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TECHNICAL NOTE 1975

EXPERIMENTAL INVESTIGATION OF TEMPERATURE RECOVERY FACTORS ON BODIES OF REVOLUTION AT SUPERSONIC SPEEDS

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Washington
October 1949

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EXPERIMENTAL INVESTIGATION OF TEMPERATURE RECOVERY FACTORS ON BODIES OF REVOLUTION AT SUPERSONIC SPEEDS

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SUMMARY

Local temperature recovery factors have been measured on two bodies of revolution, a conical body and a parabolic body at a Mach number of 2.0, and on the parabolic body at a Mach number of 1.5. Data were obtained at two Reynolds numbers with laminar boundary layers and with turbulent boundary layers induced by surface roughness.

The measured recovery factors with laminar boundary layers were constant along the length of the bodies and were independent of Mach number, Reynolds number, and body shape. However, the recovery factor apparently increased slightly with the roughness of the body surface. The recovery factors with artificially induced turbulent boundary layers were also independent of Mach number, Reynolds number, and body shape. However, the values varied slightly with the method employed to cause transition. The recovery factor for a laminar boundary layer is well represented by the square root of Prandtl number, and for a turbulent boundary layer by the cube root of Prandtl number, as predicted by theory.

INTRODUCTION

When relative motion occurs between an insulated body and air, the body assumes a higher temperature than that of the undisturbed fluid. At low relative velocities this temperature difference is small, but as the velocity approaches and exceeds that of sound the temperature difference becomes large. An accurate means of predicting the temperature attained by the surface of such a body is essential in order to anticipate the speed at which aircraft would be subject to adverse thermal effects and in order to design cooling systems to alleviate these effects at greater speeds. Regardless of the power available or the aerodynamic efficiency, the maximum speed of an aircraft without cooling is limited to the speed at which the insulated surface temperature equals the maximum allowable temperature for the structure, cargo, or occupants. If

greater speeds are to be attained, cooling is required, and the difference between the insulated surface temperature at the flight speed and the maximum allowable surface temperature is the thermal potential for the heat-transfer calculations involved in the design of the cooling equipment.

The surface temperature attained by an insulated body submerged in a fluid moving at high speeds depends on the combined effects of two phenomena: (1) A temperature rise in the boundary layer adjacent to the body resulting from the dissipation of kinetic energy as the fluid is brought to rest at the surface of the body by the action of viscous forces, and (2) a heat flow outward in the boundary layer by conduction and convection. The rate of change of temperature with time due to the dissipation of kinetic energy at any point in the boundary layer is a function of the slope of the velocity profile at that point and the kinematic viscosity and specific heat of the fluid. The rate of change of temperature with time due to conduction is a function of the slope of the temperature profile at the point and the thermal diffusivity of the fluid. The rate of change of temperature with time due to convection is a function of the vertical and horizontal velocity components and temperature gradients. Therefore, when flow is first established around the body, the temperature of the fluid in the boundary layer rises until the temperature at every point is such that the rate of change of temperature due to the dissipation of kinetic energy is equal to the rate of change of temperature due to conduction and convection. When this equilibrium condition is attained, a fixed temperature profile is established across the boundary layer at every station along the body. Thus the temperature difference across the boundary layer is a function of the magnitude of the velocity just outside the boundary layer, the velocity profile, and the kinematic viscosity and thermal diffusivity of the fluid.

The ratio of the kinematic viscosity to the thermal diffusivity is known as the Prandtl number and is the parameter that relates the boundary-layer temperature profile to the velocity profile. If the fluid in which the body is submerged has a Prandtl number of one, that is, if the kinematic viscosity is equal to the thermal diffusivity, the reduction in kinetic energy from the free stream to any point in the boundary layer is accompanied by an equivalent increase in heat energy. Thus, the total energy is constant across the boundary layer and the temperature at the surface of the body, where the fluid velocity is zero, is the total or stagnation temperature of the fluid stream regardless of the shape of the body. If the Prandtl number of the fluid is greater than one (the kinematic viscosity greater than the thermal diffusivity), energy in the form of heat accumulates near the surface of the body until the viscous dissipation effect is balanced by the effect of increased conduction and convection. In this case, when equilibrium is reached, the surface temperature is higher than

the total temperature of the stream. Conversely, if the Prandtl number is less than one, the surface temperature will be less than the total temperature of the stream. If the Prandtl number is other than one, the surface temperature varies with the local Mach number along the body and is thus a function of body shape.

For any Prandtl number the actual temperature rise across the boundary layer can be expressed as the temperature rise that would result if the fluid were brought to rest adiabatically multiplied by a coefficient known as the temperature recovery factor. (See reference 1.) In the literature, this recovery factor has been defined in several different ways by evaluating the reference temperature at various points in the fluid stream. For this investigation the recovery factor is defined as the ratio of the actual temperature difference across the boundary layer at any longitudinal station to the temperature rise that would result if the fluid were brought to rest adiabatically from the velocity existing just outside the boundary layer at that station. This definition is the one most commonly employed.

Johnson and Rubesin have reviewed the existing literature on recovery factors in reference 2. They discuss the theoretical solutions for laminar boundary layers along flat plates obtained by numerous authors and conclude that for selected values of Prandtl number from 0.72 to 1.20, for Mach numbers from 0 to 10, and for a variation of the temperature exponent of viscosity and thermal conductivity from 0.5 to 1.25, the recovery factor for laminar flow is independent of Reynolds number and Mach number and is well represented by the square root of the Prandtl number. Similarly, they discuss the recovery factor for turbulent flow along a flat plate and state that, as deduced by other investigators, it can be approximated by the cube root of Prandtl number for a fluid with constant properties.

These theoretical results have been verified experimentally for air at subsonic speeds by Hilton (reference 3) and Eckert and Weise. (See reference 4, p. I-13.) The data obtained by Eckert and Weise show that, with the onset of transition, the recovery factor increases with distance along the body and approaches as a limit the theoretical value for turbulent flow. Experimental values for turbulent flow at supersonic speeds, which are about 7 percent higher than the theoretical value, are reported by Kraus (reference 4, pp. I-12 and I-18). At supersonic speeds the relation for laminar flow has been partially substantiated by the data of Eber (reference 4, p. I-19) who measured recovery factors and heat-transfer coefficients on a series of cones at Mach numbers from 1.2 to 3.1. However, in Eber's experiments the heat-transfer data indicate that transition to turbulent flow occurred on the cones, but the measured recovery factors were constant along the cones and were equal to the value predicted by theory for laminar flow.

