CASE FILE
COPY

NACA TN 2242

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 2242

ANALYTICAL INVESTIGATION OF TURBULENT FLOW IN SMOOTH TUBES WITH HEAT TRANSFER WITH VARIABLE FLUID PROPERTIES FOR PRANDTL NUMBER OF 1

By Robert G. Deissler

Lewis Flight Propulsion Laboratory
Cleveland, Ohio

PROPERTY FAIRCHILD
ENGINEERING LIBRARY

Washington
December 1950

JAN 4 REC'D

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 2242

ANALYTICAL INVESTIGATION OF TURBULENT FLOW IN SMOOTH TUBES WITH HEAT TRANSFER WITH VARIABLE FLUID PROPERTIES FOR PRANDTL NUMBER OF 1

By Robert G. Deissler

SUMMARY

Equations were derived for the prediction of radial distributions of velocity and temperature for fully developed turbulent flow of gases in smooth tubes for the case where the variation of fluid properties due to temperature variation across the tube is considered. The equations apply for both heat addition to the gas and heat extraction or cooling. The effect of frictional heating on temperature distribution was investigated and the analysis indicated that the velocity and total-temperature profiles are similar for a fluid with a Prandtl number of 1.

By use of the equations for velocity and temperature distributions, relations were obtained among Nusselt number, Reynolds number, and friction factor for a fluid with a Prandtl number of 1. It was found that the effect of variation of gas properties across the tube on the Nusselt number and friction-factor correlations can be eliminated by evaluating the gas properties at a temperature close to the average of the wall and bulk temperatures. The Nusselt numbers and friction factors analytically predicted agree with those experimentally determined by measurement of average surface heat-transfer coefficients and friction factors in tubes for Reynolds numbers above 10,000.

INTRODUCTION

Although some work has been done on laminar flow with variable fluid properties, very little has been done on turbulent flow with variable fluid properties. In reference 1, equations are developed for velocity distributions for fully developed adiabatic turbulent flow in smooth tubes. The plots of the equations agree closely with experimental data for adiabatic turbulent velocity distributions when appropriate values of the experimental constants appearing in the equations are used.

The investigation reported herein, which was conducted at the NACA Lewis laboratory, is an extension of the analysis developed in reference 1 for adiabatic flow in smooth tubes to include flow with heat transfer. For flow with high rates of heat transfer, the temperature variation, and thus the variation of fluid properties across the tube, may be so large that it might be expected that the velocity distributions for flow with heat transfer would differ from those for flow without heat transfer. In developing the equations for turbulent flow of gases with heat transfer where the variation of gas properties is considered, the same hypotheses for the turbulent transfer of momentum are used as were found to apply to flow without heat transfer. Because of these and other assumptions made in the analysis, the equations for velocity distributions with heat transfer should be experimentally verified.

ANALYSIS

Turbulent Transfer of Heat

When turbulent flow exists through a tube, portions of the fluid move about in random fashion (reference 1). If heat transfer between the tube wall and the fluid takes place, a temperature gradient occurs across the tube and some of the portions of fluid move transversely into regions of different mean temperature. This motion produces heat transfer in addition to that which takes place by local molecular conduction.

By analogy with the law for transfer of heat by molecular conduction $q_v = -k\, dt/dy$, the equation for the transfer of heat by turbulent motion is often written in the following form (All symbols are defined in the appendix.):

$$q_t = -\rho g c_p \epsilon_h \, dt/dy \tag{1}$$

where $\rho g c_p \epsilon_h$ is comparable to the conductivity k for molecular heat transfer and ϵ_h is the coefficient of eddy diffusivity for heat with a value determined by the amount and the kind of turbulent mixing at a point. Adding the molecular heat transfer to the turbulent heat transfer gives for the total heat transfer,

$$q = -(k + \rho g c_p \epsilon_h)\, dt/dy \tag{2}$$

As in the case of momentum transfer, the analogy between molecular and turbulent heat transfer is inexact because the mechanism for heat transfer by molecular motion is somewhat different than the mechanism

for heat transfer when portions of the fluid move about in random fashion. In the case of heat transfer by molecular conduction, transfer takes place only at the instant the molecules collide; whereas in the case of turbulent heat transfer, the transfer can take place continuously by convection inasmuch as the portions of fluid are continuously in contact. This difference can, however, be absorbed in the value of ϵ_h, the value for which depends on the turbulence mechanism, so that equation (1) should still apply.

Velocity-Distribution Equations for Flow of Gases with Heat Transfer

The equation for heat transfer (equation (2)) can be rewritten as follows:

$$\frac{q}{c_p g} = \left(\frac{\mu}{Pr} + \rho\epsilon_h\right)\frac{dt}{dy} \tag{3}$$

The equation for shear stress (reference 1) is

$$\tau = (\mu + \rho\epsilon)\frac{du}{dy} \tag{4}$$

Assumptions. - The following assumptions are made in the use of equations (3) and (4) for obtaining velocity distributions with heat transfer:

1. The eddy diffusivities for momentum and heat transfer (ϵ and ϵ_h) are equal. Previous analyses for flow in tubes based on this assumption yielded heat-transfer coefficients and friction factors that agree with experiment (reference 2).

2. The expressions for eddy diffusivity that are found in reference 1 to apply to flow without heat transfer apply also to flow with heat transfer with variable fluid properties. These expressions are

$$\epsilon = n^2 uy \tag{5}$$

for flow close to the wall and the Kármán relation

$$\epsilon = \kappa^2 \frac{\left(\frac{du}{dy}\right)^3}{\left(\frac{d^2u}{dy^2}\right)^2} \tag{6}$$

for flow far from the wall, where n and κ are experimental constants having the values 0.109 and 0.36, respectively.

3. The shear stress τ and the heat transfer per unit area q can be considered uniform across the tube. It is shown in reference 1 that the shear stress can be considered, with good approximation, to be uniform across the tube for turbulent flow. In the present analysis, the heat transfer per unit surface area q is also considered uniform across the tube.

4. The static pressure can be considered constant across the tube.

5. The Prandtl number ($Pr = 1$) and specific heat can be considered constant with temperature variation.

6. The effects of frictional heating (compressibility effects) on velocity distributions can be neglected for subsonic flow (reference 1).

Relations between gas-property values and velocity. - When heat transfer takes places in a gas flowing through a tube, a temperature gradient exists across the tube and the gas properties vary from point to point. The gas properties that vary appreciably with temperature and are important in the present investigation are the density ρ, the viscosity μ, and the thermal conductivity k. The specific heat is considered constant because its variation with temperature is small compared with that of the other properties.

Dividing equation (3) by equation (4) then gives, for a Prandtl number of 1 and constant shear stress and heat transfer across the tube,

$$\frac{q_0}{c_p g \tau_0} = -\frac{dt}{du} \tag{7}$$

Integration of this equation gives

$$\frac{q_0}{c_p g \tau_0} \int_0^u du = - \int_{t_0}^t dt$$

or

$$\frac{q_0}{c_p g \tau_0} u = t_0 - t \tag{8}$$

This equation can be made dimensionless by substitution of the dimensionless quantities

$$u^+ \equiv \frac{u}{\sqrt{\tau_0/\rho_0}}$$

and

$$\beta \equiv \frac{q_0 \sqrt{\tau_0/\rho_0}}{c_p g \tau_0 t_0}$$

The equation then becomes

$$\frac{t}{t_0} = 1 - \beta u^+ \tag{9}$$

where β is the heat-transfer parameter. Equation (9) indicates that the velocity and temperature profiles are similar for a Prandtl number of 1 so that if the velocity profile is known, the temperature profile is also known.

The variation of density across the tube can then be written by using the assumption of constant static pressure across the tube and the perfect gas law as

$$\frac{\rho}{\rho_0} = \frac{t_0}{t} = \frac{1}{1 - \beta u^+}$$

or the density at any point across the tube is

$$\rho = \frac{\rho_0}{1 - \beta u^+} \tag{10}$$

The variation of the viscosity of a gas with temperature can be shown from viscosity data to be given by an equation of the form

$$\frac{\mu}{\mu_0} = \left(\frac{t}{t_0}\right)^d \tag{11}$$

The viscosity at any point across the tube is then

$$\mu = \mu_0 \left(1 - \beta u^+\right)^d \tag{12}$$

With the assumptions of constant Prandtl number and constant specific heat, the law for variation with temperature or velocity for the conductivity must be the same as that for the viscosity.

Flow close to wall. - For obtaining the equation for flow close to a smooth wall, the expression for ϵ from equation (5) is substituted into equation (4) to give

$$\tau_0 = \left(\mu + \rho u y\, n^2\right) \frac{du}{dy} \tag{13}$$

where τ is replaced by τ_0 because the shear stress is assumed not to vary with radius. In order to obtain equations that are in dimensionless form, the following well-known dimensionless quantities are used:

$$u^+ \equiv \frac{u}{\sqrt{\tau_0/\rho_0}} \tag{14}$$

$$y^+ \equiv \frac{\sqrt{\tau_0/\rho_0}}{\mu_0/\rho_0}\, y \tag{15}$$

Then

$$\frac{du}{dy} = \frac{\tau_0}{\mu_0}\frac{du^+}{dy^+} \tag{16}$$

and

$$\frac{d^2u}{dy^2} = \frac{\tau_0\sqrt{\tau_0/\rho_0}}{\mu_0^2/\rho_0}\frac{d^2u^+}{dy^{+2}} \tag{17}$$

Substituting the expressions for density and viscosity from equations (10) and (12) and also equations (14) to (16) into equation (13) in order to make it dimensionless gives

$$1 = \frac{n^2u^+y^+}{1 - \beta u^+}\frac{du^+}{dy^+} + (1 - \beta u^+)^d\frac{du^+}{dy^+}$$

By rearrangement,

$$\frac{dy^+}{du^+} - \frac{n^2u^+y^+}{1-\beta u^+} = (1 - \beta u^+)^d \tag{18}$$

Equation (18) is a first-order linear differential equation, the solution for which is, with the condition that $u^+ = 0$ when $y^+ = 0$,

$$y^+ = e^{-\frac{n^2u^+}{\beta}}(1 - \beta u^+)^{-\frac{n^2}{\beta^2}}\int_0^{u+} e^{\frac{n^2u^+}{\beta}}(1 - \beta u^+)^{\frac{n^2}{\beta^2} + d}\,du^+ \tag{19}$$

For flow close to a wall with heat transfer, equation (19) gives the relation between u^+ and y^+ for various values of the heat-transfer parameter β.

