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SPONSE OF A HELICOPTER ROTOR TO OSCILLATORY PITCH AND THROTTLE MOVEMENTS

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SUMMARY

Measurements of the drag-angle oscillation of a helicopter rotor were made with the Langley helicopter apparatus to determine both the natural frequencies and the damping required to prevent excessive drag-angle-oscillation response to various throttle and control movements.

There are two types of drag-angle oscillations: symmetrical oscillations in which all blades lag and advance together and unsymmetrical oscillations in which the blades are out of phase with each other. The symmetrical type was induced by a main-pitch oscillation or a throttle oscillation at a frequency considerably below shaft speed, and the unsymmetrical type was induced by a cyclic-pitch oscillation at a frequency near that of shaft speed.

The results show that the engine and gear-box inertias and the shaft torsional stiffness are factors influencing the frequency of the symmetrical drag-hinge oscillations, and the experimental results show good agreement with the theory used. The unsymmetrical drag-hinge-oscillation frequency is apparently unaffected by the engine and gear-box inertia and the shaft torsional stiffness but is influenced by the pylon bending stiffness, and the experimental results show fair agreement with the predicted values.

A calculation of the damping required in the drag-hinge dampers for the symmetrical case shows that the damping should be approximately 35 percent of the critical. The results further indicate care should be exercised to ascertain that no regular disturbing forces are present in the helicopter - such as a hunting pitch or throttle governor or a hunting automatic pilot - with frequencies near those of the resonant conditions.

INTRODUCTION

The question has been raised as to the possibility of exciting drag-hinge oscillations, by throttle or pitch manipulations, in helicopter rotors with the usual hinged-blade construction. The ability to predict the natural frequency of the in-plane motion is essential in rotor-stress calculations, vibration analysis, and rational design of the rotor hub, specifically the drag damper. The object of this research is the determination of the natural frequencies of the drag-hinge oscillation and the damping required to prevent excessive response to various throttle and control movements.

Unsymmetrical whirling modes, response to forced lateral vibration of the pylon, and instabilities are discussed in reference 1. However, symmetrical modes, those induced by torque oscillations either aerodynamic or engine, in general have not been considered for helicopters before. The method of analysis for the symmetrical oscillations used in this investigation is based to a large extent on the work presented in reference 2, in which the effect of engine inertia and drive-shaft torsional stiffness on the symmetrical oscillations of a hinged propeller blade was considered. This method is particularly suitable for determination of the natural frequencies of hinged helicopter rotor blades for symmetrical oscillation. A method of estimating the required drag damping for this type of oscillation, which is given in reference 3, is also included.

A test program was inaugurated to obtain data on helicopter in-plane blade motion due to throttle, main-pitch, and cyclic-pitch oscillations. Results of these tests are presented for selected conditions and compared with the theories of references 1 and 2. These tests were conducted on the helicopter tower of the Langley helicopter apparatus.

SYMBOLS

b number of blades per rotor

R blade radius, feet

r radial distance to blade element, feet

c blade section chord, feet

c_e equivalent blade chord, feet $\left(\dfrac{\int_0^R cr^2dr}{\int_0^R r^2dr}\right)$

σ rotor solidity $\left(bc_e/\pi R\right)$

d	radius of gyration of a blade about the vertical-pin axis, inches
e	distance from axis of rotation to vertical-pin axis, inches
l	distance from vertical-pin axis to center of gravity of blade, inches
s	radius of gyration of a blade about its center of mass, inches
m_b	blade mass (one blade), pound-seconds^2 per inch
m_p	effective mass of rotor hub and pylon, pound-seconds^2 per inch
M	combined mass of blades, rotor hub, and pylon, pound-seconds^2 per inch $\left(M = bm_b + m_p\right)$
q	mass ratio $\left(\dfrac{bm_b}{m_p + bm_b}\right)$
I	moment of inertia for one blade about vertical pin, pound-inch-seconds^2
I_2	combined blade mass moment of inertia about vertical pin for all blades, pound-inch-seconds^2
I_3	combined blade mass moment of inertia about blade center of gravity, including assembly inboard to the vertical-pin axis, pound-inch-seconds^2
I_4	engine inertia, pound-inch-seconds^2
I_5	gear-box and shaft inertia, pound-inch-seconds^2
I_6	combined gear-box and shaft and effective engine polar moment of inertia of rotating and reciprocating parts, pound-inch-seconds^2 $\left(I_6 = n^2\left(I_4\right) + I_5\right)$
k	drive-shaft torsional stiffness, pound-inches per radian
K	pylon bending stiffness, pounds per inch
n	gear ratio of engine to rotor shaft
C	damping coefficient, pound-foot-seconds per radian

C_{cr}	critical damping coefficient, pound-foot-seconds per radian
ω	drag-oscillation angular frequency, radians per second
ω_n	drag-oscillation natural frequency, radians per second
ω_f	whirling frequency of the unbalanced side forces, radians per second
Q	rotor-shaft torque, pound-feet
Ω	rotor angular velocity, radians per second
ζ	drag-angle oscillation amplitude, degrees
ζ_{st}	drag-angle movement for a given static pitch change, degrees

$$\Lambda_1 = \frac{e}{l\left[1 + \left(\frac{s}{l}\right)^2\right]}$$

$$\Lambda_3 = \frac{q}{2\left[1 + \left(\frac{s}{l}\right)^2\right]}$$

DESCRIPTION OF ROTOR AND DRIVE ASSEMBLY

A three-blade rotor representative of a helicopter weighing 2500 pounds was used in this investigation. The blades are plywood-covered and have NACA 23015 airfoil sections, a linear washout of 8°, and a solidity of 0.042.

A summary of the pertinent rotor and drive-assembly characteristics is as follows:

Rotor-blade characteristics:

Blade radius, feet	19
Blade twist, degrees	-8
Solidity, σ	0.042
Blade area (total, three blades), square feet	46.3
Blade section	NACA 23015
Blade weight (one blade including drag-hinge assembly), pounds	82.5
Blade moment of inertia (total, three blades) about vertical-pin axis, I_2 pound-inch-seconds^2	5760
Blade moment of inertia (total, three blades) about their center of gravity, I_3, pound-inch-seconds^2	2690
Blade center-of-gravity location from vertical-pin axis, inches	69.1
Vertical-pin-axis location from center of rotation, e, inches	9.08
Offset of center line of flapping hinge from center of rotation	0
Blade radius of gyration about vertical-pin axis, d, inches	94.5
Blade radius of gyration about its center of gravity, s, inches	64.3
Combined mass of pylon, blades, and rotor hub, M, pound-seconds^2 per inch	0.9

Drive-shaft and gear-box characteristics:

Drive-shaft torsional stiffness, k, pound-inches per radian	524,000
Pylon bending stiffness, K, pounds per inch	16,400
Gear reduction, n	6:1 and 12:1
Effective moment of inertia of engine, n^2I_4, pound-inch-seconds^2	$n^2(60)$
Moment of inertia of drive system with 6:1 gear box, I_5, pound-inch-seconds^2	2750
Moment of inertia of drive system with 6:1 gear box with reduced I_5 shaft inertia, pound-inch-seconds^2	2400
Moment of inertia of drive system with 12:1 gear box with reduced I_5 shaft inertia, pound-inch-seconds^2	2400

Damper characteristics:

Oil in damper, no bypass (unmodified), pound-foot-seconds per radian	938
Oil in damper, 0.050-inch bypass, pound-foot-seconds per radian	693
Oil in damper, 0.125-inch bypass, pound-foot-seconds per radian	122
No oil in damper, pound-foot-seconds per radian	Near 0

INSTRUMENTATION AND TEST METHODS

Quantities measured during tests included the following: Rotor angular velocity, rotor-shaft torque, rotor thrust, blade pitch angle, blade drag angle, blade flapping angle, side forces, and cyclic pitch.

All the data were obtained with a recording oscillograph so that the frequency and amplitude could be easily measured. Rotor angular velocity was obtained by a breaker installed on the shaft and leading to one of the oscillograph circuits. Rotor-shaft torque was measured by electric strain gages, as was the rotor thrust. Blade pitch angle, drag angle, flapping angle, and swash-plate position were measured by potentiometer control-position transmitters. A complete description of the recording system is contained in reference 4.

The drag-hinge dampers are used to minimize and nullify disturbances in the plane of rotation. The dampers were filled with hydraulic damper oil to approximate viscous damping, that is, a damping force directly proportional to the velocity.

Several investigations of the symmetrical oscillations were conducted with varying amounts of damping and were begun with the damper in its original form and then by varying the oil bypass to a final no-oil damping condition. The mean geometric pitch angle for these tests was adjusted to 2° and to 12.5°, on which was superimposed a sinusoidal-oscillation main pitch of $\pm 1^{\circ}$. Tests were conducted for the most part at a main-pitch angle of 12.5°. The investigation conducted at a main-pitch angle of 2° was discontinued inasmuch as the magnitude of the torque, the drag, and the other related oscillations measured was small and, therefore, made accurate measurement difficult. The oscillation was produced by an electric motor driving a system of gears and control links. Various rates of main-pitch oscillation were attained by changing the speed of the electric motor with a Variac.