The present investigation was undertaken in an attempt to clarify the large differences in the results obtained in previous investigations

and to determine whether the theoretical values of local recovery factor for flat plates can be applied to a body of revolution with a large surface pressure gradient at supersonic speeds.

NOTATION

C_R local temperature recovery factor, dimensionless

c_p specific heat at constant pressure, Btu per pound, °F

c_v specific heat at constant volume, Btu per pound, °F

g gravitational constant, 32.2 feet per second squared

H total pressure, pounds per square foot absolute

k thermal conductivity, Btu per hour, square foot, °F per foot

l over-all length of the body, feet

M Mach number, dimensionless

p static pressure, pounds per square foot absolute

Pr Prandtl number $\left(\frac{c_p\mu}{k} \times 3600\ g\right)$, dimensionless

r radius of the body at any longitudinal station, feet

T temperature, °F absolute

x axial distance from the nose of the body to any longitudinal station, feet

γ ratio of specific heats $\left(\frac{c_p}{c_v} = 1.400 \text{ for air}\right)$, dimensionless

θ shock-wave angle measured from the free-stream flow direction, degrees

μ absolute viscosity, pound-second per square foot

Subscripts

In addition, the following subscripts have been used in combination with the foregoing symbols:

a reservoir conditions

o conditions at total temperature and pressure in the free stream

1 conditions just behind the shock wave originating at the nose of a body

s local conditions at the surface of a body

v local conditions just outside the boundary layer on a body

APPARATUS AND PROCEDURE

Wind Tunnel

The tests were performed in the Ames 1- by 3-foot supersonic wind tunnel No. 1 which is of the closed-circuit continuous-operation type and is equipped with a flexible-plate nozzle that provides a variation of Mach number from 1.2 to 2.4. Reynolds number variation is accomplished by changing the absolute pressure level in the tunnel from one-fifth of an atmosphere to approximately three atmospheres. The maximum pressure obtainable varies with the Mach number and the ambient-air temperature. The water content of the air in the wind tunnel is maintained at less than 0.0001 pound of water per pound of dry air in order to make the effects of humidity on the supersonic flow negligible.

Test Bodies and Instrumentation

Two body shapes were employed in this investigation. The first was a cone 8-1/2 inches long with a 20° nose angle. The other was a body of revolution generated by rotating a segment of a parabola in such a manner that the radius at any longitudinal station was given by the relation

$$\frac{r}{l} = \frac{1}{3}\left[\frac{x}{l} - \frac{1}{2.118}\left(\frac{x}{l}\right)^2\right] \tag{1}$$

where the length l was also 8-1/2 inches. This particular shape was selected in order to provide a body with a severe pressure gradient and a length and base diameter comparable to the conical body. Both of these bodies were hollow, stainless-steel shells approximately 1/32-inch thick. Photographs of the bodies mounted in the tunnel are shown in figure 1 and drawings of their contours are shown in figure 2. These bodies were designed primarily for heat-transfer measurements, but they were also well suited for recovery-factor measurements since the shells were so thin that the rate of longitudinal thermal conduction was negligible.

Two other bodies, identical in contour to the ones previously described, were employed to determine the pressure distribution and, consequently, the Mach number distributions just outside the boundary layers on the bodies. Pressure orifices were spaced uniformly along these bodies and were connected to manometer tubes containing dibutyl

phthalate. By actually measuring the pressure distributions in this manner, the effects of stream irregularities were included and better accuracy was assured than would have been possible if the pressure distributions had been calculated.

Thermocouples were employed to measure the surface temperatures on the two bodies. They were made of iron-constantan duplex wire and were soldered into holes drilled through the shells of the bodies in the positions indicated in figure 2. These thermocouples were connected through a selector switch to a direct-reading potentiometer.

Twelve thermocouples uniformly spaced on the turbulence damping screen in the settling chamber of the wind tunnel were connected through a selector switch to another direct-reading potentiometer. These thermocouples measured the stagnation temperature of the air stream.

Test Procedure

The test procedure was identical with both the conical and parabolic bodies. Data were obtained with both bodies at a free-stream Mach number of 2.0 and with the parabolic body at a free-stream Mach number of 1.5. Measurements were also made at two nominal pressures at a Mach number of 2.0. These pressures provided Reynolds numbers of approximately 2.7 and 4.8 million based on free-stream conditions and the axial length of the bodies. The data at a Mach number of 1.5 were obtained at a Reynolds number of approximately 3.8 million. First, measurements were made with the pressure-distribution bodies and the total pressure, the static pressure in the test section, and the local pressures acting on the bodies were recorded. Then measurements were made with the temperature-distribution bodies. When the surface temperature varied by less than $1/2^{\circ}$ in 5 minutes, the air stream and the body were assumed to be in thermal equilibrium and readings were made of the local surface temperatures and the temperatures at the 12 thermocouple positions in the wind-tunnel settling chamber.

In addition to the data obtained as outlined in the foregoing paragraph, similar measurements were made employing three different techniques to produce turbulent boundary layers on the conical body. First, a ring of 0.005-inch-diameter wire was cemented around the body approximately 1-1/4 inch from the apex.[1] For the second method, the first 3/4 inch of the body was covered with lampblack. This was applied by first spraying the surface with a thin layer of clear lacquer to serve as a binder. Then a mixture of lampblack suspended in lacquer thinner was sprayed over the lacquer, providing an approximately uniform roughness of small dimensions. The third method employed a band of salt

[1]All length dimensions were measured along the surface of the cone.

crystals cemented around the body and extending from 3/4 inch to 1-1/4 inch from the apex. From the tests with the conical body, the lampblack was judged to be the most successful of the three methods and it alone was employed to obtain turbulent flow on the parabolic body.

Method of Reducing Data

The temperature recovery factor has been defined as the ratio of the actual temperature rise across the boundary layer to the temperature rise that would result if the air were brought to rest at the surface adiabatically. Since the temperature increments are measured relative to the local temperature just outside the boundary layer, the actual temperature rise is the difference T_s-T_v, and the temperature rise that would result if the air were brought to rest adiabatically is the difference T_o-T_v. Hence the recovery factor is given by the equation

$$C_R = \frac{T_s-T_v}{T_o-T_v} \tag{2}$$

The surface temperature T_s is simply the temperature measured by the thermocouples on the surface of the body. The total or stagnation temperature was found by averaging the readings from the eight thermocouples nearest the center of the settling chamber. The thermocouples were arranged in three concentric circles with four thermocouples in each circle. There was less than 2° F variation in the readings of the thermocouples in the two inner circles, but the outside four often varied as much as 5° F from the average of the eight nearer the center. This difference has been investigated (reference 5) and the temperature variation is due to conduction through the settling-chamber walls and is a function of the difference between the room temperature and the temperature of the air in the settling chamber. The test section of the tunnel has been surveyed with a temperature probe and the results show that the effective total temperature and the average reading of the eight thermocouples in the center of the settling chamber differ by less than ±0.5° F.

