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TECHNICAL NOTES

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

No. 806

STRESSES AND DEFLECTIONS IN THIN SHELLS AND CURVED PLATES
DUE TO CONCENTRATED AND VARIOUSLY DISTRIBUTED LOADING

By Raymond J. Roark
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SUMMARY

Tests made upon cylindrical and spherical shells to determine the local stresses and deflections produced by concentrated and variously distributed loading are described. The results of these tests are correlated with those of earlier experiments, and empirical formulas based on these data are proposed. Data are presented on the effect of severe prestressing upon stiffness and on the efficacy of welded lugs of various forms in transmitting a load to a thin shell without producing excessive local stresses.

INTRODUCTION

The stresses produced in thin shells having the form of surfaces of revolution have been analyzed mathematically for some cases of distributed and symmetrical loading (reference 1). The solutions are not simple, and to apply similar methods to cases of concentrated and nonsymmetrical loading would be difficult, if not impracticable. It therefore seems expedient to seek purely empirical formulas that, for such cases, express with adequate accuracy the relationship between the load, the dimensions of the shell, and the stresses and deflections produced; and it was the purpose of the investigation herein reported to make appropriate tests and to derive therefrom formulas of this kind. Tests made for a similar purpose have been described in reference 2; but the earlier investigation was limited to cylindrical shells under concentrated loading, and the expressions derived for stress and deflection were, it is believed, needlessly cumbersome.

When a thin shell, such as a pipe, a tank, or a mono-coque fuselage, is subjected to localized loading remote

from points of support, the stresses produced may, for convenience, be classified as (a) general stresses, mainly membrane stresses, due to the bridgelike action of the shell in transmitting the load to the supports; and (b) local stresses, mainly bending stresses, due to the plate-like action of the loaded part of the shell in transmitting the load to the structure as a whole. When the load is concentrated on a relatively small portion of the shell surface, the stresses at even a small distance from the point of loading become negligibly small in comparison with those adjacent thereto, and, for such loading, the local stresses alone are likely to be important.

These local stresses, and the local deflections as well, depend mainly upon the thickness and curvature of the shell and the magnitude and degree of concentration of the load and are almost independent of the general dimensions of the shell (such as span), provided the load is reasonably distant from points of support. As long, then, as these local stresses and deflections are substantially independent of the deflection - that is, bear a linear relationship to the load - it would seem that it should be possible to express them approximately by simple formulas involving only curvature and thickness of the shell wall and distance from the point of loading. For the shells tested, it was found that this linear relationship existed and that stresses and deflections could be expressed by formulas of the kind indicated.

The tests and the results are discussed under two headings: Cylindrical Shell and Spherical Shell.

This investigation, conducted at the University of Wisconsin, was sponsored by, and conducted with financial assistance from the National Advisory Committee for Aeronautics and the Wisconsin Alumni Research Foundation.

NOTATION

The following notation is used throughout:

- A area over which the load is distributed
- a half length (in x direction) of a rectangular loaded area
- b half width (in y direction) of a rectangular loaded area

- d deflection of shell wall at the middle of a given span
- K , K_b , and K_m experimentally determined coefficients for total stress, bending stress, and membrane stress, respectively
- P total applied load
- p load per unit area
- R shell radius
- r radius of circular loaded area of ring
- s_x , s_{bx} , s_{mx} total stress, bending stress, and membrane stress in x direction
- s_y , s_{by} , s_{my} total stress, bending stress, and membrane stress in y direction
- t thickness of the shell wall
- x distance measured along the x axis
- y distance measured along the y axis
- z distance measured along the z axis
- α experimentally determined coefficient for deflection

All dimensions are either in terms of R or in inches; all forces are in pounds.

CYLINDRICAL SHELL

Test Specimen, Apparatus, and Method of Testing

The specimen used was a cylindrical steel shell or pipe, 26 feet long with a mean diameter of 31.2 inches and a thickness of 0.072 inch. The material was a low alloy structural steel having a proportional limit of 31,500 pounds per square inch both parallel and normal to the direction of rolling. The modulus of elasticity was

28,300,000 and 27,050,000 pounds per square inch parallel and normal to the direction of rolling, respectively. The pipe had welded to each end a square bulkhead 3/16 inch thick with a circular opening about 30 inches in diameter to permit access to the interior. These bulkheads rested on supports, as shown in figure 1, so that the shell acted as a simple beam carrying its own weight and the applied loads.

The shell was subjected to outward loads applied at the inner surface and to inward loads applied at the outer surface. The outward loads were applied through a simple lever system. A long timber beam, inside the shell and parallel to its axis, was pinned at its outer end to one of the bulkheads. On this beam was mounted a sliding block that carried a vertical loading column, and this loading column could be placed at any desired position along the axis of the pipe by sliding the block along the beam. To the inner end of the beam was attached a rod that passed vertically downward through a small hole in the bottom of the shell. A spring dynamometer was suspended from this rod, and by means of a small self-locking winch and chain any desired load could be applied thereto. This lever system was carefully calibrated in advance over the entire range of loading contemplated.

The inward loading was accomplished by means of an ordinary automobile screwjack provided with an adjustable loading column and resting on an accurate platform scale.

Strains were measured by Huggenberger tensometers adjusted for half-inch gage lengths. Deflections were measured with a Federal dial reading directly to 1/10000 inch.

Testing Procedure and Results

Concentrated loading. - It was desired to determine first the stress produced at a given point, herein called "station," by a concentrated load applied at any adjacent point. For purposes of reference, a longitudinal line through the station was taken as the x axis, a circumferential line as the y axis, and a line at 45° - or half-way between these two - as the z axis. The positions of these reference axes are shown in figure 2. With the strain gage placed at the station, the load was applied at close intervals along each of these three axes and the longitudinal stresses s_x and the circumferential stresses

s_y which, because of symmetry, were known to be the principal stresses, were determined for each such position of the load. This procedure was carried out for each of four different stations, spaced some distance apart along the bottom element of the cylinder, at distances from the nearest point of support (the end bulkhead) varying from $1/2R$ to $6R$. Within these limits proximity to support appeared to have no appreciable influence upon the stresses at and near the point of loading; hence the results of strain measurements at the several stations were averaged.

In the first test the load was applied progressively in small increments, to ascertain the maximum load that could be applied without danger of overstressing and to ascertain whether the load-stress relationship was appreciably affected by the deflection. This relationship was found to be a linear one and, subsequently, the maximum load was applied directly.

In order to ascertain what part of the measured strain was due to bending stress and what part to membrane stress, simultaneous measurements were taken on both sides of the shell wall. Directly at the point of loading, where such readings could not be taken, readings were taken on the outer face with outward loading and on the inner face with inward loading. The first readings represented the sum of bending and membrane stresses and the second readings, the difference; and so it was possible to determine the value of each kind of stress. It was found that at and near the point of loading the membrane stress was small compared with the bending stress, being less than 10 percent of the total; for this reason it was decided that the total surface stress at any point could be regarded as a bending stress and expressed by the formula $s = KP/t^2$, K being an empirical coefficient depending upon the distance from the point of loading. The experimentally determined stresses, expressed in terms of K and as functions of the distance from the load in terms of R , are given by the curves of figures 3 and 4.

The strain measurements gave only the average stress over half-inch gage lengths, and, since the variation in stress intensity is very rapid near the point of loading, the maximum value at that point is considerably greater than this average value. It was estimated by assuming that its ratio to the average stress was equal to the corresponding ratio in the case of a centrally loaded circular flat

plate having a diameter equal to the distance between points of inflection in the shell. This method of calculation indicated a ratio of maximum to average stress of 1.61 in the case of circumferential stress and of 1.41 in the case of longitudinal stress. In the calculation of the stress for the hypothetical flat plate, the concentrated load was assumed to be distributed over a circular area of radius $0.325t$, in accordance with Westergaard's approximate formula for flat plates under point loading. (See reference 3*) It follows that the peaks of the curves of figures 3 and 4 depend somewhat upon the thickness of the shell, but, so far as the practical question of the effect of a load distributed over an area of any considerable size is concerned, the influence of t upon K would be quite negligible, at least for relatively thin shells.