Investigations were conducted for several representative shaft speeds and for three effective engine and gear-box inertias:

(a) Engine to shaft 6:1 gear reduction

(b) Engine to shaft 6:1 gear reduction with reduced shaft inertia

(c) Engine to shaft 12:1 gear reduction with reduced shaft inertia

Tests were also conducted with the 6:1 gear-reduction condition to determine the effect of engine throttle oscillation on the drag-hinge movement.

The effect of cyclic-pitch oscillations was also investigated. During these tests the main pitch was set at a geometric pitch of 12.5° and the swash plate was given a sinusoidal oscillation producing a cyclic-pitch variation of $\pm 1.5^{\circ}$.

ANALYSIS

The method of analysis is, for convenience, divided into three sections: The first section predicts the frequency of the symmetrical drag-angle oscillation, which is the one caused by main-pitch or throttle oscillation. The second section predicts the frequency of the unsymmetrical drag-angle oscillation, which is the one caused by cyclic-pitch oscillation. The third section deals with the damping required to prevent excessive response of the symmetrical oscillations. The methods of determining the physical constants used in this analysis are given in the appendix.

Symmetrical Oscillations

Most prior investigations have dealt with the unsymmetrical type of drag-hinge oscillations in which the shaft torsional stiffness and engine inertia have no effect. For the case of symmetrical oscillations of the drag hinge, however, the engine and gear-box inertia and shaft torsional stiffness, as well as the offset of the drag hinge from the center line of rotation, must be considered (reference 2) in calculating the natural frequency. The natural frequency is obtained by solving the quadratic

$$\omega_n^4 - c_1\left(t_1^2 + c_2 t_2^2\right)\omega_n^2 + c_1 t_1^2 t_2^2 = 0 \tag{1}$$

where t_1 is the frequency of engine and gear box against the shaft stiffness and

$$t_1^2 = \frac{k}{I_6}$$

$$t_2^2 = bm_b l e \Omega^2 \left[\frac{I_6 + I_3 + bm_b\left(l + e\right)^2}{I_6\left(I_3 + bm_b l^2\right) + I_3 bm_b e^2}\right]$$

$$c_1 = \frac{I_6\left(I_3 + bm_b l^2\right) + I_3 bm_b e^2}{I_3 bm_b e^2}$$

$$c_2 = \frac{I_3 + bm_b(l + e)^2}{I_6 + I_3 + bm_b(l + e)^2}$$

Equation (1) yields two pairs of natural frequencies. The higher frequency is predominantly that of the engine-hub system whereas the lower frequency is predominantly that of the hub-blade system. The lower frequency is investigated in this paper and the result is expressed as a ratio of the shaft angular velocity to the drag-angle frequency Ω/ω_n, although this ratio will change slightly with different shaft speeds.

Unsymmetrical Oscillations

The cyclic-pitch oscillation introduces a different type of drag oscillation than does the oscillation of the throttle or main pitch in that there is no apparent variation of the net torque or rotor speed. However, large side forces are present at a cyclic-pitch-oscillation rate near that of the shaft speed. In this phenomenon, no net torque or speed variation is considered to be present because each blade executes its own oscillation in such a manner that the lag of one blade compensates for the advance of the next blade and the large side forces are due to rotor unbalance caused by unsymmetrical blade displacements.

Since, for the case of unsymmetrical drag-hinge oscillations caused by movement of the cyclic pitch, no apparent variation in the net torque or rotor speed was noted, the effective engine inertia and the shaft torsional stiffness have no effect and may be considered as infinite. If this assumption is used, the frequency of the drag-hinge oscillation can be determined by the method of reference 2 for the case of infinite pylon bending stiffness by solving the equation

$$\frac{\omega_n^2}{\Omega^2} = \frac{bm_b l e}{I_3 + bm_b l^2} \qquad (2)$$

which is a special case of equation (1).

The cases for which the pylon bending stiffness is not infinite are discussed in reference 1, which presents a theoretical analysis for calculating the resonant frequency of the side forces due to unsymmetrical drag-hinge displacement. This resonant side-force frequency is called the whirling frequency.

For any given rotor, these whirling frequencies ω_f may be calculated for the case of no damping and isotropic supports by the equation

$$\frac{K}{M} - \omega_f^2\left[\Omega^2\Lambda_1 - \left(\omega_f - \Omega\right)^2\right] - \Lambda_3\omega_f^4 = 0 \tag{3}$$

The difference between the rotor speed and the whirling speed is the rate of progression of the unbalance around the center of rotation. This is also the frequency of the drag-hinge unsymmetrical oscillations. Equations (2) and (3) are seen to yield essentially the same result by the use of a value of infinity for the pylon bending stiffness. The pylon stiffness of an actual helicopter, however, is generally low; therefore, the method of reference 1 must be used.

Blade Damping

The section entitled "Symmetrical Oscillations" indicates that the rotor hub does not rotate with uniform velocity but oscillates during blade vibration. The motion of the engine as well as the blades, therefore, had to be taken into account in order to calculate the frequencies of the symmetrical oscillations accurately. Similarly, the oscillation amplitude and the damping of the entire system should be considered in order to make an accurate determination of the damping. However, since there are various sources of damping, such as engine damping, which cannot readily be determined accurately, the calculation of the air damping does not warrant the consideration of the motion of the entire system. For simplicity, in the present air- and critical-damping analysis, the hub is assumed to rotate with uniform velocity and all the vibration energy of the system appears in the blade. The blade is considered a rigid body having a single degree of freedom oscillating about the drag hinge. The engine and gear box do oscillate, however, and their friction contributes appreciably to the damping in the system.

Reference 3 (pp. 38 and 52) shows that, for torsional oscillation, the damping constant C is defined as the torque caused by an angular speed of rotation of 1 radian per second and the critical damping C_{cr} is given as

$$C_{cr} = 2I\omega_n \tag{4}$$

where I is the moment of inertia of a blade about the vertical-pin axis and ω_n is the natural frequency of the blade about the vertical-pin axis. The ratio of C/C_{cr} is called the damping factor and is the ratio that determines the amplitude of the response of the blade to a given control movement.

An estimate of the air damping may be obtained by referring to reference 3 (p. 261). For any given rotor and blade-pitch-angle setting, the steady-state relation between the torque and rotational speed is expressed by

$$Q = \text{Constant} \times \Omega^2$$

and the air damping for any particular speed is given as twice the slope of this curve at that speed:

$$C_{air} = 2\frac{dQ}{d\Omega}$$

Friction damping of the drag-hinge oscillations is caused by gear and bearing friction and hysteresis losses in the rotor head and drive assembly. The amount of damping due to these losses is not easily calculated and varies with each installation. Usual practice is to allow an empirical value that makes the calculated results coincide with the measured values (reference 3, p. 264).

RESULTS AND DISCUSSION

For convenience the results and discussion are divided into four sections - the first section pertaining to the symmetrical drag-hinge oscillations, the second section pertaining to the unsymmetrical drag-hinge oscillations, the third section pertaining to the damping required to prevent excessive response of the symmetrical drag-hinge oscillation, and the fourth section pertaining to a general discussion of the problem.

Symmetrical Oscillations

Figure 1 shows a typical record of a resonant condition of the symmetrical drag-hinge oscillations. This record shows that the resonant condition of the drag angle due to the throttle or main-pitch oscillations is accompanied by large periodic variations of the rotor torque and speed at the same frequency as the drag-angle motion and is usually excited at a main-pitch-oscillation speed considerably slower than shaft speed.

A summary of the data obtained with the various damping rates and effective engine and gear-box inertias at a rotor speed of approximately 220 rpm is presented in table I. These values are plotted in figures 2 to 4. Figure 2 shows a typical resonant diagram for the various damping conditions with an effective engine and gear-box inertia of approximately 4910 pound-inch-seconds^2. Figure 3 shows a resonant curve with only air and friction damping for an effective engine inertia of 4560 pound-inch-seconds^2, and figure 4 is the resonant diagram with the same damping but with an effective engine and gear-box inertia of approximately 11,000 pound-inch-seconds^2. With an effective engine and gear-box inertia of approximately 11,000 pound-inch-seconds^2, the ratio of the drive-shaft angular frequency to the drag-oscillation frequency is 3.70. A reduction of the inertias to approximately 4910 pound-inch-seconds^2 reduces the frequency ratio to 2.85. A further reduction of the inertias to about 4560 pound-inch-seconds^2 reduces the frequency ratio to about 2.7.

Figure 5 shows the agreement between the frequency ratios measured and the theoretical curve calculated from equation (1) for the various amounts of effective engine and gear-box inertias and a shaft speed of 23 radians per second. This figure also indicates that the effective engine and gear-box inertias are a definite factor affecting the natural frequency of the symmetrical drag-hinge oscillations due to main-pitch or throttle oscillations and must be considered in computing the frequency of these oscillations. The ratio of the shaft angular velocity to the drag-angle frequency changes only slightly with large changes in shaft angular velocity; for example, at an effective engine and gear-box inertia of 4910 pound-inch-seconds^2, a change from an angular velocity of 23 radians per second to 27 radians per second yields a change in Ω/ω_n of 2.96 to 3.16. The constants used in calculating the theoretical curve for the configuration tested were as follows: $k = 524{,}000$ pound-inches per radian, $bm_b = \frac{3(82.5)}{386} = 0.642$, $l = 69.1$ inches, $e = 9.08$ inches, $\Omega = 23$ radians per second, and $I_3 = 2690$ pound-inch-seconds^2.

