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TECHNICAL NOTE

No. 1497

A METHOD OF CYCLE ANALYSIS FOR AIRCRAFT GAS-TURBINE POWER PLANTS DRIVING PROPELLERS

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A METHOD OF CYCLE ANALYSIS FOR AIRCRAFT GAS-TURBINE POWER PLANTS DRIVING PROPELLERS

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SUMMARY

The primary purpose of this investigation is to develop a convenient and accurate method for analyzing gas-turbine cycles to aid in predicting the performance of propeller-driving power plants and the state of the gas at various stations within the power plant. The method takes into account the following factors affecting the power-plant performance:

1. Variation of pressure and temperature with altitude

2. Variation of specific heats with temperature and composition of the working fluid

3. Difference between weight flow through the compressor and weight flow through the turbine

4. Effects of ram

5. Thrust from exhaust jet

The tabulated gas properties were used to produce charts from which the power-plant data may be selected without lengthy calculations.

The secondary purpose of this investigation is to determine, by application of the method that was developed, the performance of the basic propeller-driving power plant, consisting of inlet diffuser, compressor, combustion chamber, turbine, propeller, and exhaust nozzle, and the basic power plant modified by the addition of intercooling, reheating, and regeneration. As an example of the application of the method, the performance of propeller-driving power plants was determined for a flight speed of 400 miles per hour, a ratio of turbine-inlet temperature to compressor-inlet temperature of 4.5, a compressor adiabatic efficiency of 0.85, and a turbine adiabatic efficiency of 0.90; the power plants considered were the basic power plant and the basic power plant modified by the addition of intercooling, reheating, or regeneration. In addition, the effects of changing altitude and turbine-inlet temperature of the basic power plant were determined.

From the determination of the power-plant performance, the following conclusions were drawn: The basic gas-turbine power plant can cruise with a specific fuel consumption as low as 0.37 pound of fuel per horsepower-hour. The effect of the addition of an intercooling heat exchanger or a reheating combustion chamber to the basic gas-turbine power plant is primarily to increase the specific power output with only small changes in the specific fuel consumption. The addition of reheating yields a slightly greater increase in the specific power output than the addition of intercooling. In order to obtain a reduction in the minimum specific fuel consumption by adding a regenerating heat exchanger to the basic gas-turbine power plant, the regenerating heat exchanger must have an effectiveness greater than 0.50. If the turbine-inlet temperature is increased, the specific fuel consumption is decreased and the specific power output is increased; if the ambient atmospheric temperature is increased by decreasing the altitude, the specific fuel consumption is increased and the specific power output is reduced.

INTRODUCTION

In the design of an aircraft gas-turbine propeller-driving power plant, a convenient and accurate method is desirable with which to analyze the power-plant performance over a wide range of design conditions and to determine the state of the gas at various stations within the power plant. A great amount of information has been published on the performance of gas-turbine power plants and several methods are now available for calculating the performance. (See references 1 to 10.) The performance calculations that have been made for stationary or marine power plants are not entirely suitable for use in the design of aircraft power plants because the performance has been calculated at sea level and at zero flight speed. The methods of analysis that make use of constant specific heats or a single working substance are satisfactory for making general comparisons of many kinds of power plant but these methods are not sufficiently accurate to be useful in the final design of an aircraft power plant.

The primary purpose of this investigation is to develop a convenient and accurate method for analyzing gas-turbine cycles to aid in predicting the performance of propeller-driving power plants and the state of the gas at various stations within the power plant. The method takes into account the following factors affecting the power-plant performance:

(1) Variation of pressure and temperature with altitude

(2) Variation of specific heats with temperature and composition of the working fluid

(3) Difference between weight flow through the compressor and weight flow through the turbine

(4) Effects of ram

(5) Thrust from exhaust jet

The methods of analysis neglecting these factors are simple and brief but are not sufficiently accurate to be used in the final design of an aircraft power plant. The methods of analysis that make use of tabulated gas properties are very accurate, but the process of analysis is long and laborious. The method reported is a compromise between these two extremes and makes use of charts prepared from the tabulated gas properties in order that accurate data may be selected in a comparatively short time.

The secondary purpose of this investigation is to determine the performance of several theoretical propeller-driving power plants by applying the developed method of analysis. The power plants to be considered are the basic power plant (consisting of inlet diffuser, compressor, combustion chamber, turbine, propeller, and exhaust nozzle) and the basic power plant modified by the addition of an intercooling heat exchanger, a reheating combustion chamber, or a regenerating heat exchanger. In addition, the effects of changing altitude and turbine-inlet temperature on the basic power plant are to be determined.

SYMBOLS

The following symbols are used in this analysis:

C coefficient of velocity for exhaust nozzle

e effectiveness of heat exchanger

f fuel-air ratio

F specific fuel consumption, pounds per horsepower-hour

g mass ratio, 32.17 pounds per slug

H enthalpy in total state, foot-pounds per pound

P total pressure, pounds per square foot absolute

p static pressure, pounds per square foot

R gas constant, foot-pounds per pound °R

T total temperature, °R

t static temperature, °R

V velocity relative to engine, feet per second

W work, foot-pounds per pound

γ ratio of specific heats

η efficiency

Subscripts:

ad reversible adiabatic

b combustion or burner

b' reheating combustion

c compressor

c' first, or low-pressure, stage of compressor

c" second, or high-pressure, stage of compressor

d inlet diffuser

i intercooling heat exchanger

id ideal

j jet

n net

o over all

p propeller

r regenerating heat exchanger

s shaft

t turbine

t' first, or high-pressure, stage of turbine

t" second, or low-pressure, stage of turbine

The following subscripts refer to stations in the engine:

0 free stream or entrance to inlet diffuser

1 compressor inlet or exit from inlet diffuser

2 inlet to intercooling heat exchanger

3 exit from intercooling heat exchanger

4 compressor exit or air inlet to regenerating heat exchanger

5 combustion-chamber inlet or air exit from regenerating heat exchanger

6 turbine inlet or combustion-chamber exit

7 inlet to reheating combustion chamber

8 exit from reheating combustion chamber or inlet to second-stage turbine

9 turbine exit or exhaust-gas inlet to regenerating heat exchanger

10 inlet to exhaust nozzle or exhaust-gas exit from regenerating heat exchanger

11 exit from exhaust nozzle

ANALYSIS

Description of Cycles

Several cycles of operation have been proposed for gas-turbine power plants. The simplest of these is based on the Brayton (or Joule) cycle in which the working substance is isentropically compressed, heated at constant pressure, expanded with constant entropy to the initial pressure, and then cooled at constant pressure

to the original state. The adaptation of this ideal cycle for use in an actual power plant, is herein referred to as the basic cycle. The effects of intercooling during the compression process and reheating or regeneration during the expansion process on the basic cycle are considered in the analysis of the intercooling cycle, the reheating cycle, and the regenerating cycle, respectively.

Basic cycle. - In the basic cycle, air from the free stream (station 0) is diffused so the velocity of the air is reduced from the flight speed to the velocity at the compressor inlet (station 1). (See fig. 1.) In figure 1 the solid lines represent ideal, reversible processes; the dotted lines indicate the approximate path between the end points of the actual, irreversible processes and link the ends of the ideal processes with the ends of the real processes. In the compressor, the air is compressed between states 1 and 4 (fig. 1(a)). The losses in the compressor alter the compression from the reversible path of 1 to 4id. The air leaving the compressor is burned with fuel between states 5 and 6. The pressure losses during combustion produce a greater change in entropy than the constant-pressure combustion from state 5 to 6id. The hot combustion products are expanded with losses in a turbine from state 6 to 9 and in the exhaust nozzle from state 10 to 11. The cycle is closed in the atmosphere from state 11 to 0.

Intercooling cycle. - The cycle of operation for a power plant using intercooling is represented by the diagrams in figure 1(b). The air compression is broken into two stages, states 1 to 2 and 3 to 4; and the air is cooled, states 2 to 3, between the stages, with a small loss in total pressure. Intercooling reduces the work of compression.

Reheating cycle. - The cycle of operation for a power plant using reheating is represented by the diagrams in figure 1(c). In the reheating cycle, the expansion in the turbines is broken into two stages, states 6 to 7 and 8 to 9, and the air is burned with additional fuel, states 7 to 8, between the stages. Reheating increases the work of expansion.

Regenerating cycle. - The cycle of operation for a power plant using regeneration is represented by the diagrams in figure 1(d). Part of the energy in the gas leaving the turbine is reclaimed by means of a regenerating heat exchanger. The heat transferred from the exhaust gas cools the exhaust gas between states 9 and 10 and heats the compressed air between states 4 and 5. Regeneration reduces the fuel required to reach a given turbine-inlet temperature.

Method of Calculating Engine Performance

These cycles of operation can be constructed from the following elements:

(a) Compression in the inlet diffuser

(b) Compression in the compressor

(c) Intercooling

(d) Combustion

(e) Expansion in the turbine

(f) Reheating

(g) Regeneration

(h) Expansion in the exhaust nozzle

In the following analysis each of these elements is considered separately.

Compression in the inlet diffuser. - The state of the air entering the compressor is different from the state of the free stream surrounding the power plant because compression is obtained in the inlet diffuser. The total temperature of the air leaving the inlet diffuser is expressed as

$$T_1 = t_0 + \frac{\gamma_d - 1}{\gamma_d R_d} \frac{V_0^2}{2g} \tag{1}$$

The values of γ_d and R_d were assumed equal to 1.40 and 53.3, respectively. The variation of the total temperature with altitude and speed is shown in figure 2.

If the diffusion of the air ahead of the compressor is isentropic, the total pressure at the compressor inlet may be expressed as

$$P_1 = p_0 \left(\frac{T_1}{t_0}\right)^{\frac{\gamma_d}{\gamma_d - 1}} \tag{2}$$

This relation was used to indicate the variation of total pressure with altitude and flight speed, as shown in figure 3. An actual diffuser will not produce as great a pressure rise as equation (2) indicates. The relation between the actual and the ideal pressure recoveries is expressed in several ways and each of these forms of expression is used by a different group of designers. The ideal pressure recovery is given here so that each designer may use his own method of correction.

Compression in the compressor (no intercooling). - If there is no variation in the value of γ_c, the work of compression for air may be expressed as

$$W_c = \frac{1}{\eta_{s,c}} \frac{\gamma_c}{\gamma_c - 1} R_c T_1 \left[\left(\frac{P_4}{P_1} \right)^{\frac{\gamma_c - 1}{\gamma_c}} - 1 \right] \tag{3}$$

Unfortunately, the variation of γ_c is sometimes large and this expression gives inaccurate results for a single value of γ_c over a wide range of pressure ratios and initial temperatures. In order to obtain a simple and accurate means of computing the compressor work, the data for dry air obtained from Keenan and Kaye (reference 11) were plotted in figure 4 with the ratio of change in enthalpy for an isentropic compression, or ideal compressor work, to the compressor-inlet total temperature $\Delta H_{ad,c}/T$ as a function of the pressure ratio for compressor-inlet total temperatures of 400°, 500°, and 600° R. For pressure ratios less than 10, $\Delta H_{ad,c}/T_1$ is virtually unaffected if the initial temperature varies from 400° to 600° R. For a pressure ratio as high as 100, figure 4 indicates within ±1.5 percent the variation of $\Delta H_{ad,c}/T_1$ with P_4/P_1 for initial temperatures of 400° to 600° R.

The work of compression in foot-pounds per pound of air passing through the compressor is expressed as

$$W_c = \frac{T_1}{\eta_{s,c}} \left(\frac{\Delta H_{ad,c}}{T_1} \right) \tag{4}$$

where $\Delta H_{ad,c}/T_1$ is determined from figure 4. An alternate method for determining the work of compression is given in reference 12.

The following method may be used to determine the temperature after compression: The change in enthalpy during compression is expressed as

$$H_4 - H_1 = \frac{T_1}{\eta_{ad,c}}\left(\frac{\Delta H_{ad,c}}{T_1}\right) \tag{5}$$

where $\Delta H_{ad,c}/T_1$ is determined from figure 4. The variation of the enthalpy of air with temperature (reference 11) is shown in figure 5 for a base temperature of 400° R. With this chart, the initial enthalpy may be determined when the initial temperature is known. The enthalpy of the air leaving the compressor may be determined by adding the initial enthalpy to the change in enthalpy in equation (5); the temperature of the air leaving the compressor T_4 may then be found from this final enthalpy and figure 5.

Compression with intercooling. - The addition of an intercooling heat exchanger between two stages of compression reduces the work of compression because the temperature of the air entering the second stage is lowered by cooling the air between the stages. For a given turbine-inlet temperature T_6, the net work output of the cycle is increased and the fuel input must be increased.

For a given over-all pressure ratio, the work of compression varies with the pressure ratio across the first stage of compression. By taking the partial derivative of an expression for the over-all compressor work, the work of compression is found to be a minimum if

$$\frac{P_2}{P_1} = \sqrt{\left(\frac{\eta_{c'} + e_i - 1}{\eta_{c''} + e_i - 1}\right)^{\frac{\gamma_c}{\gamma_c - 1}} \frac{P_2}{P_3}\frac{P_4}{P_1}} \tag{6}$$

where the effectiveness of the intercooling heat exchanger e_i is defined as

$$e_i = \frac{T_2 - T_3}{T_2 - T_1} \tag{7}$$

The relation expressed in equation (6) is derived in appendix A.

Because for a given turbine-inlet temperature T_6, the addition of intercooling increases both the net work output of the cycle and the fuel that must be added, the manner in which the specific fuel consumption is affected by intercooling is not immediately apparent. The pressure ratio given in equation (6) was selected to minimize the compressor work without regard for the efficiency of the cycle. By taking the partial derivative of an expression for the cycle efficiency, the cycle efficiency is found to be a maximum if

$$\frac{P_2}{P_1} = \left(\frac{b + \sqrt{b^2 - 4ac}}{2a}\right)^{\frac{\gamma}{\gamma-1}} \tag{8}$$

where

$$a = \frac{T_6}{T_1}\frac{1}{\eta_{c'}}\left[e_i - \left(1-e_i\right)\left(1-\eta_t\right)\left(\frac{1}{\eta_{c''}} - 1\right)\right] + \frac{1}{\eta_{c'}}\left(\frac{1}{\eta_{c''}} - 1\right)$$

$$- \left(\frac{P_4}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \frac{e_i(1-e_i)}{\eta_{c'}^{\,2}\,\eta_{c''}} - \frac{T_6}{T_1}\frac{\eta_t}{\eta_{c'}}\left(\frac{P_1}{P_4}\right)^{\frac{\gamma-1}{\gamma}}\left(1-e_i\right)\left(\frac{1}{\eta_{c''}} - 1\right)$$

$$b = \left(\frac{P_4}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \frac{2\,e_i}{\eta_{c'}\eta_{c''}}\left(1 - \frac{1-e_i}{\eta_{c'}}\right)$$

$$c = \left(\frac{P_4}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \frac{1}{\eta_{c''}}\left(1 - \frac{1-e_i}{\eta_{c'}}\right)\left(1 + \frac{e_i}{\eta_{c'}}\right) - \frac{T_6}{T_1}\left(\frac{P_4}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \frac{1-\eta_t}{\eta_{c''}}\left(1 - \frac{1-e_i}{\eta_{c'}}\right)$$

$$- \frac{T_6}{T_1}\frac{\eta_t}{\eta_{c''}}\left(1 - \frac{1-e_i}{\eta_{c'}}\right)$$

This relation is derived in appendix B. In the derivation of equations (6) and (8), several simplifying assumptions were made with the result that the pressure ratios indicated by these equations are in error, but the values are approximately correct.

If the single-stage compression of the basic cycle is replaced by a two-stage compression with intercooling and the product of the pressure ratios of the two-stage compression equals the pressure ratio of the single-stage compression, the efficiency of each stage of the two-stage compression must be different from the efficiency of the single-stage compression for the power-plant performance calculated in order for the intercooling cycle to be comparable with that calculated for the basic cycle. If two stages with equal adiabatic efficiencies are so arranged that the air flows successively through the two stages without any intercooling, the adiabatic efficiency of the over-all compression is lower than the adiabatic efficiency of each of the stages (reference 13). In order to give comparable results, the efficiencies of two stages must be reassigned so the change in enthalpy for a two-stage compression with intercooling of zero effectiveness will be equal to the change in enthalpy for a corresponding single-stage compression. As the effectiveness of the intercooling heat exchanger is varied from zero, the efficiencies of each of the two stages should be held constant.

The over-all pressure ratio across two compressor stages with intercooling is expressed as

$$\frac{P_4}{P_1} = \frac{P_2}{P_1}\frac{P_3}{P_2}\frac{P_4}{P_3} \tag{9}$$

Combustion. - The air entering the combustion chamber is burned with fuel to provide a high gas temperature at the turbine inlet. Because the power plant operates with a large work output and a low specific fuel consumption when the turbine-inlet temperature is high, the power plant is normally designed to operate at the highest temperature that the materials will permit. For convenience in this analysis the combustion efficiency is defined as

$$\eta_b = \frac{f_{id}}{f} \tag{10}$$

The combustion chart (fig. 6) is presented for the determination of the ideal fuel-air ratio f_{id} when the initial and final temperatures of combustion are given. This figure was prepared from the data of reference 14. The fuel was assumed to be saturated liquid octane at 60° F and the air was assumed to be composed of 79-percent nitrogen and 21-percent oxygen by volume. The maximum temperature considered is low enough to neglect the effects of dissociation. The chemical energy of octane is 2,201,618 Btu per mole and the latent heat of vaporization is 17,730 Btu per mole (reference 15). If an alternate fuel is used with a hydrogen-carbon ratio between 0.15 and 0.25, an equivalent ideal fuel-air ratio may be determined from figure 6. The ideal fuel-air ratio for the alternate fuel may be determined by multiplying the equivalent ideal fuel-air ratio by the lower heating value of octane and dividing the resulting product by the lower heating value of the alternate fuel. Reference 16 supports the accuracy of this procedure. If the hydrogen-carbon ratio is between 0.15 and 0.25, this method is correct within $1\frac{1}{2}$ percent.

When combustion occurs in a reheating combustion chamber, the gas entering the reheating combustion chamber is the product of a previous combustion. The temperature and the fuel-air ratio at the inlet of the reheating combustion chamber are known. These values can be used with figure 6 to obtain a fictitious initial temperature that would exist in a combustion chamber whose exit conditions are the same as the inlet conditions of the reheating combustion chamber. If the temperature at the exit from this theoretical combustion chamber is increased, figure 6 gives the new value of f_{id} required to reach the new exit temperature. The difference between these two fuel-air ratios $f_{id,8} - f_6$ is the amount of fuel ideally required to change $(1+f_6)$ pounds of gas from the state at the inlet of the reheating combustion chamber to $(1+f_{id,8})$ pounds of gas at the required temperature at the exit from the reheating combustion chamber. The fuel-air ratio at this point is expressed as

$$f_8 = f_6 + \frac{f_{id,8} - f_6}{\eta_b} \tag{11}$$

For some conditions at the entrance of the reheating combustion chamber, the fictitious initial temperature will be less than 500° R and will therefore be beyond the range of figure 6. The data of figure 6 may be extrapolated by assuming that the curve of constant initial temperature less than 500° R lies a constant vertical distance above the curve for an initial temperature of 500° R. The conditions

at the entrance of the reheating combustion chamber determine one point on this line of constant initial temperature and the assumption made for extrapolation determines the rest of the line.

Expansion in turbine. - After combustion, the gases are expanded in a turbine. If γ_t does not vary during the expansion, the turbine work in foot-pounds per pound of gas passing through the turbine may be expressed as

$$W_t = \eta_{s,t} \frac{\gamma_t}{\gamma_t - 1} R_t T_6 \left[1 - \left(\frac{P_9}{P_6} \right)^{\frac{\gamma_t - 1}{\gamma_t}} \right] \tag{12}$$

If γ_t varies during the expansion, equation (12) may be used to determine the turbine work as long as an appropriate value of γ_t is selected. Pinkel and Turner (reference 17) give data that are useful in choosing the value of γ_t for pressure ratios as high as 10. Data on the variation of γ with temperature and fuel-air ratio obtained from references 11 and 18 are plotted in figure 7.

