\omega_L &= 199 \text{ radians per second}
\end{align*}
\]

\( S_t \) \hspace{1cm} 33 \text{ square feet}

\( l_t \) \hspace{1cm} 13 \text{ feet}

Computations:

\[ \frac{n_z}{V_t} \quad 0.0308 \]

\[ \lambda = \frac{1875}{1614} \times \frac{13}{15} \times 8 \quad 8.05 \text{ radians per second} \]

\[ \frac{n_z}{V_L} \quad 0.0493 \]

\( q_t \) at terminal velocity \quad 1.0 \text{ radian per second}

\( q_L \) at high-speed level flight \quad 1.58 \text{ radians per second}
Forces on N.A.C.A. cowling (estimated):

Cowling dimensions:

Outside diameter .......... 51 inches
Inside diameter .......... 48 inches
Length ................. 12 inches
Projected area, frontal, $S_f$ 1.62 square feet
Projected area, plan $S_p$ 4.17 square feet

Antidrag coefficient at terminal velocity ..... 2.0

Antidrag coefficient at high speed .......... 2.0

Lift coefficient at terminal velocity .......... 0

Lift coefficient at high speed .......... 1.25

Antidrag load = $2.0 \times 1.62 \times \frac{1}{2} \rho V_t^2$ for dive pull-out
= $2.0 \times 1.62 \times \frac{1}{2} \rho V_L^2$ for abrupt pull-up

Lift force = 0 at $V_t$
Lift force = $1.25 \times 4.17 \times \frac{1}{2} \rho V_L^2$ for abrupt pull-up

Pitching moment due to unsymmetrical antidrag load:

$M = 0$ at zero lift

$M = 0.9 \times 1.62 \times \frac{50}{12} \times \frac{1}{2} \rho V_L^2$ for abrupt pull-up

Note: The foregoing calculations are for an arbitrary cowling and are included to show the relative magnitude of the cowling forces and moments.
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<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>( n_z W_e )</td>
<td>Z</td>
<td>6,960</td>
<td>6,960</td>
<td>6,960</td>
</tr>
<tr>
<td>( \frac{\lambda x_x}{c} W_e )</td>
<td>Z</td>
<td>0</td>
<td>900</td>
<td>0</td>
</tr>
<tr>
<td>Lift of cowling</td>
<td>Z</td>
<td>0</td>
<td>-473</td>
<td>0</td>
</tr>
<tr>
<td>Total Z</td>
<td>Z</td>
<td>6,960</td>
<td>7,387</td>
<td>6,960</td>
</tr>
<tr>
<td>Total Y</td>
<td>Y</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( n_x W_e )</td>
<td>X</td>
<td>0</td>
<td>870</td>
<td>870</td>
</tr>
<tr>
<td>T</td>
<td>X</td>
<td>0</td>
<td>800</td>
<td>-605</td>
</tr>
<tr>
<td>Cowling antidrag</td>
<td>X</td>
<td>0</td>
<td>208</td>
<td>603</td>
</tr>
<tr>
<td>( \frac{W_e x_a}{g} (q^a + r^2) )</td>
<td>X</td>
<td>0</td>
<td>685</td>
<td>155</td>
</tr>
<tr>
<td>Total X</td>
<td>X</td>
<td>0</td>
<td>2,543</td>
<td>1,023</td>
</tr>
</tbody>
</table>

## Moment

<table>
<thead>
<tr>
<th></th>
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<th></th>
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</thead>
<tbody>
<tr>
<td>Cowling drag</td>
<td>Y</td>
<td>0</td>
<td>-780</td>
<td>0</td>
</tr>
<tr>
<td>( I_p u r )</td>
<td>Y</td>
<td>0</td>
<td>-1,290</td>
<td>0</td>
</tr>
<tr>
<td>( I_y \lambda )</td>
<td>Y</td>
<td>0</td>
<td>-200</td>
<td>-200</td>
</tr>
<tr>
<td>Total</td>
<td>Y</td>
<td>0</td>
<td>-2,270</td>
<td>-200</td>
</tr>
<tr>
<td>( I_p w q )</td>
<td>Z</td>
<td>0</td>
<td>1,290</td>
<td>1,250</td>
</tr>
<tr>
<td>Q</td>
<td>X</td>
<td>-2,360</td>
<td>-1,180</td>
<td>300</td>
</tr>
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## Periodic couples

<table>
<thead>
<tr>
<th>Axis</th>
<th>Existing rules</th>
<th>Abrupt pull-up</th>
<th>Pull-out</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2Ip\omega q \sin^2 \psi$</td>
<td>$Z$</td>
<td>$0$</td>
<td>$2,590 \sin^2 \psi$</td>
</tr>
<tr>
<td>$2Ip\omega r \cos^2 \psi$</td>
<td>$Y$</td>
<td>$0$</td>
<td>$2,590 \cos^2 \psi$</td>
</tr>
<tr>
<td>$2Ip\omega q \sin \psi \cos \psi$</td>
<td>$Z$</td>
<td>$0$</td>
<td>$2,590 \sin \psi \cos \psi$</td>
</tr>
<tr>
<td>$2Ip\omega r \sin \psi \cos \psi$</td>
<td>$Y$</td>
<td>$0$</td>
<td>$2,590 \sin \psi \cos \psi$</td>
</tr>
</tbody>
</table>

In a comparison of the normal loads, it will be noted immediately that, owing to the $\alpha$ forces, the effective normal-load factor $n_z$ is $7,387/870$ or $8.60 \, g$, as compared with the present design-rule value of $8 \, g$. In the case of the $X$ forces, there is no fair comparison available and in the $Y$ direction in all cases the load is zero.