The magnitude of the torque oscillations superimposed upon the steady-state running torque is also of interest. The amplitude of this oscillating torque for the symmetrical drag-hinge oscillations caused by oscillating throttle or control movements can be approximately determined by multiplying the steady-state running-torque change due to a given throttle or main-pitch movement by the drag-angle amplification factor.

The higher frequency from equation (1) is usually too high to be excited by a pitch oscillation but may possibly be excited by torque oscillations caused by engine cylinder firing. For the case considered, the higher frequency is of the order of 180 radians per second; whereas the lower frequency is only approximately 7.3 radians per second.

Unsymmetrical Oscillations

Figures 6 and 7 show two typical records of the resonant condition of the unsymmetrical drag-hinge oscillations. These records show that two resonant conditions exist, one excited by a cyclic-pitch oscillation below shaft speed (fig. 6) and one excited by a cyclic-pitch oscillation above shaft speed (fig. 7). These figures also show that this resonant condition is accompanied by rotor mass unbalance with the same frequency as the cyclic-pitch motion and results in large side forces, but no apparent periodic variation in the rotor speed or net rotor torque exists. The experimental values of the unsymmetrical drag-angle frequencies yield values of Ω/ω_n of approximately 5.

A summary of the data obtained for one representative condition is presented in table II. These values are plotted in figure 8 and show the two resonant frequencies, one at 0.81 of rotor speed and the other at 1.2 of the rotor speed. For the configuration tested, the amplitude of the side forces present for the case of a cyclic-pitch oscillation of $\pm 1.5^\circ$ and at a rate faster than the shaft speed was of the order of 1550 pounds. These side forces are not seen in the records of the symmetrical drag-hinge oscillations since in this case all the blades lag and advance simultaneously. For the unsymmetrical case in which each blade executes its own oscillation, however, a large rotor unbalance (side force) due to the unsymmetrical blade displacements occurs. Also, no apparent net-torque or rotor-speed variation occurs because the blades move in such a manner that the lag of one blade compensates for the advance of the next.

The resonant frequency of these unbalanced side forces is called the whirling speed ω_f in reference 1 and can be calculated by using the theory presented therein. Figure 9 shows the agreement between the frequencies measured and the theoretical curve calculated from reference 1 and also shows the effect of static frequency $\sqrt{K/M}$ on the whirling frequencies, where for the present investigation $\Omega = 23$ radians per second, $\Lambda_1 = 0.07$, $\Lambda_3 = 0.19$, $M = 0.9$, and K is the pylon bending stiffness measured in pounds per inch. Four roots of the equation are found; the top and bottom curve are predominantly the bending frequency of the shaft, and the two center curves are predominantly the frequencies of the blade motion. The top and bottom curves have real roots which go to infinity at a value of $\sqrt{K/M}$ of infinity. The sign of the root indicates the sense of rotation of the whirling frequency. The other two roots are real for values of $\sqrt{\frac{K}{M}} < 10$, are imaginary for values of $\sqrt{\frac{K}{M}}$ between 10 and 39.6, and become real again for values of $\sqrt{\frac{K}{M}} > 39.6$. The imaginary roots indicate a region of self-excited unstable oscillations. The real roots indicate the frequencies at which stable oscillations may be excited.

For the roots which are predominantly the blade motions, a value of $\sqrt{\frac{K}{M}} > 100$ may be regarded as infinite for the purpose of estimating the whirling frequency. Lower values of $\sqrt{K/M}$ yield frequencies which converge at a point slightly less than the shaft rotational frequency at approximately $\sqrt{\frac{K}{M}} = 39.6$. The experimental points obtained from figure 8 show whirling frequencies at 1.2 and 0.81 of the shaft angular velocity. These values are plotted in figure 9 and show fair agreement with the theoretical curve (within 10 percent). In an actual helicopter with a fairly low pylon stiffness, the roots could either be real or imaginary, depending upon the values of $\sqrt{K/M}$, Λ_1, Λ_3, and Ω. For an analysis of this problem of the unsymmetrical type of drag-hinge oscillation, reference 1 should be used.

The rate of progression of the unbalance about the center of rotation is the drag-hinge unsymmetrical natural frequency and is the difference between the rotor frequency and the whirling frequency. For this configuration, the theory of reference 1 indicates a difference of 6 radians per second which yields a ratio of $\frac{23}{6} = 3.83$. If equation (2) herein, from the theory of reference 2, is used and the engine inertia and pylon stiffness are assumed to be infinite, the almost identical result of $\frac{\Omega}{\omega_n} = 3.8$ may be obtained.

Blade Damping

The total damping of the drag-hinge oscillation is composed of three parts: (1) air damping, (2) friction damping in bearings, gears, hysteresis losses, and so forth, and (3) blade damping supplied by some external mechanism. Each of these is reduced to a ratio of $\frac{C}{C_{cr}}$ since this ratio determines the amplitude of the blade response to a given control movement.

The critical damping torque per blade for the case of symmetrical oscillation (equation (4)) is

$$
\begin{aligned}
C_{cr} &= 2I\omega_n \\
&= 2 \times \frac{1920}{12} \times 7.94 \\
&= 2539 \text{ pound-foot-seconds per radian}
\end{aligned}
$$

The estimated air damping is given by the equation

$$C_{air} = 2\frac{dQ}{d\Omega}$$

where, for the rotor tested, $Q = 3020$ foot-pounds and $\Omega = 23$ radians per second. Therefore,

$$Q = 5.71\Omega^2$$

and

$$\frac{dQ}{d\Omega} = 2(5.71)\Omega$$

which gives the air-damping rotor torque as:

$$C_{air} = 2 \times 2 \times 5.71 \times 23$$

$$= 526 \text{ pound-foot-seconds per radian}$$

or, for each blade,

$$C_{air} = 175.3 \text{ pound-foot-seconds per radian}$$

The air-damping factor is

$$\frac{C_{air}}{C_{cr}} = \frac{175.3}{2539} = 0.069$$

or the air damping is approximately 7 percent of the critical.

The friction damping of the drag-hinge oscillations caused by gear and bearing friction and hysteresis losses in the rotor head and drive assembly is not easily calculated and will vary with each installation. Usual practice is to allow an empirical value that makes the calculated results coincide with the measured values. As suggested in reference 3 (p. 264), this damping value should be about 10 to 15 percent of the critical for the average installation.

Consider the maximum resonant curve in figure 2 with only air and friction damping. The height of this curve at the resonant frequency indicates that the damping present is approximately 15 percent of the critical value (reference 3, equation (32a)). The air damping for a rotor speed of 220 rpm and a main-pitch angle of 12.5° is approximately 7 percent of the critical; a friction value of about 8 percent is thereby indicated. Inspecting the next lower curve of the same figure, which represents the lowest amount of oil damping used (approximately 4 percent of the critical), shows that the total damping is about 19 percent of the critical and also indicates that the value for the friction damping is about 8 percent. In this paper, an empirical value of 10 percent is allowed for friction damping.

If the symmetrical drag-angle oscillations are to be kept from being magnified, a total damping of about 50 percent to 60 percent of the critical should be used. If the critical value and the amount of damping allowed for air and friction are known, then the external blade dampers must supply 30 to 40 percent of the critical. For purposes of calculating the required damping, assume 35 percent of the critical. Thus, for the blades under consideration, the following damping coefficient is required per blade:

$$
\begin{aligned}
C &= 0.35C_{cr} \\
&= 0.35 \times 2539 \\
&= 888 \text{ pound-foot-seconds per radian}
\end{aligned}
$$

General Discussion

In general, oscillation of the drag hinge will not occur (with the exception of ground resonance) unless some exciting force near the whirling speed ω_f or near the natural frequency ω_n of the symmetrical drag-hinge oscillation is present. Without damping, the rotor blade is very sensitive to disturbances in the plane of rotation and a pilot can, by small throttle or control movements, induce large drag-angle oscillations that might result in blade failure.

Another source of induced drag-angle oscillation is a hunting pitch or throttle governor or a hunting automatic pilot. Care should be taken to see that the natural frequency of contemplated governors or automatic pilots does not coincide with either of the two resonant conditions.

Damping of the unsymmetrical oscillation is not included in this paper, but apparently experimental verification of the damping factors given in reference 1 is a field for further research.

CONCLUSIONS

The natural frequencies and amplitudes of helicopter rotor-blade oscillations about the drag hinge were experimentally determined with the Langley helicopter apparatus (helicopter tower) by the use of several effective engine and gear-box inertias and various blade-damper settings. The experimental results have been compared with the available theory and, on the basis of the configuration tested, the following conclusions may be drawn:

1. There are two types of oscillations of the rotor blade about the drag hinge: symmetrical and unsymmetrical. The symmetrical types were induced by a throttle or main-pitch oscillation at a frequency considerably below shaft speed. The unsymmetrical types were induced by a cyclic-pitch oscillation at a frequency near that of shaft speed.

