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METHOD OF MATCHING COMPONENTS AND PREDICTING PERFORMANCE OF A TURBINE-PROPELLER ENGINE

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SUMMARY

Equations were derived for the equilibrium operation of a turbine-propeller engine in terms of parameters used in turbine- and compressor-performance maps. By use of these analytical relations, geometric, thermodynamic, and aerodynamic relations among compressor, turbine, and exhaust nozzle may be calculated. For a known compressor-performance map, the matching method described indicates some of the turbine and exhaust-nozzle design compromises that must be made when the components are combined into a turbine-propeller engine.

If the physical relations among the components are known, the matching relations may be used directly to predict engine performance over a range of operating conditions.

An illustrative example of the matching method and the performance analysis is presented for an axial-flow compressor and a single-stage turbine coupled to a constant-efficiency propeller.

INTRODUCTION

Various performance investigations of gas-turbine engines based on cycle analysis have been conducted (for example, references 1 to 5). Considerable information is available on gas-turbine-engine performance over a wide range of design conditions. Cycle analysis of a gas-turbine engine may be used to determine some of the performance parameters of the individual components required for a design-operating condition but is inadequate for describing (1) the type and the size of component suitable for incorporation in an actual engine, and (2) the off-design performance of an actual engine.

Matching methods must be used as an aid in proper engine design. A matching method may be defined as a process for selecting suitable geometric relations among and within the various components of an engine designed to operate at specified conditions.

Several methods (references 6 to 10) are available for predicting equilibrium operating characteristics of gas-turbine engines if the performance characteristics of the compressor and the turbine are given. These methods, however, are not applied to the problem of selecting the turbine diameter and blade length for operation with a selected compressor.

An investigation was conducted at the NACA Lewis laboratory to develop a simplified analytical method of matching, which will aid in the determination of the geometric design of the turbine and the exhaust nozzle to be mated with a selected compressor in order to provide a suitable combination of components for use in a turbine-propeller engine. After the components are matched, the performance of the engine for a range of operating conditions may be predicted by means of the equilibrium equations used for matching the engine, if the component performance characteristics are known.

The development of the method is followed by an example in which performance maps of a turbine-propeller engine are calculated for three values of ram pressure ratio and for a range of ratios of exhaust-nozzle to turbine-inlet area.

MATCHING ANALYSIS

Basic Equations

Matching of the components for the coupled-type gas-turbine engine considered herein is based on the following three fundamental relations that apply to both turbojet and turbine-propeller engines (fig. 1).

1. A fixed relation must exist between the rotor speeds of the compressor and the turbine. For the case considered, where the compressor and the turbine are directly coupled, the rotor speeds are equal.

2. The mass flow of air through the compressor plus the fuel flow minus any air bled for cooling or other uses is equal to the gas flow through the turbine, which, in turn, is equal to the gas flow through the exhaust nozzle.

3. The power developed by the turbine must equal the sum of the compressor and shaft powers, where the shaft power is equal to the sum of the propeller-shaft power, accessory power, and power consumed in reduction-gear and bearing friction.

1267 When the corrected parameters developed in reference 11 and the symbols and the stations, as defined in appendix A and figure 1, respectively, are used these fundamental relations may be analytically expressed in the following manner:

For equal compressor and turbine rotor speeds,

$$\frac{U_{t,m}}{\sqrt{\theta_3}} = \frac{U_{c,o}}{\sqrt{\theta_1}} \sqrt{\frac{T_1}{T_3}} \frac{D_{t,m}}{D_{c,o}} \tag{1}$$

When any change in mass flow due to the addition of fuel or the bleedoff of air is neglected,

$$\frac{w\sqrt{\theta_3}}{A_3\delta_3} = \frac{w\sqrt{\theta_1}}{A_1\delta_1} \frac{P_1}{P_2} \frac{P_2}{P_3} \sqrt{\frac{T_3}{T_1}} \frac{A_1}{A_4} \frac{A_4}{A_3} \tag{2a}$$

$$\frac{w\sqrt{\theta_6}}{A_6\delta_6} = \frac{w\sqrt{\theta_3}}{A_3\delta_3} \frac{P_3}{P_5} \cdot \frac{P_5}{P_6} \sqrt{\frac{T_6}{T_3}} \frac{A_4}{A_6} \frac{A_3}{A_4} \tag{2b}$$

$$\frac{w\sqrt{\theta_6}}{A_6\delta_6} = \frac{P_s}{\sqrt{T_s}} \left[\frac{2g}{R} \frac{\gamma_t}{(\gamma_t - 1)}\right]^{\frac{1}{2}} \left[\left(\frac{p_6}{P_6}\right)^{\frac{2}{\gamma_t}} - \left(\frac{p_6}{P_6}\right)^{\frac{\gamma_t + 1}{\gamma_t}}\right]^{\frac{1}{2}} \tag{2c}$$

If turbine power is equated to the sum of shaft and compressor power

$$\frac{T_3 - T_5}{T_3} = \frac{c_{p,c}}{c_{p,t}} \frac{T_1}{T_3} \left(\frac{T_2 - T_1}{T_1}\right) \left(1 + \frac{shp}{chp}\right) \tag{3}$$

Combinations of the previous equations that eliminate the temperature-ratio term T_3/T_1 are useful in matching the components of a gas-turbine engine. The turbine weight-flow parameter multiplied by blade-speed parameter $wU_{t,m}/A_4\,\delta_3$ is obtained by multiplying each side of equation (1) by the corresponding sides of equation (2a) and then transposing the area-ratio term A_4/A_3:

$$\frac{wU_{t,m}}{A_4\delta_3} = \frac{w\sqrt{\theta_1}}{A_1\delta_1}\,\frac{U_{c,o}}{\sqrt{\theta_1}}\,\frac{P_1}{P_2}\,\frac{P_2}{P_3}\,\frac{D_{t,m}}{D_{c,o}}\,\frac{A_1}{A_4} \tag{4}$$

The following relation is obtained by dividing each side of equation (3) by the square of each side of equation (1) and rearranging terms:

$$\psi_t = S\left(\frac{D_{c,o}}{D_{t,m}}\right)^2\left(1 + \frac{shp}{chp}\right) \tag{5}$$

where ψ_t, the turbine-pressure coefficient, is defined as

$$\psi_t = \frac{gJc_{p,t}T_s\left(\frac{T_3 - T_5}{T_3}\right)}{\left(\frac{U_{t,m}}{\sqrt{\theta_3}}\right)^2}.$$

and represents the ratio of the change in tangential velocity across the turbine rotor to the blade speed at the mean diameter

$$\frac{C_{\omega,4} + C_{\omega,5}}{U_t}$$

The compressor slip factor S is defined as

$$S = \frac{gJc_{p,c}T_s\left(\frac{T_2 - T_1}{T_1}\right)}{\left(\frac{U_{c,o}}{\sqrt{\theta_1}}\right)^2}$$

Component-Performance Maps

The parameters $\frac{w\sqrt{\theta_1}}{A_1\delta_1}$, $\frac{P_2}{P_1}$, $\frac{T_2 - T_1}{T_1}$, and $\frac{U_{c,o}}{\sqrt{\theta_1}}$ are used to plot the compressor-performance map and $\frac{wU_{t,m}}{A_4\delta_3}$, $\frac{P_3}{P_5}$, $\frac{T_3 - T_5}{T_3}$, and ψ_t are used for the turbine-performance map because they condense component-performance data for approximately similar dynamic conditions of operation and they may be conveniently utilized in the matching equations given. The effects of Reynolds number and variable γ on the component-performance maps are neglected for simplicity. Large changes in the value of γ should be accounted for in the performance parameters given above. For example, if cold flow tests are used for determining turbine-performance maps, γ and R should be introduced into the turbine parameters (references 8, 9, and 11).

A typical axial-flow-compressor performance map is shown in figure 2. Performance maps for three single-stage turbines of different rotor-outlet angles are shown in figure 3. The turbine-stage velocity diagram is shown in figure 4. The angle of swirl is positive when the absolute velocity of the gases leaving the turbine has a tangential component opposite in direction to that of the moving blades. These turbine maps were replotted from a series of calculated maps presented in reference 12. The turbine weight-flow parameter multiplied by blade-speed parameter $wU_{t,m}/A_4\delta_3$ eliminates the turbine-inlet-temperature term and spreads out the plots of turbine characteristics for stator choking conditions.

For the compressor map, the weight-flow parameter is based on a unit area and the constant-speed curves are designated by lines of corrected tip speed in order to make the map independent of compressor size. The performance of the turbine stage is based on conditions at the pitch diameter so that the maps are also independent of size. The components may therefore be scaled in size for the purpose of matching. The performance maps are assumed to remain unchanged with change in component size.

When the component-performance maps uncorrected for the effect of Reynolds number are used, the relations among compressor, combustion chamber, and turbine given in equations (1), (2a), and (3) and in their combinations, equations (4) and (5), are independent of altitude, ram pressure ratio, and exhaust-nozzle size. The relations between the turbine and the exhaust nozzle are given by equations (2b) and (2c) and may be related to the operating condition of the engine by means of the following relation:

$$\frac{P_6}{p_6} = \frac{P_1}{p_0}\frac{P_2}{P_1}\frac{P_3}{P_2}\frac{P_5}{P_3}\frac{P_6}{P_5}\frac{p_0}{p_6} \qquad (6)$$

Combined Turbine and Exhaust-Nozzle Characteristics

For a given turbine operating at constant inlet conditions, the proportionment of power between the propeller and the jet is dependent on exhaust-nozzle size. Determination of the combined performance map of the turbine and the exhaust nozzle is therefore desirable.

When adiabatic expansion in the turbine stator is assumed,

$$T_3 = T_4$$

Rearranging the terms of equation (2b) gives

$$\frac{A_6}{A_4} = \frac{\dfrac{w\sqrt{\theta_4}}{A_4\delta_4}}{\dfrac{w\sqrt{\theta_6}}{A_6\delta_6}} \frac{P_4}{P_3} \frac{P_3}{P_6} \sqrt{\frac{T_6}{T_3}} \tag{7}$$

The value of the weight-flow parameter $w\sqrt{\theta}/A\delta$ for choking flow from equation (2c) is given by

$$\frac{w\sqrt{\theta}}{A\delta} = \left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}} \sqrt{\frac{2\gamma}{\gamma+1} \frac{g}{R}} \frac{P_s}{\sqrt{T_s}} \tag{8}$$

For the hot turbine gases, a convenient value for the average value of γ_t was chosen:

$$\gamma_t = 4/3$$

Letting

$$R = 53.35$$

$$\frac{w\sqrt{\theta}}{A\delta} = 48.62$$

for choking flow.

Introducing the choking value of the weight-flow parameter m' into equation (7) and rearranging terms yields

$$\frac{A_6}{A_4}\left(\frac{m'}{m}\right)_4 \frac{P_3}{P_4} = \frac{48.62}{\dfrac{w\sqrt{\theta_6}}{A_6\delta_6}} \frac{P_3}{P_6}\sqrt{\frac{T_6}{T_3}} \tag{9}$$

The parameters on the right side of this equation can be shown to be mainly a function of P_3/P_6 and P_3/p_6. From equation (2c), $w\sqrt{\theta_6}/A_6\delta_6$ is seen to be a function only of p_6/P_6 and

$$\frac{p_6}{P_6} = \frac{\dfrac{P_3}{P_6}}{\dfrac{P_3}{p_6}} \tag{10}$$

In order to eliminate the necessity of implicitly solving equation (2c), figure 5 is included so that the ratio of total to static pressure at the nozzle may be obtained by inspection. If negligible heat losses are assumed from the turbine outlet to the exhaust nozzle,

$$T_5 = T_6$$

Approximate values of turbine efficiency may be used in the expression of $\sqrt{T_6/T_3}$ because the effect of turbine efficiency is small. For example, if P_3/P_5 is equal to 2.5 and γ_t is 4/3,

$$\sqrt{\frac{T_6}{T_3}} = \sqrt{\frac{T_5}{T_3}} = \sqrt{1 - \frac{T_3 - T_5}{T_3}} = \sqrt{1 - \eta_t\left[1 - \left(\frac{1}{\dfrac{P_3}{P_6}\dfrac{P_6}{P_5}}\right)^{\frac{\gamma_t - 1}{\gamma_t}}\right]} \tag{11}$$

$$\sqrt{\frac{T_6}{T_3}} = 0.898 \quad \text{for} \quad \eta_t = 0.95$$

$$\sqrt{\frac{T_6}{T_3}} = 0.909 \quad \text{for} \quad \eta_t = 0.85$$

The effect of tail-pipe and exhaust-nozzle losses are also small, as may be seen from the following example where

1267

$$\eta_t = 0.90$$

$$\frac{P_3}{P_6} = 2.5$$

$$\sqrt{\frac{T_6}{T_3}} = \sqrt{\frac{T_5}{T_3}} = \sqrt{1 - 0.90\left[1 - \left(\frac{1}{2.5\,\frac{P_6}{P_5}}\right)^{\frac{1}{4}}\right]}$$

$$= 0.904 \text{ for no losses}$$

$$= 0.914 \text{ for 10-percent loss in tail pipe and exhaust nozzle}$$

A general map for turbine and exhaust-nozzle characteristics was plotted from equation (9). Values of P_3/P_6 were plotted against P_3/p_6 for various values of $\frac{A_6}{A_4}\left(\frac{m'}{m}\right)_4 \frac{P_3}{P_4}$, as shown in figure 6.

For the $\sqrt{T_6/T_3}$ term, a turbine efficiency of 0.86 was assumed, and tail-pipe and exhaust-nozzle losses were assumed to be 5 percent. For stator-choking conditions, the ratio $(m'/m)_4$ is 1.0.

For nonchoking conditions, values of $\frac{w\sqrt{\theta_4}}{A_4\delta_4}$ vary with the type of turbine and its operation so that no fixed relation to the ratios already given exists. For no losses in the stator passages, P_3/P_4 is 1.0. The curves were not extended to the point (1,1) because at that point the assumption of constant percentage loss in tail pipe and exhaust nozzle cannot apply. All the curves approach the line $\frac{A_6}{A_4}\left(\frac{m'}{m}\right)_4 \frac{P_3}{P_4} = \infty$, however, as they approach point (1,1) because the ratio $(m'/m)_4$ approaches infinity.

1267

The lines of maximum total-pressure ratio shown in figure 6 indicate choking conditions in the turbine stator and in the turbine-rotor annulus downstream of the turbine rotor. These values may be obtained from continuity considerations across the turbine.

$$\frac{w\sqrt{\theta_5}}{A_5\delta_5} = \frac{w\sqrt{\theta_4}}{A_4\delta_4}\,\frac{P_4}{P_3}\,\frac{P_3}{P_5}\sqrt{\frac{T_5}{T_3}}\,\frac{A_4}{A_5} \tag{12}$$

When the weight-flow parameters for choking flow are canceled and the terms rearranged,

$$\frac{A_5}{A_4}\,\frac{P_3}{P_4}\,\frac{P_5}{P_6} = \frac{P_3}{P_6}\sqrt{1 - \eta_t\left[1 - \left(\frac{1}{\frac{P_3}{P_6}\,\frac{P_6}{P_5}}\right)^{\frac{\gamma_t-1}{\gamma_t}}\right]} \tag{13}$$

The relation between $\frac{P_3}{P_6}$ and $\frac{A_5}{A_4}\,\frac{P_3}{P_4}\,\frac{P_5}{P_6}$ for $\eta_t = 0.86$ and $P_6/P_5 = 0.95$ in the $\sqrt{T_5/T_3}$ term has been plotted in figure 7.

The maximum value of P_3/P_6 for any value of $\frac{A_6}{A_4}\left(\frac{m'}{m}\right)_4\frac{P_3}{P_4}$ will be found on the curve of $M_6 = 1.0$ in figure 6. This curve represents the maximum corrected turbine power if the stator is choked, that is, $(m'/m)_4 = 1.0$. On this curve, the value of $\frac{A_6}{A_4}\left(\frac{m'}{m}\right)_4\frac{P_3}{P_4}$ is equal to $\frac{A_5}{A_4}\,\frac{P_3}{P_4}\,\frac{P_5}{P_6}$ for the same value of P_3/P_6 and $\sqrt{T_6/T_3}$. (See equations (9) and (13).) The maximum value of P_3/P_6 for any value of $\frac{A_6}{A_4}\left(\frac{m'}{m}\right)_4\frac{P_3}{P_4}$ can therefore be found from figure 7.

If the value of $\frac{A_5}{A_4}\,\frac{P_3}{P_4}\,\frac{P_5}{P_6}$ is less than the value of $\frac{A_6}{A_4}\left(\frac{m'}{m}\right)_4\frac{P_3}{P_4}$, the value of P_3/P_6 used for determining maximum turbine pressure ratio P_3/P_5 cannot be greater than that shown in figure 7 for the specified value of $\frac{A_5}{A_4} \frac{P_3}{P_4} \frac{P_5}{P_6}$. The actual value of P_3/P_6 will increase, however, with P_3/P_6 for a particular engine according to the continuity relations shown by the solid curves of figure 6 because the total-pressure losses in the tail pipe and the exhaust nozzle will increase. These losses may be estimated from knowledge of the maximum turbine pressure ratio P_3/P_5 and the operating point of the engine on the solid curves of figure 6

$$\frac{P_5}{P_6} = \frac{\dfrac{P_3}{P_6}}{\dfrac{P_3}{P_5}}$$

The map of figure 6 may be applied to any type and size turbine, single or multistage, and exhaust nozzle. If station 6 is interpreted as being identical to station 5 at the turbine-rotor outlet, the map may be used to represent turbine characteristics in general. It is especially useful in determining: (1) the ratio of exhaust-nozzle to turbine-flow area for matching an engine, (2) the engine performance for a fixed-area exhaust nozzle, and (3) engine performance when the engine is equipped with a variable-area exhaust nozzle.

A more accurate turbine- and exhaust-nozzle-performance map for a specific turbine may be obtained from a plot of equation (2b) expressed in terms of the turbine parameters shown on the turbine-performance map.

$$\frac{w\sqrt{\theta_6}}{A_6\delta_6} = \frac{w\,U_{t,m}}{A_4\delta_3}\left[\frac{\psi_t}{gJc_{p,t}\,T_s}\left(\frac{T_3}{T_3 - T_5}\right)\right]^{\frac{1}{2}} \frac{P_3}{P_5}\,\frac{P_5}{P_6}\sqrt{\frac{T_6}{T_3}}\,\frac{A_4}{A_6} \qquad (14)$$

For any selected point on the turbine map, values of $w\,U_{t,m}/A_4\delta_3$, ψ_t, $(T_3-T_5)/T_3$, and P_3/P_5 are known and may be introduced into equation (14). For a known value of tail-pipe and exhaust-nozzle loss, the remaining variables in equation (14) are A_6/A_4 and $w\sqrt{\theta_6}/A_6\delta_6$. For a selected value of A_6/A_4, the value of the weight-flow parameter may be evaluated.