In the case of applied moments the present rules recognize only the engine torque, which is used with a factor of safety of 2. Actually it is easily seen that several other applied moments are present, owing to gyroscopic action and the $\alpha$ moments, which will tend to add sufficient load to make the present requirements unconservative.

A separate set of couples, which varies periodically in magnitude, is present for the actual case and is of sufficient magnitude to induce severe vibrations in the structure of an airplane. As explained in the text, however, these vibrations can be eliminated through the use of a 3-blade propeller.

The cowling force coefficients were estimated from wind-tunnel tests and, as may be seen, the resulting forces are of such magnitude and direction as to warrant their consideration.

Langley Memorial Aeronautical Laboratory, National Advisory Committee for Aeronautics, Langley Field, Va., December 10, 1936.
Moments of inertia of airplanes, engines, and propellers:

1. Airplanes:

Reference 6 gives a very complete set of tests on airplanes to determine the principal moments of inertia for an airplane. Quite an accurate method is outlined in this reference.

Figure 2 will serve for approximate calculations of conventional military airplanes. It should not be used as an accurate method but as a rough approximation.

2. Engines:

The following table gives the results of some tests by the Bureau of Aeronautics, U.S. Navy, for the moments of inertia of single-row radial engines:

<table>
<thead>
<tr>
<th>Engine</th>
<th>Weight (lb)</th>
<th>$I_{Y_e}$ (slug-ft.$^2$)</th>
<th>$I_{Z_e}$ (slug-ft.$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>R-1340-12</td>
<td>765</td>
<td>28.15</td>
<td>27.55</td>
</tr>
<tr>
<td>R-1820-04</td>
<td>939</td>
<td>37.85</td>
<td>35.85</td>
</tr>
<tr>
<td>GR-1820-82</td>
<td>942</td>
<td>39.80</td>
<td>39.87</td>
</tr>
</tbody>
</table>

As no similar data are available for twin-row or in-line engines, the following approximations are suggested:

(a) The radial engine, either single-row or twin-row, can be assumed to be replaced by a homogeneous disk of the same mass and diameter and a thickness equal to the distance between the front of the forward cylinders to the back of the rear row of cylinders.

(b) The in-line or V-type engine can be assumed to be replaced by a rectangular homogeneous body of the same over-all dimensions as the engine.
These moments of inertia may be calculated by the method outlined in reference 6.

3. Propellers:

Figure 3 gives the results of (unpublished) tests to determine the moments of inertia of propellers. The moment of inertia of the crankshaft (reference 7) is about 10 percent or less of the propeller moment of inertia and, for the purpose of this paper, should be multiplied by the gear ratio and added to the propeller moment of inertia.

REFERENCES


### TABLE I

<table>
<thead>
<tr>
<th>Axis</th>
<th>Force parallel to axis</th>
<th>Moment about axis</th>
<th>Velocity</th>
<th>Acceleration</th>
<th>Moments of inertia</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Linear</td>
<td>Angular</td>
<td>Linear (g-units)</td>
<td>Angular</td>
</tr>
<tr>
<td>X</td>
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<td>L</td>
<td>u</td>
<td>p</td>
<td>n_X</td>
</tr>
<tr>
<td>Y</td>
<td>Y</td>
<td>M</td>
<td>v</td>
<td>q</td>
<td>n_Y</td>
</tr>
<tr>
<td>Z</td>
<td>Z</td>
<td>N</td>
<td>w</td>
<td>r</td>
<td>n_Z</td>
</tr>
</tbody>
</table>

- $x$, distance along $X$ axis from c.g. of airplane to c.g. of propeller
- $x_a$, distance along $X$ axis from c.g. of airplane to c.g. of engine
- $x_a$, distance along $X$ axis from c.g. of airplane to c.g. of propeller-engine combination
- $y$, distance along $Y$ axis from c.g. of airplane to c.g. of propeller
- $y_a$, distance along $Y$ axis from c.g. of airplane to c.g. of engine
- $y_a$, distance along $Y$ axis from c.g. of airplane to c.g. of propeller-engine combination
- $z$, distance along $Z$ axis from c.g. of airplane to c.g. of propeller
- $z_a$, distance along $Z$ axis from c.g. of airplane to c.g. of engine
- $z_a$, distance along $Z$ axis from c.g. of airplane to c.g. of propeller-engine combination
- $I_p$, polar moment of inertia of propeller
- $\omega$, angular velocity of propeller in radians per second
- $T$, thrust
- $Q$, torque
- $\rho$, radius of gyration

Subscript "a" refers to airplane

Subscript "e" refers to engine
Figure 1. Angular velocity of an airplane in pitch.
Figure 2.- Radii of gyration of single-engined military tractor biplanes.
Note: Numbers are those of propeller blade.

Figure 3: Polar moment of inertia. 

(Diameter)^5, ft.^5