STRENGTH ANALYSIS OF STIFFENED THICK BEAM WEBS

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A previously published method for the strength analysis of stiffened beam webs, with particular attention to computing crippling failure of the uprights, has been revised and extended to apply to beams with ratios of applied shear to buckling shear less than 2.5. A comparison of this revised method with the results of tests of thick-web beams is presented. The results in this paper concerning the procedures for calculating the critical shear stresses and for predicting forced crippling failure in the uprights supersede NACA TN No. 1364. Formulas and graphs applying to the parts of the strength-analysis method which have been revised are presented.

INTRODUCTION

Published methods for strength analysis of stiffened beam webs are of doubtful accuracy for beams with thick webs. Kuhn and Peterson suggested in reference 1 that the methods of that paper be limited to beams with ratios of web depth to web thickness greater than 200 but less than 1500 and ratios of upright to web thickness greater than 0.6. At that time there were very little experimental data to check the accuracy of these strength-analysis methods when applied to thicker webs, and the data that were available indicated some possibility that the strength-analysis methods of reference 1 would not be satisfactory for thicker webs.

The present investigation was undertaken to determine the accuracy that might be expected from the strength-analysis formulas of reference 1 when applied to beams with ratios of web depth to web thickness of approximately 115. As a result of this investigation some parts of the method in reference 1 were modified in order to obtain a method of strength analysis which would be satisfactory for thick-web beams as well as for thin-web beams similar to those of reference 1. The present paper, therefore, supersedes the sections of reference 1 which give the procedures for calculating the critical shear stresses and for predicting forced crippling failure in the uprights.
SYMBOLS

A

cross-sectional area, square inches

\( Cr \)

rivet factor \( \left( \frac{\text{Net area along line of holes}}{\text{Gross area along line of holes}} \right) \)

E

Young's modulus, ksi

I

moment of inertia, inches^4

P

force, kips

Q

static moment about neutral axis of parts of cross section as specified by subscript, inches^3

R

coefficient of edge restraint (see formula (2))

S

transverse shear force, kips

d

spacing of uprights, inches

e

distance from median plane of web to centroid of (single) upright, inches

h

depth of beam, inches (see Special Combinations)

k

diagonal-tension factor

t

thickness, inches (used without subscript signifies thickness of web)

\( \rho \)

centroidal radius of gyration of cross section of upright about axis parallel to web, inches (no sheet should be included)

\( \sigma \)

normal stress, ksi

\( \tau \)

shear stress, ksi

\( \eta \)

plasticity reduction factor (ratio of critical shear stress in the plastic region to the critical shear stress that would be obtained if the material were wholly elastic)

Subscripts:

F

flange

U

upright
W web
calc calculated
cr critical
e effective
g gross section of web
max maximum
meas measured
ult ultimate

Special Combinations:

\( P_u \) internal force in upright, kips

\( d_c \) clear width between uprights (measured between rivet lines on single uprights, measured between edges of uprights for double uprights), inches

\( h_c \) clear depth between flanges, inches

\( h_e \) depth of beam measured between centroids of flanges, inches

\( h_u \) length of upright measured between centroids of upright-to-flange rivet patterns, inches

\( k_{ss} \) theoretical buckling coefficient for plates with simply supported edges

\( R_{d}, R_{h} \) restraint coefficients for edges of sheet along flanges and upright, respectively (If \( d_c > h_c \), substitute \( h_c \) for \( d_c \), \( d_c \) for \( h_c \), \( R_d \) for \( R_h \), and \( R_h \) for \( R_d \)).

\( \sigma_0 \) "basic" allowable stress for forced crippling of uprights (valid for stresses in upright material below proportional limit in compression), ksi

\( \omega_d \) flange flexibility factor \( \left(0.7d \sqrt{\frac{t}{(I_c + I_T)h_c}}\right) \), where \( I_c \) and \( I_T \) are moments of inertia, about their own axis perpendicular to web, of compression flange and tension flange, respectively.
TEST SPECIMENS

The test specimens were 24S-T3 aluminum-alloy beams with a ratio of web depth to web thickness of approximately 115. The ratios of stiffener spacing to web depth were approximately 0.25 and 0.70. Both single-upright and double-upright beams were tested.

Each beam was given a code designation which parallels the designation used in reference 1. For example beam V-12-4S has the following meaning:

- V designates the present series of tests (series I, II, III, and IV were published in reference 1)
- 12 is the approximate depth of the beam in inches
- 4 is the number of the beam within the series
- S stands for single uprights (D for double uprights)

The nominal dimensions of the beams and the details of the construction are shown in figure 1. The actual properties of each beam are given in table 1.

