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VELOCITY GAINED AND ALTITUDE LOST IN RECOVERIES FROM INCLINED FLIGHT PATHS

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SUMMARY

A series of charts is given showing the variation of the velocity gained and the altitude lost in dive pull-outs with the initial indicated air speed and the dive angle. The effects of the maximum load factor, the drag parameter K, the initial attitude, and the type of recovery on the velocity gained and the altitude lost are also considered.

The results were obtained from a step-by-step solution of the equations of motion in which mean values of the air density and the airplane drag coefficient were used. The load-factor variation with time is arbitrarily specified in various ways to simulate pull-out procedures, some of which might be encountered in flight.

INTRODUCTION

The determination of the velocity-altitude relations of an airplane recovering from a dive has been the subject of a number of investigations. Most of these investigations have been analytical in nature and have consisted of presentations of methods and approximations for solving the equations of motion. As the methods have become more exact, the equations have become longer and more involved until finally the step-by-step solution is approached. Even in the more lengthy and exact methods it is still necessary to make assumptions regarding the time variation of some of the quantities involved. In none of the analytical investigations, however, have results been presented in such a form that items of immediate practical interest, such as the maximum velocity gained and the altitude lost in dive pull-outs, can be readily determined for different airplane types.

The matter of speed gained in the recovery from a nose-down attitude has recently become of some importance owing to the fact that the specified limiting diving speeds for the large low-load-factor airplane are only a relatively small amount above the top speed. Thus for this class of airplane, if a steep nose-down attitude results from an emergency operation, the question arises as to whether or not a recovery could be made within the limits of wing strength without exceeding the specified limiting speed.

In order to answer this question, several examples are given, in which the values were obtained from a step-by-step solution of the basic equations of motion. These equations are then slightly simplified and a series of general charts is presented that gives a close approximation of the altitude lost and the velocity gained during various types of dive recovery for different values of the drag parameter K $\left(\text{that is, } \frac{C_D g}{W/S}\right)$.

EQUATIONS OF MOTION

On the assumption that the pitching inertia is zero, one of the three equations of motion is eliminated while only a slight error is introduced. This reduced system of equations is

$$W \cos \gamma - C_L \frac{\rho}{2} V^2 S = \frac{W}{g} V \frac{d\gamma}{dt} = \frac{W}{g} \frac{V^2}{R} \qquad (1)$$

$$W \sin \gamma - (C_{D_o} + C_{D_i}) \frac{\rho}{2} V^2 S = \frac{W}{g} \frac{dV}{dt} \qquad (2)$$

where

W airplane weight

γ flight-path angle from horizontal plane

C_L lift coefficient

C_{D_o} parasite-drag coefficient

C_{D_i} induced-drag coefficient

ρ mass density of air

V air speed

S wing area

g acceleration due to gravity

t time

R instantaneous flight-path radius

If the mass density of the air ρ is assumed to have some mean value $\bar{\rho}$ throughout the maneuver and if the parasite-drag coefficient and the induced-drag coefficient are combined into a total or resultant drag coefficient, the value of which may be found from a lift and drag polar of the airplane as it is being flown, a simplification of equations (1) and (2) becomes

$$g \cos \gamma - n(t)\, g = \frac{V^2}{R} \qquad (3)$$

$$g \sin \gamma - \frac{C_D \bar{\rho} V^2 g}{2\, W/S} = \frac{dV}{dt} \qquad (4)$$

where $n(t)$ is the load-factor variation with time,

$$n = \frac{C_L \frac{\bar{\rho}}{2} V^2}{W/S}$$

and C_D is the resultant drag coefficient.

Before equations (3) and (4) can be solved, it is necessary to specify, or to know in advance, the time variation of the load factor in the recovery. Although the exact manner of the load-factor variation is unpredictable, since it depends upon the pilot's reactions to circumstances or accelerations, the maximum value of the load factor is fairly well defined because it is governed by the limitations either of the pilot or of the airplane structure.

For the purpose of illustration, three types of load-factor variation $n(t)$ are considered. (See fig. 1.)

Type 1 remains constant with time; this type is a practical impossibility but represents a definite limit that gives minimum velocity gained and minimum altitude lost. Type 2 varies with time in the manner shown by the dashed line of figure 1 and is in qualitative agreement with the normal pull-out procedure in which the load factor is reduced after a maximum value is reached. Type 3 varies with time in the same manner as type 2 until the maximum load factor is obtained, after which it remains constant. Type 3 represents a variation of the load factor that might occur when the danger of striking the ground or exceeding the limiting diving velocity of the airplane is imminent.

In order to show some quantitative results, a step-by-step solution of equations (3) and (4) has been made by the use of the supplementary relation

$$R d\gamma \doteq V dt \qquad (5)$$

for several dive recoveries (see table I) of an airplane with the lift-drag polar shown in figure 2. In all these cases an average air density ρ equal to 0.0020 slug per cubic foot, corresponding to an average altitude of 5800 feet, was used. Table I shows for each recovery the type of load-factor variation, the initial air speed and altitude, the maximum load factor imposed on the airplane, the maximum velocity gained, and the altitude lost during the maneuver.

In the use of equations (3) and (4) it is necessary to have either the actual airplane polar (as in fig. 2) or to construct a polar in which the induced drag is properly taken into account. Thus, aspect ratio might be considered as an additional variable. Experience gained in solving a number of examples of this sort indicates that the actual drag variation is of slight importance as far as the desired results are concerned, that is, the evaluation of the maximum velocity and the altitude loss in the recovery. In fact, results identical with those listed in table I could have been obtained by the use of a properly chosen constant average drag coefficient C_D. Although the results obtained by the use of such a drag coefficient would agree, as regards altitude loss and maximum velocity gained, with those obtained for variable C_D, there would be no point-to-point agreement in the computed flight paths or velocities.

In accordance with the foregoing reasoning, equations (3) and (4) could be written as follows:

$$g \cos \gamma - n(t)\, g = \frac{V^2}{R} \qquad (6)$$

$$g \sin \gamma - K \frac{\rho}{2} V^2 = \frac{dV}{dt} \qquad (7)$$

where $K = \dfrac{\overline{C}_D g}{W/S}$.

In equations (6) and (7) the term K is the only term in which any definite characteristic of the airplane itself exists and, for this reason, the assumption that various airplanes could be grouped according to certain K $\left(\text{or } \dfrac{\overline{C}_D g}{W/S}\right)$ values, which would apply for the airplane during a given pull-out, was indicated. It must be appreciated that a given airplane may have a definite range of K values, depending on the actual airplane flight condition, the initial air speed, the type of load-factor variation, and the maximum load factor obtained during the recovery. In other words, the K value for a given case is a function of the airplane polar and of the portion of the polar that is traversed during the pull-out.

Because the parameter K to be used for any given case is dependent upon the value of the mean drag coefficient $\overline{C}_D$ that will apply during the recovery, the average drag coefficient could be defined by the following equation:

$$\overline{C}_D = C_{D_1} + F(C_{D_2} - C_{D_1}) \qquad (8)$$

where

C_{D_1} drag coefficient existing at time pull-out is started

C_{D_2} drag coefficient corresponding to lift coefficient necessary to give required load factor at initial indicated velocity

F approximate weighing factor that includes the effect of velocity gained and time spent in attaining level flight

The following estimated values of the weighing factor F seem to apply well to the types of load-factor variation considered:

Type of pull-out	F
1	0.90
2	.60
3	.75

It is appreciated that no single value of F will satisfy all airplane pull-out conditions for any given type of load-factor variation. The foregoing values have been selected as the ones that give the closest agreement in all cases for each type of variation.

In order to derive a series of general charts by which the altitude lost and the velocity gained can be determined, a number of step-by-step computations were made using equations (5), (6), and (7). In these computations a mean value of air density $\bar{\rho}$ of 0.0020 slug per cubic foot corresponding to an altitude of 5800 feet was used together with three arbitrarily selected values of K (0.015, 0.030, and 0.060) taken to represent, respectively: an extremely clean heavily loaded airplane making a recovery from high velocity at a fairly low load factor, a clean normally loaded airplane making a recovery from a fairly high velocity at a medium load factor, and a clean normally loaded airplane making a recovery from a low velocity at a fairly high load factor. It was felt that this range of K values would be sufficient to cover most present-day transport airplanes, provided that the stall angle of the polar was not approached during the recoveries $(0 < C_L < 1.1)$. It is obvious that, if the large values of C_D associated with $C_{L_{max}}$ were used, values of K larger than those given would be obtained. Later examples, however, will show that even for this case the choice of the largest K will give good results.

For the computations, increments of time were so chosen that the corresponding increment in flight-path angle $\Delta\gamma$ always fell within a range of 3° to 8°. Thus, on the average , about 18 points were used to establish the velocity variation from the assumed initial flight-path angle until the horizontal was reached. These computations yielded, for each case considered, two values of practical interest, namely, the maximum velocity gained and the maximum altitude lost in the dive recovery. These values were then plotted in the form of general charts.