There is no simple direct method available for measuring static temperature just outside the boundary layer on the bodies, but it is related to the total temperature and the local Mach number just outside the boundary layer by the relation

$$\frac{T_o}{T_v} = 1 + \frac{\gamma-1}{2} M_v^2 \tag{3}$$

The local Mach number can be calculated by the method of characteristics,

or, as was done in this investigation, determined from pressure-distribution measurements. The equation

$$\frac{p_s}{H_1} = \left(1 + \frac{\gamma-1}{2} M_v^2\right)^{-\frac{\gamma}{\gamma-1}} \tag{4}$$

relates the local Mach number to the pressure as measured at the surface of the body. The pressure H_1 is the total pressure behind the bow shock wave. Calibrations of the wind tunnel have proved that the flow is essentially isentropic and hence the total pressure of the free stream H_o is very nearly equal to the static pressure in the settling chamber H_a. After the bow shock-wave angle is determined and from the known nose angle of the body and the free-stream Mach number, the ratio of the total pressures across the bow shock wave may be found by the equation

$$\frac{H_1}{H_o} = \left[\frac{(\gamma+1)\ M_o^2\ \sin^2\theta}{(\gamma-1)\ M_o^2\ \sin^2\theta + 2}\right]^{\frac{\gamma}{\gamma-1}} \left[\frac{2\ \gamma\ M_o^2\ \sin^2\theta - (\gamma-1)}{\gamma+1}\right]^{-\frac{1}{\gamma-1}} \tag{5}$$

This pressure ratio may be found more directly from the known nose angle of the body by use of the tables and charts of reference 6.

Accuracy

The potentiometers and thermocouples employed for the surface-temperature measurements were calibrated and were found to be accurate to ±0.25° F. The total temperature measurements were subject to the same uncertainty plus a possible error due to the use of the average of the eight temperatures measured at different locations in the wind-tunnel settling chamber. It is estimated that the over-all uncertainty of the total temperature measurements was in the order of ±1° F. This error does not affect the comparison between data obtained at the same Mach number and total pressure because the total temperature distribution was essentially unchanged when these two parameters were held constant.

The determination of local Mach number was subject to an uncertainty of ±0.01 as determined by repeating the pressure-distribution measurements and the calibration for free-stream Mach number in the wind-tunnel test section. However, this uncertainty does not affect the accuracy of comparison between tests made on the same body at the same free-stream Mach number because the Mach number distribution along the body was determined once for each free-stream Mach number and the results were used in the reduction of all the recovery-factor data obtained at the same test conditions.

Under these conditions, the relative accuracy of successive values of recovery factor measured on the same body at the same Mach number is in the order of ±0.2 percent in the range of total temperature in which the tests were conducted. However, the effects of averaging the eight total temperature measurements and the accuracy of the Mach number distributions along the bodies caused the over-all accuracy of the measurements to be in the order of ± 1 percent.

All the recovery-factor measurements were repeatable within the relative accuracy ±0.2 percent. It is reasonable to conclude, therefore, that apparent differences in recovery factor due to varying surface roughness or the methods of inducing turbulence are reliable to ±0.2 percent of the total recovery factor because all these measurements were made at essentially the same total pressures and Mach numbers.

Conduction in the shells of the bodies could have affected the accuracy of the experimental measurements. This effect was minimized by making the shells as thin as possible. In the case of the cone, there was no conduction effect since the surface temperature was constant along its length. In the case of the parabolic body, the surface temperature varied along the length and the resulting heat conduction caused a small error in the data. The magnitude of this error was calculated considering an element of the surface and the conduction and convection effects. The maximum possible error was found to be less than 0.01° F and therefore has been neglected in the consideration of experimental accuracy.

RESULTS AND DISCUSSION

Measurements to determine temperature recovery factors for laminar boundary layers and for turbulent boundary layers induced by the addition of surface roughness on two bodies of revolution were made at a free-stream Mach number of 2.0 and at nominal total pressures of 14 and 28 pounds per square inch absolute. The flow over the smooth bodies was expected to be laminar because of the relatively low Reynolds numbers of the tests, and also because no abrupt pressure variations that might cause transition were evident along the body length. These expectations were confirmed by liquid-film tests and schlieren observations which also showed that the boundary layer became turbulent when roughness was added. The local Mach number distributions are shown in figure 3, the surface temperature distributions in figure 4, and the recovery factors in figure 5. In addition, recovery-factor data for the parabolic body at a free-stream Mach number of 1.5 and a nominal total pressure of 18 pounds per square inch absolute are shown in figure 6.

The theoretical curves shown in figures 5 and 6 are based on a Prandtl number of 0.715. This value, obtained from the available literature, is apparently the most acceptable and most accurate value

for dry air at 70° F. The theoretical recovery factors are therefore 0.846 and 0.894 for laminar and turbulent boundary layers, respectively. It can be seen that, within the accuracy of the experiment, the measured recovery factors are independent of the shape of the body or the Reynolds number and are constant along the length of the bodies as would be predicted by application of the theory for flat plates. A comparison of figure 5(b) and figure 6 reveals that the measured recovery factor for a laminar boundary layer on the parabolic body was approximately 1 percent higher at a free-stream Mach number of 2.0 than it was at a Mach number of 1.5. This difference is of the same order of magnitude as the over-all uncertainty of the measurements. Therefore, it is not possible to determine if there is a small effect of Mach number on the temperature recovery factor. However, in view of the small difference in recovery factor shown by available data at low subsonic speeds and that obtained at a Mach number of 2.0 in the present tests, it appears logical to conclude that for practical purposes the recovery factor is constant over this speed range.