In this analysis a chart was constructed to permit the ideal turbine work to be read directly for pressure ratios up to 32. The chart was constructed by using equation (12) and figure 7; the value of γ_t was selected from figure 7 at the mean temperature between the inlet total temperature and the ideal exit total temperature for the expansion. An estimate of the ideal exit total temperature was obtained by choosing a value of γ_t at the temperature T_6 and the corresponding fuel-air ratio. The average of the estimated ideal exit total temperature and the inlet total temperature is the first approximation of the true mean temperature; a second approximation is unnecessary. The value of R_t was determined from the relation

$$R_t = \frac{53.30 + 60.9f}{1 + f} \tag{13}$$

This relation was obtained from equation (21) of reference 17 and is valid for the products of combustion of octane and dry air when the fuel-air ratio is not greater than stoichiometric and combustion is complete.

The ideal turbine work $W_t/\eta_{s,t}$ was determined from equation (12) for a range of turbine-inlet temperature from 800° to 3000° R, for pressure ratios from 1.5 to 32, and for fuel-air ratios of 0.010, 0.040, and 0.067. The results of this evaluation are plotted in figure 8. These results were checked with the tables of the gas properties given in reference 17. The maximum disagreement between the two results was less than 0.5 percent of the turbine work.

The temperature of the gas leaving the turbine may be determined from the enthalpy drop across the turbine. The change in enthalpy during expansion is expressed

$$H_6 - H_9 = \eta_{ad,t}\left(\frac{W_t}{\eta_{s,t}}\right) \tag{14}$$

The value of $W_t/\eta_{s,t}$ may be determined from figure 8. The enthalpy of the gas is plotted against the temperature in figure 9 from data given in reference 18. The base temperature for this chart is 540° R, which is 140° R greater than that of figure 5. The enthalpy H_6 may be determined from figure 9 at the temperature T_6; H_9 may then be determined from equation (14) and T_9 may be found from figure 9.

Reheating during expansion. - The addition of reheating to the basic cycle results in the production of an increased quantity of work for each pound of air that passes through the power plant. By taking the partial derivative of an expression for the work of a two-stage expansion with reheating, the work is found to be a maximum if

$$\frac{P_7}{P_6} = \sqrt{\left(\frac{\eta_{t''}}{\eta_{t'}}\frac{T_8}{T_6}\right)^{\frac{\gamma_t}{\gamma_t-1}}\frac{P_9}{P_6}\frac{P_7}{P_8}} \tag{15}$$

This expression is derived in appendix C.

Equation (15) was derived to maximize the work output of the power plant without regard for the specific fuel consumption. By taking the partial derivative of an expression for the cycle efficiency, the cycle efficiency is found to be a maximum if

$$\frac{P_7}{P_6} = \left(\frac{y - \sqrt{y^2 - 4xz}}{2x}\right)^{\frac{\gamma}{\gamma - 1}} \tag{16}$$

where

$$x = \eta_{t'} \frac{T_6}{T_1}\left(\frac{T_1}{T_8} + \eta_{t''} - 1\right)$$

$$y = 2\,\eta_{t'}\,\eta_{t''}\,\frac{T_6}{T_1}\left(\frac{P_9}{P_6}\right)^{\frac{\gamma-1}{\gamma}}$$

$$z = \eta_{t''}\left(\frac{P_9}{P_6}\right)^{\frac{\gamma-1}{\gamma}}\left\{\eta_{t'}\,\frac{T_6}{T_1} + \frac{T_8}{T_1} - 1 - \frac{1}{\eta_c}\left[\left(\frac{P_6}{P_9}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]\right\}$$

Equation (16) is derived in appendix D. In the derivation of equations (15) and (16), several simplifying assumptions were made with the result that the pressure ratios indicated by these equations are in error, but the values are approximately correct.

If the single-stage expansion of the basic cycle is replaced by a two-stage expansion with reheating and the product of the pressure ratios of the two-stage expansion equals the pressure ratio of the single-stage expansion, the efficiency of each stage of the two-stage expansion must be different from the efficiency of the single-stage expansion in order that the power-plant performance calculated for the reheating cycle may be comparable with that calculated for the basic cycle. If two expansion stages with equal adiabatic efficiencies are so arranged that the air flows successively through the two stages without any reheating between the stages, the adiabatic efficiency of the over-all expansion is higher than the adiabatic efficiency of each of the stages. In order to give comparable results, the efficiencies of the two stages must be reassigned so the change in enthalpy for a two-stage expansion without reheating is equal to the change in enthalpy for a corresponding single-stage expansion. This adjustment in efficiency is very similar to the change in efficiency for a two-stage compression.

The over-all pressure ratio across two turbines with reheating is expressed as

$$\frac{P_9}{P_6} = \frac{P_7}{P_6}\frac{P_8}{P_7}\frac{P_9}{P_8} \tag{17}$$

Regeneration. - When the cycle makes use of regeneration, consideration must be given to the effects of the regenerating heat exchanger on the temperatures of the gas entering the exhaust nozzle and the air entering the combustion chamber. The effectiveness of the heat exchanger is conveniently defined as

$$e_r = \frac{T_5 - T_4}{T_9 - T_4} \tag{18}$$

The temperature T_4 is determinable from the compressor calculations. The value of T_9 may be estimated from the homologous basic cycle, and this approximate value of T_9 may be substituted in equation (18) and T_5 may be computed for the assumed value of e_r. By means of the calculation methods previously outlined, a second approximation of T_9 may be determined; a third approximation is unnecessary. With this correct value of T_9, a new value of T_5 may be found from equation (18), and f_6 may in turn be determined from figure 6.

The temperature of the exhaust gases leaving the regenerating heat exchanger may be determined by applying the principle of the conservation of energy. When there are no heat losses from the regenerating heat exchanger,

$$H_9 - H_{10} = \frac{H_5 - H_4}{1 + f} \tag{19}$$

The quantity H_4 may be determined from equation (5). The value of H_5 may be found from T_5 and figure 5, and the magnitude of H_9 may be determined from equation (14). The value of T_{10} may then be determined from equation (19) and figure 9. The two plots of enthalpy as a function of temperature (figs. 5 and 9) were not combined because they have different base temperatures and minor differences in the composition of dry air.

Expansion in exhaust nozzle. - The gases leaving the power plant are expanded in an exhaust nozzle and given a rearward velocity to aid in propelling the airplane. The exhaust jet velocity is given as

$$V_j = C\sqrt{2g\frac{\gamma_j}{\gamma_j-1}R_tT_{10}\left[1-\left(\frac{p_{11}}{P_{10}}\right)^{\frac{\gamma_j-1}{\gamma_j}}\right]} \tag{20}$$

Equation (20) was evaluated for various values of γ_j and p_{11}/P_{10}, and the results are plotted in figure 10 with $\frac{V_j}{C\sqrt{T_{10}}}$ as a function of γ_j and P_{10}/p_{11}. The value of P_{10}/p_{11} may be determined by by considering all the pressure ratios in the power plant:

$$\frac{P_{10}}{p_{11}} = \frac{P_1}{p_0}\frac{P_4}{P_1}\frac{P_5}{P_4}\frac{P_6}{P_5}\frac{P_9}{P_6}\frac{P_{10}}{P_9} \tag{21}$$

For a cycle without regeneration

$$\frac{P_5}{P_4} = \frac{P_{10}}{P_9} = 1 \tag{22}$$

The work of propulsion from the jet in foot-pounds per pound of air entering the compressor is expressed as

$$W_j = \frac{V_0}{g}\left[(1+f)\,V_j - V_0\right] \tag{23}$$

Over-all power-plant performance. - The net thrust work of the power plant in foot-pounds per pound of air entering the compressor is expressed as

$$W_n = \left[(1+f)\,W_t - W_c\right]\eta_p + W_j \tag{24}$$

The specific fuel consumption in pounds of fuel per net thrust horsepower-hour is expressed as

$$F = 1.98 \times 10^6 \frac{f}{W_n} \tag{25}$$

Sample calculations are given in appendix E for each cycle.

DISCUSSION OF CYCLE ANALYSES

Application of Analysis

The foregoing method of analysis was used to analyze the performance of gas-turbine power plants at their design points. Four cycles of operation were considered: (1) the basic cycle, (2) the intercooling cycle, (3) the reheating cycle, and (4) the regenerating cycle. A flight speed of 400 miles per hour was selected. One set of calculations was run with the altitude varying from sea level to 30,000 feet in order to show the effect of altitude; another set of calculations was run with the turbine-inlet temperature varying from 2000° to 2500° R to show the effect of peak cycle temperature. The other calculations were made for an altitude of 30,000 feet and a turbine-inlet temperature of 2000° R corresponding to a ratio of turbine-inlet temperature to compressor-inlet temperature of 4.5. The compressors were assumed to have over-all adiabatic and shaft efficiencies of 0.85 and 0.84, respectively, and the turbines were assumed to have over-all adiabatic and shaft efficiencies of 0.90 and 0.89, respectively. The assumed velocity coefficient of the exhaust nozzle was 0.97. The combustion efficiency was assumed to be 0.90, and a propeller efficiency of 1.00 was assigned to obtain a conservative estimate of the equivalent shaft work. A list of the cycles analyzed and a summary of the operating conditions are presented in table I.

It was assumed that the combustion takes place with no loss in total pressure. This assumption results in values for the specific power output and the specific fuel consumption that are from 2 to 5 percent higher and lower than the specific power output and specific fuel consumption, respectively, of a power plant with a total-pressure loss of 5 percent during combustion. The flow through the intercooling and regenerating heat exchangers was assumed to take place with no loss in total pressure or heat. The drag produced by a change in momentum of the cooling air passing through the intercooling heat exchanger was not considered.

The ratio of the total pressures across the compressor was assigned equal to the ratio of the total pressures across the turbines. This assumption, in combination with the assumption of no pressure losses in the combustion chambers, intercooling heat exchanger, and regenerating heat exchanger, fixes the pressure ratio across the exhaust nozzle at the same value as the pressure ratio across the inlet diffuser. Preliminary calculations showed that this condition gives a specific fuel consumption 3 to 5 percent higher than the consumption obtained with the optimum exhaust-jet velocity. Reference 10 presents data that support this statement. With the same pressure ratio across the exhaust nozzle and inlet diffuser, the jet work is generally a small part of the net work. For over-all pressure ratios greater than 4.0, the jet work is less than 12 percent of the net work.

It was necessary to reassign the compressor efficiencies when intercooling was used and to reassign the turbine efficiencies when reheating was used. The reassigned efficiencies were selected by trial and error. In assigning the new efficiencies, the expansion of compression was divided into two stages without reheating or intercooling between the stages. The efficiencies of the first stage were kept equal to the efficiencies of the second stage, and the values of these efficiencies were varied until a set of values was found for which the work of the two-stage process was equal to the work of the one-stage process. When reheating or intercooling was added between the stages, the efficiencies were not changed from the efficiencies that were found by trial and error.

The curves in this report do not represent the performance of an actual power plant under variable operating conditions. In an actual power plant, pressure losses will occur in the inlet diffuser, the combustion chamber, the exhaust nozzle, the intercooling heat exchanger, and the regenerating heat exchanger. These pressure losses, in addition to reducing the output of the power plant and raising the specific fuel consumption, change the pressure ratios at which the maximum specific power output and the minimum specific fuel consumption are obtained. In all cases the performance curves show the best possible results for the design operating conditions.

Basic Cycle

The high specific power output and the low specific fuel consumption that result from operating a gas-turbine power plant at high altitudes are clearly shown in figure 11. The increase in the specific power output as the altitude is increased with a corresponding

decrease in ambient atmospheric temperature partly compensates for the low air flow through the engine at high altitude. At an altitude of 30,000 feet, a gas-turbine power plant operating on the basic cycle can have a minimum specific fuel consumption of 0.37 pound of fuel per horsepower-hour (fig. 11(b)).

The familiar gains in power and economy to be expected for operation at high maximum cycle temperatures are shown in figure 12. Appreciable gains in fuel economy with high turbine-inlet temperatures are obtained only at very high pressure ratios (fig. 12(b)). The chief advantage in high turbine-inlet temperatures is the large specific power output. These improvements in performance indicate the value of developing the means of operating gas-turbine power plants at high turbine-inlet temperatures.

Intercooling Cycle

The intercooling cycle was first evaluated with equation (6) for the pressure ratio of the first stage of compression to give the maximum specific power output for the cycle. Because no pressure loss was assumed through the intercooling heat exchanger and equal efficiencies were assigned, the maximum specific power output is obtained when the pressure ratio across the first stage is the square root of the over-all pressure ratio. Figure 13 shows that the addition of an intercooling heat exchanger with an effectiveness of 0.50 will increase the peak specific power output 12 percent over the peak specific power output of the basic cycle. The change in the specific fuel consumption produced by the addition of an intercooling heat exchanger is negligible.

For intercooling at the point of lowest specific fuel consumption, the pressure ratio of the first stage was evaluated from equation (8), which gave pressure ratios for the first stage less than the square root of the over-all pressure ratio. A small improvement in specific fuel consumption over that of the basic cycle is shown in figure 14(b). Because the difference between the specific fuel consumptions with intercooling for the greatest specific power output and with intercooling for the lowest specific fuel consumption is small, the specific power output may be maximized with little regard for the specific fuel consumption.

Because the effects of power-plant weight and size have not been considered in the analysis, it cannot be definitely concluded whether a power plant with intercooling will perform better than one without intercooling. For a temperature ratio T_6/T_1 of 4.5, a compressor adiabatic efficiency of 0.85, and a turbine adiabatic

efficiency of 0.90, these calculations indicate that, even with an intercooling heat exchanger with no pressure losses, the possible reduction in specific fuel consumption is very small. If the power plant is to produce a given net power, the addition of an intercooling heat exchanger will decrease the weight of the compressor, the turbine, and the combustion chamber because the increase in specific power output will allow a reduction in the air flow through each of these components, and it will add to the power plant the weight of the intercooling heat exchanger with its attendant ducting. The intercooling heat exchanger selected should permit the airplane to have the smallest weight of power plant and fuel. In order to be a practical addition to the basic cycle, the intercooling heat exchanger and its ducts must have a weight less than the reduction in weight of the remainder of the engine. Intercooling in combination with regeneration may be more advantageous.

Reheating Cycle

In order to obtain the maximum specific power output for the cycle with reheating, the pressure ratio of the first stage of expansion is determined from equation (15). With equal two-stage efficiencies assigned and no pressure loss assumed during combustion, the maximum output is obtained when the pressure ratio across the first stage is equal to the square root of the over-all pressure ratio. The peak specific power output of the reheating cycle with reheating at the point for maximum specific power output is 33 percent greater than the peak specific power output of the basic cycle (fig. 15); this added output is, however, obtained at the expense of some loss in fuel economy.

When equation (16) was used to determine the pressure ratio of the first stage of expansion, the performance equations were evaluated with reheating at the point for best fuel economy. The performance curves in figure 15 show a 15 percent gain in the maximum specific power output over the basic cycle, and for pressure ratios greater than 17, a slight reduction in the specific fuel consumption. At pressure ratios less than 17, the specific fuel consumption is slightly higher with reheating at the point for the best fuel economy than for the basic cycle. This incongruity is the result of errors introduced by the simplifying assumptions made in the derivation of equation (16).

This analysis indicates that the principal effect of the addition of reheating to the basic cycle is to increase the output of the cycle. A supplementary analysis has indicated that if the pressure ratio of the first stage of expansion is carefully chosen, the

specific power output may be 25 percent greater for the reheating cycle than for the basic cycle and the specific fuel consumption will not be changed.

When reheating is added to the basic cycle and the air flow through the power plant is not changed, the weight of the power plant will increase because a second combustion chamber must be added and the turbine following the reheating combustion chamber must be enlarged to accommodate the increased volume flow. The power-plant components ahead of the reheating combustion chamber are unchanged. In order to be a desirable addition to the basic cycle, reheating must produce an increase in the specific power output that is proportionately greater than the increase in engine weight.

Regenerating Cycle

The results of the regenerating-cycle analysis are shown in figure 16. The curves were not extended beyond a pressure ratio of about 17 because at higher pressure ratios the exhaust temperature of the turbine is lower than the discharge temperature of the compressor and a reversal of heat flow occurs in the regenerating heat exchanger. For a pressure ratio greater than 17, the addition of regeneration to the basic cycle results in an increase in the specific fuel consumption.

Large reductions in the specific fuel consumption are obtained at low pressure ratios, but the specific power output at a given pressure ratio is only slightly affected by the addition of regeneration. The reduction in the output is principally due to the reduced temperature at the entrance of the exhaust nozzle, which in turn reduces the thrust obtained from the exhaust jet. The low fuel-air ratio accompanying regeneration slightly reduces the mass flow through the turbine and the exhaust nozzle and increases the value of γ_t, both of which reduce the output of the turbine and the jet. For a temperature ratio T_6/T_1 of 4.5, a compressor adiabatic efficiency of 0.85, and a turbine adiabatic efficiency of 0.90, the regenerating heat exchanger must have an effectiveness greater than 0.50 to obtain a minimum specific fuel consumption for the regenerating cycle lower than the minimum specific fuel consumption for the basic cycle.

The design pressure ratio of a power plant normally is between the pressure ratio for maximum specific power output and the pressure ratio that produces the minimum specific fuel consumption; the

design pressure ratio of a power plant operating on the regenerating cycle will be low compared with the other power plants. The low design pressure ratio results in a low specific weight for the compressor and the turbine, but the power plant will also be burdened with the weight of a regenerating heat exchanger and its ducting. The regenerating heat exchanger must pass a large volume of gas at a high temperature, which the light metals cannot withstand. The regenerating heat exchanger will therefore be both larger and heavier than a comparable intercooling heat exchanger.

Neglecting the pressure losses in the regenerating heat exchanger, as was done in this analysis, will affect both the compressed air entering the combustion chamber and the hot exhaust gas entering the exhaust nozzle. In order to obtain high heat-transfer coefficients and thereby limit the weight of the regenerating heat exchanger, the percentage loss in total pressure in the regenerating heat exchanger will be made sufficiently large to have a significant effect on the power-plant performance. For this reason, the performance of a power plant operating on the regenerating cycle will depart by a greater amount from the performance predicted by this analysis than will the performance of the three other cycles. Regeneration combined with reheating or intercooling may appear more promising.

CONCLUSIONS

A method was developed for analyzing gas-turbine cycles to aid in predicting the performance of propeller-driving power plants and the state of the gas at various stations within the power plant. The method takes into account the effects of ram, altitude, variation of specific heats, increase in weight flow due to the addition of fuel, and thrust from the exhaust jet.

As an example of the application of the method, the performance of propeller-driving power plants was determined for a flight speed of 400 miles per hour, a ratio of turbine-inlet temperature to compressor-inlet temperature of 4.5, a compressor adiabatic efficiency of 0.85, and a turbine adiabatic efficiency of 0.90; the following conclusions were derived:

1. The basic gas-turbine power plant can cruise with a specific fuel consumption as low as 0.37 pound of fuel per horsepower-hour.

2. The effect of the addition of intercooling or reheating to the basic cycle is primarily to increase the specific power output with only small changes in the specific fuel consumption. The reheating cycle yields a greater specific power output than the intercooling cycle.

3. In order to obtain a minimum specific fuel consumption for the regenerating cycle lower than that for the basic cycle, the regenerating heat exchanger must have an effectiveness greater than 0.50.

4. When the turbine-inlet temperature is increased, the specific power output is increased and the specific fuel consumption is decreased; if the ambient atmospheric temperature is increased by decreasing the altitude, the specific power output is reduced and the specific fuel consumption is increased.

Flight Propulsion Research Laboratory,
National Advisory Committee for Aeronautics,
Cleveland, Ohio, July 24, 1947.