The value of $\frac{P_6}{p_6} = \frac{\frac{P_3}{p_6}}{\frac{P_3}{P_6}}$ may be obtained from figure 5. Thus, the relation between A_6/A_4 and P_3/p_6 may be obtained for any point on the turbine map.

Letting $A_6/A_4 = 2.0$ and using the parameters from various points on the turbine map of figure 3(b) in equation (14) gave the relation between P_3/P_6 and P_3/p_6, as shown by figure 8. For these points, a 5-percent loss in total pressure through the tail pipe and exhaust nozzle was assumed.

For comparison of the two methods of obtaining the combined turbine and exhaust-nozzle map, a curve at $A_6/A_4 = 2.0$ was plotted on figure 8 by means of the methods used to obtain figure 6. For the solid line of figure 8, choking conditions were assumed in the turbine stator and thus $(m'/m)_4 = 1.0$. A constant value of P_4/P_3 equal to that for sonic flow was assumed. The turbine maps of reference 12 are based on a stator loss coefficient $\lambda = 0.1$. The nozzle loss per pound of gas is given by $\frac{1}{2}\lambda C_4^{\,2}$, where C_4 is the jet velocity issuing from the stator nozzles at the pitch line. The following relation was used to obtain total-pressure loss across the stator:

$$\frac{P_4}{P_3} = \left\{1 + \lambda\left[1 - \left(\frac{P_4}{p_4}\right)^{\frac{\gamma_t-1}{\gamma_t}}\right]\right\}^{\frac{\gamma_t}{\gamma_t-1}} \tag{15}$$

The ratio of total to static pressure is given by

$$\frac{P_4}{p_4} = \left(1 + \frac{\gamma_t-1}{2} M_4^{\,2}\right)^{\frac{\gamma_t}{\gamma_t-1}} \tag{16}$$

For sonic flow, $M_4 = 1.0$ and

$$\frac{P_4}{p_4} = \left(\frac{\gamma_t+1}{2}\right)^{\frac{\gamma_t}{\gamma_t-1}} = 1.853 \tag{16a}$$

and

$$\frac{P_4}{P_3} = 0.935$$

A comparison of the points obtained from the turbine map and the curve obtained from continuity considerations with choking conditions in the turbine stator shows that the curve may be used in place of a curve drawn through the points taken from the turbine map. The curves of figure 6 will therefore be used in place of those obtained from the specific turbine maps.

Method of Matching

The basic equations and the simplifying assumptions given in the previous sections can be used to determine the physical dimensions of a gas-turbine engine if the performance maps of the compressor and the turbine are known. The type of compressor and the conditions of operation at the design point (turbine-inlet temperature, any two compressor-performance parameters, and ambient conditions) must be selected first by the engine designer. Inasmuch as the design conditions will affect the complete performance map of the engine, a subsequent performance analysis should be made to indicate the wisdom in the choice of design conditions. After the conditions of operation at the design point have been selected, the relations among the physical dimensions of the engine, namely, ratio of turbine-outlet to stator-throat area, ratio of turbine to compressor area, ratio of exhaust-nozzle to turbine area, and ratio of compressor to turbine diameter, can be determined from the following matching procedure.

Selected conditions of operation. - The selection of a point on the compressor map that corresponds to engine-design conditions fixes compressor-tip speed, pressure ratio, weight-flow parameter, and slip factor at the design point.

1267

For maximum-power operation, approximately rated speed of the compressor should be selected in order to give the highest pressure ratio and weight flow within mechanical-design limits. Although minimum fuel consumption for the engine may sometimes be obtained by selecting the higher pressure ratios nearest the compressor surge point for design operation, particularly in axial-flow compressors, operating too close to the surge limit is undesirable for several reasons: (1) The surge point is inexactly defined for the compressor, (2) at the design-turbine-inlet temperature, a change in altitude or engine speed may place the compressor operating point in the surge region, and (3) acceleration is impossible at the design speed without producing operation in the surge region.

Turbine-inlet temperatures are limited by material stress and life considerations.

Determination of turbine-area ratio. - Results of an analysis made in appendix B will aid in the selection of the turbine-area ratio A_4/A_5. Maximum values of turbine-power parameter based on the turbine-outlet area $\frac{hp}{A_5 P_3 \sqrt{T_3}} \left(\frac{m'}{m}\right)_5$ are obtained for a turbine-area-ratio parameter $\frac{A_4}{A_5} \frac{m_4}{m_5} \frac{P_4}{P_3}$ of approximately 0.4, the exact value depending on turbine efficiency. (See fig. 9.)

Maximum corrected turbine power $\frac{hp}{A_5 P_3 \sqrt{T_3}}$ is obtained when the turbine-outlet annulus is choked because $(m'/m)_5$ is then equal to 1.0. For this condition, the required area ratio A_4/A_5 is approximately represented by the value of $\frac{A_4}{A_5} \frac{m_4}{m_5} \frac{P_4}{P_3}$.

In order to obtain increasing corrected power with increasing flight speeds, it is desirable to select an area ratio A_4/A_5 near the values of $\frac{A_4}{A_5} \frac{m_4}{m_5} \frac{P_4}{P_3}$ indicated by the maximum power line (approximately 0.4). In this manner, the geometry of the turbine may be made such that maximum corrected turbine power is available at maximum flight speed.

For turbines having a constant-area annular passage and operating at flow conditions such that $\frac{m_4}{m_5}\frac{P_4}{P_3} = 1.0$, stator-outlet angles of 20° to 30° may be used with less than 2.2-percent loss in maximum corrected power because the horsepower change is small with changes in area ratio in the region of maximum power. 1267

Determination of ratio of turbine- to compressor-flow area. - For maximum corrected turbine power per unit flow area, the turbine must operate near stator-choking conditions. Although the stator may not actually be choked, the turbine weight-flow parameter for most design operating conditions may be approximated by

$$\frac{w\sqrt{\theta_4}}{A_4\delta_4} = 48.62$$

From equation (2a), the ratio of turbine to compressor-flow area may be solved for

$$\frac{A_4}{A_1} = \frac{\dfrac{w\sqrt{\theta_1}}{A_1\delta_1}\sqrt{\dfrac{T_3}{T_1}}}{\dfrac{w\sqrt{\theta_4}}{A_4\delta_4}\dfrac{P_4}{P_3}\dfrac{P_2}{P_1}\dfrac{P_3}{P_2}} \tag{17}$$

The value of T_3/T_1 is obtained from the design turbine-inlet temperature and a knowledge of ambient and ram conditions. Values of $w\sqrt{\theta_1}/A_1\delta_1$ and P_2/P_1 are obtained from the design point on the compressor map. The combustion-chamber pressure ratio P_3/P_2 and turbine-stator-loss ratio P_4/P_3 may be estimated for simplicity.

At this point it is desirable to calculate the centrifugal stresses to be encountered at the root of the turbine-rotor blades. The stress at the root of a blade of constant length is given (reference 13) by

$$s_b = \frac{\rho_b\,\omega^2 D_{t,o}^{\;2}}{8g}\left[1 - \left(\frac{D_i}{D_o}\right)_t^2\right]\Phi \tag{18}$$

where Φ is the stress correction factor for different blade-area distributions and represents the ratio of the stress in a tapered blade to the stress in a parallel-sided blade. Equation (18) may be rewritten as

$$s_b = \frac{\rho_b \omega^2}{2\pi g} \frac{\pi}{4} \left(D_o^{\,2} - D_i^{\,2}\right)_t \Phi \tag{19}$$

For the turbine having a constant axial-passage height, the rotor-outlet flow area A_5 is approximately equal to the annular passage area for low rotor-outlet swirl angles; thus

$$s_b = \frac{\rho_b \omega^2}{2\pi g} A_5 \Phi \tag{20}$$

For rotor-outlet swirl angles up to 14°, the flow area perpendicular to the flow differs from the flow area perpendicular to the axial direction by not more than 3 percent. The equation for blade stress may be rewritten for the directly coupled turbine and compressor in terms of quantities already known

$$s_b = \frac{\rho_b \Phi}{2g} \left[1 - \left(\frac{D_i}{D_o}\right)_1^2\right] \theta_1 \left(\frac{U_{c,o}}{\theta_1}\right)^2 \frac{A_4}{A_1} \frac{A_5}{A_4} \tag{21}$$

If the centrifugal stresses are higher than allowable, another point must be selected on the compressor map or design conditions must be altered.

<u>Determination of ratio of exhaust-nozzle to turbine-inlet flow area.</u> - For a particular flight condition and the engine operating at constant conditions upstream of the turbine, one value of exhaust-nozzle area exists that permits optimum division of power between the propeller and the jet. The exact optimum nozzle size is, however, dependent on the values of turbine, propeller, and jet efficiencies. (See references 14 and 15.)

For optimum division of power between propeller and jet for all ram conditions, the following approximation may be used (references 1 and 15):

$$\left(\frac{P_6}{p_6}\right)_{opt} = \frac{P_1}{p_0} \tag{22}$$

For ram pressure ratios up to those causing choking in the exhaust nozzle,

$$\frac{P_3}{p_6} = \frac{P_1}{p_0}\frac{P_2}{P_1}\frac{P_3}{P_2} \tag{23}$$

1267

When equations (22) and (23) are combined

$$\left(\frac{P_3}{P_6}\right)_{opt} = \frac{P_2}{P_1}\frac{P_3}{P_2} \tag{24}$$

Thus the value of $(P_3/P_6)_{opt}$ may be determined for a particular compressor pressure ratio and combustion-chamber loss ratio and lines of constant $(P_3/P_6)_{opt}$ may be drawn on figure 6. The variation of $\frac{A_6}{A_4}\left(\frac{m'}{m}\right)_4 \frac{P_3}{P_4}$ with $\left(\frac{P_6}{p_6}\right)_{opt}$ may then be determined. A plot of this result indicates the exhaust-nozzle area required for optimum power division at various ram pressure ratios for the compressor and combustion-chamber pressure ratio selected.

For take-off conditions, equation (23) may be used to determine the value of $(P_3/p_6)_T$ for flow conditions up to choking in the exhaust nozzle. The line of constant $(P_3/p_6)_T$ may be drawn on figure 6 for selected values of compressor and combustion-chamber pressure ratio at take-off. The variation of $(P_3/P_6)_T$ or $(P_3/P_5)_T$ may then be plotted against the area-ratio parameter.

These results show that increasing the exhaust-nozzle area permits a greater pressure decrease across the turbine for take-off but a small exhaust nozzle is required for optimum power division at high flight speeds. For a fixed exhaust-nozzle area, compromises in performance at take-off and for a range of flight speeds must be made. For the area ratio selected, the variation of P_3/P_6 with P_3/p_6 should be plotted. This curve will represent the equilibrium operating condition for the matched engine. The selection of area ratio automatically determines the value of total-pressure ratio across the turbine P_3/P_5 for a known value of tail-pipe and exhaust-nozzle loss ratio P_5/P_6.

Determination of ratio of turbine to compressor diameter. - Because the turbine pressure ratio is known, a point on the turbine map is determined when the diameter ratio $D_{t,m}/D_{c,o}$ is selected. (See equation (4) or (26).)

$$\frac{wU_{t,m}}{A_4\delta_3} = \frac{w\sqrt{\theta_4}}{A_4\delta_4}\,\frac{P_4}{P_3}\,\frac{U_{t,m}}{\sqrt{\theta_3}} \tag{25}$$

From equation (1)

$$\frac{wU_{t,m}}{A_4\delta_3} = \frac{w\sqrt{\theta_4}}{A_4\delta_4}\,\frac{P_4}{P_3}\,\frac{U_{c,o}}{\sqrt{\theta_1}}\sqrt{\frac{T_1}{T_3}}\,\frac{D_{t,m}}{D_{c,o}} \tag{26}$$

By use of the turbine maps such as shown in figure 3, the turbine-outlet swirl angle may be determined for each value of $D_{t,m}/D_{c,o}$.

For some designs, a decrease in turbine wheel weight results in a decrease in wheel diameter (see reference 13). At low values of $D_{t,m}/D_{c,o}$, however, the swirl angle is high. For a fixed blade speed, stator-exit angle, and jet velocity leaving the stator, the work done by the turbine per pound of gas may generally be made large by selecting a high positive value of swirl angle. Large values of swirl angle, however, are usually undesirable if the gases are to be expanded in an exhaust nozzle because the tangential component of velocity cannot generally be utilized for obtaining forward thrust.

After a value of swirl angle for design conditions is selected, the rotor-outlet angle β_5 must be chosen. High values of β_5 will give the smallest turbine diameter. For a vortex-flow design, however, when constant axial velocity from root to tip of the blade is assumed, the turbine blading approaches impulse at the root of the blade as β_5 is increased.

Impulse conditions at the root of the blade may be obtained by equating the relative velocity entering the rotor to the relative velocity leaving the rotor. (See appendix C.) The ratio of mean turbine diameter to rotor diameter at the blade root for impulse conditions at the blade root is given by

$$\left(\frac{D_m}{D_i}\right)_t^2 = \frac{\left[\left(\frac{C_{a,5}}{C_{a,4}}\right)_m^2 - 1\right]\tan^2 \alpha_{4,m} + 2\left(\frac{U_t}{C_{\omega,4}}\right)_m\left[\left(\frac{C_{\omega,5}}{C_{\omega,4}}\right)_m + 1\right]}{\left[1 - \left(\frac{C_{\omega,5}}{C_{\omega,4}}\right)_m^2\right]} \tag{27}$$

1267

All ratios on the right side of equation (27) including $\tan^2 \alpha_{4,m}$ are obtained from the vector diagram at the pitch section of the turbine. The vector diagram at the pitch section may be drawn because the angles α_4, β_5, and swirl, are known together with turbine-blade pitch speed $U_{t,m}$ and absolute velocity issuing from the stator nozzle C_4. The nozzle-jet velocity is given by

$$C_4 = \sqrt{\frac{2\gamma_t}{\gamma_t - 1}\, gRT_3 \left[1 - \left(\frac{p_4}{P_4}\right)^{\frac{\gamma_t - 1}{\gamma_t}}\right]} \tag{28}$$

The value of p_4/P_4 may be obtained from reference 12 for a point on the turbine map because the degree of reaction, the nozzle loss coefficient, and the total-to-static pressure ratio across the turbine P_3/p_5 are given. The value of C_4 for sonic flow in the stator nozzles is given by

$$C_4 = \sqrt{\frac{2\gamma_t}{\gamma_t + 1}\, gRT_3} \tag{28a}$$

The hub-to-tip-diameter ratio is then given by

$$\left(\frac{D_i}{D_o}\right)_t = \frac{1}{2\left(\frac{D_m}{D_i}\right)_t - 1} \tag{29}$$

1267

When the design swirl angle is selected, the value of $D_{t,m}/D_{c,o}$ is fixed for each value of β_5. The hub-to-tip ratio of the turbine is then given by

$$\left(\frac{D_i}{D_o}\right)_t = \frac{1 - \dfrac{\dfrac{A_5}{A_1}\left[1 - \left(\dfrac{D_i}{D_o}\right)_1^2\right]}{4\left(\dfrac{D_{t,m}}{D_{c,o}}\right)^2}}{1 + \dfrac{\dfrac{A_5}{A_1}\left[1 - \left(\dfrac{D_i}{D_o}\right)_1^2\right]}{4\left(\dfrac{D_{t,m}}{D_{c,o}}\right)^2}} \tag{30}$$

The ratio A_5/A_1 is assumed to represent approximately the ratio of annular passage area enclosing the rotor blades to the annular passage area at the compressor inlet. The ratio $(D_i/D_o)_t$ may then be plotted against $D_{t,m}/D_{c,o}$ from equation (30) for the particular swirl angle selected and the curve compared with the curve of impulse at the blade root as given by equations (27) and (29). A compromise must then be made in the selection of $D_{t,m}/D_{c,o}$ between rotor weight and the degree of reaction because higher actual turbine efficiencies may be obtained by selecting blading with a positive reaction at the root. (See reference 16.)

The selection of $D_{t,m}/D_{c,o}$ then fixes the design value of rotor turning angle $\beta_4 + \beta_5$ and the design point on the turbine map. Thus the turbine pressure coefficient ψ_t and total-temperature ratio across the turbine T_3/T_5 are determined for design conditions.

If the exhaust-nozzle area was not fixed, equations (4) and (5) could be used to determine the relations between shaft horsepower, turbine diameter, and turbine performance parameters. The turbine aerodynamic factors must then be compromised with engine power and the size and the weight of the turbine wheel, propeller, and reduction gear for the flight condition considered.

The value of the turbine weight-flow parameter and pressure-ratio loss across the stator should now be checked against the assumed choking values by means of equation (31) using the parameters for the design point on the turbine map:

$$\frac{w\sqrt{\theta_4}}{A_4\delta_4}\,\frac{P_4}{P_3} = \frac{\dfrac{wU_{t,m}}{A_4\delta_3}}{\sqrt{\dfrac{gJc_{p,t}T_s}{\psi_t}\left(\dfrac{T_3 - T_5}{T_3}\right)}} \tag{31}$$

If the results of the solution of equation (31) indicate that flow conditions much below choking are obtained at design conditions, the matching process should be repeated with the new value of

$\frac{w\sqrt{\theta_4}}{A_4\delta_4}\frac{P_4}{P_3}$ from equation (31). 1267

PERFORMANCE ANALYSIS

When the physical dimensions of the engine are known or determined by the matching procedure given in the previous section, use of the basic equations (1) to (6) and the component performance maps completely determines engine performance at all operating conditions. The selection of a point on the compressor curve determines compressor speed, pressure ratio, weight flow, and slip factor. From the compressor and combustion-chamber characteristics, the pressure at the turbine inlet can be established. The assumption of a constant combustion-chamber pressure-drop ratio is usually sufficient for most performance analyses.

Once the turbine-inlet pressure is known, the ratio of total pressure across the turbine can be obtained from the curve relating total-pressure ratio to total-to-static pressure ratio for the known ratio of exhaust nozzle to turbine area. (See fig. 6.)

The total-pressure ratio of the turbine P_3/P_5 and the turbine parameter $wU_{t,m}/A_4\delta_3$ determined from equation (4) fix all the turbine values for the particular point selected on the compressor map. Corrected temperature drop, pressure coefficient, and outlet swirl angle may now be obtained from the turbine-performance map. Shaft power, jet thrust, and specific fuel consumption can be calculated from the equations derived in appendix D.

DETAILED PROCEDURE

Matching

The step-by-step process for matching a turbine and an exhaust nozzle to a selected compressor type is presented with an illustrative example. In each step given, the general procedure will be followed by the specific procedure for the illustrative engine chosen.

