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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE

No. 1098

EFFECT OF CHANGE IN CROSS SECTION UPON STRESS DISTRIBUTION IN CIRCULAR SHELL-SUPPORTED FRAMES

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SUMMARY

Bending-stress and bending-moment coefficients are presented for application to design of circular shell-supported fuselage frames of variable cross section for applied radial load, applied moment, and applied tangential load. A ring which is properly reinforced in the region of maximum bending moment is shown to have a structural efficiency higher than that of a similar ring of uniform cross section. The structural efficiency is given in terms of the relative weights of the reinforced and nonreinforced rings.

INTRODUCTION

Solutions to the problem of the analysis of circular shell-supported frames are usually given in general form or in terms of a frame with a constant moment of inertia. In the analysis in reference 1, however, the bending moments have been computed for a circular ring in which the moment of inertia of one-half the ring has been increased for applied radial load and applied moment (fig. 1). The moment of inertia was increased in the analysis in reference 1 by the factors 2, 3, 4, 5, and 10. The purpose of the present paper is to extend the analysis of circular rings of variable cross section and to show how a ring may be proportioned so that a strength-weight ratio higher than that of a similar ring of constant cross section is obtained (fig. 2).

SYMBOLS

M	bending moment at any point in ring
c	distance from neutral axis of ring cross section to extreme fibre of ring cross section
P_a	applied radial load
M_a	applied moment
T_a	applied tangential load
R	radius of ring
C_M	bending-moment coefficient; nondimensional term in the bending-moment equations: $M = C_M P_a R$, $M = C_M T_a R$, and $M = C_M M_a$
C_σ	bending-stress coefficient $\left(\text{assumed equal to } \frac{C_M}{r}\right)$
σ	bending stress
σ_{all}	allowable bending stress
A_o	cross-sectional area of nonreinforced ring
I	moment of inertia at any station of ring cross section
I_R	moment of inertia of ring cross section at a region with reinforcement
I_o	moment of inertia of ring cross section at a region with no reinforcement
r	ratio of moments of inertia $\left(I_R/I_o\right)$
W_R	weight of reinforced ring
W_o	weight of nonreinforced ring
ρ	radius of gyration of ring cross section
w	specific weight of ring material

θ station of change in cross section, degrees; measured in clockwise direction from point 180° from point of applied load

φ station of bending moment and bending stress, radians; measured in same manner as θ

η, ξ bending factors determined by manner of reinforcement

K factor of proportionality

FORMULAS AND BENDING FACTORS

By a least-work analysis, or any other standard method of frame analysis, the bending moment in a radially loaded shell-supported circular frame of constant cross section can be shown to be given by

$$M = \frac{P_a R}{\pi}\left(\frac{\varphi}{2}\sin\varphi + \frac{\cos\varphi}{4} - \frac{1}{2}\right)$$

if the shear stresses in the shell are distributed according to the engineering theory of bending.

The same method may be used to compute the bending moment in a ring in which a reinforcement of uniform cross section is placed symmetrically about the point of applied load. The equation for bending moment involving a radial load then becomes

$$M = \frac{P_a R}{\pi}\left(\frac{\varphi}{2}\sin\varphi + \xi\cos\varphi - \eta\right)$$

The bending factors η and ξ are determined by a given combination of r and θ and are presented in table 1. Bending factors for values of r and θ not given in table 1 may be determined by interpolation.

Values of the bending factor η for a ring of variable cross section loaded by a couple are given in table 2. The bending-moment equations are

$$M = \frac{M_a}{2\pi} (\varphi - \eta \sin \varphi)$$

where

$$0 \leqq \varphi \leqq \pi$$

and

$$M = \frac{M_a}{2\pi} (\varphi - \eta \sin \varphi - 2\pi)$$

where

$$\pi \leqq \varphi \leqq 2\pi$$

For a tangentially loaded ring of constant cross section the bending-moment equations are

$$M = \frac{T_a R}{4\pi} \left[3 \sin \varphi - 2\varphi (\cos \varphi + 1)\right]$$

where

$$0 \leqq \varphi \leqq \pi$$

and

$$M = \frac{T_a R}{4\pi} \left[3 \sin \varphi + (4 - 2\varphi) (\cos \varphi + 1)\right]$$

where

$$\pi \leqq \varphi \leqq 2\pi$$

A separate set of equations for a tangentially loaded ring of variable cross section was not derived.