CHARTS

Figure 3 shows, for the smallest value of the parameter $(K = 0.015)$, the variation of the velocity gained and the altitude lost with the initial indicated velocity for load factors of 2, 3, 4, 6, and 8 and initial flight-path angles γ_0 of 90°, 75°, 60°, and 45°. Similar results are given in figures 4 and 5 for the medium (0.030) and the largest (0.060) values of K, respectively. In figures 3 to 5 the load factor is assumed to be constant with time (type 1, fig. 1).

Similarly, the results for the second type of load-factor variation considered (type 2, fig. 1) are given in figures 6, 7, and 8. Figure 9 gives results for the third type of load-factor variation (type 3, fig. 1), only one initial dive angle of 90° being considered.

DISCUSSION

The charts (figs 3 to 8) indicate, in general, that less velocity is gained in the recovery as the load factor is increased and that, for a given case, a certain initial value of air speed exists where the velocity gained is a maximum. There are, owing to the assumptions made, two initial velocities from which no speed increase would be experienced during the pull-out, namely, zero velocity and terminal velocity. The first limit is purely analytical because it implies that infinitely large values of C_L are obtainable and, for this reason, the curves on the charts (figs. 3 to 9) are not continued to zero but are arbitrarily cut off at 100 miles per hour. The second limit, although a more practical one, has been determined for each case on the assumption that the drag coefficient is not a function of velocity. The terminal velocities are therefore indicated by the intersection $n = 0$ of the curve and the zero abscissa line and are the so-called nominal values.

Although not explicitly given by the charts, the computed results show that, for a given case, the maximum velocity occurs earlier in the pull-out as the initial velocity is increased. This result follows from the fact that, where the initial velocity is near the terminal velocity, the maximum velocity occurs very near the start of the recovery after which the velocity will decrease. Thus, the velocity increment and the altitude loss given by the charts do not correspond in time but represent maximum values reached sometime during the pull-out.

A comparison of the charts for the three types of load-factor variation shows that less velocity is gained and less altitude is lost when more of the area under the upper horizontal line of figure 1 is included, that is, when the maximum acceleration is attained as soon as possible and then maintained. For minimum values of velocity gained and altitude lost, it is particularly desirable to add area under the $n(t)$ curve near the beginning of the manouvor where a relatively lower velocity in combination with a given acceleration will result in a greater flight-path curvature. In practice, however, the inertia of the airplane operates to prevent the inclusion of all this area with the result that, even with an instantaneous control operation, the time required to reach a maximum acceleration is of the order of 1.5 or 2 seconds.

It can be seen from the charts that, in general, the altitude loss sustained in recoveries increases with the initial speed, with the steepness of the dive, and also with a decrease in the load factor.

In order to obtain an idea of the effect of using a constant instead of the actual variation of drag coefficient in the solution for speed gained and altitude lost, the results listed in table I are compared with similar results obtained from the charts. These comparisons are shown in table II.

In the column of K values in table II, two values are given for pull-outs 1 and 4; the lower value was obtained from an extrapolated value of the lift-drag polar inserted in equation (8) and the upper value is that of the highest K value given in the charts. The tabulated values of velocity increment, altitude loss, and maximum velocity were obtained with this highest available value (0.060) of K. Even in these cases the discrepancy between this approximation and the more accurate solution is only

8.4 miles per hour and 37 feet altitude for pull-out 1 and 5.8 miles per hour and 46 feet altitude for pull-out 4. These values represent, in general, an error of about 4 percent in the maximum velocity and the altitude lost. This result emphasizes the fact that the average drag coefficient need be known to only a fairly low degree of accuracy, particularly in this range, in order to obtain satisfactory results from the charts.

Each of the pull-outs illustrated in tables I and II was so chosen as to represent fairly extreme conditions in order to indicate the maximum errors involved from the use of a constant average drag coefficient throughout the recovery. These conditions of maximum error in order of importance are: (1) where the greatest change in drag coefficient occurs, that is, a high-load-factor pull-out from a low speed; (2) where the most velocity is gained and the most altitude lost, that is, a low-load-factor pull-out from a low speed; (3) where the longest time is involved in accomplishing the maneuver, that is, a low-load-factor pull-out from a high speed; and (4) where the high load factor is taken in conjunction with the high initial velocity.

A load factor of 6 was taken for several of the pull-outs in order to exaggerate the conditions that might be expected if the airplane of figure 2 is used, although it was appreciated that this value was far above the design load factor of present-day transport aircraft. Even with this high load factor, the accuracy obtainable from the charts is considered quite satisfactory. For this reason, there is an indication that the charts might be used for categories of aircraft that would include some of the more maneuverable airplanes of the present day whose mission might call for diving maneuvers.

In general, it may be noticed that: (a) more altitude is lost and more speed is gained as the type of load-factor variation proceeds in the order 1, 3, and 2; and (b) the initial attitude has a considerable effect on both the velocity gained and the altitude lost. Inasmuch as the attitude can be only roughly estimated by the pilot, its effect may become almost as important as the variation in load factor in correlating the results obtained from the charts with the values obtained from flight tests.

Langley Memorial Aeronautical Laboratory,
National Advisory Committee for Aeronautics
Langley Field, Va., September 16, 1941.

TABLE I

VARIOUS DIVE RECOVERIES FOR THE AIRPLANE REPRESENTED BY FIGURE 2

(These results were obtained by use of a variable drag coefficient)

Pull-out	Type of load-factor variation	Initial indicated velocity (mph)	Initial flight-path angle (deg)	Maximum load factor (g)	Initial altitude (ft)	Velocity increment (mph)	Altitude loss (ft)	Maximum indicated velocity (mph)
1	3	200	90	6	6,500	31.1	963	231.1
2	3	260	90	6	6,800	38.9	1,464	298.9
3	3	390	90	6	8,000	44.8	2,928	434.8
4	2	200	90	6	6,500	35.2	1,054	235.2
5	2	390	90	3	19,000	165.3	19,242	555.3
6	1	200	90	3	7,000	69.9	1,755	269.9
7	3	200	90	3	7,200	85.5	2,174	285.5
8	3	390	90	3	10,000	98.6	6,862	488.6

TABLE II

COMPARISON OF RESULTS FROM CHARTS WITH THOSE OBTAINED IN TABLE I

Pull-out	Type of load-factor variation	Initial indicated velocity (mph)	Initial flight-path angle (deg)	Maximum load factor (g)	Initial altitude (ft)	Parameter, K	Velocity increment (mph)		Altitude loss (ft)		Maximum indicated velocity (mph)	
							From charts	From table I	From charts	From table I	From charts	From table I
1	3	200	90	6	6,500	{0.060 .288	39.5	31.1	1,000	963	239.5	231.1
2	3	260	90	6	6,800	.060	34.5	38.9	1,500	1,464	294.6	298.9
3	3	390	90	6	8,000	.0295	45.3	44.8	3,080	2,928	435.3	434.8
4	2	200	90	6	6,500	{.060 .234	41	35.2	1,100	1,054	241.0	235.2
5	2	390	90	3	19,000	.0228	164.5	165.3	18,810	19,242	554.5	555.3
6	1	200	90	3	7,000	.0553	73	69.9	1,921	1,755	273.0	269.9
7	3	200	90	3	7,200	.0496	76.8	85.5	2,130	2,174	276.8	285.5
8	3	390	90	3	10,000	.0231	97.3	98.6	6,953	6,862	487.3	488.6

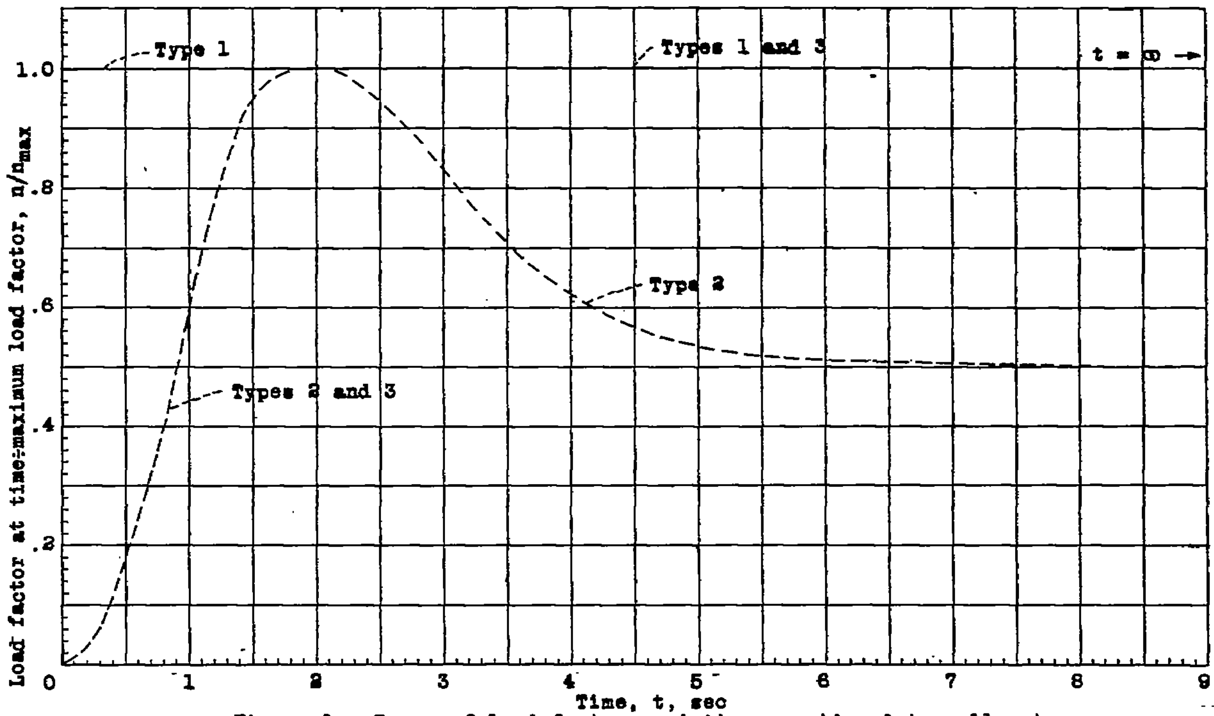


Figure 1.- Types of load-factor variation considered in pull-out.