The differences between the experimental values and the theoretical values are less than the ±1 percent uncertainty of the measurements in all cases except for the laminar boundary layer on the conical body, in which case the experimental value is 1.5 percent higher than theory. The surface of the conical body was visibly rougher than that of the parabolic body, suggesting the possibility that the condition of the surface might affect the laminar recovery factor. Further evidence that this might be the case is shown in figure 7. The data in best agreement with theory were obtained during an earlier investigation with the parabolic body at a free-stream Mach number of 2.2. At this time the body was new and the surface had a mirror-like finish. The second set of data was obtained during the present investigation at a Mach number of 2.0 and, although the surface was carefully polished by hand, it was obviously dull in comparison with the original finish. The third set of data was also obtained at a Mach number of 2.0 after a scratch had been accidentally made around the nose of the body. This scratch was approximately 1 inch from the tip and was so slight that it was barely visible. All three sets of data shown in figure 7 were obtained at the same Reynolds number and with identical instrumentation. Since theory predicts no change with Mach number and is substantiated by experiment, it is logical to expect that the 0.2 difference in Mach number between the old and new measurements would have no effect. The differences between the three sets of data are within the ±1 percent uncertainty in the over-all measurements; however, as discussed previously, differences between measurements made under identical conditions should be considerably more reliable. In view of these considerations, the data shown in figure 7 suggest that the recovery factor varied with the roughness of the surface even though the boundary layer remained laminar. This effect warrants further investigation with careful and quantitative control of surface roughness. The introduction of this parameter in the analysis of recovery factors may explain the variations in the results obtained by various investigators.

The data for turbulent boundary layers presented in figures 5 and 6 were obtained with the lampblack on the tips of the bodies and are in good agreement with theory. However, it has been suggested that data obtained with artificially induced turbulence may vary with the method employed. As a check on this possibility, the three different types of roughness described previously were tested on the conical body at a Mach number of 2.0 and at Reynolds numbers of approximately 2.7 and 4.8 million. The results obtained are shown in figure 8. The lampblack and the salt band produced turbulent flow at both Reynolds numbers, but the wire ring was effective only at the higher Reynolds number. The data shown in figure 8 were all obtained at a Reynolds number of 4.8 million and indicate that the type of roughness employed to induce turbulence does, to a limited extent, determine the magnitude of the recovery factor. However, the data from all three methods appear to be converging to a single value at the base of the body. The lampblack method was selected as the most satisfactory means of inducing turbulence for the investigation because it produced the smallest variation in recovery factor along the length of the body and because it was effective at both Reynolds numbers.

CONCLUSIONS

The following conclusions are based on the results obtained from tests of a conical body and a parabolic body at supersonic velocities and are applicable at least in the range of test Mach numbers and Reynolds numbers. Within the accuracy of the experiment (±1 percent):

1. The local temperature recovery factor is constant along a body of revolution moving through air at supersonic speeds and is independent of Mach number, Reynolds number, and body shape if the boundary layer is laminar. However, the recovery factor apparently increases slightly with the roughness of the body surface.

2. For laminar boundary layers, the recovery factor is well represented by the square root of Prandtl number.

3. The local temperature recovery factor on a body of revolution moving at supersonic speeds with an artificially induced turbulent boundary layer is constant and independent of Mach number, Reynolds number, and shape of the body. However, the values obtained vary slightly with the method employed to cause transition.

4. For fully developed, turbulent boundary layers, the recovery factor is well represented by the cube root of Prandtl number.

Ames Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Moffett Field, Calif., Aug. 26, 1949.

REFERENCES

1. Goldstein, S. (Editor): Modern Developments in Fluid Dynamics. Vol. II, Sec. 269, Oxford, The Clarendon Press, 1938, pp. 627-631.

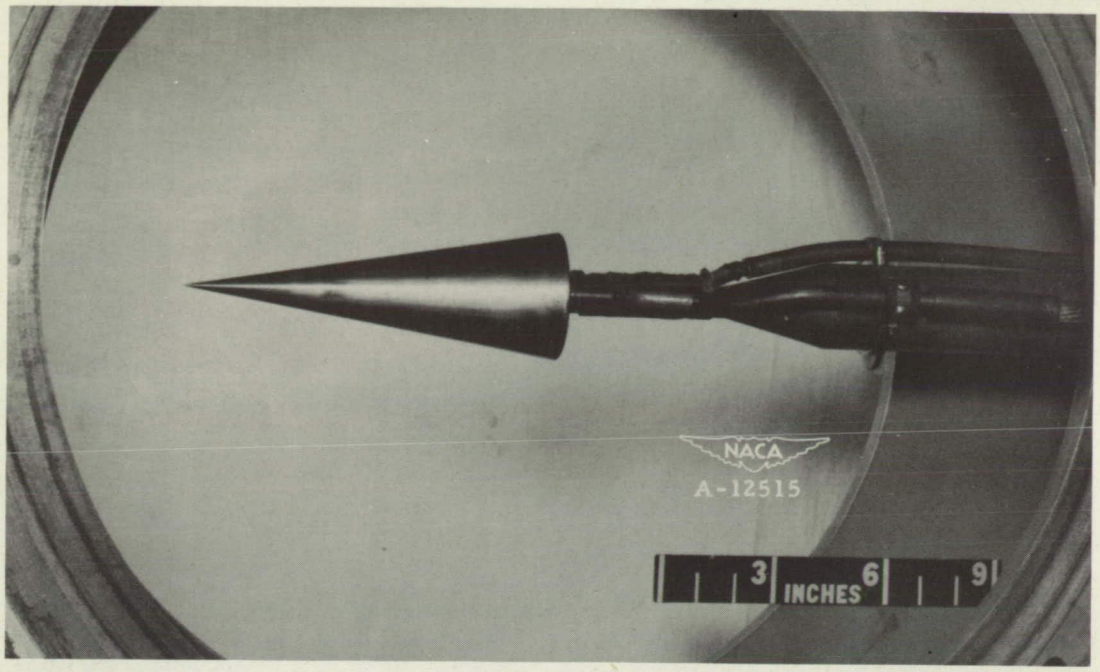
2. Johnson, H. A., and Rubesin, M. W.: Aerodynamic Heating and Convective Heat Transfer - Summary of Literature Survey. ASME Paper No. 48-A-39 presented at the annual meeting, New York, N.Y. November 28 - December 3, 1948, of the American Society of Mechanical Engineers.

3. Hilton, W. F.: Thermal Effects on Bodies in an Air Stream. Proceedings of the Royal Society, Series A., vol. 168, October 1938, pp. 43-57.