2. The engine and gear-box inertia and shaft torsional stiffness are factors affecting the frequency of the symmetrical drag-hinge oscillations. The theory used in this analysis takes these factors into consideration and the experimental results show good agreement with the predicted values. This type of drag oscillation is characterized by large torque and rotor-speed oscillations along with the drag-angle motion.

3. The assumption of constant rotor speed is apparently valid in determining the whirling and natural drag-hinge frequencies of the unsymmetrical blade oscillation caused by periodic cyclic-pitch movements. For infinite pylon stiffness, the two theories presented predict approximately the same results but yield natural unsymmetrical drag-hinge frequencies somewhat higher than those determined experimentally. The discrepancy of the drag-angle frequency itself is rather large, but the discrepancy in the whirling frequency, which determines the drag-angle frequency and which is the one that must be avoided, is only of the order of 10 percent and is considered in fair agreement. This type of drag-angle movement is characterized by large oscillating side forces, but no apparent rotor-speed or net-torque variation exists.

4. The drag-hinge damping added to the system to prevent resonance caused by main-pitch or throttle movements should be approximately 35 percent of the calculated critical.

5. Care should be exercised in the design of the helicopter to ascertain that no regular disturbing forces are present - such as a hunting pitch or throttle governor or a hunting automatic pilot - with frequencies near those of the resonant conditions.

Langley Aeronautical Laboratory
National Advisory Committee for Aeronautics
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APPENDIX

DETERMINATION OF PHYSICAL CONSTANTS

Blade moment of inertia was determined by suspending the blade from its vertical-pin axis, swinging the blade, and determining the time of a single oscillation in seconds. If the blade constants and the time for a single oscillation are known, the moment of inertia is then given by the formula

$$I = \frac{m_b l g t^2}{\pi^2}$$

where g is the gravitational constant and t is the time for one-half cycle.

A similar procedure was used in the determination of the engine and gear-box inertias. A bar with a weight attached to one end and the other end attached to the crankshaft of the engine (or gear-box shaft) was deflected and the time for a single oscillation determined. Making the proper corrections for bar and weight and substituting in the moment-of-inertia equation, yield the engine and gear-box inertias.

The shaft torsional stiffness (in.-lb/radian) was determined by applying a known torque to one end of the shaft and measuring the angular deflection.

The pylon bending stiffness was calculated by considering the pylon as a cantilever beam rigidly supported. The stiffness of the pylon can be calculated from its known dimensions by the following formula:

$$K = \frac{3EI_c}{L^3}$$

where L is the length of the pylon, I_c is the moment of inertia of the cross section of the pylon, and E is the modulus of elasticity for the material.

The mass of the pylon was considered as that part of the shaft above the top support. This mass was calculated by using the dimensions of the shaft. The mass of the rotor head and blades was determined by weighing. The combined mass of blades, rotor hub, and pylon yielded a value of

$$M = \frac{\text{Weight}}{g}$$

$$= \frac{347}{386}$$

$$= 0.9 \text{ pound-seconds}^2 \text{ per inch}$$

The damping constants for the various oil bypass openings were determined by measuring the force necessary to move the damper at several different speeds.

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TABLE I

SUMMARY OF DATA OBTAINED FOR SYMMETRICAL DRAG-ANGLE

OSCILLATIONS CAUSED BY SINUSOIDAL MOVEMENT

OF THE MAIN PITCH

Case I.- Engine and gear-box inertia of 4910 pound-inch-seconds^2 and rotor speed of 220 rpm

Symbol (see fig. 2)	ζ (deg)	$\frac{\zeta}{\zeta_{st}}$	$\frac{\text{Shaft speed}}{\text{Main-pitch-oscillation speed}}$
Full oil damping			
○	1.30	0.541	2.35
	1.43	.596	2.40
	1.69	.705	2.50
	2.02	.842	2.55
	1.95	.812	2.65
	1.62	.670	2.70
	1.43	.596	2.80
	1.30	.541	2.90
	1.49	.623	3.05
	1.82	.758	3.10
	1.95	.812	3.22
	1.62	.670	3.50
	1.82	.758	3.55
	1.75	.730	3.65
	1.95	.812	3.80
	1.62	.677	3.80
	1.62	.677	4.60
	1.88	.784	4.70
0.050-inch bypass			
□	1.43	0.595	1.20
	1.49	.623	1.30
	1.30	.541	2.10
	1.30	.541	2.20
	1.95	.812	2.30
	1.95	.812	2.45
	2.60	1.082	2.55
	2.99	1.200	2.67
	2.06	.859	2.70
	1.95	.812	2.80
	2.47	1.030	2.90
	2.21	.920	3.13
	2.06	.859	3.55
	2.79	1.160	2.75
	2.88	1.200	3.00
	2.88	1.200	3.12
	2.33	.970	3.60

Symbol (see fig. 2)	ζ (deg)	$\frac{\zeta}{\zeta_{st}}$	$\frac{\text{Shaft speed}}{\text{Main-pitch-oscillation speed}}$
0.125-inch bypass			
◇	2.14	0.893	2.00
	2.08	.867	2.30
	3.25	1.350	2.50
	3.64	1.520	2.60
	4.22	1.760	2.75
	4.94	2.050	2.75
	4.87	2.030	2.85
	6.17	2.575	2.90
	6.50	2.710	2.95
	5.40	2.240	3.10
	5.07	2.115	3.20
	4.55	1.895	3.25
	4.16	1.730	3.30
	3.25	1.353	3.50
	2.99	1.247	3.90
No oil damping			
△	2.27	0.947	1.40
	2.34	.975	1.80
	2.86	1.190	2.30
	2.25	1.353	2.35
	4.42	1.840	2.50
	3.64	1.520	2.60
	4.55	1.896	2.65
	5.00	2.085	2.72
	6.05	2.520	2.76
	6.50	2.700	2.80
	7.15	2.980	2.82
	7.02	2.930	2.90
	7.67	3.200	2.90
	7.93	3.300	3.00
	6.83	2.845	3.00
	5.20	2.170	3.10
	4.23	1.760	3.35
	3.25	1.353	3.75
	2.08	.867	4.20
	8.16	3.400	2.90

TABLE I - Concluded

SUMMARY OF DATA OBTAINED FOR SYMMETRICAL DRAG-ANGLE

OSCILLATIONS CAUSED BY SINUSOIDAL MOVEMENT

OF THE MAIN PITCH - Concluded

Case II.- Engine and gear-box inertia of 4560 pound-inch-seconds^2, rotor speed of 220 rpm, and no oil damping

Symbol (see fig. 3)	ζ (deg)	$\frac{\zeta}{\zeta_{st}}$	$\frac{\text{Shaft speed}}{\text{Main-pitch-oscillation speed}}$
◺	1.44	0.60	1.35
	1.51	.63	1.39
	1.44	.60	1.45
	1.68	.70	1.85
	1.90	.79	2.42
	3.31	1.38	2.57
	3.16	1.30	2.63
	7.30	3.04	2.64
	8.15	3.40	2.69
	6.05	2.52	2.70
	4.60	1.92	2.80
	5.00	2.08	2.80
	6.15	2.54	2.83
	5.70	2.36	2.85
	5.12	2.13	2.87
	4.10	1.72	2.88
	3.86	1.61	3.00
	3.43	1.43	3.07
	3.31	1.38	3.19
	3.10	1.29	3.32
	2.77	1.14	3.45
	2.70	1.12	3.55
	2.56	1.06	3.65
	2.42	1.01	3.77
	2.30	.96	4.90
	2.24	.94	6.80

Case III.- Engine and gear-box inertia of 11,000 pound-inch-seconds^2, rotor speed of 200 rpm, and no oil damping

Symbol (see fig. 4)	ζ (deg)	$\frac{\zeta}{\zeta_{st}}$	$\frac{\text{Shaft speed}}{\text{Main-pitch-oscillation speed}}$
◗	2.88	1.20	8.75
	2.50	1.04	5.40
	2.28	.95	4.50
	2.56	1.07	4.25
	5.10	2.12	4.05
	4.95	2.06	3.95
	5.18	2.16	3.95
	5.66	2.36	3.90
	5.56	2.32	3.85
	6.53	2.72	3.70
	5.92	2.47	3.70
	6.48	2.70	3.60
	5.52	2.30	3.58
	5.04	2.10	3.55
	5.71	2.38	3.55
	4.40	1.84	3.45
	4.60	1.92	3.40
	4.37	1.82	3.32
	3.60	1.50	3.30
	4.80	2.00	3.25
	4.32	1.80	3.10
	3.74	1.56	3.10
	3.41	1.42	2.90
	3.36	1.40	3.05
	2.69	1.12	2.80
	2.06	.86	2.80
	2.11	.88	1.30
	1.97	.82	1.20

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TABLE II

SUMMARY OF DATA OBTAINED FOR UNSYMMETRICAL DRAG-ANGLE OSCILLATIONS CAUSED BY A SINUSOIDAL OSCILLATION OF THE CYCLIC PITCH

[Rotor speed, 220 rpm; blade damper, 0.125-inch bypass]