APPENDIX A

PRESSURE RATIOS OF COMPONENT COMPRESSORS FOR MINIMUM WORK OF COMPRESSION WHEN INTERCOOLING IS USED

In order to determine the pressure ratio across each of the compressors the following assumptions were made:

1. The adiabatic and the shaft efficiencies are equal for each compressor.

2. There is no variation in the value of γ_c.

$$W_c = \frac{\gamma_c}{\gamma_c - 1} R_c \frac{T_1}{\eta_{c'}} \left[\left(\frac{P_2}{P_1} \right)^{\frac{\gamma_c - 1}{\gamma_c}} - 1 \right] + \frac{\gamma_c}{\gamma_c - 1} R_c \frac{T_3}{\eta_{c''}} \left[\left(\frac{P_4}{P_3} \right)^{\frac{\gamma_c - 1}{\gamma_c}} - 1 \right] \tag{26}$$

$$\eta_{c'} = \frac{\left(\frac{P_2}{P_1} \right)^{\frac{\gamma_c - 1}{\gamma_c}} - 1}{\frac{T_2}{T_1} - 1} \tag{27}$$

$$e_i = \frac{T_2 - T_3}{T_2 - T_1} \tag{7}$$

By combining equations (7) and (27),

$$\frac{T_3}{T_1} = \frac{\left(\frac{P_2}{P_1} \right)^{\frac{\gamma_c - 1}{\gamma_c}} \left(1 - e_i \right)}{\eta_{c'}} + \frac{\eta_{c'} + e_i - 1}{\eta_{c'}} \tag{28}$$

By combining equations (26) and (28),

$$W_c = \frac{\gamma_c}{\gamma_c - 1} R_c \frac{T_1}{\eta_{c'}} \left\{ \left[\left(\frac{P_2}{P_1}\right)^{\frac{\gamma_c - 1}{\gamma_c}} - 1 \right] \frac{\eta_{c''} + e_i - 1}{\eta_{c''}} \right.$$

$$\left. + \frac{\eta_{c'} + e_i - 1}{\eta_{c''}} \left(\frac{P_4}{P_1} \frac{P_2}{P_3} \frac{P_1}{P_2}\right)^{\frac{\gamma_c - 1}{\gamma_c}} + \frac{1 - e_i}{\eta_{c''}} \left(\frac{P_4}{P_1} \frac{P_2}{P_3}\right)^{\frac{\gamma_c - 1}{\gamma_c}} - \frac{\eta_{c'}}{\eta_{c''}} \right\}$$

All of the variables except W_c are assumed to be independent of P_2/P_1. When W_c is a minimum, $\partial W_c/\partial(P_2/P_1) = 0$, and

$$\frac{P_2}{P_1} = \sqrt{\left(\frac{\eta_{c'} + e_i - 1}{\eta_{c''} + e_i - 1}\right)^{\frac{\gamma_c}{\gamma_c - 1}} \frac{P_2}{P_3} \frac{P_4}{P_1}} \tag{6}$$

APPENDIX B

PRESSURE RATIOS OF COMPONENT COMPRESSORS FOR MAXIMUM CYCLE EFFICIENCY WHEN INTERCOOLING IS USED

In order to determine the pressure ratio across each of the compressors the following assumptions were made:

1. The value of γ is constant for the cycle.

2. The weight of fuel passing through the turbine is neglected.

3. The adiabatic efficiency is equal to the shaft efficiency for the turbine and for each of the compressors.

4. There is no pressure loss through the combustion chamber.

5. The jet work is zero.

$$W_t = \frac{\gamma}{\gamma-1} R\, T_6 \left(1 - \frac{T_9}{T_6}\right) \tag{29}$$

$$W_c = \frac{\gamma}{\gamma-1} R\, T_1 \left(\frac{T_2}{T_1} - 1\right) + \frac{\gamma}{\gamma-1} R\, T_3 \left(\frac{T_4}{T_3} - 1\right) \tag{30}$$

$$\eta_o = \frac{\frac{T_6}{T_1}\left(1 - \frac{T_9}{T_6}\right) - \frac{T_3}{T_1}\left(\frac{T_4}{T_3} - 1\right) - \left(\frac{T_2}{T_1} - 1\right)}{\frac{T_6}{T_1} - \left(\frac{T_4}{T_3}\frac{T_3}{T_1}\right)} \tag{31}$$

$$1 - \frac{T_9}{T_6} = \eta_t \left[1 - \left(\frac{P_1}{P_4}\right)^{\frac{\gamma-1}{\gamma}}\right] \tag{32}$$

$$\frac{T_2}{T_1} - 1 = \frac{1}{\eta_{c'}}\left[\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right] \tag{33}$$

$$\frac{T_4}{T_3} - 1 = \frac{1}{\eta_{c''}}\left[\left(\frac{P_4}{P_3}\right)^{\frac{\gamma-1}{\gamma}} - 1\right] \tag{34}$$

$$e_i = \frac{T_2 - T_3}{T_2 - T_1} \tag{7}$$

By combining equations (29), (30), (31), (32), (33), (34), and (7),

$$\eta_o = \frac{\frac{T_6}{T_1}\eta_t\left[1-\left(\frac{P_1}{P_5}\right)^{\frac{\gamma-1}{\gamma}}\right] - \left\{\frac{1-e_i}{\eta_{c'}}\left[\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}-1\right]+1\right\}\frac{1}{\eta_{c''}}\left[\left(\frac{P_4}{P_1}\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}}-1\right] - \frac{1}{\eta_{c'}}\left[\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}-1\right]}{\frac{T_6}{T_1} - \left\{1+\frac{1}{\eta_{c''}}\left[\left(\frac{P_4}{P_1}\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}}-1\right]\right\}\left\{\frac{1-e_i}{\eta_{c'}}\left[\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}-1\right]+1\right\}} \tag{35}$$

All of the variables except η_o are assumed to be independent of P_2/P_1. When η_o is a maximum, $\partial\eta_o/\partial(P_2/P_1) = 0$, and

$$\frac{P_2}{P_1} = \left(\frac{b + \sqrt{b^2 - 4ac}}{2a}\right)^{\frac{\gamma}{\gamma-1}} \tag{8}$$

where

$$a = \frac{T_6}{T_1}\frac{1}{\eta_{c'}}\left[e_i - \left(1-e_i\right)\left(1-\eta_t\right)\left(\frac{1}{\eta_{c''}} - 1\right)\right] + \frac{1}{\eta_{c'}}\left(\frac{1}{\eta_{c''}} - 1\right)$$

$$- \left(\frac{P_4}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \frac{e_i(1-e_i)}{\eta_{c'}{}^2\,\eta_{c''}} - \frac{T_6}{T_1}\frac{\eta_t}{\eta_{c'}}\left(\frac{P_1}{P_4}\right)^{\frac{\gamma-1}{\gamma}}\left(1-e_i\right)\left(\frac{1}{\eta_{c''}} - 1\right)$$

$$b = \left(\frac{P_4}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \frac{2\,e_i}{\eta_{c'}\eta_{c''}}\left(1 - \frac{1-e_i}{\eta_{c'}}\right)$$

$$c = \left(\frac{P_4}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \frac{1}{\eta_{c''}}\left(1 - \frac{1-e_i}{\eta_{c'}}\right)\left(1 + \frac{e_i}{\eta_{c'}}\right) - \frac{T_6}{T_1}\left(\frac{P_4}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \frac{1-\eta_t}{\eta_{c''}}\left(1 - \frac{1-e_i}{\eta_{c'}}\right)$$

$$- \frac{T_6}{T_1}\frac{\eta_t}{\eta_{c''}}\left(1 - \frac{1-e_i}{\eta_{c'}}\right)$$

APPENDIX C

PRESSURE RATIOS OF COMPONENT TURBINES FOR MAXIMUM TURBINE WORK WHEN REHEATING IS USED

In order to determine the pressure ratio across each of the turbines, the following assumptions were made:

1. The value of γ_t is constant.

2. The adiabatic efficiency is equal to the shaft efficiency for each of the turbines.

3. The weight of fuel passing through the turbines may be neglected.

$$W_t = \eta_{t'} \frac{\gamma_t}{\gamma_t - 1} R_t T_6 \left[1 - \left(\frac{P_7}{P_6}\right)^{\frac{\gamma_t - 1}{\gamma_t}}\right] + \eta_{t''} \frac{\gamma_t}{\gamma_t - 1} R_t T_8 \left[1 - \left(\frac{P_9}{P_8}\right)^{\frac{\gamma_t - 1}{\gamma_t}}\right] \tag{36}$$

$$W_t = \eta_{t'} \frac{\gamma_t}{\gamma_t - 1} R_t T_6 \left\{1 - \left(\frac{P_7}{P_6}\right)^{\frac{\gamma_t - 1}{\gamma_t}} + \frac{\eta_{t''}}{\eta_{t'}} \frac{T_8}{T_6} \left[1 - \left(\frac{P_9}{P_6} \frac{P_6}{P_7} \frac{P_7}{P_8}\right)^{\frac{\gamma_t - 1}{\gamma_t}}\right]\right\}$$

All of the variables except W_t are assumed to be independent of P_7/P_6. When W_t is a maximum, $\partial W_t/\partial(P_7/P_6) = 0$, and

$$\frac{P_7}{P_6} = \sqrt{\left(\frac{\eta_{t''}}{\eta_{t'}} \frac{T_8}{T_6}\right)^{\frac{\gamma_t}{\gamma_t - 1}} \frac{P_9}{P_6} \frac{P_7}{P_8}} \tag{15}$$

APPENDIX D

PRESSURE RATIOS OF COMPONENT TURBINES FOR MAXIMUM CYCLE EFFICIENCY WHEN REHEATING IS USED

In order to determine the pressure ratio across each of the turbines, the following assumptions were made:

1. The value of γ is constant.

2. The weight of fuel passing through the turbines is neglected.

3. The adiabatic efficiency is equal to the shaft efficiency for the compressor and for each of the turbines.

4. There is no pressure loss in the combustion chambers.

5. The jet work is zero.

$$W_c = \frac{1}{\eta_c} \frac{\gamma}{\gamma - 1} RT_1 \left[\left(\frac{P_4}{P_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right] \tag{37}$$

$$W_t = \eta_{t'} \frac{\gamma}{\gamma - 1} RT_6 \left[1 - \left(\frac{P_7}{P_6} \right)^{\frac{\gamma-1}{\gamma}} \right] + \eta_{t''} \frac{\gamma}{\gamma - 1} RT_8 \left[1 - \left(\frac{P_9}{P_8} \right)^{\frac{\gamma-1}{\gamma}} \right] \tag{38}$$

$$\frac{T_4}{T_1} = 1 + \frac{\left(\frac{P_4}{P_1} \right)^{\frac{\gamma-1}{\gamma}} - 1}{\eta_c} \tag{39}$$

$$\frac{T_7}{T_6} = 1 - \eta_{t'} \left[1 - \left(\frac{P_7}{P_6} \right)^{\frac{\gamma-1}{\gamma}} \right] \tag{40}$$

$$\eta_o = \frac{W_t - W_c}{\frac{\gamma}{\gamma-1} R \left(T_6 - T_4 + T_8 - T_7\right)} \tag{41}$$

When equations (37), (38), (39), (40), and (41) are combined,

$$\eta_o = \frac{\eta_{t'} \frac{T_6}{T_1}\left[1 - \left(\frac{P_7}{P_6}\right)^{\frac{\gamma-1}{\gamma}}\right] + \eta_{t''} \frac{T_8}{T_1}\left[1 - \left(\frac{P_9}{P_6}\frac{P_6}{P_7}\frac{P_7}{P_8}\right)^{\frac{\gamma-1}{\gamma}}\right] - \frac{1}{\eta_c}\left[\left(\frac{P_4}{P_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]}{\eta_{t'} \frac{T_6}{T_1}\left[1 - \left(\frac{P_7}{P_6}\right)^{\frac{\gamma-1}{\gamma}}\right] + \frac{T_8}{T_1} - 1 - \frac{1}{\eta_c}\left[\left(\frac{P_4}{P_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]}$$

By hypothesis, $P_7 = P_8$, $P_9 = P_1$, and $P_5 = P_6$. All of the variables except η_o are assumed independent of P_7/P_6. When η_o is a maximum, $\partial\eta_o/\partial(P_7/P_6) = 0$, and

$$\frac{P_7}{P_6} = \left(\frac{y - \sqrt{y^2 - 4xz}}{2x}\right)^{\frac{\gamma}{\gamma-1}} \tag{16}$$

where

$$x = \eta_{t'} \frac{T_6}{T_1}\left(\frac{T_1}{T_8} + \eta_{t''} - 1\right)$$

$$y = 2\eta_{t'} \eta_{t''} \frac{T_6}{T_1}\left(\frac{P_9}{P_6}\right)^{\frac{\gamma-1}{\gamma}}$$

$$z = \eta_{t''}\left(\frac{P_9}{P_6}\right)^{\frac{\gamma-1}{\gamma}}\left\{\eta_{t'} \frac{T_6}{T_1} + \frac{T_8}{T_1} - 1 - \frac{1}{\eta_c}\left[\left(\frac{P_6}{P_9}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]\right\}$$

APPENDIX E

SAMPLE CALCULATION FOR GAS-TURBINE POWER PLANT

Basic Cycle

The following conditions were assigned for calculations of the basic cycle:

Compressor pressure ratio, P_4/P_1	10.0
Maximum temperature, T_6, °R	2000
NACA standard altitude, ft	30,000
Flight speed, V_0, mph	400
C	0.97
$\eta_{ad,c}$	0.85
$\eta_{ad,t}$	0.90
η_b	0.90
η_p	1.00
$\eta_{s,c}$	0.84
$\eta_{s,t}$	0.89
P_1/p_0	P_{10}/p_{11}
P_6/P_5	1.00

From figure 2, for 400 miles per hour and 30,000 feet

$$T_1 = 440^\circ \text{ R}$$

From figure 3

$$p_0 = 4.37 \text{ lb/sq in.}$$

$$P_1 = 5.50 \text{ lb/sq in.}$$

$$P_1/p_0 = 5.50/4.37 = 1.26$$

From figure 4 at a pressure ratio of 10

$$\Delta H_{ad,c}/T_1 = 173$$

By substituting this value in equation (4),

$$W_c = (440/0.84)\ 173 = 90,600 \text{ ft-lb/lb air}$$

From equation (5)

$$H_4 - H_1 = (440/0.85)\ 173 = 89,500$$

From figure 5

$$H_4 = 7800 + 89,500 = 97,300 \text{ ft-lb/lb air}$$

$$T_4 = T_5 = 917^\circ \text{ R}$$

From figure 6 for $T_5 = 917^\circ$ R and $T_6 = 2000^\circ$ R

$$f_{id} = 0.0157$$

By substituting this value in equation (10)

$$f = 0.0157/0.90 = 0.0174$$

From figure 8 at a pressure ratio of 10 and $T_6 = 2000^\circ$ R

$$W_t/\eta_{s,t} = 187,000$$

$$W_t = 187,000\ (0.89) = 166,400 \text{ ft-lb/lb gas}$$

When equation (14) is used

$$H_6 - H_9 = 187,000\ (0.90) = 168,300$$

From figure 9

$$H_6 = 302,000$$

$$H_9 = 302,000 - 168,300 = 133,700 \text{ ft-lb/lb gas}$$

Therefore,

$$T_9 = 1214^\circ \text{ R}$$

From figure 10 at a pressure ratio $P_{10}/p_{11} = 1.26$

$$V_j/C\sqrt{T_9} = 28.0$$

$$V_j = (28.0)\ (0.97)\sqrt{1214} = 946 \text{ ft/sec}$$

By substituting in equation (23),

$$W_j = (587/32.2)\left[(1.0174)(946) - 587\right]$$

$$= 6800 \text{ ft-lb/lb air}$$

By using equation (24),

$$W_n = \left[(1.0174)(166,400) - 90,600\right](1.00) + 6800$$

$$= 85,500 \text{ ft-lb/lb air}$$

$$= 155.4 \text{ hp-sec/lb air}$$

From equation (25)

$$F = 1.98 \times 10^6 (0.0174/85,500) = 0.403$$

Intercooling Cycle

The following conditions were assigned for the intercooling cycle:

Compressor pressure ratio, P_4/P_1	10.0
Maximum temperature, T_6, °R	2000
NACA standard altitude, ft	30,000
Flight speed, V_0, mph	400
C	0.97
e_i	0.50
$\eta_{ad,t}$	0.90
η_b	0.90
η_p	1.00
$\eta_{s,t}$	0.89
P_1/p_0	P_{10}/p_{11}
P_2/P_3	1.00
P_6/P_5	1.00

The compressor adiabatic efficiencies should be equal and should be chosen so that if no intercooling takes place between the two stages, the change in enthalpy during compression is the same as the change in enthalpy of a single compression having the compressor adiabatic efficiency equal to 0.85. After the adiabatic efficiencies have been chosen for the two stages, the shaft efficiencies 0.01 less than the adiabatic efficiencies will be chosen. The value of P_2/P_1 will be so chosen that the specific power output will be maximized.

The following values are obtained from the calculations of the basic cycle:

$$T_1 = 440^\circ \text{ R}$$

$$P_1/p_0 = 1.26$$

From equation (6)

$$P_2/P_1 = \sqrt{10.0} = 3.16$$

In order to determine $\eta_{ad,c'}$, and $\eta_{ad,c''}$, try $\eta_{ad,c'} = \eta_{ad,c''} = 0.873$.

From figure 4 at a pressure ratio of 3.16,

$$\Delta H_{ad,c'}/T_1 = H_{ad,c''}/T_3 = 72.5$$

By substituting this value in equation (5)

$$H_2 - H_1 = (440/0.873)\ 72.5 = 36{,}500 \text{ ft-lb/lb air}$$

From figure 5

$$H_2 = 7800 + 36{,}500 = 44{,}300 \text{ ft-lb/lb air}$$

$$T_2 = 637^\circ \text{ R}$$

With an intercooler effectiveness of zero

$$T_2 = T_3 = 637^\circ \text{ R}$$

From equation (5)

$$H_4 - H_3 = (637/0.873)\ 72.5 = 52{,}900 \text{ ft-lb/lb air}$$

For intercooling of zero effectiveness

$$(H_4 - H_3) + (H_2 - H_1) = 52{,}900 + 36{,}500 = 89{,}400 \text{ ft-lb/lb air}$$

If this value for $H_4 - H_1$ is compared with the value of $H_4 - H_1$ in the basic cycle, it will be found to be very nearly the same; therefore, the value for $\eta_{ad,c'}$ and $\eta_{ad,c''}$ that was arbitrarily selected is satisfactory.

The compressor efficiencies are

$$\eta_{ad,c'} = \eta_{ad,c''} = 0.873$$

$$\eta_{s,c'} = \eta_{s,c''} = 0.863$$

When intercooling is used with an effectiveness of 0.50, the conditions in the first stage of the compression do not change.

$$T_2 = 637^\circ \text{ R}$$

$$\Delta H_{ad,c'}/T_1 = 72.5$$

$$W_{c'} = (440/0.863)\ 72.5 = 37{,}000 \text{ ft-lb/lb air}$$

From equation (7)

$$0.50 = (637-T_3)/(637-440)$$

$$T_3 = 539^\circ \text{ R}$$

$$\Delta H_{ad,c''}/T_3 = 72.5$$

From equation (4)

$$W_{c''} = (539/0.863)\ 72.5 = 45{,}300 \text{ ft-lb/lb air}$$

From equation (5)

$$H_4 - H_3 = (539/0.873)\ 72.5 = 44{,}800 \text{ ft-lb/lb air}$$

From figure 5

$$H_4 = 26{,}000 + 44{,}800 = 70{,}800 \text{ ft-lb/lb air}$$

$$T_4 = 776^\circ \text{ R}$$

$$W_c = W_{c'} + W_{c''}$$

$$W_c = 37{,}000 + 45{,}300 = 82{,}300 \text{ ft-lb/lb air}$$

The remainder of the calculations for the intercooling cycle are carried out in the same manner as for the basic cycle.

Reheating Cycle

The following conditions were assigned for calculations of the reheating cycle:

Compressor pressure ratio, P_4/P_1,	10.0
Maximum temperature, $T_6 = T_8$, °R	2000
NACA standard altitude, ft	30,000
Flight speed, V_0, mph	400
C	0.97
$\eta_{ad,c}$	0.85
η_b	0.90
η_p	1.00
$\eta_{s,c}$	0.84
P_1/p_0	P_{10}/p_{11}
$P_6/P_5 = P_8/P_7$	1.00

The turbine adiabatic efficiencies should be equal and should be chosen so that if no reheating takes place between the two stages, the change of enthalpy during the expansion is the same as the change of enthalpy of a single expansion having a turbine adiabatic efficiency equal to 0.90. After the adiabatic efficiencies have been chosen for the two stages, shaft efficiencies 0.01 less than the adiabatic efficiencies will be chosen. The pressure ratio P_7/P_6 will be so chosen that the specific power output will be maximized.