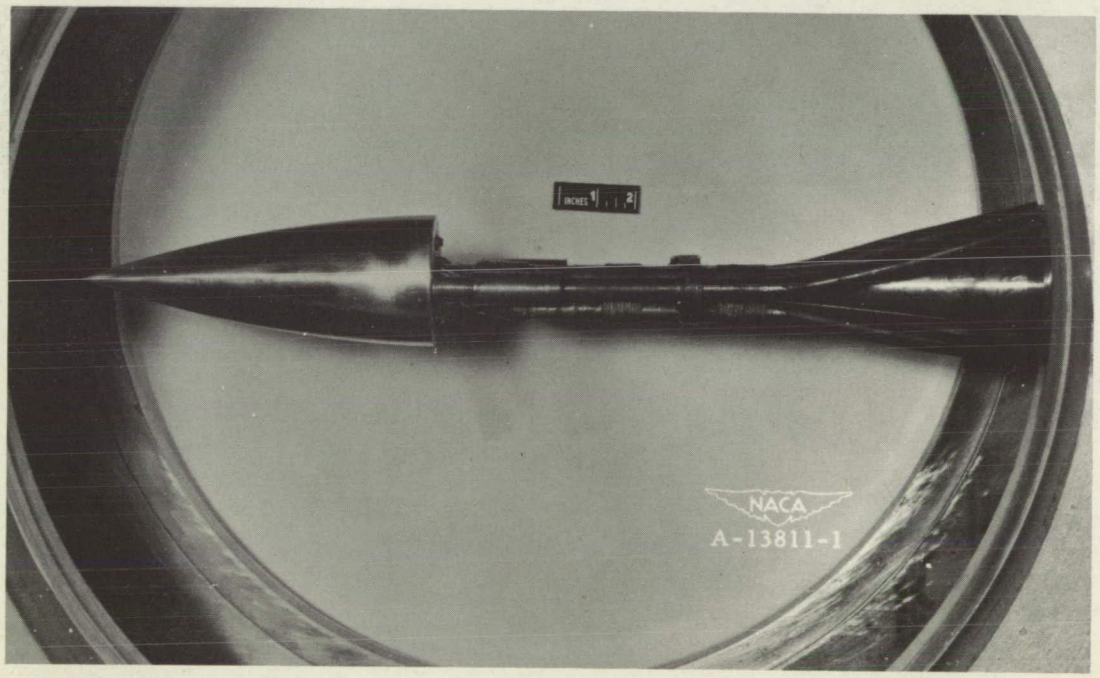
4. Johnson, H. A., Rubesin, M. W., Sauer, F. M., Slack, E. G., and Possner, L.: A Design Manual for Determining the Thermal Characteristics of High Speed Aircraft. AAF TR 5632, Air Materiel Command, Army Air Forces, Sept. 1947.

5. Goldstein, David L., and Scherrer, Richard: Design and Calibration of a Total-Temperature Probe for Use at Supersonic Speeds. NACA TN 1885, 1949.

6. The Staff of the Ames 1- by 3-Foot Supersonic Wind-Tunnel Section: Notes and Tables for Use in the Analysis of Supersonic Flow. NACA TN 1428, 1947.



(a) Conical body.



(b) Parabolic body.

Figure 1.– The test bodies installed in the test section of the Ames 1– by 3–foot supersonic wind tunnel No. 1.

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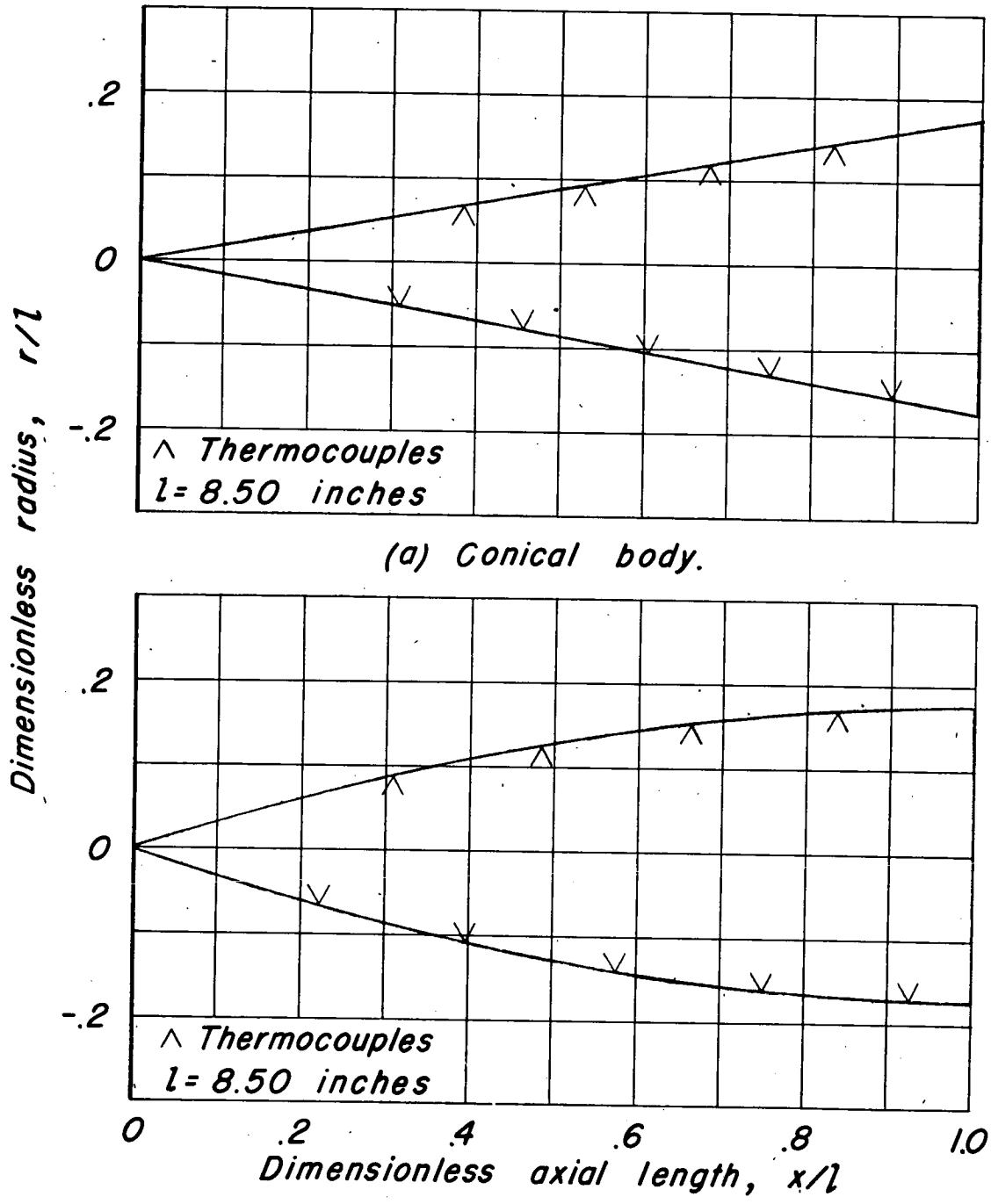

Figure 2.- Contours of the two bodies of revolution showing the thermocouple locations.