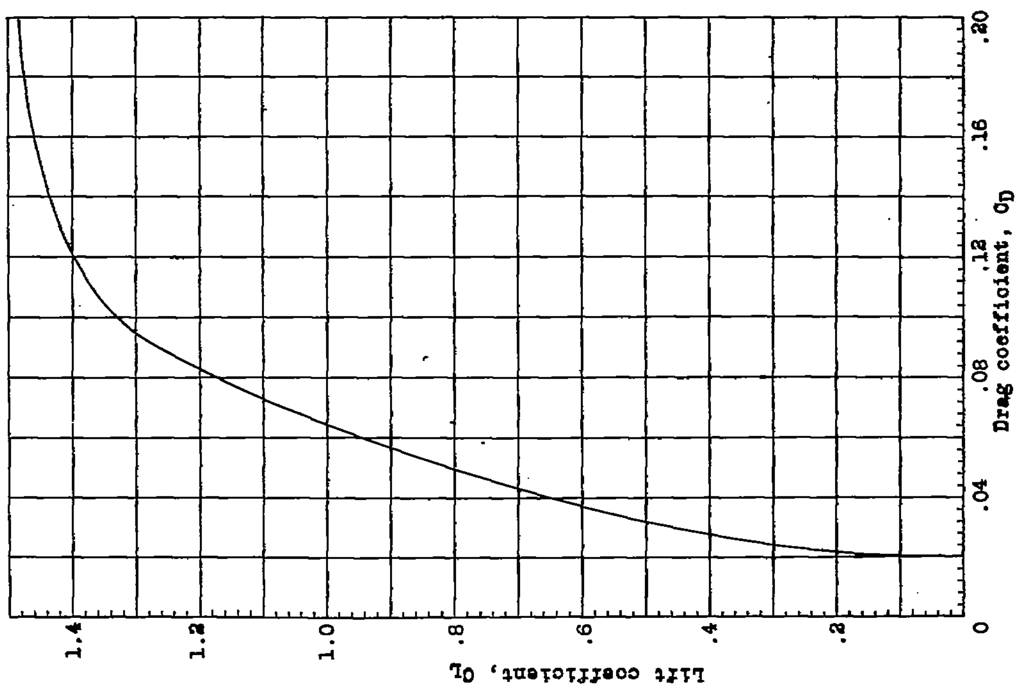


Figure 2.- Lift and drag polar for a typical transport airplane. Weight, 45,000 pounds; wing area, 1500 square feet.

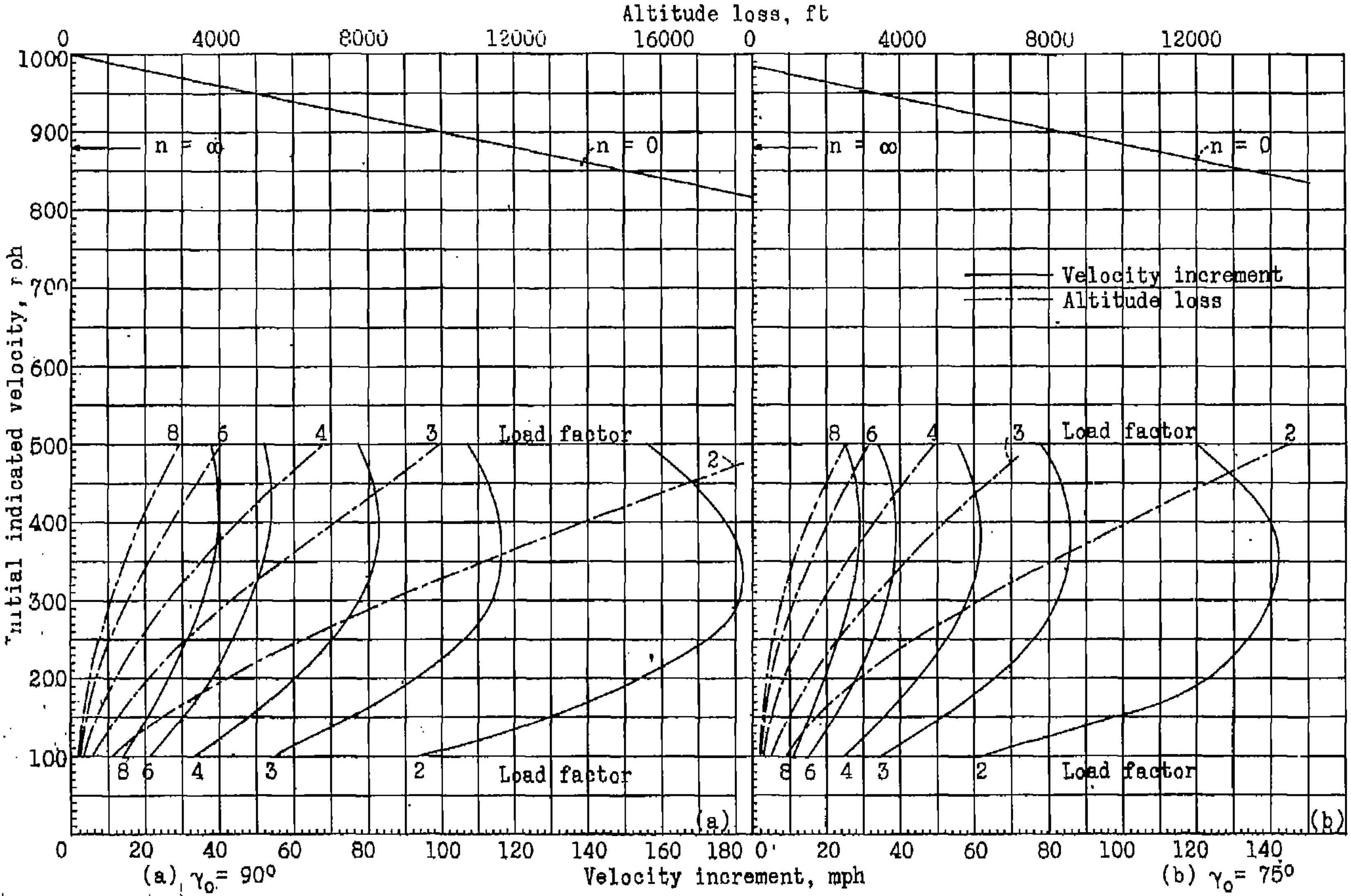
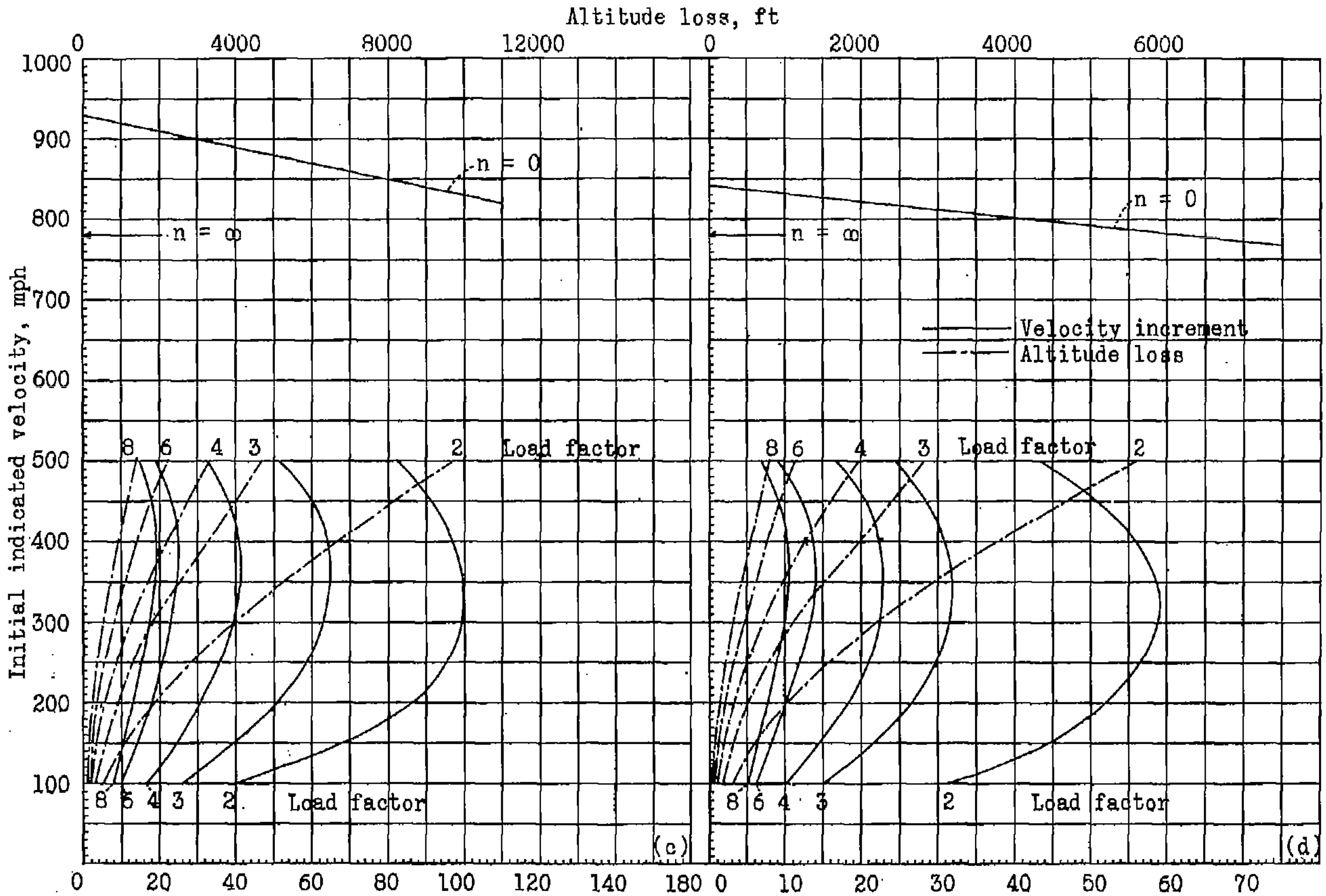


