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CORRUGATED METAL DIAPHRAGMS

FOR AIRCRAFT PRESSURE-MEASURING INSTRUMENTS

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SUMMARY

A large number of corrugated diaphragms of beryllium copper, phosphor bronze, and Z-nickel, having geometrically similar outlines but of various diameters and thicknesses, were formed by hydraulic pressing. The apparatus and the technique used in the manufacture, the testing, and the heat treatment are described.

The shape of the diaphragms was such that the central deflections were nearly proportional to the differential pressures up to deflections of 2 percent of the diameter. The pressure-deflection characteristics of the various diaphragms were correlated with the thickness, the diameter, and the elastic properties by dimensional analysis to obtain formulas and charts applicable to the design of similar diaphragms. The formula

$$\frac{FX}{PD} = 2.25 \times 10^5 \left(\frac{t \times 10^3}{D}\right)^{-1.52}$$

holds for values of $\frac{t \times 10^3}{D}$ over the range from 1 to 4. Outside this range, the exponent is larger. In the formula, t, D, and X refer to the thickness, the diameter, and the central deflection of the diaphragm, respectively; P refers to the differential pressure causing the deflection X; and F, a function of the elastic moduli, has the values 18.9, 17.5, and 28.4 × 10^6 pounds per square inch for beryllium copper, phosphor bronze, and Z-nickel, respectively.

For comparison, some data are presented for flat diaphragms and for corrugated diaphragms differing slightly from the standard design. The use of the experimental results in the selection or the design of corrugated diaphragms is briefly discussed.

INTRODUCTION

Metallic diaphragms, singly or in pairs as capsules, find a wide use in pressure-measuring instruments of many types. The compactness, the simplicity, and the cheapness of diaphragms as well as the wide range of load, sensitivity, and deflection characteristics available recommend their use, especially in aeronautic instruments. There are, however, certain limitations in practice that confront the designer who wishes to make use of diaphragms in any given application. For one thing, there are no rational design formulas to predict the behavior of corrugated diaphragms. Even for the limiting case of flat disks, where theory has led to the development of performance equations (references 1 and 2), convenient charts, tables, or other aids to design are not yet available. The use of corrugated disks makes possible larger deflections and affords a control of the shape of the load-deflection curve. It is possible to set theoretical limits to the gain in deflection that can be achieved by corrugating disks (reference 1), but there is no way of predicting, except in a general qualitative fashion, the performance to be expected for a given type of corrugation. This practical difficulty is serious because it is always desirable to use the diaphragm best suited to the specific application; the selection of the best form of diaphragm by the trial and rejection method may involve considerable development expense.

Diaphragm instruments are subject to various errors arising from the imperfect elastic properties of the diaphragm materials, evident in the phenomena of hysteresis, drift, and aftereffect. Hysteresis is the discrepancy in a load cycle between the load-deflection curves for increasing and decreasing pressures. Drift is the change in deflection, at a constant differential pressure, that occurs with time. Aftereffect is the difference in zero position before and after a cycle of loading. For many applications, especially in aeronautic instruments, the errors arising from these sources must be less than 0.1 percent of the maximum deflection. This relatively high standard of performance is achieved by limiting the load to values that will not stress the material to near its yield point, by using those materials with the best elastic properties, by following closely a satisfactory technique of manufacture coupled with selection on the basis of inspection, and in some measure by calibrating at a rate corresponding to the manner of use.

With imperfect knowledge of the distribution of stress in diaphragms under load and of the laws governing drift, hysteresis, and aftereffect, experience alone can tell which material, which shape, or which technique of preparation, will result in producing the diaphragms with the most desirable properties.

It is apparent that experimental investigation is required to obtain performance data concerning the many possible types of corrugation, to check the existing theories, to furnish an empirical basis for design formulas, and to guide the development of more general theories.

In the present investigation, a start has been made on some phases of the general problem. Apparatus permitting control of clamping and forming pressures was constructed for forming diaphragms of various size and thickness. The diaphragms were made with similar outlines, in order that the data on various sizes could be easily used in developing design formulas by dimensional analysis. Diaphragms were made of beryllium copper, phosphor bronze, and Z-nickel.

The deflections of each diaphragm were measured under increasing and decreasing pressures. Typical data are shown graphically. The load-deflection data thus obtained were correlated by dimensional analysis to obtain a general formula that applies to the particular corrugation outline used.

Less complete data available on diaphragms with other corrugation shapes have also been correlated in this manner to obtain some indication of the effect of changes in shape. Some work was done on the deflection characteristics of diaphragms subjected to concentrated central loads.

This investigation has been carried out with the cooperation and financial assistance of the National Advisory Committee for Aeronautics.

MANUFACTURE AND HEAT TREATMENT OF THE DIAPHRAGMS

Manufacture of Corrugated Diaphragms

Corrugations may be formed in diaphragms by any of the usual metal-working methods: spinning, hydraulic pressing, or mechanical stamping. Spinning appears to introduce more variations in the finished diaphragms than the other methods and requires more skill and time. Mechanical stamping or pressing generally requires two dies and a means of holding the edge of the diaphragm. With hydraulic pressing only one die is required, which may also serve to clamp the edge of the diaphragm. On the basis of these and other considerations the method of hydraulic pressing was chosen for this work.

Four dies were made, with dimensionally similar outlines or corrugations as shown in figures 1 and 2. The effective diameters D of the diaphragms formed with these dies were 1-1/2, 2 2-1/2, and 3 inches. The rim width was one-fourth inch for all sizes. The outside diameter was the same for all the dies so that they could be used interchangeably with the one base.

The apparatus used for making the diaphragms included: a hand-operated hydraulic press of 18 tons capacity; a hydraulic pump, also hand operated; the dies; a base for the die; paper gaskets; dental dam rubber; and circular metal blanks. The press was used to clamp the die and its base and was also used in the course of testing the diaphragms. Figure 3 is a photograph of the press with the testing apparatus. When the press was used for making the diaphragms, the micrometer tip A, seen projecting below the upper platen in figure 3, could be screwed up out of the way. The hydraulic pump was connected by small pressure tubing to the base 5 (fig. 2). In operation, the clamping force was made greater than the hydraulic force exerted by the pump in forming the diaphragm. The difference between these two forces was the effective clamping force. The clamping pressure was seldom less than 10,000 pounds per square inch of rim area. The forming pressures used were from 1,000 to 9,500 pounds per square inch, depending on the size and the thickness of the diaphragm.

The die rested in its base with the blank between them. The pressure fluid was conducted to the lower side of the blank through the hole in the die base shown in figure 1. Annular rings (1/32 inch thick) were laid in the base to center the blank and the gasket. The die was automatically centered by the cylindrical projecting rim of the base. Each die had vent holes (see figs. 1 and 2) to allow the air between the blank and the die to escape.

Since blanks of several diameters were needed and facilities for stamping them from the metal strip were not available, the technique worked out for cutting them on a lathe is of some practical interest. The metal strip was cut with snips or cutter into squares or other shapes slightly larger than the size desired. A number of these pieces were piled together as a pack. The pack was held in a lathe between a plane cutting face held on the chuck and a pressure pad of the same diameter as the desired blanks, which was mounted by a ball bearing to the tailstock. The ball bearing allowed sufficient pressure to be applied by means of the tailstock screw to hold the pieces of strip firmly while they were being cut with a sharp tool and also permitted the work to turn at as high a speed as was necessary for smooth cutting. The slight roughness sometimes noticed on the edges of the blanks could be quickly removed by drawing the blank through emery cloth held in the fingers. Including a thicker brass plate between the pressure pad and the pack, and cutting both, was found to result in smoother edges.

Paper gaskets were found to be necessary to prevent leakage between the blank and the base at high forming pressures. They were conveniently made on the lathe by the same technique as was used in cutting the metal blanks.

In the formation of a diaphragm, the procedure was as follows: the paper gasket and the blank were placed on the pressure base; the die was placed on top; the die and the pressure base together were placed in the press and clamped with a force sufficient to overcome the hydraulic forming force and to clamp the edge of the diaphragm with a pressure up to 10,000 pounds per square inch; then hydraulic pressure up to 9,500 pounds per square inch, depending on the thickness, was applied. After the pressures were released, the formed diaphragm was removed. The pressures necessary to force the beryllium copper blanks fully into the die are approximately those given in table I. Usually these pressures were exceeded by 40 to 50 percent, just to be sure of fully forming the diaphragms. A number of diaphragms were made at lower pressures. These diaphragms were not fully formed and their load-deflection characteristics differed from the typical, as discussed later in the section on Results. A few diaphragms were made with a thin rubber sheet (dental dam, 0.012 inch thick) between the blank and the die; these diaphragms also had characteristics different from those of the fully formed diaphragms.

A number of fully formed diaphragms were made in two stages, first using the rubber backing and then completing the forming without the backing. This method gave a more uniform thickness for the various corrugations. By this "two-stage" method, diaphragms could be made of harder material than could be formed directly in one stage.

Most of the diaphragms were made from rolled sheets of phosphor bronze and beryllium copper. The nominal thicknesses were 2, 4, 5, 8, and 13 thousandths of an inch for Be-Cu; and 2, 3, 4, and 6 thousandths for phosphor bronze.

For further comparison, a few diaphragms were made of 0.006-inch and 0.008-inch Z-nickel, a comparatively new corrosion-resistant alloy containing more than 98 percent nickel. Like beryllium copper, it can be formed when in a soft condition, and hardened by subsequent heat treatmemt. The Z-nickel was obtained through the courtesy of the International Nickel Company, who also heat treated the diaphragms made of this material.

Most of the beryllium copper was in the annealed, or dead-soft, condition. This material, except for the 0.008-inch thickness, and the 0.002-inch thickness in the large diameters, showed no tendency to break or tear while being formed. Some diaphragms were made of quarter-hard beryllium copper, by the two-stage forming technique. Lower clamping pressures were used in the first stage, to permit slight pulling in of the edges. The rubber allowed a more even stretching of the metal, as was evident in the resulting thickness of the final diaphragms. The variations in thickness measured at the various corrugations were as much as 15 percent in diaphragms formed in one stage; whereas, in those formed in two stages, the corresponding variation was seldom more than 5 percent.

A preliminary heat treatment, described in the next section, was required to soften the phosphor bronze so that it could be fully formed in the dies.

The Z-nickel in the soft condition was found to be too hard to form in one stage and was therefore formed in two stages. Even with this method, the half-hard material could not be formed in the dies. The harder materials could probably be used with dies having shallower corrugations or less abrupt bends.

Heat Treatment and Seasoning

The phosphor bronze was hard-rolled and could not be fully formed in the dies, breaking or tearing when less than half formed. Experimentation showed that this material could be sufficiently softened to allow full forming in one stage in all the dies. The procedure was to put the phosphor-bronze blanks, surrounded with carbon in a small cast-iron chamber, in a furnace at 425° C. for 2-1/2 hours and then to allow the chamber to cool in the air for 1 hour before removing the blanks.

After forming, the phosphor-bronze diaphragms showed pronounced drift and aftereffect, which could be removed by seasoning, i.e., by repeated working, or by heating, or by a combination of both. Experiment showed that 250, or more, cycles of loading were necessary to reduce the aftereffect to less than 10^{-5} inches following a deflection of 0.02 of the diameter, if no heating were given the diaphragms. Heat treatment similar to the initial softening treatment with furnace temperatures of 200° C. or higher reduced the necessary number of loading cycles. After the diaphragms were given this type of heat treatment with the furnace at 300° C., no working was necessary to get repeatable load-deflection data and the stiffness was not noticeably affected; this treatment was therefore made a part of the standard procedure. The phosphor bronze was always heated in carbon to avoid the scaling due to oxidation, which occurred in air even at 300° C.