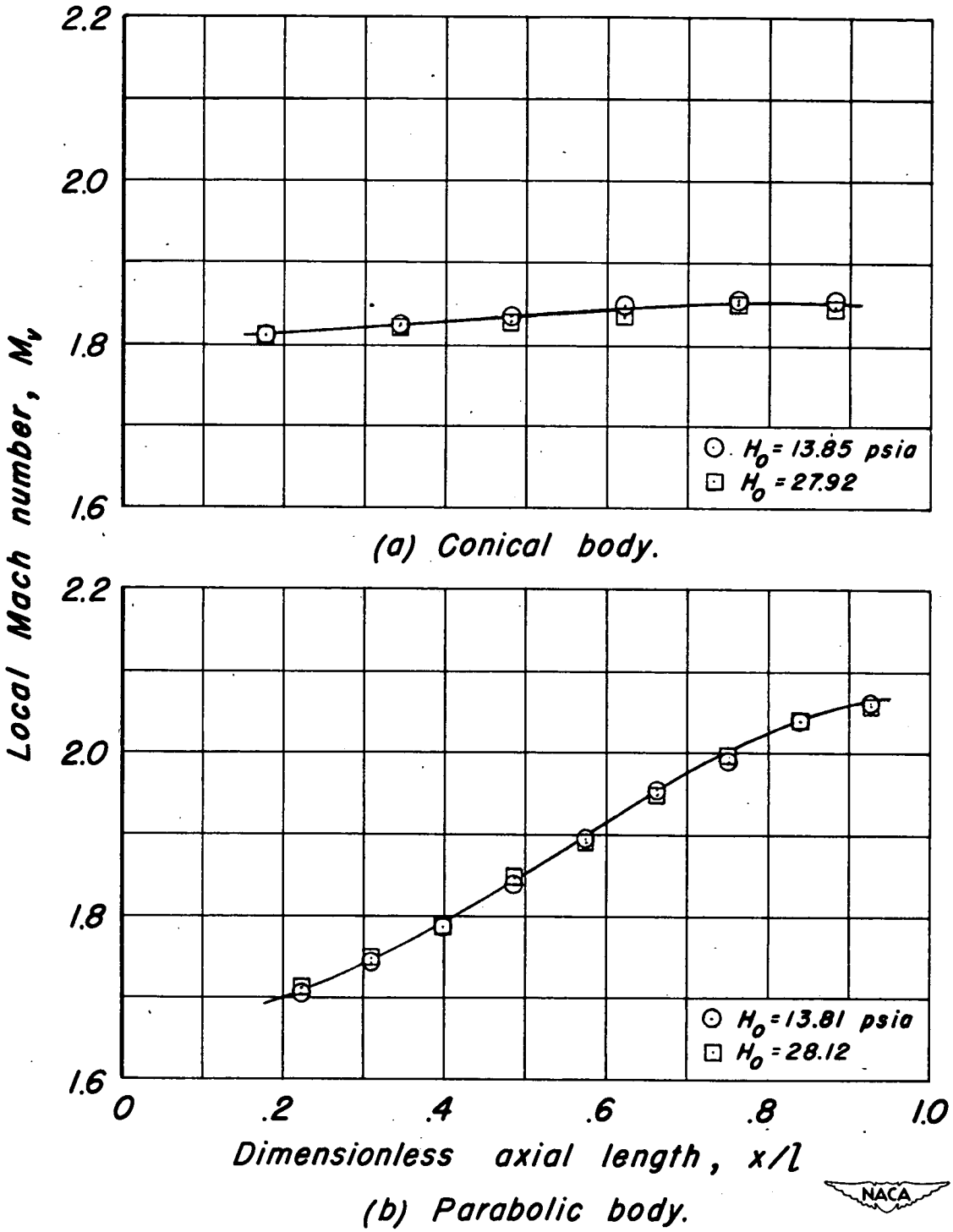

Figure 3.– Variation of Mach number just outside the boundary layer with axial length at a free-stream Mach number of 2.0.