Figure 3a,b,c,d.- Velocity gained and altitude lost in dive pull-outs for various initial flight-path angles. Type 1 load-factor variation. K = 0.015.



(c) $\gamma_o = 60^\circ$ (d) $\gamma_o = 45^\circ$

Figure 3.- Concluded.

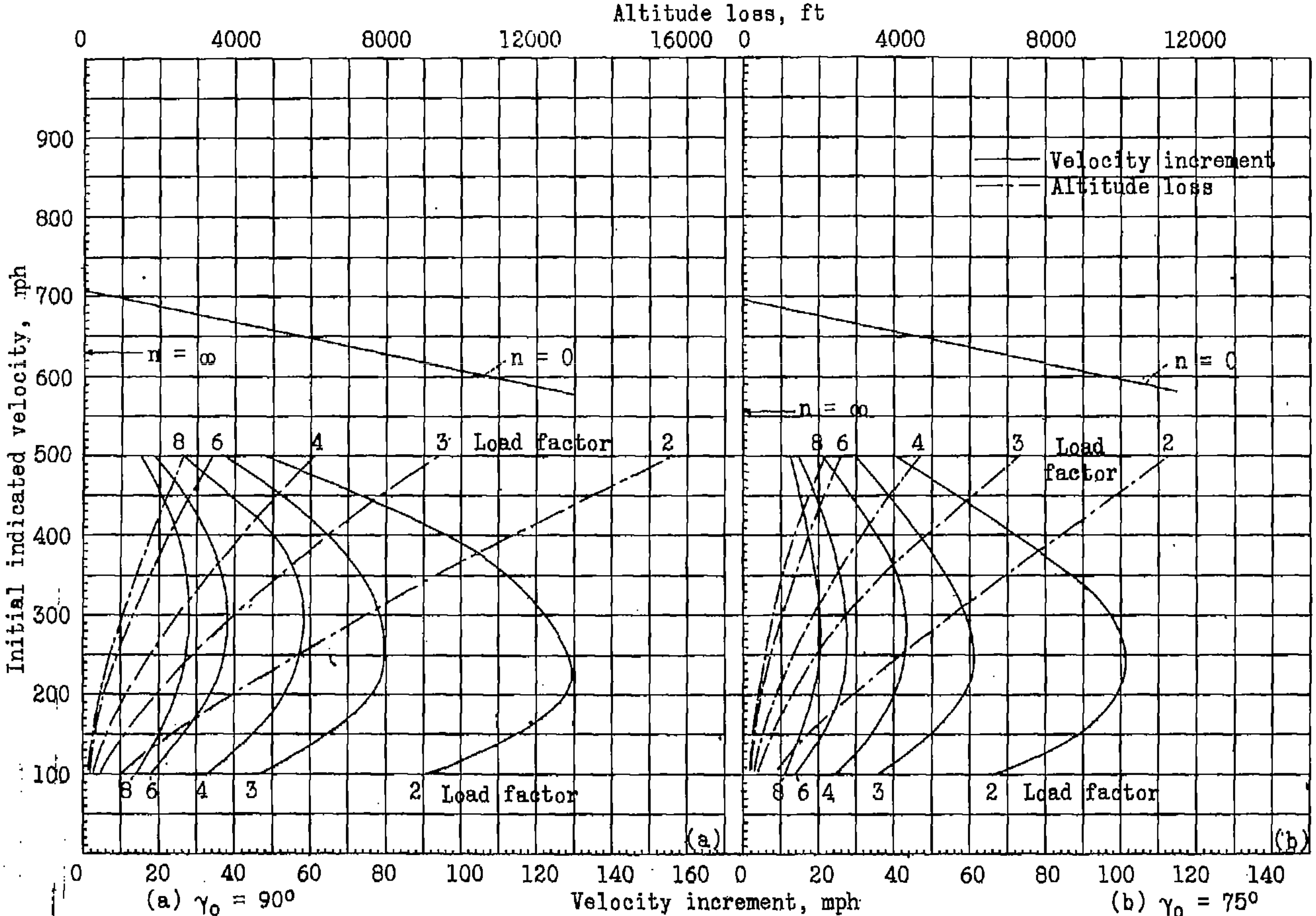
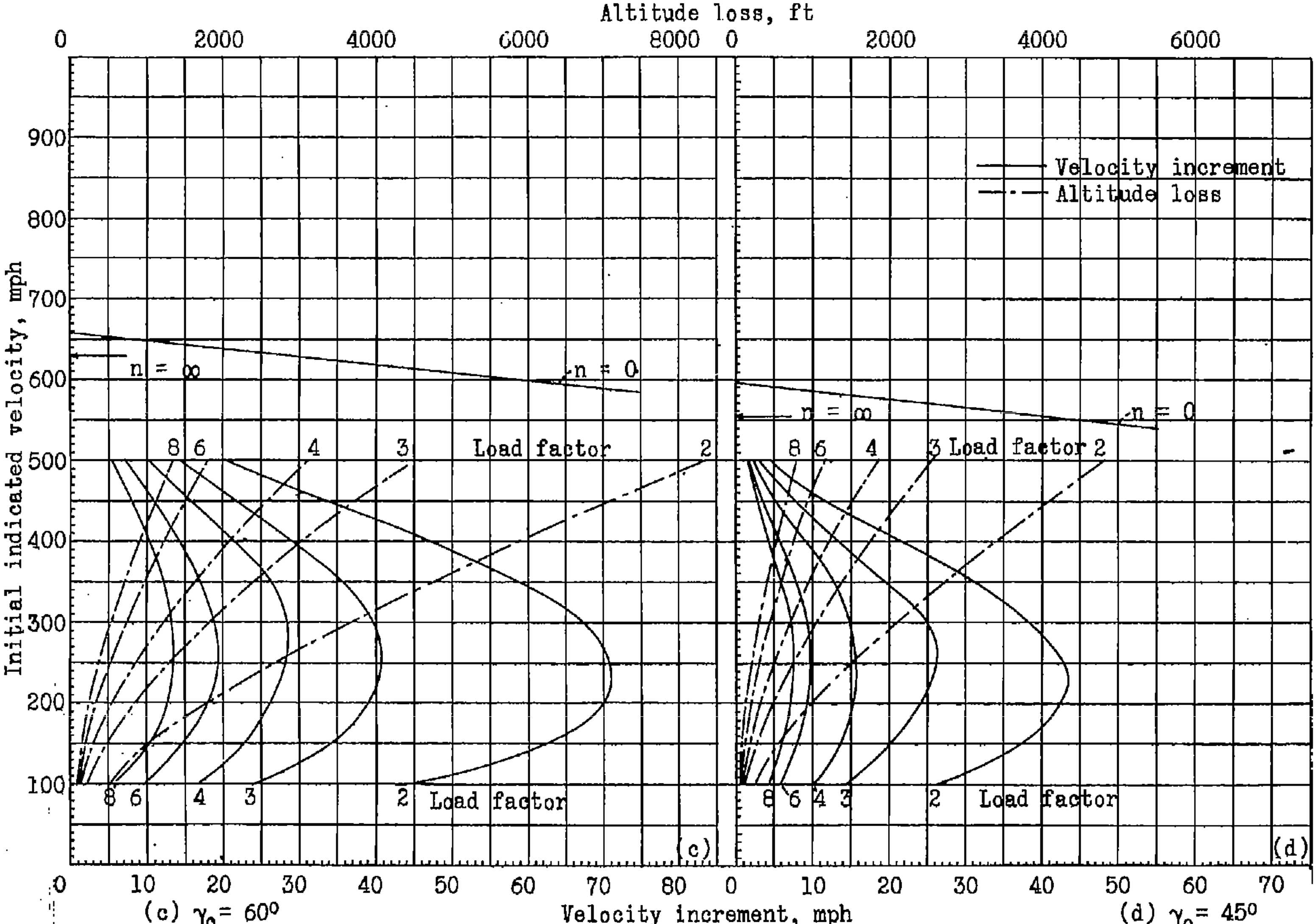


Figure 4a,b,c,d.- Velocity gained and altitude lost in dive pull-outs for various initial flight-path angles. Type 1 load-factor variation. K = 0.030.