After being formed, the beryllium-copper diaphragms were heat treated by being placed in an electric furnace at 300° C., and kept there for 1 hour. Clamping of the diaphragms during heat treatment was found to be unnecessary and actually undesirable because it caused distortion. The heating was done in air. A very thin oxide film formed on the surface; this film could be easily removed by hydrochloric acid, if desired. The oxide film gave a dark color to the diaphragm but, inasmuch as it did not change the performance, it was usually not removed, except at the center, as a preliminary to soldering on the center reinforcing disks or to provide a clean electrical contact for the micrometer point.

Composition and Hardness of the Diaphragm Materials

The compositions of the beryllium copper and the phosphor bronze were determined by chemical analysis of the scrap clippings and turnings obtained in cutting the blanks from the strips. The average percentages of the various alloying elements were as follows:

	Beryllium copper (percent)	Phosphor bronze (percent)
Copper	97.3	95
Phosphorus	-	0.18 to 0.29
Beryllium	2.3	-
Tin	-	4.6
Iron	.03	.01
Nickel	.26	-
Lead	.01	-

The results of hardness measurements taken on these materials in various conditions are given in table II. The heat treatments referred to in the table are the ones described in the previous section. The hardness measurements were made on a Knoop indentation machine, which is well adapted for measurements of thin materials (reference 3). Loads ranging from 50 to 1,000 grams were used, depending on the thickness of the sheet. The indentation numbers in table II may be considered roughly equivalent to the Brinell numbers that might be obtained on much thicker specimens of the same materials.

Center Reinforcing and Mounting Disks

Most diaphragms on which measurements were taken were equipped with reinforcing disks as a part of the regular manufacturing process. The disks were of copper or phosphor bronze 0.03 inch thick. They were soldered on after heat treatment of the diaphragms, in the center of the convex side. Their diameters were $D/4$, the same as that of the uncorrugated central area.

The purpose of the disks was to stabilize the central part and to make the construction correspond to what is usually necessary in practice where an indicating mechanism, contacts, or mounting connections are fastened to the center of the diaphragm.

Etching

The object of making diaphragms of various diameters and thicknesses was to obtain data on diaphragms over a large range of values of the ratio of thickness to diameter, t/D. In order to extend the range covered by the available thicknesses and diameters and to obtain data for intermediate values, some diaphragms were made thinner by etching their inner surfaces with dilute nitric acid. The diaphragms so treated seemed to be almost exactly similar in performance to unetched diaphragms having corresponding values of t/D. This result is shown later in figure 9.

One diaphragm was reduced from 0.005 to 0.001 inch in thickness by repeated etching, measurements being taken after each of eight reductions in thickness.

TESTING THE DIAPHRAGMS

Practically all of the previous workers with diaphragms have noted the necessity for uniform clamping of the edges of diaphragms under test. The characteristics of diaphragms depend considerably on the initial stresses, especially radial tensions, in the material. When comparative data are obtained, care must therefore be taken that the method of clamping used either introduces no stresses at all or stresses different diaphragms in the same way and by the same amount. For this reason, a method of testing was evolved in which a hydraulic press is used to clamp the diaphragms. The consistent results obtained indicate that this method obviates many difficulties.

The testing apparatus may be seen in figure 3. The steel pressure chamber B has four annular steps to fit the four different diameters. Steel cylinders such as C of 1/4-inch wall thickness, which is the width of the rims of the diaphragms, were constructed with inside diameters equal to the effective diameters D of the diaphragms.

For the tests, a greased paper gasket was placed on the annular step in the pressure chamber, the diaphragm was placed on it, the cylinder was seated on the rim of the diaphragm, and the assembly was centered on the lower platen of the press. When the lower platen was raised, the cylinder clamped the diaphragm directly under the micrometer A mounted in a hole through the upper platen.

The pressure on the diaphragm was applied by means of hand-operated air pump (not shown) and was measured by water or mercury manometers, M. The accuracy of the pressure measurements was better than 0.5 millimeter on the manometers. The micrometer, held by an insulating Bakelite bushing in the upper platen, was connected to a graduated wheel E, 12.5 inches in diameter, the rim of which was ruled with 500 divisions. The micrometer screw had 40 threads to the inch, so that each division or unit on the wheel corresponded to a movement of 5×10^{-5} inches. Readings were estimated to tenths of a wheel unit and could be repeated with about this precision. The accuracy of measurement is, of course, not so great as the sensitivity indicated but, for measurement of drift or aftereffect, the sensitivity of the measuring apparatus is more important than the absolute accuracy. An electric circuit was arranged so that, when the micrometer touched the diaphragm, current flowed in a galvanometer, moving a spot of light reflected from the galvanometer mirror. Thus contact could be visually determined. A voltage of only 1-1/2 to 3 volts was used, and the current was limited by a 0.5 megohm resistance to avoid sparking or arcing. The micrometer was fitted with a small rounded tip of steel; the surface of the diaphragm or reinforcing disk was cleaned and polished over the central area where the micrometer touched.

In order to obtain the pressure-deflection data on a diaphragm, the zero reading was first taken with no pressure, then the micrometer was screwed up, a small pressure was applied, and the micrometer was screwed down to make contact. Manometer and micrometer readings were then recorded, and the process was repeated at a somewhat higher pressure. Alternatively, it was possible by careful adjustment of the pressure to obtain a desired deflection to within a few tenths of a wheel unit (approximately 10^{-5} inches).

No appreciable drift or hysteresis occurred within a range of deflection up to 0.02 of the diameter after the diaphragms had been deflected a few times. Therefore much of the testing could be done by stepwise loading or by releasing the pressure after each test point.

As a quick method of estimating the deflections at which drift would be significant, the pressure was released after each test point and the zero reading was obtained. The load limit was taken as the pressure that caused an increase in the zero reading of more than 10^{-5} inches.

The pressure chamber was designed so that the diaphragm could be tested with either side up. Since the outline is not symmetrical about the plane of the rim, the load-deflection characteristics would be expected to be different in the two directions. A few diaphragms were tested in both directions, but most of the diaphragms were tested with the pressure applied only to the convex side.

In the tests with concentrated loading at the center, loads were applied to the centers of diaphragms by a rod carried by a balance beam, resting on a knife edge 10 inches from the center of the diaphragms. The beam extended over the diaphragm through a slot in the wall of the cylinder. With this arrangement, pressure loads and central loads could be simultaneously applied in opposite directions. The pressure necessary to neutralize the deflection due to a small central load is the same as the pressure which, acting alone, would give the same deflection as the central load alone. This relation is not necessarily true for large loads owing to the difference in stress distribution under the two types of loading.

The temperature of the apparatus was not closely controlled, but the room temperature was usually within a few degrees of 23° C. and had a negligible variation during any one test. After the pressure was changed, it was necessary to wait half a minute or so for the temperature and pressure of the air in the pressure chamber to attain equilibrium.

The thickness of each diaphragm was measured in each corrugation, usually at four or eight equally spaced points. A micrometer fitted with ball contact points was used for these measurements.

EXPERIMENTAL RESULTS

Although most of the testing consisted in obtaining load-deflection data within the elastic limit, a number of tests were made on one or more diaphragms to obtain some knowledge of other properties of the diaphragms.

Pressure-Deflection Curves

The relationship between pressure and deflection for the various sizes, thicknesses, and kinds of diaphragms was the main object of investigation, and involved the testing of hundreds of diaphragms. The results of each individual test cannot conveniently be given in detail; nor would any particular benefit be obtained from voluminous tables or group of graphs giving such data. Typical curves of the deflection at the center against pressure applied on the convex side are given in figure 4 for several different diaphragms of beryllium copper, all of which had reinforcing disks at the center, as previously described. These curves show qualitatively the effect of variations in thickness and diameter and the progressive change in stiffness with the ratio of thickness to diameter, t/D. The analysis of all the pressure-deflection data will be treated in the next section.

The pressure-deflection curves were found to be nearly straight lines over a considerable range of loading. The deviations from the straight lines through the 0.02D points, expressed as percentages of 0.02D, are plotted in figure 5 against the ratio of the deflection X to the diameter D for a few diaphragms. The average maximum deviation for all the diaphragms was less than 1 percent, within the standard range of deflection. There seems to be a tendency for the load-deflection curves to be slightly concave upward (less stiff at higher pressures) for diaphragms of small values of t/D, and to be concave downward for the larger values of t/D. The data are not very coherent, and further work is required on this point.

The diaphragms were normally tested with pressure applied to the convex side. When loaded on the concave side, the deflections were also nearly linearly proportional to pressure, with slightly different slopes. The deviation of the pressure-deflection curves from a straight line drawn through the origin and the $X = 0.02D$ point (for loading on the convex side) is shown in figure 6, with deviation plotted as percentage of 0.02D. The diaphragms were somewhat more flexible when loaded on the concave side.

The stiffness of the partly formed diaphragms was less than that of the fully formed ones, and the deviations of their pressure-deflection curves from linearity was greater. The stiffness of those formed with a rubber backing was also less than for the fully formed ones but, for some diaphragms, notably the 0.004- by 2-inch size in beryllium copper, the rubber backing gave a more nearly linear characteristic.

The deflection of 0.02D is taken as a standard because it represents a safe deflection for all the materials tested and because the load-deflection curves deviate somewhat more from linearity over a larger deflection range. Over this range the hysteresis was not detectable, being less than 10^{-5} inches for all diaphragms, and the pressure-deflection curves could be repeated quite exactly.

The pressure-deflection curves for a group of different diaphragms of the same thickness, diameter, and material varied from the average for the group by 2 percent or less. The average pressure-deflection performance of the various sizes is shown in table III. The values of pressure given in the table are the averages for from two to six similar diaphragms.

Load Limit

A number of diaphragms were deflected until they failed to return to within about 10^{-5} inches of their zero positions when the differential pressure was reduced to zero. The average deflection-diameter ratio for this amount of aftereffect was slightly more than 0.02 for phosphor bronze diaphragms, and 0.04 or higher for those of beryllium copper. These values are not exactly the same for different values of the thickness-diameter ratio of the diaphragms, but more data will have to be obtained to establish the exact dependence. The data thus far obtained are from diaphragms having t/D ratios less than 4×10^{-3}. Further studies will be of value in relating the useful or permissible deflection to the values of elastic limit, as obtained in tensile tests, or to hardness measurements.

The decrease in hardness of the phosphor bronze when heated preliminary to forming the diaphragms (table II) may account for the relatively low load limit obtained for this material. With die shapes that do not require such severe stretching of the metal, it could be used without softening, thus obtaining a better tensile strength and a higher load limit. In this connection, it has more recently been found that the diaphragms may be formed by the two-stage method from blanks subjected to a heat treatment of the kind described earlier with a furnace temperature of only 390° C.