This study of the effect of purely concentrated loading reproduced, to a certain extent, work done in the earlier investigation reported in reference 2, and the results are in reasonably good agreement with those previously obtained.

Loading uniformly distributed over a rectangular area.-

From the curves of figures 3 and 4, the diagrams of figures 5 and 6 were constructed. These diagrams show contour lines of influence surfaces for s_y and s_x ; an ordinate to one of these surfaces at any point represents the stress produced at the station $x = 0$, $y = 0$ by a load applied at the point in question. The stress produced at $x = 0$, $y = 0$ by a unit load p uniformly distributed over any area A is obviously $\Sigma KpdA$, which is proportional to the volume under the influence surface and within the area A . It is therefore possible, by the graphical integration of such volumes, to determine approximately the circumferential and longitudinal stresses produced by any given distribution of load. This procedure has been carried out for the case of a load uniformly distributed over a rectangular area of length $2a$ and width $2b$. The calculations were limited to areas so small that local stresses, comparatively independent of span and method of support, would become excessive before the general membrane stresses became important.

The results of these computations are given in tables I and II. The values in the upper horizontal row ($b = 0$) represent stresses due to a line load uniformly distributed along an element of the cylinder, the values in the

*Out of print. Available for reference in libraries.

first vertical column ($a = 0$) represent the stresses due to a line load uniformly distributed along a portion of the circumference, and other values represent the stresses due to a load uniformly distributed over an area of the indicated length and width. By linear interpolation, the effect of a load distributed over any rectangular area within the limits of the table can be found.

These coefficients are, of course, approximate and are probably more nearly correct for small areas than for relatively large ones. They were checked by determining the stresses produced by a load applied through padded templates of various widths, cut to fit the curvature of the shell and placed at various longitudinal distances from the station. The area under the $s-x$ curve for such loading gave values of K corresponding to $a = x$ and $b =$ the half width of the loaded arc. Values of K determined in this way were in reasonably good agreement with those found from the volumes under the influence surfaces.

Loading on a circular area and on a circle.- By means of circular blocks cut to fit the inner surface of the shell and thickly padded with sponge rubber, the maximum stresses due to a uniform load on a circular area were determined. The results are shown in figure 7. In the same way, the effect of a load uniformly distributed along a concentric circle or ring was determined, and the results are shown in figure 8. In the case of a circular area the maximum stresses were found to occur at the center; in the case of a ring the maximum longitudinal and circumferential stresses were found to occur at the ends and the sides of the ring, respectively.

Loading through welded lugs.- The usual method of introducing a load or a reaction into a thin shell is through a welded or riveted lug or clip. There seems to be no very definite agreement among metal fabricators as to the best design for such attachments, nor any very definite information as to the stresses produced by loads thus applied. There was, therefore, little information to go on in designing the experimental forms tested in the present investigation. Of the infinite variety of shapes and sizes that might be used, the eight shown in figure 9 and the three additional ones described in table III were adopted. It was desired to ascertain something concerning the effect of length in the case of longitudinal and circumferential plate lugs, and it was thought

that the tapered forms might, by deflecting slightly, cause less severe stresses at the tips than the stiffer rectangular forms. The circular rings represent conditions that might be expected to occur when tubular stress members are welded directly to a shell.

All the lugs were electrically welded to the exterior of the shell and loaded inwardly. Longitudinal and circumferential stresses were determined at the ends of the flat lugs and at the ends and the sides of the rings. The results are given in table III; they are irregular and doubtless reflect the influence of distortion due to welding, as well as the influence of the form and the size of the lugs. The welds were made with care by an experienced operator, and the distortion that occurred was probably typical of what might be generally expected in thin metal. The amount of this distortion was more or less proportional to the amount of welding, and this fact appears to have offset in great measure the advantage that might have been expected from an increase in lug dimensions. Particularly was this true in the case of the larger rings, where the distortion was very marked and took the form of a flattening of the shell which might be expected to reduce greatly its resistance to inward loading.

It is interesting to compare the stresses produced by loads applied through the attached lugs with the maximum stresses that would accompany the application of equal loads uniformly distributed along lines or circles of corresponding length or size. These stresses, computed by means of tables I and II and figure 8, are therefore included in table III.

It would be incorrect to assume that either the experimentally determined values of K for welded lugs or the computed values of K for line loading are applicable to riveted lugs. In the case of a riveted lug or member, the distortion of the shell produced by welding would be absent and the loading would be distributed, not along a line, but over an area of at least sufficient width to permit the riveting operation. It is possible that the stresses produced by the imposition of a thrust through a riveted connection could be, in some instances, approximately determined by assuming the pressure to be distributed uniformly over the entire area of contact and by taking (from table II or fig. 7 or 8) a value of K corresponding to the shape and the size of that area. But there is usually likelihood of higher than average press-

ures at the edges and corners of such a connection, with consequent high local stresses. Reliable information concerning the stresses to be expected at riveted connections could be secured only from tests made on the type of lug and joint in question.

Deflection.- Measurements were made of the midspan deflection produced by a concentrated load, relative to points at the extremities of a longitudinal span of varying magnitude. Such measurements were made in the earlier tests mentioned, and an empirical formula was proposed which fitted the results of those tests reasonably well. This formula was cumbersome, however; so an attempt was made to find a simple expression of the form $d = \alpha \frac{P}{E} \frac{R^n}{t^{n+1}}$, with α a coefficient dependent upon the span, that would fit the results of the older tests as well as the present ones. The formula adopted was $d = \alpha \frac{P}{Et} \left(\frac{R}{t}\right)^{1.2}$, with values of α as follows:

span/R =	0.1	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0	3.0	4.0
$\alpha =$	0.07	0.15	0.25	0.32	0.36	0.39	0.42	0.44	0.46	0.48	0.50	0.59	0.66

The deflection is due partly to bending and partly to membrane strain and the relative amounts of strain energy accounted for in these two ways varies with the span and with the deflection. Hence, it is not possible to express accurately the deflection by a simple formula such as that given except for comparatively small ranges of the R/t and span/R ratios. The old and new tests combined covered a range of R/t ratios from 36 to 217 and of span/R ratios up to 4. The values of α were adjusted for close agreement with the present tests ($R/t = 217$), but in no case do the measured deflections found in the earlier tests ($R/t = 36$ and 89) differ by more than about 20 percent from deflections computed by the formula.

In the tests used to determine α , the point of loading was distant at least $2R$ from the nearest bulkhead, but proximity to support outside the span was found to have little effect upon deflection, and the deflection at the middle of a span having a bulkhead at one or both ends can be found approximately by the formula given.

Effect of initial overloading.- It is the practice of

some metal fabricators to produce by overstressing an initial distortion at the place where a concentrated load is to be applied to a plate or shell. This effect enables the structure to resist subsequent loading almost wholly by membrane stress, and there is consequently a gain in strength and stiffness. In order to ascertain something concerning the effectiveness of this procedure, load-deflection diagrams were obtained for initial and repeated heavy loading, both inwardly and outwardly applied, the load being concentrated at a point to make possible the greatest degree of local overstressing with the least load. The results of these tests are shown in figure 10. For subsequent loadings (after the second) the load-deflection curve was practically identical with that for the first repetition.

It is apparent from the load-deflection diagrams that the shell can withstand, without excessive deformation, far more load when loaded outwardly than when loaded inwardly. This result is what would be expected because in the one case the deformation results in a form better adapted to membrane resistance while in the other the flattening of the shell has the opposite effect. It is interesting to note that the diagram for inward loading has somewhat the same characteristics as a load-deflection curve for a Belleville spring, showing a similar temporary decrease in stiffness at a deflection that corresponds to a form making the membrane stresses least effective.

It is apparent from the diagrams of figure 10 that, if circumstances permit initial overloading, the ability of the shell to resist a subsequently applied load can be greatly increased thereby.