RELATIVE WEIGHT OF A REINFORCED FRAME AND A NONREINFORCED FRAME

The bending-moment diagram for a loaded circular frame often shows the bending moment reaching a maximum at several points, the greatest value usually occurring at the point of applied load. The bending-moment diagram for a circular frame with a single radial load contains four maximum points (fig. 3). For a circular frame with single radial load, the magnitude of the largest moment is more than twice that of the next largest moment and would lead to an inefficient design if it were used in designing a ring of constant cross section. Greater structural efficiency may be realized by reinforcing the ring in the region of applied load so that the resulting bending stresses are of approximately equal magnitude at points of maximum bending moment. The desired redistribution of bending stresses may be achieved by varying either the size of reinforcement or the length of reinforcement or by varying both (fig. 3).

The structural efficiency of a reinforced ring may be determined by comparing its weight with that of a similar ring of constant cross section capable of carrying the same load at the same allowable stress. The weight of the ring is assumed to be proportional to the moment of inertia of the ring; that is, the reinforcement is assumed to consist of doubler plates added to the flanges of the ring and to result in a negligible change in depth of cross section. The relative weight of a reinforced ring as compared with the weight of a nonreinforced ring may be found in the following manner:

In the equation for bending stresses $\sigma = \frac{Mc}{I}$, substitute σ_{all} for σ, $C_M P_a R$ for M, and $rA_o\rho^2$ for I and solve for A_o. Thus

$$\sigma_{all} = \frac{Mc}{I} = \frac{C_M P_a Rc}{rA_o\rho^2}$$

$$A_o = \frac{C_M}{r}\frac{P_a Rc}{\rho^2\sigma_{all}}$$

The weight of the reinforced ring will then be

$$W_R = 2\left[(\pi - \theta)r + \theta\right](RA_o w)$$

$$= 2\left[(\pi - \theta)r + \theta\right]\left(\frac{P_a R^2 wc}{\rho^2\sigma_{all}}\frac{C_M}{r}\right)$$

Let

$$K = \frac{2P_a R^2 wc}{\rho^2\sigma_{all}}$$

and

$$C_\sigma = \frac{C_M}{r}$$

Then

$$W_R = KC_\sigma \left[(\pi - \theta)r + \theta\right]$$

where C_σ is the bending-stress coefficient for the reinforced ring in question. For a nonreinforced ring $(r = 1)$ the equation for weight becomes

$$W_o = KC_\sigma \pi$$

where C_σ is the bending-stress coefficient for a nonreinforced ring. The structural efficiency of a reinforced ring can then be found from its relative weight by

$$\text{Relative weight} = \frac{W_R}{W_o}$$

A relative weight greater than unity indicates that the reinforced ring is heavier, and therefore less efficient, than a nonreinforced ring. A relative weight less than unity indicates that the reinforced ring is lighter and more efficient than a nonreinforced ring. A comparison of the relative weights is given in table 3.

A ring for which the bending-moment diagram contains maximum ordinates of widely varying magnitudes may be reinforced in a number of ways in order to reduce its relative weight with an improvement in structural efficiency. This improvement can be made for a ring with a single applied radial load or a single applied moment (figs. 3 and 4). For a nonreinforced ring in which the maximum bending moments are of approximately equal magnitude, a condition of relatively high structural efficiency already exists and there is little room for improvement, as in the case of a ring with a single tangential load (fig. 5). It is advisable, therefore, to be guided by the shape of the bending-moment diagram for a nonreinforced ring in designing the proper reinforcement for a lighter and more efficient reinforced ring, particularly when a condition of combined loading exists. Methods of obtaining lighter rings may be suggested by the bending-moment diagram for the nonreinforced ring. Ease of fabrication, method of attachment, and other practical problems can then be considered. A number of different designs may possibly lead to a highly efficient frame, but the lightest and most efficient frame probably will be one in which the maximum bending stresses are of equal or nearly equal magnitude.

CONCLUDING REMARKS

A ring which is properly reinforced in the region of maximum bending moment is shown to have a structural efficiency higher than that of a similar ring of uniform cross section. The structural efficiency is given in terms of the relative weights of the reinforced and non-reinforced rings.