Effect of Center Reinforcing

Without the central reinforcing disks, the deflection of the diaphragms may be thought of as somewhat equivalent to the deflection of the reinforced diaphragm plus the deflection of a flat diaphragm the size and thickness of the original flat central area. The main effect of omitting the disk is to increase the deflection, somewhat more at small than at large deflections. Some effect is probably also due to stresses introduced in soldering the disks. The results of several tests showed that the reinforcing disks, with some exceptions, increased the stiffness by 10 percent or more and apparently tended to make the load-deflection curve slightly concave upward, that is, to increase the stiffness more at low deflections or loads. Because of the scatter of the results, no quantitative conclusions can be drawn. For most applications, however, the disks are necessary for mounting mechanisms or contacts and in this work were necessary to remove the uncertainty of zero position of the flat part, which acts like a flat diaphragm.

Snap-Action Diaphragms

In tests of some 0.004- by 2.5-inch Be-Cu diaphragms without reinforcing disks, an interesting snap action was obtained. This result is shown in figure 7 by a curve for one of the diaphragms. As the pressure was increased (on the convex side) the deflection was fairly linear up to a certain pressure, where the deflection increased suddenly by about 0.01 inch, then again increased linearly. With decreasing pressures, the diaphragms snapped in the other direction but at a considerably lower pressure, resulting in a loop in the load-deflection curve.

This snap action is apparently due to a dishing of the central area, perhaps due to strains not relieved by heat treatment. When reinforcing disks were soldered on, the snap action, of course, disappeared, but the stiffness was the same as before, indicating that except for the snap action, the central area had contributed but little to the deflection.

Deflection Traverse

The deflections of the various corrugations were determined for several diaphragms. The central deflection was measured at various differential pressures as usual. Pressure-deflection data were then obtained with the diaphragm positioned in the test apparatus so that the micrometer was over one of the corrugations. This process was repeated for each corrugation. From these data, corresponding deflections at the various corrugations for a given pressure could be obtained. Figure 8 shows the deflections at the various corrugations for one diaphragm. Corresponding curves for other diaphragms, although not exactly similar, were somewhat like the one shown.

Thickness Variations

The blanks from a given sheet of material usually varied in thickness not more than a few percent, and the variations between measurements made on a given blank were usually negligible. After the diaphragms were formed, the thickness measured at the various corrugations differed by as much as 20 percent owing to the uneven stretching of the material, the outer corrugations being the thinnest. When the diaphragms were formed in two stages, the variations in thickness were much less, usually not more than 5 percent.

Concentrated Central Loads

For a comparison of the effects of concentrated central loads and uniformly distributed pressure loads on diaphragms, it is convenient to use the dimensionless ratio of the central load to the product of the projected area of the diaphragm and the pressure that gives the same central deflection as the central load.

This ratio was determined for diaphragms of various thickness and diameters by applying central loads, on the concave side, and adjusting the pressure on the convex side to reduce the deflection to zero. The values of the ratio thus obtained are given in table IV. The concentrated loads and the compensating pressures were varied up to the values that alone would give a deflection of 0.005D, without markedly affecting the value of the ratios.

ANALYSIS OF RESULTS

For problems involving the correlation of as many variables as relate to diaphragms, the most fruitful method of attack is that of dimensional analysis (reference 4). Some discussion of the use of this method in diaphragm problems has been given by Hersey (reference 5) without, however, any application to experimental data.

Correct information can be derived from dimensional analysis, only by taking account of all the parameters of the problem. In an attempt to find an expression, or formula, relating the deflection of a diaphragm to the pressure applied, it is obvious from experience that the diameter, the thickness, and the elastic properties of the material will need to be considered. It is also to be expected that the relationship between these variables will be different for differently shaped diaphragms.

Let X be the deflection at the center of the diaphragm, t its thickness, D its diameter, p the pressure (uniformly applied), and G and E, respectively, the rigidity and Young's modulus of the material. Then, for diaphragms of similar outline, the relationship may be indicated, as

$$X = \phi\,(p,\ D\ \ t,\ E,\ G) \qquad (1)$$

where ϕ indicates the functional relationship, the form of which is to be determined. If the diaphragm has a concentrated load as well as pressure acting on it, the load L would have to be included. Also, if the material is not homogeneous and isotropic, E and G might be different in different directions or in the different parts. More important, the thickness of a diaphragm is seldom exactly uniform, since the process of manufacture may introduce considerable variations owing to stretching of the material. It may reasonably be expected, however, that the variations in these (assumed) constants will be similar for diaphragms of geometrically similar contours, formed in the same manner. As a first approximation, then, the form given in equation (1) will hold for similarly formed diaphragms as long as the material follows Hooke's Law.

The dimensionless ratios formed by various combinations of the parameters in equation (1) are also functionally related (reference 4) so that the equation may be written

$$\frac{X}{D} = \phi\left(\frac{t}{D},\ \frac{p}{E},\ \sigma\right) \qquad (2)$$

where Poisson's ratio

$$\sigma = \frac{E - 2G}{2G}$$

The problem is now to determine the form of the function ϕ, by analyzing the experimental data. As described in preceding sections, diaphragms having geometrically similar outlines were made of various materials in a variety of thicknesses and diameters, and their deflection characteristics were determined; that is, a number of corresponding values of X and p were measured for each diaphragm.

The contour shape of the diaphragms was such that the pressure-deflection curves (fig. 4) were very nearly straight lines over a considerable range. The existence of this linear relationship permits a simplification. Since $X/D = k\ p/E$ for a given diaphragm then, in general, for diaphragms of similar shape:

$$\frac{X}{D} = \frac{p}{E}\ f\left(\frac{t}{D},\ \sigma\right) \tag{3}$$

If, in this equation, the values of all the parameters were known, values of $\frac{t}{D}$ against $\frac{X}{D}\frac{E}{p}$ could be plotted for diaphragms of one material to determine the form of the function f for a given value of σ; then two such plots for different materials could be compared to determine the dependence of the function f on σ. Unfortunately, only X, p, and D can be said to be known exactly. The value of t, as mentioned before, is different for different corrugations of the same diaphragm, and the problem of determining effective values from the measured values remains. Also, the values of σ and of E for the diaphragm may be quite different from the values obtained on test specimens.

In the theory of elastic deflection of flat plates, σ enters mainly in a factor $(1 - \sigma^2)$, which may be combined with E. Its effect, other than this, is less than 2 percent for a variation of σ from 0.25 to 0.3 (reference 2). This fact does not prove, but does suggest, that the influence of σ on performance of a corrugated diaphragm may be approximately the same.

In view of these considerations, the equation might be expected to apply fairly accurately to diaphragms of these different materials when written as

$$\frac{X}{D} = \frac{p}{E/(1 - \sigma^2)}\ f\ (t/D) \tag{4}$$

The modulus $E/(1 - \sigma^2)$ is called the "plate modulus," and is designated by the symbol F.

The values of the modulus of elasticity E for beryllium copper, as obtained from handbooks and from manufacturer's data, vary from 16×10^6 to 18.9×10^6 pounds per square inch. (See reference 6, pp. 171-179.) The modulus depends on the amount of cold working and heat treatment. Data given by the Beryllium Corporation, for instance, indicate 18.0×10^6 pounds per square inch as the value for the soft (quenched) material, and 18.9×10^6 pounds per square inch when heat treated from the soft state; 17.2×10^6 pounds per square inch when worked four numbers hard from the soft state; and 18.4×10^6 pounds per square inch when heat treated after the working. The working of the metal that occurs in the stretching necessary to form the diaphragm certainly hardens it but probably by different amounts in different parts. The same thing probably occurs with phosphor bronze. Values of E for it are given as about 14.5×10^6 to 15×10^6 pounds per square inch.

In view of the complexity and the uncertainty connected with the elastic properties, it appeared best to assume an arbitrary value of the plate modulus for beryllium copper and to use the experimental data to determine the form of $f(t/D)$ in equation (4); then to determine effective values of F for diaphragms of other materials that would give agreement at some value of t/D; and, finally, to see whether the same variation with t/D occurs with all materials.

In the problem of determining the effective values of thickness from measurements taken on the diaphragms, two things must be known: first, what is the relative contribution of different annular zones of the diaphragm to the total deflection; and, second, how does the deflection vary with thickness, other things being equal? The first question is answered by deflection traverse measurements, such as are shown in figure 8. From such data, it was determined that the contributions of the zones between the letters (fig. 8) were approximately in the ratio 5, 4, 3, 2, 1, starting at a.

From an initial analysis of the data on beryllium-copper diaphragms, using the initial thicknesses of the blanks, it appeared that the ratio (t/D) entered in the equation as about the -1.47 power; the equation being

$$\frac{X}{D} = 2.6 \times 10^5 \frac{P}{F} \left(\frac{t \times 10^3}{D}\right)^{-1.47} \tag{5}$$

where F was taken as 10^9 mm Hg (18.9×10^6 lb. per sq. in.).

This equation, as it stands, is applicable to the design of similar diaphragms, made in the same way as those on which the data were obtained, over the range of t/D from 0.001 to 0.005. The use of the equation assumes that the variations in thickness due to stretching will be similar for various diaphragms formed in the same way. Actually, the ratio of the initial thickness to effective thickness (defined later) varied from 1.18 to 1.10 (average about 1.15) for diaphragms made in one forming operation and from 1.12 to 1.09 (average 1.10) for those made in two stages.

It is, of course, possible to make diaphragms with this same outline by other methods, such as stamping, or spinning, or by two-stage hydraulic forming as described, which result in different variations of thickness. It is desirable to deduce an equation that will represent the characteristics of the shape, rather than of the method of forming. To do this, it is necessary to use an effective thickness.

Having, from equation (5), the approximate dependence of deflection on t/D as the -1.5 power and, from the deflection traverse data (fig. 8), the relative contributions of the various corrugations, the effective thickness, t_e, is theoretically given by the relation:

$$t_e^{-1.5} = \frac{5t_{ab}^{-1.5} + 4t_{bc}^{-1.5} + 3t_{cd}^{-1.5} + 2t_{de}^{-1.5} + t_{ef}^{-1.5}}{5 + 4 + 3 + 2 + 1}$$

Inasmuch as the greatest variation of thickness in any one diaphragm was about 20 percent and the power to be used is only 1.5, the maximum error introduced by using merely the weighted average is only of the order of 1 percent. The effective thicknesses were therefore computed from the micrometer measurements by straight averaging of four or more measurements on a given corrugation and by weighted averaging of the averages for the five zones (shown in fig. 8) using 5, 4, 3, 2, 1 as the relative weights. The thicknesses measured at the indicated points were taken as representative of the average thickness throughout the zones. Each zone consisted of a half-corrugation, with the exception of the outer one, which was taken to include the offsetting corrugation.

With the values of t/D computed in this way, the load-deflection data were plotted on a log-log chart. The data first obtained extended over the range from 0.001 to 0.005 for t/D, and were found to be represented by a formula of the type of (5). It was to be expected, on theoretical grounds, that this type of equation could not be safely extrapolated, since an extrapolation to higher values of t/D would soon indicate a smaller stiffness than the initial stiffness of flat diaphragms. (See fig. 11.) The initial deflection of a flat diaphragm involves no tensile stressing, only pure bending; and, since the corrugations lower the bending stresses at large deflections only at the cost of introducing initial tensile stresses, the corrugated diaphragms would be expected to be stiffer than the flat ones are initially. Data beyond the range then covered were obtained from a few diaphragms 1.5 inches in diameter made of beryllium copper 0.013 inch thick ($t/D \times 10^3 = 8.4$). One of these diaphragms was etched to obtain data on intermediate values of t/D. The range was also extended to lower values of t/D by etching diaphragms with initially low values of t/D.