SPHERICAL SHELL

Test Specimen, Apparatus, and Method of Testing

The specimen used was a shell in the form of an almost complete hemisphere, with a radius of 15 inches and a thickness of 0.0586 inch. This shell had welded to its rim a stiffening ring made of a 2- by 2 $\frac{1}{2}$ - by $\frac{1}{4}$ -inch angle. The material was a CR steel having a yield point of about 48,000 pounds per square inch and an ultimate strength of 58,000 pounds per square inch. The shell was mounted on a wooden frame that permitted it to be held in

any desired position and was loaded, both inwardly and outwardly, by means of the column and jack apparatus that was used in testing the cylinder. Figure 11 shows the shell arranged for outward loading.

Strains were measured by means of Huggenberger tensometers adjusted for half-inch gage lengths. Deflections were measured with a Federal dial reading directly to 1/10000 inch.

Testing Procedure and Results

Concentrated loading.- Essentially the same procedure was followed as in the tests of the cylindrical shell. Four stations, symmetrically situated about halfway between the pole and the rim of the shell, were selected. A meridian through any given station was taken as the x axis, a great circle normal thereto as the y axis, and a great circle at 45° thereto as the z axis. The positions of these reference axes are shown in figure 12. Strains in the x direction due to a load applied at intervals along each of the three axes were measured at each station and, by means of inward and outward loading and instruments set on inner and outer surfaces, data were obtained that made it possible to distinguish between bending and membrane stresses.

The membrane stresses were found to be of sufficient importance to warrant separate consideration, and the following formula was adopted for the total surface stress at any given distance from the load:

$$s = s_b + s_m = K_b \frac{P}{t^2} + K_m \frac{P}{Rt}$$

where the subscripts b and m refer to bending and membrane stresses and K_b and K_m are empirical coefficients depending upon the distance from the point of loading. The experimentally determined stresses, expressed in terms of K_b and K_m , are given by the curves of figures 13 and 14.

By the use of the same procedure as in the case of the cylindrical shell, the bending stress at the point of loading was estimated to be 2.39 times as great as the mean stress measured over the 1/2-inch gage length.

Loading on a circular area and on a circle.- From the curves of figures 13 and 14, the contour lines of influence surfaces for both s_p and s_m were constructed; they are shown in figures 15 and 16. By the use of the methods outlined for the cylinder, the principal stresses produced by any load symmetrically disposed about a point can be found from these diagrams. Stresses were found for two cases: that of a load uniformly distributed over a concentric circular area and that of a load uniformly distributed along a concentric ring. The results are given in table IV. These results were checked by direct measurement of the stresses produced by loading through circular blocks and through rings; pads or gaskets of sponge rubber were used to secure as nearly as possible the assumed uniform distribution of pressure. The agreement found was reasonably good.

Deflection.- Deflections produced by progressively increased outwardly applied concentrated loading were measured for 4- and 8-inch spans in order to ascertain the nature of the load-deflection relationship. The load-deflection curve, shown in figure 15, indicates a linear relationship. The deflections for an 8-inch span were almost exactly the same as for the 4-inch span, a fact consistent with the very rapid fade-out of stresses.

As in the case of the cylinder, the relative influence on deflection of bending strains and of membrane strains would be expected to vary with the R/t ratio and with the span; with only one test specimen available, it was not possible to establish a formula for deflection. Further tests, covering a range of R/t values, would be necessary before this formula could be established.

It is interesting to note that, although the spherical shell was only about eight-tenths as thick as the cylindrical shell, its deflection measured over a 4-inch span was less, being 0.052 inch (for a load of 100 lb.) as against 0.057 inch for the cylinder. This result indicates the greater effectiveness of membrane stresses in a shell of double curvature. As in the case of the cylinder when tested under outward loading, the computed maximum stress corresponding to the load at which the rate of deflection showed an appreciable increase was considerably greater than the elastic limit of the material. Also as in the case of the cylinder, the deflection increased at a very slowly increasing rate for higher loads, and the shell proved

capable of sustaining, without excessive deformation, a load very much greater than that at which the deflection rate showed an appreciable increase.

University of Wisconsin,
Madison, Wisconsin, December 1940.

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TABLE I.-- VALUES OF K FOR MAXIMUM s_y PRODUCED BY A LOAD UNIFORMLY
 DISTRIBUTED OVER A RECTANGULAR AREA OF HALF-LENGTH a AND HALF-
 WIDTH b , CYLINDRICAL SHELL

[Dimensions in terms of shell radius R]

$a \backslash b$	0	0.05	0.10	0.15	0.20	0.30	0.40	0.50	0.60	0.70	1.0
0	2.21	1.10	0.78	0.62	0.52	0.40	0.34	0.30	0.27	0.24	0.20
.025	1.10	.83	.65	.53	.45	.36	.31	.27	.25	.22	.19
.050	.78	.64	.54	.45	.39	.33	.28	.25	.23	.20	.17
.10	.48	.43	.38	.34	.30	.26	.23	.21	.19	.17	.15
.15	.33	.31	.28	.26	.24	.20	.18	.17	.15	.14	.12
.20	.24	.23	.21	.19	.18	.16	.14	.13	.12	.11	.10
.30	.15	.14	.13	.12	.11	.094	.083	.075	.070		
.40	.096	.09	.084	.076	.070	.057					
.50	.068										

TABLE II.- VALUES OF K FOR MAXIMUM ϵ_x PRODUCED BY A LOAD UNIFORMLY DISTRIBUTED OVER A RECTANGULAR AREA OF HALF-LENGTH a AND HALF-WIDTH b, CYLINDRICAL SHELL

[Dimensions in terms of shell radius R]

a \ b	0	0.05	0.10	0.15	0.20	0.30	0.40	0.50	0.60	0.70	1.0
0	1.89	0.82	0.56	0.44	0.37	0.30	0.25	0.22	0.20	0.18	0.15
.025	1.20	.69	.49	.40	.34	.28	.24	.21	.19	.17	.14
.050	.87	.60	.44	.36	.31	.26	.23	.19	.18	.16	.13
.10	.54	.44	.34	.28	.26	.22	.19	.17	.16	.14	.12
.15	.38	.33	.25	.22	.20	.18	.16	.15	.14	.13	.10
.20	.28	.25	.19	.17	.16	.15	.14	.13	.12	.11	.092
.30	.17	.16	.12	.11	.10	.096	.091	.089	.084	.080	.068
.40	.12	.11	.080	.071	.066	.062	.050	.059			
.50	.081										

TABLE III.- VALUES OF K FOR MAXIMUM STRESSES PRODUCED BY A LOAD APPLIED THROUGH A WELDED LUG AND FOR CORRESPONDING LINE LOADING, CYLINDRICAL SHELL

Designation	Type of lug	Lug loading		Line loading	
	Description	s_y	s_x	s_y	s_x
a	Longitudinal plate (fig. 9)	0.37	0.35	0.75	0.54
b	Longitudinal plate (fig. 9)	.23	.17	.75	.54
c	Longitudinal plate (fig. 9)	.31	.35	.39	.29
d	Longitudinal plate (fig. 9)	.21	.29	.39	.29
e	Transverse plate (fig. 9)	.24	.28	.45	.51
f	Transverse plate (fig. 9)	.40	.38	.45	.51
g	Transverse plate (fig. 9)	.40	.30	.14	.16
h	Transverse plate (fig. 9)	.36	.29	.14	.16
i	Circular ring, diam., 0.08R	.44	.19	.50	.46
j	Circular ring, diam., 0.22R	.45	.18	.17	.14
k	Circular ring, diam., 0.43R	.36	.13	.06	.05

TABLE IV.- VALUES OF K_b AND K_m FOR MAXIMUM STRESS
 PRODUCED BY A LOAD UNIFORMLY DISTRIBUTED OVER A
 CIRCULAR AREA, AND ALONG A CONCENTRIC CIRCLE,
 SPHERICAL SHELL

r/R	Load on area of radius r		Load on circle of radius r	
	K_b	K_m	K_b	K_m
0	1.68	39	1.68	39
.01	1.06	30	.84	35
.02	.70	25	.40	29
.05	.46	21	.18	23
.04	.28	18	.12	19
.06	.09	14	.07	13
.08	.03	11	.04	8
.10	.01	10	.02	4.5