Langley Memorial Aeronautical Laboratory
National Advisory Committee for Aeronautics
Langley Field, Va., April 15, 1946

REFERENCE

1. Stieda, W.: Statics of Circular-Ring Stiffeners for Monocoque Fuselages. NACA TM No. 1004, 1942.

VALUES OF BENDING FACTORS FOR A SINGLE RADIAL LOAD

θ (deg)	90		120		135		150		165	
r	η	ξ	η	ξ	η	ξ	η	ξ	η	ξ
1	0.500	0.250	0.500	0.250	0.500	0.250	0.500	0.250	0.500	0.250
2	.502	.297	.536	.325	.545	.336	.542	.331	.526	.302
3	.513	.327	.564	.376	.572	.387	.562	.370	.537	.323
4	.525	.351	.585	.414	.590	.421	.574	.393	.542	.334
5	.537	.372	.602	.444	.603	.444	.582	.409	.546	.341
7	.559	.409	.626	.486	.620	.476	.592	.428	.550	.349
8	.569	.424	.635	.502	.626	.487	.595	.434	.552	.352
10	.587	.452	.650	.527	.635	.503	.600	.443	.553	.356

TABLE 2

VALUES OF BENDING FACTOR η FOR A MOMENT LOAD

θ (deg) / r	90	120
1	2.000	2.000
2	1.758	1.785
3	1.637	1.700
4	1.563	1.655
5	1.515	1.625
10	1.405	1.574

TABLE 3

COMPARISON OF RELATIVE WEIGHTS

Radial load										
r \ θ (deg)	90		120		135		150		165	
	Maximum C_σ	W_R/W_o	Maximum C_σ	W_R/W_o	Maximum C_σ	W_R/W_o	Maximum C_σ	W_R/W_o	Maximum C_σ	W_R/W_o
1	0.239	1.000	0.239	1.000	0.239	1.000	0.239	1.000	0.239	1.000
2	.127	.798	.137	.765	.140	.733	.139	.678	.142	.643
3	.089	.746	.100	.696	.102	.639	.099	.552	.152	.740
4	.083	.867	.080	.666	.080	.589	.083	.520	.157	.820
5	.079	.993	.067	.650	.067	.558	.090	.624	.160	.893
7	.072	1.207	.051	.635	.053	.551	.098	.819	.164	1.029
8	.069	1.298	.048	.667	.051	.585	.101	.913	.165	1.094
10	.063	1.454	.043	.724	.048	.652	.105	1.096	.167	1.222

Moment				
r \ θ (deg)	90		120	
	Maximum C_σ	W_R/W_o	Maximum C_σ	W_R/W_o
1	0.500	1.000	0.500	1.000
2	.250	.750	.250	.667
3	.167	.667	.167	.556
4	.125	.625	.125	.500
5	.100	.600	.110	.512
10	.050	.549	.109	.908

Tangential load		
r \ θ (deg)	90	
	Maximum C_σ	W_R/W_o
1	0.0640	1.000
2	.0354	0.832

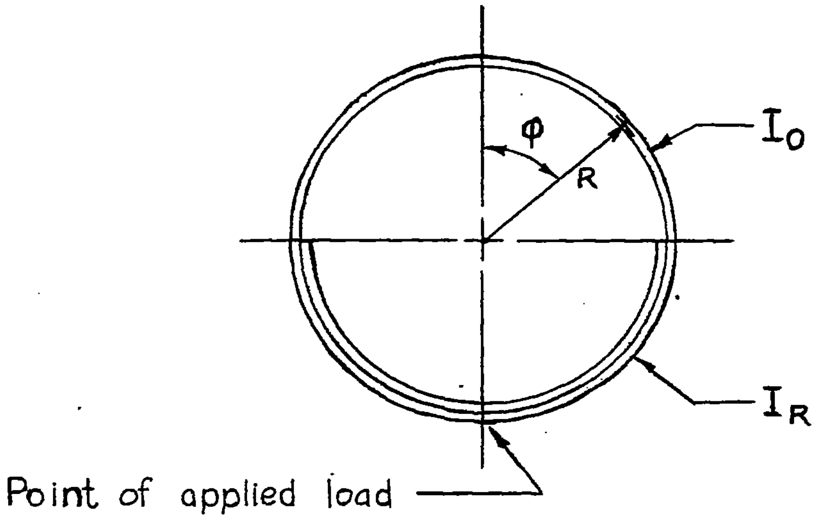

Figure 1.- Circular frame reinforced over half the circumference.