In figure 9, which presents the data on Be-Cu diaphragms, the points for the etched diaphragms are marked with filled circles, triangles, or crosses. The same character applies to a series of measurements made on one diaphragm after successive etchings. As the figure shows, the points representing these successive stages fall nicely on the curve formed by other points representing unetched diaphragms and other etched diaphragms. The etching technique is apparently reliable, at least for beryllium copper, and is quite simple and rapid. It is believed that further use of it will greatly facilitate the work of obtaining similar data on diaphragms of other shapes.

This extension of the range confirmed the suspicion that a simple equation of the form of (5) would not be valid over a longer range of t/D values. The equation

$$\frac{F}{P}\frac{X}{D} = 2.25 \times 10^5 \left(\frac{t \times 10^3}{D}\right)^{-1.52} \qquad (6)$$

represents the data over the range of $\frac{t \times 10^3}{D}$ from 0.9 to 4.0 with fair accuracy (where, for beryllium copper, $F = E/(1 - \sigma^2)$ is taken as 18.9×10^6 lb. per sq. in.). A large number of design problems, especially in aircraft instruments, will be solved by diaphragms in this range. In general, of course, the use of charts of the type of figure 9 is preferable because the curve in the figure cannot be expressed by a simple formula.

In figure 10 the curve of figure 9 is repeated, with the data for phosphor bronze and Z-nickel diaphragms. The value of $F = 17.5 \times 10^6$ pounds per square inch was assumed for the phosphor bronze and $F = 28.4 \times 10^6$ pounds per square inch, for the Z-nickel, in order to have the points fall as nearly as possible on the curve for beryllium copper.

Although the data here are not as coherent or as numerous as those on beryllium copper, the same general trend is apparent. Equation (6) can therefore be used for these materials, with somewhat less exactness. It may reasonably be expected that the curve of figure 9 will hold approximately for these materials over its full range of values of t/D, and even for other materials with appropriate values of F.

As mentioned previously, it is not known just what values should be taken for σ and E for these materials. Further work is necessary to determine the average value of the elastic constants of diaphragms of various materials. Meanwhile, for practical design work it is sufficient to use the values of F here assumed, when using the charts.

DESIGN OF DIAPHRAGMS

The curves of figures 9 and 10 are repeated as curve (1) in figure 11. Other curves useful in estimating performance limits of possible diaphragms are included in the same figure. Curve (2) is drawn from data on diaphragms formed with a thin rubber backing. This change in effective die shape resulted in more flexible diaphragms. The deflections were nearly proportional to load over the 0.02D range but the deviations from proportionality seemed to depend on thickness, the thinner diaphragms made this way becoming more flexible, and the thicker ones less flexible, at the larger deflections. The use of a thicker backing gave diaphragms having a still greater initial flexibility, but the deflection of these diaphragms was linear over only a very small range, all of them becoming less flexible at the larger deflections.

Curves (3), (4), and (5) represent approximate data supplied through the courtesy of Manning, Maxwell & Moore, Inc., on diaphragms made by them. The diaphragms were of beryllium copper 2-1/2 inches in effective diameter, with a reinforced flat in the center of a 5/16-inch diameter. The corrugations were circular arcs of approximately 60° with suitable radii for forming two or five complete corrugations. The corrugations themselves were symmetrical about the plane of the rim, that is, the outline was not offset, or concave, as was the N.B.S. outline (fig. 1) in which the two and a half corrugations of 60° arcs were offset so that three of the arc centers were in the plane of the rim.

Curves (3) and (4) are for all values of deflection up to 0.02D, while curve (5) is only for the value 0.03D. The load-deflection characteristics of these diaphragms are not as nearly linear as those of the N.B.S. outline.

Without more extended data, it may reasonably be assumed that curves similar to (1), (2), (3), (4), and (5) could be drawn to represent other shapes of diaphragms provided that their pressure-deflection curves are straight lines. As an indication of what limits of flexibility might be obtained, curves (6) and (7) are shown. Curve (6) represents the initial part of the computed pressure-deflection data for flat diaphragms. It is valid only for relatively small deflections, up to about 0.4 of the thickness. Curve (7) represents the computed deflection-pressure ratio for flat diaphragms deflected to $X = 0.02D$. This curve is valid only for this value of the deflection. Other curves, for deflections less than 0.02D, would lie between curve (6) and curve (7).

The data for plotting these curves were obtained from the approximate equation for flat diaphragms. (See reference 2.)

$$\frac{X}{t} + 0.5\left(\frac{X}{t}\right)^3 = \frac{3}{256}\frac{P}{F}\left(\frac{D}{t}\right)^4 \qquad (7)$$

For initial deflections, $\left(\frac{X}{t}\right)^3$ may be neglected. Multiplying the remaining terms by t/D:

$$\frac{X}{D} = \frac{3}{256}\,\frac{P}{F}\left(\frac{D}{t}\right)^3$$

or

$$10^{-4}\,\frac{F}{P}\,\frac{X}{D} = 1{,}170\left(\frac{1000t}{D}\right)^{-3} \qquad (8)$$

This equation is represented by curve (6) of figure 11. Equation (7) may be written in the form

$$10^{-4}\,\frac{F}{P}\,\frac{X}{D} = \frac{1170}{\left(\frac{1000t}{D}\right)^3 + 5 \times 10^5 \left(\frac{X}{D}\right)^2 \left(\frac{1000t}{D}\right)} \qquad (9)$$

For $\frac{X}{D} = 0.02$, this reduces to

$$10^{-4}\,\frac{F}{P}\,\frac{X}{D} = \frac{1170}{\left(\frac{1000t}{D}\right)^3 + 200\left(\frac{1000t}{D}\right)} \qquad (10)$$

which is the equation represented by curve (7).

The initial flexibility of a flat plate is presumably the limiting flexibility for stable diaphragms of a given t/D ratio. In general, a diaphragm may be rated on its flexibility and the range of pressure over which it gives the desired relations between pressure and deflection. It is seen from figure 11 that, even at the higher values of t/D, none of the diaphragms represented by curve (1) is more than half as flexible as a corresponding flat diaphragm is initially, as indicated by the ratio of ordinates of curves (1) and (6), for a given t/D. The flexibility is much greater, however, than for a flat diaphragm having a deflection of 0.02D and, for larger deflections, it would be even more so.

Curve (7), for flat diaphragms deflected to 0.02D, approaches curve (6) at high values of (t/D) because curve (6) applies up to deflections of about 0.4t, which for the larger values of t/D approaches more and more closely to equality with 0.02D.

Since there are five variables (X, t, P, D, F) to be considered (or four for each material), it is possible, in general, to represent only a limited part of the relations on a two-dimensional graph as in figure 11. The five quantities may always be combined into three dimensionless ratios $\frac{X}{D}$, $\frac{t}{D}$, and $\frac{P}{F}$, allowing the relations to be represented by a three-dimensional surface. Only in the event that there is a simple analytic relation between the ratios can these variables be combined to permit a general representation on a two-dimensional chart. Thus curves (1), (2), (3), and (4) (fig. 11) are general over a limited range; i.e., up to $X = 0.02D$, because of the linear pressure-deflection characteristics, whereas, curve (6) is valid only over a much smaller range. Curves (5) and (7) are valid for only one value of X/D, and therefore are not general representations of diaphragm behavior. A set of such curves, of t/D against P/F for various X/D values, may be very useful in representing data or in constructing design charts for diaphragms for which X/D is not a simple function of pressure.

It is not necessary that a linear relation exist between X/D and P/E in order that the performance data may be depicted on a two-dimensional chart. If a diaphragm design gave a pressure-deflection curve for a given value of t/D represented by any relation such that $f\left(\frac{X}{D}, \frac{P}{E}\right) =$ constant, then a two-dimensional plot of $f\left(\frac{X}{D}, \frac{P}{E}\right)$ against t/D would give a general representation of the performance over the range of X/D for which the function f was valid.

It will usually be possible to express the pressure-deflection curves of diaphragms useful in instruments at least approximately by some simple equation, and to portray the general relations by some curve of the type of those in figure 11. Such curves, once established by experiment, furnish the direct path to solutions of design problems.

The usual diaphragm design problem is to find the shape, the diameter, the thickness, and the material for a diaphragm that will give a certain deflection when subjected to a certain pressure. In general, there will be further restrictions, such as arise in specifying the shape of the pressure-deflection curve, the permissible drift, a limit to the allowable diameter, etc.

If a linear pressure-deflection characteristic is desired, the diaphragm shape shown in figure 1 will be suitable. Fixing the value of D fixes the ratio X/D. If this value is greater than 0.02, and drift is to be small, beryllium copper or other material with a high ratio of yield strength to elastic modulus should be used. Having chosen the material, F is fixed and the value of $\frac{F}{P}\frac{X}{D}$ can be computed. Suitable values of F to be used with these charts have been given for the phosphor bronze, the beryllium copper, and the Z-nickel used in these tests. Comparative data have not yet been obtained for other materials.

Using the chart (fig. 9), the value of t/D and thus of t is determined. The value of t thus obtained is the effective thickness. This value is, of course, less than the thickness of the blanks to be used because stretching must occur during forming. The ratio of initial to effective thickness depends on the method of manufacture. For the diaphragms formed by the two-stage method, which was found to give the most consistent results, this ratio was about 1.10. For the diaphragms formed in one stage, the ratio was about 1.15. By use of the appropriate ratio the desired blank thickness is found, thus completing the solution.

If close tolerances are set on the performance, there may be some adjustment needed after the results of the first design are obtained. The flexibility can be adjusted by using a slightly different thickness of material. Both the flexibility of the diaphragm and the deviations of its pressure-deflection curve from linearity will be affected by the center reinforcement, by slight changes in shape. In most cases, there is the fortunate opportunity of adjusting the mechanism related to the diaphragm to remove small discrepancies in the diaphragm performance. The data contained in table IV on the relative effect of central loads is of value in selecting control springs to obtain the desired changes in stiffness. This selection is, of course, of no help in overcoming elastic defects; they can be remedied only by the choice of proper materials and methods of manufacture. If the design conditions permit the use of a larger diameter, the resulting lower value of X/D will help to limit the stresses to a safe value.

The curves presented here represent only a start on the empirical work necessary to cover the range of practical diaphragm shapes. Correlations of stresses, load limits, hysteresis, and drift, with the variables of shape, size, and material are yet to be investigated. The correlation of performance of single diaphragms with clamped edges with that of capsules with free edges is another important phase, as yet untouched.

It is believed, however, that the curves will be of value in design, specifically where linear deflection curves are desired and, in general, in enabling the designer to predict by comparison the approximate performance of diaphragms of other shapes.

National Bureau of Standards,
Washington, D. C., October 13, 1939.

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TABLE I

Pressures Necessary for Full Forming
of Be-Cu Diaphragms

Original thickness of blank (in.)	Diameter of diaphragm (in.)	Approximate forming pressure (lb./sq.in.)
0.002	2.5	1,250
	2.0	1,250
	1.5	1,500
.0036	2.5	2,000
	2.0	2,500
	1.5	3,000
	3.0	2,000
.005	2.5	2,500
	2.0	3,500
	1.5	4,500
.008	2.5	4,000
	2.0	5,000
	1.5	6,000
.013	1.5	9,500

TABLE II

Hardness of Diaphragm Materials

Thickness in 0.001 inch	Condition	Indentation number $H = W/A$[1] (Knoop indenter)
	Phosphor bronze	
0.002	As received	175
.003	do.	177
.004	do.	198
.006	do.	200
.008	do.	222
.004	Heat-treated at 425° C.	105
.006	do.	118
	Beryllium copper	
0.002	Heat-treated at 300° C.	336
.004	do.	351
.005	do.	335
.008	do.	317

[1]W is the weight in kilograms and A is the projected area of the indentation in square millimeters.