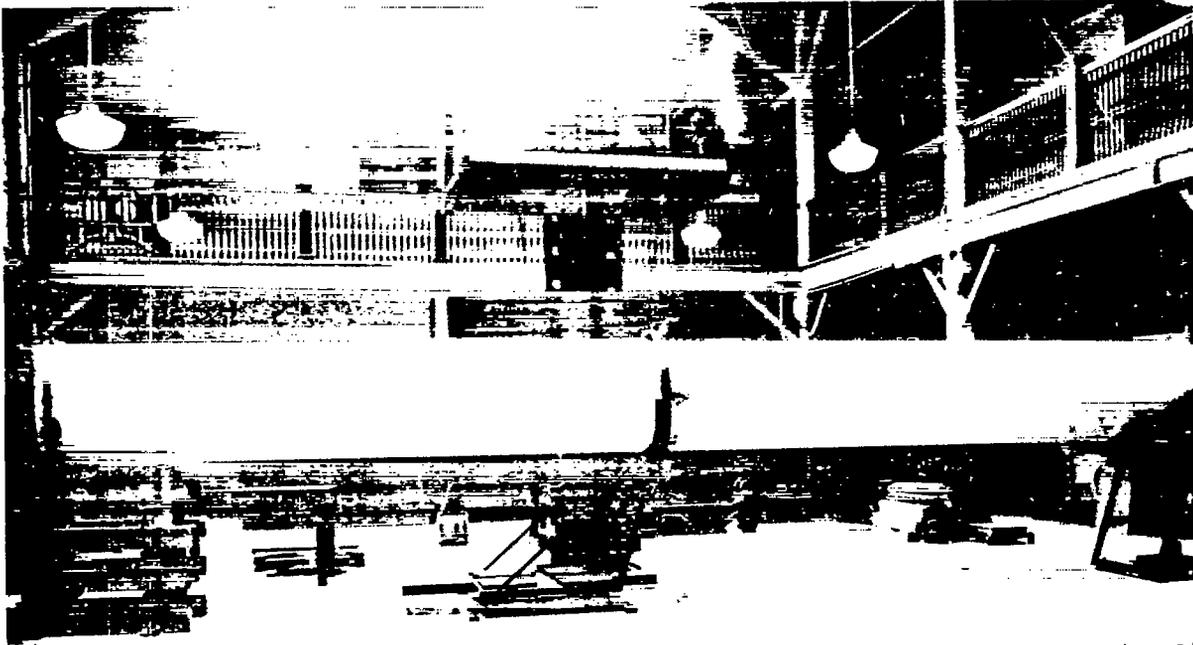


Figure 1.- Cylindrical shell
used in tests.

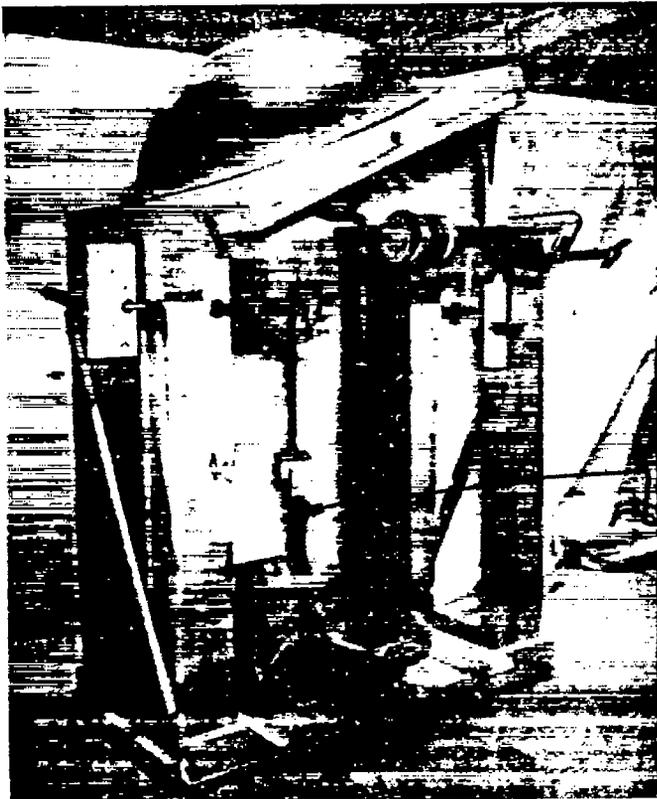


Figure 11.- Spherical shell
used in tests.

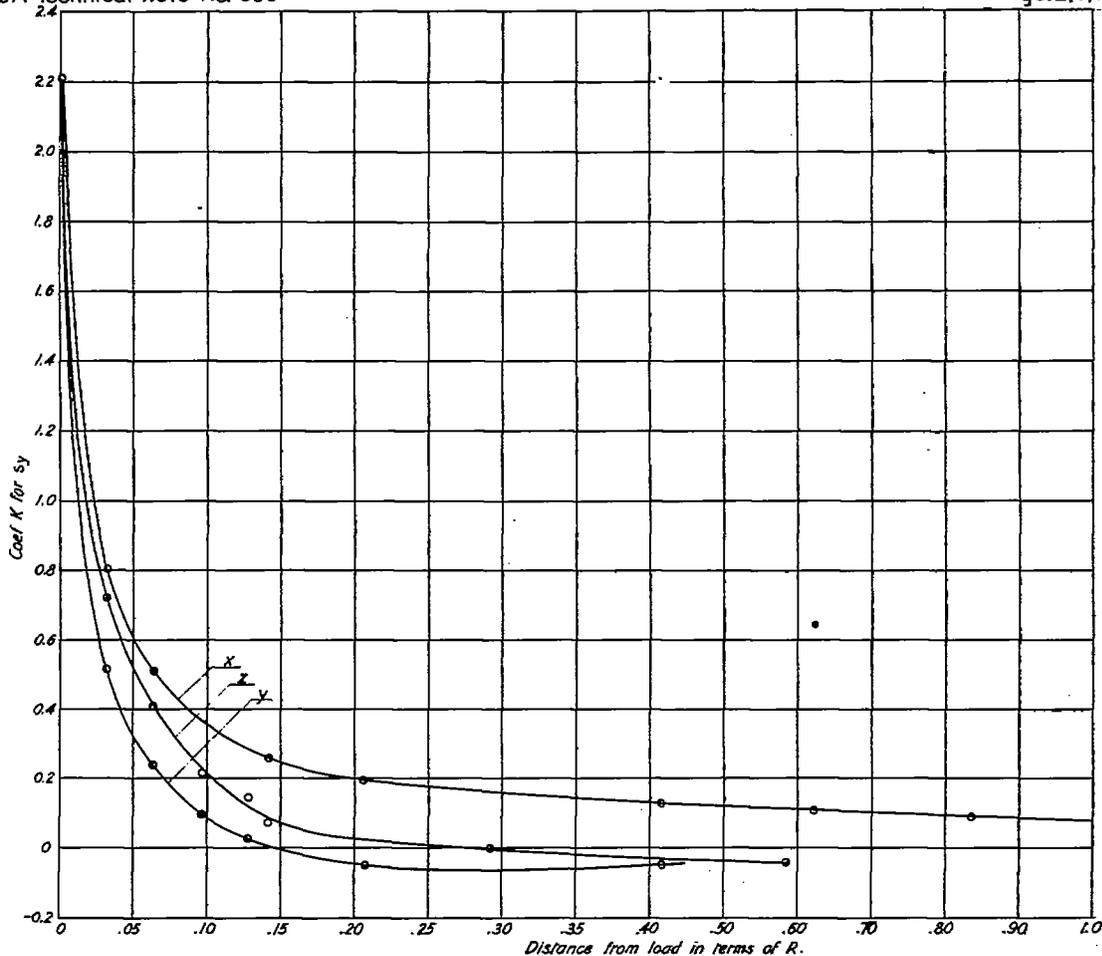


FIG. 3 - Variation of circumferential stress s_y with distance from point of loading measured longitudinally (x), circumferentially (y), and at 45° (z). Cylindrical shell.