As an example of the matching procedure, the components of a typical turbine-propeller engine schematically shown in figure 1 will be matched to determine the relations among the physical dimensions. Maps representative of typical axial-flow compressor and single-stage turbine performance are shown in figures 2 and 3. The turbine-stage velocity diagram is shown in figure 4. The turbine performance maps were obtained from the methods presented in reference 12. The curves of figure 3(b) were extended to higher values of pressure ratio and corrected temperature drop by assuming a constant turbine efficiency of 0.86 and stator-choking conditions. The maximum pressure ratio obtained with the compressor chosen is less than 4.0 (fig. 2). Cycle analysis indicates that for high performance a turbine-propeller engine should have a compressor with a pressure ratio higher than this value. Inasmuch as the purpose herein is only to demonstrate procedure and to indicate qualitative results, this particular map was chosen because it covered almost the complete range of engine operation. A combustion-chamber pressure loss of 5 percent and a combined tail-pipe and exhaust-nozzle pressure loss of 5 percent will be assumed for all operating conditions.

The matching procedure outlined in a previous section is expanded here to aid in the determination of the physical dimensions of the engine and the component performance parameters for design conditions.

<u>Selection of operating conditions.</u> -

(1) Select over-all conditions for engine-design operation.

Example: The engine will be designed for take-off or maximum-power operation at sea-level static conditions.

(2) Determine ambient pressure and temperature for step (1).

Example: $p_0 = 2117$ pounds per square foot

$t_0 = 518.4^{\circ}$ R

(3) Determine ram pressure ratio. In terms of flight speed V_0 and inlet-ducting total-pressure-loss ratio P_1/P_0

1267

$$\frac{P_1}{p_0} = \frac{P_1}{P_0}\left(1 + \frac{\gamma_c - 1}{2\gamma_c gR}\frac{V_0^2}{t_0}\right)^{\frac{\gamma_c}{\gamma_c - 1}} \qquad (32)$$

Example: Assume $\frac{P_1}{p_0} = 1.0$

(4) Calculate T_1. If the process is assumed to be adiabatic to the compressor inlet,

$$T_1 = t_0\left(\frac{\frac{P_1}{p_0}}{\frac{P_1}{P_0}}\right)^{\frac{\gamma_c - 1}{\gamma_c}}$$

Example: $T_1 = t_0 = 518.4^\circ$ R

(5) Select a compressor map.

Example: The axial-flow compressor map shown in figure 2 was selected.

(6) Select design point on compressor map.

Example: For approximately rated compressor speed, a pressure ratio lower than the maximum permissible ratio was selected.

$$\frac{U_{c,o}}{\sqrt{\theta_1}} = 971 \text{ feet per second}$$

$$\frac{P_2}{P_1} = 3.25$$

$$\frac{w\sqrt{\theta_1}}{A_1\delta_1} = 37.4$$

$$\frac{T_2 - T_1}{T_1} = 0.50$$

1267

(7) Calculate S from the definition of compressor slip factor.

$$\text{Example:} \quad S = \frac{32.2 \times 778 \times 0.24 \times 518.4 \times 0.50}{(971)^2} = 1.65$$

(8) Select design turbine-inlet temperature.

Example: Assume $T_3 = 2000^\circ$ R

(9) Select losses in combustion chamber, stator, and tail pipe, assuming choking flow in the stator.

$$\text{Example:} \quad \text{Assume} \quad \frac{P_3}{P_2} = \frac{P_6}{P_5} = 0.95$$

For the turbine maps of reference 12, the losses across the stator may be computed from equations (15) and (16). For simplicity, sonic flow from the stator nozzles was assumed. Thus,

$$\frac{P_4}{P_3} = 0.935$$

Determination of turbine-area ratio. -

(10) Select a value of A_4/A_5 from figure 9.

Example: The turbine maps of reference 12 are based on a constant-area annular passage. A stator-outlet angle of 20° was chosen from consideration of figure 9:

$$\sin \alpha_4 = \frac{A_4}{A_5} = 0.342 \tag{33}$$

The turbine-rotor outlet annulus and the exhaust nozzle are assumed to be unchoked so that the selection of the ratio A_4/A_5 places the design-point operation in the region of the maximum turbine-power parameter and yet allows increasing corrected turbine power with increasing flight speeds.

Determination of ratio of turbine to compressor flow area. -

(11) Calculate A_4/A_1 from equation (17) using the choking value of weight-flow parameter.

Example:

$$\frac{A_4}{A_1} = \frac{37.4\sqrt{\frac{2000}{518.4}}}{48.62 \times 0.935 \times 3.25 \times 0.95} = 0.524$$

(12) Check centrifugal stresses in the turbine blade by means of equation (21).

Example: When the turbine-blade material is assumed to have a density of 540 pounds per cubic foot,

$$\frac{s_b}{\Phi} = \frac{540}{64.4}\left[1 - (0.50)^2\right](971)^2 \frac{0.524}{0.342}$$

$$= 9{,}080{,}000 \text{ pounds per square foot}$$

or

63,100 pounds per square inch

From reference 13, it may be seen that the blade stress will have a reasonable value for turbine design if Φ is decreased by tapering the blade. The selected values for design operation are therefore acceptable.

Determination of ratio of exhaust-nozzle to turbine-inlet flow area. -

(13) Compute $(P_3/P_6)_{opt}$ for optimum-power division from equation (24).

Example: $(P_3/P_6)_{opt} = 3.25 \times 0.95 = 3.088$

(14) Draw line of $(P_3/P_6)_{opt}$ on figure 6.

Example: The line of $(P_3/P_6)_{opt} = 3.088$ was located on figure 6.

(15) Compute $(P_6/p_6)_{opt}$ from equation (10).

1267

Example:

$$\left(\frac{P_6}{p_6}\right)_{opt} = \frac{\frac{P_3}{p_6}}{3.088}$$

For each point of intersection of the constant $\left(\frac{P_3}{P_6}\right)_{opt}$ line with the $\frac{A_6}{A_4}\left(\frac{m'}{m}\right)_4 \frac{P_3}{P_4}$ curves, $\frac{P_3}{p_6}$ is known.

(16) Plot P_1/p_0 against $\frac{A_6}{A_4}\left(\frac{m'}{m}\right)_4 \frac{P_3}{P_4}$ for optimum power division.

Example: For the illustrative engine, the relations for optimum-power division are shown in the bottom curve of figure 10.

(17) Compute $(P_3/p_6)_T$ at take-off conditions from equation (23).

Example:

$$\left(\frac{P_3}{p_6}\right)_T = 1.0 \times 3.25 \times 0.95 = 3.088$$

(18) Draw line of constant $(P_3/p_6)_T$ on figure 6.

Example: The line of $(P_3/p_6)_T = 3.088$ was located on figure 6.

(19) Plot $\left(\frac{P_3}{P_6}\right)_T$ against $\frac{A_6}{A_4}\left(\frac{m'}{m}\right)_4 \frac{P_3}{P_4}$ for take-off.

Example: The intersection of the constant line of $\left(\frac{P_3}{p_6}\right)_T = 3.088$ with the $\frac{A_6}{A_4}\left(\frac{m'}{m}\right)_4 \frac{P_3}{P_4}$ curves determines the values of $\left(\frac{P_3}{P_6}\right)_T$ to be plotted, as shown by the upper curve of figure 10. 1267

(20) Select $\frac{A_6}{A_4}\left(\frac{m'}{m}\right)_4 \frac{P_3}{P_4}$ from a consideration of the plots of steps (16) and (19).

Example: A compromise was made in the choice of a fixed exhaust-nozzle area by choosing one close to the optimum value for a range of flight speeds but choosing one that would not decrease the turbine pressure ratio at take-off conditions to abnormally low values. Such a compromise was made, as shown in figure 10, by selecting the area-ratio parameter corresponding to a turbine pressure ratio of 2.5. Thus for

$$\left(\frac{P_3}{P_6}\right)_T = 2.631$$

$$\frac{A_6}{A_4}\left(\frac{m'}{m}\right)_4 \frac{P_3}{P_4} = 3.25$$

(21) Compute $\frac{A_6}{A_4}$, assuming $\left(\frac{m'}{m}\right)_4 \frac{P_3}{P_4}$ to be choking values.

Example: $\frac{A_6}{A_4} = 3.25 \times 0.935 = 3.04$

(22) Construct curve of P_3/P_6 against P_3/p_6, as shown in figure 6 for the known value of A_6/A_4.

Example: The curve of figure 11 was constructed by the method presented in the section Combined Turbine and Exhaust-Nozzle Characteristics for

$$\frac{A_6}{A_4} = 3.04$$

$$\left(\frac{m'}{m}\right)_4 = 1.0$$

1267 and

$$\frac{P_4}{P_3} = 0.935$$

for stator-nozzle choking conditions.

(23) Compute P_3/p_6 from equation (23).

Example:

$$\frac{P_3}{p_6} = 1.0 \times 3.25 \times 0.95 = 3.088$$

(24) By using the curve of step (22), locate the design point and P_3/P_6 for P_3/p_6 from step (23).

Example: From figure 11

$$\frac{P_3}{P_6} = 2.631$$

(25) If the design point is located to the right of $M_6 = 1.0$ on figure 6, then p_0/p_6 may be computed from equation (6), letting

$$\frac{P_6}{p_6} = 1.853$$

Example: Because the design point is located to the left of $M_6 = 1.0$ on figure 6,

$$\frac{p_0}{p_6} \doteq 1.0$$

(26) Recalculate P_3/p_6 for the known value of p_0/p_6 from

$$\frac{P_3}{p_6} = \frac{P_1}{p_0}\frac{P_2}{P_1}\frac{P_3}{P_2}\frac{p_0}{p_6}$$

Example: $$\frac{P_3}{p_6} = 3.088$$

(27) Calculate

$$\frac{P_3}{P_5} = \frac{P_3}{P_6}\frac{P_6}{P_5}$$

Example: $$\frac{P_3}{P_5} = 2.631 \times 0.95 = 2.5$$

Determination of ratio of turbine to compressor diameter. -

(28) Assume several values of $D_{t,m}/D_{c,o}$.

Example: Several values of $D_{t,m}/D_{c,o}$ were assumed for each of three values of β_5. (See turbine maps (fig. 3).) For example, let

$$\frac{D_{t,m}}{D_{c,o}} = 1.0$$

for

$$\beta_5 = 32.5$$

(29) Calculate $wU_{t,m}/A_4\delta_3$ from equation (26).

Example:

$$\frac{wU_{t,m}}{A_4\delta_3} = \frac{48.62 \times 0.935 \times 971}{\sqrt{2000/518.4}}\frac{D_{t,m}}{D_{c,o}}$$

$$= 22{,}470 \text{ for } D_{t,m}/D_{c,o} = 1.0$$

(30) Locate point on turbine map for several values of β_5 and determine outlet swirl angle.

Example: On figure 3(b), for

$$\frac{wU_{t,m}}{A_4\delta_3} = 22{,}470$$

and

$$\frac{P_3}{P_5} = 2.5$$

$$\psi_t = 2.55$$

and swirl angle = 30°.

(31) Plot turbine-outlet swirl angle and ψ_t against $D_{t,m}/D_{c,o}$ for several values of β_5.

Example: Turbine design-performance values were plotted on figure 12 for three values of β_5.

(32) Select a value of swirl angle for design conditions.

Example: For take-off conditions, a swirl angle of 9° was arbitrarily chosen.

(33) Draw vector diagrams for the pitch section and the selected swirl angle for each value of β_5.

Example: Three vector diagrams were drawn, each with the outlet swirl angle equal to 9° for values of β_5 of 30°, 32.5°, and 35°. Other known quantities are:

$$\alpha_4 = 20^\circ \quad \text{(assumed constant)}$$

$$C_4 = \sqrt{\frac{2\left(\frac{4}{3}\right)}{\frac{4}{3} + 1} 32.2 \times 53.35 \times 2000} \quad \text{(assumed constant)}$$

$$= 1981 \text{ feet per second}$$

$$U_{t,m} = 971 \frac{D_{t,m}}{D_{c,o}}$$

from equation (1).

(34) Calculate $(D_i/D_o)_t$ for impulse at the root of the turbine blade from equations (27) and (29).

Example: From values obtained on the three pitch-section vector diagrams, $(D_i/D_o)_t$ for impulse conditions was calculated by means of equations (27) and (29).

1267

(35) Calculate $(D_i/D_o)_t$ for several values of $D_{t,m}/D_{c,o}$ from equation (30).

Example: Three values of $D_{t,m}/D_{c,o}$ were used in equation (30) with

$$\frac{A_5}{A_1} = \frac{0.524}{0.342} = 1.532$$

and

$$\left(\frac{D_i}{D_o}\right)_1 = 0.50$$

(36) Plot $(D_i/D_o)_t$ for the actual blade and the blade having impulse at the root against $D_{t,m}/D_{c,o}$.

Example: The results of steps (34) and (35) are plotted in figure 13.

(37) Plot rotor turning angle $\beta_4 + \beta_5$ on same plot as step (36).

Example: Rotor turning angle was obtained from the vector diagram by adding β_4 and β_5 and is plotted in figure 13.

(38) Select $D_{t,m}/D_{c,o}$ as a compromise between rotor weight and degree of reaction.

Example: Although a design value of β_5 of 35° could be chosen for accelerating flow through the blades, better actual turbine efficiencies could probably be obtained by selecting blading with a greater amount of reaction at the root.

Let

$$\frac{D_{t,m}}{D_{c,o}} = 1.322$$

1267

(39) From former plots for the selected value of $D_{t,m}/D_{c,o}$, find design values of rotor turning angle, turbine pressure coefficient, hub-to-tip ratio of turbine blade, and turbine rotor-blade-outlet angle.

Example: From figures 12 and 13 for

$$\frac{D_{t,m}}{D_{c,o}} = 1.322$$

$$\beta_4 + \beta_5 = 81^\circ$$

$$\psi_t = 1.5$$

$$\left(\frac{D_i}{D_o}\right)_t = 0.718$$

$$\beta_5 = 32.5^\circ$$

(40) Find design value of $wU_{t,m}/A_4\delta_3$ from equation (26).

Example:

$$\frac{wU_{t,m}}{A_4\delta_3} = 22{,}470 \times 1.322 = 29{,}700$$

(41) Find $\dfrac{T_3 - T_5}{T_3}$ from turbine map or from the equation

$$\frac{T_3 - T_5}{T_3} = \frac{\dfrac{\psi_t}{c_{p,t}}\left(\dfrac{U_{c,o}}{\sqrt{\theta_1}}\right)^2\left(\dfrac{D_{t,m}}{D_{c,o}}\right)^2}{gJ\left(\dfrac{T_3}{\theta_1}\right)} \tag{34}$$

Example:

$$\frac{T_3 - T_5}{T_3} = \frac{\frac{1.5}{0.274}\,(971)^2\,(1.322)^2}{32.2 \times 778 \times 2000} = 0.18$$

1267

(42) If the parameters for the design point on the turbine map are inserted in equation (31) and $\frac{w\sqrt{\theta_4}}{A_4\delta_4}\frac{P_4}{P_3}$ is much below the choking value, the matching process should be repeated with the new value from equation (31).

Example:

$$\frac{w\sqrt{\theta_4}}{A_4\delta_4}\frac{P_4}{P_3} = \frac{29{,}700}{\sqrt{\frac{32.2 \times 778 \times 0.274 \times 518.4 \times 0.18}{1.5}}}$$

$$= 45.4$$

Because this value is approximately equal to that for choking conditions, the assumption of choking conditions in the stator is verified.

(43) Calculate design shaft power per unit compressor area, net thrust per unit exhaust-nozzle area, specific fuel consumption, and other performance parameters from equations given in appendix D.

Example: The design parameters found in the previous steps were introduced into equations of appendix D.

(44) The engine may now be scaled to size to give the required design power.

Example: The value of the shaft-power parameter $shp/A_1\delta_1\sqrt{\theta_1}$ is approximately 2000 horsepower per square foot. If 2000 horsepower is desired at take-off conditions, the compressor-inlet flow area should be scaled to 1 square foot. The areas and the diameters of the turbine and the exhaust nozzle may now be calculated inasmuch as the ratios have already been determined.

1267

Performance Analysis

For the performance analysis, the following quantities must be known or assumed:

$$\frac{P_3}{P_2}, \quad \frac{P_4}{P_3}, \quad \frac{P_6}{P_5}, \quad \frac{A_4}{A_1}, \quad \frac{A_6}{A_4}, \quad \frac{D_{t,m}}{D_{c,o}}$$

The compressor and turbine performance maps must also be available.

Example: A performance analysis is made for the engine matched in the previous example

$$\frac{P_3}{P_2} = 0.95$$

$$\frac{P_4}{P_3} = 0.935$$

$$\frac{P_6}{P_5} = 0.95$$

$$\frac{A_4}{A_1} = 0.524$$

$$\frac{A_6}{A_4} = 3.04$$

$$\frac{D_{t,m}}{D_{c,o}} = 1.322$$

The axial-flow-compressor map of figure 2 is used.

The turbine map used is taken from reference 12 for α_4 of 20° and β_5 of 32.5°.

The original curves were extended to higher values of pressure ratio and corrected temperature drop by assuming a constant turbine efficiency of 0.86 and choking conditions in the turbine stator.

1267

The turbine maps of reference 12 can be used to represent conditions in the turbine at the design point for the turbine-propeller engine. In general, however, at off-design conditions the relative rotor-inlet angle β_4 will not correspond to the design value. The incidence losses were not included in the turbine maps of reference 12. Nevertheless, figure 3(b) will be assumed to represent actual turbine performance at all conditions of engine operation.

The following procedure may be used to determine the equilibrium performance of the turbine-propeller engine:

(a) Select ram pressure ratio or flight Mach number and inlet-ducting total-pressure-loss ratio

$$\frac{P_1}{p_0} = \frac{P_1}{P_0}\left(1 + \frac{\gamma_c - 1}{2} M_0^2\right)^{\frac{\gamma_c}{\gamma_c - 1}} \tag{35}$$

Example: Assume $\frac{P_1}{p_0} = 1.5$

(b) Select point on compressor map and find values of

$$\frac{w\sqrt{\theta_1}}{A_1\delta_1}, \quad \frac{P_2}{P_1}, \quad \frac{U_{c,o}}{\sqrt{\theta_1}}, \quad \frac{T_2 - T_1}{T_1}$$

Example: From figure 2, a point was selected

$$\frac{U_{c,o}}{\sqrt{\theta_1}} = 777$$

$$\frac{P_2}{P_1} = 2.25$$

$$\frac{w\sqrt{\theta_1}}{A_1\delta_1} = 26.0$$

$$\frac{T_2 - T_1}{T_1} = 0.335$$

1267

(c) Calculate S from definition of slip factor.

Example: $S = 1.731$

(d) Compute

$$\frac{P_3}{P_1} = \frac{P_2}{P_1}\frac{P_3}{P_2}$$

Example: $\frac{P_3}{P_1} = 2.14$

(e) Construct curve of P_3/P_6 against P_3/p_6 as shown in figure 6 or 11 for the known value of A_6/A_4.

Example: Figure 11 represents the curve of equilibrium operation for the matched engine.