The following values are obtained from the calculations of the basic cycle:

$$T_1 = 440^\circ \text{ R}$$

$$P_1/p_0 = 1.26$$

$$W_c = 90{,}600 \text{ ft-lb/lb air}$$

$$T_4 = T_5 = 917^\circ \text{ R}$$

$$f_6 = f_7 = 0.0174$$

From equation (15),

$$P_6/P_7 = \sqrt{10.0} = 3.16$$

In order to determine $\eta_{ad,t'}$ and $\eta_{ad,t''}$, try $\eta_{ad,t'} = \eta_{ad,t''} = 0.880$

From figure 8 at a pressure ratio of 3.16, $f_6 = 0.0174$, and $T_6 = 2000^\circ$ R,

$$W_{t'}/\eta_{s,t'} = 107{,}500 \text{ ft-lb/lb gas}$$

From equation (14)

$$H_6 - H_7 = 107{,}500\ (0.880) = 94{,}600 \text{ ft-lb/lb gas}$$

As in the basic cycle

$$H_6 = 302{,}000$$

$$H_7 = 302{,}000 - 94{,}600 = 207{,}400 \text{ ft-lb/lb gas}$$

From figure 9

$$T_7 = 1565^\circ \text{ R}$$

From figure 8 at a temperature of 1565° R, $f_7 = 0.0174$, and at a pressure ratio of 3.16

$$W_{t''}/\eta_{s,t} = 83{,}400 \text{ ft-lb/lb gas}$$

$$H_7 - H_9 = 83{,}400\ (0.880) = 73{,}400 \text{ ft-lb/lb gas}$$

$$(H_6 - H_7) + (H_7 - H_9) = 94{,}600 + 73{,}400 = 168{,}000 \text{ ft-lb/lb gas}$$

If this value for $H_6 - H_9$ is compared with the value of $H_6 - H_9$ in the basic cycle, it will be found to be very nearly the same; therefore, the values for $\eta_{ad,t'}$ and $\eta_{ad,t''}$ that were arbitrarily chosen are satisfactory.

The turbine efficiencies are

$$\eta_{ad,t'} = \eta_{ad,t''} = 0.880$$

$$\eta_{s,t'} = \eta_{s,t''} = 0.870$$

When reheating is used between the stages, the conditions in the first stage of the expansion do not change.

$$T_7 = 1565^\circ \text{ R}$$

$$W_{t'}/\eta_{s,t'} = 107{,}500 \text{ ft-lb/lb gas}$$

$$W_{t'} = 107{,}500\ (0.870) = 93{,}500 \text{ ft-lb/lb gas}$$

The method of finding the fuel-air ratio after the reheating combustion chamber (see discussion of combustion in Analysis section of this report) is as follows: The conditions at the entrance to the reheating combustion chamber ($T_7 = 1565^\circ$ R, $f_7 = 0.0174$) correspond to a fictitious initial temperature of less than 500° R at the entrance to the imaginary combustion chamber. The point that describes the conditions in the imaginary combustion chamber may be located on figure 6 because T_7 and f_7 are known, and the distance of this point above the line of 500° R initial temperature may be determined. On the vertical line of 2000° R discharge temperature at a point that is an equal distance above the line of 500° R initial temperature,

$$f_{id,8} = 0.0244$$

From equation (11)

$$f_8 = 0.0174 + \frac{0.0244 - 0.0174}{0.90}$$

$$f_8 = 0.0252$$

From figure 8

$$W_{t''} = 107{,}500\ (0.870) = 93{,}500 \text{ ft-lb/lb gas}$$

$$W_t = W_{t'} + W_{t''} = 93{,}500 + 93{,}500 \text{ ft-lb/lb gas}$$

$$W_t = 187{,}000 \text{ ft-lb/lb gas}$$

$$H_8 - H_9 = 107{,}500\ (0.880) = 94{,}600 \text{ ft-lb/lb gas}$$

From figure 9 at a temperature T_8 of 2000° R, and $f_8 = 0.0252$

$$H_8 = 306{,}000 \text{ ft-lb/lb gas}$$

$$H_9 = 306{,}000 - 94{,}600 = 211{,}400 \text{ ft-lb/lb gas}$$

From figure 9

$$T_9 = 1574^\circ \text{ R}$$

The rest of the calculations for the reheating cycle are carried out in the same manner as those for the basic cycle.

Regenerating Cycle

The following conditions were assigned for calculations of the regenerating cycle:

Compressor pressure ratio, P_4/P_1,	10.0
Maximum temperature, T_6, $^\circ$R	2000
NACA standard altitude, ft	30,000
Flight speed, V_0, mph	400
C	0.97
e_r	0.50
$\eta_{ad,c}$	0.85
$\eta_{ad,t}$	0.90
η_b	0.90
η_p	1.00
$\eta_{s,c}$	0.84
$\eta_{s,t}$	0.89
P_1/p_0	P_{10}/p_{11}
$P_5/P_4 = P_6/P_5 = P_{10}/P_9$	1.00

The following values are obtained from the calculations of the basic cycle:

$$T_1 = 440^\circ \text{ R}$$

$$P_1/p_0 = 1.26$$

$$W_c = 90{,}600 \text{ ft-lb/lb air}$$

$$T_4 = 917^\circ \text{ R}$$

The temperature T_9 may be estimated by assigning it equal to T_9 of the basic cycle as a first approximation.

$$T_9 = 1214^\circ \text{ R}$$

Substituting into equation (18)

$$0.50 = \frac{T_5 - 917}{1214 - 917}$$

$$T_5 = 1065^\circ \text{ R}$$

From figure 6 for $T_5 = 1065^\circ$ R and $T_6 = 2000^\circ$ R

$$f_{id} = 0.0139$$

Substituting into equation (10)

$$f = \frac{0.0139}{0.90} = 0.0154$$

From figure 8 at a pressure ratio of 10 and $T_6 = 2000^\circ$ R

$$W_t/\eta_{s,t} = 187,000$$

$$W_t = 187,000\ (0.89) = 166,400 \text{ ft-lb/lb gas}$$

From equation (14)

$$H_6 - H_9 = 187,000\ (0.90) = 168,300 \text{ ft-lb/lb gas}$$

From figure 9

$$H_6 = 302,000 \text{ ft-lb/lb gas}$$

$$H_9 = 302,000 - 168,300 = 133,700 \text{ ft-lb/lb gas}$$

$$T_9 = 1214^\circ \text{ R}$$

The difference between the fuel-air ratio of the regenerating cycle and that of the basic cycle has a negligible effect on the exit turbine temperature T_9. A second approximation is therefore unnecessary. From figure 5 at $T_5 = 1065^\circ$ R and $T_4 = 917^\circ$ R

$$H_5 = 126,000 \text{ ft-lb/lb air}$$

$$H_4 = 97,300 \text{ ft-lb/lb air}$$

Substituting in equation (19)

$$133{,}700 - H_{10} = \frac{126{,}000 - 97{,}300}{1.0154}$$

$$H_{10} = 105{,}400 \text{ ft-lb/lb gas}$$

$$T_{10} = 1077^{\circ} \text{ R}$$

The rest of the calculations for the regenerating cycle are carried out in the same manner as those for the basic cycle.

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TABLE I - A TABULATION OF THE CYCLES ANALYZED AND THE ASSUMED CONDITIONS

Cycle	Altitude (ft)	Air velocity (ft/sec)	Compressor adiabatic efficiency, $\eta_{ad,c}$	Compressor shaft efficiency, $\eta_{s,c}$	Turbine adiabatic efficiency, $\eta_{ad,t}$	Turbine-shaft efficiency, $\eta_{s,t}$	Combustion efficiency, η_b	Turbine-inlet temperature, T_6 (°R)	Intercooler effectiveness, e_i	Regenerator effectiveness, e_r	Reheat turbine-inlet temperature, T_8 (°R)	Results primarily show the effects of:
Basic	0 15,000 30,000	586	0.85	0.84	0.90	0.89	0.90	2000	---	---	---	Altitude
Basic	30,000	586	0.85	0.84	0.90	0.89	0.90	2000 2250 2500	---	---	---	Turbine-inlet temperature
Intercooling	30,000	586	(1)	(1)	0.90	0.89	0.90	2000	0 .50 1.00	---	---	Intercooling at the point for greatest power output
Intercooling	30,000	586	(1)	(1)	0.90	0.89	0.90	2000	0 .50 1.00	---	---	Intercooling at the point for lowest specific fuel consumption
Reheating	30,000	586	0.85	0.84	(2)	(2)	0.90	2000	---	---	2000	Reheating during expansion
Regenerating	30,000	586	0.85	0.84	0.90	0.89	0.90	2000	---	0 .25 .50 .75 1.00	---	Regeneration

(1) Efficiencies of compressor stages with intercooling chosen somewhat higher to give performance equivalent to single stage or basic cycle at same over-all pressure ratio.

(2) Efficiencies of turbine stages with reheating chosen somewhat lower to give performance equivalent to single stage of basic cycle at same over-all pressure ratio.

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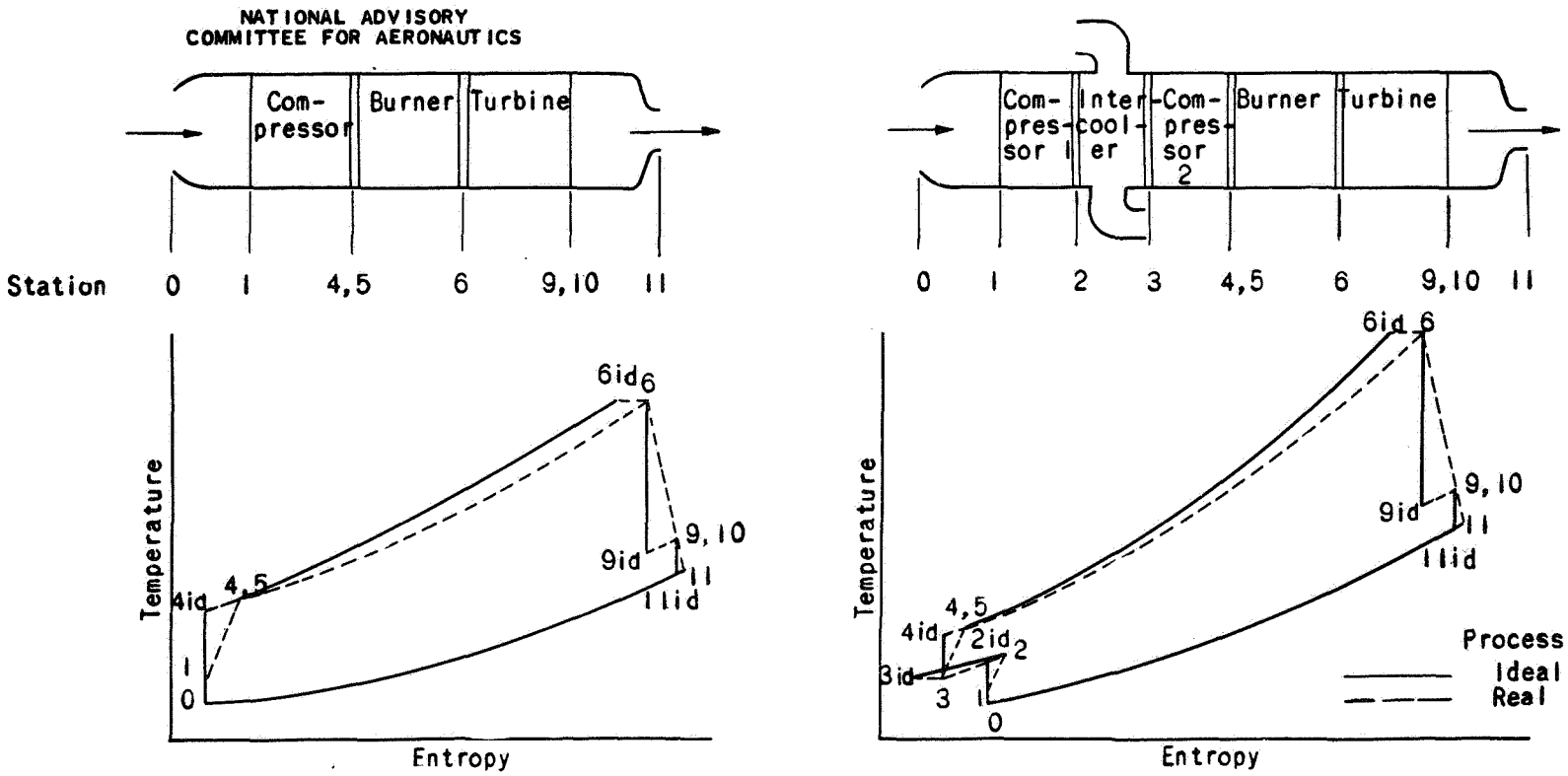
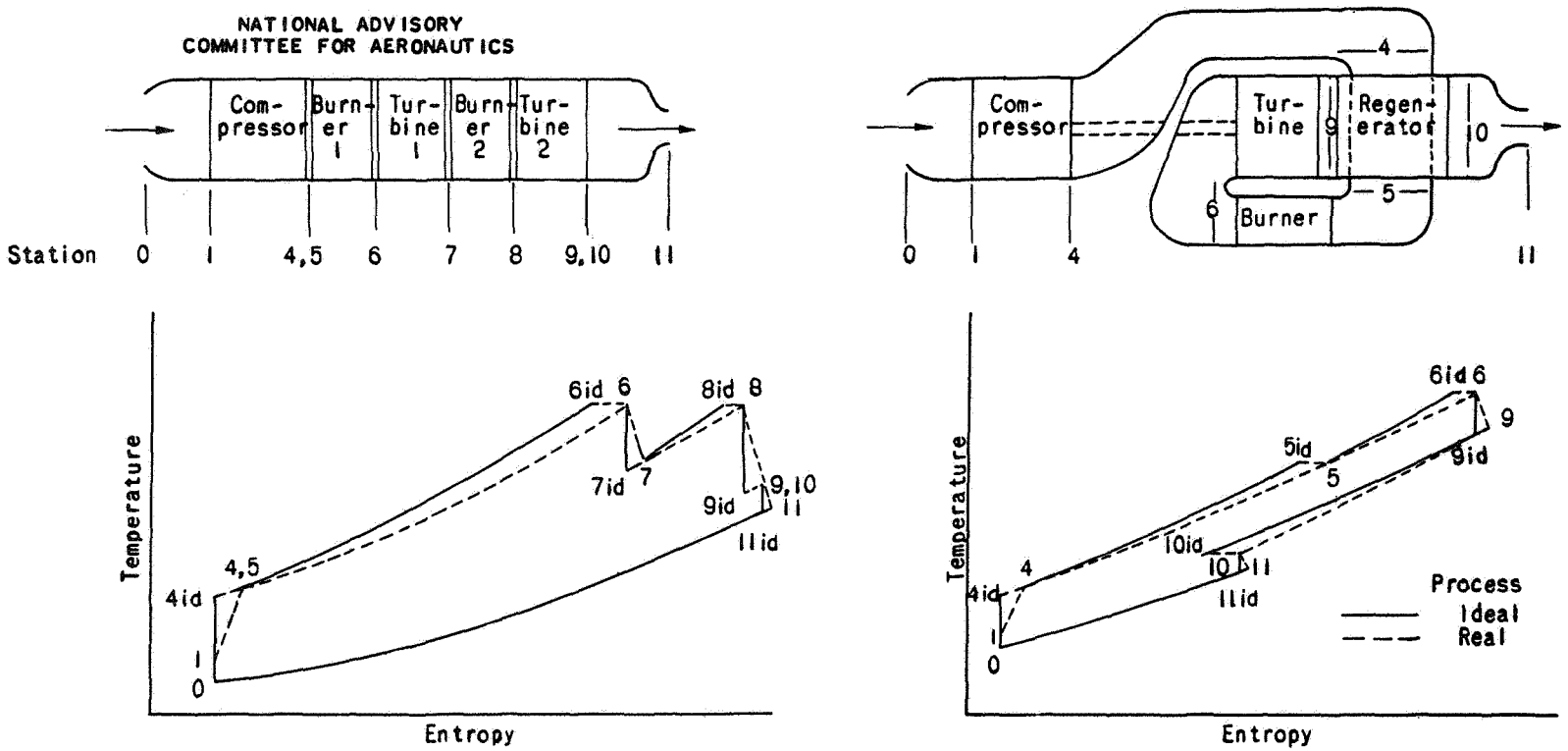


(a) Basic cycle.

(b) Intercooling cycle.

Figure 1. - Temperature-entropy diagram of gas-turbine cycles for propeller-driving power plant.



(c) Reheating cycle.

(d) Regenerating cycle.

Figure 1. - Concluded. Temperature-entropy diagram of gas-turbine cycles for propeller-driving power plant.

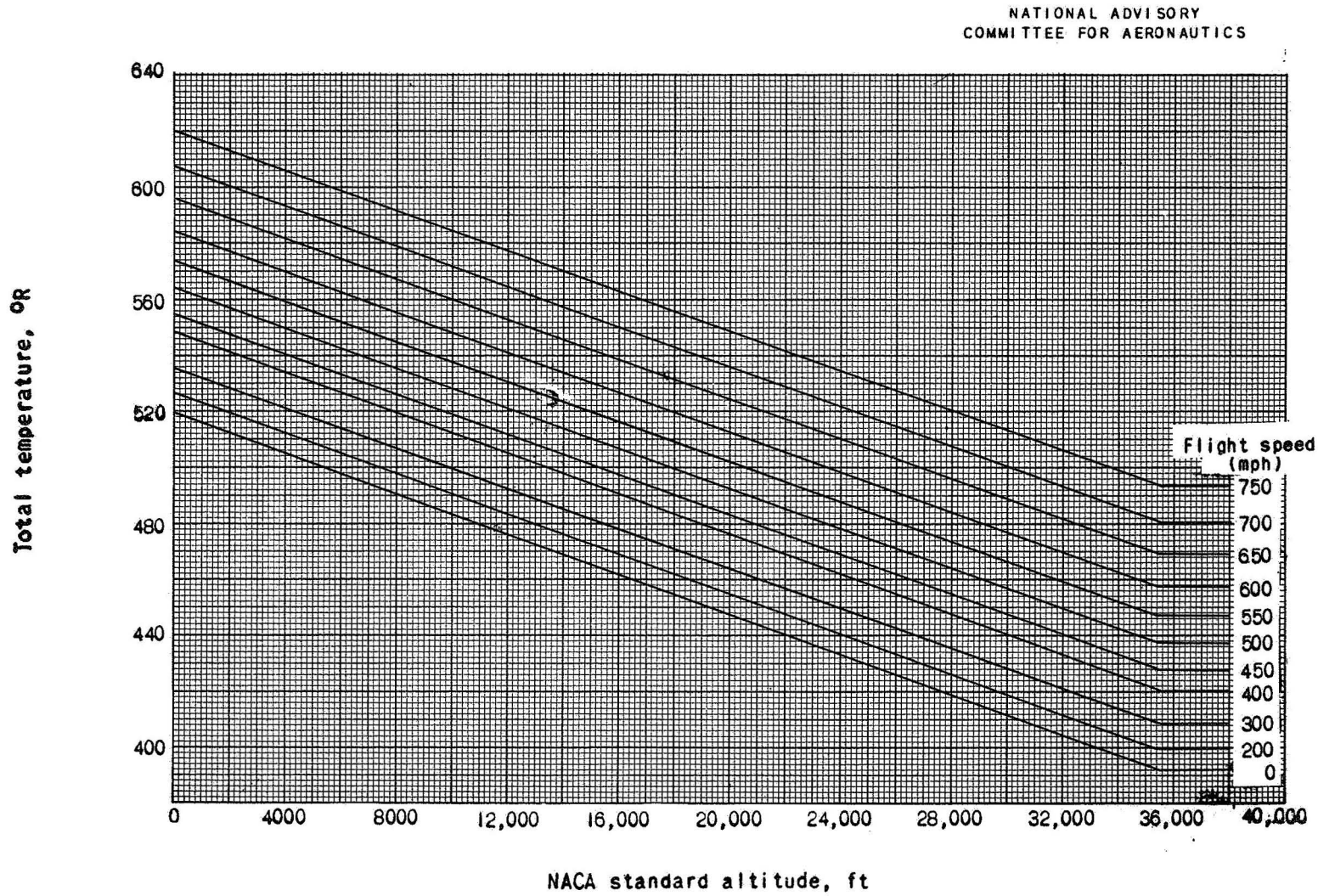


Figure 2. - Variation of compressor-inlet total temperature with altitude and speed.