TABLE III

Pressure-Deflection Data for Diaphragms of National Bureau of Standards Outline 1 Loaded on the Convex Side

Diameter D (in.)	0.02D (in.)	Blank thickness (in.)	Pressure corresponding to deflection of 0.02D (mm Hg)	(lb./sq.in.)
		Beryllium copper		
1.5	0.03	0.0022	129	2.50
		.0036	284	5.50
		.0051	458	8.89
		.0079	991	19.2
2.0	.04	.0022	96	1.86
		.0036	195	3.78
		.0051	313	6.07
		.0079	581	11.25
2.5	.05	.0022	52	1.01
		.0036	120	2.32
		.0051	191	3.70
		.0079	376	7.28
3.0	.06	.0050	142	2.75
		Phosphor bronze		
1.5	0.03	0.0024	136	2.63
		.0033	223	4.31
		.0042	313	6.05
		.0064	605	11.7
2.0	.04	.0024	95	1.84
		.0033	157	3.04
		.0042	235	4.54
		.0064	420	8.12
2.5	.05	.0024	58	1.12
		.0033	94	1.82
		.0042	141	2.72
		.0064	240	4.64
3.0	.06	.0033	72	1.39
		.0042	110	2.12
		.0064	199	3.85

TABLE IV

Values of the ratio of concentrated load to distributed load (projected area of the diaphragm times the differential pressure) for equal central deflections

(B = beryllium copper; P = phosphor bronze)

Diaphragm diameter (in.)	Approximate thickness of diaphragms (in.)						
	0.002	0.003	0.004	0.005	0.006	0.008	0.013
1.5	0.39P	0.41P	0.39P	0.42B	0.41P	0.39B	0.31B
			.38B				.35B
2.0	.44P	.45P	.43P	.44B	.44P	.41B	-
	.42B		.44B				
2.5	.43P	.42P	.45P	.44B	.44P	.44B	-
3.0	-	.46P	.46P	.44B	.41P	-	-
				.44B	.44P		

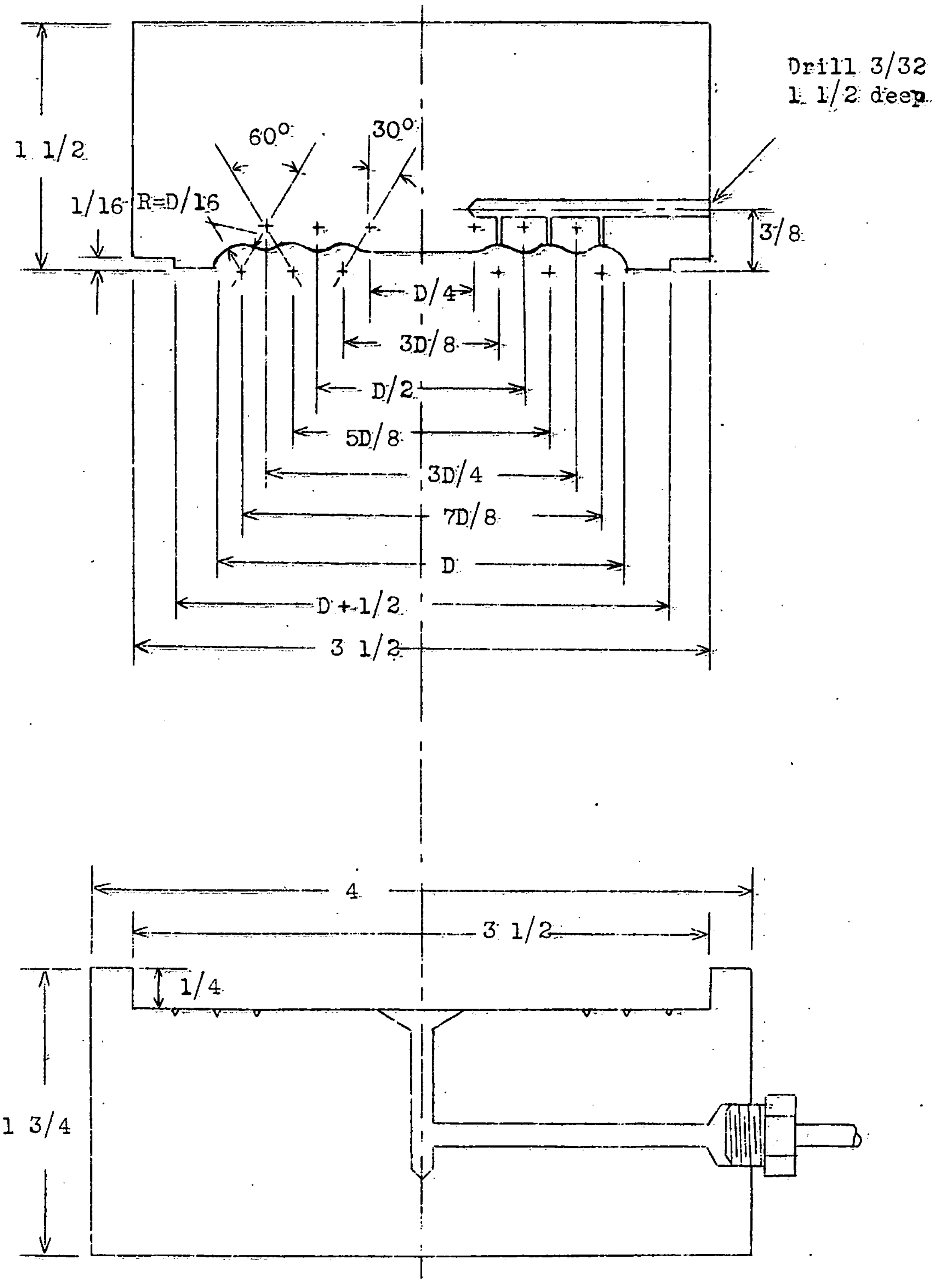


Figure 1.- Cross-sectional outline of die and pressure base used in forming diaphragms. The four dies used had geometrically similar outlines, differing only in the values of D, which were 1 1/2, 2, 2 1/2, and 3 inches.

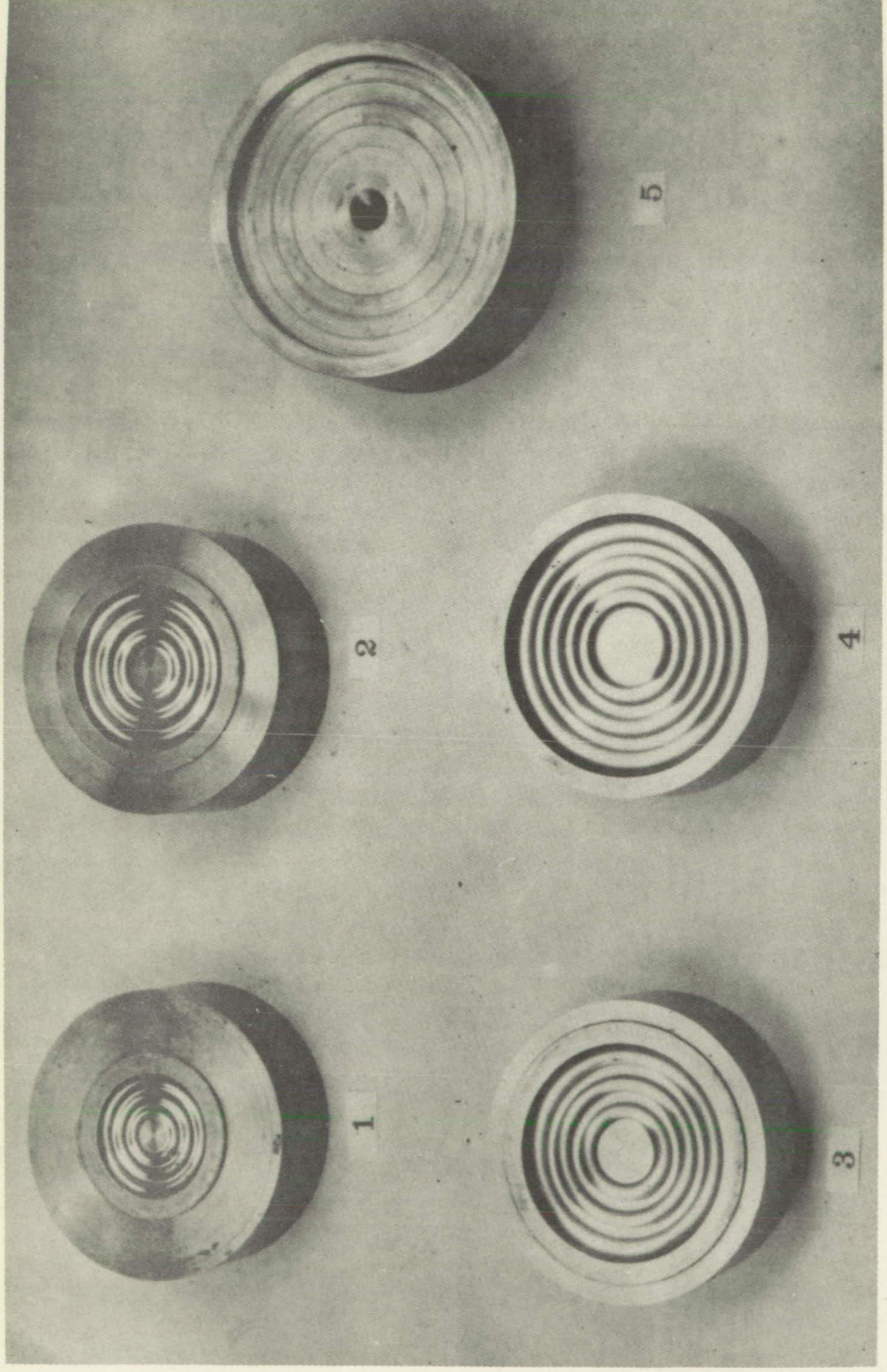
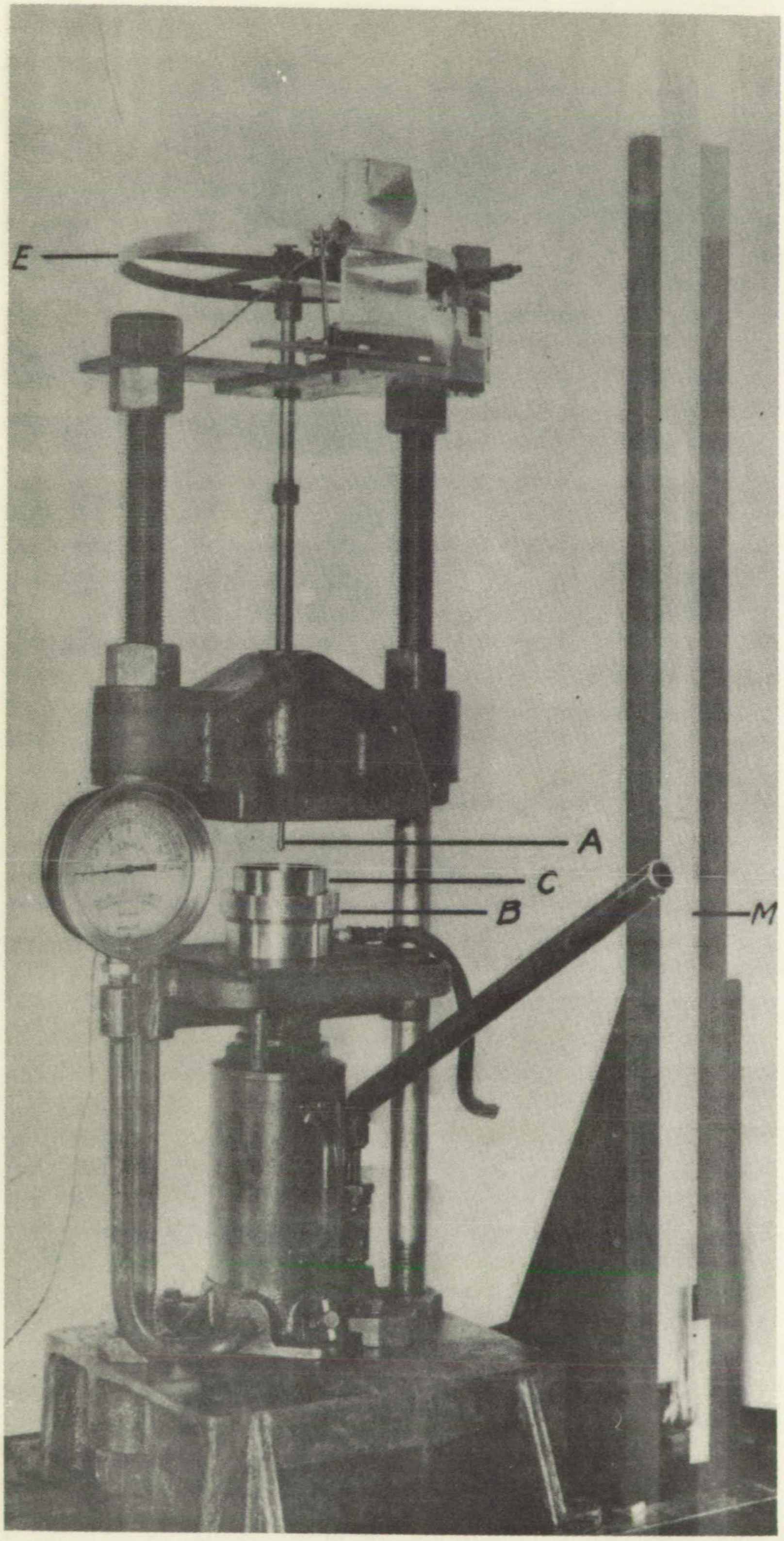