(f) Compute

$$\frac{P_3}{p_0} = \frac{P_1}{p_0}\frac{P_2}{P_1}\frac{P_3}{P_2}$$

Example: $\frac{P_3}{p_0} = 3.21$

(g) By using the value of P_3/p_0 from step (f) on the P_3/p_6 scale for the curve of step (e), find P_3/P_6 for the curve of constant A_6/A_4.

Example: $\frac{P_3}{P_6} = 2.71$

from figure 11.

(h) If the value of P_3/p_0 lies to the right of $M_6 = 1.0$, p_0/p_6 may be found from equation (6), letting $P_6/p_6 = 1.853$ and P_3/P_6 equal the maximum value for that area ratio.

Example: Because the operating point on figure 6 or 11 lies to the left of $M_6 = 1.0$,

$$\frac{p_0}{p_6} = 1.0$$

1267

(i) Calculate

$$\frac{P_3}{p_6} = \frac{P_1}{p_0}\frac{P_2}{P_1}\frac{P_3}{P_2}\frac{p_0}{p_6}$$

Example: $$\frac{P_3}{p_6} = 3.21$$

(j) Compute

$$\frac{P_3}{P_5} = \frac{P_3}{P_6}\frac{P_6}{P_5}$$

Example: $$\frac{P_3}{P_5} = 2.57$$

(k) Compute

$$\frac{P_6}{p_6} = \frac{P_3/p_6}{P_3/P_6}$$

Example: $$\frac{P_6}{p_6} = 1.18$$

(l) Compute

$$\frac{P_6}{p_0} = \frac{P_6/p_6}{p_0/p_6}$$

Example: $$\frac{P_6}{p_0} = 1.18$$

1267

(m) Compute $wU_{t,m}/A_4\delta_3$ from equation (4).

Example: $$\frac{wU_{t,m}}{A_4\delta_3} = 23,940$$

(n) From the turbine map, locate the operating point from knowledge of P_3/P_5 and $wU_{t,m}/A_4\delta_3$.

Example: Operating point on figure 3(b) was found for

$$\frac{P_3}{P_5} = 2.57$$

and

$$\frac{wU_{t,m}}{A_4\delta_3} = 23,940$$

(o) Find operating values of $\frac{T_3 - T_5}{T_3}$, ψ_t, and swirl angle from the turbine map.

Example: $$\frac{T_3 - T_5}{T_3} = 0.182$$

$$\psi_t = 2.35$$

and

$$\text{swirl angle} = 28^\circ$$

(p) Calculate the ratio of turbine to compressor power from equation (5)

$$\left(\frac{\text{chp} + \text{shp}}{\text{chp}}\right) = \frac{\psi_t}{S}\left(\frac{D_{t,m}}{D_{c,o}}\right)^2$$

Example: $$1 + \frac{\text{shp}}{\text{chp}} = 2.375$$

(q) Calculate the required ratio of turbine-inlet temperature to compressor-inlet temperature from equation (3)

$$\frac{T_3}{T_1} = \frac{\dfrac{c_{p,c}}{c_{p,t}}\left(\dfrac{T_2 - T_1}{T_1}\right)\left(1 + \dfrac{shp}{chp}\right)}{\dfrac{T_3 - T_5}{T_3}}$$

1267

Example: $\frac{T_3}{T_1} = 3.847$

(r) Calculate values of horsepower, thrust, specific fuel consumption, and other performance parameters from appendix D.

Example: The values of the parameters calculated in the previous steps were introduced into the equations of appendix D to obtain other performance parameters at the selected operating point. Compressor-inlet-ducting losses were neglected and a constant propeller efficiency of 0.85 was assumed.

PERFORMANCE CHARACTERISTICS OF ENGINE SELECTED AS ILLUSTRATIVE EXAMPLE

Engine Performance with Fixed-Area Exhaust Nozzle

The results of the performance analysis for the compressor and turbine selected may be plotted on the compressor performance map. Figure 14 shows the range of operation on the compressor map from zero shaft power to a corrected turbine-inlet temperature of 2000° R for ram pressure ratios of 1.0, 1.2, and 1.5. From 72 to 107 percent of rated speed, the ram pressure ratio has little effect on the line of constant corrected turbine-inlet temperature.

From 72 to 100 percent of rated speed, the slip factor for a constant corrected turbine-inlet temperature does not change more than 8 percent.

The results may be plotted in a form that shows the available shaft horsepower per unit compressor-inlet flow area at the design turbine-inlet temperature and compressor-tip speed (fig. 15). For the fixed-area exhaust nozzle, the available shaft horsepower is nearly doubled by changing the flight speed from 0 to 600 miles per hour at sea level. The shaft horsepower available at an altitude of 15,000 feet and a flight speed of 400 miles per hour is approximately equal to the static sea-level value.

1267 Because the compressor surge line is approximately parallel to the line of constant T_3/θ_1 over the usual range of operating speeds (fig. 14), the engine is essentially limited by constant corrected turbine-inlet-temperature operation. Altitude operation as well as high turbine-inlet temperatures may raise the value of T_3/θ_1 to such high values that compressor surge may be encountered. From figure 15, it may be seen that if the design values of temperature and engine speed are maintained, compressor surge will be encountered at altitude. If this altitude limit is to be raised, another design point on the compressor map must be selected or the engine power must be decreased by reducing turbine-inlet temperature. From figure 14, it may be seen that the selection of a lower value of design pressure ratio places the line of $T_3/\theta_1 = 2000^\circ$ R farther away from the surge line and therefore raises the altitude limit at which surge is encountered.

For a fixed engine operating at all flight conditions and exhaust-nozzle sizes, equation (5) shows that a straight line can be drawn through a plot of ψ_t/S against shp/chp. For a turbojet engine operating with a centrifugal compressor (approximately constant slip factor), a line of constant ψ_t represents the engine operating line on the turbine map.

Engine Performance with Variable-Area Exhaust Nozzle

If it is assumed that the engine is used at flight conditions corresponding to a constant ram pressure ratio, investigation of the possibility of increasing the total thrust horsepower of the unit at constant ram pressure ratio by varying the nozzle area of the engine is desirable. Accordingly, engine performance has been calculated at a ram pressure ratio of 1.5 for a range of ratios of exhaust-nozzle to turbine-inlet area and is presented in figure 16. In this figure, the power parameter is plotted in terms of equivalent shaft horsepower, the sum of the shaft power and the jet power corrected for propulsive efficiency, assuming a constant propeller efficiency of 0.85.

Inspection of the curves shows that at a constant corrected compressor speed, in general, if the ratio of exhaust-nozzle area to turbine-throat area is reduced to the smallest value shown in order to increase the jet power, the corrected equivalent shaft horsepower is decreased from a maximum value. If the ratio of exhaust-nozzle to turbine area is increased to the highest values shown in order to increase the shaft power, the corrected equivalent shaft horsepower is also decreased from a maximum value.

The design-area ratio lies in the region of maximum corrected equivalent shaft horsepower. In the regions of maximum corrected equivalent shaft horsepower, however, the area ratio does not greatly affect the value of corrected equivalent shaft horsepower, especially at lower than rated speed. For example, from figure 16(a) at the corrected compressor-tip speed of 971 feet per second, a change in area ratio from 2.5 to 4.0 varies the corrected equivalent shaft horsepower by 4.2 percent over this range. Reference 17 shows that for the general case of an engine with varying propeller and turbine efficiencies, a two- or three-position-area control can provide adequate optimum-power proportionment. The area ratio determined as satisfactory for the ram pressure ratio considered may therefore be used over a range of operating conditions depending on propeller characteristics.

CONCLUDING REMARKS

For a particular compressor-performance map chosen by the engine designer, a simplified systematic method has been developed to show some of the compromises necessary in the selection of a turbine to be mated with the compressor. The results of component matching may be used to obtain the predicted power-per-unit size of engine. When the engine power is selected, the absolute size of each component is fixed and can be determined from the results of the component matching. The method of performance analysis provided enables the engine designer to predict the performance of the overall engine design and some of the practical limitations, such as compressor surge and maximum turbine pressure ratio.

Lewis Flight Propulsion Laboratory,
National Advisory Committee for Aeronautics,
Cleveland, Ohio, October 19, 1949.

1267

APPENDIX A

SYMBOLS

The following symbols are used in this report:

A	effective area perpendicular to flow, sq ft
C	absolute velocity of gas in turbine, ft/sec
chp	compressor horsepower
$c_{p,b}$	average specific heat at constant pressure for combustion chamber, Btu/(lb)($^{\circ}$R)
$c_{p,c}$	average specific heat at constant pressure for compressor, Btu/(lb)($^{\circ}$R)
$c_{p,t}$	average specific heat at constant pressure for turbine and exhaust nozzle, Btu/(lb)($^{\circ}$R)
D	rotor diameter, ft
eshp	equivalent shaft horsepower
F_j	jet thrust, lb
F_n	net thrust, lb
g	acceleration due to gravity, 32.2 ft/sec^2
h	lower heating value of fuel, 18,500 Btu/lb
hp	horsepower
J	mechanical equivalent of heat, 778 ft-lb/Btu
jhp	jet horsepower
M	Mach number
m	weight-flow parameter, $w\sqrt{\theta}/A\delta$
m'	choking value of weight-flow parameter
P	total pressure, lb/sq ft absolute

p static pressure, lb/sq ft absolute

R gas constant, ft-lb/(lb)($^{\circ}$R)

r radius

S compressor slip factor, $\dfrac{gJc_{p,c}\,T_s\left(\dfrac{T_2-T_1}{T_1}\right)}{\left(\dfrac{U_{c,o}}{\sqrt{\theta_1}}\right)^2}$ 1267

s_b turbine-rotor-blade stress, lb/sq ft

shp shaft horsepower

T total temperature, $^{\circ}$R

t static temperature, $^{\circ}$R

thp thrust horsepower

U blade velocity, ft/sec

V_j jet velocity, ft/sec

V_0 flight speed, ft/sec

W gas velocity relative to turbine rotor, ft/sec

w air flow, lb/sec

w_f fuel flow, lb/hr

α_4 turbine-stator-outlet angle

α_5 turbine-rotor-outlet angle

β_4 turbine-rotor-inlet angle relative to rotor blade

β_5 turbine-rotor-outlet angle relative to rotor blade

γ_c ratio of specific heats in compressor

γ_t ratio of specific heats in turbine and exhaust nozzle

δ total pressure divided by NACA standard sea-level pressure (14.7 lb/sq in. absolute)

η polytropic turbine efficiency

1267 η_b combustion efficiency

η_t adiabatic turbine efficiency

η_p propeller efficiency

θ total temperature divided by NACA standard sea-level temperature (518.4° R)

λ stator loss coefficient

ρ density of gas, slugs/cu ft

ρ_b turbine-rotor-blade density, lb/cu ft

Φ stress correction factor

ψ_t turbine pressure coefficient, $\dfrac{gJc_{p,t}\, T_s \left(\dfrac{T_3-T_5}{T_3}\right)}{\left(\dfrac{U_{t,m}}{\sqrt{\theta_3}}\right)^2}$

ω angular velocity, radians/sec

Subscripts:

0 ambient air

1 compressor inlet

2 combustion-chamber inlet

3 turbine-stator inlet

4 turbine-stator outlet

5 turbine-rotor outlet

6	exhaust-nozzle throat
a	axial direction
c	compressor
i	inner dimension with respect to rotor-blade height
m	mean dimension with respect to rotor-blade height
o	outer dimension with respect to rotor-blade height
opt	optimum
s	NACA standard pressure or temperature
T	take-off
t	turbine
ω	tangential direction

1267

1267

APPENDIX B

CORRECTED TURBINE-HORSEPOWER VARIATION WITH TURBINE-AREA RATIO AND EFFICIENCY

Turbine-power variation with area ratio and efficiency. - From continuity across the turbine

$$\frac{A_4}{A_5}\,\frac{A_3}{A_4}\,\frac{\dfrac{w\sqrt{\theta_3}}{A_3\delta_3}}{\dfrac{w\sqrt{\theta_5}}{A_5\delta_5}} = \frac{\dfrac{P_5}{P_3}}{\sqrt{\dfrac{T_5}{T_3}}} \tag{B1}$$

When $T_3 = T_4$ and the weight-flow parameter is represented by m

$$\frac{A_4}{A_5}\,\frac{m_4}{m_5}\,\frac{P_4}{P_3} = \frac{\left(1 - \dfrac{T_3-T_5}{\eta_t\, T_3}\right)^{\frac{\gamma_t}{\gamma_t-1}}}{\left(1 - \dfrac{T_3 - T_5}{T_3}\right)^{\frac{1}{2}}} \tag{B2}$$

From the definition of turbine horsepower

$$hp = \frac{wJc_{p,t}(T_3-T_5)}{550} \tag{B3}$$

$$\frac{hp}{A_5P_3\sqrt{T_3}} = \frac{Jc_{p,t}\sqrt{T_s}}{550\ P_s}\,\frac{w\sqrt{\theta_3}}{A_4\ \delta_3}\,\frac{A_4}{A_5}\,\frac{T_3-T_5}{T_3} \tag{B4}$$

Multiplying both sides by the ratio of the weight-flow parameter at choking to the actual weight-flow parameter at station 5 yields

$$\frac{hp}{A_5 P_3 \sqrt{T_3}} \left(\frac{m'}{m}\right)_5 = \frac{Jc_{p,t}\ m'_5 \sqrt{T_s}}{550\ \ P_s} \frac{A_4}{A_5} \frac{m_4}{m_5} \frac{P_4}{P_3} \frac{T_3-T_5}{T_3} \tag{B5}$$

Letting

$$\gamma_t = 4/3$$

$$c_{p,t} = 0.274$$

$$m'_5 = 48.62$$

$$\frac{hp}{A_5 P_3 \sqrt{T_3}} \left(\frac{m'}{m}\right)_5 = 0.203 \frac{A_4}{A_5} \frac{m_4}{m_5} \frac{P_4}{P_3} \frac{T_3-T_5}{T_3} \tag{B6}$$

For various values of adiabatic turbine efficiency η_t the relation between $\frac{T_3-T_5}{T_3}$ and $\frac{A_4}{A_5} \frac{m_4}{m_5} \frac{P_4}{P_3}$ is known from equation (B2). Thus the relation between $\frac{hp}{A_5 P_3 \sqrt{T_3}} \left(\frac{m'}{m}\right)_5$ and $\frac{A_4}{A_5} \frac{m_4}{m_5} \frac{P_4}{P_3}$ may be plotted for various values of turbine efficiency, as shown in figure 9.

Maximum-power curve. - The curve of maximum corrected power may be derived by the use of the polytropic turbine efficiency η.

$$\frac{P_5}{P_3} = \left(\frac{T_5}{T_3}\right)^{\frac{\gamma_t}{(\gamma_t-1)\ \eta}} \tag{B7}$$

Inserting equation (B7) into equation (B1) yields,

$$\frac{A_4}{A_5} \frac{m_4}{m_5} \frac{P_4}{P_3} = \left(\frac{T_5}{T_3}\right)^{\frac{\gamma_t}{(\gamma_t-1)\ \eta} - \frac{1}{2}} \tag{B8}$$

1267

For $\gamma_t = 4/3$

$$\frac{A_4}{A_5}\,\frac{m_4}{m_5}\,\frac{P_4}{P_3} = \left(\frac{T_5}{T_3}\right)^{\frac{8-\eta}{2\eta}}$$

$$= \left(1 - \frac{T_3-T_5}{T_3}\right)^{\frac{8-\eta}{2\eta}}$$

$$\frac{T_3-T_5}{T_3} = 1 - \left(\frac{A_4}{A_5}\,\frac{m_4}{m_5}\,\frac{P_4}{P_3}\right)^{\frac{2\eta}{8-\eta}} \tag{B9}$$

Inserting this value into equation (B6) yields

$$\frac{hp}{A_5P_3\sqrt{T_3}}\left(\frac{m'}{m}\right)_5 = 0.203\,\frac{A_4}{A_5}\,\frac{m_4}{m_5}\,\frac{P_4}{P_3}\left[1 - \left(\frac{A_4}{A_5}\,\frac{m_4}{m_5}\,\frac{P_4}{P_3}\right)^{\frac{2\eta}{8-\eta}}\right] \tag{B10}$$

In order to find the maximum corrected power, this expression may be differentiated with respect to the area-ratio term and the resulting expression set equal to zero.

$$\frac{d\left[\frac{hp}{A_5P_3\sqrt{T_3}}\left(\frac{m'}{m}\right)_5\right]}{d\left(\frac{A_4}{A_5}\,\frac{m_4}{m_3}\,\frac{P_4}{P_3}\right)} = 0.203\left\{\left[1 - \left(\frac{A_4}{A_5}\,\frac{m_4}{m_5}\,\frac{P_4}{P_3}\right)^{\frac{2\eta}{8-\eta}}\right] - \frac{2\eta}{8-\eta}\left(\frac{A_4}{A_5}\,\frac{m_4}{m_5}\,\frac{P_4}{P_3}\right)^{\frac{2\eta}{8-\eta}}\right\} = 0 \tag{B11}$$

$$\frac{A_4}{A_5}\,\frac{m_4}{m_5}\,\frac{P_4}{P_3} = \frac{1}{\left(1 + \frac{2\eta}{8-\eta}\right)^{\frac{8-\eta}{2\eta}}} \quad \text{for maximum power} \tag{B12}$$

Because the term $\frac{2\eta}{8-\eta}$ is known as a function of $\frac{A_3}{A_4} \frac{m_3}{m_4} \frac{P_4}{P_3}$, it may be substituted in equation (B10) and the line of maximum power may be drawn as shown in figure 9.

1267

APPENDIX C

FREE-VORTEX FLOW CONDITIONS

For a turbine-rotor blade with accelerating flow (positive reaction) at the pitch section, the problem is to determine the hub diameter at which impulse conditions occur.