Flow at a distance from wall. - For obtaining the equation for flow at a distance from a wall, the viscous shear stress is assumed

negligible compared with the turbulent shear stress. Substituting the expression for ϵ from equation (6) into equation (4) and substituting τ_0 for τ (assumption 3) gives

$$\tau_0 = \kappa^2 \rho \frac{\left(\frac{du}{dy}\right)^4}{\left(\frac{d^2u}{dy^2}\right)^2} \tag{20}$$

Substituting equations (10), (16), and (17) into equation (20) gives

$$\frac{d^2u^+}{dy^{+2}} = -\frac{\kappa}{\sqrt{1 - \beta u^+}}\left(\frac{du^+}{dy^+}\right)^2 \tag{21}$$

where the minus sign was chosen in taking the square root in order to make κ positive. After one integration, equation (21) becomes

$$\log_e\left(C_1 \frac{du^+}{dy^+}\right) = 2\kappa \frac{\sqrt{1 - \beta u^+}}{\beta}$$

or

$$e^{\frac{2\kappa}{\beta}\sqrt{1 - \beta u^+}} = C_1 \frac{du^+}{dy^+} \tag{22}$$

or

$$dy^+ = C_1 e^{-\frac{2\kappa}{\beta}\sqrt{1 - \beta u^+}} du^+ \tag{23}$$

Equation (23) can be integrated to yield

$$y^+ = \frac{C_1\beta}{2\kappa^2} e^{-\frac{2\kappa}{\beta}\sqrt{1 - \beta u^+}}\left(\frac{2\kappa}{\beta}\sqrt{1 - \beta u^+} + 1\right) + C$$

As the wall is approached, the velocity gradient becomes very large compared with that at a distance from the wall so that dy^+/du^+ approaches zero as y^+ approaches zero. By substituting dy^+/du^+ from equation (22)

for $C_1 e^{-\frac{2\kappa}{\beta}\sqrt{1 - \beta u^+}}$ in the preceding equation and letting dy^+/du^+ equal zero when y^+ and u^+ equal zero, C equals zero so that

$$y^+ = \frac{C_1\beta}{2\kappa^2} e^{-\frac{2\kappa}{\beta}\sqrt{1 - \beta u^+}} \left(\frac{2\kappa}{\beta}\sqrt{1 - \beta u^+} + 1\right) \tag{24}$$

For flow at a distance from a wall with heat transfer, equation (24) gives the relation between u^+ and y^+ for various values of the heat-transfer parameter β. For $\beta = 0$, equation (24) is indeterminate. For this case β is set equal to zero in equation (21) before integrating, and the well-known logarithmic equation is obtained:

$$u^+ = \frac{1}{\kappa} \log_e y^+ + C$$

Temperature Distributions with Frictional Heating

In reference 1 it is shown that the effect of compressibility or frictional heating on the generalized velocity distribution is small; it was therefore neglected in the preceding section (assumption 6). In this section, the effect of frictional heating on temperature distributions is investigated and the relation between the velocity and temperature distributions for a Prandtl number of 1 is determined. The effect of frictional heating in a boundary layer has been previously investigated by Kalikhman (reference 3).

In a gas flowing at high velocity, there is, in addition to heat transferred from the wall, heat generated in the fluid by friction. It is known that the rate of conversion of flow energy into turbulent and heat energy per unit volume is $\tau\, du/dy$. In the present analysis, the assumption is made that all the flow energy converted to turbulent energy at a point is transformed into heat at the same point, so that the rate of generation of heat per unit volume is $(\tau\, du/dy)/J$. For flow over a flat surface, this expression is equal to the rate of change of heat transfer perpendicular to the surface with distance from the surface, or

$$\frac{dq}{dy} = \frac{1}{J}\tau\frac{du}{dy} \tag{25}$$

This equation is applied to flow in a tube inasmuch as for the region close to the wall, where the greatest changes of velocity and temperature with distance occur, the wall can be considered flat. As in the case of velocity distributions, the shear stress is assumed constant across the tube (assumption 3) and equation (25) becomes, on integration,

$$\frac{1}{J}\tau_0 \int_0^u du = \int_{q_0}^q dq$$

or

$$q = q_0 + \frac{1}{J}\tau_0 u \tag{26}$$

In obtaining equation (8) in the preceding section, the frictional heating was neglected and the heat transfer was considered constant across the tube. If the frictional heating is not neglected, equation (8) can be written as

$$\frac{q}{c_p g \tau_0} = -\frac{dt}{du}$$

On substituting the value for q from equation (26) in this equation, there results

$$\frac{q_0}{c_p g \tau_0} + \frac{\tau_0 u}{J\, c_p g \tau_0} = -\frac{dt}{du}$$

When this equation is written in dimensionless form,

$$\beta + 2\alpha u^+ = -\frac{d(t/t_0)}{du^+} \tag{27}$$

where α is the compressibility parameter, which was used in reference 1. The total temperature is defined as

$$T \equiv t + \frac{u^2}{2gJc_p}$$

or

$$\frac{T}{T_0} = \frac{t}{t_0} + \alpha u^{+2} \tag{28}$$

Differentiation gives

$$d\left(\frac{T}{T_0}\right) = d\left(\frac{t}{t_0}\right) + 2\alpha u^+ du^+$$

Substituting the value for $d(t/t_0)$ from this equation into equation (27) gives

$$\beta\ du^+ = -\ d\left(\frac{T}{T_0}\right)$$

Integration results in

$$\beta \int_0^{u^+} du^+ = -\int_1^{\frac{T}{T_0}} d\left(\frac{T}{T_0}\right)$$

or

$$\beta u^+ = 1 - \frac{T}{T_0} \tag{29}$$

If T^+ is defined as

$$T^+ \equiv \frac{(T_0 - T)\ c_p g \tau_0}{q_0 \sqrt{\tau_0/\rho_0}} = \frac{1 - \frac{T}{T_0}}{\beta} \tag{30}$$

equation (29) becomes

$$u^+ = T^+ \tag{31}$$

where T^+ will be called the total-temperature parameter. Equations (30) and (31) indicate that the velocity and total-temperature profiles for compressible flow with heat transfer are similar for a Prandtl number of 1. This conclusion was also reached by Kalikhman (reference 3).

Equations for Nusselt Number, Reynolds Number, and Friction Factor

Bulk temperature. - For obtaining equations for the Nusselt number, Reynolds number, and friction factor in terms of the dimensionless parameters used in the preceding sections, an expression for the bulk total temperature is first obtained. The bulk total temperature at a cross section in the tube is defined as the temperature the fluid would assume in a mixing tank placed immediately downstream of the cross section. Thus, the bulk total temperature is given by

$$T_b = \frac{\int_0^A T\rho u dA}{\int_0^A \rho u dA}$$

or

$$T_b = \frac{\int_0^{r_0} T\rho u \left(r_0 - y\right) dy}{\int_0^{r_0} \rho u \left(r_0 - y\right) dy} \tag{32}$$

Equation (32) can be written in dimensionless form as

$$T_b^+ = \frac{\int_0^{r_0^+} T^+ \frac{\rho}{\rho_0} u^+(r_0^+ - y^+)\, dy^+}{\int_0^{r_0^+} \frac{\rho}{\rho_0} u^+(r_0^+ - y^+)\, dy^+} \tag{33}$$

where

$$T_b^+ \equiv \frac{1}{\beta}\left(1 - \frac{T_b}{T_0}\right)$$

and

$$r_0^+ \equiv \frac{\sqrt{\tau_0/\rho_0}}{\mu_0/\rho_0} r_0$$

From equations (28) and (29),

$$\frac{t}{t_0} = 1 - \beta u^+ - \alpha u^{+2}$$

or

$$\frac{\rho}{\rho_0} = \frac{1}{1 - \beta u^+ - \alpha u^{+2}} \tag{34}$$

This equation differs from equation (10) because the effects of compressibility were neglected in obtaining equation (10). Substituting equations (31) and (34) into equation (33) gives T_b^+ for a Prandtl number of 1:

$$T_b^+ = \frac{\displaystyle\int_0^{r_0^+} \frac{u^{+2}(r_0^+ - y^+)}{1 - \beta u^+ - \alpha u^{+2}} dy^+}{\displaystyle\int_0^{r_0^+} \frac{u^+(r_0^+ - y^+)}{1 - \beta u^+ - \alpha u^{+2}} dy^+} \tag{35}$$

Nusselt number. - The heat-transfer coefficient h is defined as

$$h \equiv \frac{q_0}{T_0 - T_b} \tag{36}$$

and the Nusselt number with the thermal conductivity based on the wall temperature is

$$Nu_0 \equiv \frac{hD}{k_0} \tag{37}$$

By use of the definitions of T_b, r_0^+, Pr, and h, the expression for the Nusselt number can be written

$$Nu_0 = \frac{2\, r_0^+ \, Pr}{T_b^+} \tag{38}$$

The Nusselt number with the thermal conductivity based on the wall temperature is given by equation (38). It may be desired to base the conductivity on another temperature, perhaps the static bulk temperature; in order to do so, the ratio of bulk-to-wall static temperature can be obtained from

$$\frac{t_b}{t_0} = \frac{\displaystyle\int_0^{r_0^+} \frac{t}{t_0} \frac{\rho}{\rho_0} u^+(r_0^+ - y^+)\, dy^+}{\displaystyle\int_0^{r_0^+} \frac{\rho}{\rho_0} u^+(r_0^+ - y^+)\, dy^+}$$

Substituting

$$\frac{t}{t_0} = \frac{\rho_0}{\rho}$$

and equation (34), this expression becomes

$$\frac{t_b}{t_0} = \frac{\displaystyle\int_0^{r_0^+} u^+(r_0^+ - y^+)\, dy^+}{\displaystyle\int_0^{r_0^+} \frac{u^+(r_0^+ - y^+)\, dy^+}{1 - \beta u^+ - \alpha u^{+2}}} \tag{39}$$

With the assumptions of constant Prandtl number and constant specific heat, the law for the variation with temperature of the conductivity must be the same as the law for the variation with temperature of the viscosity or

$$\frac{k_b}{k_0} = \left(\frac{t_b}{t_0}\right)^d \tag{40}$$

The conductivity at any temperature between the wall and bulk temperature can be found in a similar manner.

Reynolds number. - The Reynolds number with fluid properties evaluated at the wall temperature is defined as

$$Re_0 \equiv \frac{\rho_0 u_b D}{\mu_0} \tag{41}$$

where

$$u_b \equiv \frac{\int_0^{r_0} u(r_0 - y)\, dy}{\int_0^{r_0} (r_0 - y)\, dy}$$

This equation written in dimensionless form with the denominator integrated becomes

$$\frac{u_b}{\sqrt{\tau_0/\rho_0}} \equiv u_b^+ = \frac{2}{r_0^{+2}} \int_0^{r_0^+} u^+(r_0^+ - y^+)\, dy^+ \tag{42}$$

Substituting equation (42) in equation (41) and making use of the definition of r_0^+ give

$$Re_0 = 2u_b^+ r_0^+ \tag{43}$$

The Reynolds number with the fluid properties evaluated at the bulk temperature is, from equations (41) and (43),

$$Re_b = 2u_b^+ r_0^+ \left(\frac{t_0}{t_b}\right)\left(\frac{t_0}{t_b}\right)^d \tag{44}$$

where t_0/t_b is calculated from equation (39).