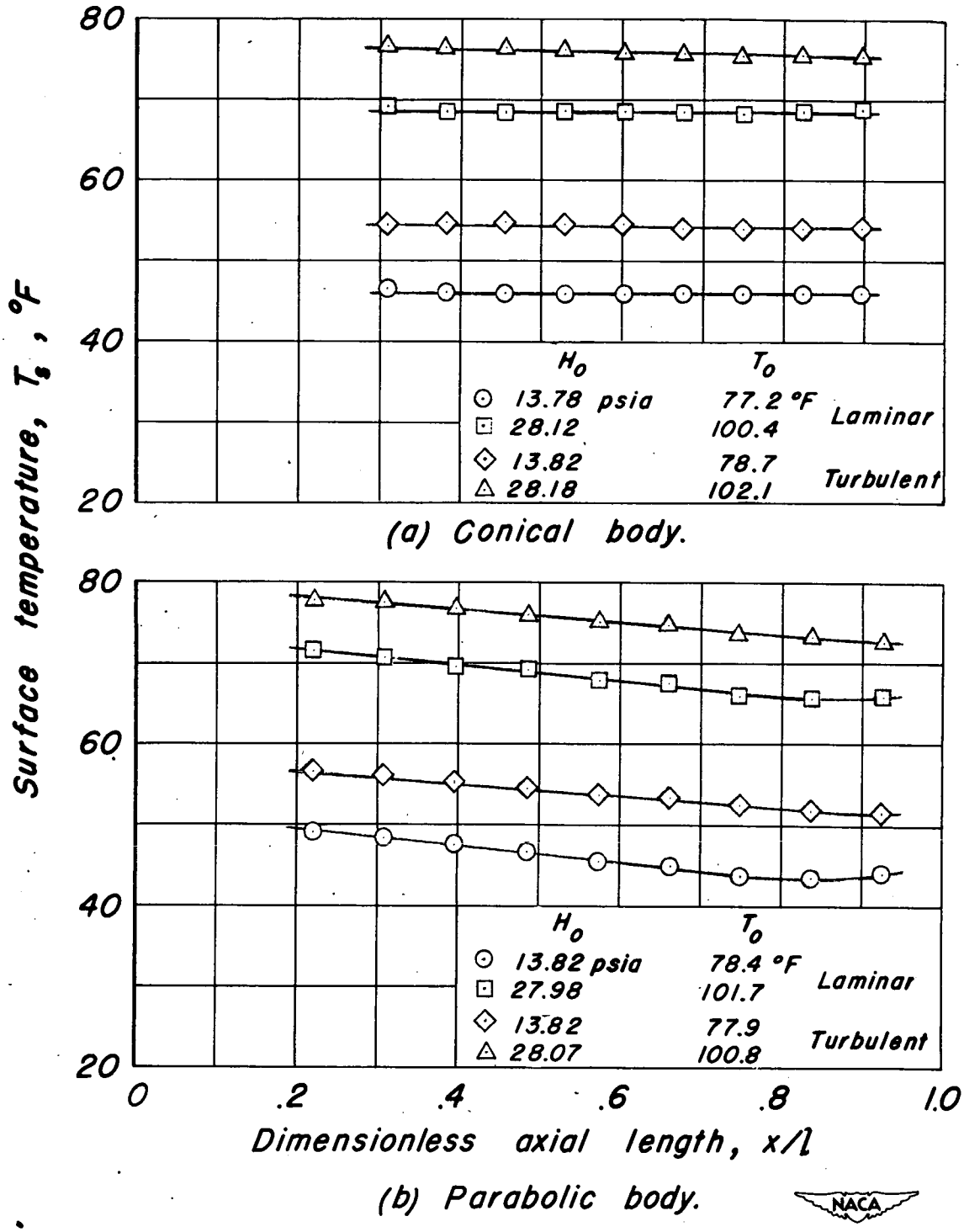

Figure 4.— Variation of surface temperature with axial length at a free-stream Mach number of 2.0.

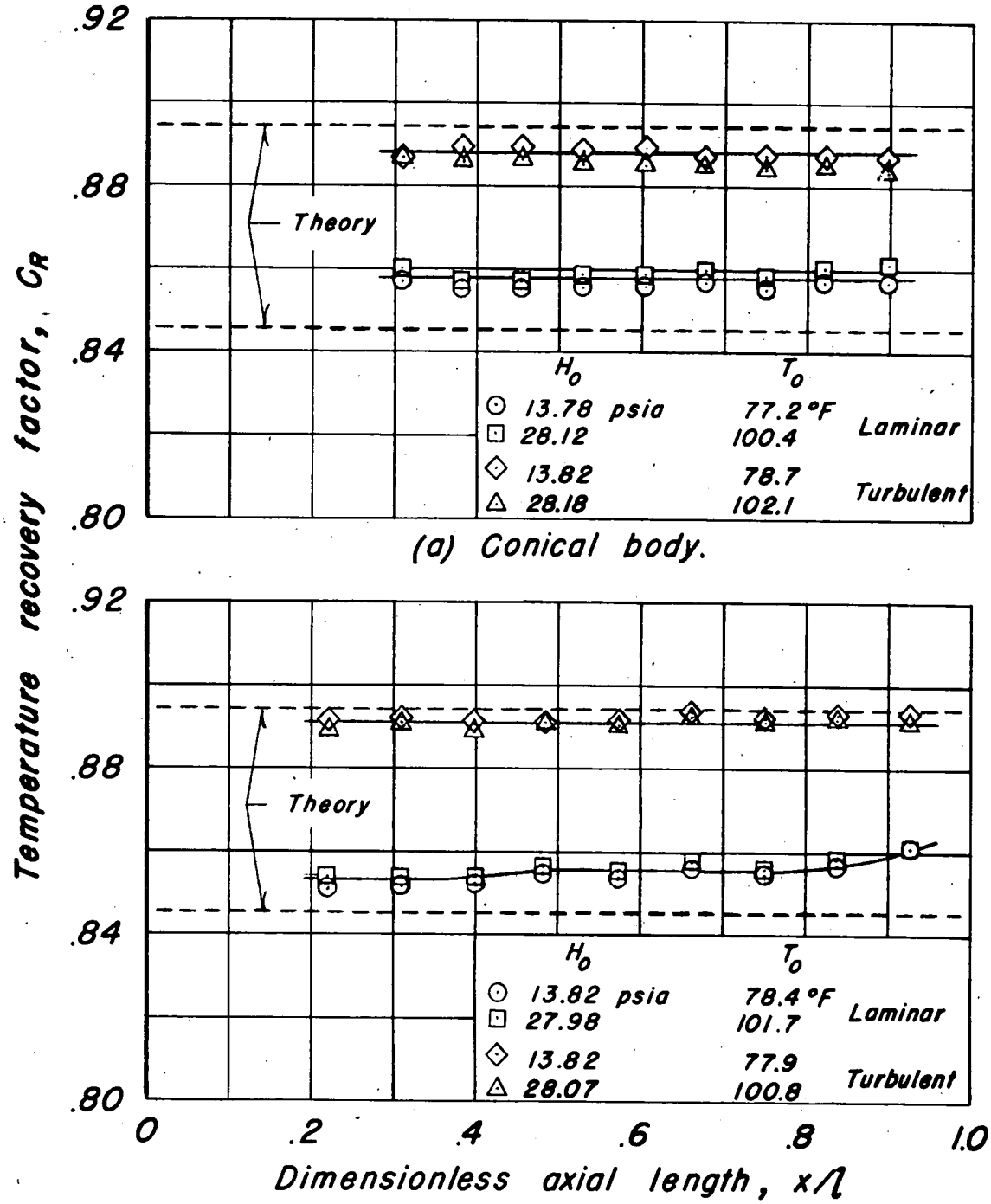

Figure 5.— Variation of recovery factor with axial length at a free-stream Mach number of 2.0.

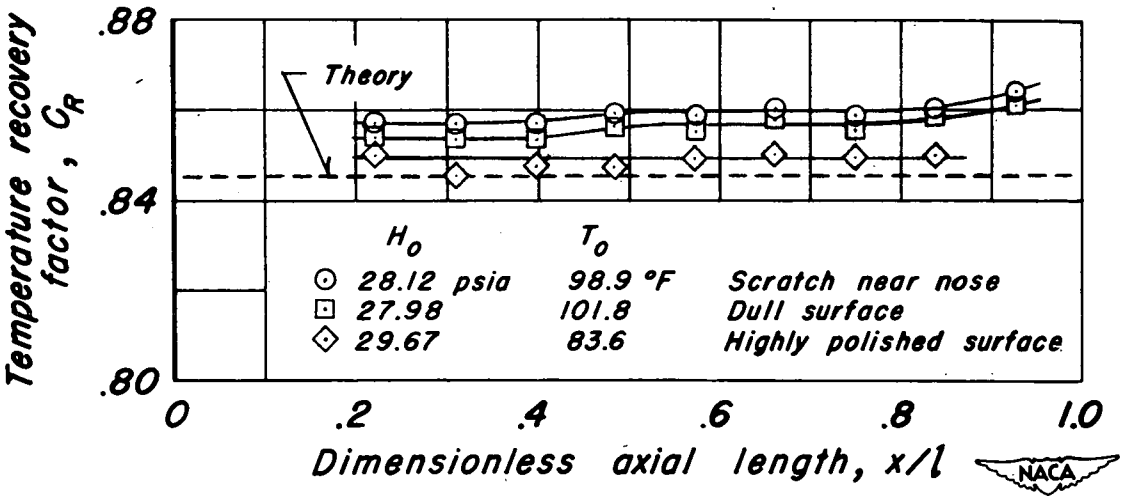

Figure 7.- The effect of surface roughness on the laminar recovery factor for the parabolic body at constant Reynolds number.