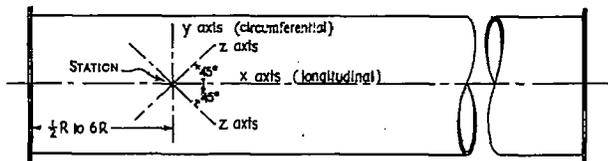


FIG. 2 - Reference axes for tests of cylindrical shell. Figure shows the bottom surface of the shell. The x axis was common to all stations, but each station had its own y and z axes.

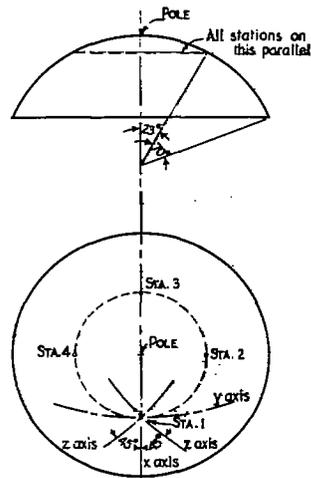


FIG. 12 - Reference axes for tests of spherical shell. Each station had x, y, and z axes as shown for Sta. 1.

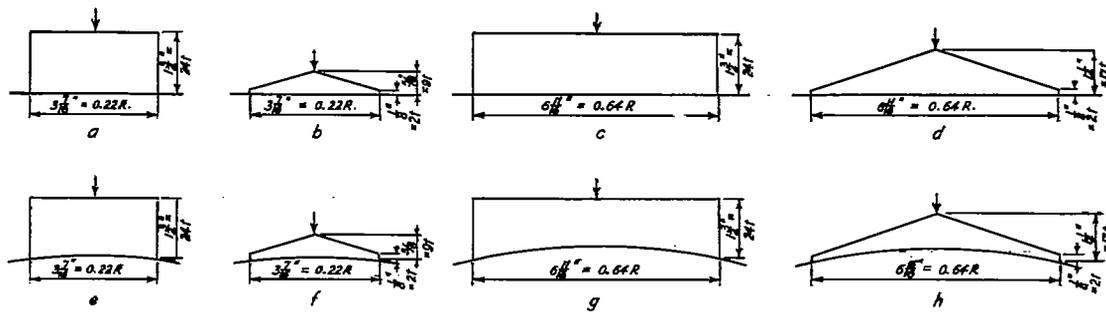


FIG. 9 Form and dimensions of flat lugs. All lugs $\frac{3}{8}$ " (3t) thick.

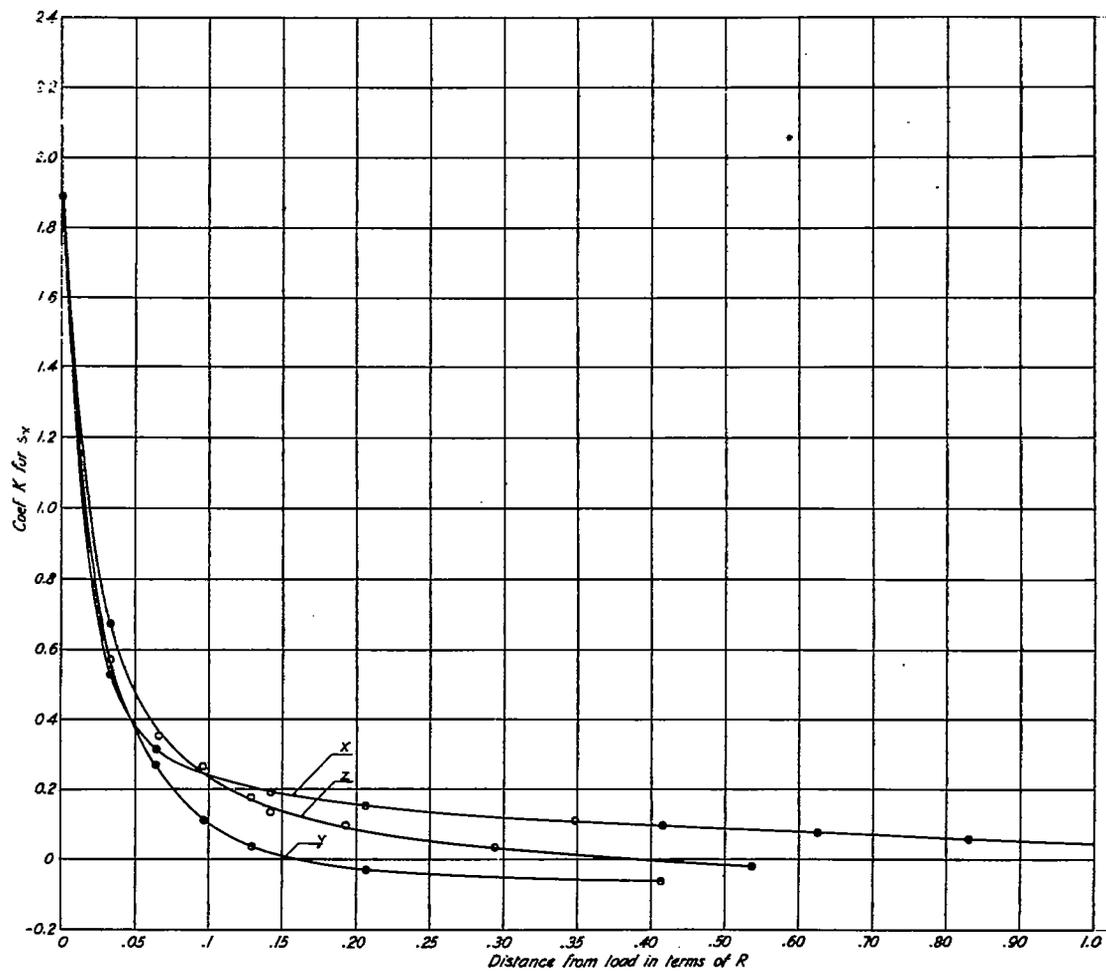


FIG. 4- Variation of longitudinal stress s_y with distance from point of loading measured longitudinally (x), circumferentially (y), and at 45° (z). Cylindrical shell.

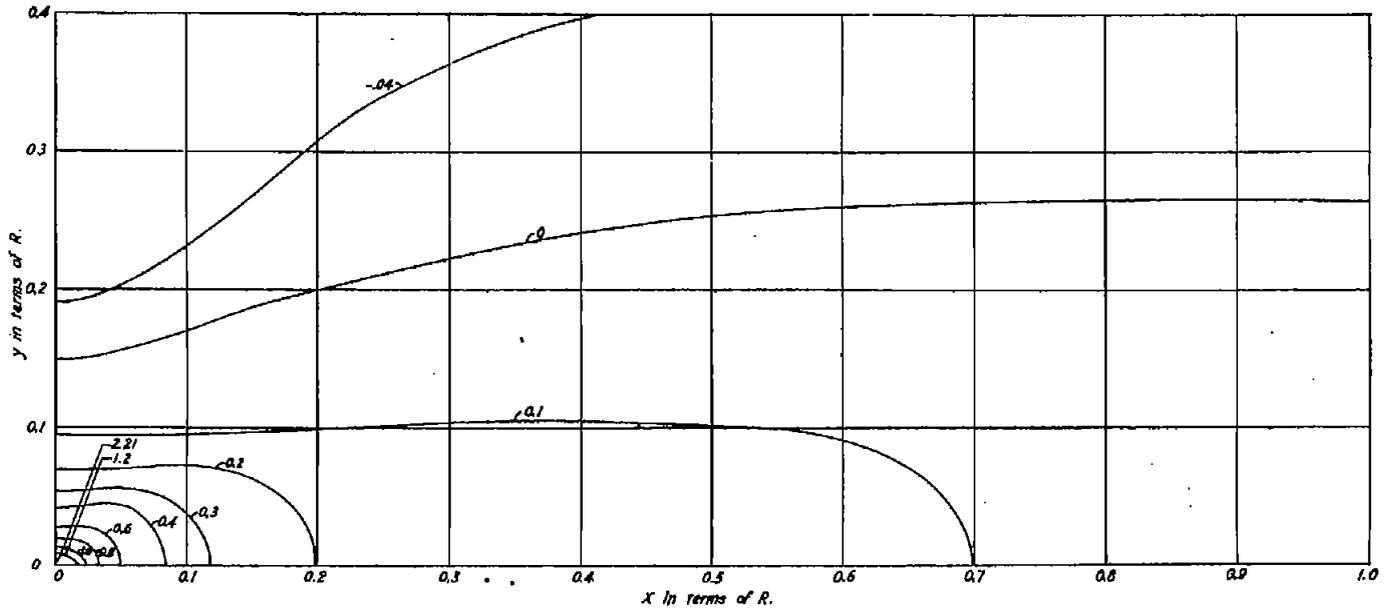


Fig. 5 Contour lines of influence surface for coef. K for s_y . Cylindrical shell.