Friction factor. - The friction factor with the fluid properties evaluated at the wall temperature is defined as

$$f_0 \equiv -\frac{D(dp/dx)_{fr}}{2\rho_0 u_b^2} = \frac{2\tau_0}{\rho_0 u_b^2} \tag{45}$$

This equation becomes, in dimensionless form,

$$f_0 = \frac{2}{u_b^{+2}} \tag{46}$$

where u_b^+ is calculated from equation (42). Another expression for f_0 can be obtained from equations (38), (43), and (46) if it is assumed that $u^+ = T^+$ (Reynolds analogy). This expression is

$$f_0 = \frac{2Nu_0}{Re_0} \tag{47}$$

From equations (45) and (46), it is evident that the friction factor with properties based on the bulk temperature is

$$f_b = \frac{2}{u_b^{+2}} \frac{t_b}{t_0} \tag{48}$$

Expressions for Nusselt number, Reynolds number, and friction factor based on any temperature between the surface and bulk temperature can also be obtained by using the ratio of bulk-to-wall temperature from equation (39).

RESULTS AND DISCUSSION

Velocity Distributions

The equations for velocity distributions close to a wall and at a distance from a wall for flow with heat transfer (equations (19) and (24)) are plotted in figure 1. In this figure, positive values of the heat-transfer parameter β indicate heat addition to the gas, whereas

negative values indicate heat extraction or cooling. The values for the constants ($n = 0.109$ and $\kappa = 0.36$), which were found in reference 1 from the experimental data for flow without heat transfer, are used for plotting the equations. In reference 1, it was found that the equation derived for flow close to a wall without heat transfer agrees closely with the data for $0 < y^+ < 26$ and that the equation derived for flow at a distance from a wall without heat transfer fits the data for $y^+ > 26$. For plotting the present equations for flow with heat transfer, the same limits of applicability for the equations for flow close to a wall and at a distance from a wall are used. It can be seen from the figure that the exact point of intersection of the curves representing the two equations is not critical, especially for high positive values of β, inasmuch as the slopes of the two curves at their intersection do not differ greatly. The constant of integration C_1 in equation (24) was found for each value of β by substituting at $y^+ = 26$ the value of u^+ obtained from equation (19).

For plotting equation (19), the value of the integral was numerically calculated by Simpson's rule. The exponent d in the equation was found from viscosity data for air to have an average value of 0.68 for temperatures between 0° and 2000° F. Although this value was specifically obtained for air, the values of d for most other common gases do not vary greatly from this value, so that the curves plotted in figure 1 should be applicable to most common gases.

The plots of the equations in figure 1 indicate that increasing the rate of heat addition, as is evidenced by increasing the heat-transfer parameter β, causes a considerable decrease in values of u^+ for constant y^+, as compared with those for adiabatic flow, whereas the opposite effect is indicated for heat extraction. A flattening of the velocity profile in the central portion of the tube is indicated for high rates of heat addition and a peaking of the profile at the center of of the tube is indicated for heat extraction.

These changes in profile shape are opposite to those for laminar flow of a gas; the explanation is that for turbulent flow, the turbulent shear stress, which is governed by the density, is more important in the central portion of the tube than is the viscous shear stress. The variation with temperature of the density is opposite to that of viscosity, so that the changes in profile shape in the center of the tube for turbulent flow caused by heat addition or extraction are opposite to those for laminar flow.

Nusselt Numbers

In figures 2 to 5, predicted Nusselt numbers are plotted against Reynolds numbers for a fluid with a Prandtl number of 1 for various values of the heat-transfer parameter β and the compressibility parameter α. In figure 2, the Nusselt numbers and Reynolds numbers are plotted with the gas properties (density, viscosity, and thermal conductivity) evaluated at the bulk temperature. The Nusselt numbers for this case are obtained by dividing equation (38) by the expression for k_b/k_0 given by equation (40); $T_b{}^+$ is calculated from equation (35) and t_b/t_0 is calculated from equation (39), where the values of the integrals in these equations are numerically calculated and $r_0{}^+$, α, and β are held constant for each integration. The relations between u^+ and y^+ required in equations (35) and (39) are obtained from the plots of equations (19) and (24) in figure 1. The Reynolds numbers are obtained from equation (44), where $u_b{}^+$ is calculated from equation (42) and t_b/t_0 is calculated from equation (39). The exponent d is taken as 0.68 in both equations (40) and (44) because the viscosity and the thermal conductivity are assumed to follow the same law for variation with temperature. For obtaining various values of Nusselt number and Reynolds number for given values of α and β, the tube-radius parameter $r_0{}^+$ is allowed to vary arbitrarily.

The curves plotted in figure 2, which are for the case where α equals 0 or where compressibility effects caused by high velocities are neglected, indicate a decrease in Nusselt number with increasing positive values of the heat-transfer parameter β. Increasing values of the heat-transfer parameter β correspond, in general, to increasing values of the ratio of wall-to-bulk temperature t_0/t_b; the curves shown correspond to a range of ratios of wall-to-bulk temperature of from 1 to about 7 for heating and from 1 to about 0.4 for cooling. The same trends for heating have been experimentally observed to determine over-all heat-transfer coefficients in tubes with a range of ratios of surface-to-bulk temperature of from 1 to about $3\frac{1}{2}$ (references 4 to 8). The dashed line in figure 2 represents the mean of experimental data for moderate temperature differences (reference 9) and is seen to agree, in general, with the predicted line for $\beta = 0$.

For heat addition to the gas, figure 3(a) shows predicted Nusselt numbers plotted against Reynolds numbers for two values of the heat-transfer parameter β and for $\alpha = 0$ with the gas properties in the Nusselt and Reynolds numbers evaluated at a film temperature between the wall and bulk temperatures defined by

$$t_{0.4} \equiv 0.4\ (t_0 - t_b) + t_b \tag{49}$$

or

$$\frac{t_{0.4}}{t_0} = 0.4 + 0.6\ \frac{t_b}{t_0}$$

For obtaining the values for Nusselt and Reynolds numbers for figure 3(a), the same equations were used as were used for figure 2 except that t_b/t_0 was replaced by $t_{0.4}/t_0$ in the equations. The plot in figure 3(a) indicates that the effects of β, or of ratio of wall-to-bulk temperature, is practically eliminated for heat addition when the gas properties are evaluated at the film temperature as defined. In the experimental work described in references 4 to 8, it was found that the effects of ratio of wall-to-bulk temperature on the Nusselt numbers were practically eliminated when the fluid properties were evaluated at either the wall or the average film temperature. In the present analysis, however, where the maximum ratio of wall-to-bulk temperature was about twice that in the experimental work, separation of the curves was found when the properties were evaluated at the wall temperature.

The curve from figure 3(a), together with experimental data from reference 8, is shown in figure 3(b). The data points were calculated in reference 8 by evaluating the gas properties at the average of the wall and bulk temperatures; substantially the same results are obtained if the properties are evaluated at $t_{0.4}$ for the range of ratios of wall-to-bulk temperature used in the tests. The data, which were obtained for air, were corrected to a Prandtl number of 1 by dividing the Nusselt numbers by the empirical correction $Pr^{0.4}$. The data are seen to agree substantially with the predicted line for heating for Reynolds numbers greater than 10,000. The deviation of the data from the predicted line at low Reynolds numbers is probably caused by transitional effects in the fluid flow that were not considered in the analysis. It should be mentioned that the analytical and experimental results are not strictly comparable because the analytical results are for local heat-transfer coefficients at a cross section in the tube, whereas the experimental results give average heat-transfer coefficients for a whole tube.

Variations of predicted Nusselt numbers with Reynolds numbers for flow with heat extraction or cooling are shown in figure 3(c) with the properties evaluated at a film temperature defined by

$$t_{0.6} \equiv 0.6\ (t_0 - t_b) + t_b$$

It is seen that use of this temperature eliminates the effects of β on the Nusselt numbers for heat extraction.

A correlation for both heating and cooling in figure 3(d) shows Nusselt numbers with the gas properties evaluated at the average of the wall and bulk temperatures $t_{0.5}$ (This temperature is also the average of $t_{0.4}$ and $t_{0.6}$.). By use of this temperature for evaluating the fluid properties, the maximum separation of the curves is found to be about 9 percent.

In figures 4 and 5 are shown predicted Nusselt numbers plotted against Reynolds numbers for flow with heat addition where the compressibility effects associated with high subsonic velocities are considerable ($\alpha = 0.00025$). For plotting these curves, the velocity-distribution equations (equations (19) and (24)), which were derived for $\alpha = 0$, were used because the effects of compressibility on velocity distributions were found in reference 1 to be small for subsonic flow. As in the case of flow for $\alpha = 0$, the use of the static film temperature $t_{0.4}$, defined by equation (49), for evaluating the gas properties practically eliminates the effect of the heat-transfer parameter β on the Nusselt numbers for heat addition. Part of the slight difference between the plots in figures 3 and 5 is probably caused by assuming that $\alpha = 0$ for the velocity-distribution equations for both plots.

The heat-transfer coefficients used in figures 2 to 5 are defined by equation (36) and are based on the difference between the wall and bulk total temperatures. Although strictly speaking equation (36) should be used only for fluids with a Prandtl number of 1, it can be used for real gases except for cases where the temperature differences are very small and the Mach numbers are high (reference 10).

Friction Factors

Predicted friction factors plotted against Reynolds numbers for various values of the heat-transfer parameter β, with $\alpha = 0$, are shown in figures 6 and 7. In figure 6, the friction factors are calculated from equation (48) where u_b^+ and t_b/t_0 are calculated as before from equations (42) and (39). As in the case of Nusselt numbers, a decrease in friction factor is indicated for high positive values of β or of ratio of wall-to-bulk temperature when the gas properties in the friction factors and Reynolds numbers are evaluated at the bulk temperature. These trends have also been experimentally observed (reference 5). The dashed line in figure 6 represents the mean of experimental data for isothermal flow (reference 11) and agrees in general with the predicted line for Reynolds numbers above 10,000.

Friction factors are plotted against Reynolds numbers in figure 7(a) for heat addition with the gas properties evaluated at the film temperature defined by equation (49), indicating that, as in the case of Nusselt numbers, evaluating the gas properties at that temperature practically eliminates the effects of β or of ratio of wall-to-bulk temperature for flow with heat addition. In figure 7(b), which is plotted for flow with heat extraction, it is indicated that the use of $t_{0.6}$ for evaluating the gas properties eliminates the effects of β or of variable gas properties on the friction factors for heat extraction. The friction-factor correlation for both heating and cooling of figure 7(c) indicates that, as in the case of Nusselt numbers, the maximum separation of the curves for various values of β is about 9 percent when the gas properties are evaluated at the average of the wall and bulk temperatures.

SUMMARY OF RESULTS

From an analytical investigation of fluids flowing through smooth tubes with heat transfer, the following results were obtained:

1. The equations for the prediction of velocity distributions for fully developed turbulent flow of gases in smooth tubes with heat transfer indicated that variation of fluid properties across the tube causes a flattening of the velocity profile for heat addition and a peaking of the profile at the center of the tube for heat extraction.

2. The investigation of the effect of frictional heating on temperature distributions indicated that, for a fluid with a Prandtl number of 1, the velocity and total-temperature profiles are similar.

3. The equations for Nusselt number, Reynolds number, and friction factor for a fluid with a Prandtl number of 1 for the case where the variation of fluid properties across the tube is large predicted trends that agree with those experimentally obtained for Reynolds numbers greater than 10,000.