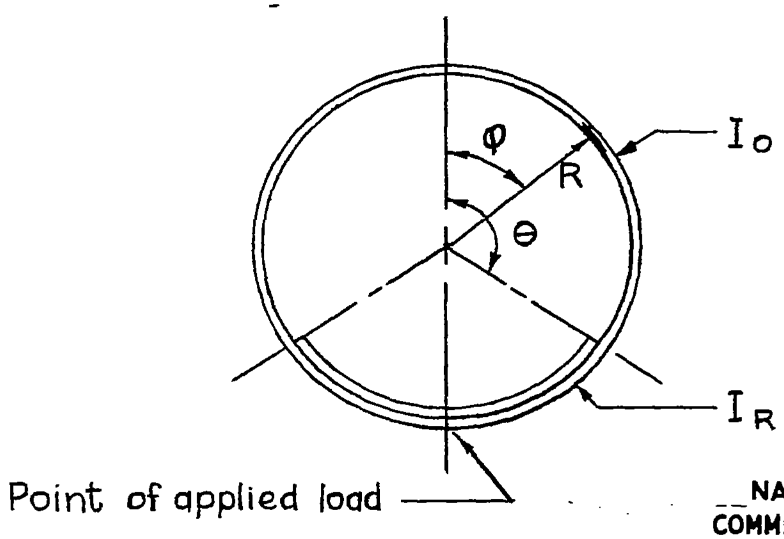

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Figure 2.- Circular frame with reinforcement of variable length.