The criterions for impulse conditions will be (fig. 4):

$$W_4 = W_5 \tag{C1}$$

For vortex flow, from root to tip of the blade,

$$C_{\omega}r = \text{constant} \tag{C2}$$

$$C_{a,4} = \text{constant} \tag{C3}$$

$$C_{a,5} = \text{constant} \tag{C4}$$

For the blade rotating at constant speed, from root to tip

$$\frac{U}{r} = \text{constant} \tag{C5}$$

From the vector diagram (fig. 4)

$$W_4^{\;2} = (C_{\omega,4}-U)^2 + C_{a,4}^{\;2} \tag{C6}$$

$$W_5^{\;2} = (C_{\omega,5}+U)^2 + C_{a,5}^{\;2} \tag{C7}$$

Equating equation (C6) to equation (C7) and expanding yields

$$(C_{a,5}^{\;2} - C_{a,4}^{\;2}) + 2U\,(C_{\omega,5} + C_{\omega,4}) + (C_{\omega,5}^{\;2} - C_{\omega,4}^{\;2}) = 0 \tag{C8}$$

$$\left[\left(\frac{C_{a,5}}{C_{a,4}}\right)^2 - 1\right]\left(\frac{C_{a,4}}{C_{\omega,4}}\right)^2 + 2\,\frac{U}{C_{\omega,4}}\left(\frac{C_{\omega,5}}{C_{\omega,4}} + 1\right) + \left[\left(\frac{C_{\omega,5}}{C_{\omega,4}}\right)^2 - 1\right] = 0 \tag{C9}$$

When the radius at the root of the blade r_i is introduced,

1267

$$\left[\left(\frac{C_{a,5}\ r}{C_{a,4}\ r}\right)_i^2 - 1\right]\left(\frac{C_{a,4}}{C_{\omega,4}\ r}\right)_i^2 r_i^2 + 2\left(\frac{U}{r}\right)_i \frac{1}{(C_{\omega,4}\ r)_i}\left[\left(\frac{C_{\omega,5}\ r}{C_{\omega,4}\ r}\right)_i + 1\right] r_i^2 + \left(\frac{C_{\omega,5}\ r}{C_{\omega,4}\ r}\right)_i^2 - 1 = 0 \tag{C10}$$

Because

$$(C_\omega r)_i = (C_\omega r)_m \tag{C11}$$

$$\left(\frac{U}{r}\right)_i = \left(\frac{U}{r}\right)_m \tag{C12}$$

$$C_{a,i} = C_{a,m} \tag{C13}$$

$$\left[\left(\frac{C_{a,5}\ r}{C_{a,4}\ r}\right)_m^2 - 1\right]\left(\frac{C_{a,4}}{C_{\omega,4}}\frac{1}{r}\right)_m^2 r_i^2 + 2\left(\frac{U}{r}\right)_m \frac{1}{(C_{\omega,4}\ r)_m}\left[\left(\frac{C_{\omega,5}\ r}{C_{\omega,4}\ r}\right)_m + 1\right] r_i^2 + \left[\left(\frac{C_{\omega,5}\ r}{C_{\omega,4}\ r}\right)_m^2 - 1\right] = 0 \tag{C14}$$

$$\left[\left(\frac{C_{a,5}}{C_{a,4}}\right)_m^2 - 1\right]\left(\tan^2 \alpha_4\right)_m \left(\frac{r_i}{r_m}\right)^2 + 2\left(\frac{U}{C_{\omega,4}}\right)_m \left(\frac{r_i}{r_m}\right)^2\left[\left(\frac{C_{\omega,5}}{C_{\omega,4}}\right)_m + 1\right] + \left[\left(\frac{C_{\omega,5}}{C_{\omega,4}}\right)_m^2 - 1\right] = 0 \tag{C15}$$

1267

$$\left[1 - \left(\frac{C_{\omega,5}}{C_{\omega,4}}\right)_m^2\right]\left(\frac{r_m}{r_i}\right)^2 = \left[\left(\frac{C_{a,5}}{C_{a,4}}\right)_m^2 - 1\right](\tan^2 \alpha_4)_m + 2\left(\frac{U}{C_{\omega,4}}\right)_m \left[\left(\frac{C_{\omega,5}}{C_{\omega,4}}\right)_m + 1\right] \tag{C16}$$

$$\left(\frac{D_m}{D_i}\right)^2 = \frac{\left[\left(\frac{C_{a,5}}{C_{a,4}}\right)_m^2 - 1\right](\tan^2 \alpha_4)_m + 2\left(\frac{U}{C_{\omega,4}}\right)_m \left[\left(\frac{C_{\omega,5}}{C_{\omega,4}}\right)_m + 1\right]}{\left[1 - \left(\frac{C_{\omega,5}}{C_{\omega,4}}\right)_m^2\right]} \tag{C17}$$

The ratio of turbine pitch diameter to hub diameter for impulse conditions at the root of the blade can therefore be calculated in terms of the ratios obtained from the vector diagram drawn for the pitch section of the blade.

The hub-to-tip diameter ratio is then given by

$$\left(\frac{D_i}{D_o}\right)_t = \frac{1}{2\left(\frac{D_m}{D_i}\right)_t - 1} \tag{C18}$$

APPENDIX D

EQUATIONS FOR HORSEPOWER, THRUST, AND SPECIFIC FUEL CONSUMPTION

1267

Shaft horsepower. - The relation between shaft power and compressor power can be rewritten from equation (5) as

$$shp = chp \left[\frac{\psi_t}{S} \left(\frac{D_{t,m}}{D_{c,o}} \right)^2 - 1 \right] \tag{D1}$$

Compressor horsepower can be shown to equal

$$\frac{chp}{A_1 \delta_1 \sqrt{\theta_1}} = \frac{J}{550} \frac{w\sqrt{\theta_1}}{A_1 \delta_1} c_{p,c} T_s \left(\frac{T_2 - T_1}{T_1} \right) \tag{D2}$$

If this expression is used, the shaft-power parameter can be written as

$$\frac{shp}{A_1 \delta_1 \sqrt{\theta_1}} = \frac{J}{550} \frac{w\sqrt{\theta_1}}{A_1 \delta_1} \left[\psi_t \left(\frac{D_{t,m}}{D_{c,o}} \right)^2 \left(\frac{U_{c,o}}{\sqrt{\theta_1}} \right)^2 \frac{1}{gJ} - c_{p,c} T_s \frac{T_2 - T_1}{T_1} \right] \tag{D3}$$

Jet thrust. - The equation for jet thrust can be written

$$F_j = \rho_6 A_6 V_j^2 + (p_6 - p_0) A_6 \tag{D4}$$

If the density and the jet velocity are replaced by equivalent expressions in terms of pressure and temperature, the jet-thrust parameter can be written

$$\frac{F_j}{A_6 \delta_1} \left(\frac{P_1}{p_0} \right) = P_s \left(\frac{p_6}{P_6} \right) \frac{P_6}{p_0} \left\{ \left(\frac{2\gamma_t}{\gamma_t - 1} \right) \left[\left(\frac{P_6}{p_6} \right)^{\frac{\gamma_t - 1}{\gamma_t}} - 1 \right] + \left(1 - \frac{p_0}{p_6} \right) \right\} \tag{D5}$$

For conditions less than critical at the nozzle throat, the jet-thrust parameter reduces to

1267

$$\frac{F_j}{A_6\ \delta_1}\left(\frac{P_1}{p_0}\right) = P_s\left(\frac{2\gamma_t}{\gamma_t - 1}\right)\left[\left(\frac{P_6}{p_0}\right)^{\frac{\gamma_t - 1}{\gamma_t}} - 1\right] \tag{D6}$$

and for choking conditions at the exhaust-nozzle throat

$$\frac{F_j}{A_6\ \delta_1}\left(\frac{P_1}{p_0}\right) = P_s\left[2\left(\frac{2}{\gamma_t + 1}\right)^{\frac{1}{\gamma_t - 1}}\frac{P_6}{p_0} - 1\right] \tag{D7}$$

Values of jet-thrust parameter as a function of the ratio of the total pressure at the nozzle to ambient pressure are given for one value of γ_t in figure 17.

Net thrust. - The net thrust from the jet nozzle is equal to the jet thrust minus the inlet momentum losses.

$$F_n = F_j - \frac{w}{g} V_0 \tag{D8}$$

If the flight velocity is replaced by its equivalent value in terms of ram pressure ratio and compressor-inlet losses are neglected, the net-thrust parameter can be expressed as

$$\frac{F_n}{A_1\delta_1} = \frac{F_j}{A_1\delta_1} - \frac{w\sqrt{\theta_1}}{A_1\delta_1}\left[\frac{2\ RT_s\ \gamma_c}{g\ (\gamma_c - 1)}\right]^{\frac{1}{2}}\left[1 - \left(\frac{p_0}{P_1}\right)^{\frac{\gamma_c - 1}{\gamma_c}}\right]^{\frac{1}{2}} \tag{D9}$$

Thrust horsepower. - The total thrust horsepower is the sum of the propulsive powers of the propeller and the jet. If all the shaft power is assumed available for the propeller

$$thp = \eta_p\ shp + jhp \tag{D10}$$

$$thp = \eta_p\ shp + \frac{F_n\ V_0}{550} \tag{D11}$$

Corrected thrust horsepower per square foot of compressor-inlet area can be expressed as follows:

$$\frac{\mathrm{thp}}{A_1\delta_1\sqrt{\theta_1}} = \frac{\eta_p\ \mathrm{shp}}{A_1\delta_1\sqrt{\theta_1}} + \frac{1}{550}\left(\frac{2\gamma_c\ gRT_s}{\gamma_c - 1}\right)^{\frac{1}{2}}\left[1 - \left(\frac{p_0}{P_1}\right)^{\frac{\gamma_c - 1}{\gamma_c}}\right]^{\frac{1}{2}}\frac{F_n}{A_1\delta_1} \qquad \text{(D12)}$$

Equivalent shaft horsepower. - The equivalent shaft horsepower of a turbine-propeller engine is equal to the sum of the propeller shaft power plus the jet power corrected for propulsive efficiency. If all the shaft power is assumed available for the propeller,

$$\frac{\mathrm{eshp}}{A_1\delta_1\sqrt{\theta_1}} = \frac{\mathrm{shp}}{A_1\delta_1\sqrt{\theta_1}} + \frac{1}{\eta_p\ 550}\left(\frac{2\ \gamma_c gRT_s}{\gamma_c - 1}\right)^{\frac{1}{2}}\left[1 - \left(\frac{p_0}{P_1}\right)^{\frac{\gamma_c - 1}{\gamma_c}}\right]^{\frac{1}{2}}\frac{F_n}{A_1\delta_1} \qquad \text{(D13)}$$

Specific fuel consumption. - The specific fuel consumption of a turbine-propeller engine can be calculated on the basis of the thrust horsepower or the equivalent shaft horsepower. The weight of the fuel flow is the same for either case and can be obtained from a consideration of the temperature rise during combustion.

$$w_f h \eta_b = w c_{p,b}\ (T_3 - T_2)(3600) \qquad \text{(D14)}$$

$$\frac{w_f}{A_1\delta_1\sqrt{\theta_1}} = \frac{w\sqrt{\theta_1}}{A_1\delta_1}\ \frac{1}{\theta_1}\ \frac{c_{p,b}}{\eta_b\ h}\ (T_3 - T_2)\ (3600) \qquad \text{(D15)}$$

$$= 3600\ \frac{w\sqrt{\theta_1}}{A_1\delta_1}\ \frac{c_{p,b}}{(0.26)\ \eta_b\ h}\ (0.26)\left[\frac{T_3}{\theta_1} - T_s\left(\frac{T_2 - T_1}{T_1} + 1\right)\right] \qquad \text{(D16)}$$

1267

The thrust-horsepower specific-fuel-consumption parameter, $\frac{w_f}{thp} \eta_b \left(\frac{0.26}{C_{p,b}}\right)$ can be obtained from equations (D12) and (D16); the equivalent shaft-horsepower specific-fuel-consumption parameter can be obtained from equations (D13) and (D16).

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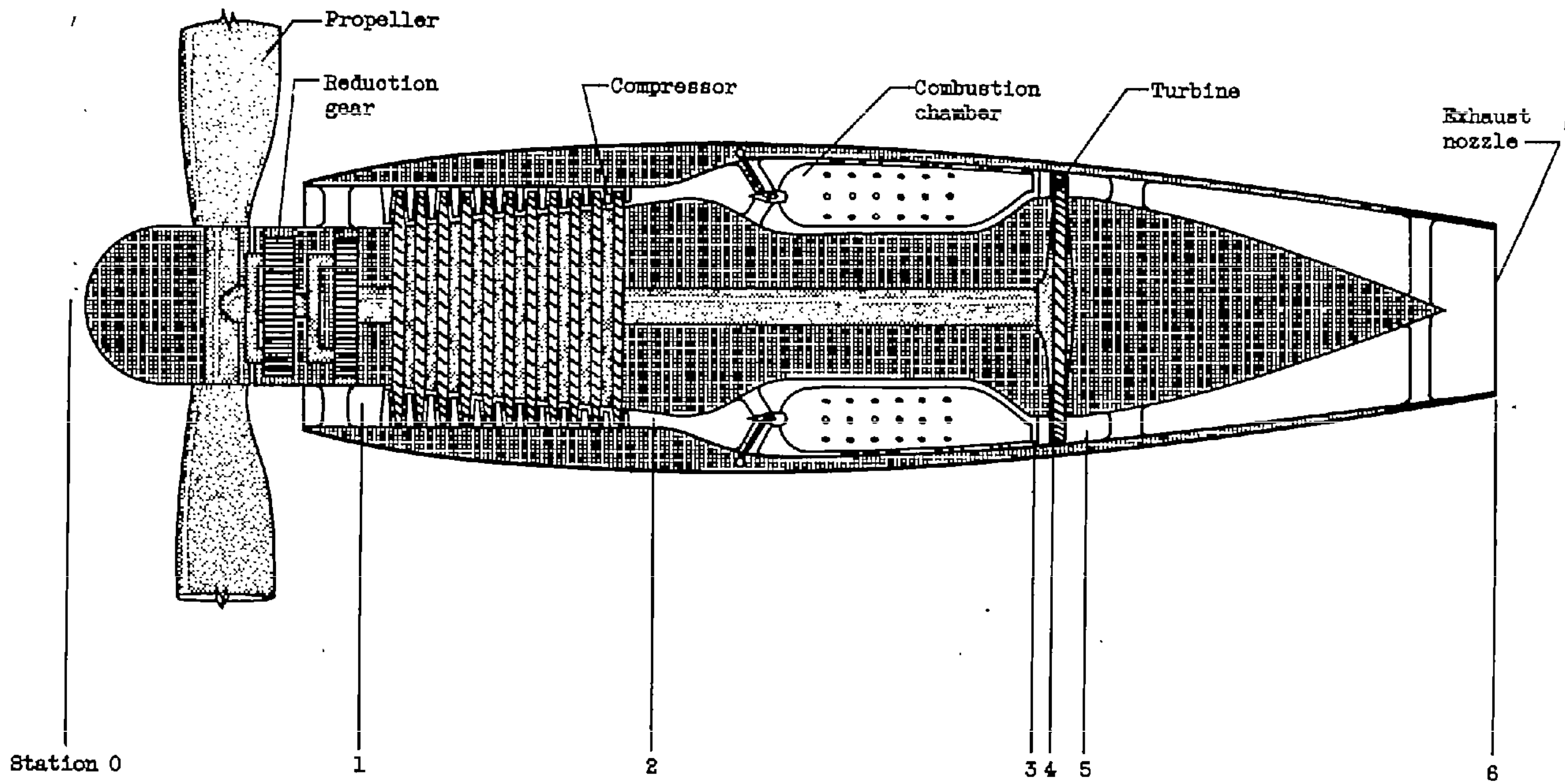

Figure 1. - Turbine-propeller engine showing stations.

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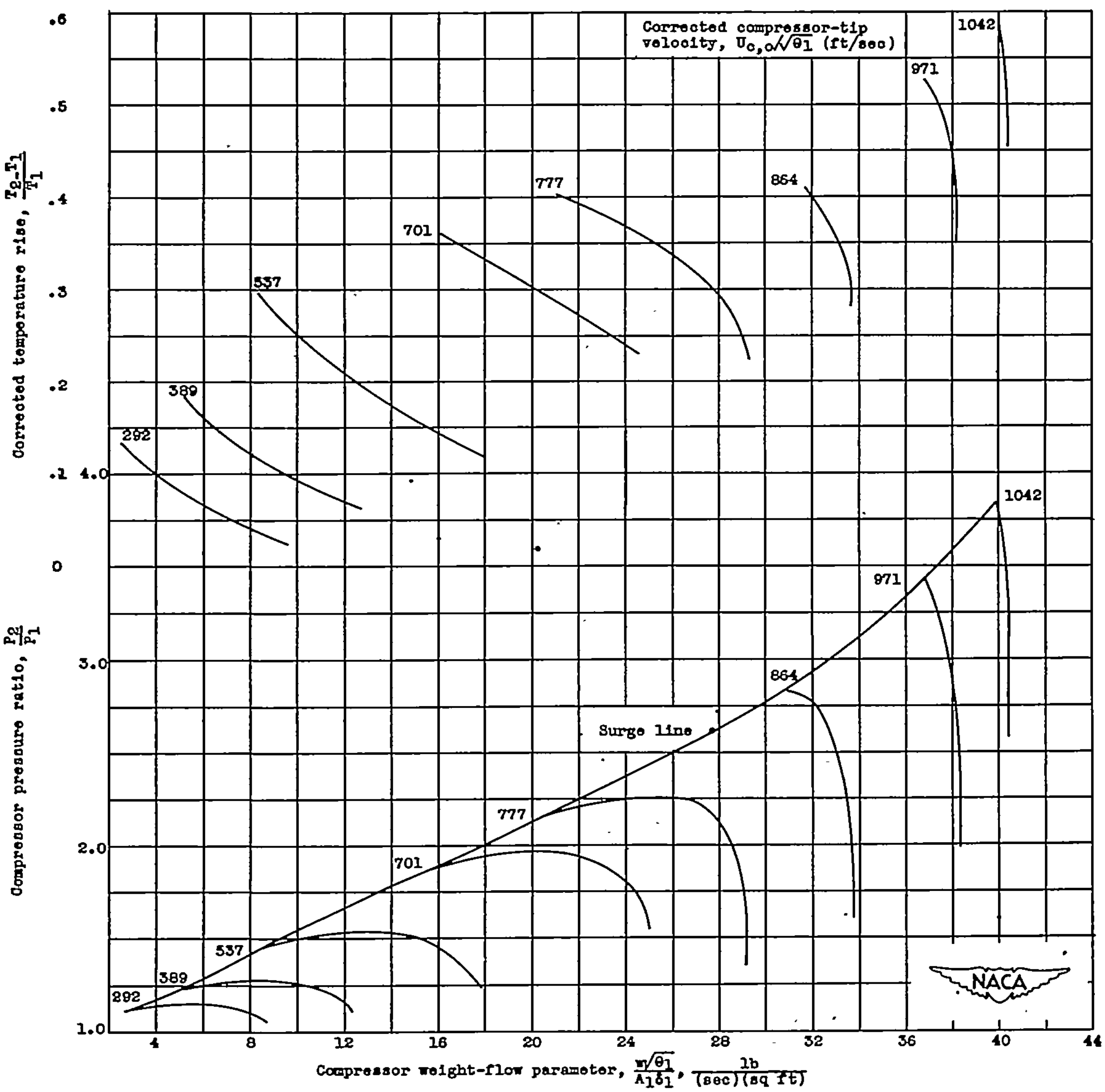

Figure 2. - Performance map for 10-stage axial-flow compressor. Hub-to-tip-diameter ratio at compressor inlet, 0.50.