4. The analysis indicated that the effect of variation of fluid properties across the tube on the Nusselt number and the friction-factor correlations for both heating and cooling can be eliminated by evaluating the fluid properties at a temperature close to the average of the wall and bulk temperatures.

Lewis Flight Propulsion Laboratory,
National Advisory Committee for Aeronautics,
Cleveland, Ohio, July 28, 1950.

APPENDIX - SYMBOLS

The following symbols are used in the report:

A	cross-sectional area based on inside diameter of tube, sq ft
C, C_1	constants of integration
c_p	specific heat of fluid at constant pressure, Btu/(lb)(°F)
D	inside diameter of tube, ft
d	exponent that describes variation of viscosity of fluid with temperature
g	acceleration due to gravity, 32.2 ft/sec^2
h	heat-transfer coefficient $\frac{q_0}{T_0 - T_b}$ for compressible fluid or $\frac{q_0}{t_0 - t_b}$ for incompressible fluid, Btu/(sec)(sq ft)(°F)
J	mechanical equivalent of heat, 778 ft-lb/Btu
k	thermal conductivity of fluid, (Btu)(ft)/(sec)(sq ft)(°F)
k_b	thermal conductivity of fluid evaluated at t_b, (Btu)(ft)/(sec)(sq ft)(°F)
k_0	thermal conductivity of fluid evaluated at wall, (Btu)(ft)/(sec)(sq ft)(°F)
$k_{0.4}$, $k_{0.5}$, $k_{0.6}$	thermal conductivity of fluid evaluated at t_0, $t_{0.4}$, $t_{0.5}$, and $t_{0.6}$, respectively, (Btu)(ft)/(sec)(sq ft)(°F)
n	constant
q	total rate of heat transfer toward tube center per unit area, Btu/(sec)(sq ft)

q_t rate of turbulent heat transfer toward tube center per unit area, Btu/(sec)(sq ft)

q_v rate of viscous heat transfer toward tube center per unit area, Btu/(sec)(sq ft)

q_0 total rate of heat transfer at wall toward tube center per unit area, Btu/(sec)(sq ft)

r_0 inside tube radius, ft

T total temperature, $^{\circ}R$

T_b bulk or average total temperature of fluid at cross section of tube, $^{\circ}R$

T_0 or t_0 absolute wall temperature, $^{\circ}R$

t absolute static temperature, $^{\circ}R$

t_b bulk or average static temperature of fluid at cross section of tube, $^{\circ}R$

$t_{0.4}$ film temperature, $0.4\ (t_0 - t_b) + t_b$, $^{\circ}R$

$t_{0.5}$ film temperature, $0.5\ (t_0 - t_b) + t_b$, $^{\circ}R$

$t_{0.6}$ film temperature, $0.6\ (t_0 - t_b) + t_b$, $^{\circ}R$

u time average velocity parallel to axis of tube, ft/sec

u_b bulk or average velocity at cross section of tube, ft/sec

y distance from tube wall, ft

ϵ coefficient of eddy diffusivity for momentum, sq ft/sec

ϵ_h coefficient of eddy diffusivity for heat, sq ft/sec

κ Kármán constant

μ absolute viscosity of fluid, (lb)(sec)/(sq ft)

μ_b absolute viscosity of fluid evaluated at t_b, (lb)(sec)/(sq ft)

μ_0 absolute viscosity of fluid at wall, (lb)(sec)/(sq ft)

$\mu_{0.4}$, $\mu_{0.5}$, $\mu_{0.6}$ absolute viscosity of fluid evaluated at $t_{0.4}$, $t_{0.5}$, and $t_{0.6}$, respectively, (lb)(sec)/(sq ft)

ρ mass density, lb-sec^2/ft^4

ρ_b bulk or average density at cross section of tube, lb-sec^2/ft^4

ρ_0 mass density of fluid at wall, lb-sec^2/ft^4

$\rho_{0.4}$, $\rho_{0.5}$, $\rho_{0.6}$ density of fluid evaluated at $t_{0.4}$, $t_{0.5}$, and $t_{0.6}$, respectively, lb-sec^2/ft^4

τ shear stress in fluid, lb/sq ft

τ_0 shear stress in fluid at wall, lb/sq ft

Subscript:

fr on friction pressure gradient

Dimensionless groups:

α compressibility or frictional heating parameter, $\dfrac{\tau_0}{2gJc_pT_0\rho_0}$

β heat-transfer parameter, $\dfrac{q_0\sqrt{\tau_0/\rho_0}}{c_pg\tau_0t_0}$

f_0 friction factor, $-\dfrac{D(dp/dx)_{fr}}{2\rho_0 u_b^2} = \dfrac{2\tau_0}{\rho_0 u_b^2}$

f_b friction factor with density evaluated at t_b, $-\dfrac{D(dp/dx)_{fr}}{2\rho_b u_b^2} = \dfrac{2\tau_0}{\rho_b u_b^2}$

$f_{0.4}$, $f_{0.5}$, $f_{0.6}$ friction factor with density evaluated at $t_{0.4}$, $t_{0.5}$, and $t_{0.6}$, respectively

Nu_b	Nusselt number with thermal conductivity evaluated at t_b, $\frac{hD}{k_b}$
Nu_0	Nusselt number with thermal conductivity evaluated at t_0
$Nu_{0.4}$, $Nu_{0.5}$, $Nu_{0.6}$	Nusselt number with thermal conductivity evaluated at $t_{0.4}$, $t_{0.5}$, and $t_{0.6}$, respectively
Pr	Prandtl number, $\frac{c_p \mu g}{k}$
Re_b	Reynolds number with density and viscosity evaluated at t_b, $\frac{\rho_b u_b D}{\mu_b}$
Re_0	Reynolds number with density and viscosity evaluated at t_0
$Re_{0.4}$, $Re_{0.5}$, $Re_{0.6}$	Reynolds number with density and viscosity evaluated at $t_{0.4}$, $t_{0.5}$, and $t_{0.6}$, respectively
r_0^+	tube-radius parameter, $\frac{\sqrt{\tau_0/\rho_0}}{\mu_0/\rho_0} r_0$
T^+	total-temperature parameter, $\frac{(T_0 - T)c_p g \tau_0}{q_0 \sqrt{\tau_0/\rho_0}}$
T_b^+	bulk total-temperature parameter, $\frac{1}{\beta}\left(1 - \frac{T_b}{T_0}\right)$
u^+	velocity parameter, $\frac{u}{\sqrt{\tau_0/\rho_0}}$
u_b^+	bulk velocity parameter, $\frac{u_b}{\sqrt{\tau_0/\rho_0}}$
y^+	wall-distance parameter, $\frac{\sqrt{\tau_0/\rho_0}}{\mu_0/\rho_0} y$

REFERENCES

1. Deissler, Robert G.: Analytical and Experimental Investigation of Adiabatic Turbulent Flow in Smooth Tubes. NACA TN 2138, 1950.

2. von Kármán, Th.: The Analogy between Fluid Friction and Heat Transfer. Trans. A.S.M.E., vol. 61, no. 8, Nov. 1939, pp. 705-710.

3. Kalikhman, L. E.: Heat Transmission in the Boundary Layer. NACA TM 1229, 1949.

4. Humble, Leroy V., Lowdermilk, Warren H., and Grele, Milton: Heat Transfer from High-Temperature Surfaces to Fluids. I - Preliminary Investigation with Air in Inconel Tube with Rounded Entrance, Inside Diameter of 0.4 Inch, and Length of 24 Inches. NACA RM E7L31, 1948.

5. Lowdermilk, Warren H., and Grele, Milton D.: Heat Transfer from High-Temperature Surfaces to Fluids. II - Correlation of Heat-Transfer and Friction Data for Air Flowing in Inconel Tube with Rounded Entrance. NACA RM E8L03, 1949.

6. Sams, Eldon W., and Desmon, Leland G.: Heat Transfer from High-Temperature Surfaces to Fluids. III - Correlation of Heat-Transfer Data for Air Flowing in Silicon Carbide Tube with Rounded Entrance, Inside Diameter of 3/4 Inch, and Effective Length of 12 Inches. NACA RM E9D12, 1949.

7. Lowdermilk, Warren H., and Grele, Milton D.: Influence of Tube-Entrance Configuration on Average Heat-Transfer Coefficients and Friction Factors for Air Flowing in an Inconel Tube. NACA RM E50E23, 1950.

8. Desmon, Leland G., and Sams, Eldon W.: Correlation of Forced-Convection Heat-Transfer Data for Air Flowing in Smooth Platinum Tube with Long-Approach Entrance at High Surface and Inlet-Air Temperatures. NACA RM E50H23, 1950.

9. Brown, Aubrey I., and Marco, Salvatore M.: Introduction to Heat Transfer. McGraw-Hill Book Co., Inc., 1942, p. 91.

10. McAdams, William H., Nicolai, Lloyd A., and Keenan, Joseph H.: Measurements of Recovery Factors and Coefficients of Heat Transfer in a Tube for Subsonic Flow of Air. NACA TN 985, 1945.