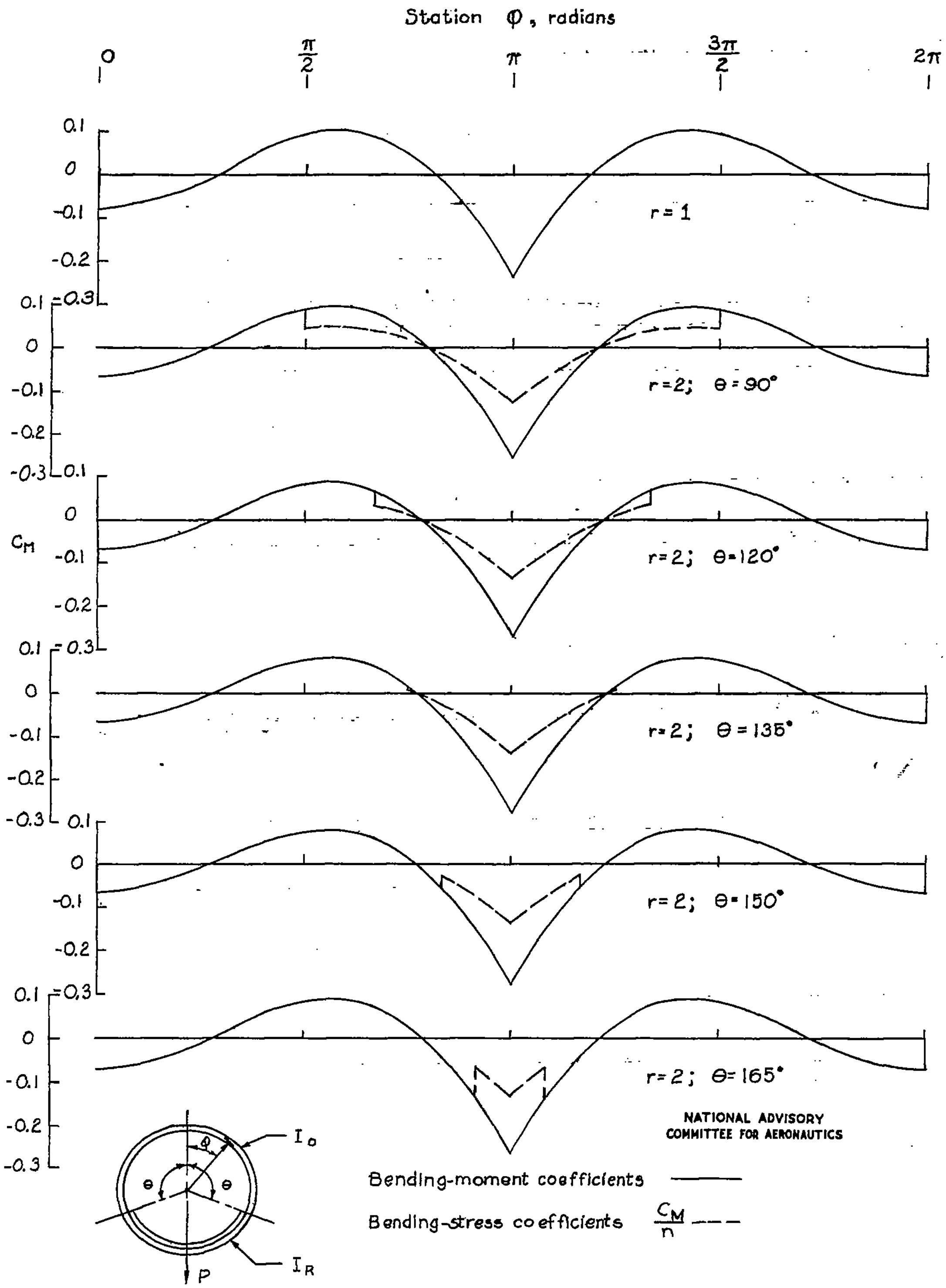

Figure 3.- Bending-moment and bending-stress coefficients for an applied radial load. Positive moment indicates tension on inner fibres of ring.