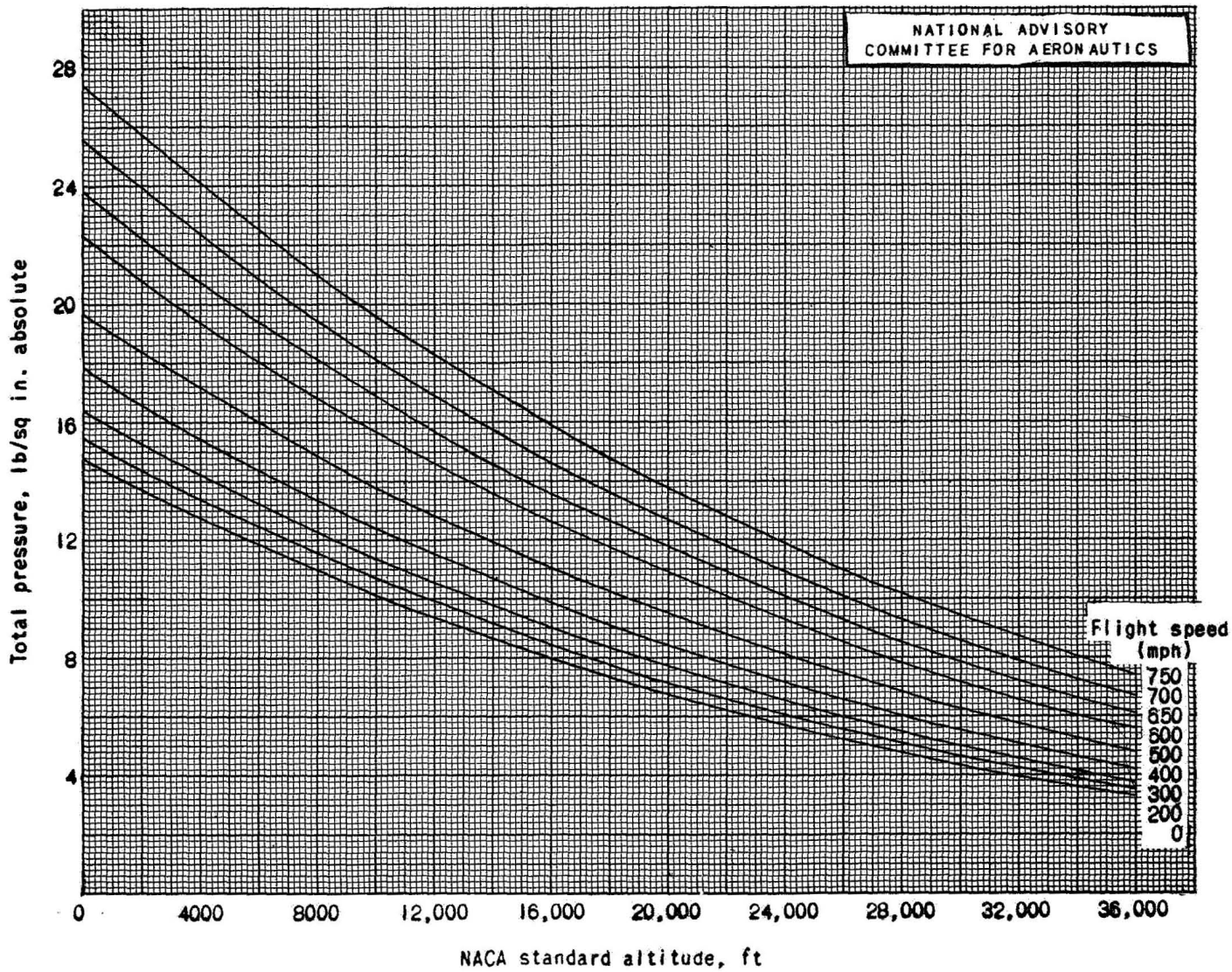


Figure 3. - Variation of compressor-inlet total pressure with altitude and speed.

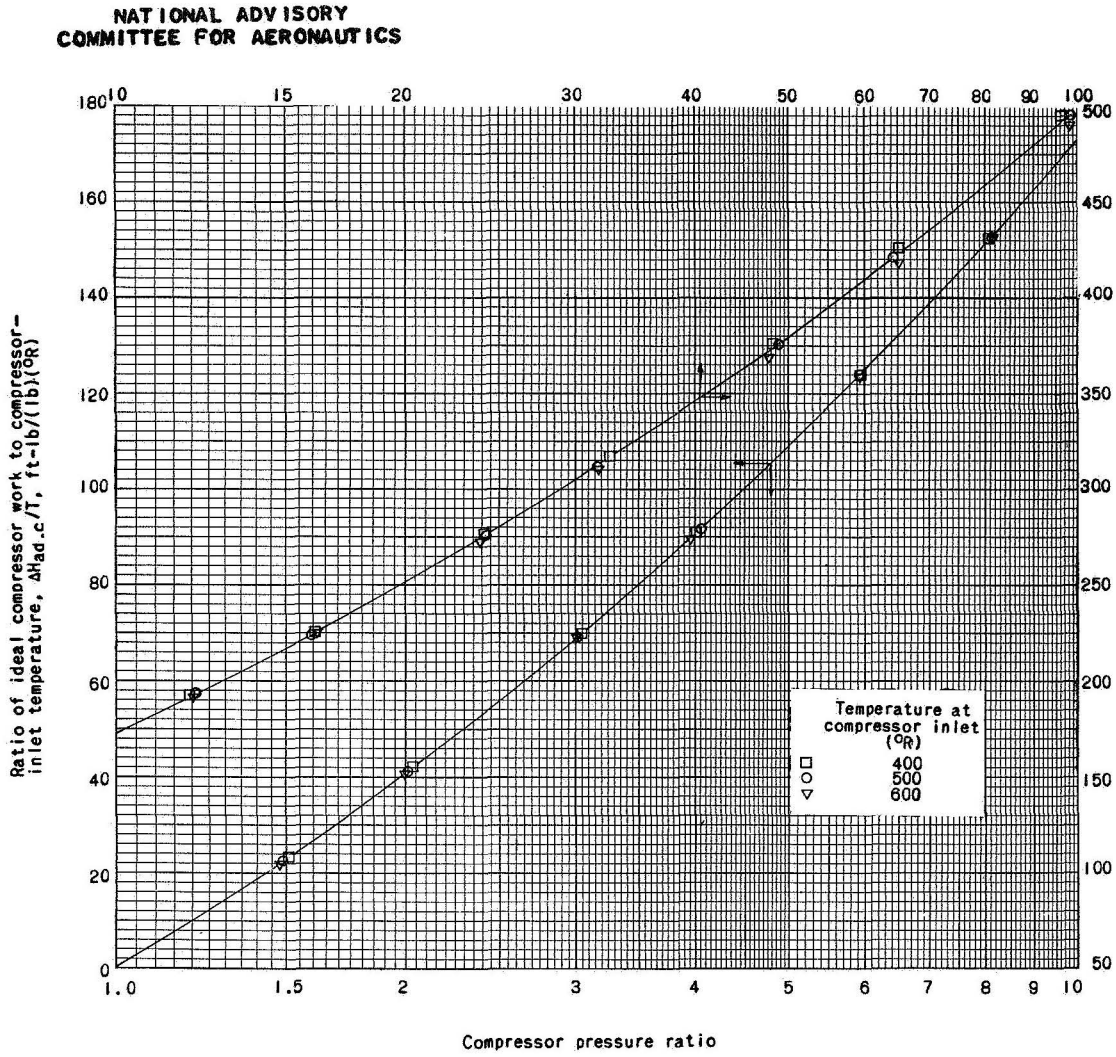


Figure 4. - Chart for evaluating isentropic compressor work for initial temperatures from 400° to 600° R and pressure ratios of 1.0 to 100. (Data from reference 11.)

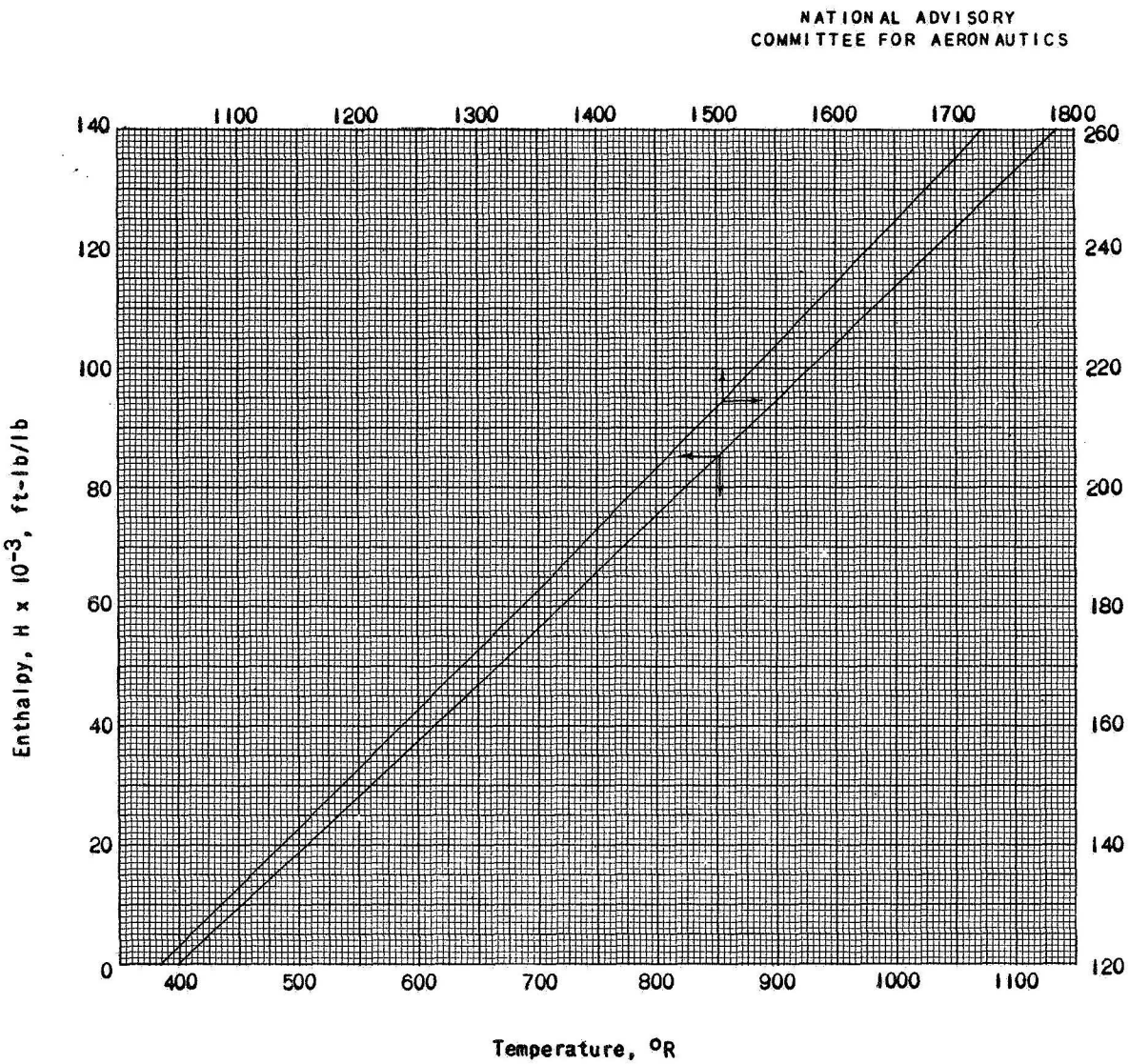


Figure 5. - Variation of enthalpy of air with temperature. (Data from reference 11.) Base temperature, H = 0 at 400° R.

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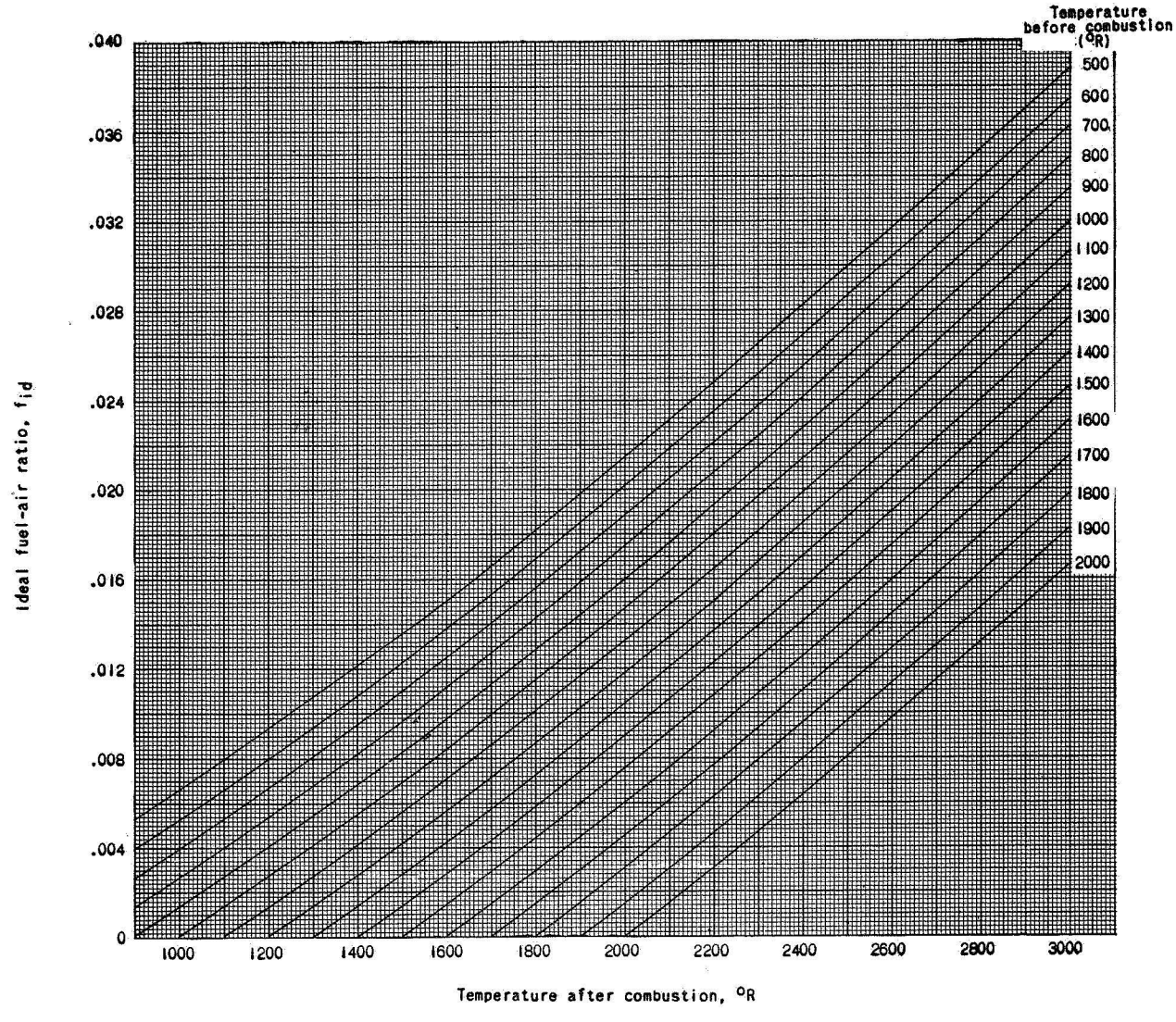

Figure 6. - Variation of ideal fuel-air ratio with the temperatures before and after combustion for the complete combustion of octane with dry air. (Data from reference 14.) Composition of air assumed 79-percent nitrogen, 21-percent oxygen by volume. Dissociation neglected.

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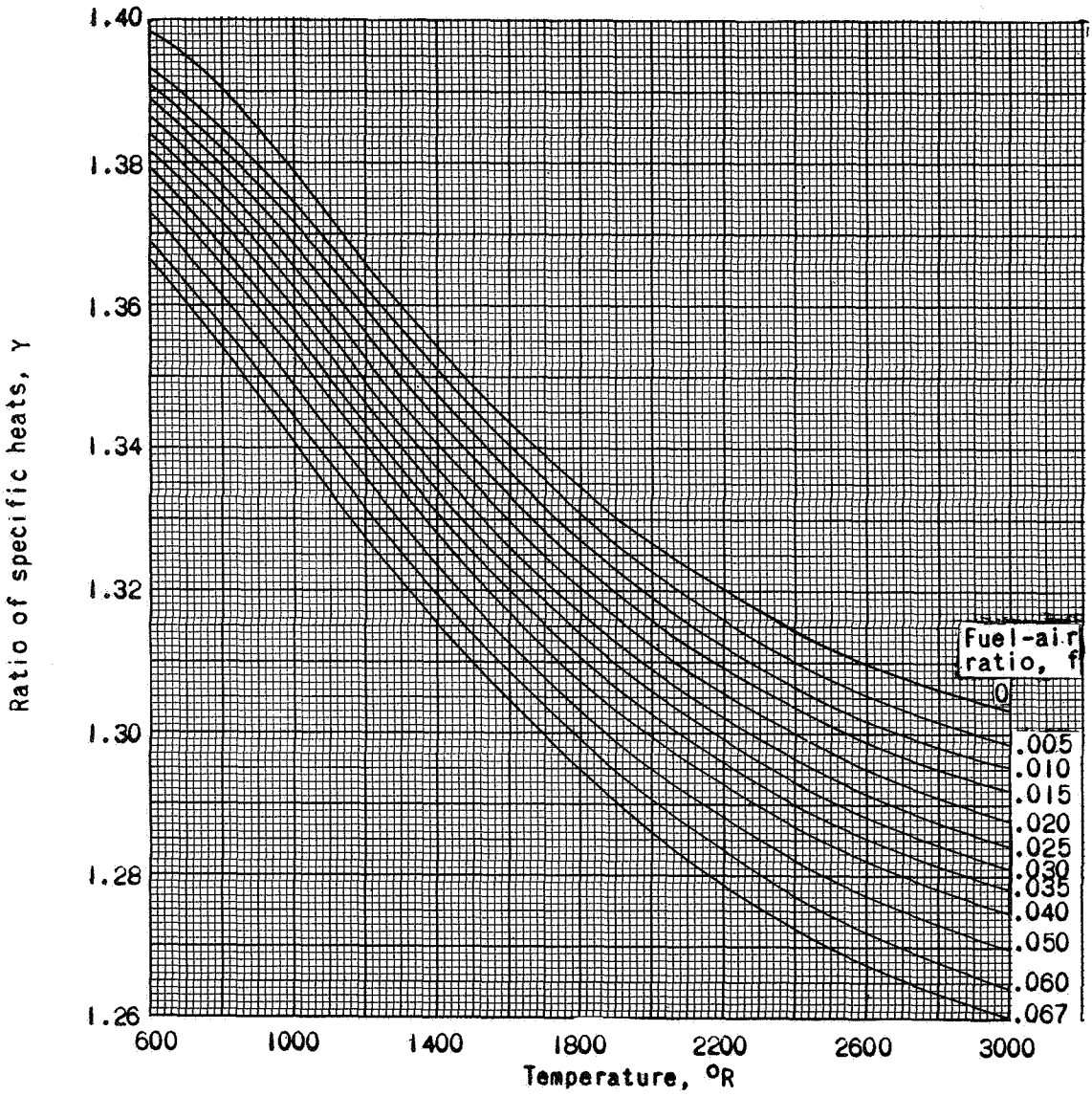


Figure 7. - Variation of specific-heat ratio γ with temperature and fuel-air ratio for the complete combustion of octane with air. Data for air (f = 0) from reference 11. Data for other curves from reference 18.