Symbol (see fig. 8)	ζ (deg)	Side-force amplification factor	$\frac{\text{Shaft speed}}{\text{Cyclic-pitch-oscillation speed}}$
○	0.98	1.9	2.20
	1.95	5.0	1.60
	3.65	10.0	1.42
	3.65	11.7	1.25
	3.70	12.5	1.23
	4.22	12.5	1.23
	4.55	12.5	1.23
	4.68	12.2	1.17
	3.90	8.5	1.12
	3.90	9.6	1.08
	3.25	10.0	1.04
	3.12	9.3	1.04
	----	8.0	1.00
	1.95	9.0	.98
	3.78	11.5	.92
	4.74	18.5	.88
	4.87	19.0	.85
	5.85	34.6	.81
	5.00	19.0	.80
	2.92	10.0	.75
	1.62	6.5	.70
	----	3.7	.65
	1.62	2.5	.62
	----	1.0	3.25

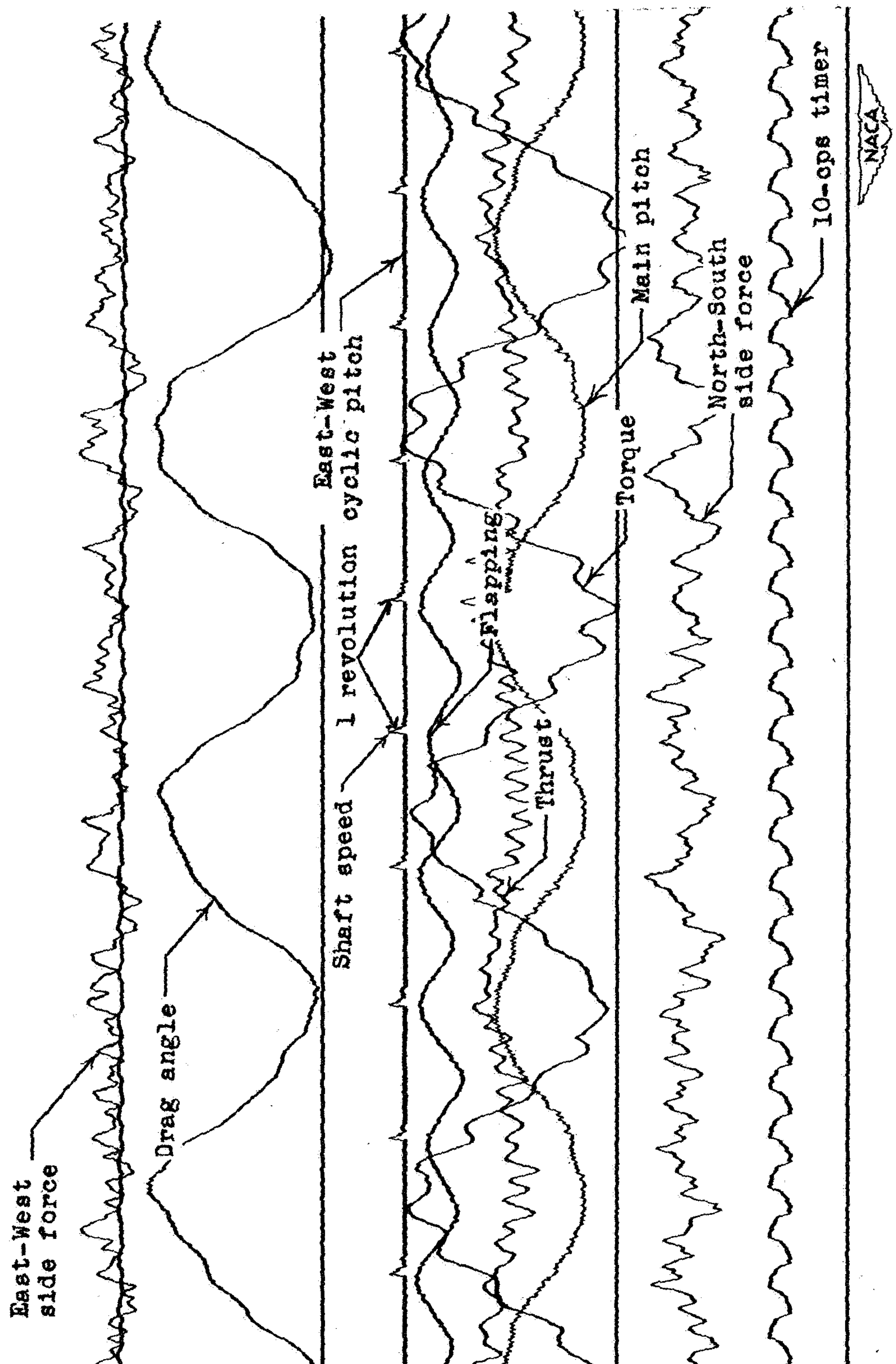

Figure 1.– Typical oscillograph record of symmetrical drag-angle oscillations caused by main-pitch oscillation with no blade dampers.

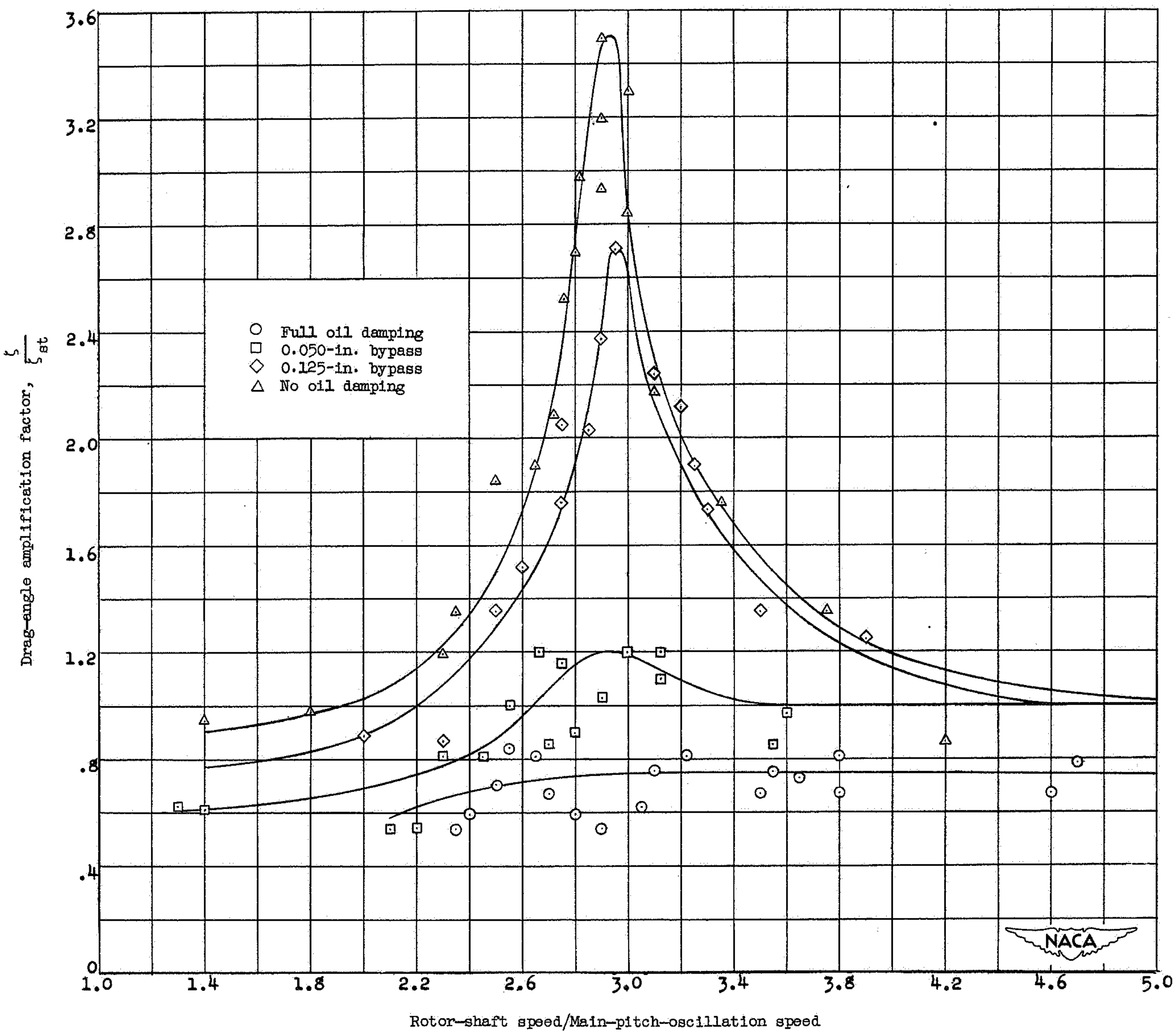

Figure 2.- Resonant diagram of symmetrical drag-angle oscillations at rotor speed of 220 rpm with engine and gear-box inertia of 4910 pound-inch-seconds^2 and various amounts of blade damping.