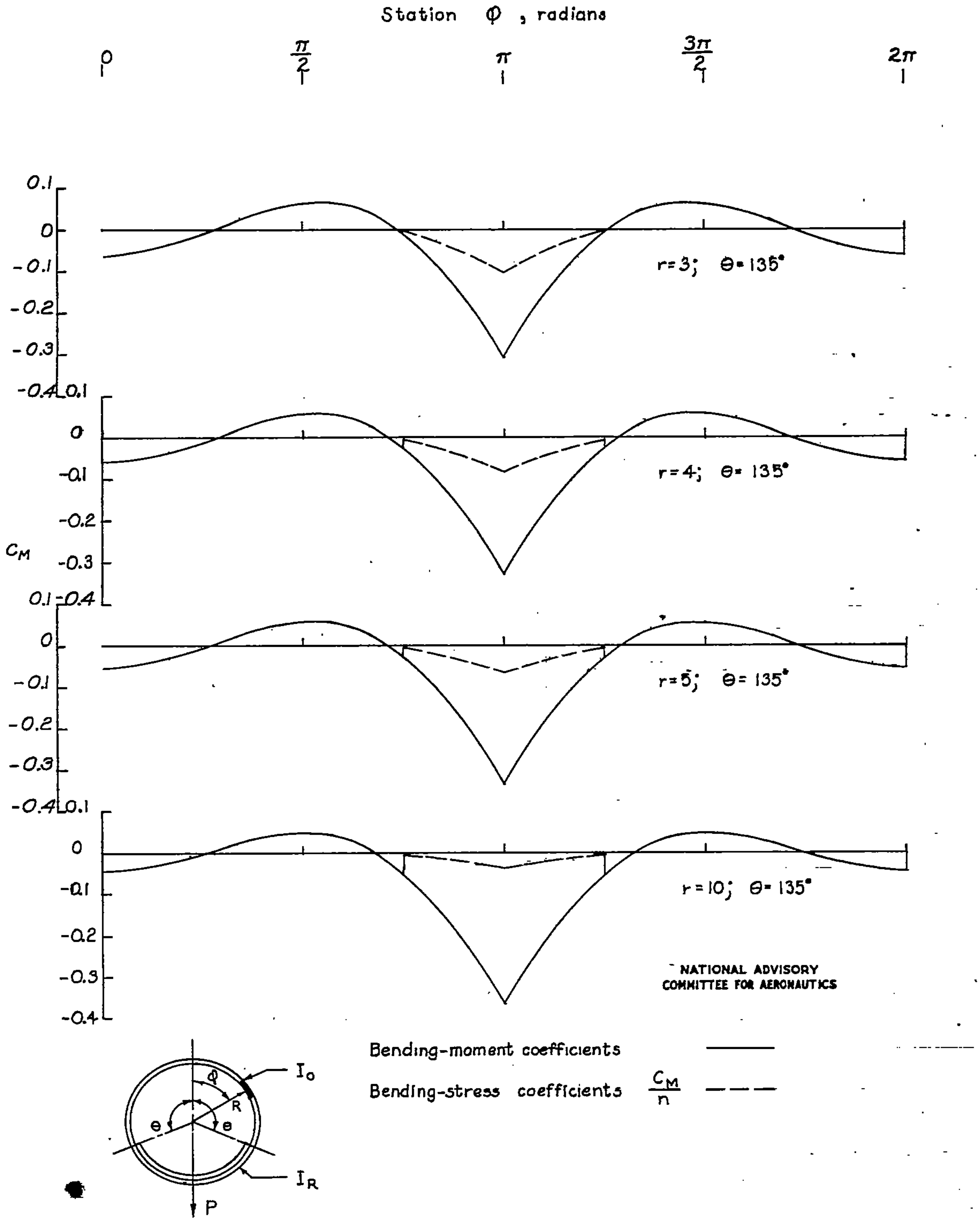

Figure 3.- Concluded.

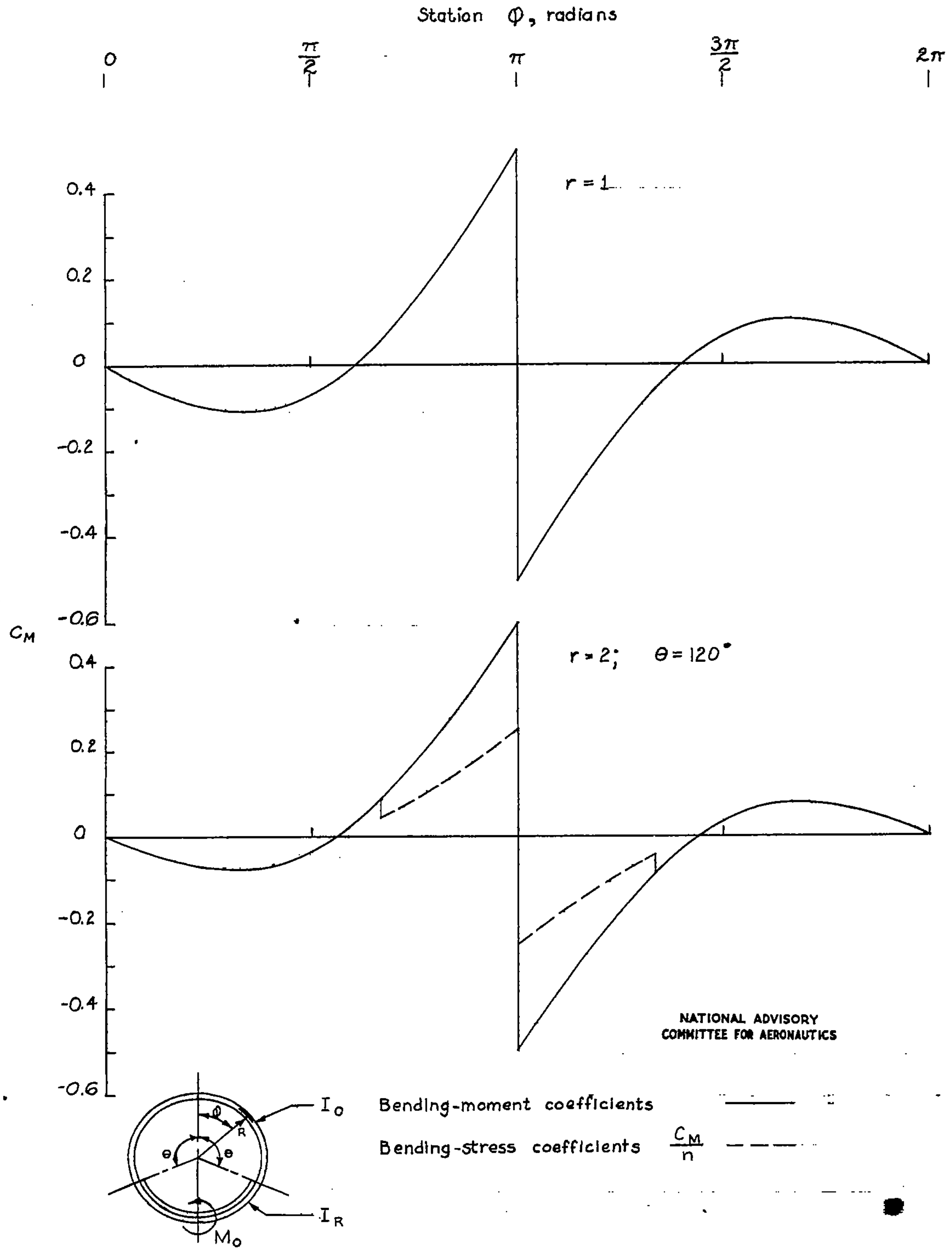

Figure 4.- Bending-moment and bending-stress coefficients for an applied moment load. Positive moment indicates tension on inner fibres of ring.