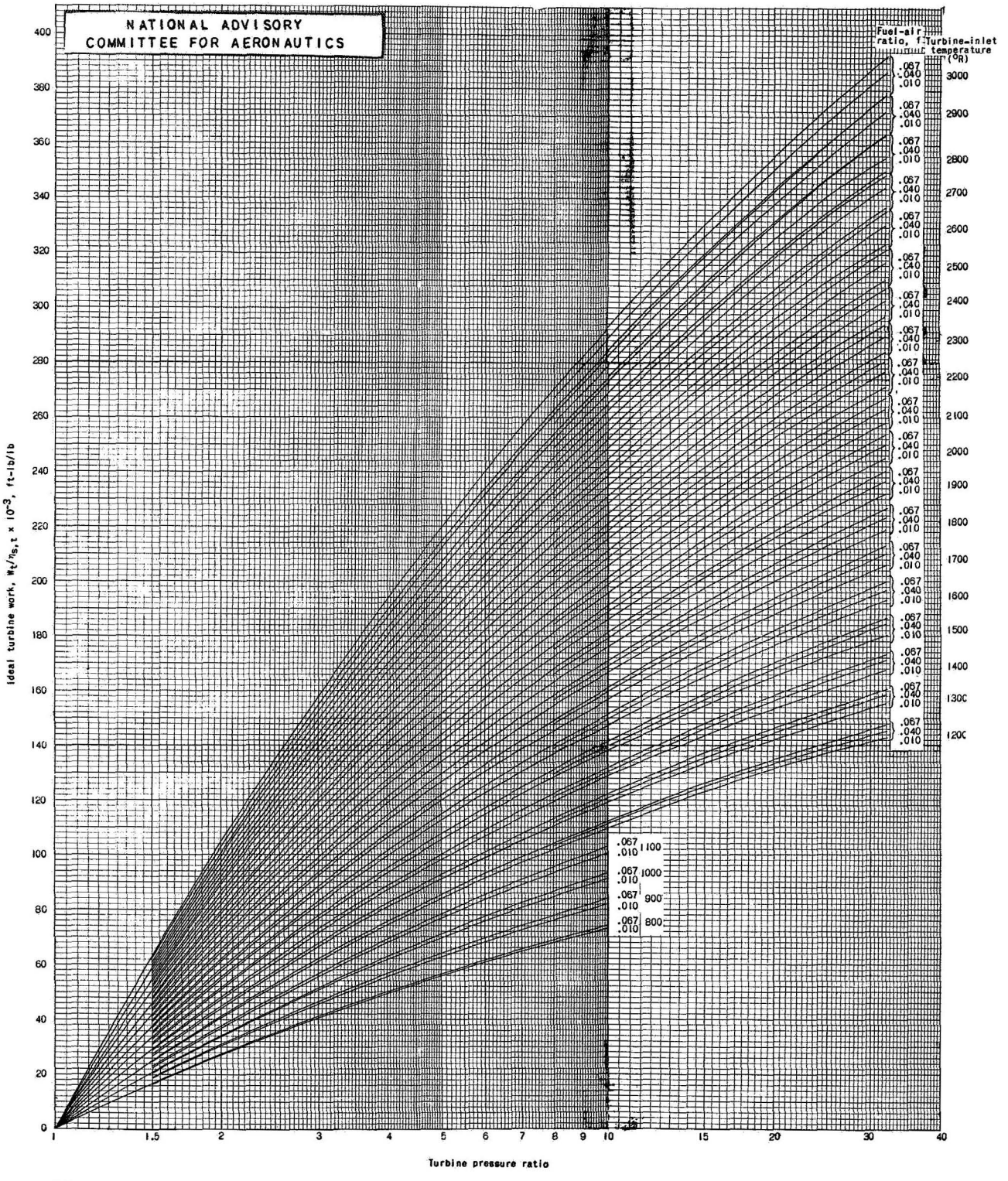


Figure 8. - Ideal turbine work for various turbine-inlet temperatures, fuel-air ratios, and turbine pressure ratios. (A 17 in. by 22 in. print of this chart is enclosed.)

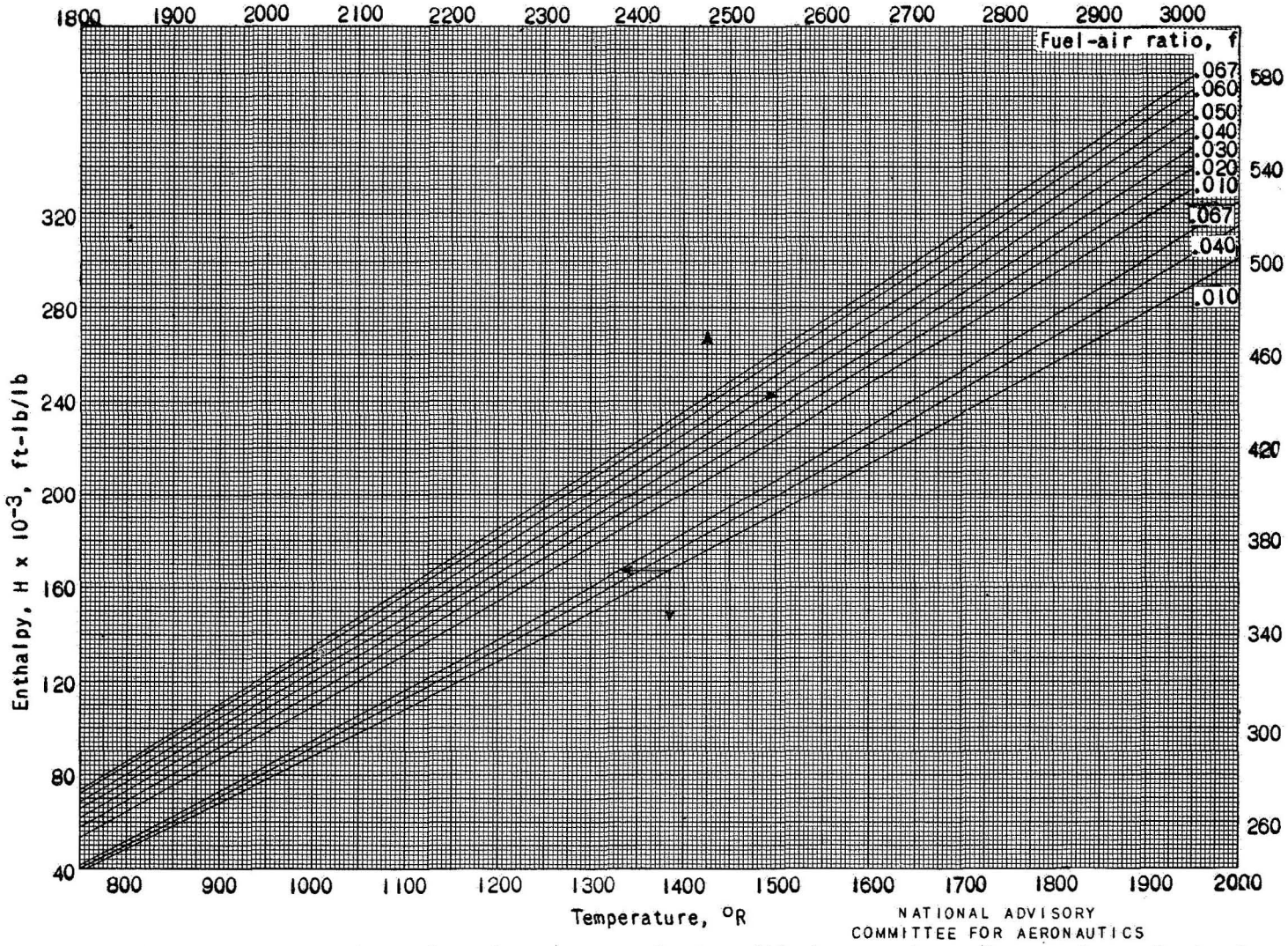


Figure 9. - Variation of enthalpy of combustion products with temperature for various fuel-air ratios. (Data from reference 18.) Base temperature, H = 0 at 540° R.

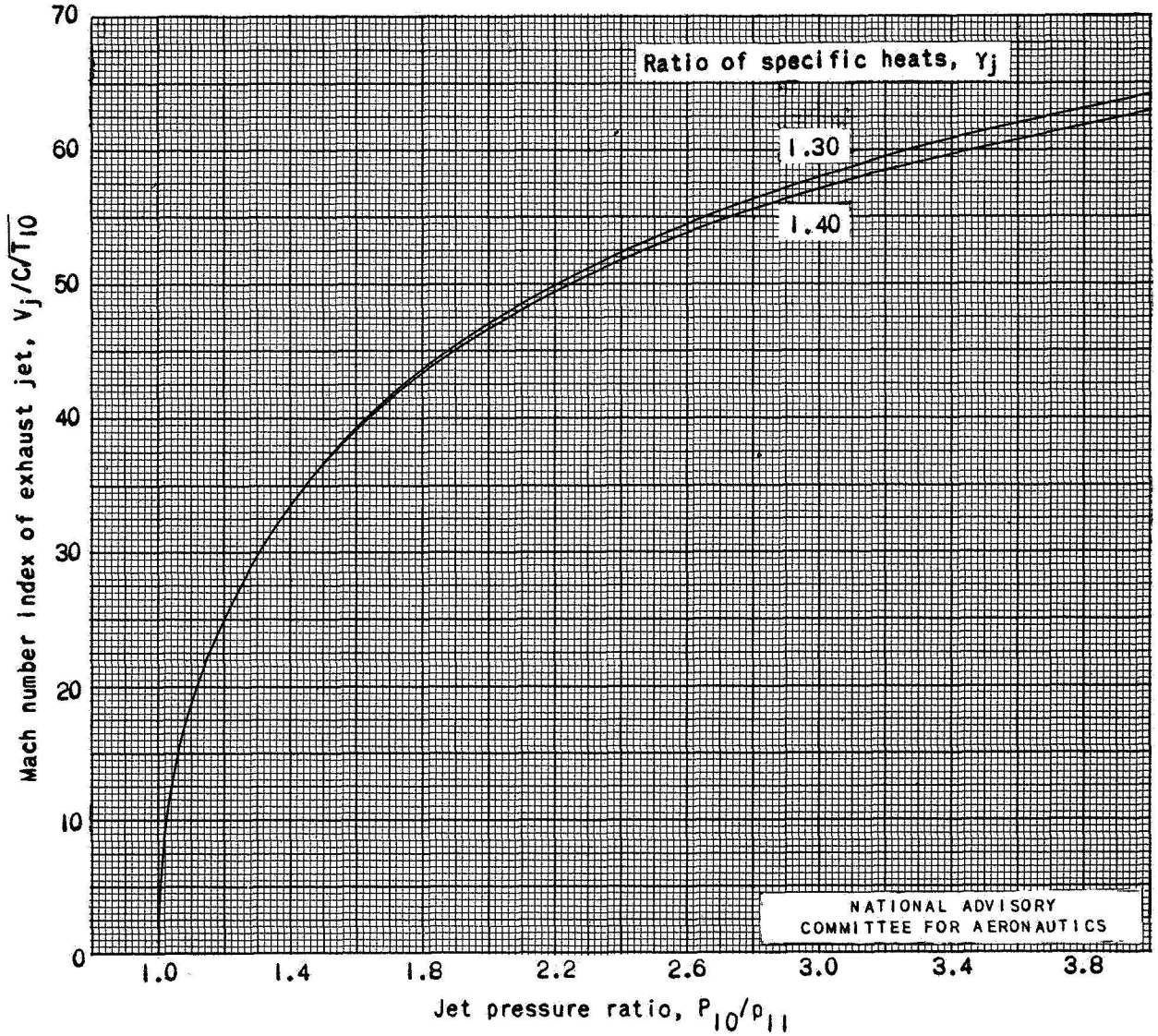
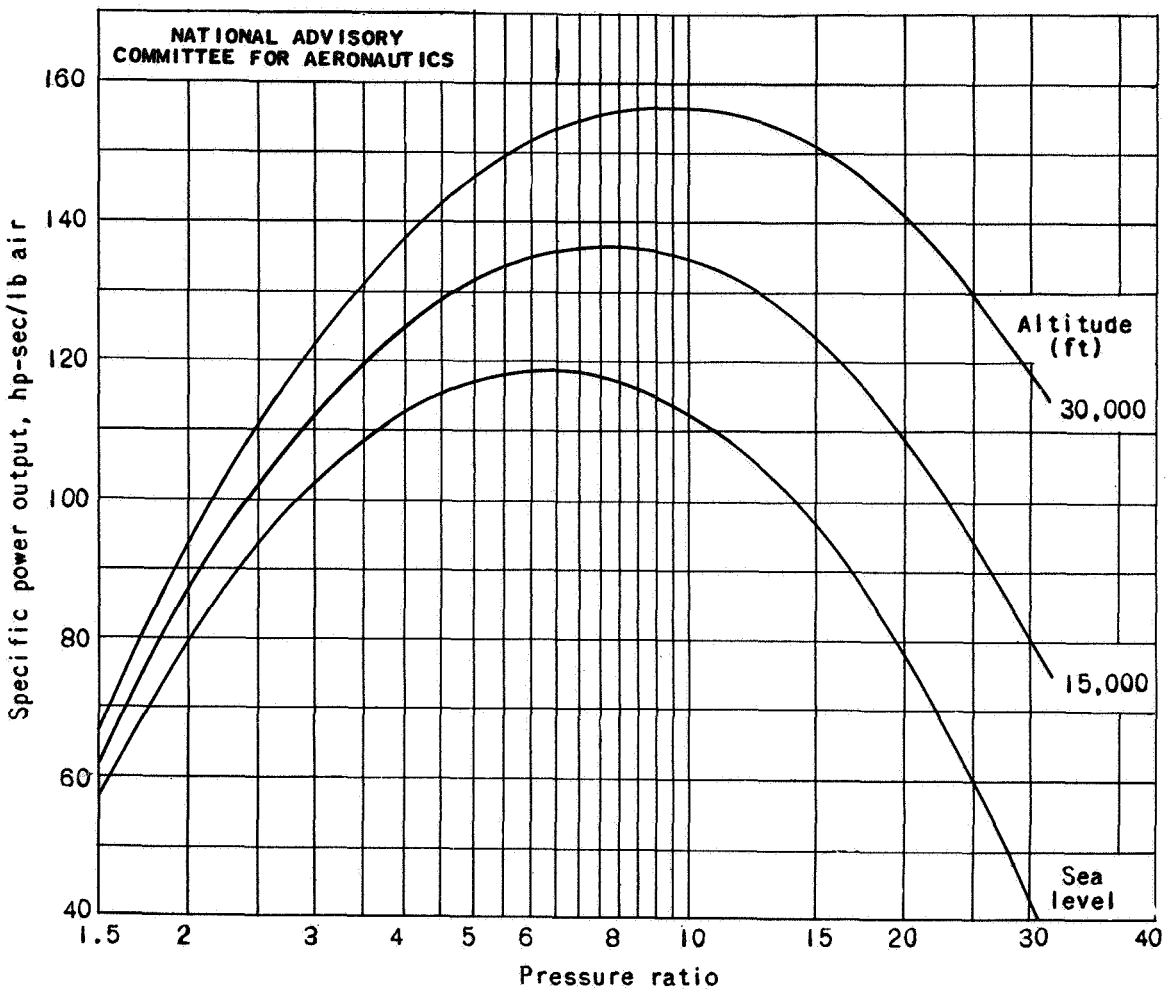
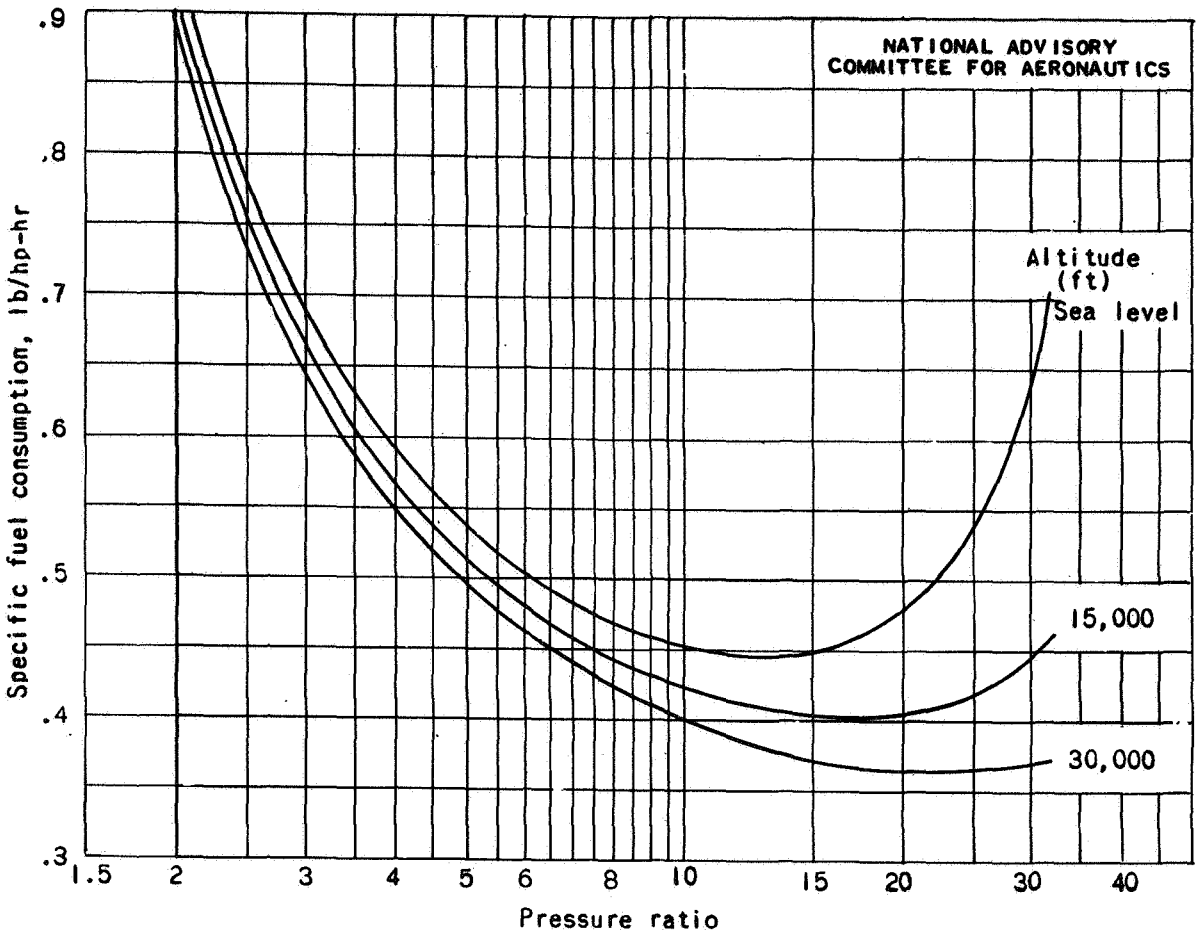


Figure 10. - Variation of Mach number index of exhaust jet with jet pressure ratio.



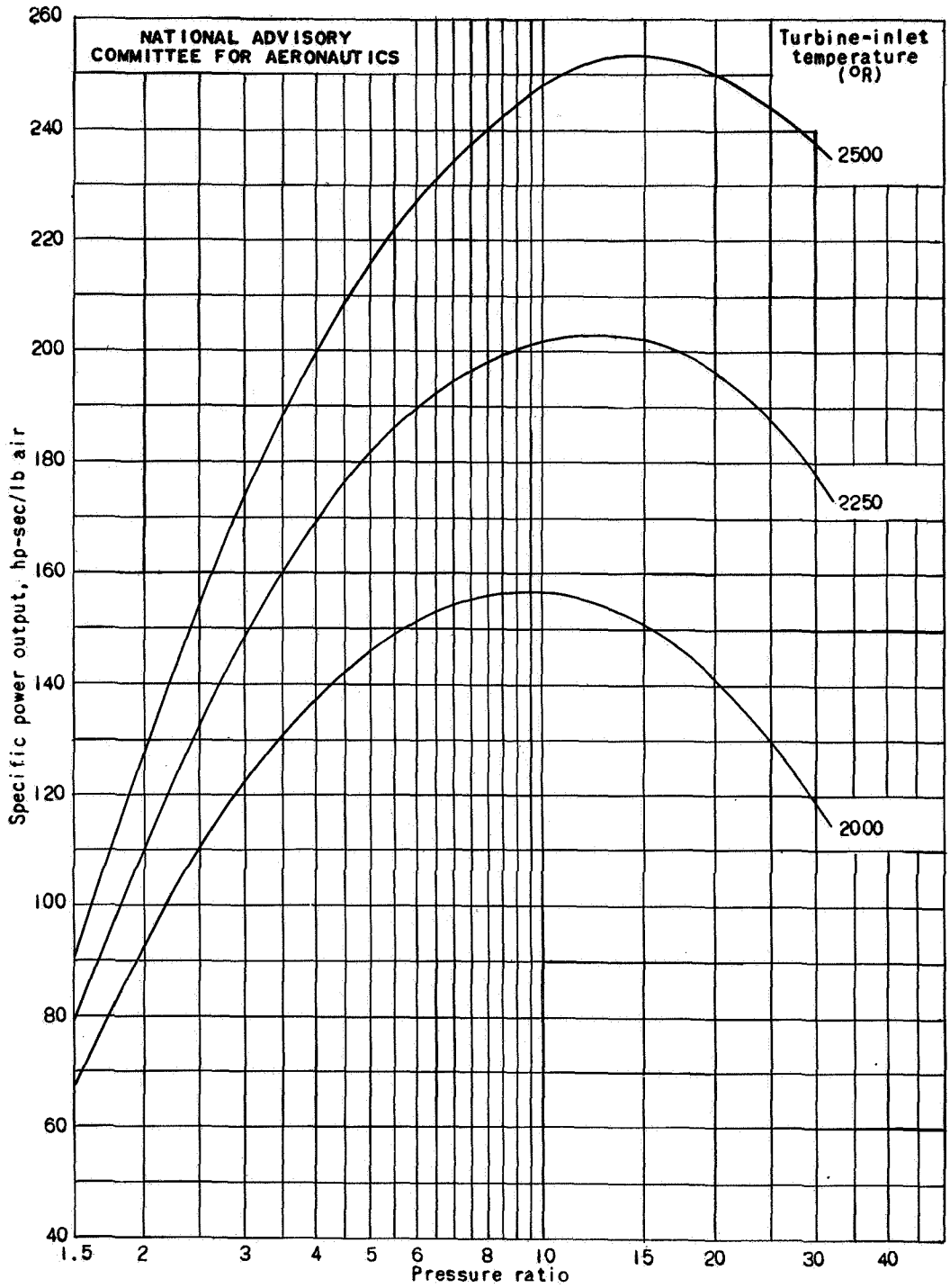
(a) Specific power output.