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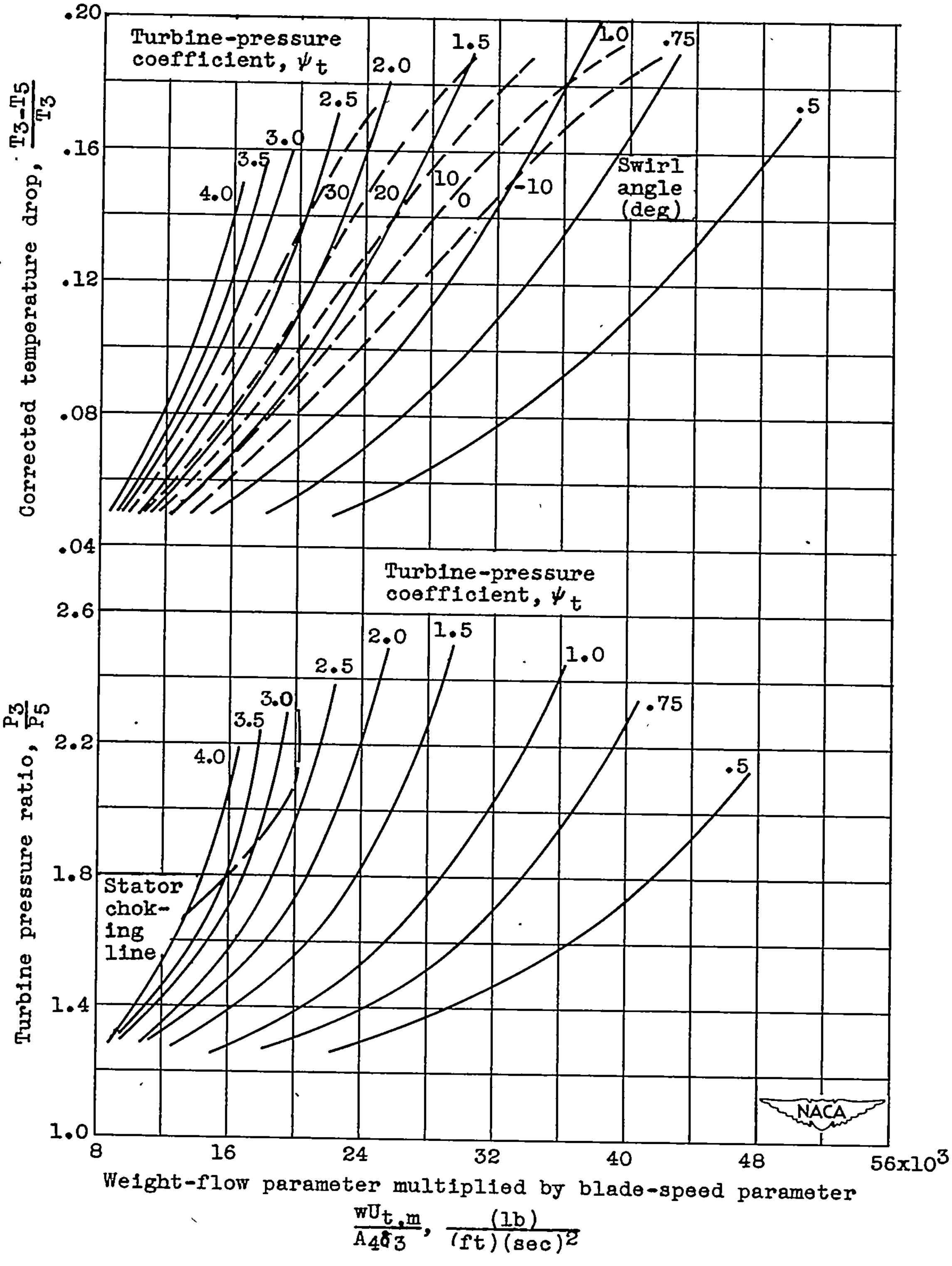
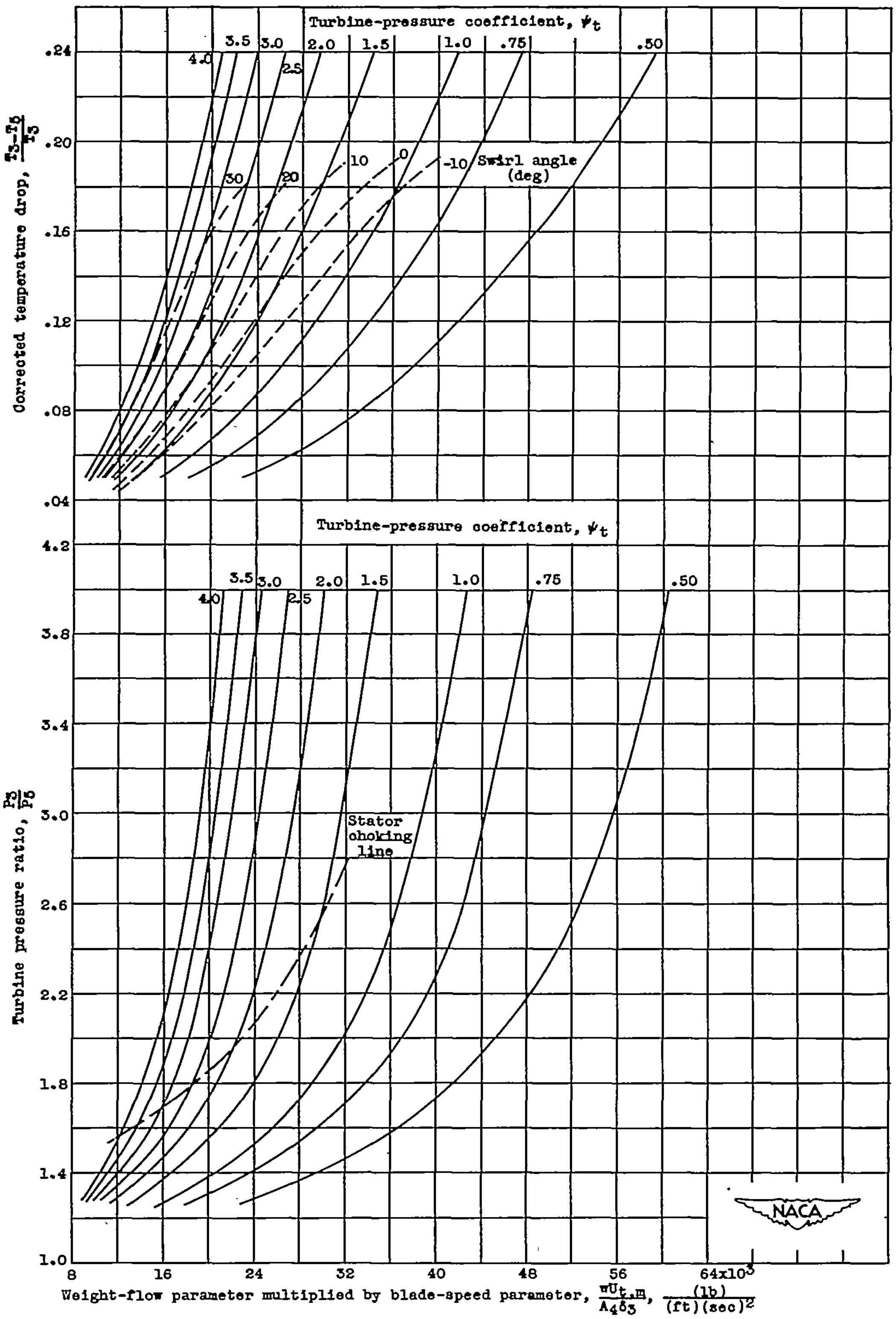


(a.) Turbine-rotor-outlet angle, 30°.

Figure 3. - Performance map for single-stage turbine. Constant annular-passage area. Turbine-stator-outlet angle, 20°.



(b) Turbine-rotor-outlet angle, 32.5°.

Figure 3. - Continued. Performance map for single-stage turbine. Constant annular-passage area. Turbine-stator-outlet angle, 20°.

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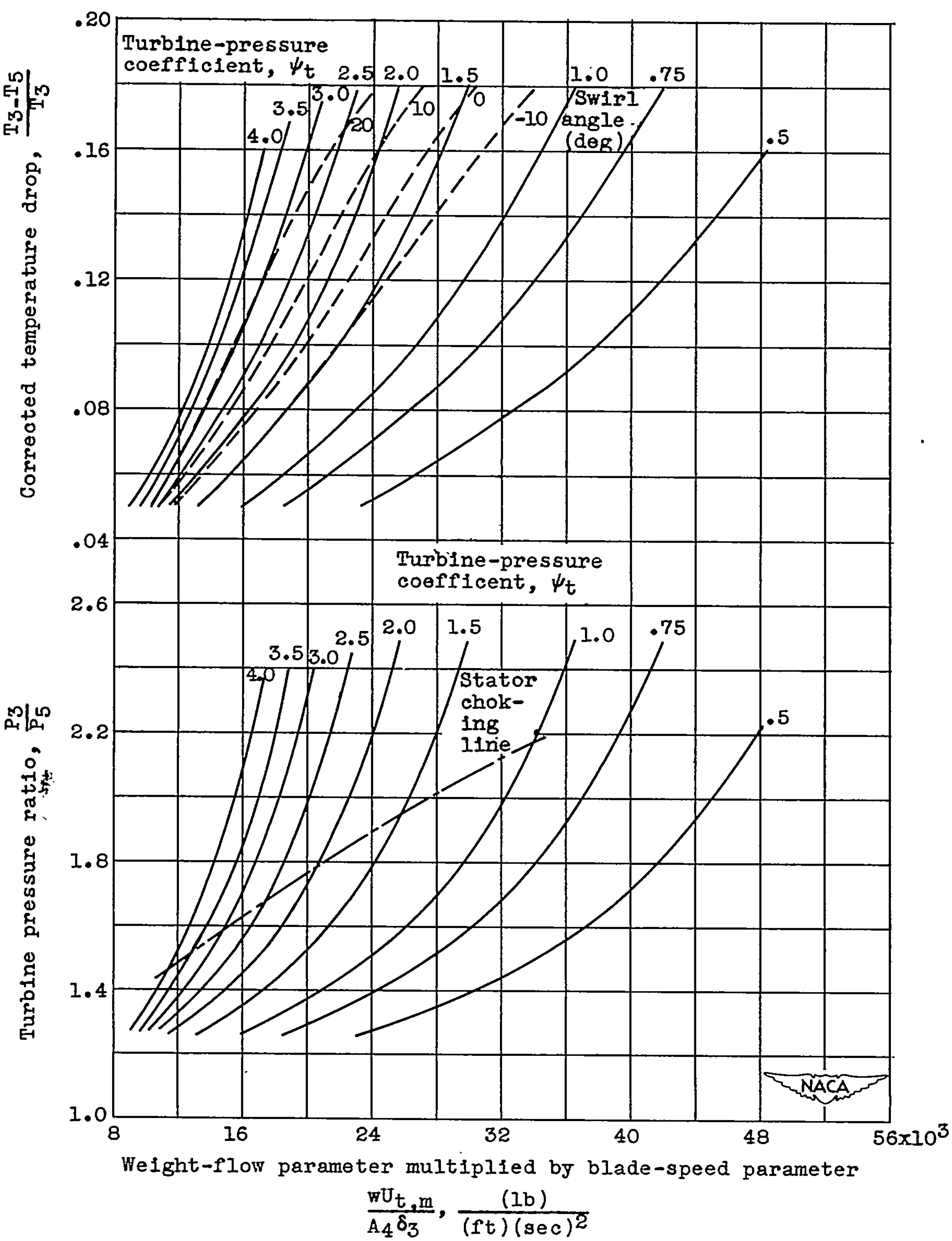

(c) Turbine-rotor-outlet angle, 35°.

Figure 3. - Concluded. Performance map for single-stage turbine. Constant annular-passage area. Turbine-stator-outlet angle, 20°.

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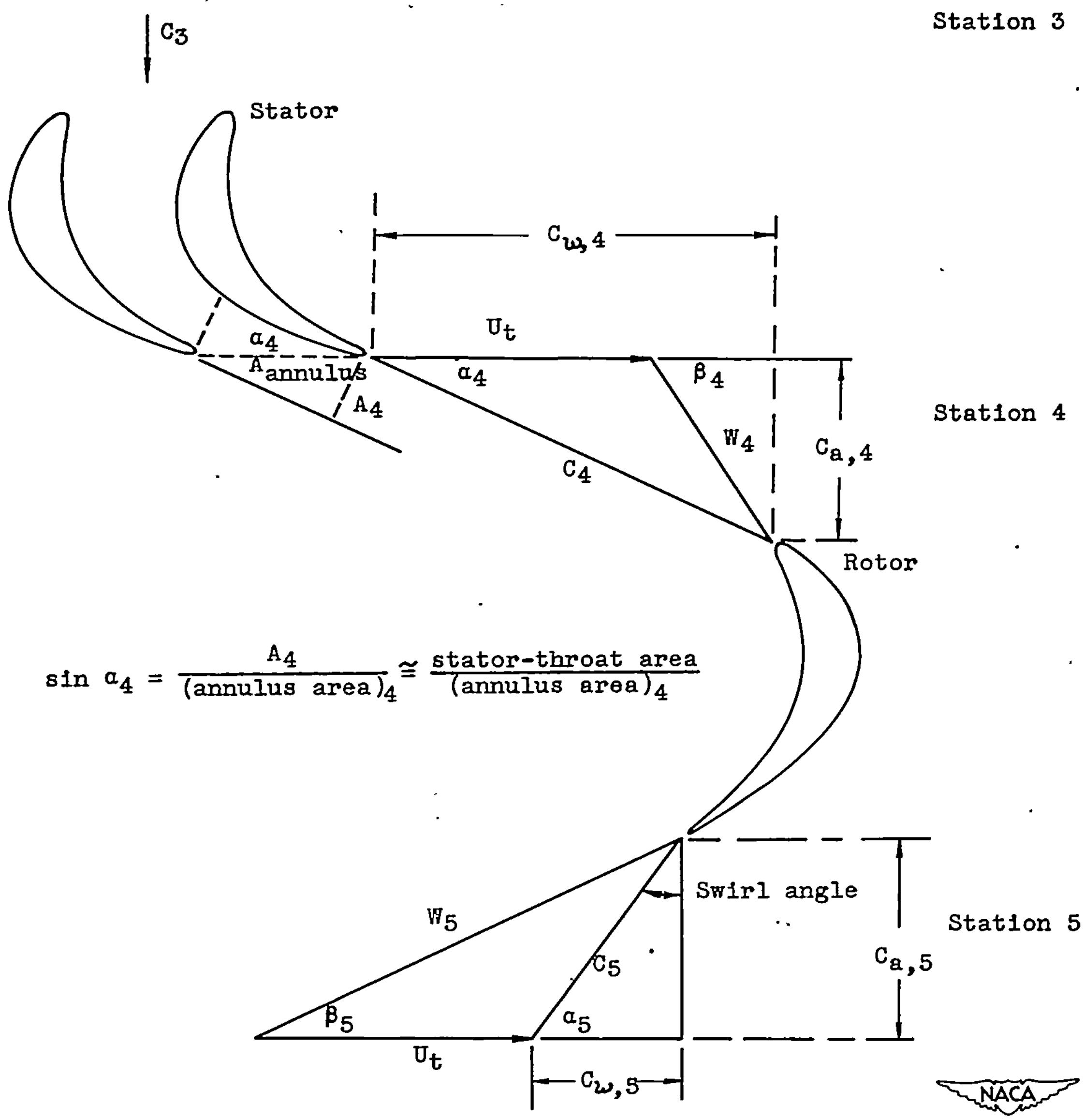

Figure 4. - Turbine-stage velocity diagram.

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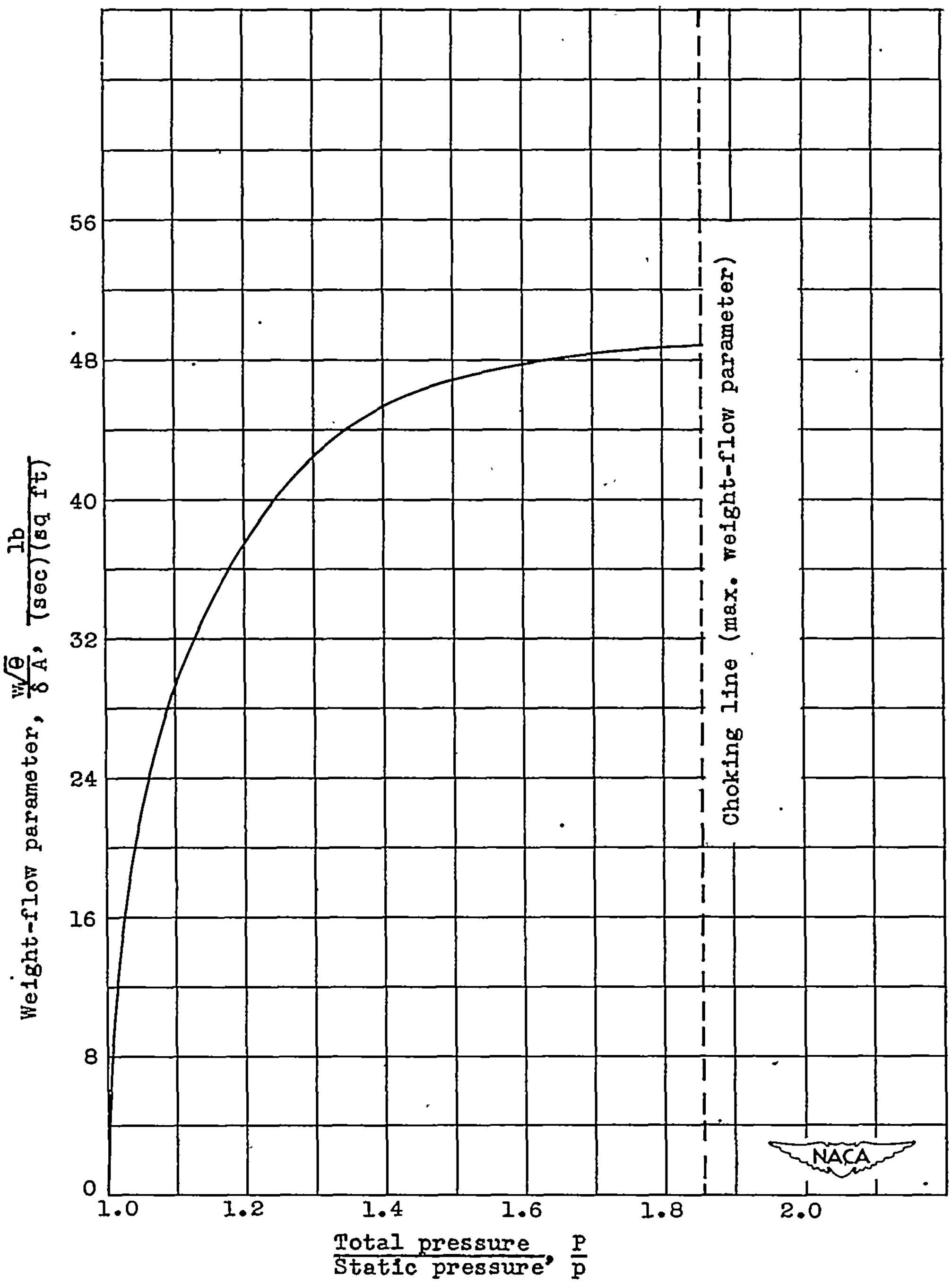