11. McAdams, William H.: Heat Transmission. McGraw-Hill Book Co., Inc., 2d ed., 1942, p. 118.

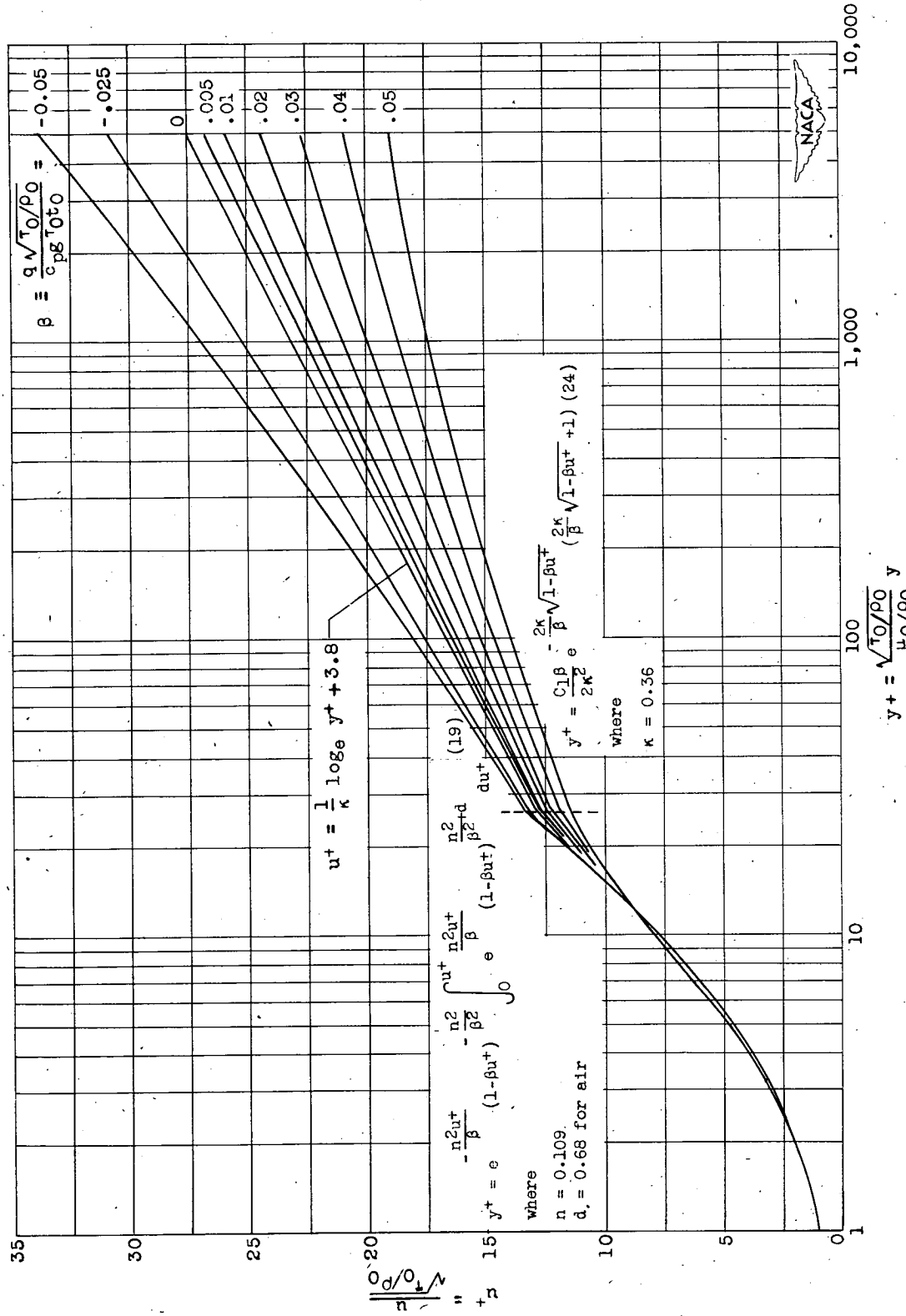


Figure 1. - Generalized velocity distribution for flow of gases with heat transfer at Prandtl number of 1.

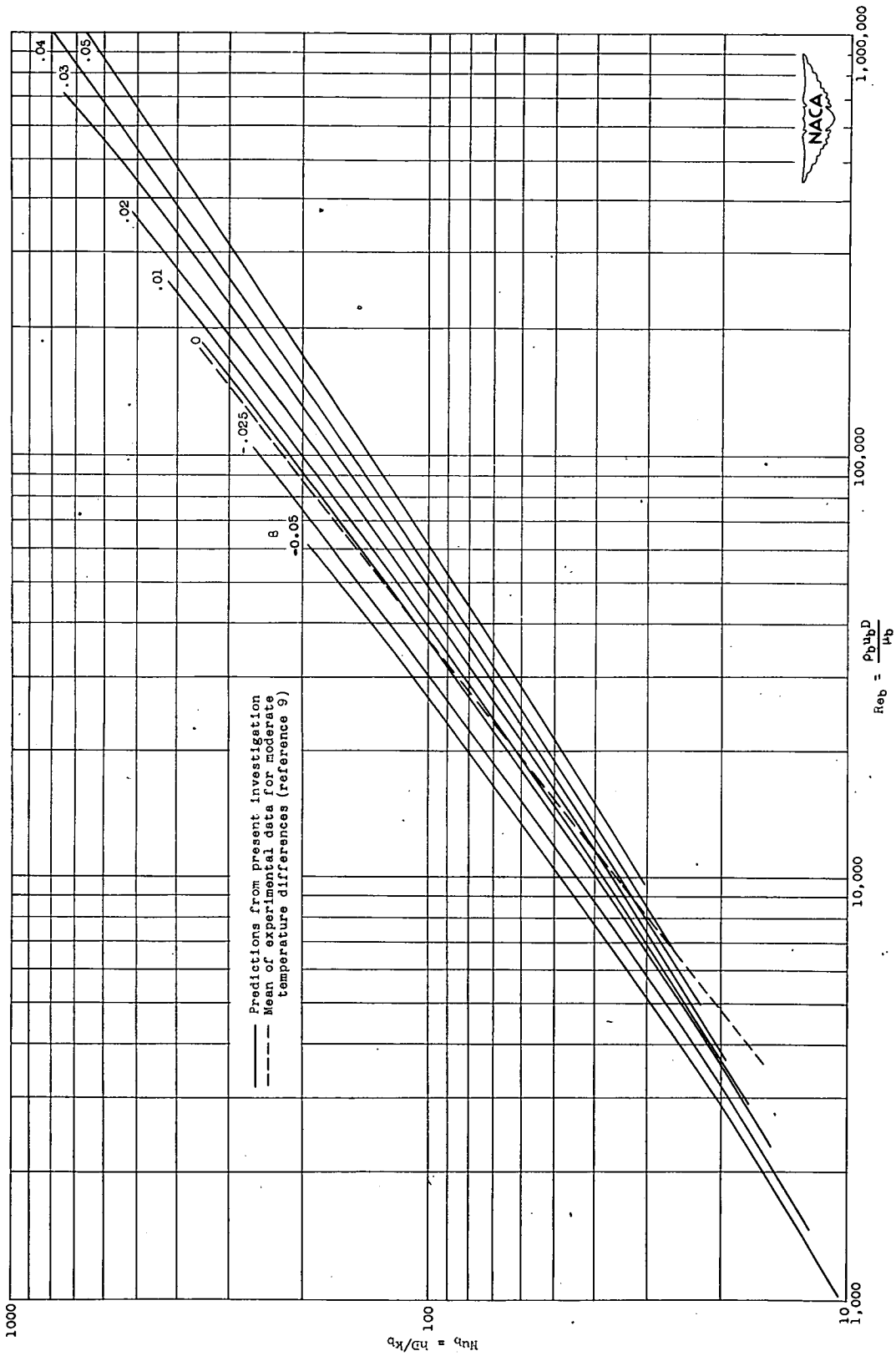
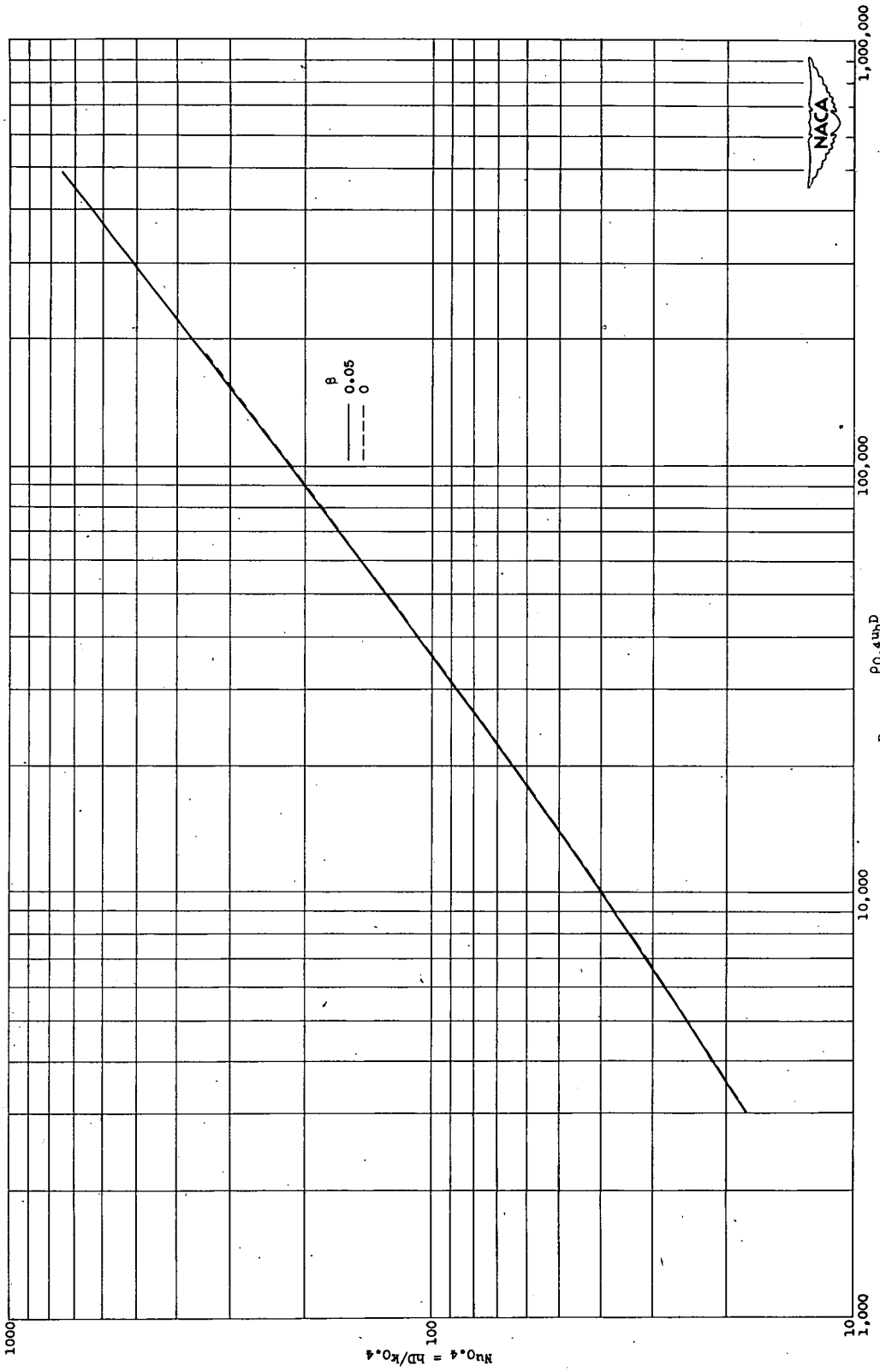
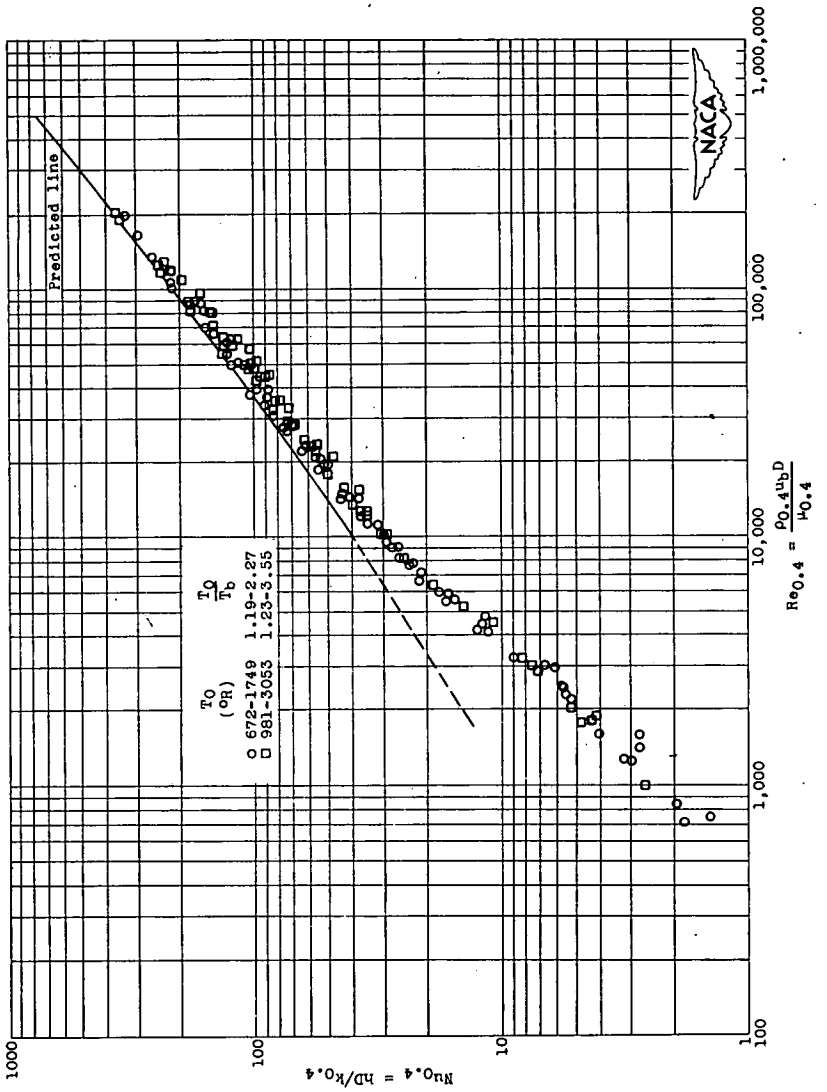


Figure 2. - Variation of Nusselt number with Reynolds number for flow of gases with gas properties evaluated at static bulk temperature. Compressibility parameter α, 0; Prandtl number, 1. Positive and negative values of β indicate heat addition and extraction, respectively.