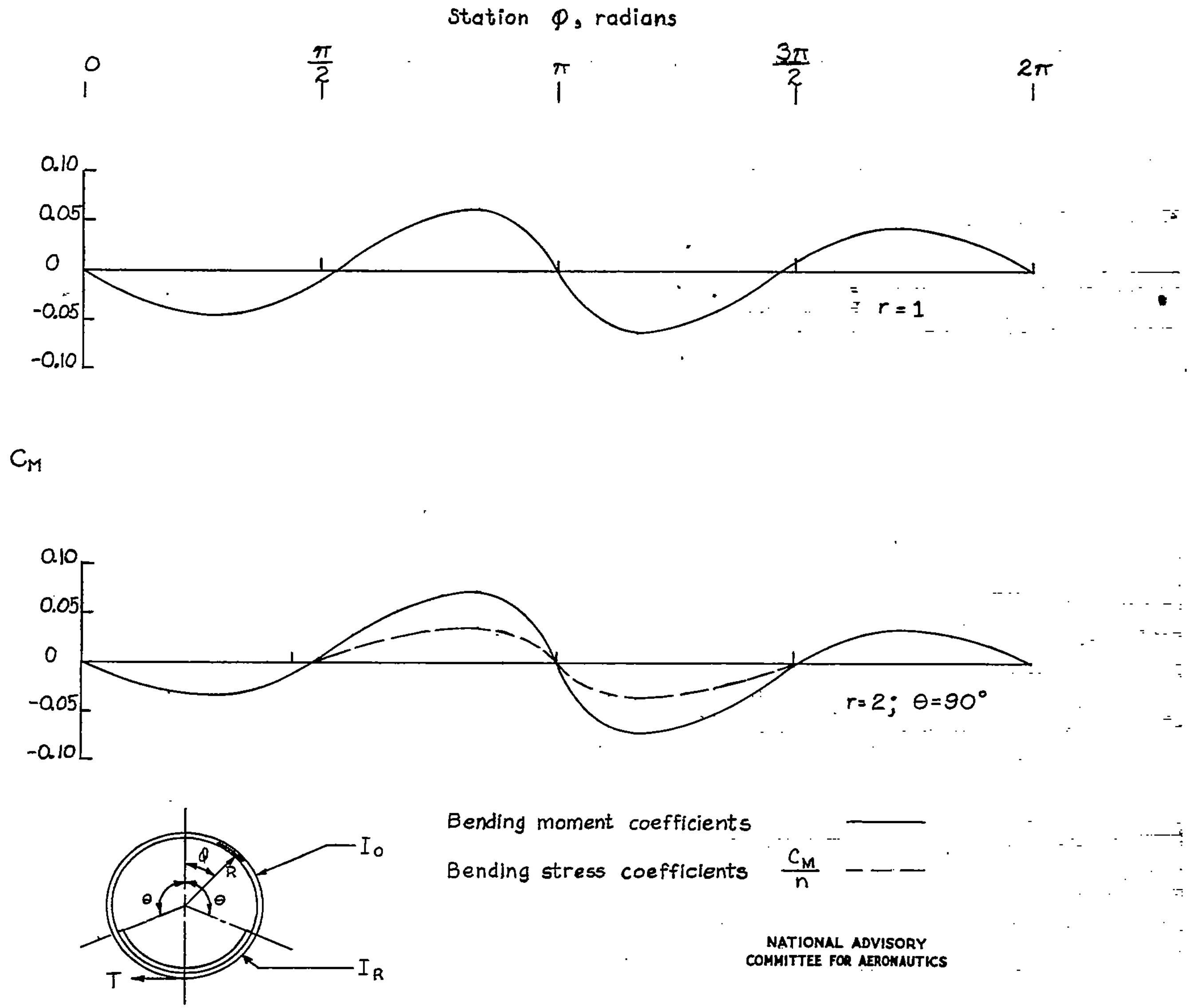

Figure 5.- Bending-moment and bending-stress coefficients for an applied tangential load. Positive moment indicates tension on inner fibres of ring.