Figure 5. - Variation in weight-flow parameter with change in ratio of total pressure to static pressure.

$$\frac{w\sqrt{\theta}}{\delta A} = 93\sqrt{\frac{2g}{R}\left(\frac{\gamma}{\gamma-1}\right)}\left[\frac{1}{\left(\frac{P}{p}\right)^{\frac{2}{\gamma}}} - \frac{1}{\left(\frac{P}{p}\right)^{\frac{\gamma+1}{\gamma}}}\right]^{\frac{1}{2}};\ \gamma = 4/3.$$

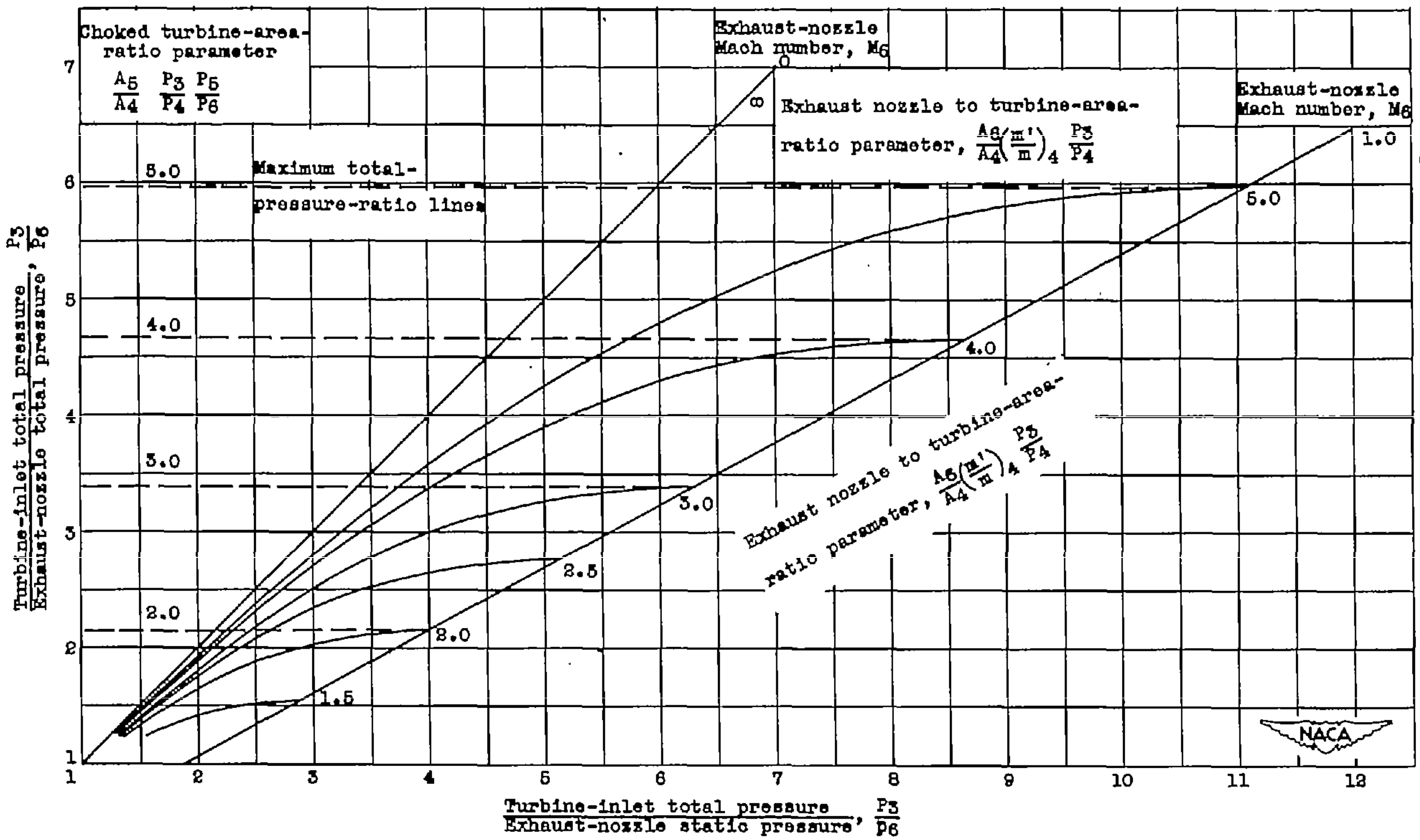

Figure 6. - Map of combined turbine and exhaust-nozzle characteristics. Turbine efficiency, 0.86; ratio of exhaust-nozzle to turbine-rotor-outlet total pressure, 0.95.

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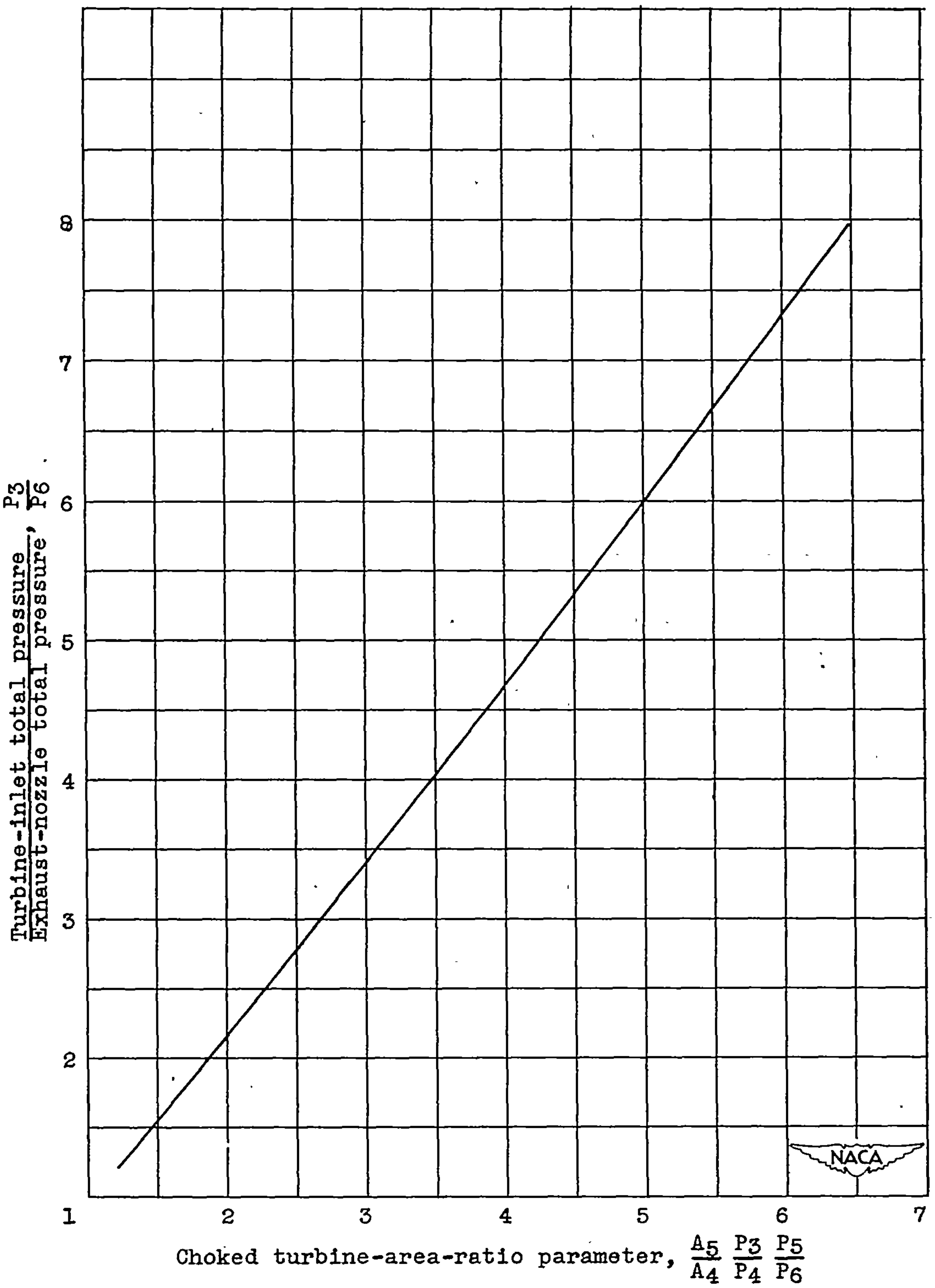

Figure 7. - Variation of maximum available ratio of turbine to exhaust-nozzle total pressure with choked turbine-area-ratio parameter. Turbine efficiency, 0.86; ratio of exhaust-nozzle to turbine-rotor-outlet total pressure, 0.95.

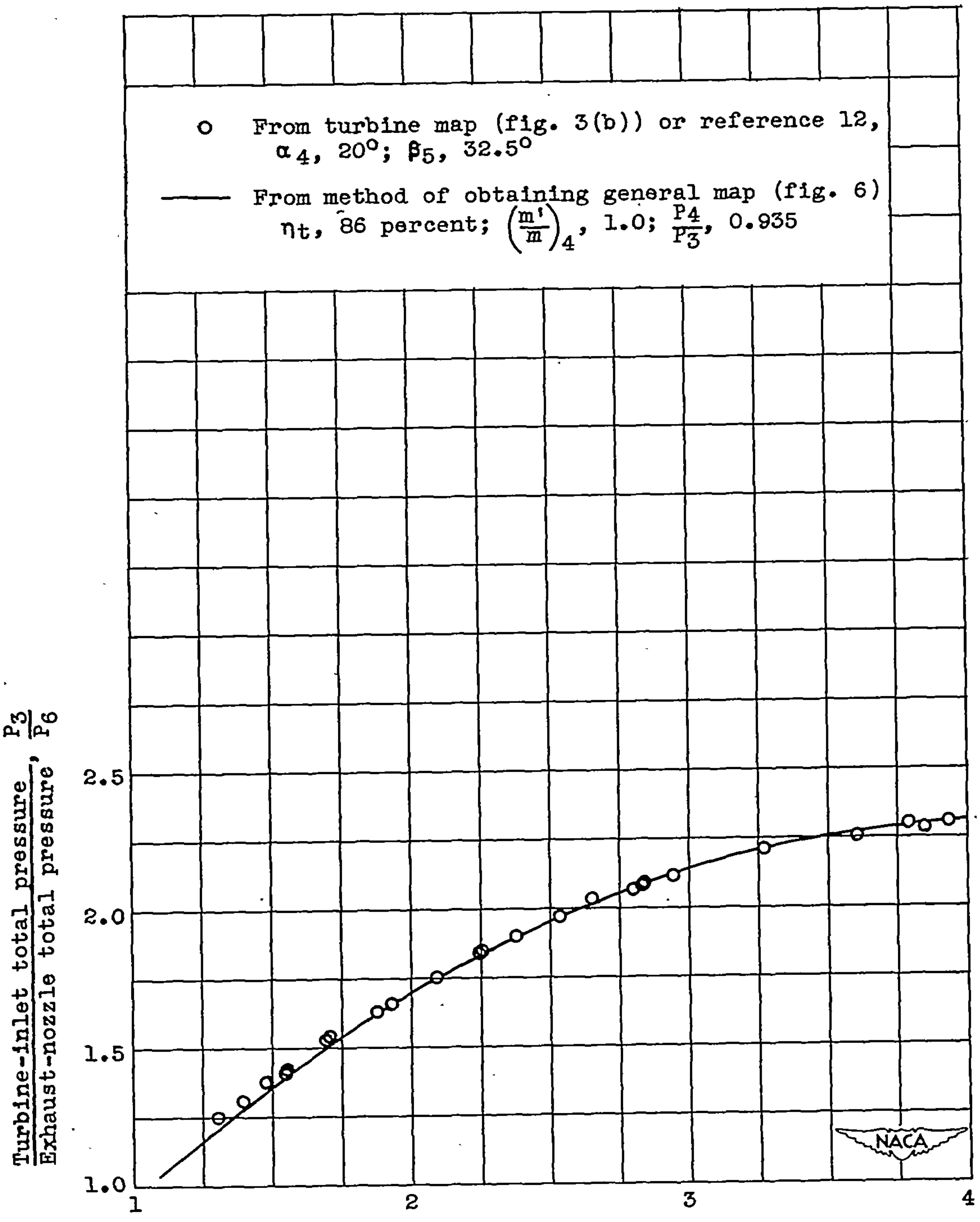

Figure 8. - Comparison of combined turbine and exhaust-nozzle performance calculated by two methods. Ratio of exhaust-nozzle-throat to turbine-stator-outlet area, 2.0; turbine and jet-nozzle ratio of specific heats, 4/3; ratio of exhaust-nozzle-throat to turbine-rotor-outlet total pressure, 0.95.

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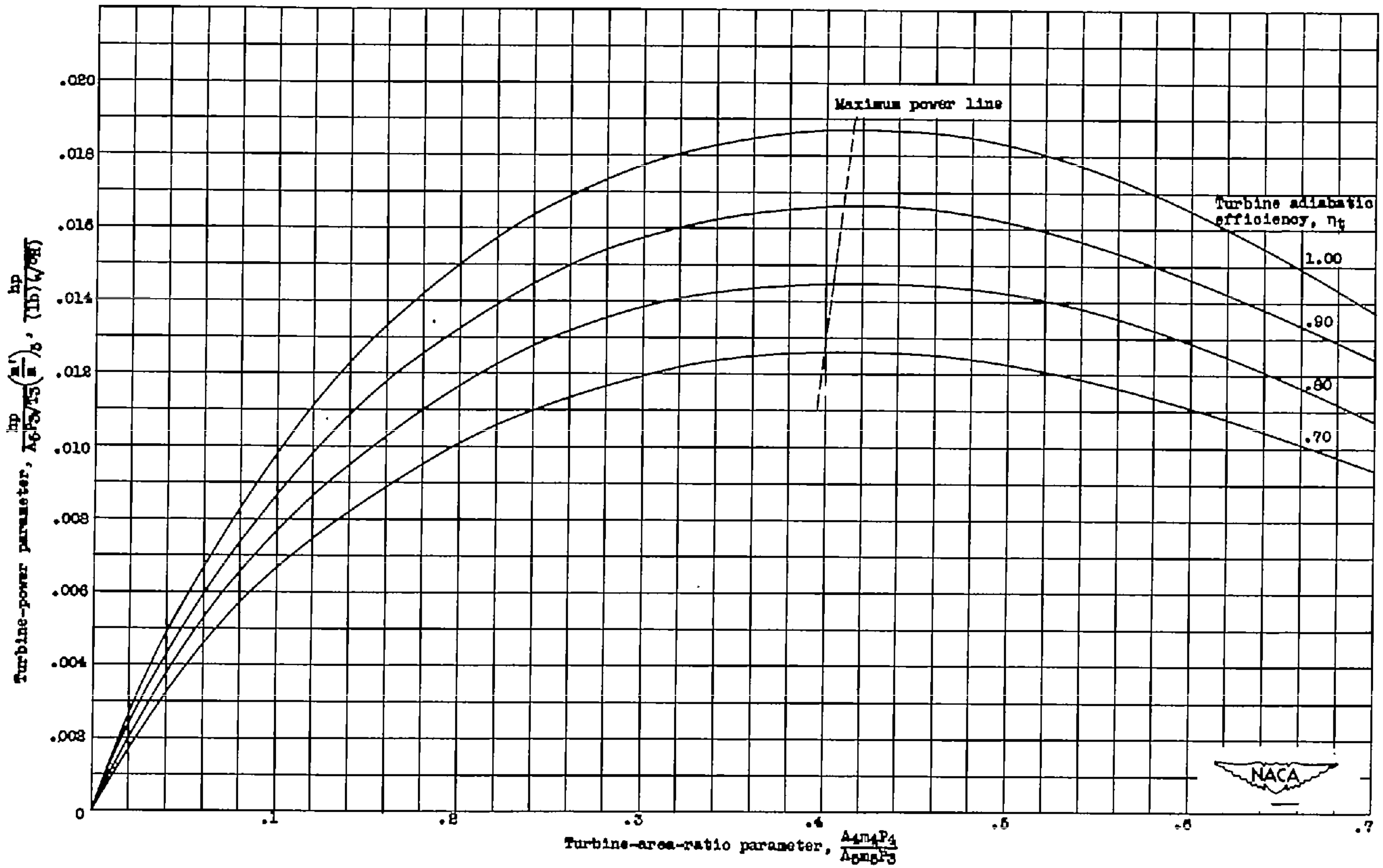

Figure 9. - Variation of turbine-power parameter with turbine-area-ratio parameter and turbine adiabatic efficiency. Ratio of turbine and jet-nozzle specific heats, 4/3; turbine and exhaust-nozzle specific heat, 0.274; choking value of turbine-rotor-outlet weight-flow parameter, 48.62.