The specimens were tested as simply supported beams in the jig shown in figure 2, which supported the beams laterally but did not restrain the bending of the beam. The flanges of the beam were supported by closely spaced vertical bars resting on rollers (not visible in the photograph) that allowed each bar to move parallel to the plane of the web as the beam deflected.

TEST PROCEDURE

Buckling loads for the web were determined by visual observation of the webs and by measuring the strains in the uprights with resistance-type-wire strain gages. There should not be any strain in the uprights until the critical shear stress is reached; however, because the webs had slight initial eccentricities, some strain in the uprights usually occurred as soon as any load was applied. The buckling load was determined by plotting the measured strain in the uprights against the shear load on the beam; the point at which the load-strain plot for the upright departed from a straight line was taken as an indication of buckling in the web. The critical shear stress in the web was computed from this buckling load by the formula
This formula gives the average shear stress in the web according to the engineering theory of bending.

RESULTS AND DISCUSSION

The results of the investigation are shown in table 2. Experimental buckling loads and failing loads are recorded. The failures were either forced crippling of the uprights or web rupture. Analysis of the present tests by the methods of reference 1 indicated that critical shear stresses and allowable upright stresses predicted by these methods were not sufficiently accurate for beams with thin uprights \( \left( \frac{tU}{t} < 1.3 \right) \) and thick webs \( \left( \frac{h}{t} \approx 1.5 \right) \). Methods, which give satisfactory results for thick-web beams as well as for beams similar to those of reference 1 \( \left( \frac{tU}{t} > 0.6 \right) \) and \( 200 < \frac{h}{t} < 1500 \), are discussed for computing the critical shear stresses and the allowable upright stresses.

Critical Shear Stress

The formula for the critical shear stress of the web was given in reference 1 as

\[
\tau_{cr} = \frac{s_w Q_F}{It} \left( 1 + \frac{2Q_w}{3Q_F} \right)
\]

(1)

A plot of this equation for a panel with four simply supported edges \( (R_h \text{ and } R_d \text{ equal to 1.0}) \) is shown as figure 3. A comparison of the experimental critical shear stresses with the critical shear stresses computed by formula (2), using the restraints \( R \) given by the empirical curves in reference 1, indicated that the values of \( R \) given in reference 1 are satisfactory for webs with double uprights, but are too high for webs with single uprights and \( \frac{tU}{t} < 1.3 \). Values of the restraint coefficient \( R_h \) for single uprights were computed from the experimental
critical shear stresses and were used to establish the curve shown in figure 4. The curve for double uprights shown in figure 4 is the same as the curve given in reference 1.

If the critical shear stress computed by formula (2) is beyond the elastic range of the material, the stress must be corrected for the reduced value of the modulus. In reference 1 critical stresses in the plastic range were obtained by drawing tangents to the elastic curve from \( \tau_{\text{ult}} \) at \( \frac{d_c}{t} = 0 \). These curves are shown in reference 1 for a panel with simply supported edges. In order to obtain the critical shear stress in the plastic range for any other set of edge conditions or any other material, a separate set of curves must be drawn.

Reference 2 presents a method of computing from the stress-strain curve of the material the plasticity reduction factor \( \eta \), which is the ratio of the critical shear stress in the plastic region to the critical shear stress that would be obtained if the material were wholly elastic; that is,

\[
\tau_{\text{cr}}(\text{plastic}) = \eta \tau_{\text{cr}}(\text{elastic})
\]

If formula (2) is substituted for \( \tau_{\text{cr}}(\text{elastic}) \), the expression for the critical shear stress in the plastic region is

\[
\tau_{\text{cr}} = \eta k_{\text{ss}} E \left( \frac{t}{d_c} \right)^2 \left[ R_h + \frac{1}{2} \left( R_d - R_h \right) \left( \frac{d_c}{h_c} \right)^3 \right]
\]

The critical shear stress in the plastic range may be obtained by computing \( \tau_{\text{cr}}/\eta \) from formula (3) and then reading \( \tau_{\text{cr}} \) from figure 5, which shows \( \tau_{\text{cr}} \) as a function of \( \tau_{\text{cr}}/\eta \). If the critical shear stress computed from formula (3) is plotted as a function of \( \frac{d_c}{t} \), the curve in the plastic range is practically a straight line and intersects \( \eta = 0 \) at \( \tau = 39 \text{ ksi} \). Formula (3), for practical purposes, gives the same line as that obtained by drawing a tangent to the elastic curve from \( \tau_{\text{ult}} \), because \( \tau_{\text{ult}} \) is between 37 ksi and 42 ksi. (See reference 3.)