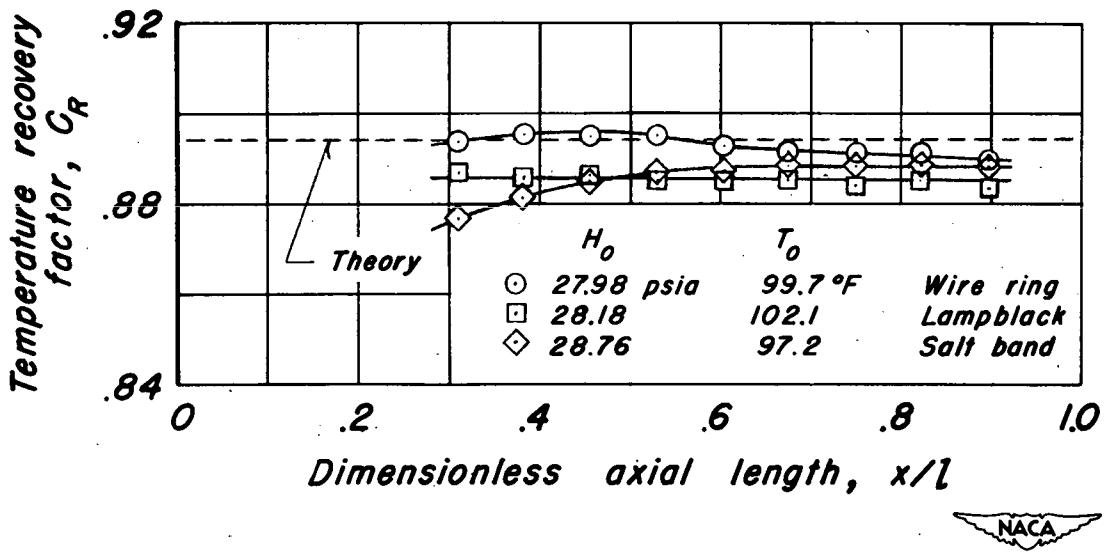

Figure 8.- The effect of various methods of inducing turbulence on the turbulent recovery factor for the conical body at constant Reynolds number.

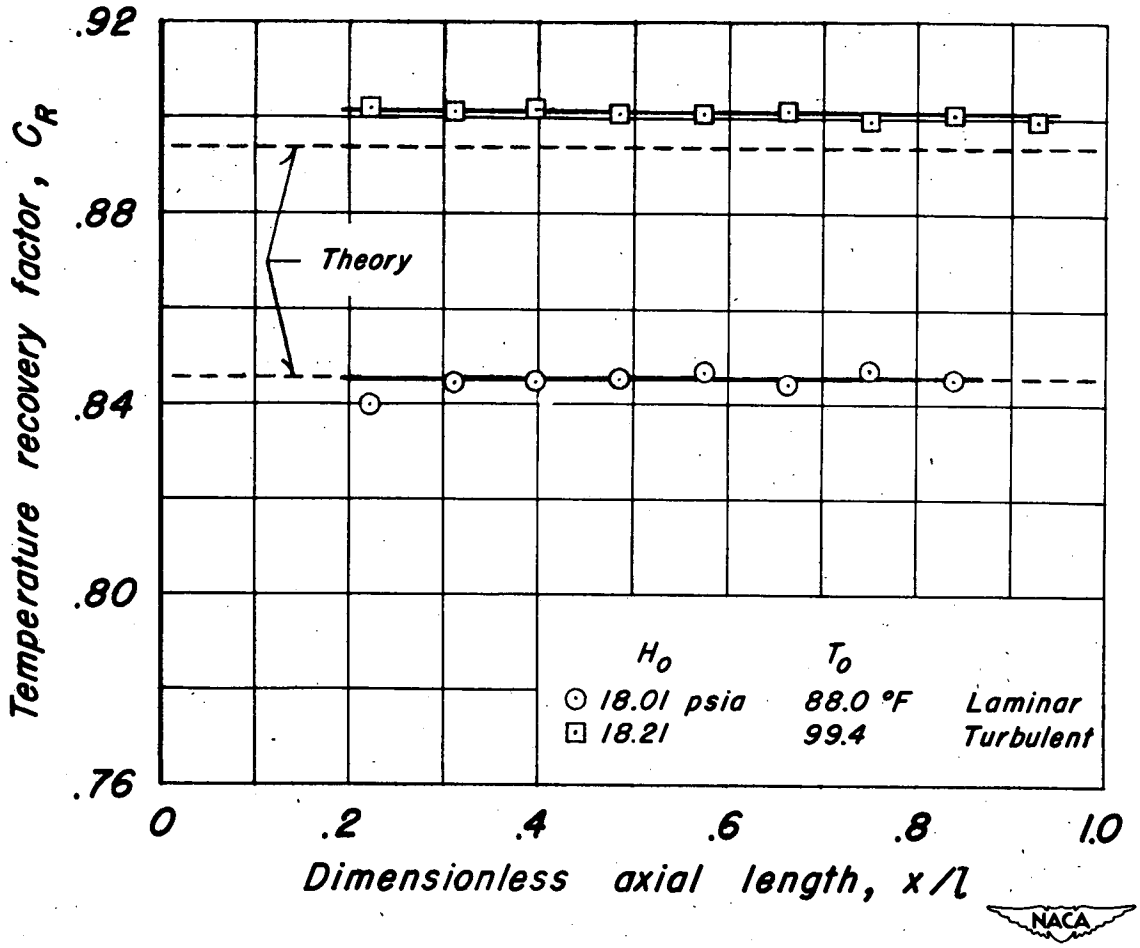

Figure 6.— Variation of recovery factor with axial length for the parabolic body at a free-stream Mach number of 1.5.