Figure 2.- Photograph of dies 1 to 4 and the base for the dies, 5.



The diaphragms were clamped in the pressure chamber B, by the cylinder C, when the lower platen was raised. The pressures were measured by the manometer M. The deflections were measured by the micrometer A, which was turned by the graduated wheel E.

Figure 3.- View of the press used in making and testing the diaphragms, with testing apparatus,

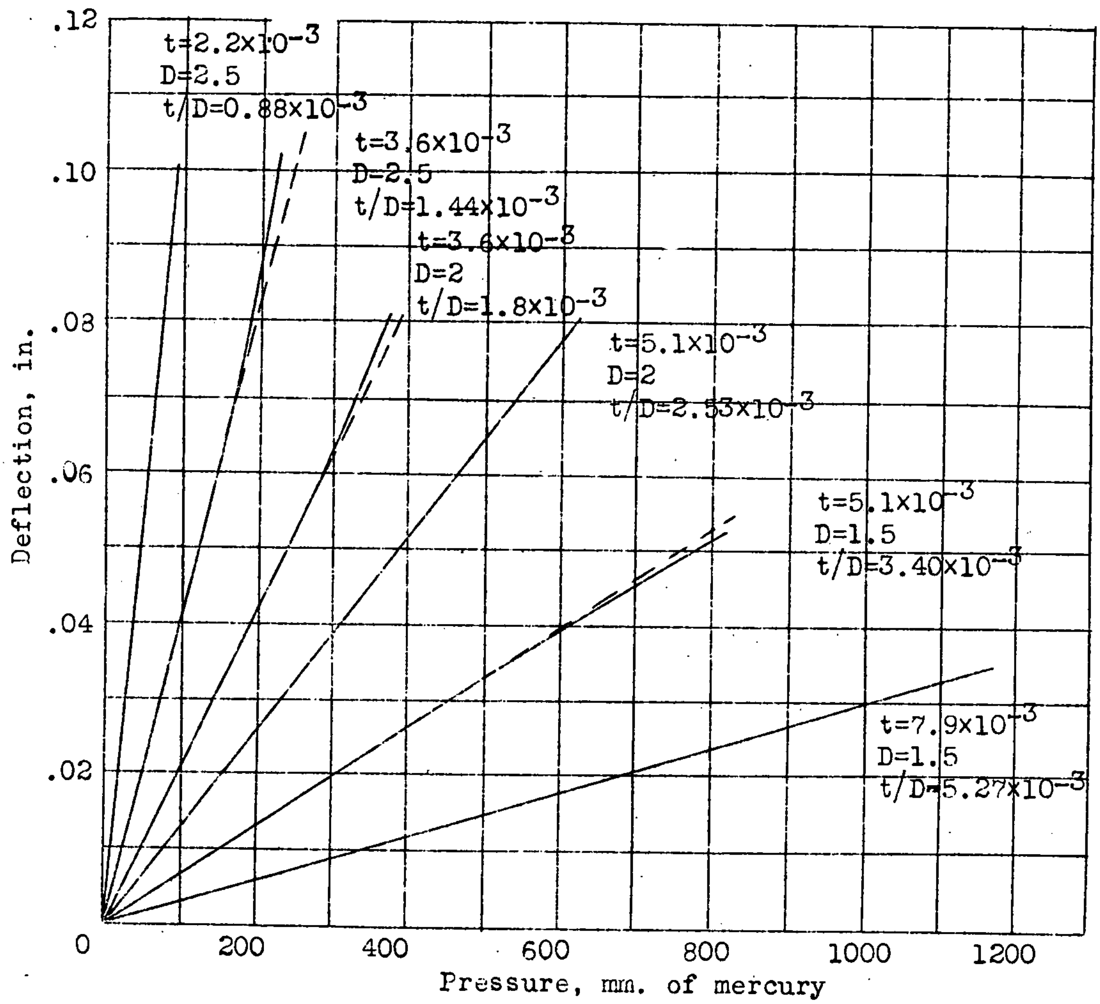


Figure 4.- Pressure-deflection curves for several beryllium copper diaphragms. The dotted lines are extensions of the straight lines through the X = 0.02D points. The indicated thickness t and diameters D are in inches.

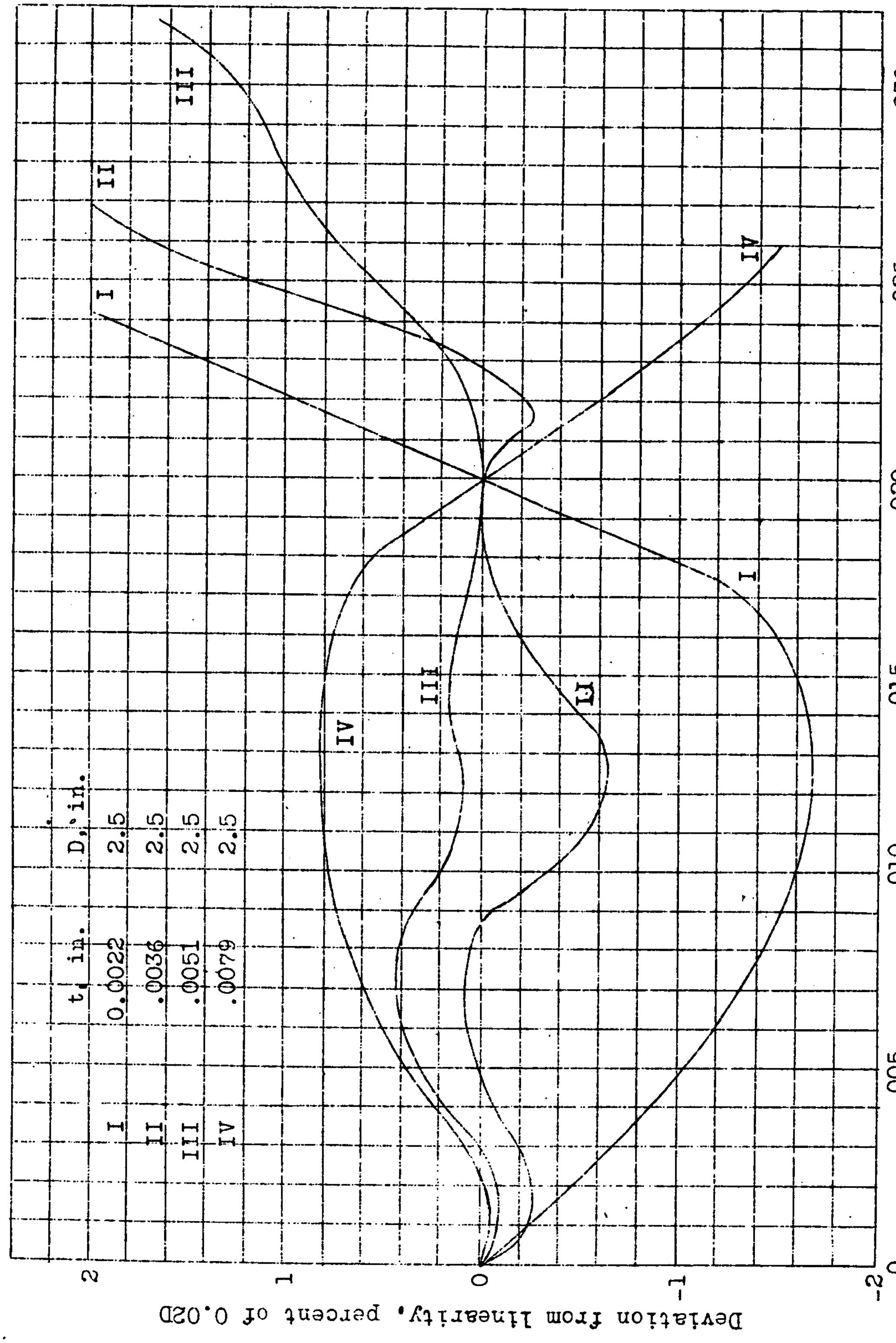


Figure 5.- The deviations from linearity of pressure-deflection curves for several beryllium copper diaphragms. The deviations from the straight line passing through the origin and the deflection X = 0.02D are plotted as percentages of 0.02D.