(c) $\gamma_0 = 60°$ (d) $\gamma_0 = 45°$

Figure 4.- Concluded.

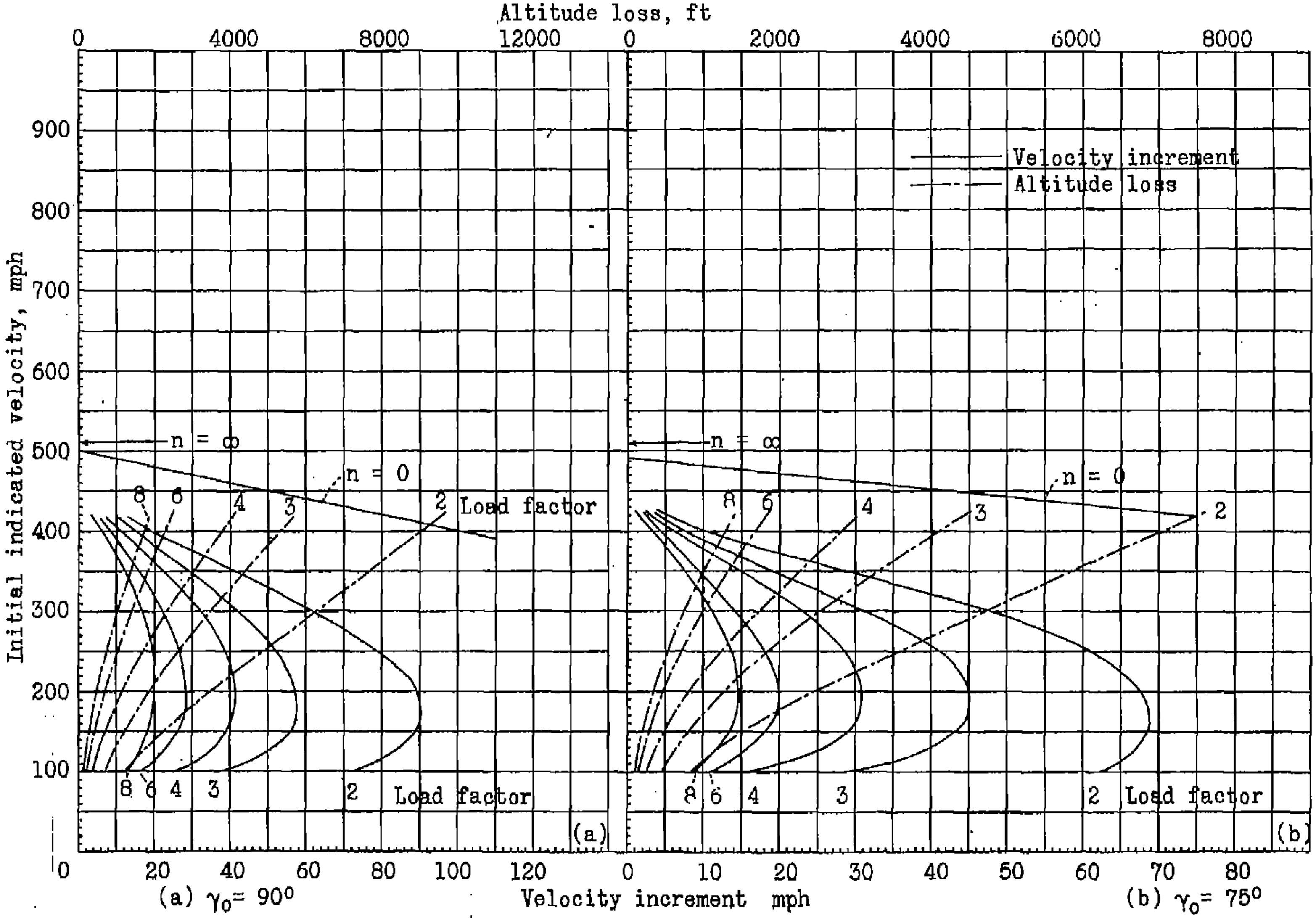


Figure 5a,b,c,d.- Velocity gained and altitude lost in dive pull-outs for various initial flight-path angles. Type 1 load-factor variation. K = 0.060.

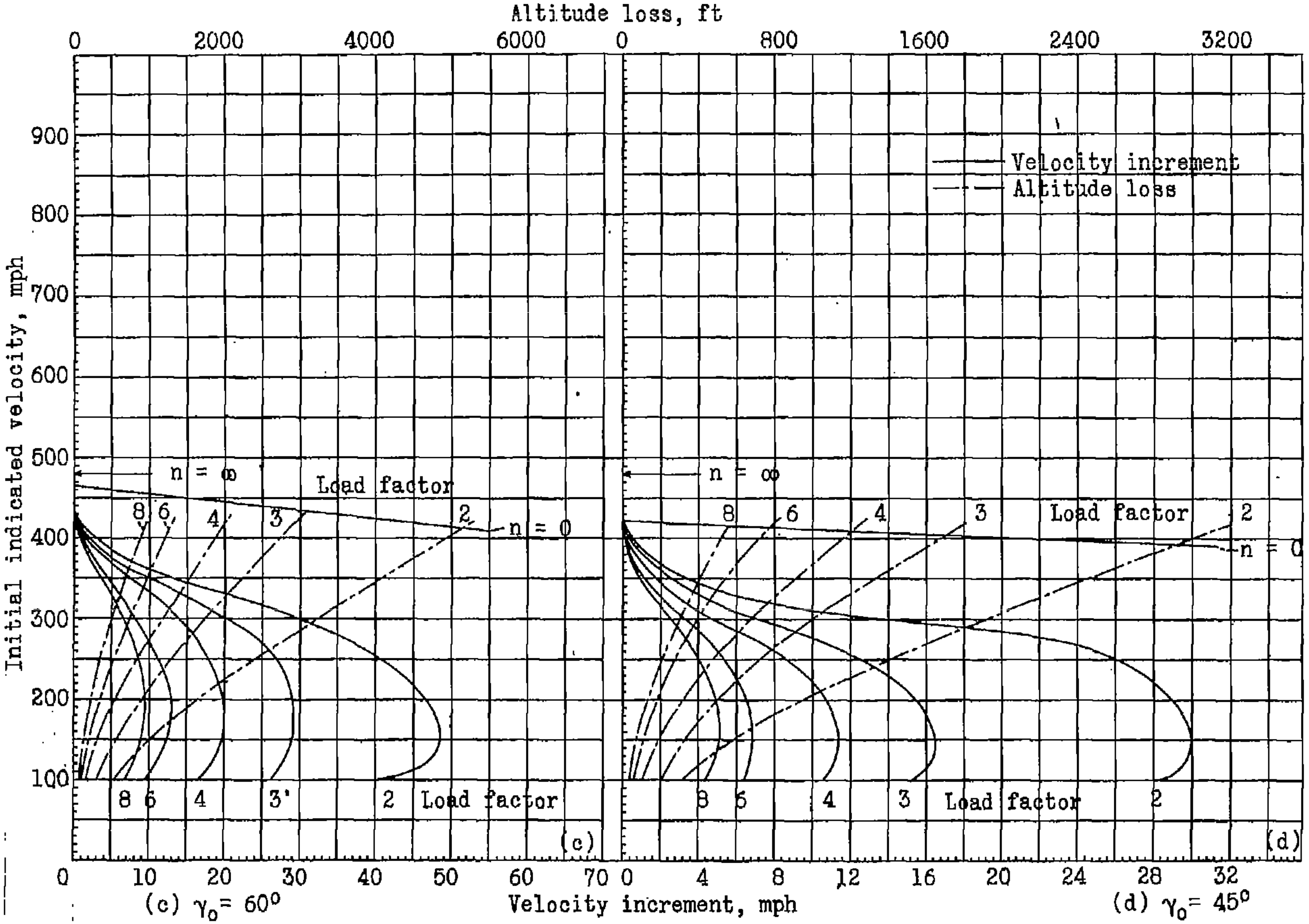


Figure 5.- Concluded.