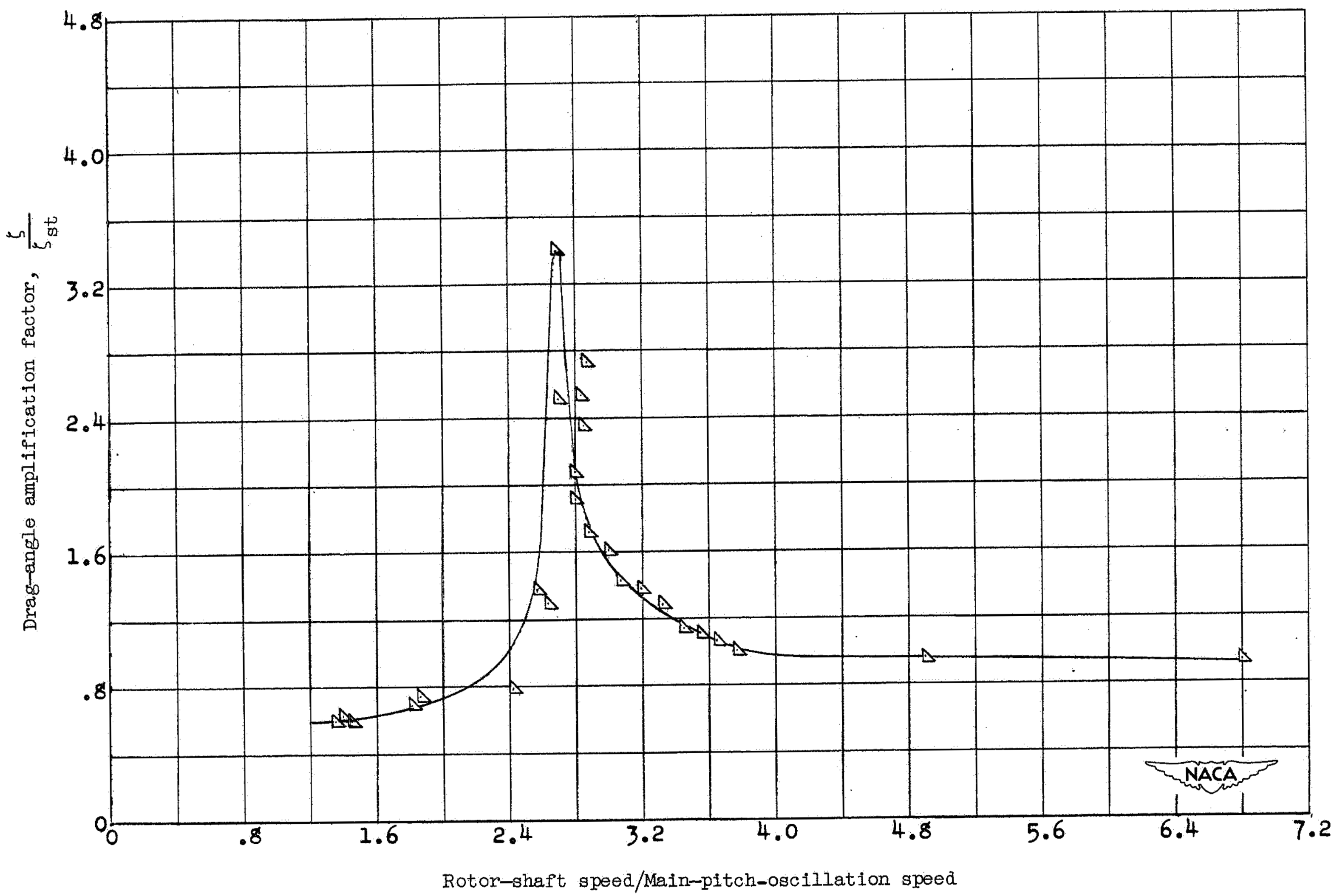

Figure 3.- Resonant diagram of symmetrical drag-angle oscillations at rotor speed of 220 rpm with engine and gear-box inertia of 4560 pound-inch-seconds^2 and no oil damping.

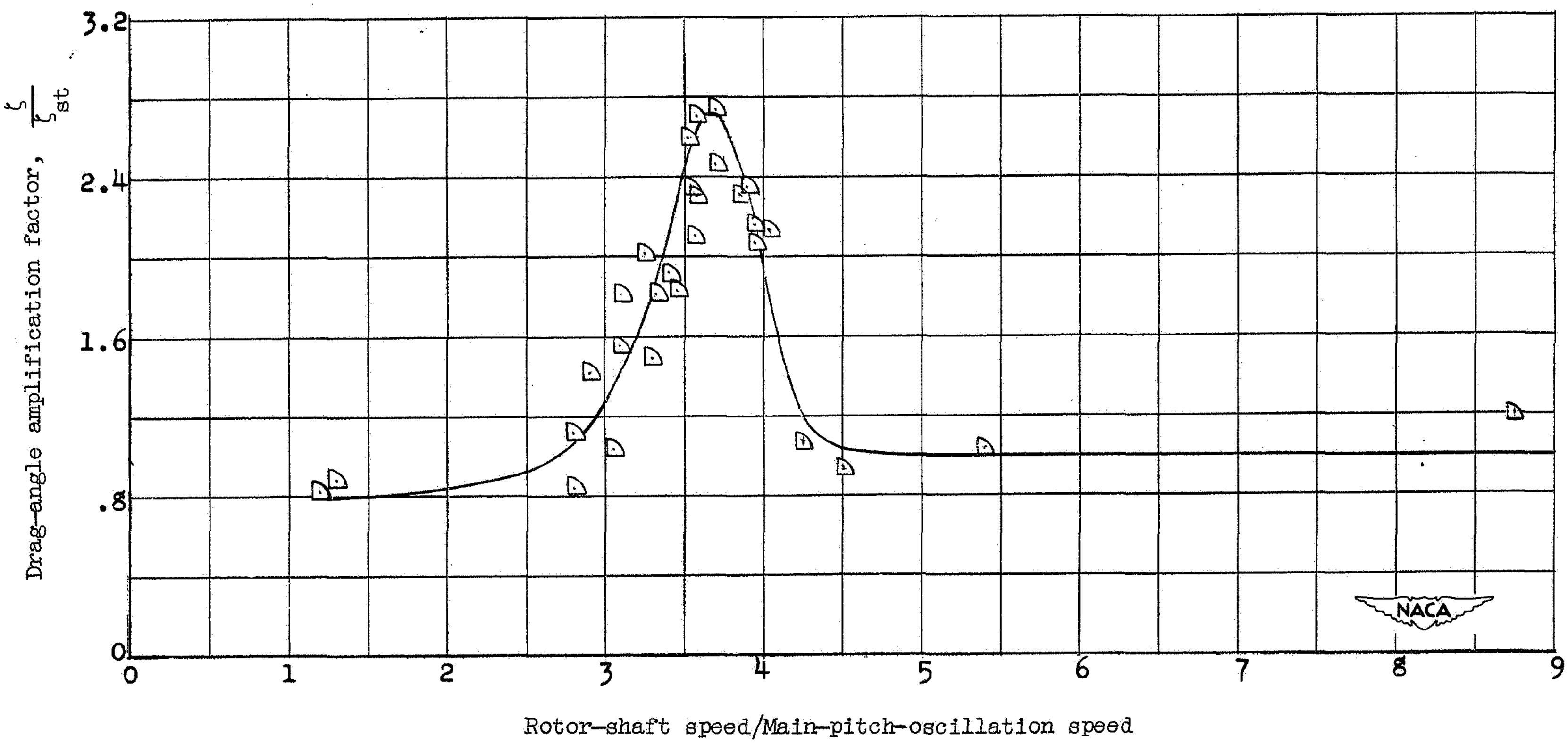

Figure 4.- Resonant diagram of symmetrical drag-angle oscillations at rotor speed of 200 rpm with engine and gear-box inertia of 11,000 pound-inch-seconds^2 and no oil damping.