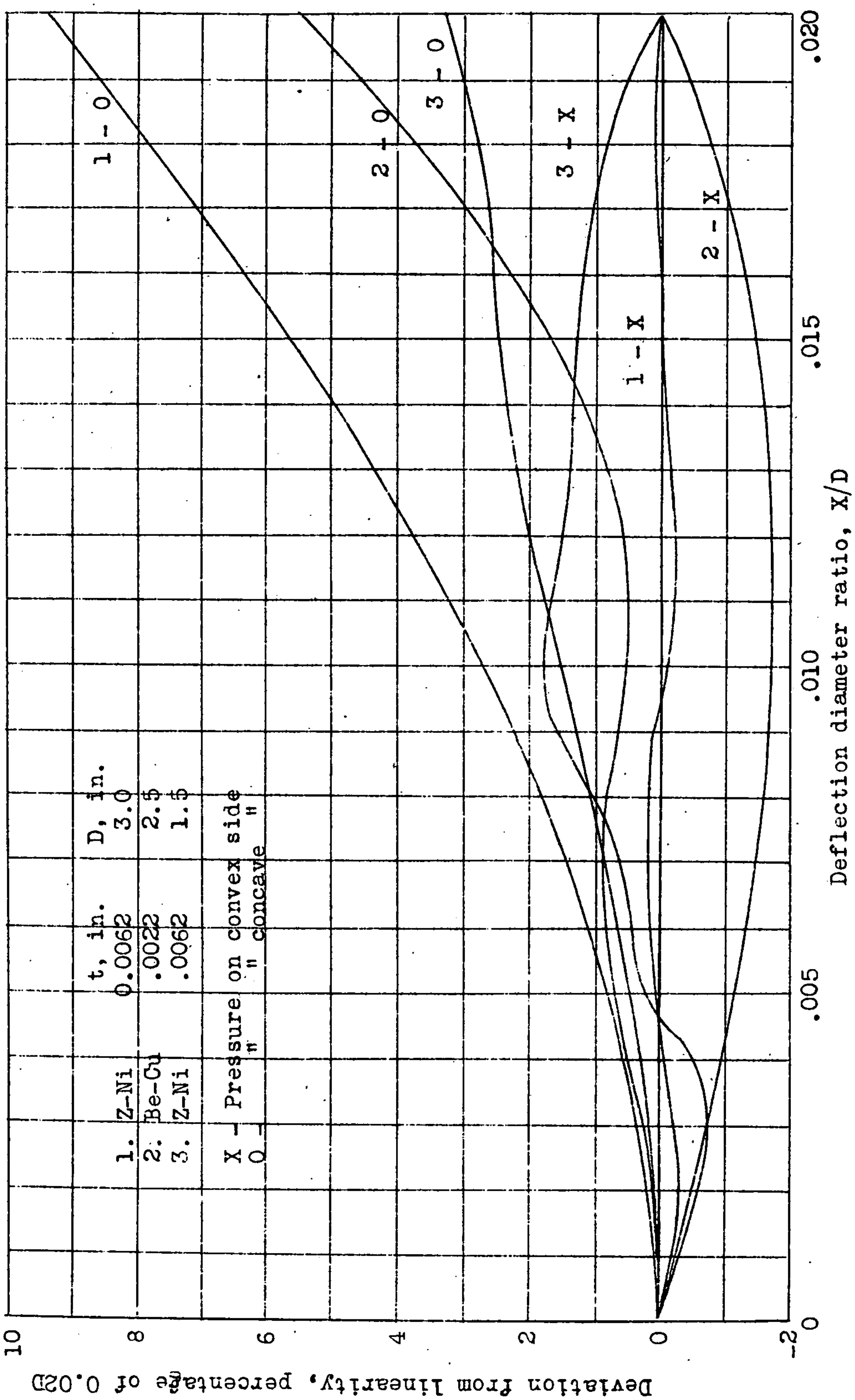


Figure 6.- The deviations from linearity of pressure deflection curves for several diaphragms loaded in opposite directions. The reference line for the deviations passes through the origin and the X = 0.02D point of the pressure-deflection curve for pressure applied to the convex side of the diaphragm.

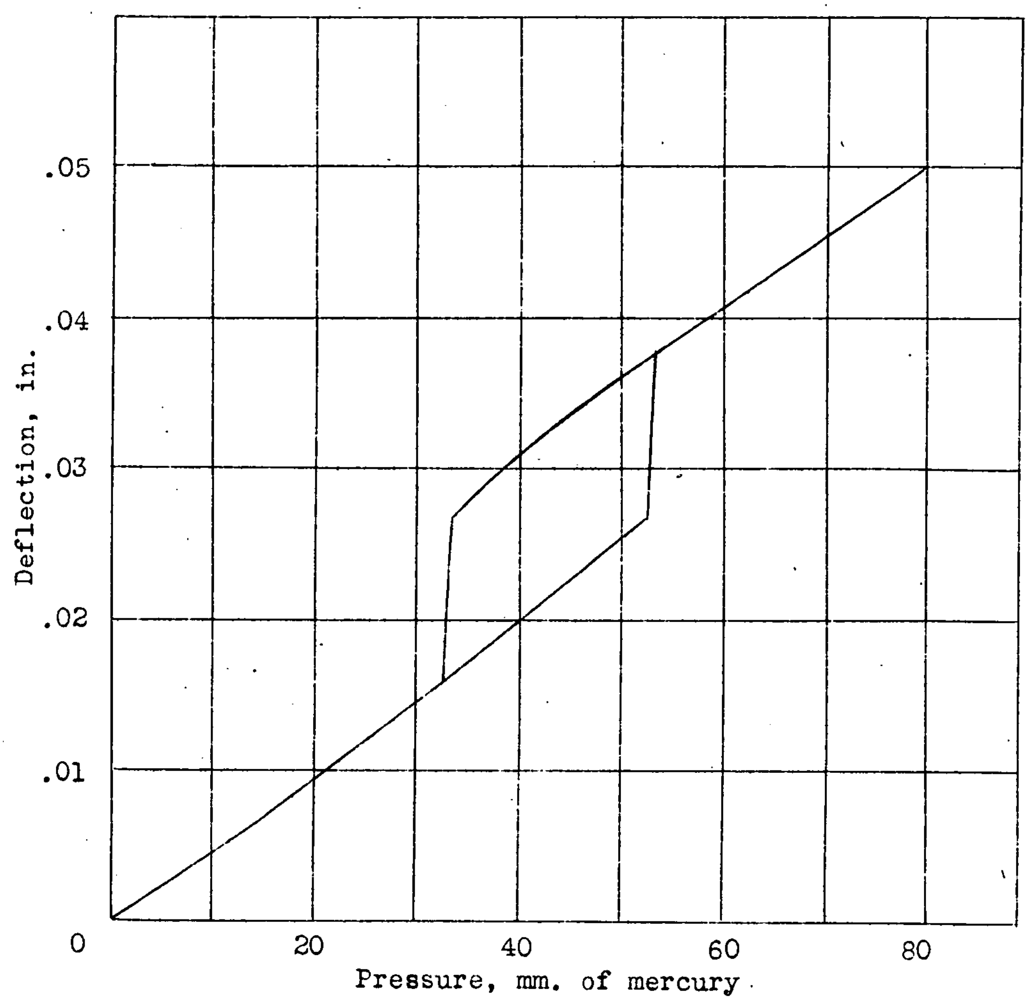


Figure 7.- Pressure-deflection curve for a snap-action diaphragm with no center reinforcement. The diaphragm was 2.5 inches in diameter and was made of beryllium copper 0.0036 inch thick.

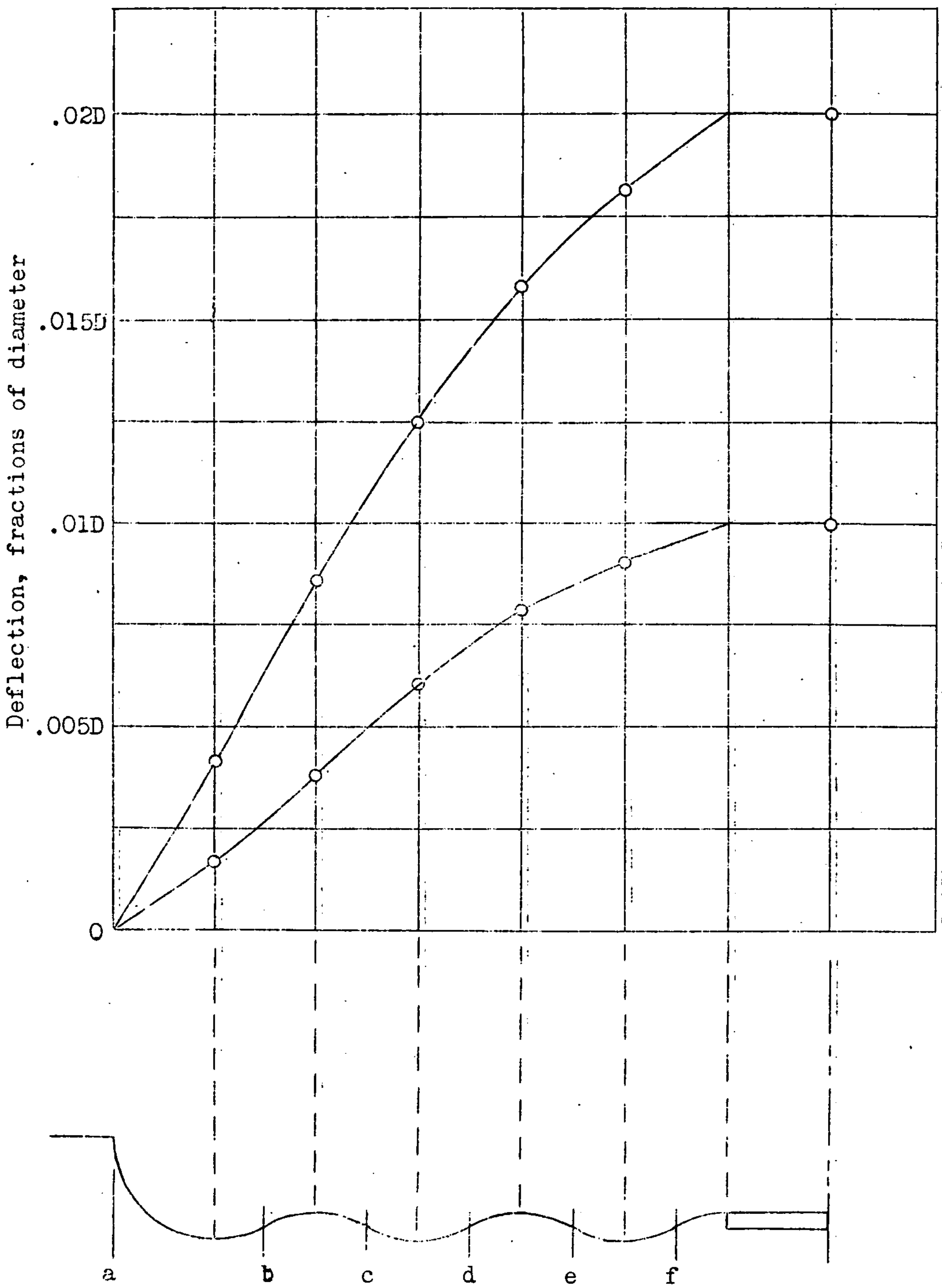


Figure 8.- Relative deflections at the several corrugations of a diaphragm for two values of central deflection. The points represent deflections measured at the corresponding corrugations shown on the outline below the curves.