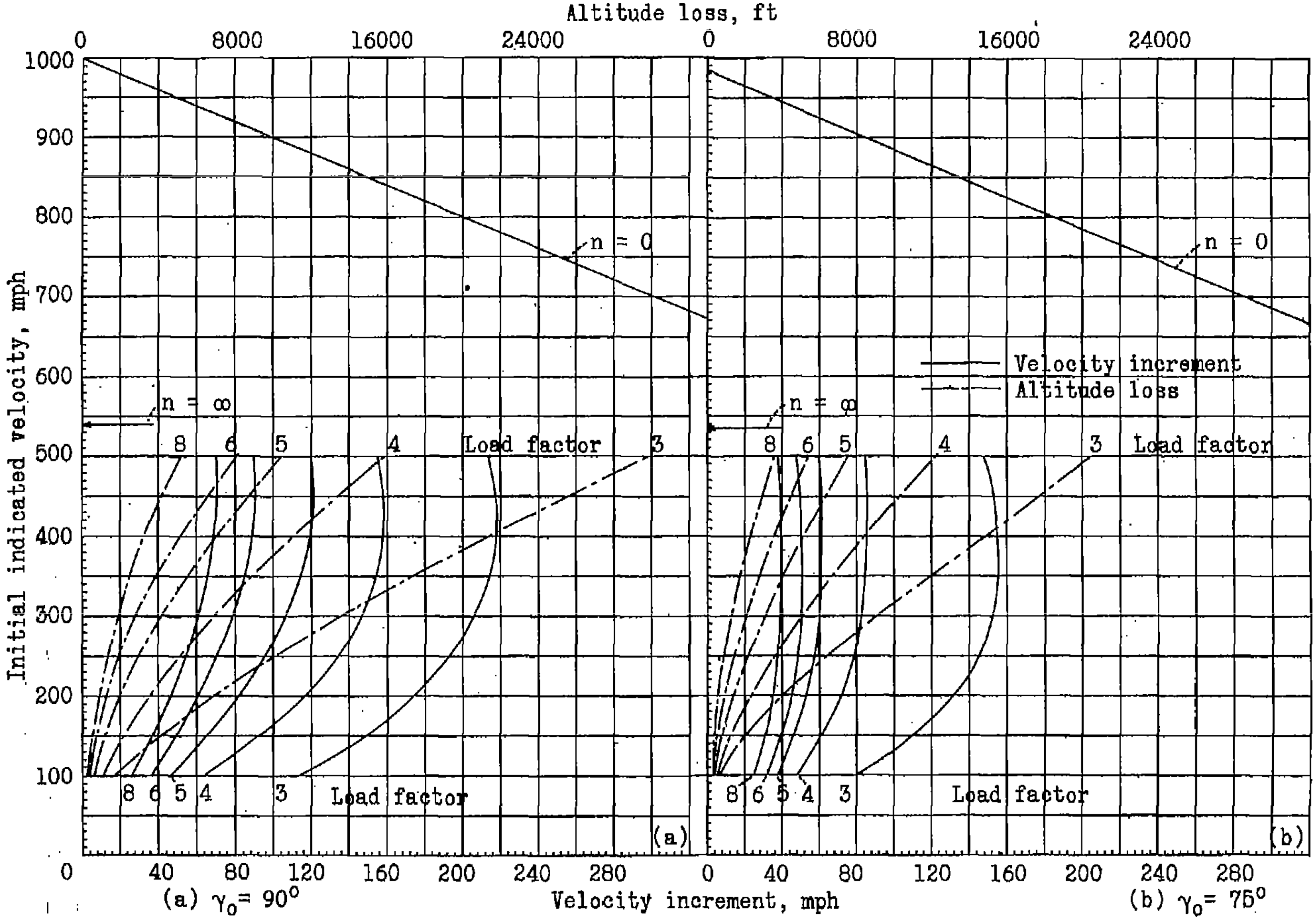
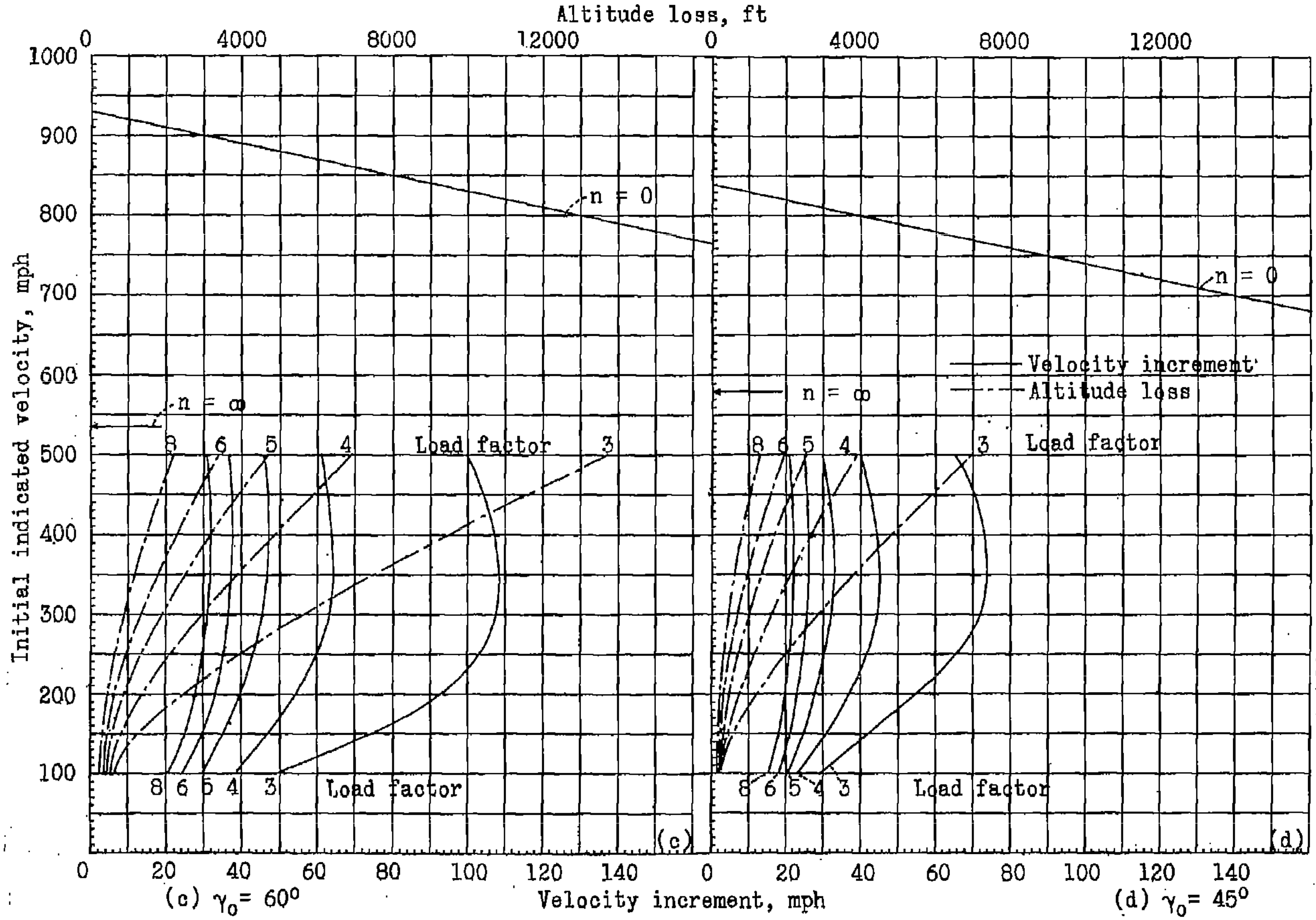


Figure 6a,b,c,d.- Velocity gained and altitude lost in dive pull-outs for various initial flight-path angles. Type 2 load-factor variation. K = 0.015.



(c) $\gamma_0 = 60^0$

(d) $\gamma_0 = 45^0$

Figure 6.- Concluded.