The calculated critical shear stresses based on restraint \( R \) obtained from figure 4 and the measured critical shear stresses for both single-upright and double-upright beams are shown in table 2. The ratios of
measured critical stress to calculated critical stress vary from 0.77 to 1.21. These calculated values of $\tau_{cr}$ are probably adequate for the purpose of determining the diagonal-tension factor $k$; better results probably cannot be obtained so long as the restraint $R$ is represented only as a function of $t_u/t$, because representing $R$ by this function only is an extreme simplification of a complex problem.

The application of formulas (2) and (3) and the restraint curves of figure 4 to beams with thin uprights may give critical shear stresses lower than those that would be obtained if the presence of the uprights were disregarded entirely and if $\tau_{cr}$ were computed for a web bounded by the flanges and the root and tip bays of the beam. This result was obtained because the value of $\tau_{cr}$ for a panel between two uprights was assumed to be the same as the value of $\tau_{cr}$ for an individual panel bounded by edge members of the same size as the flanges and uprights. Actually, the adjacent panels in the beam have an appreciable effect on one another. In beam V-12-12S the value of $\tau_{cr}$ obtained by disregarding the uprights was higher than the value obtained by assuming that the uprights divided the web into separate panels. The observed $\tau_{cr}$ was 61 percent greater than the calculated $\tau_{cr}$ if the web was assumed to be divided into separate panels by the uprights, but only 21 percent greater than the calculated $\tau_{cr}$ if the presence of the uprights was disregarded entirely. In practice $\tau_{cr}$ must be calculated by both methods for beams with thin uprights and the higher value used, because the ratios $t/d_e$ and $d/h$ will be different for the two conditions and because no general rules seem to exist that predict which method would give the higher value of $\tau_{cr}$.

**Forced Crippling Failure in Uprights**

Four types of failure of uprights are discussed in reference 1; but only one, forced crippling failure of the uprights, was observed in the present tests. General elastic instability failure of the web and uprights seems to be the only other type of upright failure likely to occur in thick-web beams.

The shear buckles in the web force the buckling of the upright in the leg attached to the web. The amount of the forced crippling (buckling) depends upon the relative sturdiness of the upright and web. In reference 1 formulas for forced crippling were based on the parameter $k \sqrt{t_u/t}$. If $k$ was less than 0.5, an effective value of $k$ was used. Use of this parameter in the present test of thick-web beams indicated that it was not satisfactory. The allowable stresses were too low if $k$ itself
was used and too high if the effective $k$ was used. Much better agreement with the present tests of thick-web beams and about the same agreement for the tests in reference 1 were obtained by using the parameter $k^3(tu)^{-1}$. This parameter also eliminated the necessity for using an effective value of $k$.

Figure 6 is a plot of the values of $\sigma_{U_{\text{max}}}$ computed from the loads on the beams at failure, for all the single-upright beams of the present investigation and for all the single-upright beams shown in figures 22 and 23 of reference 1. (The beams shown in figs. 22 and 23 of reference 1 represent about 90 beams tested by four manufacturers and 32 beams tested by NACA.) The stresses $\sigma_{U_{\text{max}}}$ were computed with the aid of the analysis chart of figure 7. This chart covers the low range of $\Delta y_{\text{p}}/dt$ and $\tau/\tau_{\text{cr}}$ that is not shown in the analysis charts of reference 1. The points shown in figure 6 are fairly evenly distributed about the average curve

$$\sigma_o = 32k^3(tu)^{-1}$$

The curve recommended for design is given by the formula

$$\sigma_o = 26k^3(tu)^{-1}$$

and is about 20 percent below the average curve. Only two points fall definitely below this design curve. The lowest of these points

\[
\text{the point at } \sigma_{U_{\text{max}}} = 10.6 \text{ ksi and } k^3(tu)^{-1} = 0.63
\]