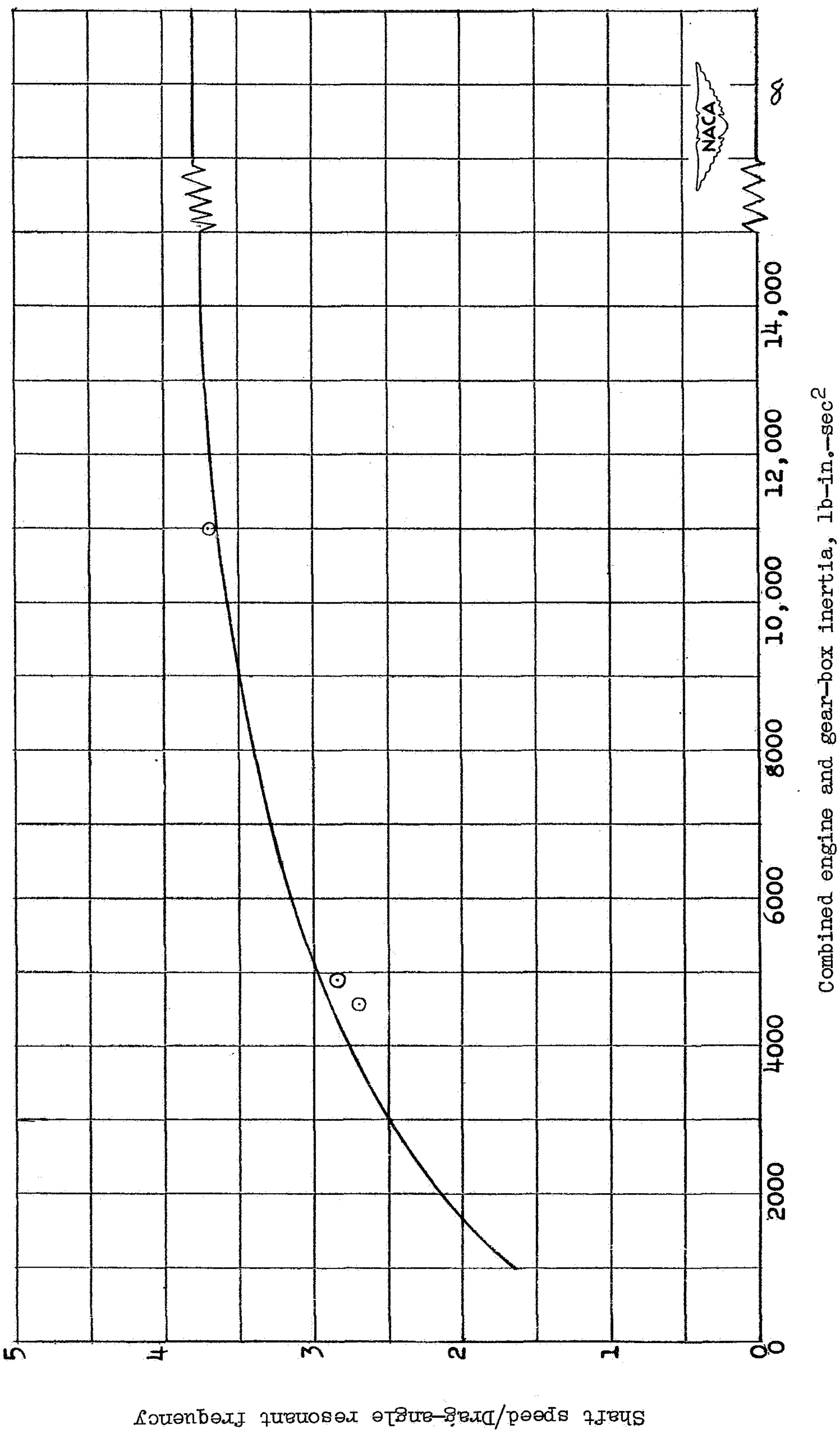

Figure 5.- Comparison of experimental results with theory of reference 2 for a rotor speed of 220 rpm for the case of symmetrical oscillations.

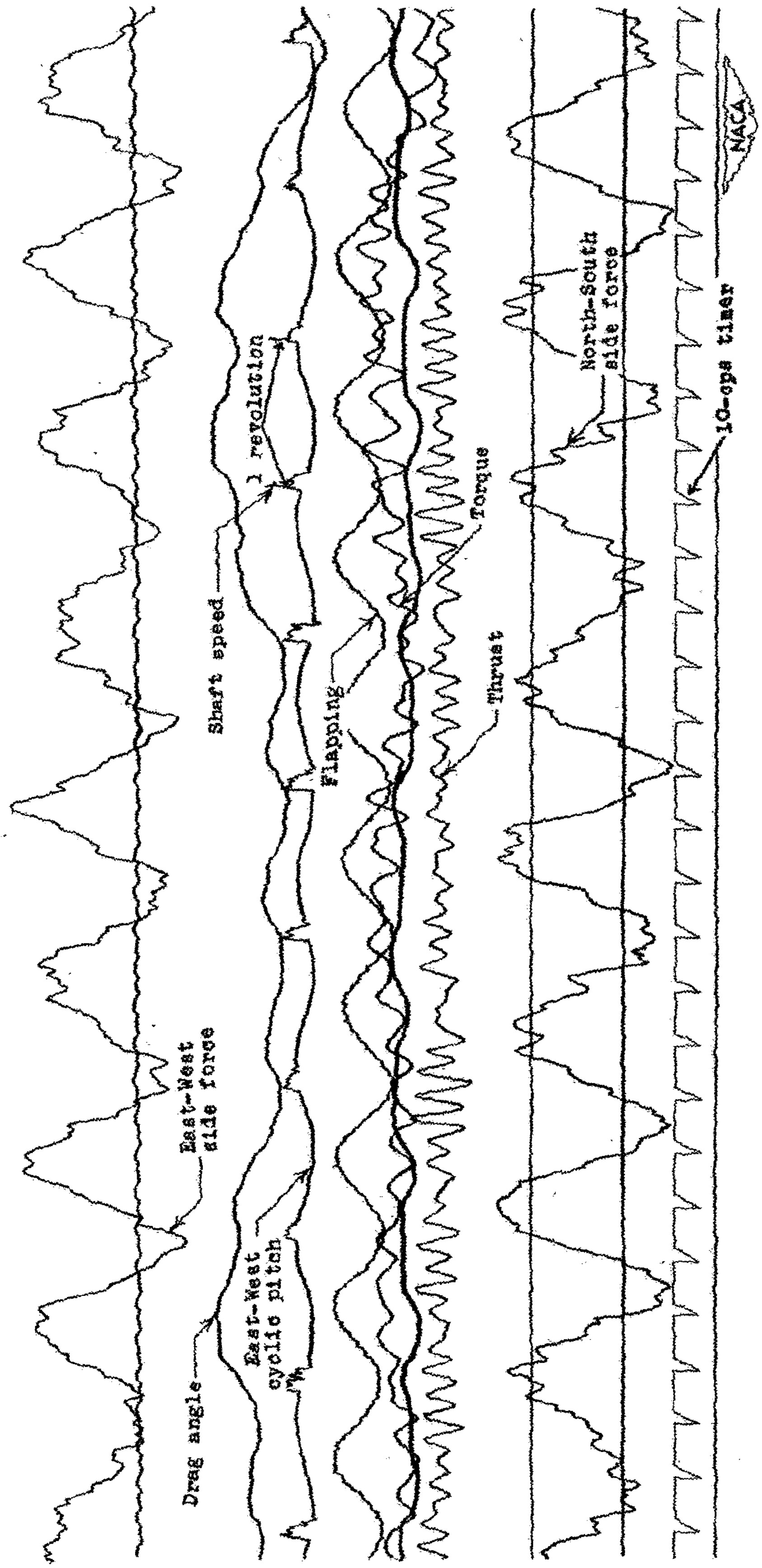

Figure 6.- Typical oscillograph record of unsymmetrical drag-angle oscillations with 0.125-inch bypass in blade damper excited by a cyclic-pitch frequency below shaft speed.

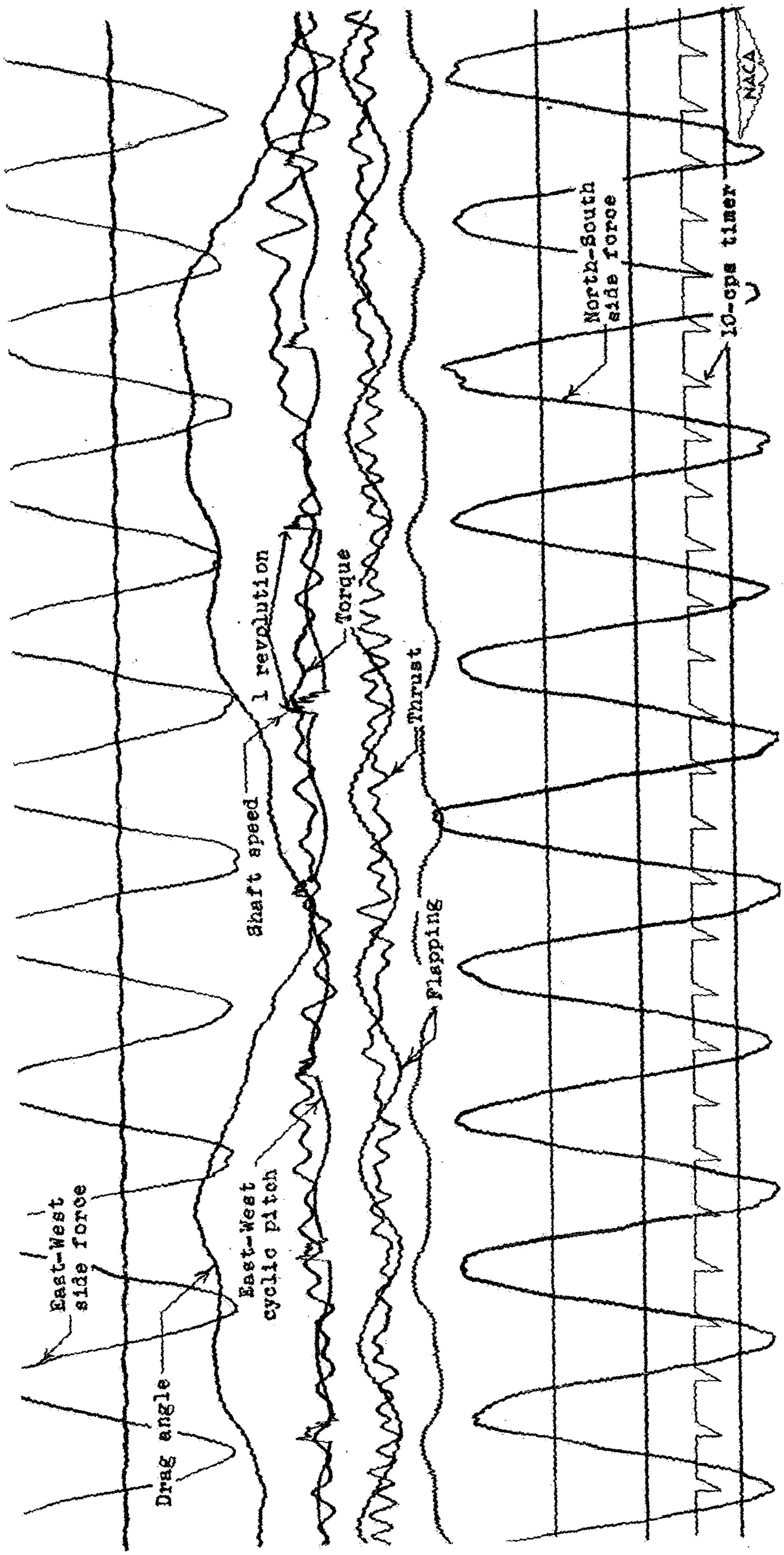

Figure 7.- Typical oscillograph record of unsymmetrical drag-angle oscillations with 0.125-inch bypass in blade damper excited by a cyclic-pitch frequency above shaft speed.