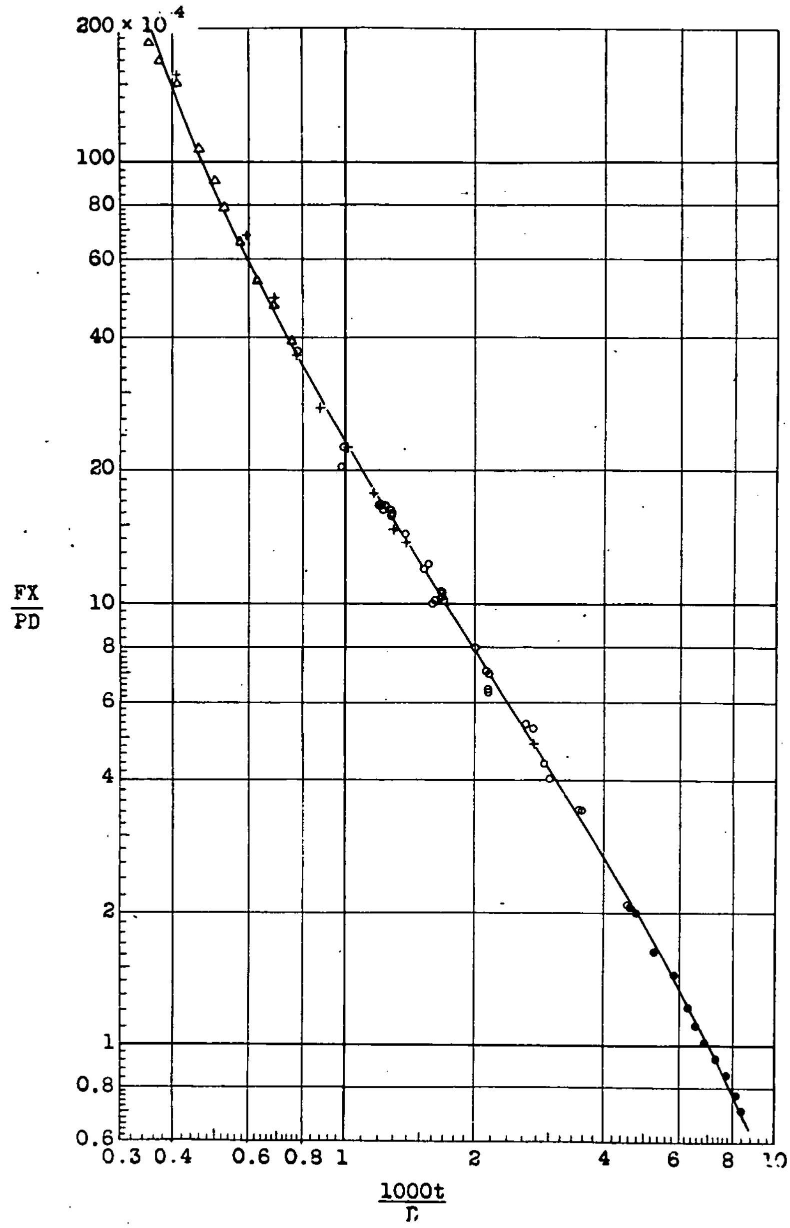


Figure 9.- The pressure-deflection data on beryllium-copper diaphragms plotted to show the relation between the dimensionless ratios FX/PD and t/D. The effective value of F (approximately $E/1 - \sigma^2$) was taken as 18.9×10^6 pounds per square inch. (10^9 mm Hg). The points marked with open circles represent different diaphragms. Those marked with crosses, triangles, or solid dots represent the series of measurements on three different diaphragms after successive etchings.

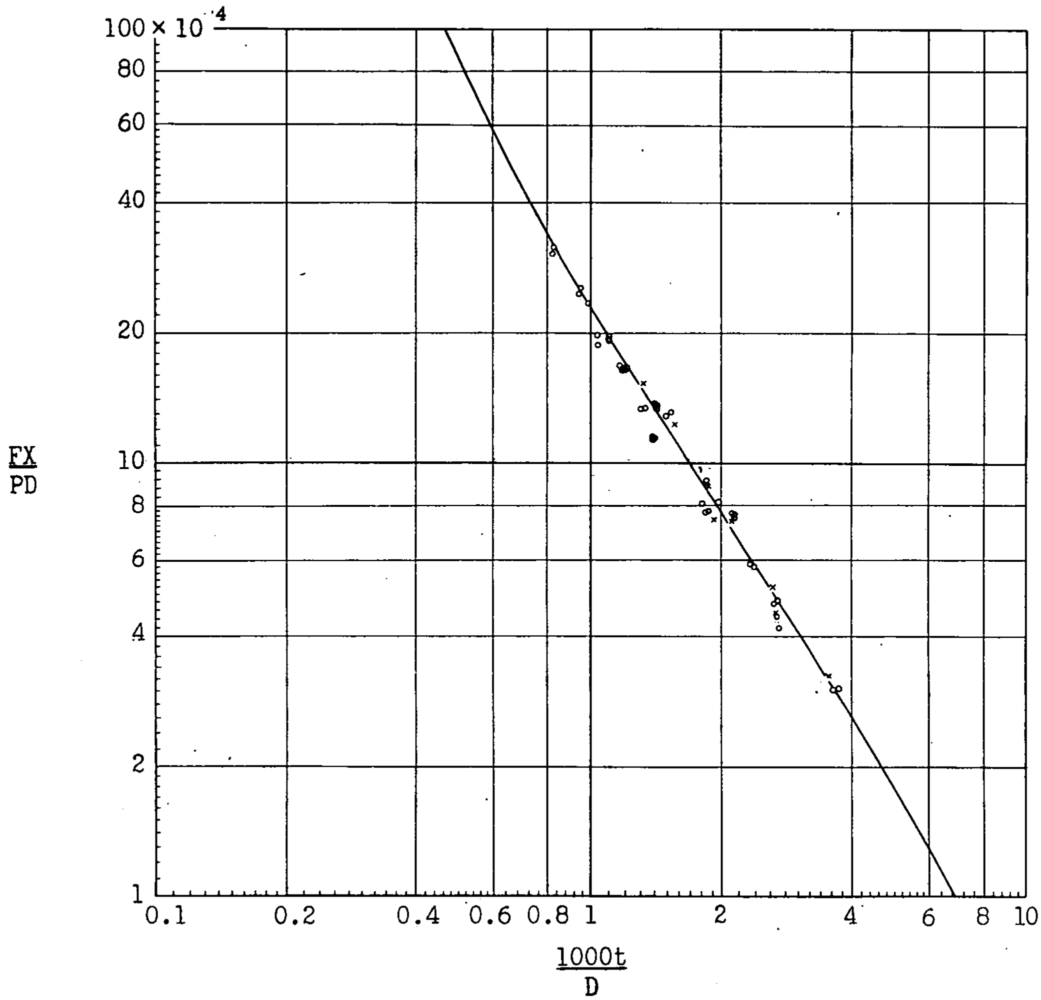


Figure 10.- Data on phosphor-bronze and Z-nickel diaphragms plotted as in figure 9. In order to have the curves for these materials coincide with that for beryllium-copper in figure 9, the effective values of F (approximately $E / 1 - \sigma^2$) were taken as 17.5×10^6 and 28.4×10^6 pounds per square inch for phosphor-bronze and Z-nickel, respectively. The open circles represent the phosphor-bronze diaphragms; the crosses, those of Z-nickel.

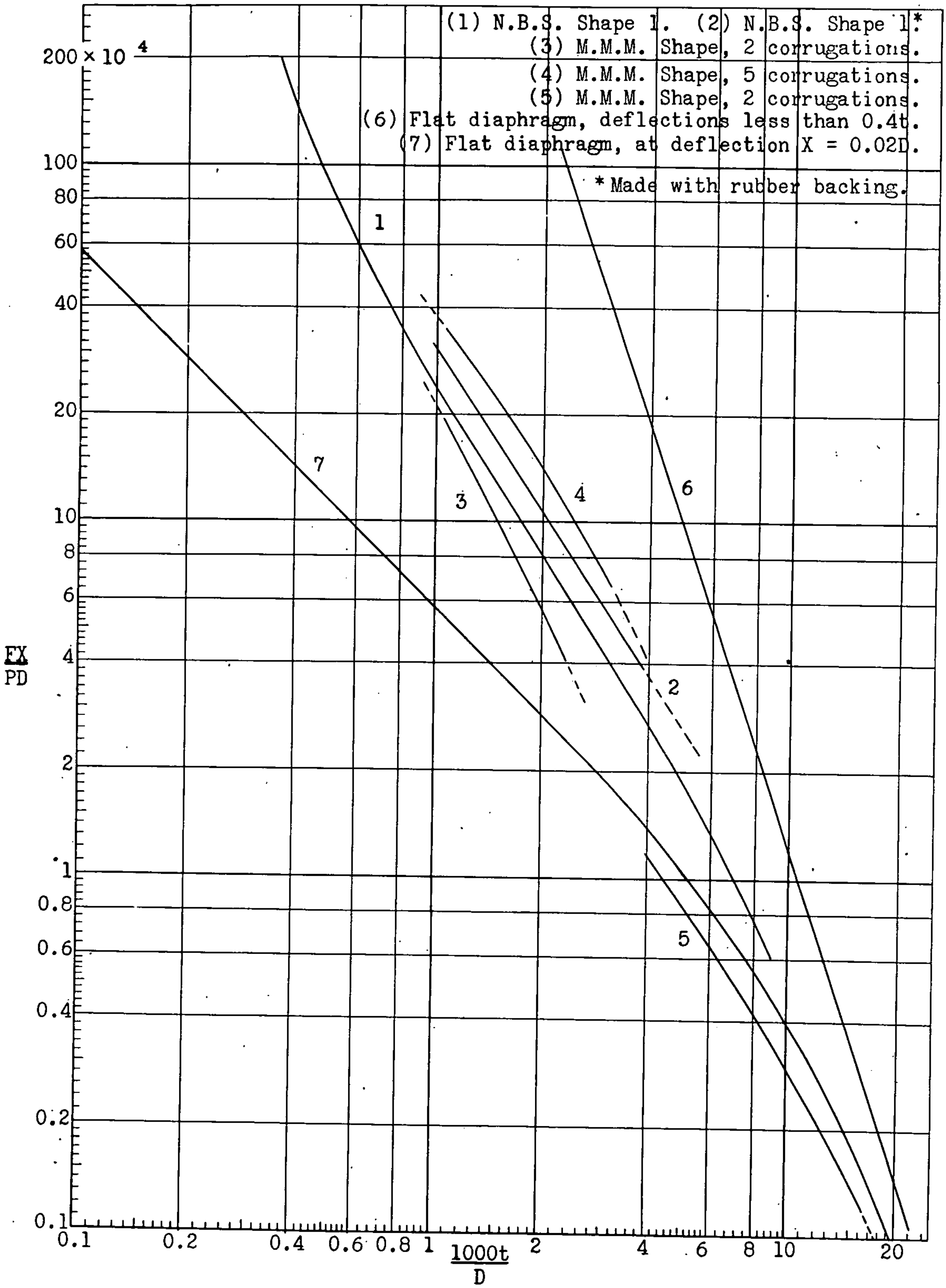


Figure 11.- Comparison of different diaphragm shapes. The curves show the relation between the dimensionless ratios $\frac{FX}{PD}$ and $\frac{t}{D}$ for various types of diaphragms.