was computed from the failing load for one of the manufacturer's beam tests. The NACA constructed and tested a duplicate of the beam tested by the manufacturer. In the NACA investigation a local buckle developed in the outstanding leg of one of the uprights at a load approximately 11 percent above the failing load given by the manufacturer; the NACA beam continued to carry load until the load was about 73 percent above the failing load given by the manufacturer. At this load two local buckles developed in each stiffener and the edges of the stiffeners started to crack at these buckles. No detail information about the behavior of the manufacturer's beams was furnished but the behavior was probably similar to that observed in the NACA test; the manufacturer might have interpreted the first buckle in the upright as failure and made no further attempt to apply more load.
The upper curve on figure 6 is 20 percent above the average curve. One of the points in the present series of tests falls above the curve, and most of those from reference 1 that were more than 20 percent above the average curve when the parameter \( k \sqrt{u/t} \) was used are still above the 20-percent line in figure 6.

Figure 8 shows all the data now available for double uprights using the parameter \( k \left( \frac{tU}{t} \right)^{\frac{1}{3}} \). The formula for the average curve for double uprights is

\[
\sigma_0 = 27k \left( \frac{tU}{t} \right)^{\frac{1}{3}}
\]  

(5a)

The formula for the recommended design curve for double uprights is

\[
\sigma_0 = 21k \left( \frac{tU}{t} \right)^{\frac{1}{3}}
\]  

(5b)

None of the tests points for the beams with double uprights is below the recommended design curve and only one point is more than 20 percent above the average curve.

In the present tests the ratio of the actual failing loads to the predicted failing load ranged from 0.90 to 1.14.

In reference 1 it was suggested that the formula for computing the effective area of single uprights

\[
A_{Ue} = \frac{A_U}{1 + (\frac{e}{\rho})^2}
\]

might not be satisfactory for thick-web beams, because the simplifying assumptions implied by this formula may not be justified. These implied assumptions are:

(a) The eccentricity \( e \) of the load on the upright is constant

(b) The ratio \( e/\rho \) is not changed appreciably if the contribution of the web to the effective cross section of the upright is neglected
Assumption (a) is plausible if the uprights are very closely spaced because the web then moves with the uprights (reference 4). Assumption (b) would not seem to be justified for thick-web beams; however, for low values of the ratio $\tau/\tau_{cr}$ a large difference in the total effective area of the upright causes only a small change in $\sigma_U$ and, therefore, satisfactory results are obtained. A study of the analysis chart in figure 7 will help to explain this fact; the curves approach a vertical line as the ratio $\tau/\tau_{cr}$ decreases.

Web Failures

The average nominal shear stress in the web was computed by formula (1); the peak value of the nominal shear stress in the web for predicting web rupture was computed by the formulas of reference 1. Critical shear stresses were computed from formulas (2) and (3) by means of the restraint coefficients given in figure 4. The allowable values of the peak shear stress in the web, which are shown in figure 9, were obtained from reference 5. The values are based on tests of long webs subjected to loads approximating pure shear and contain an allowance for the rivet factor; this factor may be included because tests have shown that the ultimate shear stress on the gross section is almost constant in the normal range of rivet factor ($C_r > 0.6$).

In the six thick-web beams which failed by web rupture, the ratio of actual failing load to predicted failing load ranged from 0.92 to 1.18. This degree of accuracy is approximately the same as that obtained for the thinner beams ($200 < h/t < 1500$) discussed in reference 1. All these comparisons are based on actual material properties.

CONCLUDING REMARKS

The methods of predicting the critical shear stresses, forced crippling failures of the uprights, and rupture of the webs presented are applicable to stiffened beam webs with ratios of web depth to web thickness between 115 and 1500. The accuracy of these methods is about the same as that of the methods presented in NACA TN No. 1364 which were applicable only to beams with ratios of web depth to web thickness between 200 and 1500.

Langley Aeronautical Laboratory
National Advisory Committee for Aeronautics
Langley Air Force Base, Va., December 16, 1948
REFERENCES


   Part II - Sheet Metal Girders with Spars Resistant to Bending. Oblique Uprights - Stiffness. NACA TM No. 605, 1931.
   Part III - Sheet Metal Girders with Spars Resistant to Bending. The Stress in Uprights - Diagonal Tension Fields. NACA TM No. 606, 1931.