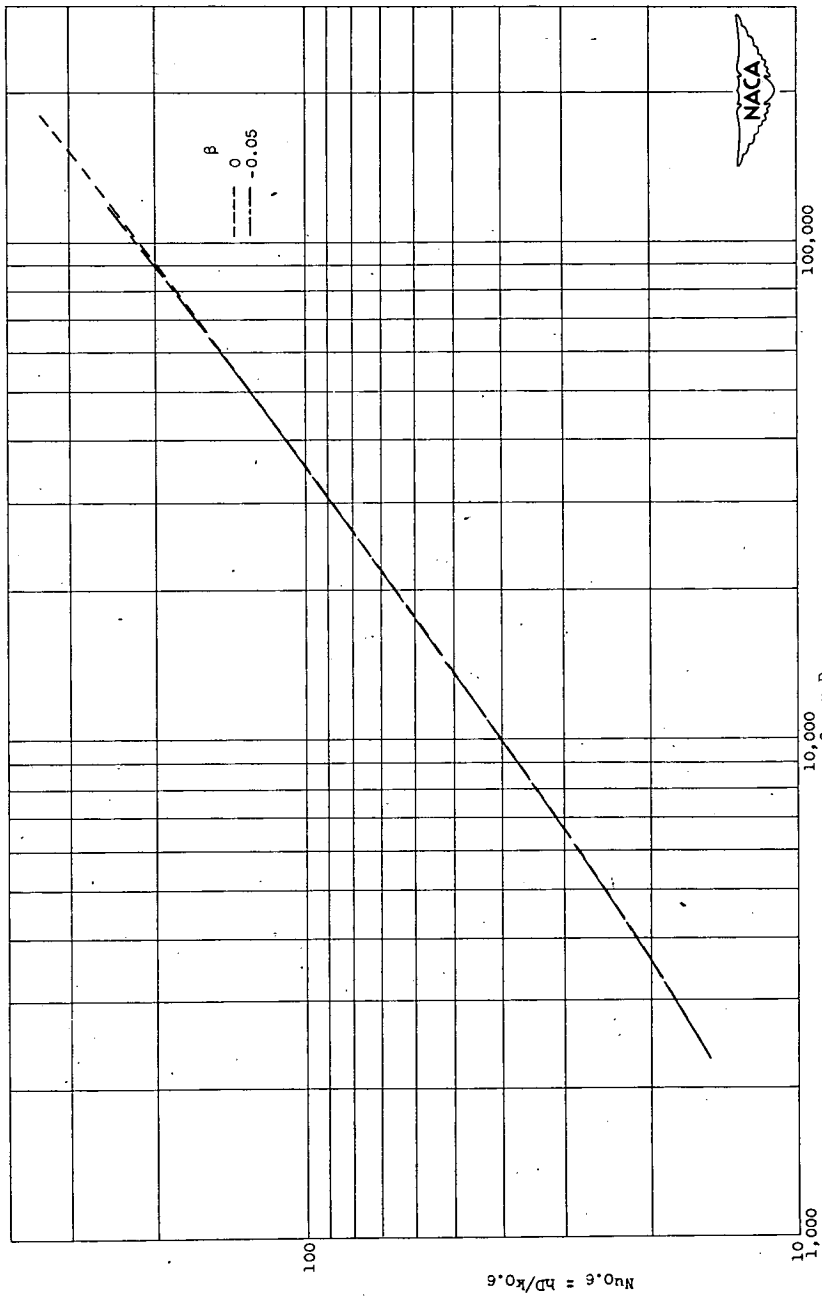
(a) Evaluation of gas properties at $t_{0.4} = 0.4(t_0 - t_b) + t_b$; heat addition.

Figure 3. - Variation of Nusselt number with Reynolds number for flow of gases. Compressibility parameter α, 0; Prandtl number, 1.



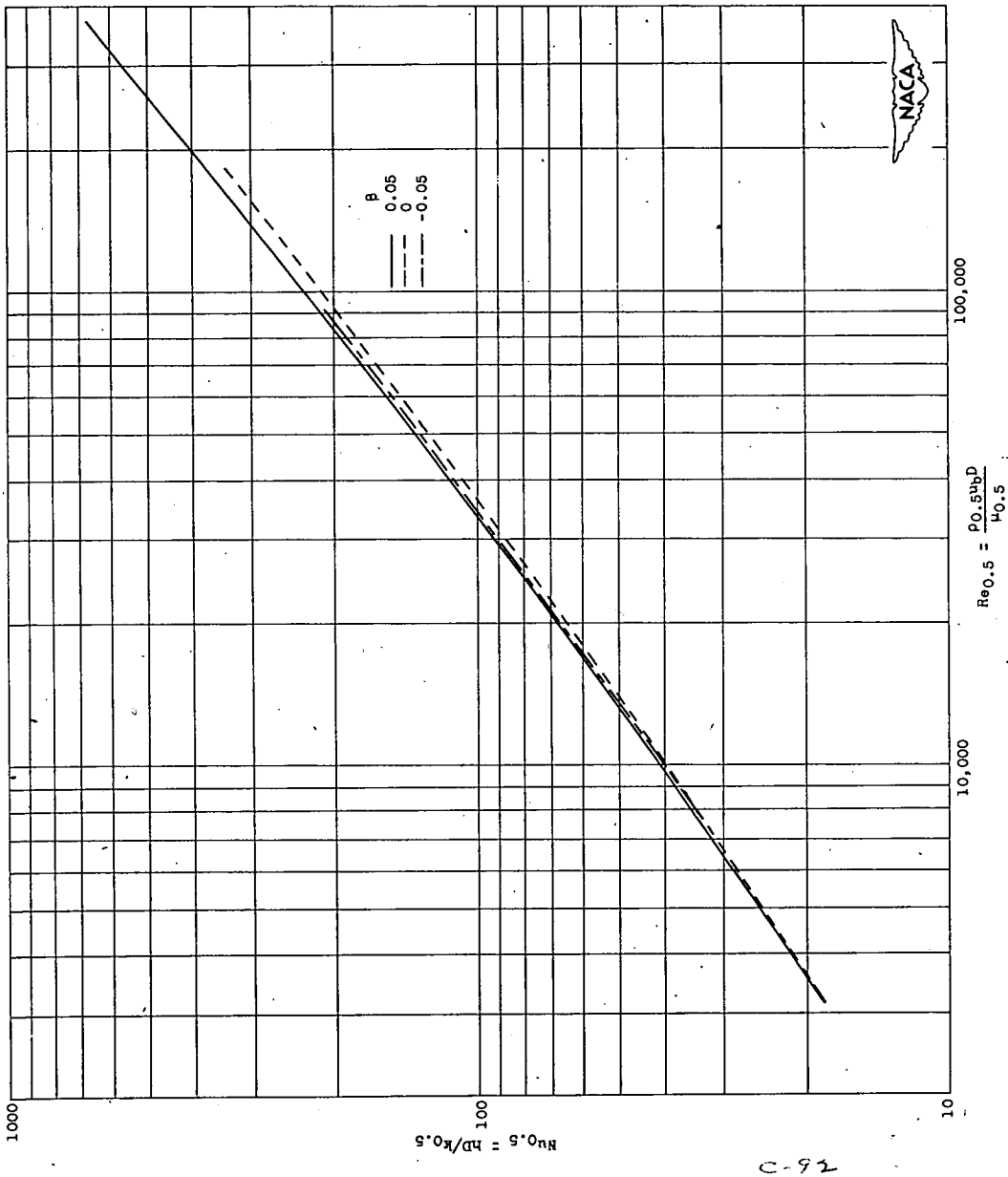
(b) Evaluation of gas properties at $t_{0.4} = 0.4(t_0 - t_b) + t_b$; heat addition; data from reference 8.

Figure 3. - Continued. Variation of Nusselt number with Reynolds number for flow of gases. Compressibility parameter α, 0; Prandtl number, 1.



(c) Evaluation of gas properties at $t_{0.6} = 0.6(t_0 - t_b) + t_b$; heat extraction.

Figure 3. - Continued. Variation of Nusselt number with Reynolds number for flow of gases. Compressibility parameter α, 0; Prandtl number, 1.



(d) Evaluation of gas properties at $t_{0.5} = 0.5(t_0 - t_b) + t_b$; heat addition and extraction. Variation of Nusselt number with Reynolds number for flow of gases. Compressibility parameter α, 0; Prandtl number, 1.

Figure 3. - Concluded.