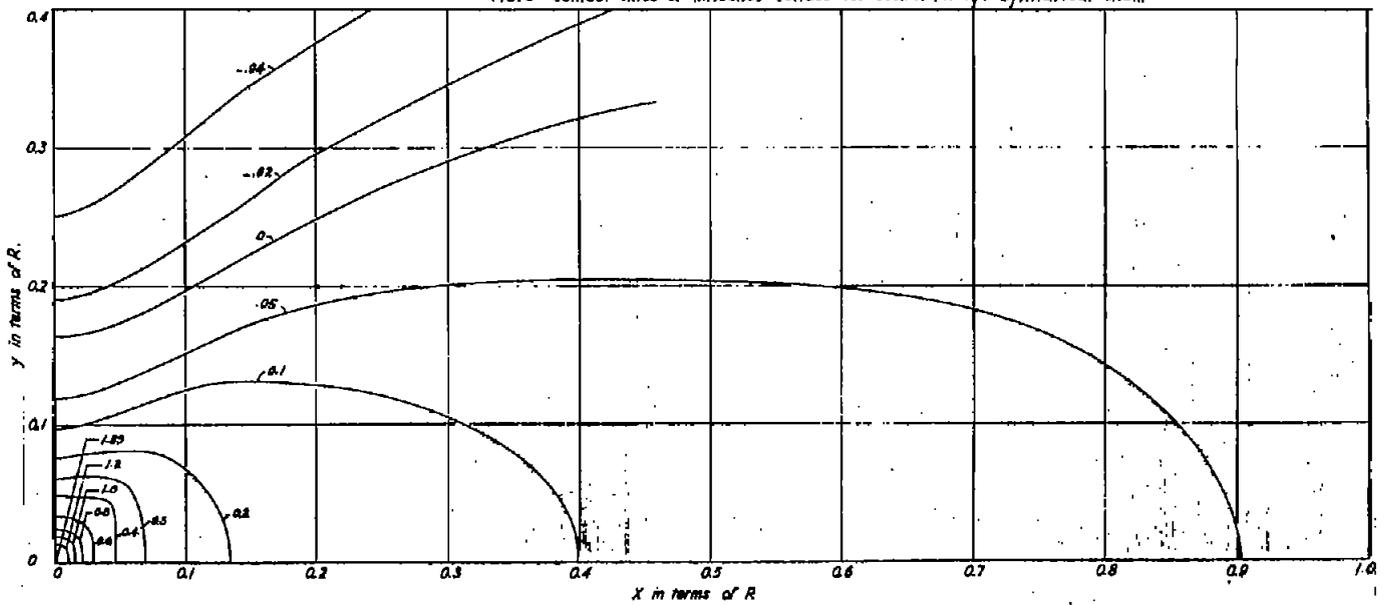


Fig. 6 - Contour lines of influence surface for coef. K for s_x . Cylindrical shell.

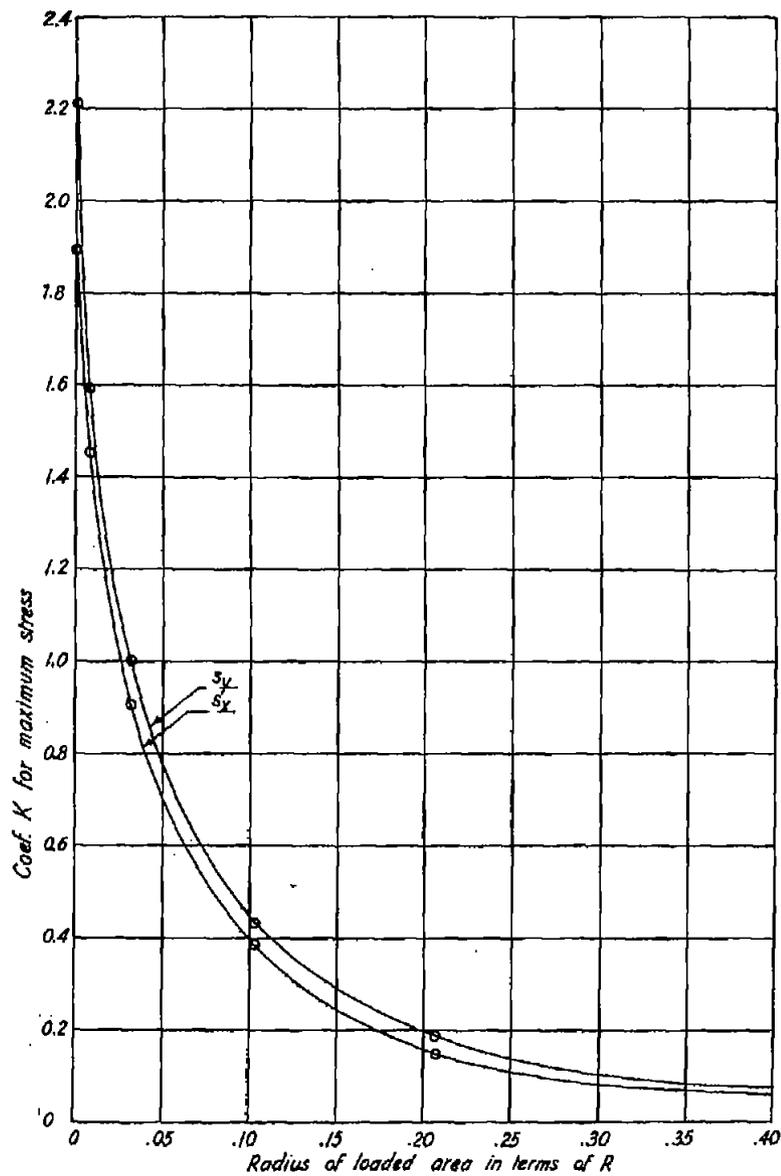


FIG. 7.- Maximum stresses due to load uniformly distributed over a circular area. Cylindrical shell.