### TABLE 1.— PROPERTIES OF TEST BEAMS

<table>
<thead>
<tr>
<th>Beam</th>
<th>$h_0$ (in.)</th>
<th>$h_U$ (in.)</th>
<th>$t$ (in.)</th>
<th>$d$ (in.)</th>
<th>Uprights (in.)</th>
<th>$A_U$ (sq in.)</th>
<th>$A_{Ue}$ (sq in.)</th>
<th>$A_t$</th>
<th>$A_{Ue}$ (sq in.)</th>
<th>Flanges ($c/s$)</th>
<th>$c_d$</th>
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</thead>
<tbody>
<tr>
<td>V-12-1D</td>
<td>11.38</td>
<td>10.88</td>
<td>0.1000</td>
<td>2.75</td>
<td>$\frac{1}{2} \times \frac{1}{2} \times 0.0666$</td>
<td>0.1260</td>
<td>0.1260</td>
<td>0.458</td>
<td>0.458</td>
<td>$\frac{1}{4} \times \frac{1}{4} \times \frac{3}{16}$</td>
<td>0.58</td>
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<tr>
<td>V-12-2S</td>
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<td>10.88</td>
<td>0.1005</td>
<td>2.75</td>
<td>$\frac{1}{2} \times \frac{1}{2} \times 0.0690$</td>
<td>0.0640</td>
<td>0.0626</td>
<td>0.233</td>
<td>0.096</td>
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<td>0.58</td>
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<td>V-12-3D</td>
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<td>10.88</td>
<td>0.1110</td>
<td>2.75</td>
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<td>0.0948</td>
<td>0.342</td>
<td>0.342</td>
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<td>V-12-4S</td>
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<td>10.75</td>
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<td>0.0239</td>
<td>0.171</td>
<td>0.086</td>
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<td>V-12-5D</td>
<td>11.75</td>
<td>10.75</td>
<td>0.1115</td>
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<td>0.2546</td>
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<td>0.912</td>
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<td>V-12-6S</td>
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<td>10.75</td>
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<td>0.413</td>
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<td>V-12-7B</td>
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<td>V-12-8S</td>
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<td>0.2860</td>
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<td>0.399</td>
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<td>V-12-10S</td>
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<td>0.2340</td>
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<td>V-12-13D</td>
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<td>10.50</td>
<td>0.1000</td>
<td>7.00</td>
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<td>0.1214</td>
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<td>0.173</td>
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<td>0.1007</td>
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<td>0.1622</td>
<td>0.220</td>
<td>0.220</td>
<td>$2 \times 2 \times \frac{7}{16}$</td>
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### Table 2: Test Data and Results

<table>
<thead>
<tr>
<th>Beam</th>
<th>(\tau_{cr,calc}) (ksi)</th>
<th>(\tau_{cr,meas}) (ksi)</th>
<th>(P_{ult,calc}) (kips)</th>
<th>(\tau_{ult} = \frac{P_{ult}}{2he_b}) (ksi)</th>
<th>(\tau_{ult} = \frac{P_{ult}}{(\tau_{cr})_{calc}})</th>
<th>(k)</th>
<th>(P_{1}) (kips)</th>
<th>(P_{2}) (kips)</th>
<th>Observed Failure</th>
<th>(P_{ult}/P_{1})</th>
<th>(P_{ult}/P_{2})</th>
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</thead>
<tbody>
<tr>
<td>V-12-1D</td>
<td>28.2</td>
<td>23.3</td>
<td>74.6</td>
<td>31.9</td>
<td>1.13</td>
<td>0.028</td>
<td>75.0</td>
<td>80.0</td>
<td>Forced crippling</td>
<td>0.99</td>
<td>0.93</td>
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<tr>
<td>V-12-2D</td>
<td>22.0</td>
<td>17.0</td>
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*a*For web failure.

*b*For forced crippling failure.

*c*\(P_{ult}\) is the lowest one of the predicted loads \(P_{1}\) or \(P_{2}\).

*d*\(P_{ult}\) is that predicted load (\(P_{1}\) or \(P_{2}\)) which corresponds to the observed type of failure.
Figure 1. Dimensions of test beams.
Figure 2.— Test jig.
Figure 3.—Buckling stresses $\tau_{cr}$ for plates with simply supported edges. $E = 10,600$ ksi.
Figure 4.—Empirical restraint coefficients for calculating web buckling stress.

Figure 5.—Relationship of $\tau_{cr}$ to $\frac{\tau_{cr}}{\eta}$ for 24S-T3 aluminum-alloy sheet.
Figure 6.— Stresses in single uprights at failure caused by forced crippling.
Figure 7.—Diagonal-tension analysis chart.
Figure 8.—Stresses in double uprights at failure caused by forced crippling.
Figure 9.- Average ultimate shear stresses in the gross section.