C-92

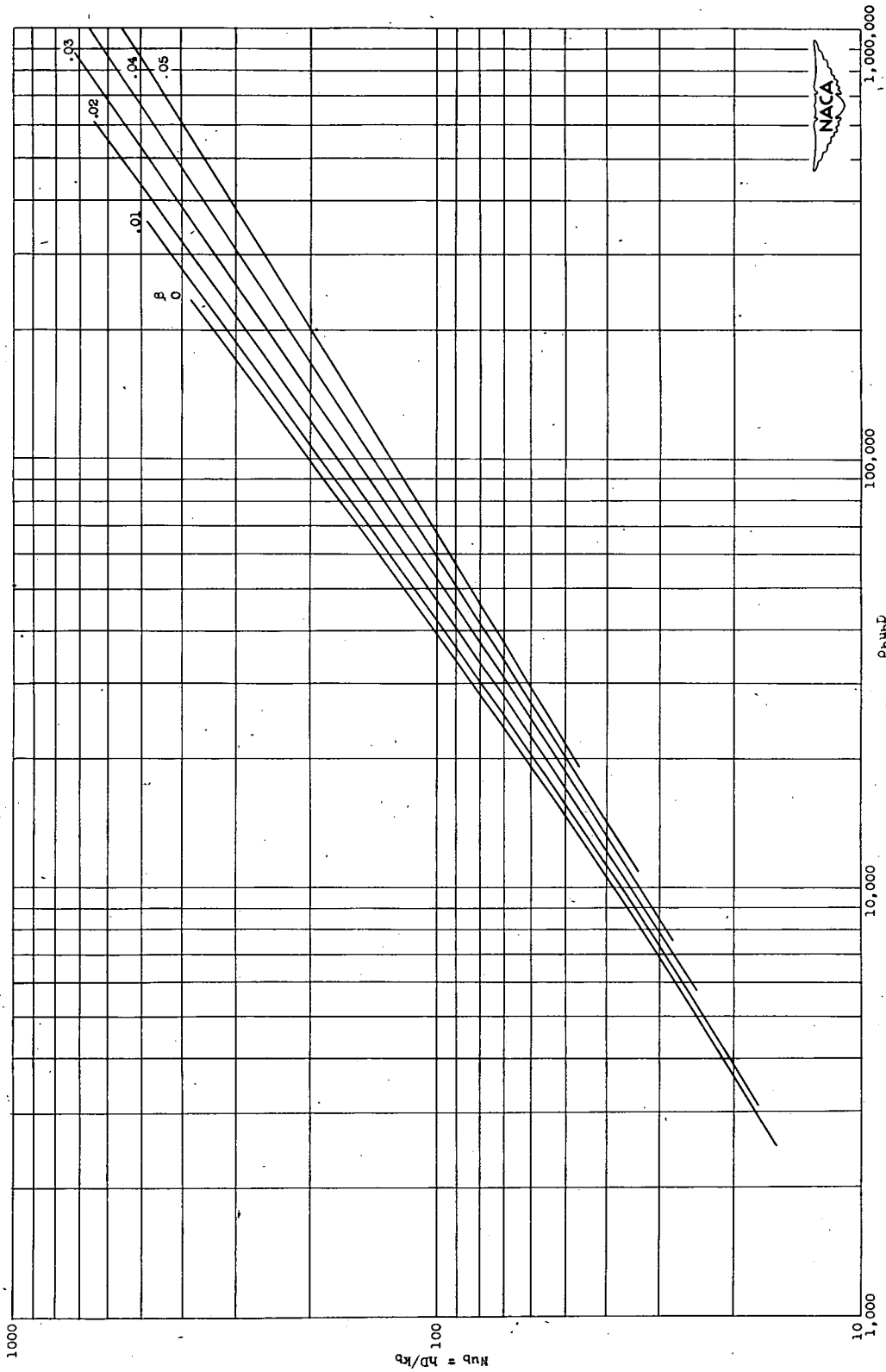


Figure 4. - Variation of Nusselt number with Reynolds number for flow of gases at high velocities with heat addition with gas properties evaluated at static bulk temperature. Compressibility parameter α, 0.00025; Prandtl number, 1.

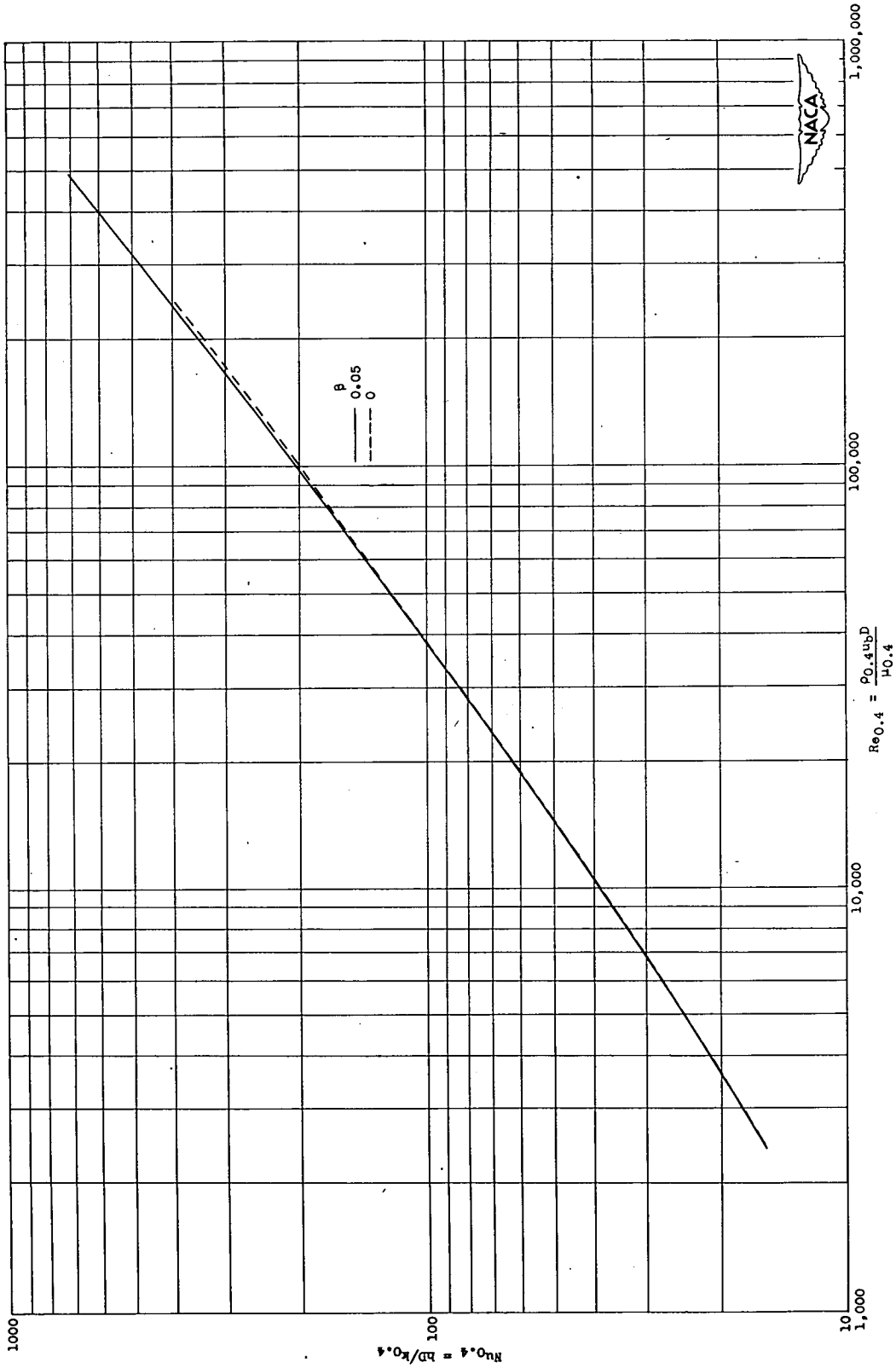


Figure 5. - Variation of Nusselt number with Reynolds number for flow of gases at high velocities with heat addition with gas properties evaluated at $t_{0.4} = 0.4(t_0 - t_b) + t_b$. Compressibility parameter α, 0.00025; Prandtl number, 1.