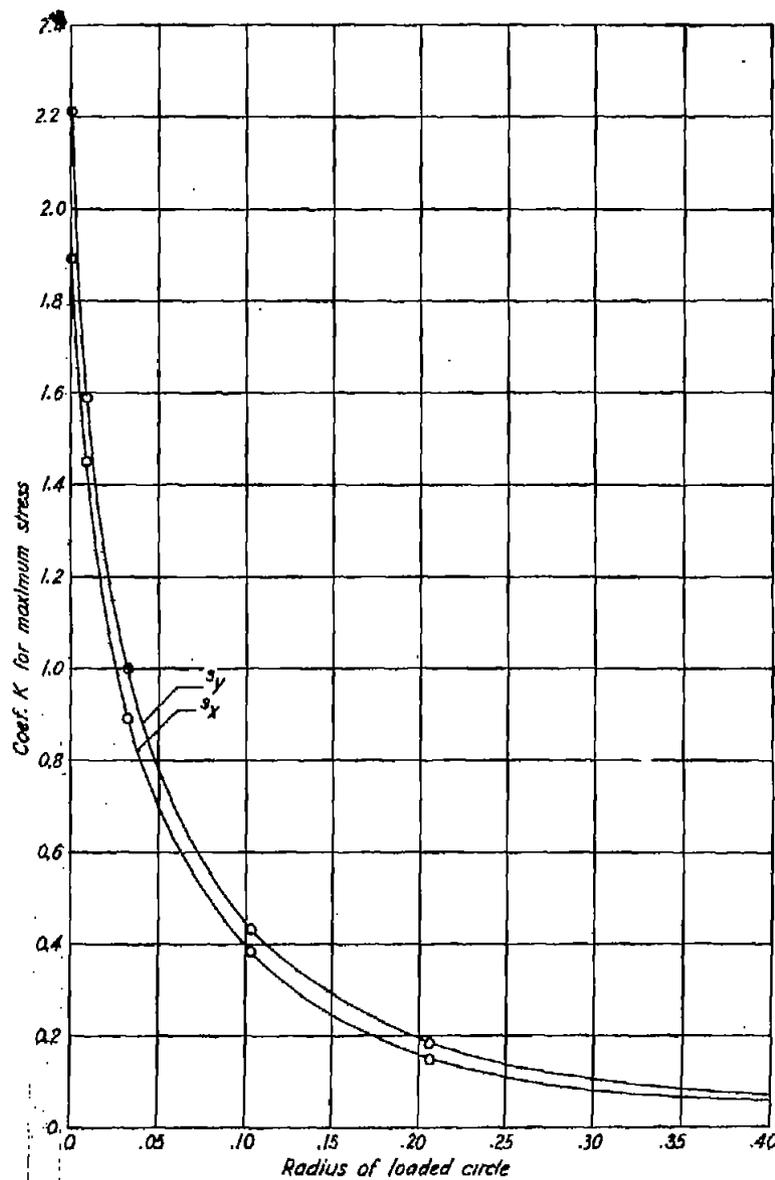


FIG. 8.- Maximum stresses due to load uniformly distributed on a circle. Cylindrical shell.

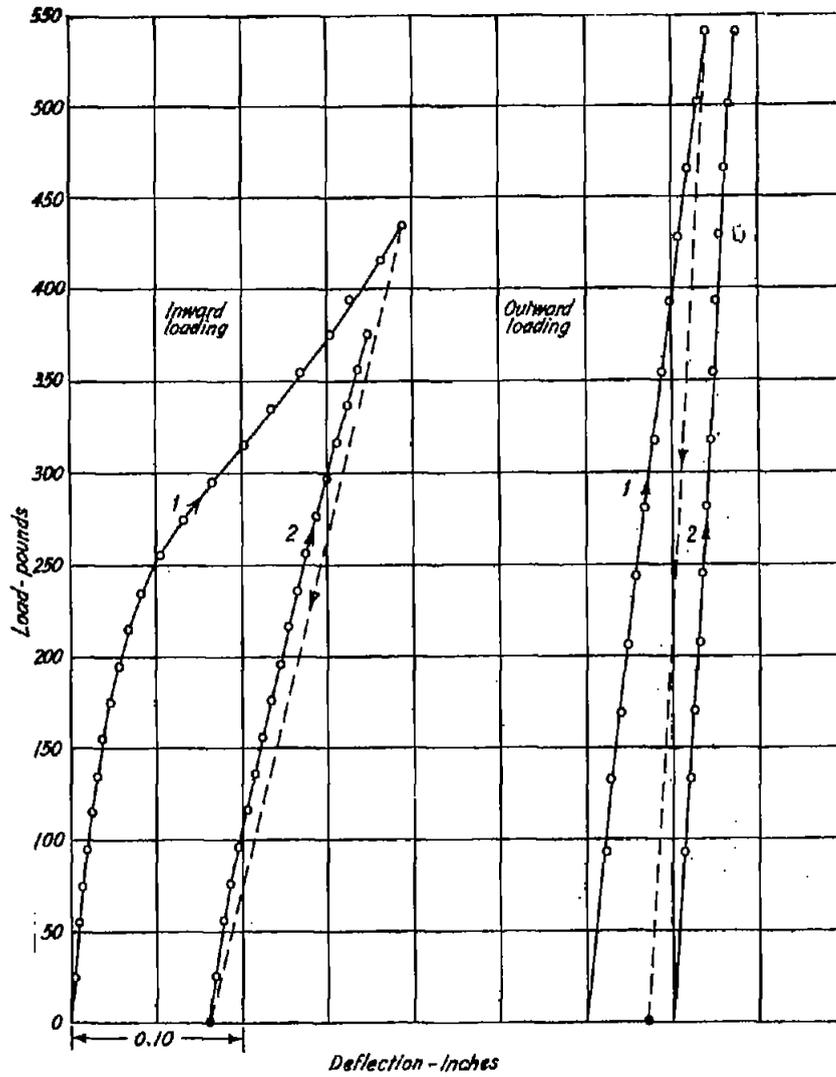


FIG. 10.- Load-deflection diagrams for initial (1) and repeated (2) loading. Cylindrical shell, concentrated load, 8-inch span.

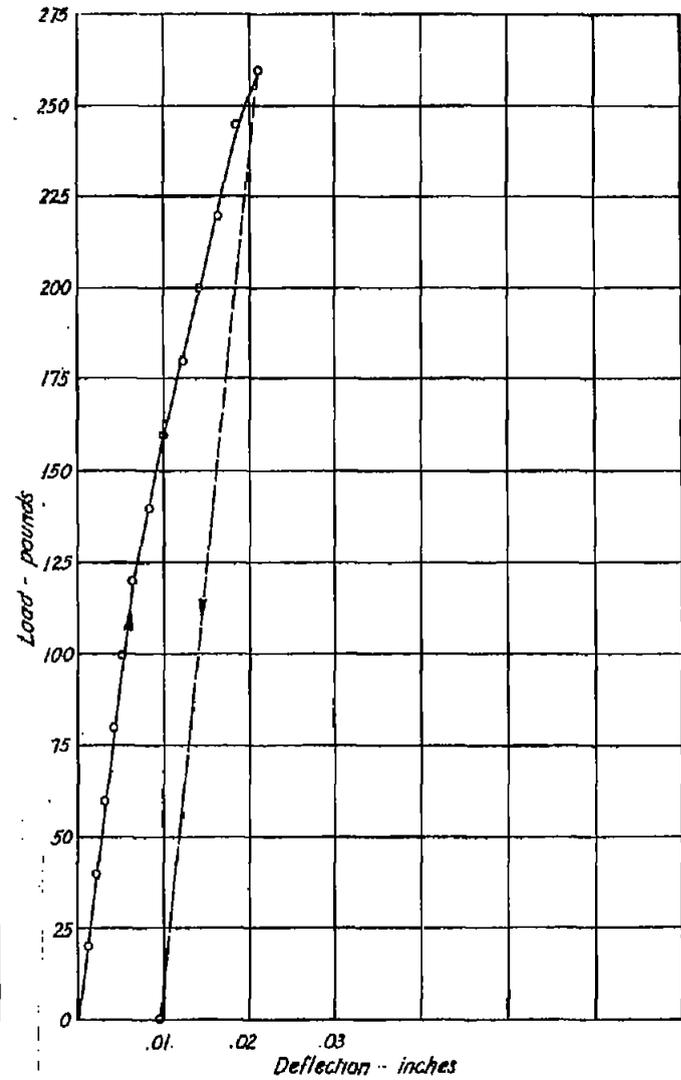


FIG. 17.- Load-deflection diagram for initial loading. Spherical shell, concentrated load, 4-inch span, outward loading.