Figure 11. - Performance of basic cycle at various NACA standard altitudes. Maximum temperature, 2000° R; flight speed, 400 miles per hour; $\eta_{ad,c}$, 0.85; $\eta_{s,c}$, 0.84; $\eta_{ad,t}$, 0.90; $\eta_{s,t}$, 0.89; η_b, 0.90; C, 0.97; no pressure loss through burner.



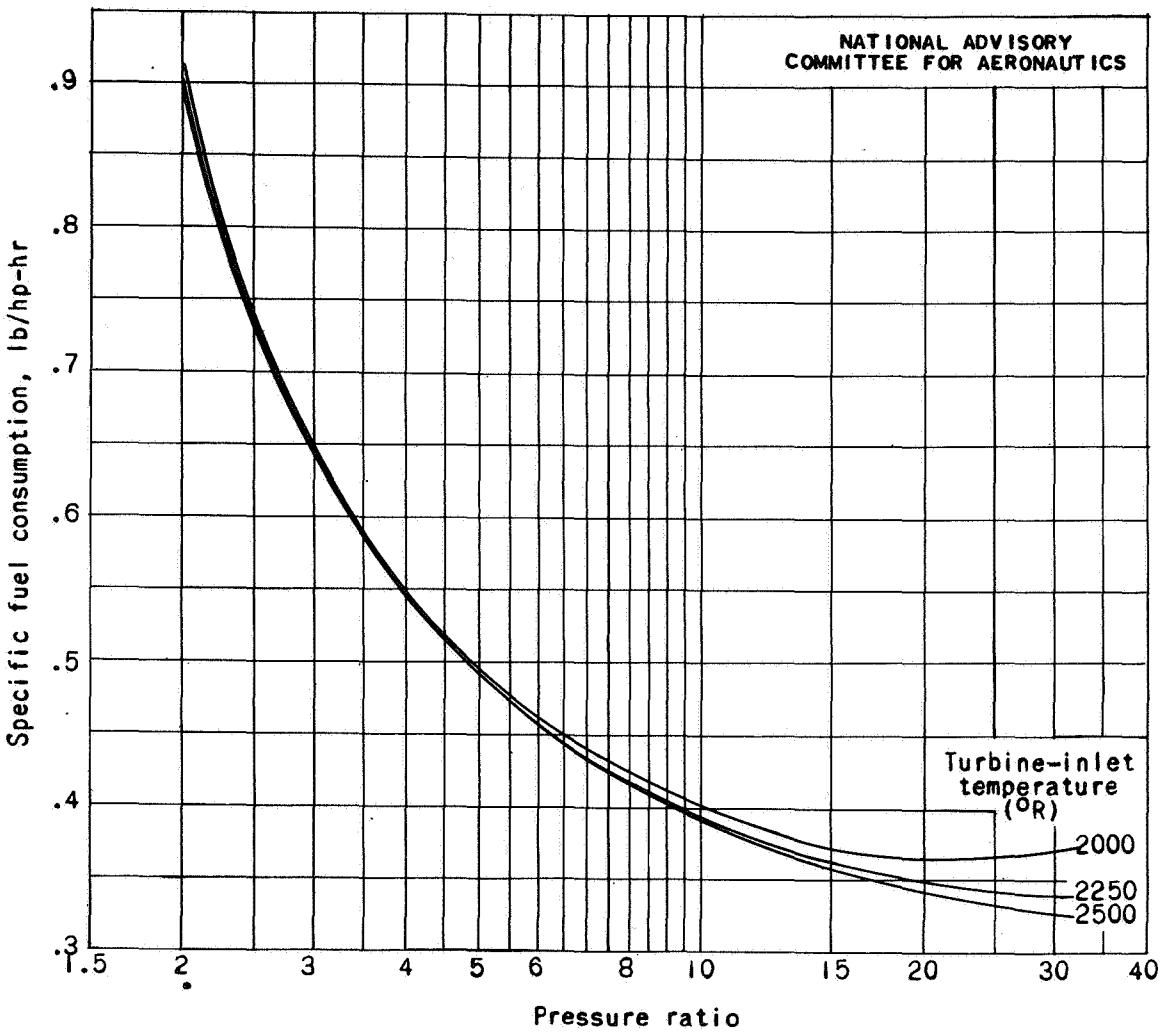
(b) Specific fuel consumption.

Figure 11. - Concluded. Performance of basic cycle at various NACA standard altitudes. Maximum temperature, 2000° R; flight speed, 400 miles per hour; $\eta_{ad,c}$, 0.85; $\eta_{s,c}$, 0.84; $\eta_{ad,t}$, 0.90; $\eta_{s,t}$, 0.89; η_{lb}, 0.90; C,0.97; no pressure loss through burner.



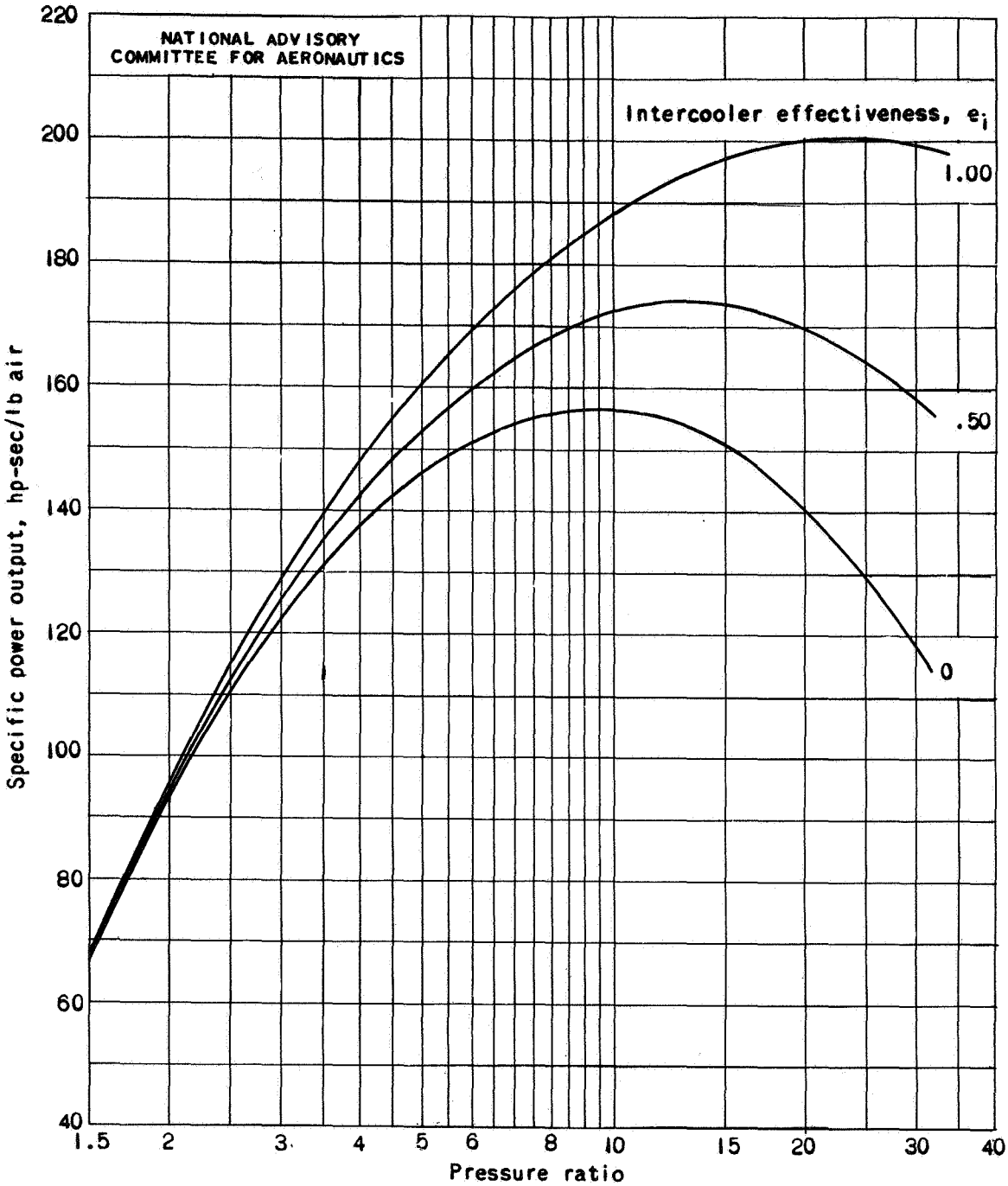
(a) Specific power output.

Figure 12. - Performance of basic cycle at various peak temperatures. NACA standard altitude, 30,000 feet; flight speed, 400 miles per hour; $\eta_{ad,c}$, 0.85; $\eta_{s,c}$, 0.84; $\eta_{ad,t}$, 0.90; $\eta_{s,t}$, 0.89; η_b, 0.90; C, 0.97; no pressure loss through burner.



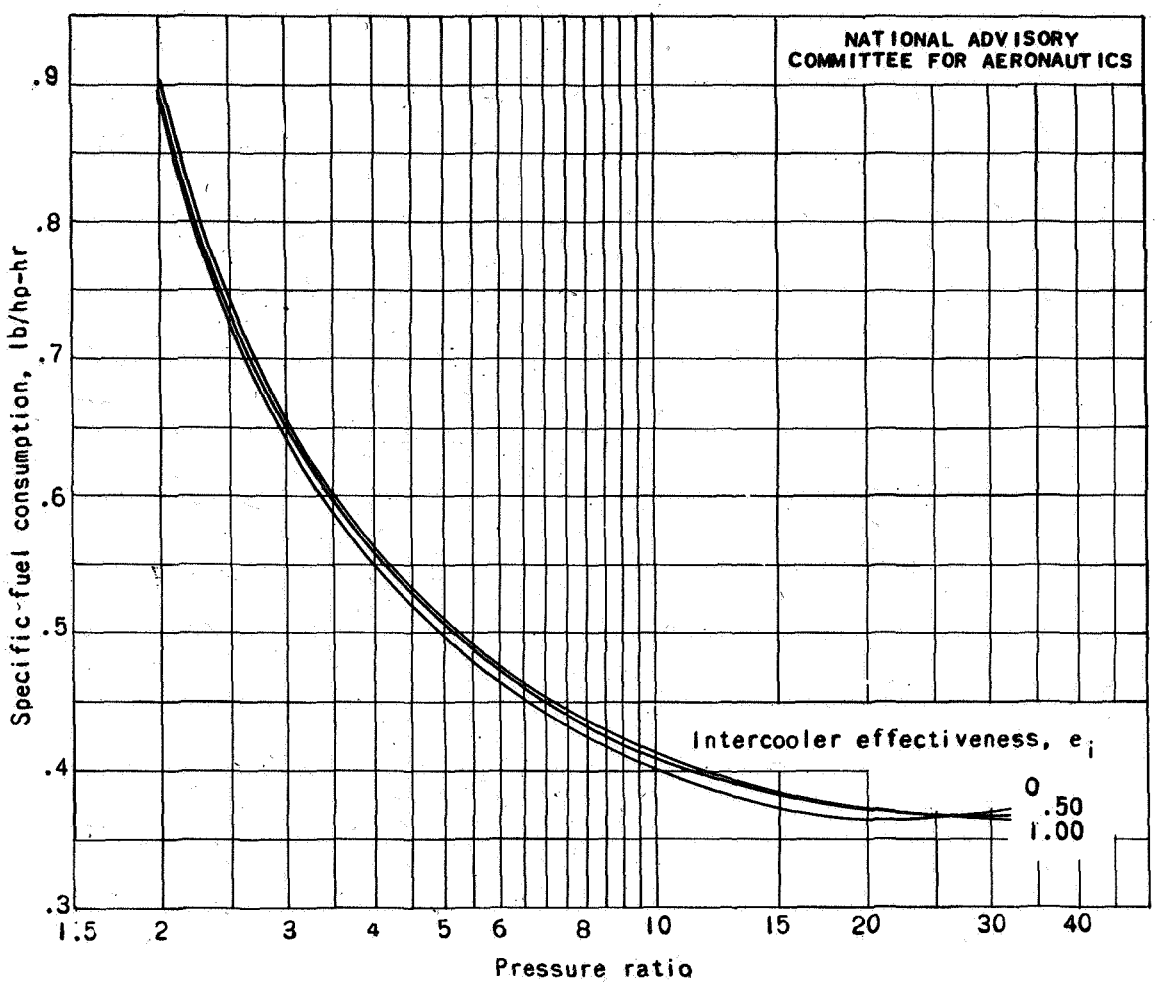
(b) Specific fuel consumption.

Figure 12. - Concluded. Performance of basic cycle at various peak temperatures. NACA standard altitude, 30,000 feet; flight speed, 400 miles per hour; $\eta_{ad,c}$, 0.85; $\eta_{s,c}$, 0.84; $\eta_{ad,t}$, 0.90; $\eta_{s,t}$, 0.89; η_b, 0.90; C, 0.97; no pressure loss through burner.



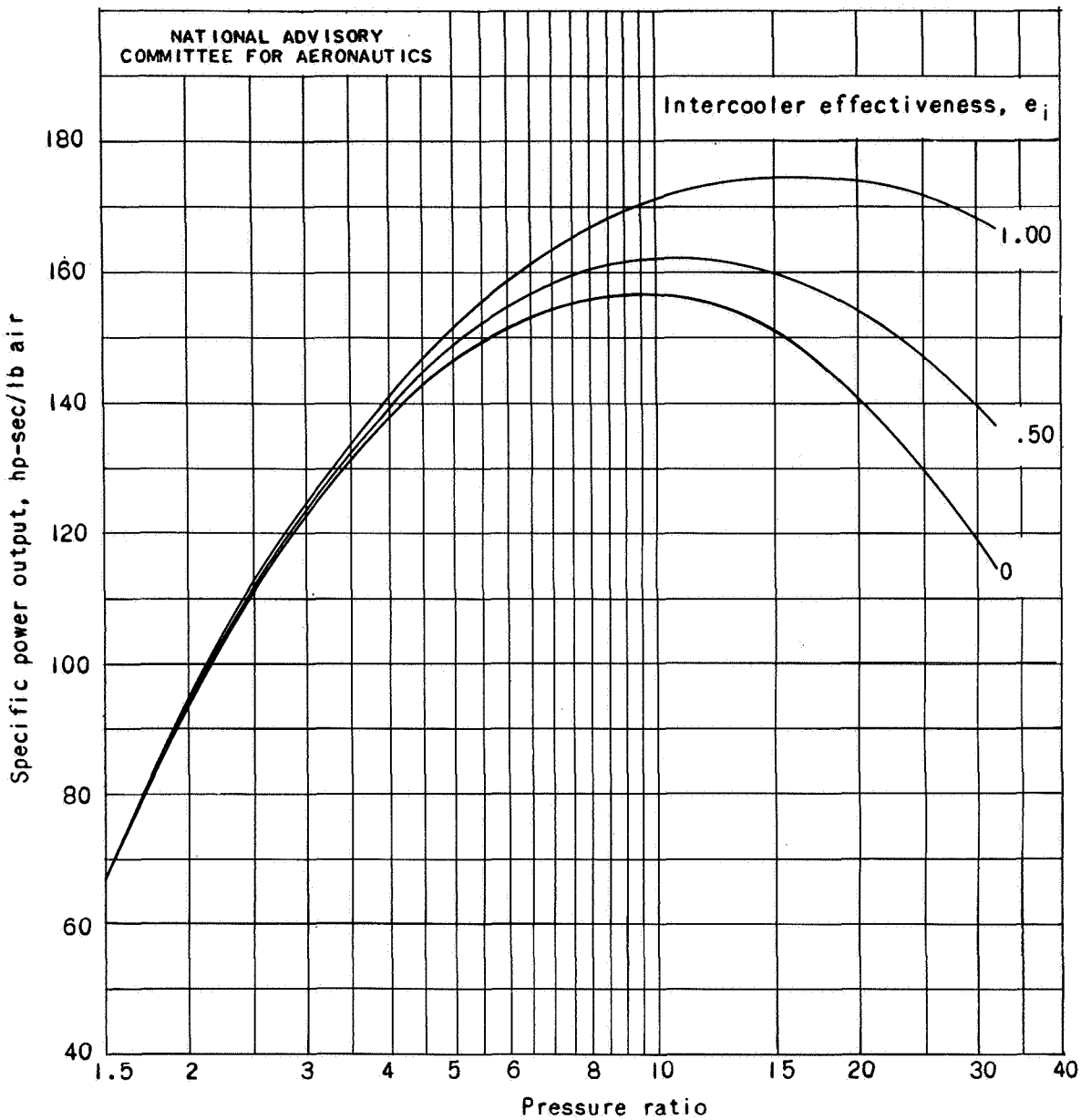
(a) Specific power output.

Figure 13. - Performance of intercooling cycle with intercooling at point for greatest power output. Maximum temperature, 2000° R; NACA standard altitude, 30,000 feet; flight speed, 400 miles per hour; $\eta_{ad,c}$, 0.85; $\eta_{s,c}$, 0.84; $\eta_{ad,t}$, 0.90; $\eta_{s,t}$, 0.89; η_b, 0.90; C, 0.97; no pressure loss through intercooler or burner.