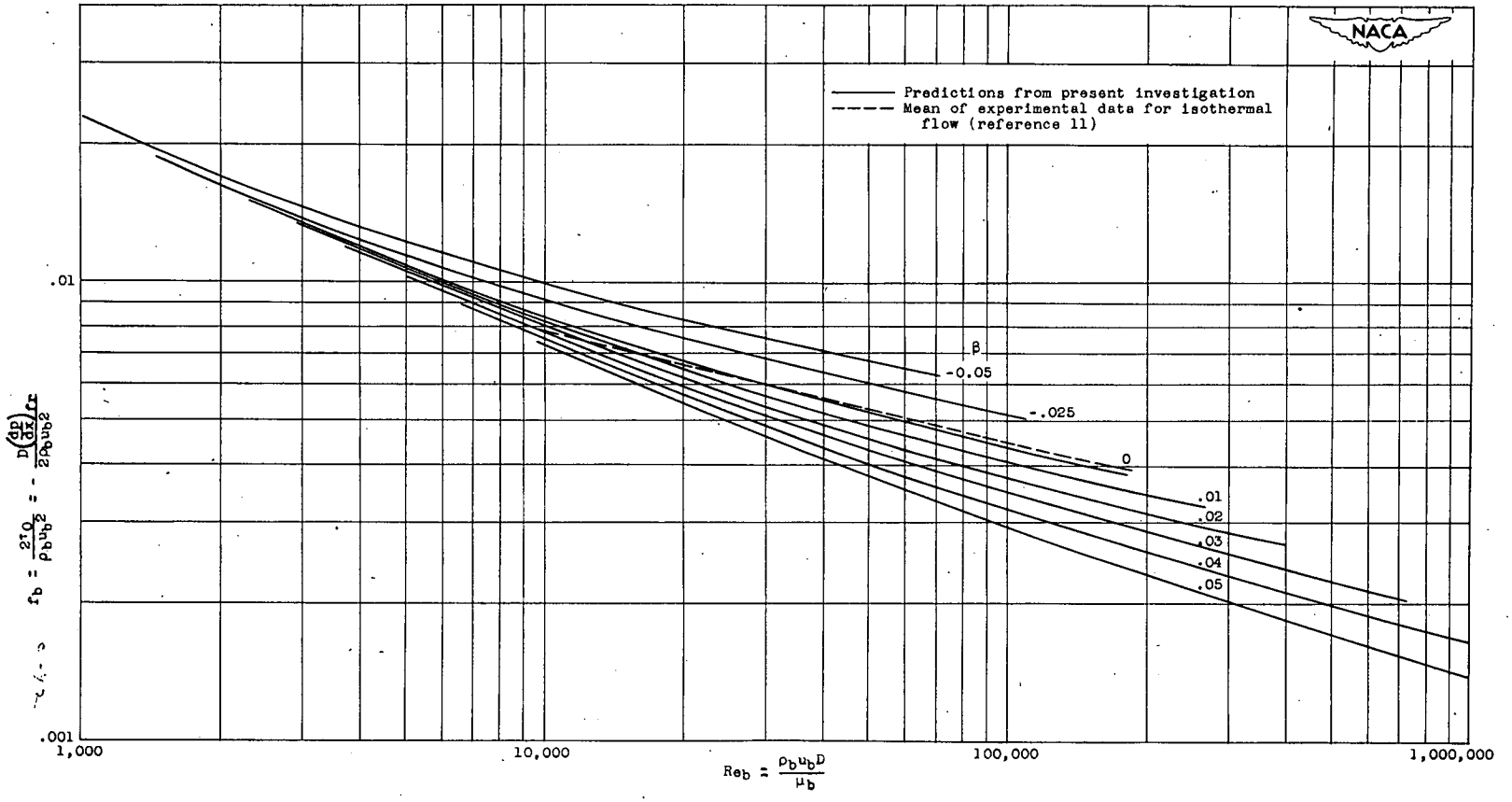
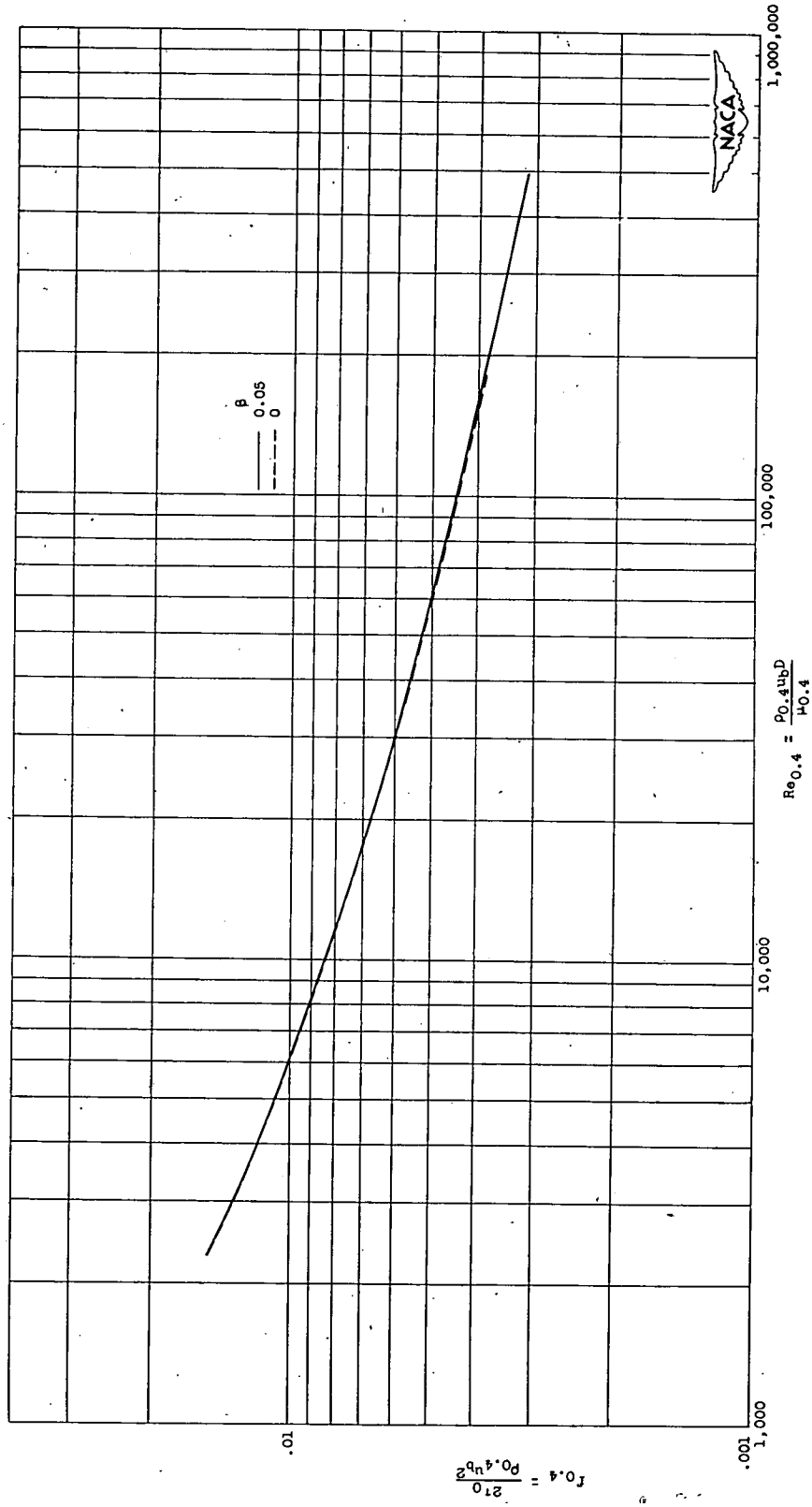
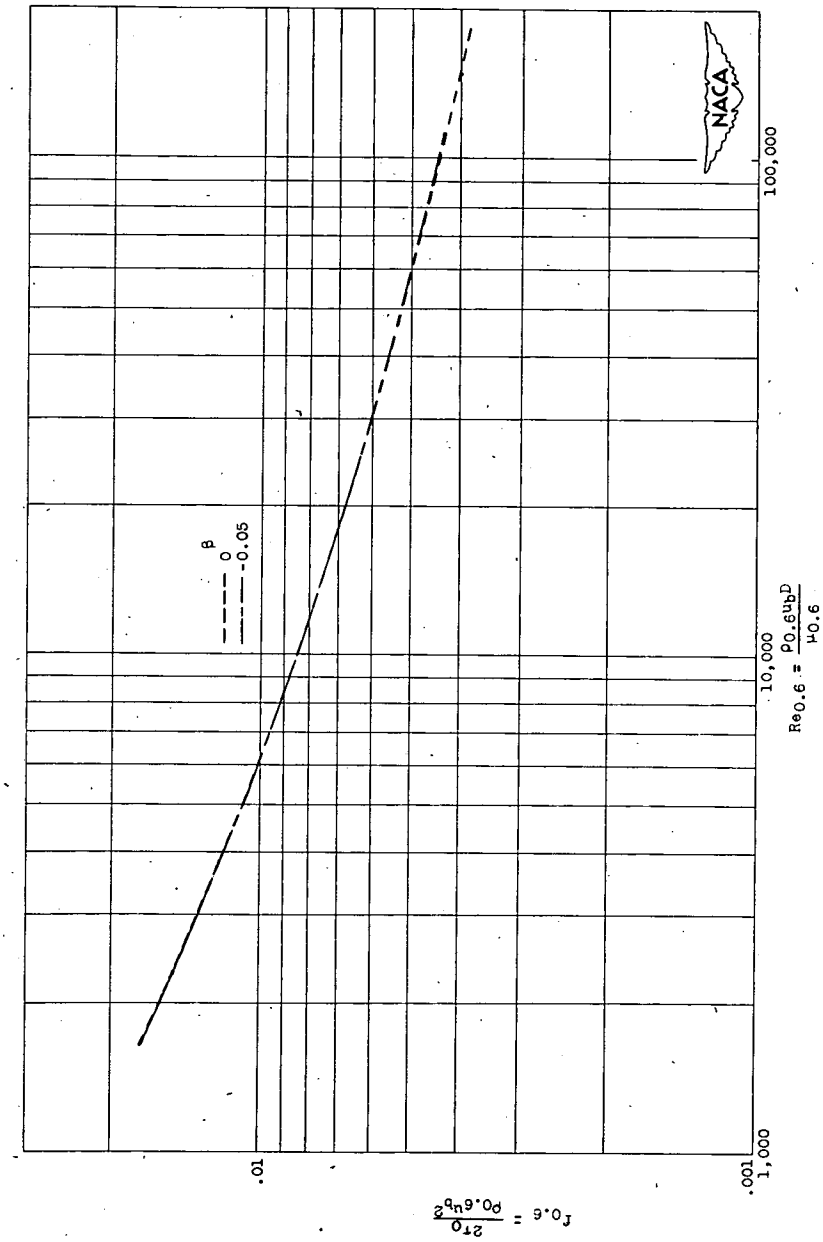


Figure 6. - Variation of friction factor with Reynolds number for flow of gases with gas properties evaluated at static bulk temperature. Compressibility parameter α, 0; Prandtl number, 1. Positive and negative values of β indicate heat addition and extraction, respectively.



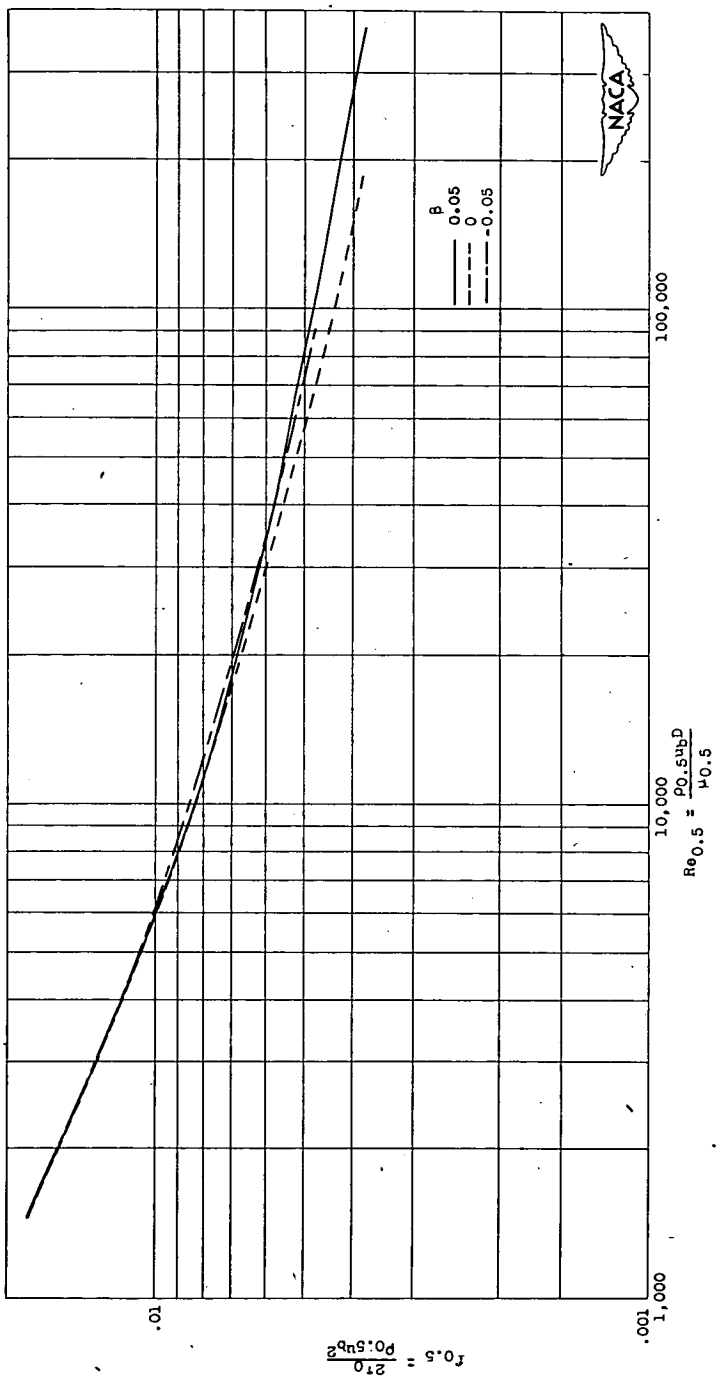
(a) Evaluation of gas properties at $t_{0.4} = 0.4(t_0 - t_b) + t_b$; heat addition.

Figure 7. - Variation of friction factor with Reynolds number for flow of gases. Compressibility parameter α, 0; Prandtl number, 1.



(b) Evaluation of gas properties at $t_{0.6} = 0.6(t_0 - t_b) + t_b$; heat extraction.

Figure 7. - Continued. Variation of friction factor with Reynolds number for flow of gases. Compressibility parameter α, 0; Prandtl number, 1.



(c) Evaluation of gas properties at $t_{0.5} = 0.5(t_0-t_b) + t_b$; heat addition and heat extraction.

Figure 7. - Concluded. Variation of friction factor with Reynolds number for flow of gases. Compressibility parameter α, 0; Prandtl number, 1.