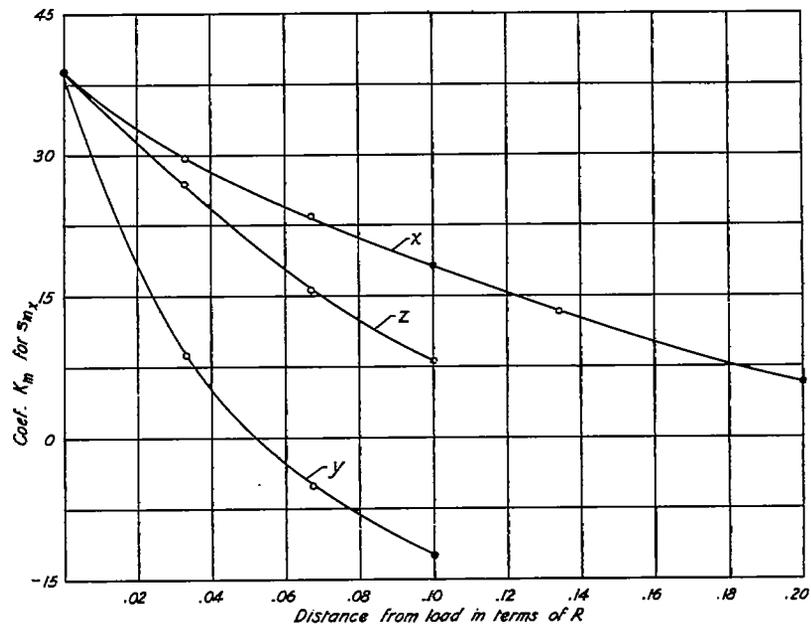


FIG. 14 - Variation of membrane stress s_{mx} with distance x, y and z from point of loading. Spherical shell.

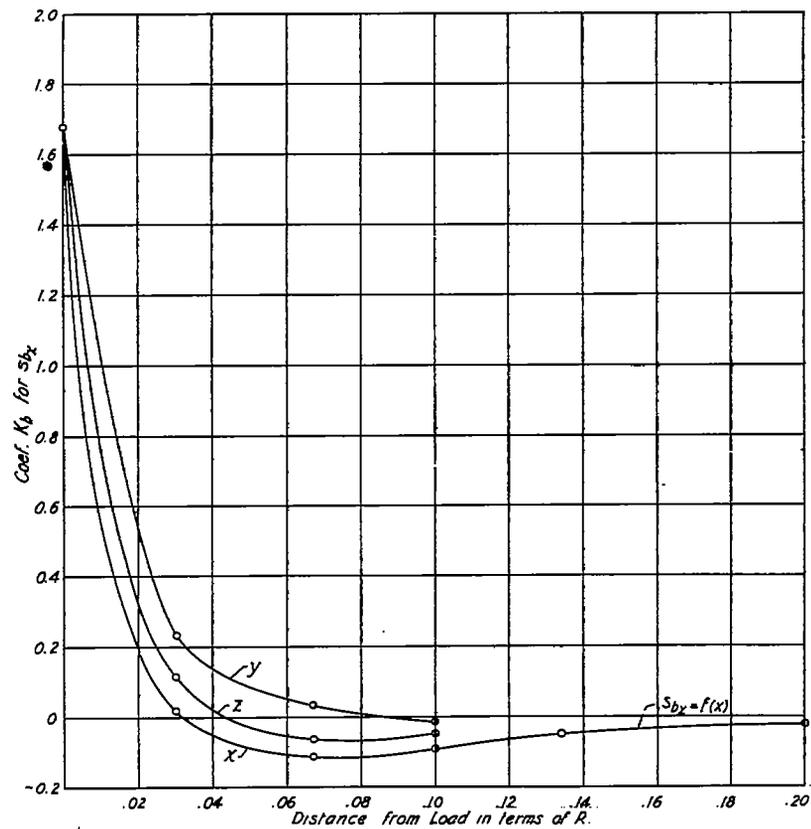


FIG. 13 - Variation of bending stress s_{bx} with distance x, y and z from point of loading. Spherical shell.

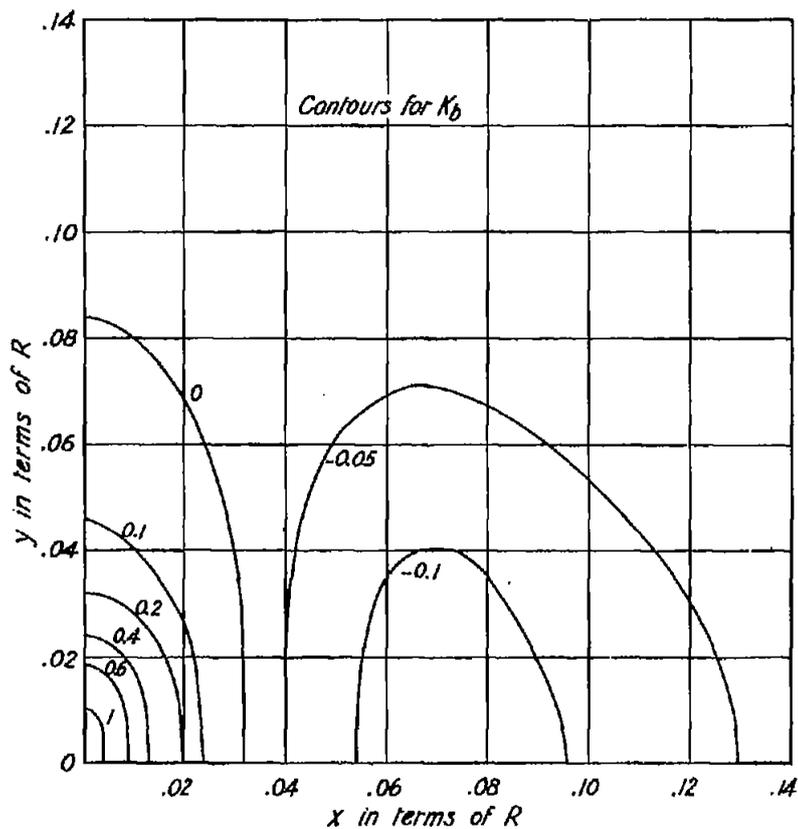


FIG. 15 - Contour lines of influence surface for coef. K_b for s_{b_x} . Spherical shell.

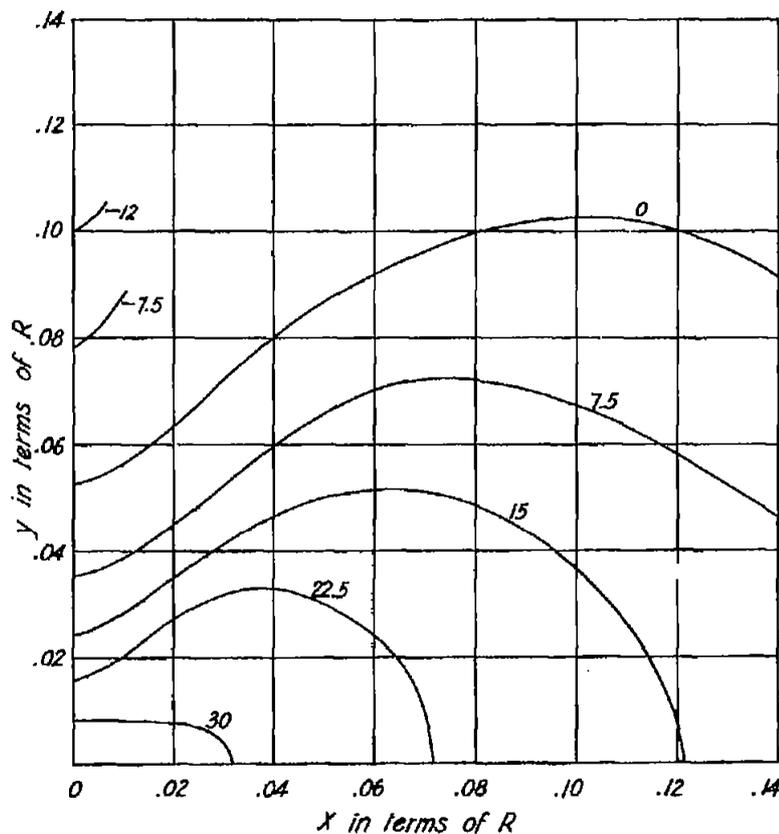


FIG. 16 - Contour lines of influence surface for coef. K_m for s_{m_x} . Spherical shell.