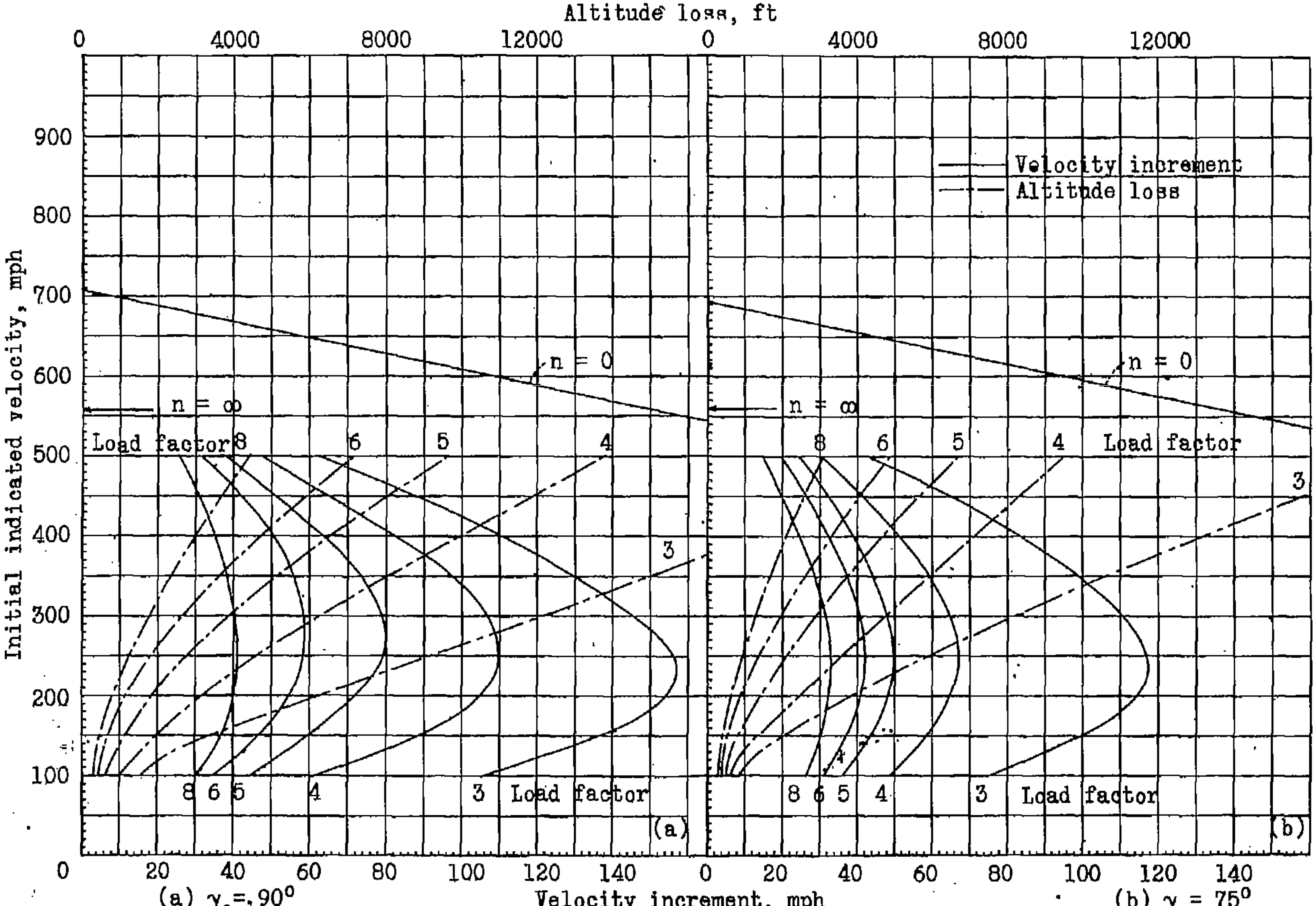


Figure 7a,b,c,d,- Velocity gained and altitude lost in dive pull-outs for various initial flight-path angles. Type 2 load-factor variation. K = 0.030.

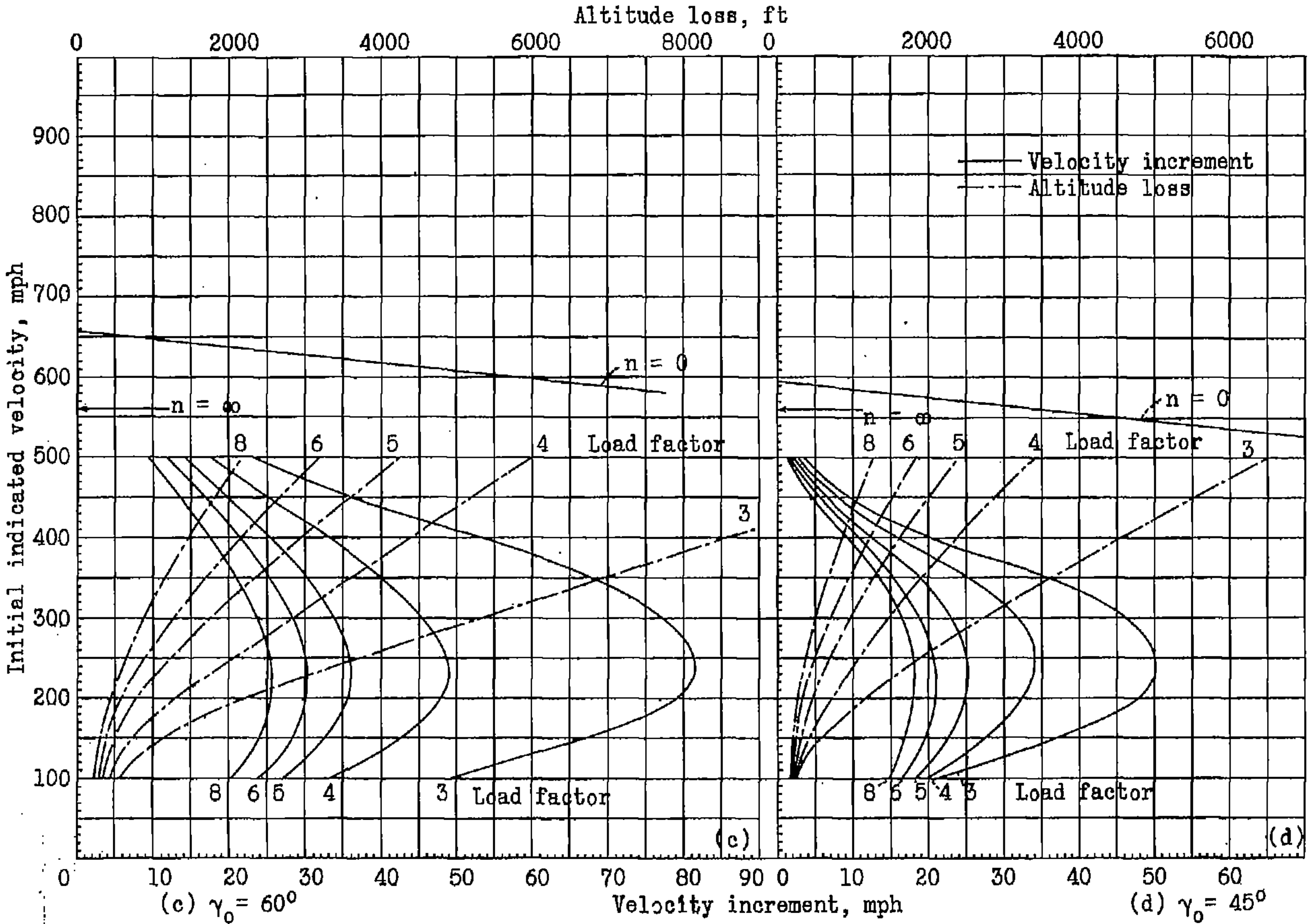


Figure 7.- Concluded.

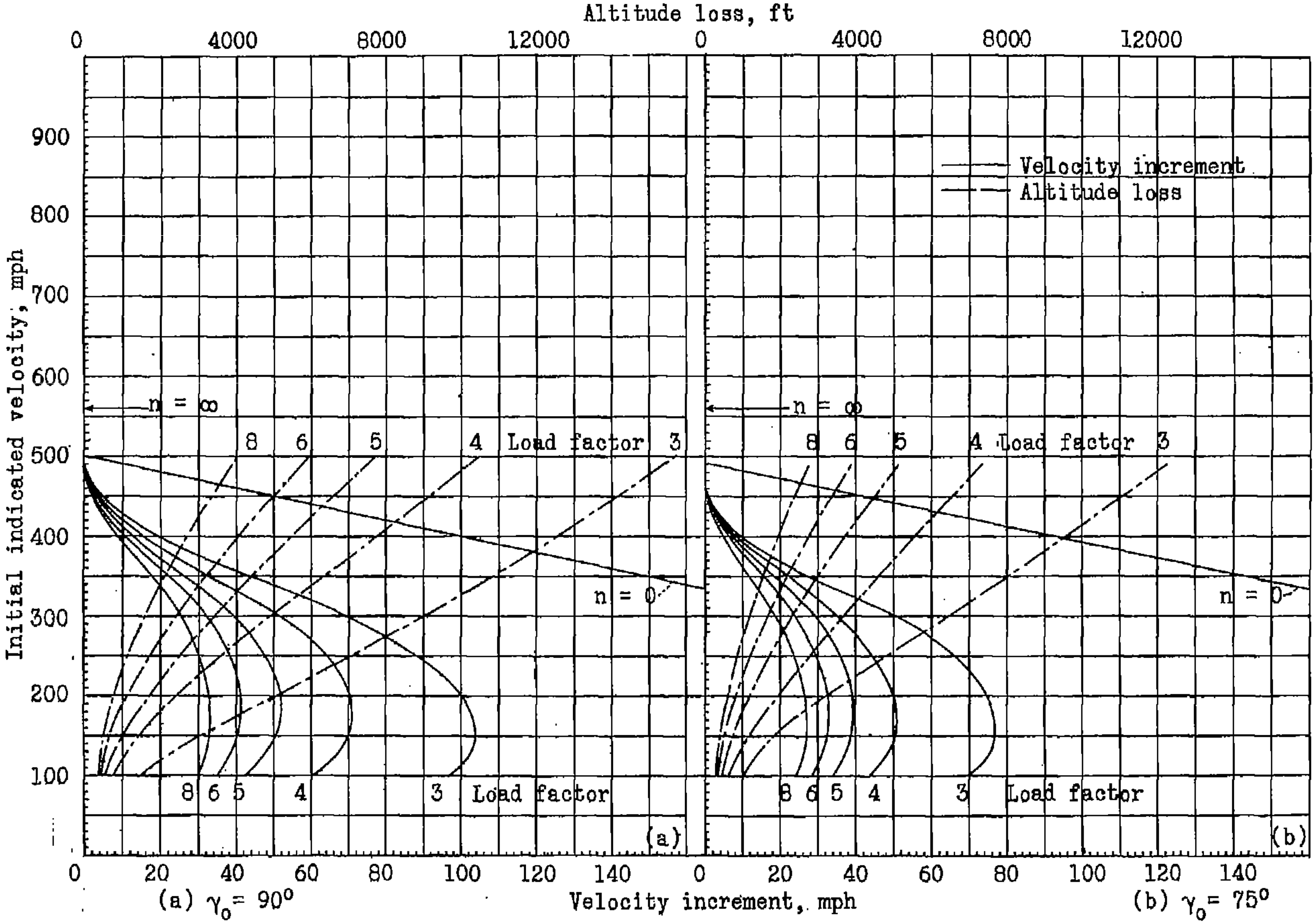


Figure 8a,b,c,d.- Velocity gained and altitude lost in dive pull-outs for various initial flight-path angles. Type 2 load-factor variation. K = 0.060.