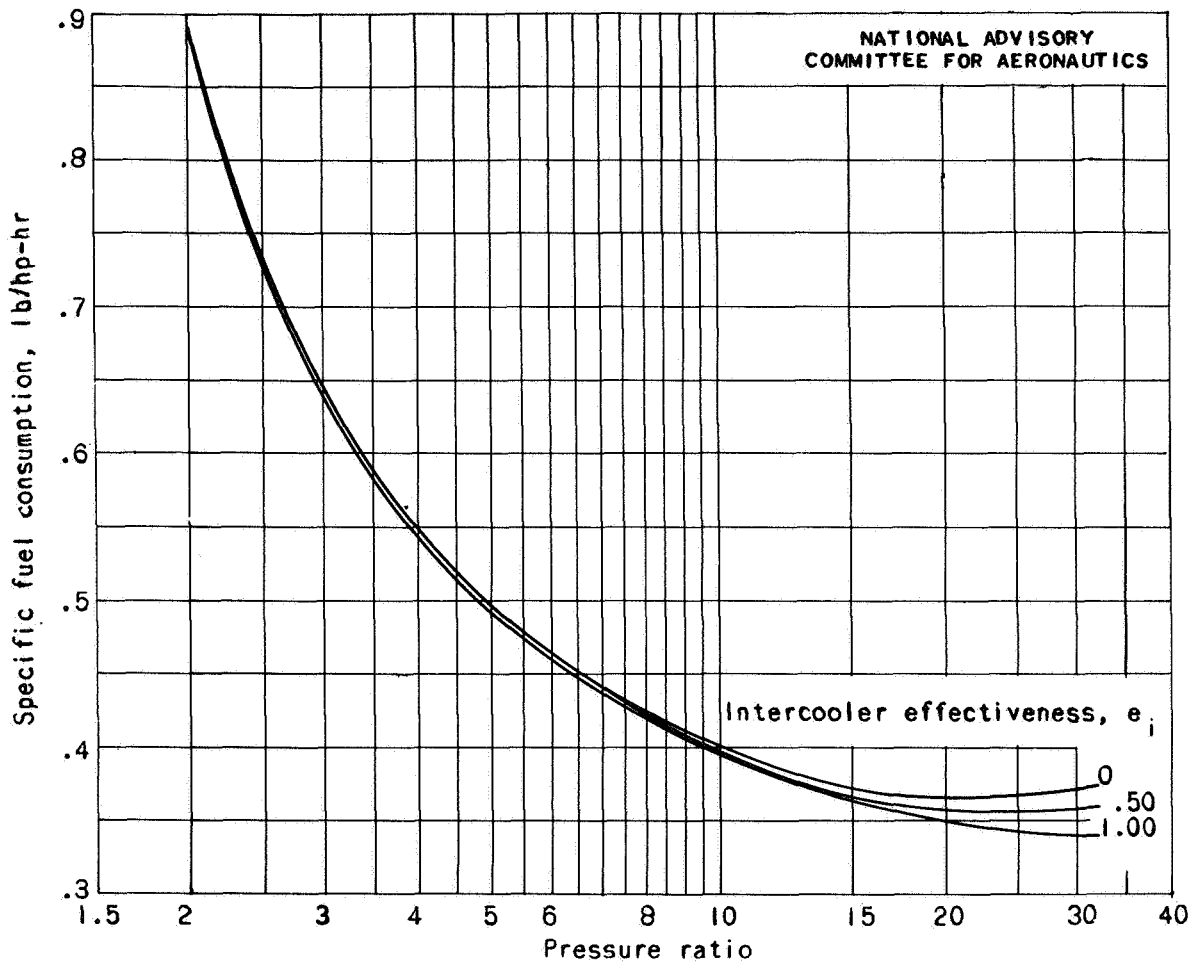
(b) Specific fuel consumption.

Figure 13. - Concluded. Performance of intercooling cycle with intercooling at point for greatest power output. Maximum temperature, 2000° R; NACA standard altitude, 30,000 feet; flight speed, 400 miles per hour; $\eta_{ad,c}$, 0.85; $\eta_{s,c}$, 0.84; $\eta_{ad,t}$, 0.90; $\eta_{s,t}$, 0.89; η_b, 0.90; C, 0.97; no pressure loss through intercooler or burner.



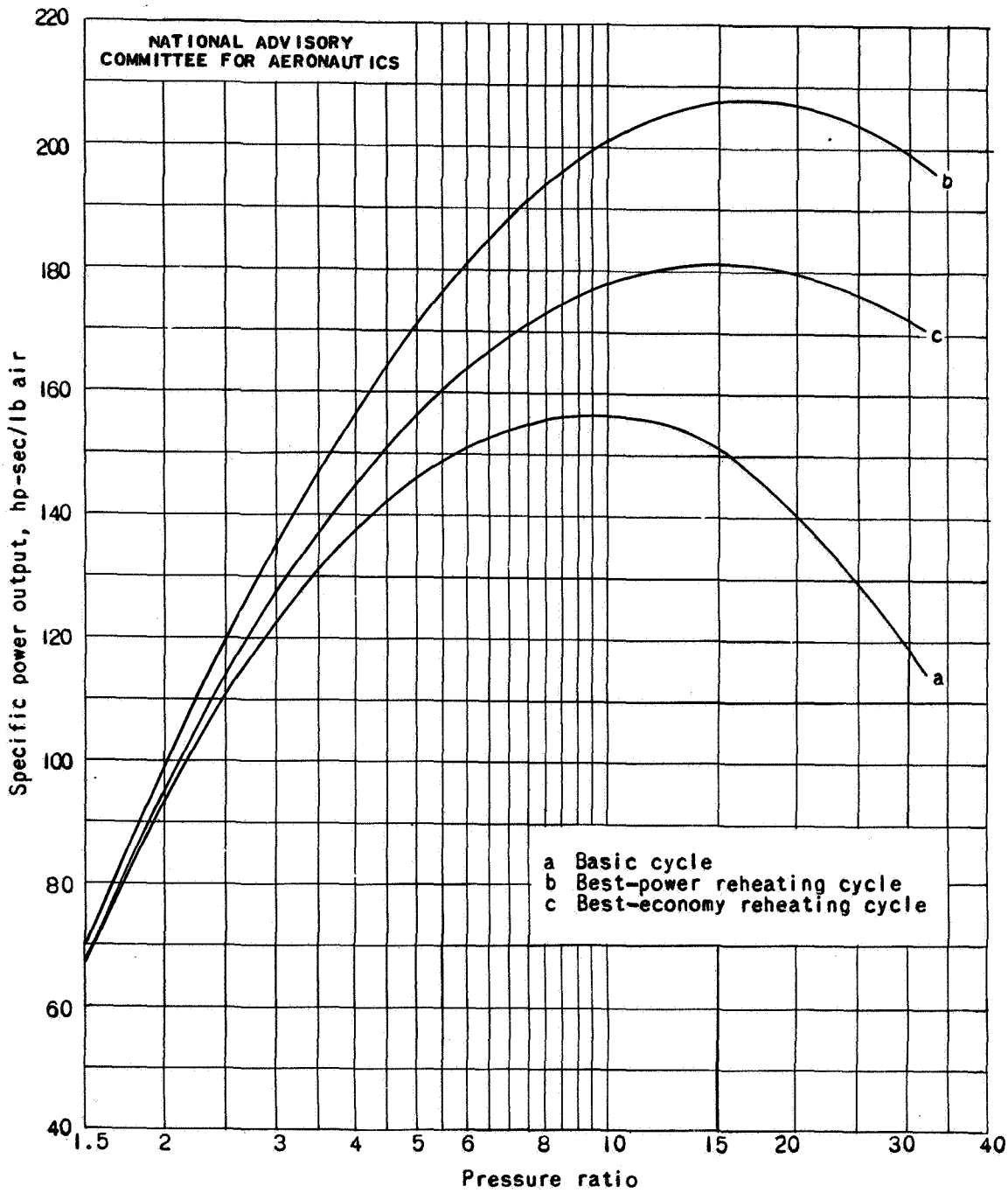
(a) Specific power output.

Figure 14. - Performance of intercooling cycle with intercooling at point of lowest specific fuel consumption. Maximum temperature, 2000° R; NACA standard altitude, 30,000 feet; flight speed, 400 miles per hour; $\eta_{ad,c}$, 0.85; $\eta_{s,c}$, 0.84; $\eta_{ad,t}$, 0.90; $\eta_{s,t}$, 0.89; η_b, 0.90; C, 0.97; no pressure loss through intercooler or burner.



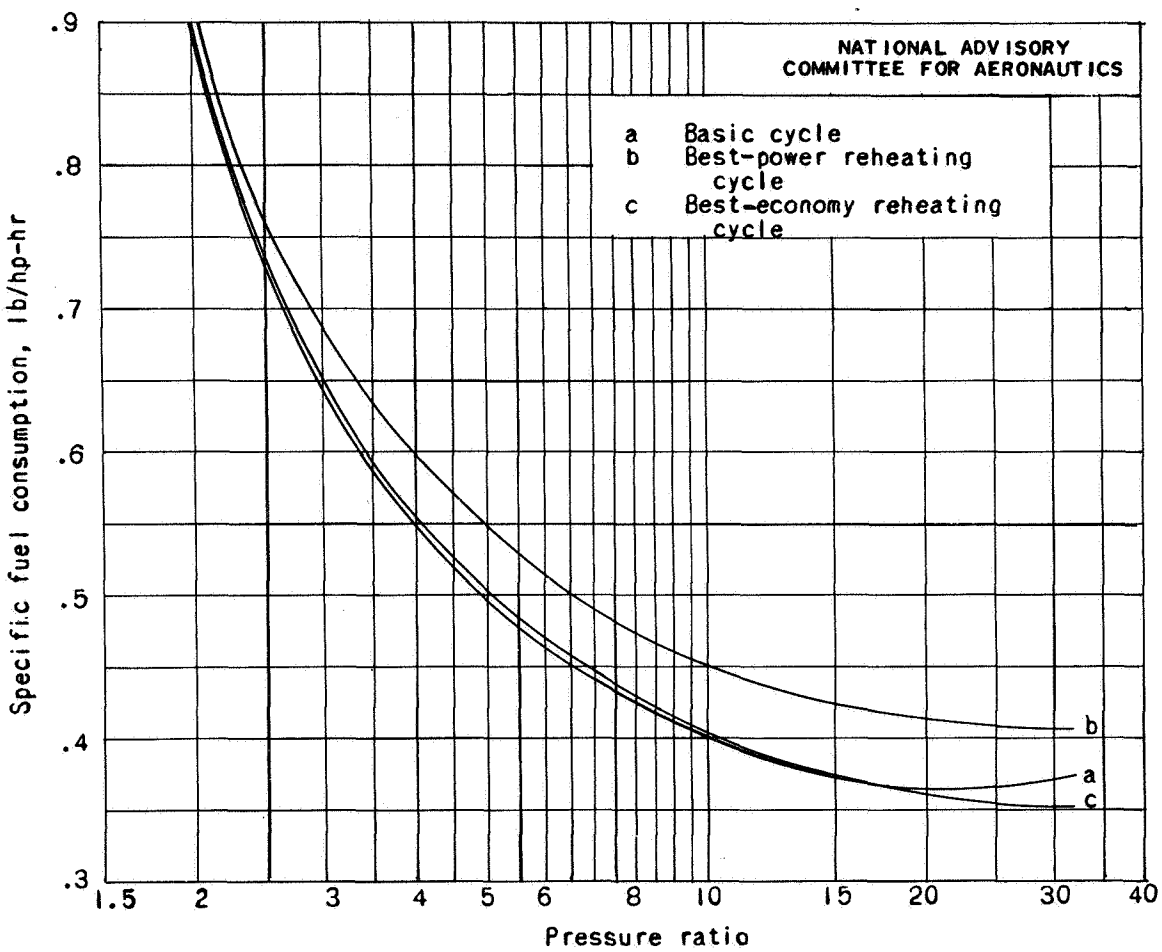
(b) Specific fuel consumption.

Figure 14. - Concluded. Performance of intercooling cycle with intercooling at point of lowest specific fuel consumption. Maximum temperature, 2000° R; NACA standard altitude, 30,000 feet; flight speed, 400 miles per hour; $\eta_{ad,c}$, 0.85; $\eta_{s,c}$, 0.84; $\eta_{ad,t}$, 0.90; $\eta_{s,t}$, 0.89; η_b, 0.90; C, 0.97; no pressure loss through intercooler or burner.



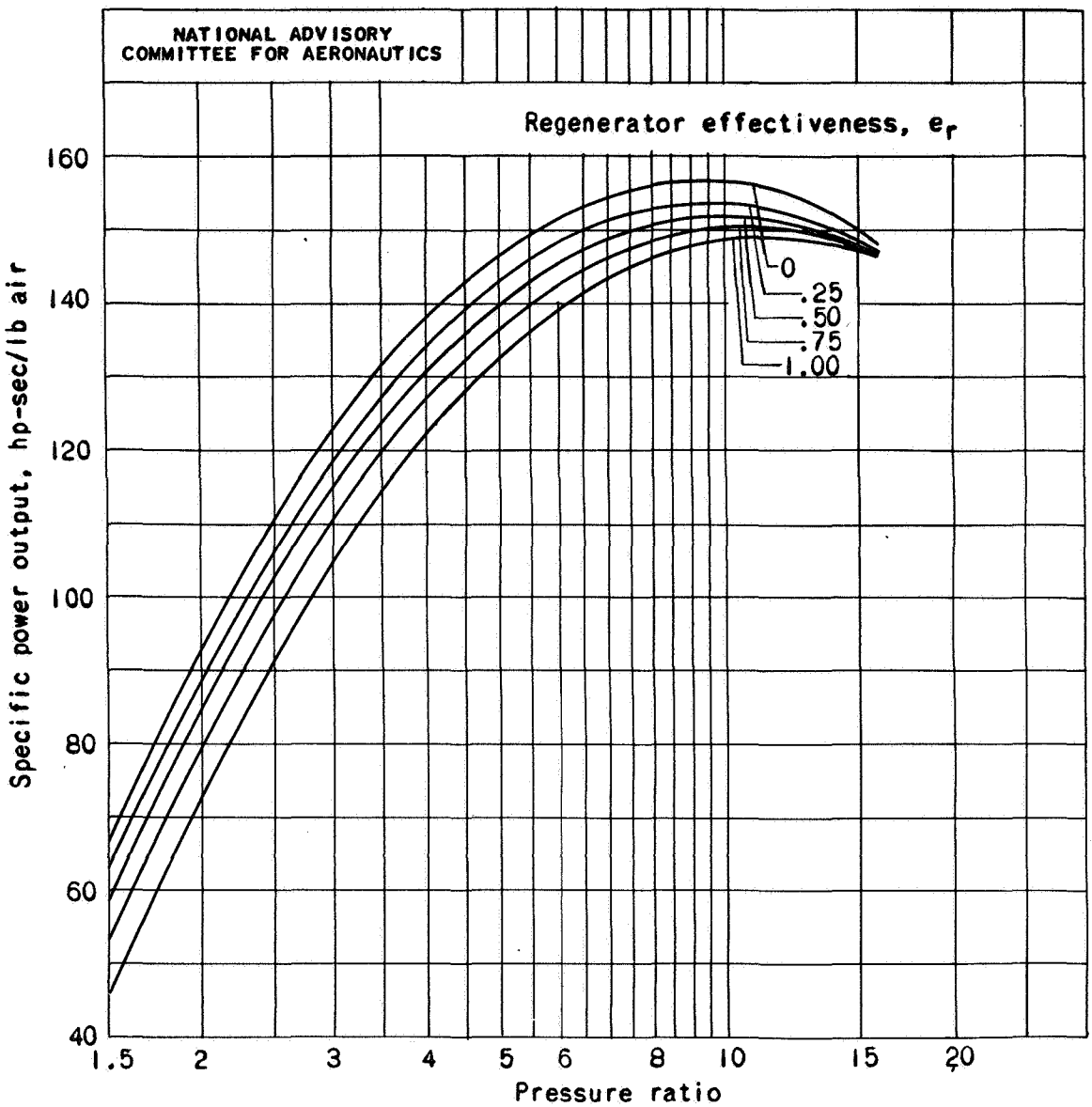
(a) Specific power output.

Figure 15. - Performance of reheating cycle with reheating at various points during expansion. Maximum temperature, 2000° R; NACA standard altitude, 30,000 feet; flight speed, 400 miles per hour; $\eta_{ad,c}$, 0.85; $\eta_{s,c}$, 0.84; $\eta_{ad,t}$, 0.90; $\eta_{s,t}$, 0.89; η_b, 0.90; C, 0.97; no pressure loss through reheater or burner.



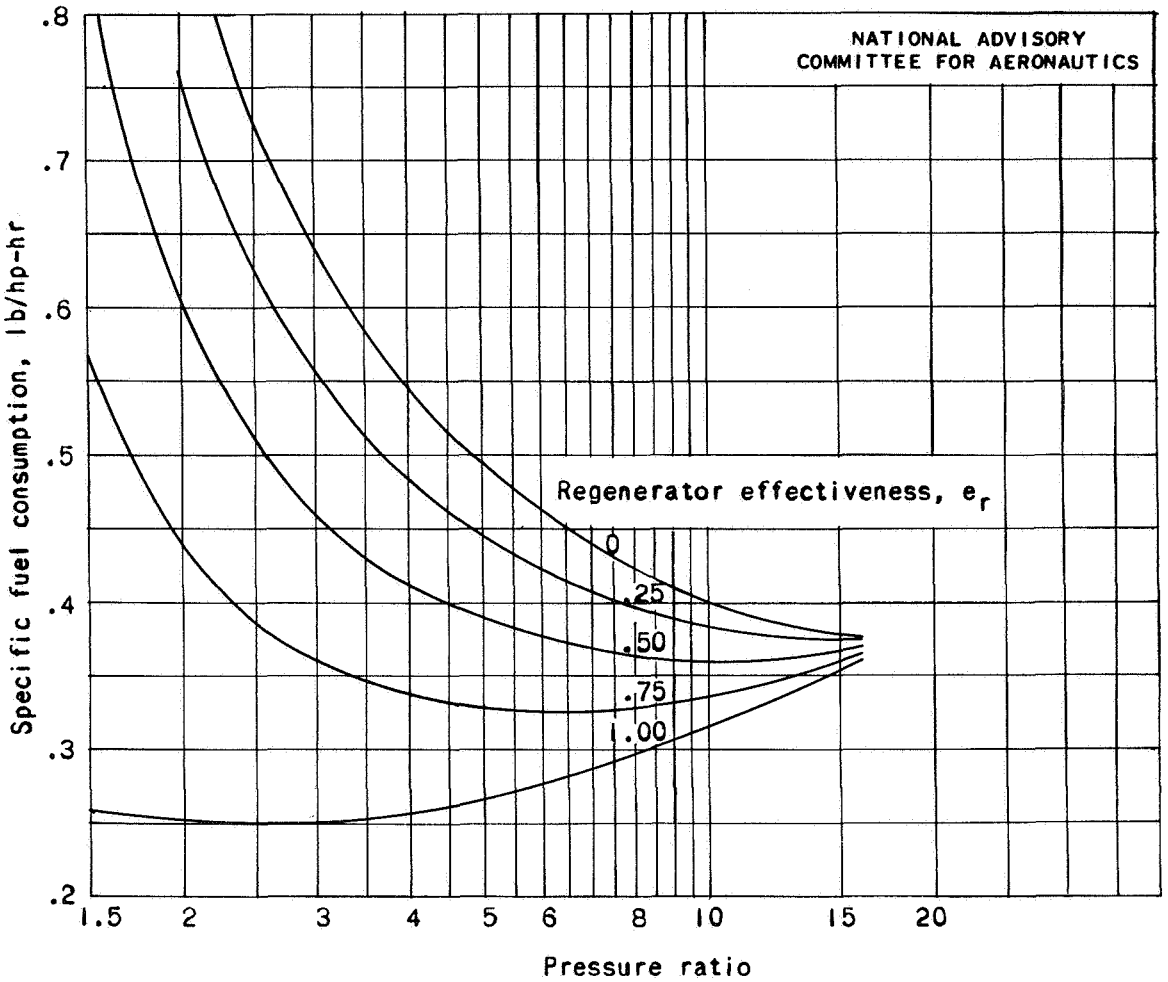
(b) Specific fuel consumption.

Figure 15. - Concluded. Performance of reheating cycle with reheating at various points during expansion. Maximum temperature, 2000° R; NACA standard altitude, 30,000 feet; flight speed, 400 miles per hour; $\eta_{ad,c}$, 0.85; $\eta_{s,c}$, 0.84; $\eta_{ad,t}$, 0.90; $\eta_{s,t}$, 0.89; η_b, 0.90; C, 0.97; no pressure loss through reheater or burner.



(a) Specific power output.

Figure 16. - Performance of regenerating cycle. Maximum temperature, 2000° R; NACA standard altitude, 30,000 feet; flight speed, 400 miles per hour; $\eta_{ad,c}$, 0.85; $\eta_{s,c}$, 0.84; $\eta_{ad,t}$, 0.90; $\eta_{s,t}$, 0.89; η_b, 0.90; C, 0.97; no pressure loss through regenerator or burner.



(b) Specific fuel consumption.

Figure 16. - Concluded. Performance of regenerating cycle. Maximum temperature, 2000° R; NACA standard altitude, 30,000 feet; flight speed, 400 miles per hour; $\eta_{ad,c}$, 0.85; $\eta_{s,c}$, 0.84; $\eta_{ad,t}$, 0.90; $\eta_{s,t}$, 0.89; η_b, 0.90; C, 0.97; no pressure loss through regenerator or burner.