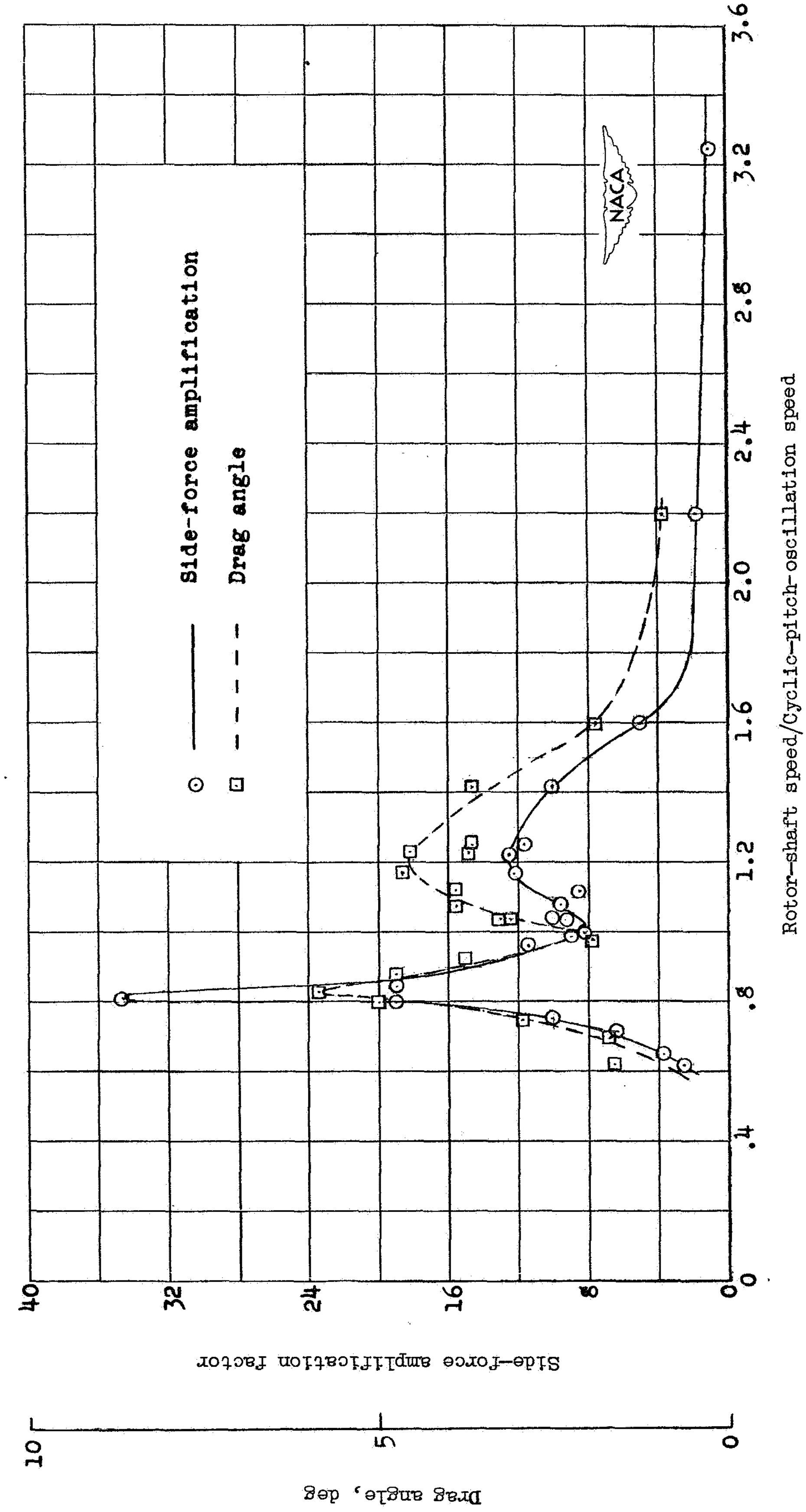

Figure 8.– Resonant diagram of side forces and unsymmetrical drag-angle oscillations with a 0.125-inch bypass in the blade damper.

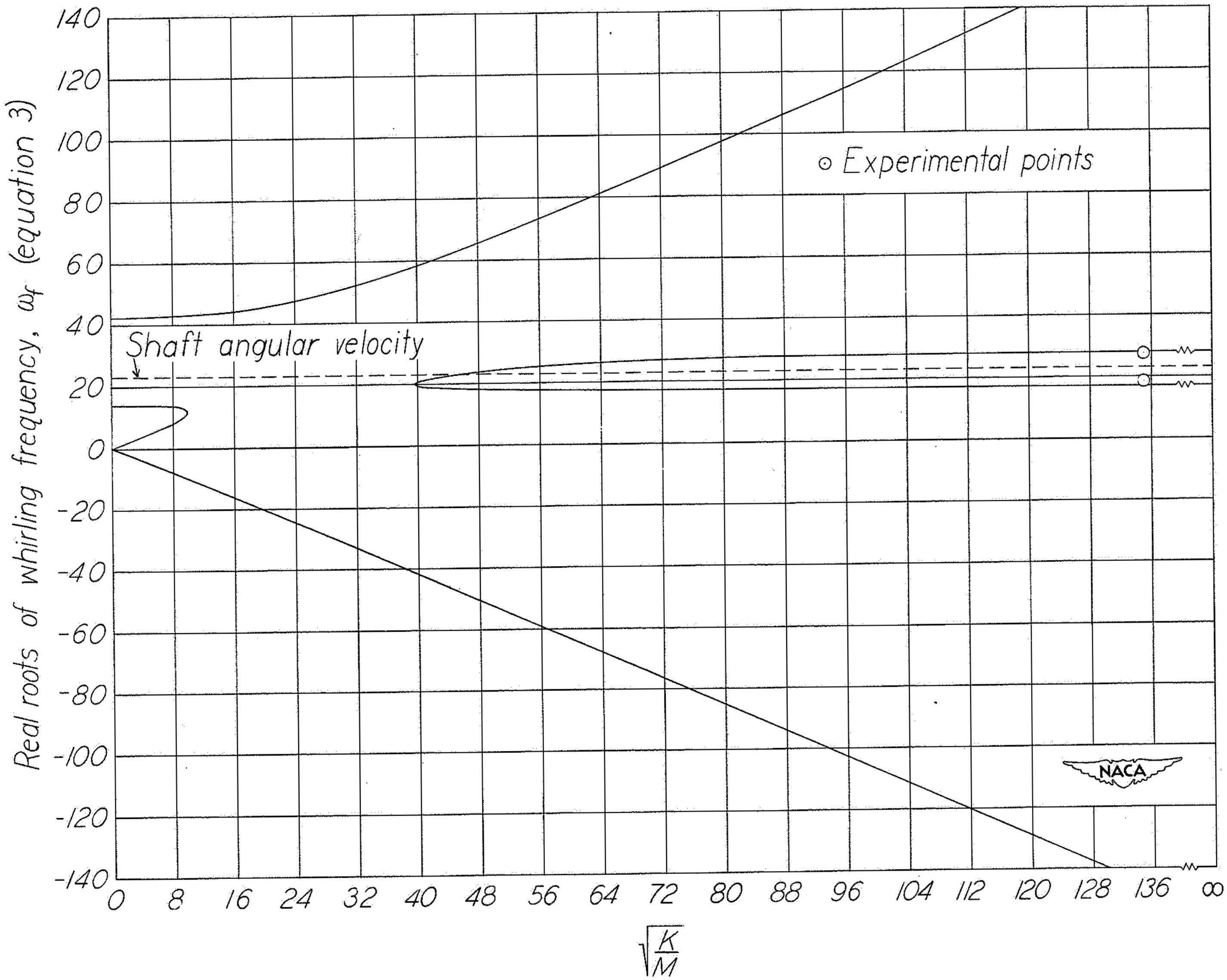

Figure 9.– Comparison of experimental results with theory of reference 1 and effect of pylon stiffness and mass on whirling frequencies at a shaft speed of 23 radians per second.