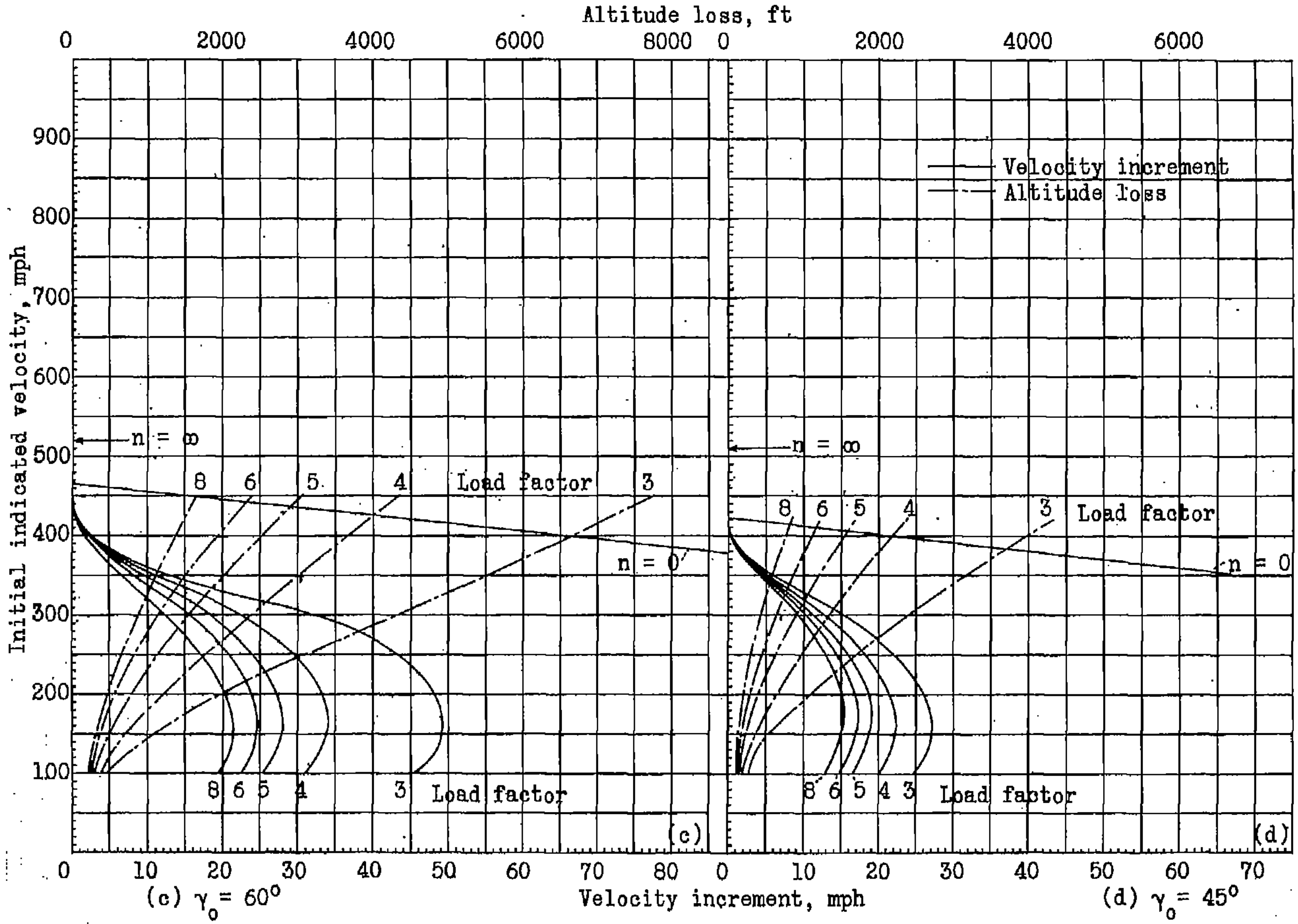


Figure 8.- Concluded.

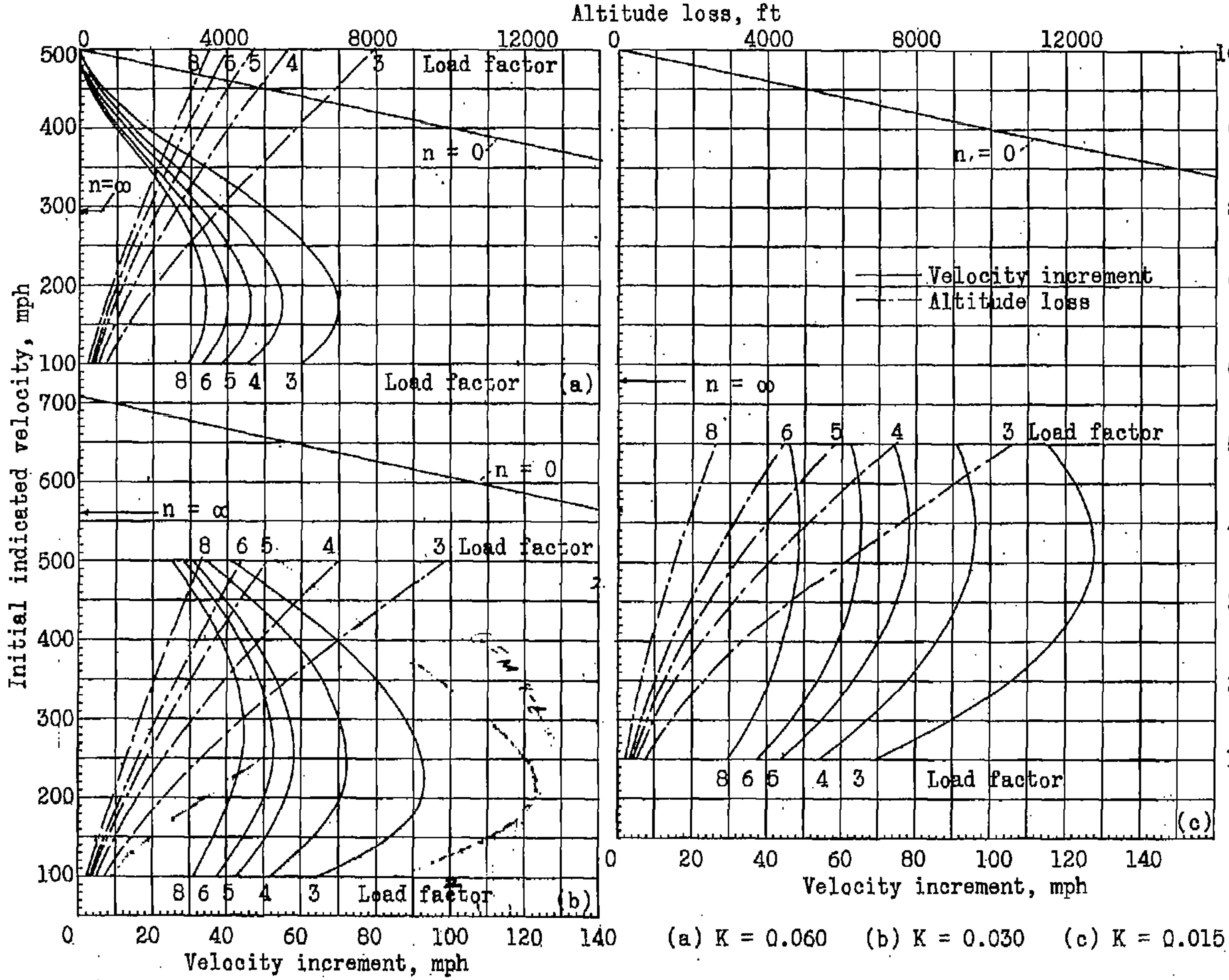


Figure 9a,b,c.- Velocity gained and altitude lost in dive pull-outs for an initial 90° flight-path angle. Type 3 load-factor variation.