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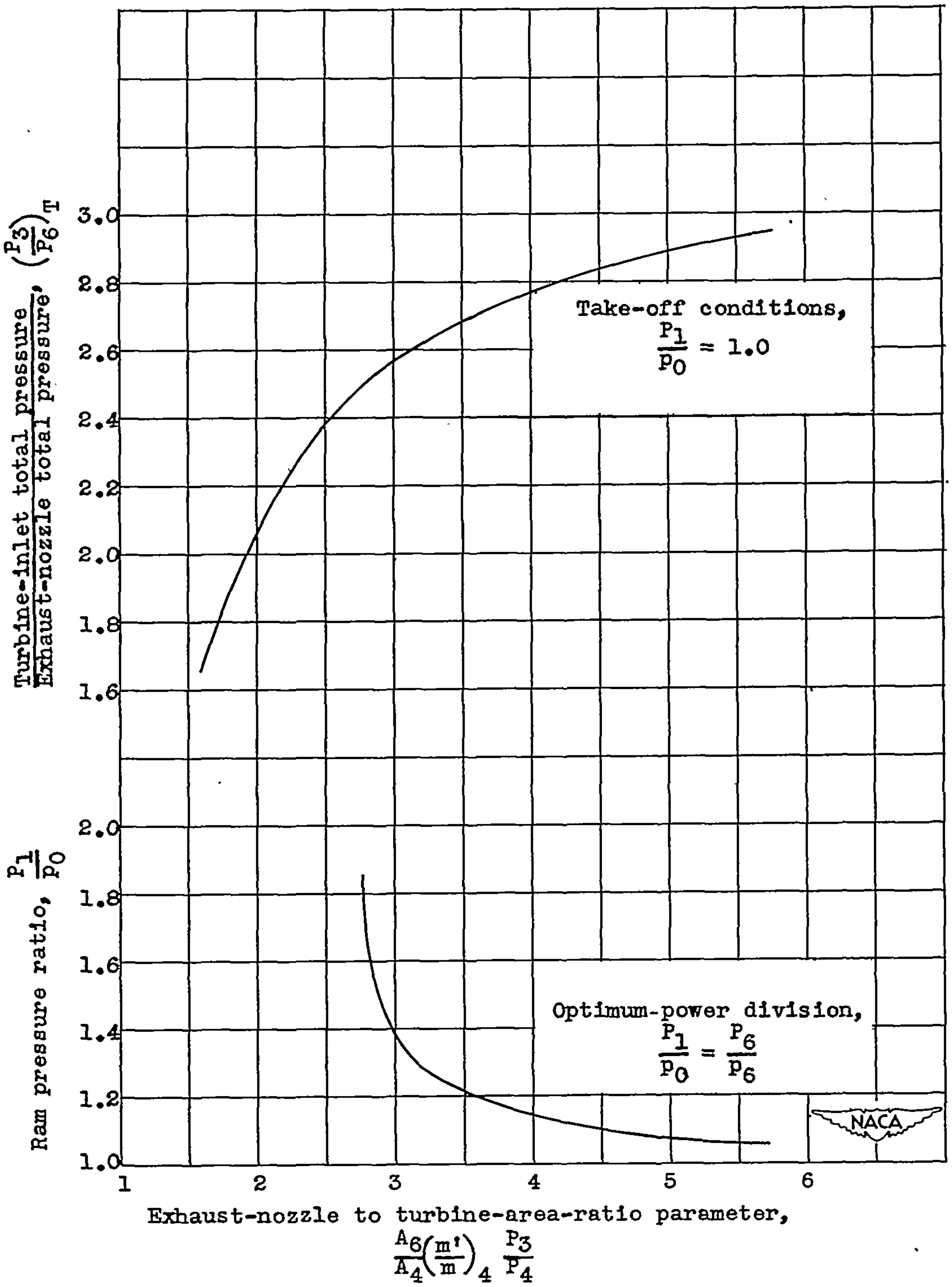

Figure 10. - Variation of take-off ratio of turbine-inlet to exhaust-nozzle total pressure and optimum ram pressure ratio with exhaust-nozzle to turbine-area-ratio parameter. Engine of illustrative example; compressor pressure ratio, 3.25; combustion-chamber total-pressure ratio, 0.95.

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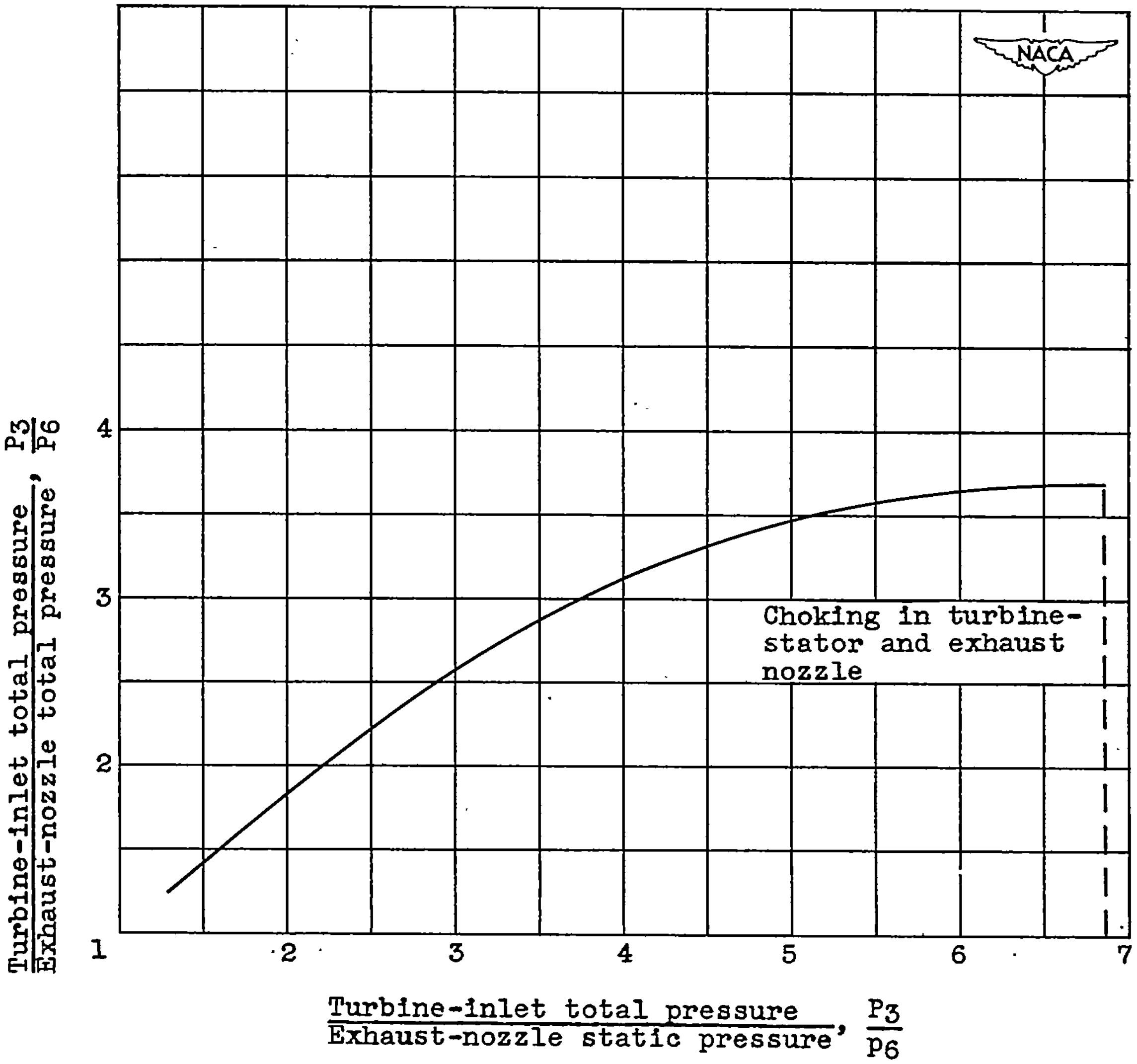
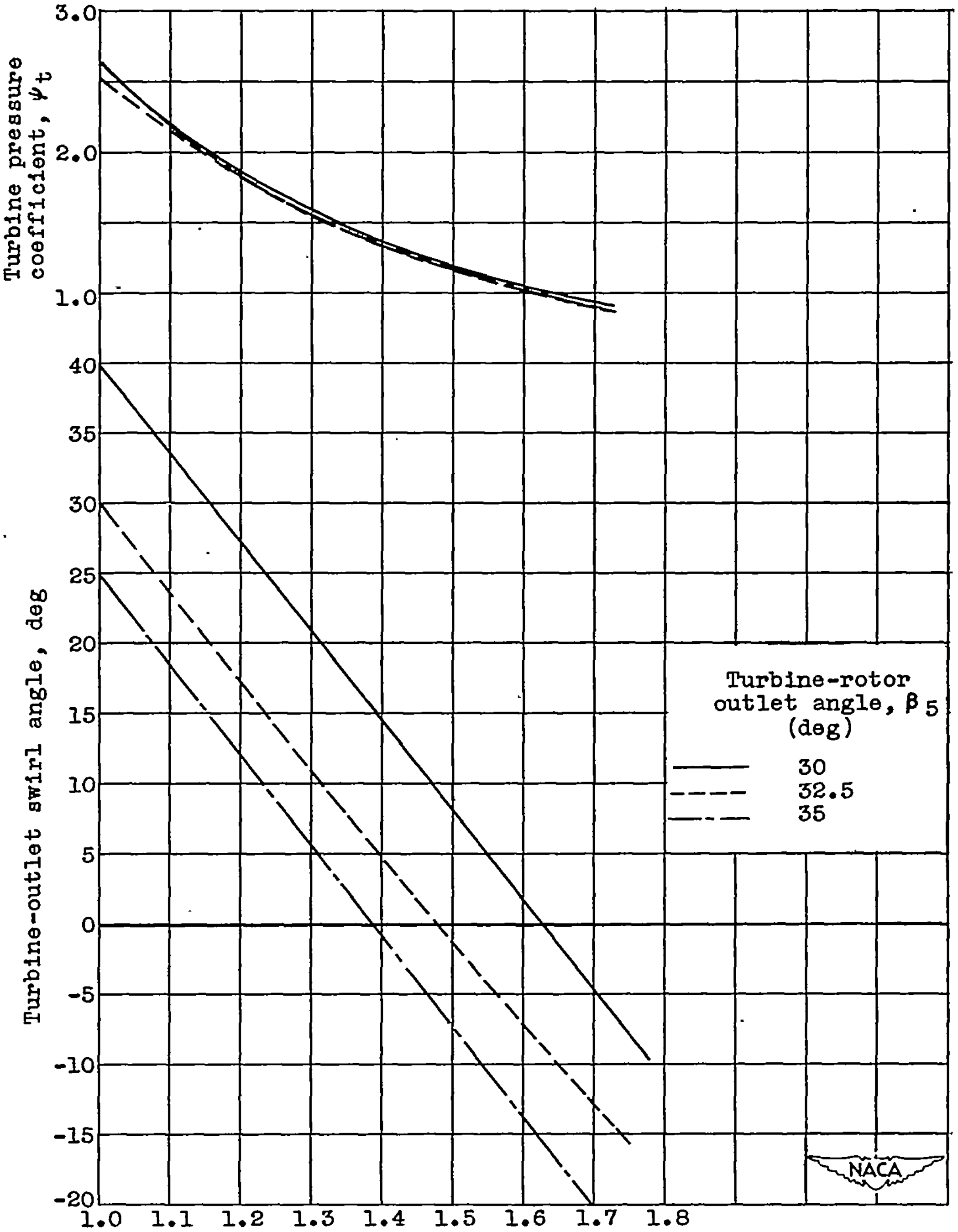


Figure 11. - Variation of ratio of turbine-inlet to exhaust-nozzle total pressure with ratio of turbine-inlet total to exhaust-nozzle static pressure at ratio of exhaust-nozzle-throat to turbine-stator-outlet area of 3.04. Engine of illustrative example; turbine-stator-outlet ratio of choking value of weight-flow parameter to weight-flow parameter, 1.0; ratio of turbine-stator-outlet to turbine-stator-inlet total pressure, 0.935; turbine efficiency, 86 percent.



$\frac{\text{Turbine-pitch diameter}}{\text{Compressor-tip diameter}}, \frac{D_{t,m}}{D_{c,o}}$

Figure 12. - Effect of turbine diameter on rotor-outlet swirl angle and turbine pressure coefficient for design conditions of illustrative example. Turbine-stator outlet angle, 20°; turbine total-pressure ratio, 2.5; turbine-inlet total temperature, 2000° R; compressor-tip velocity, 971 feet per second.

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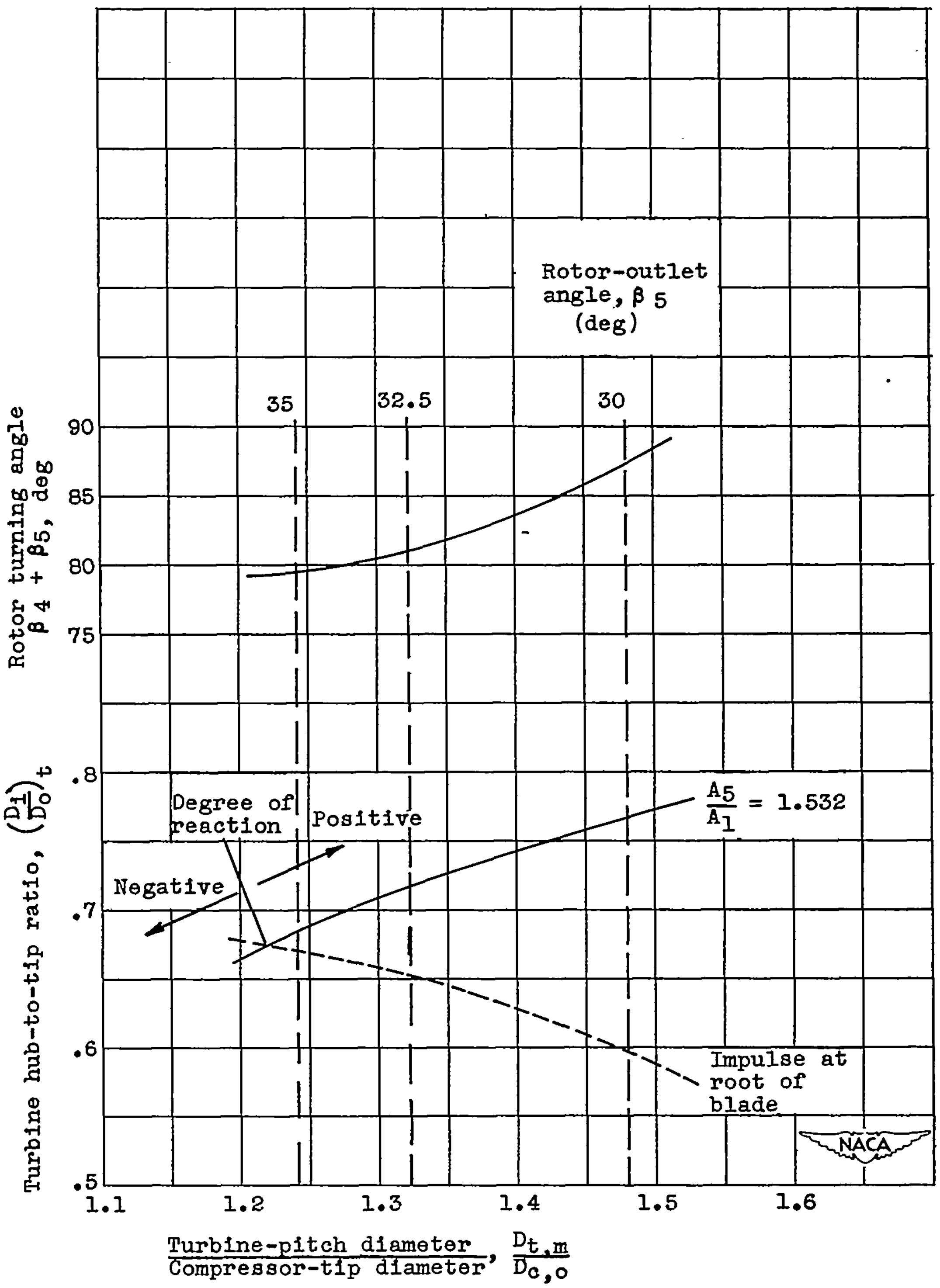


Figure 13. - Effect of ratio of turbine to compressor diameter on degree of reaction at root of turbine blade for outlet swirl angle of 9°. Engine of illustrative example; turbine-stator outlet angle, 20°; turbine-inlet total temperature, 2000° R; compressor-tip velocity, 971 feet per second.

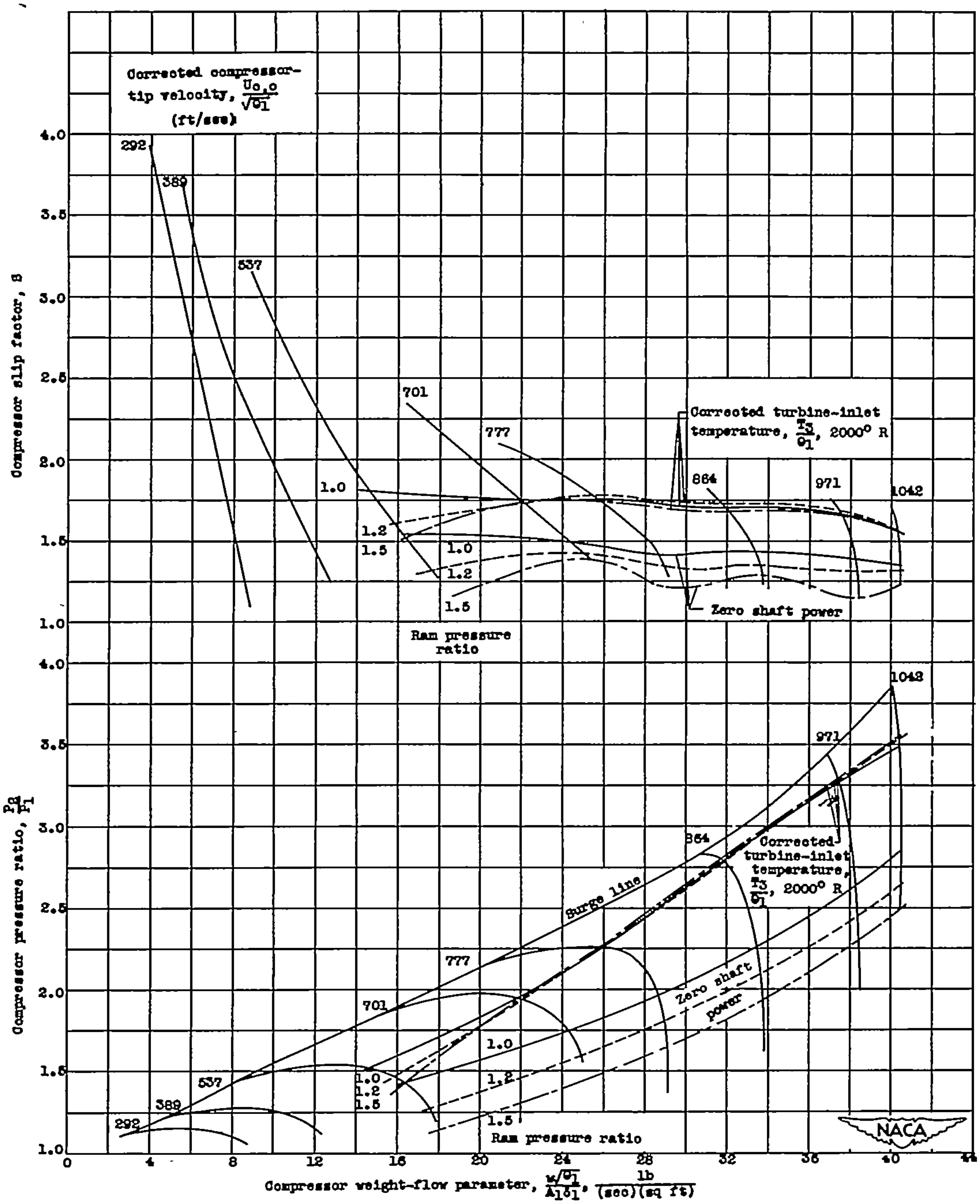


Figure 14. - Range of operation of turbine-propeller engine on compressor performance map; constant exhaust-nozzle area.

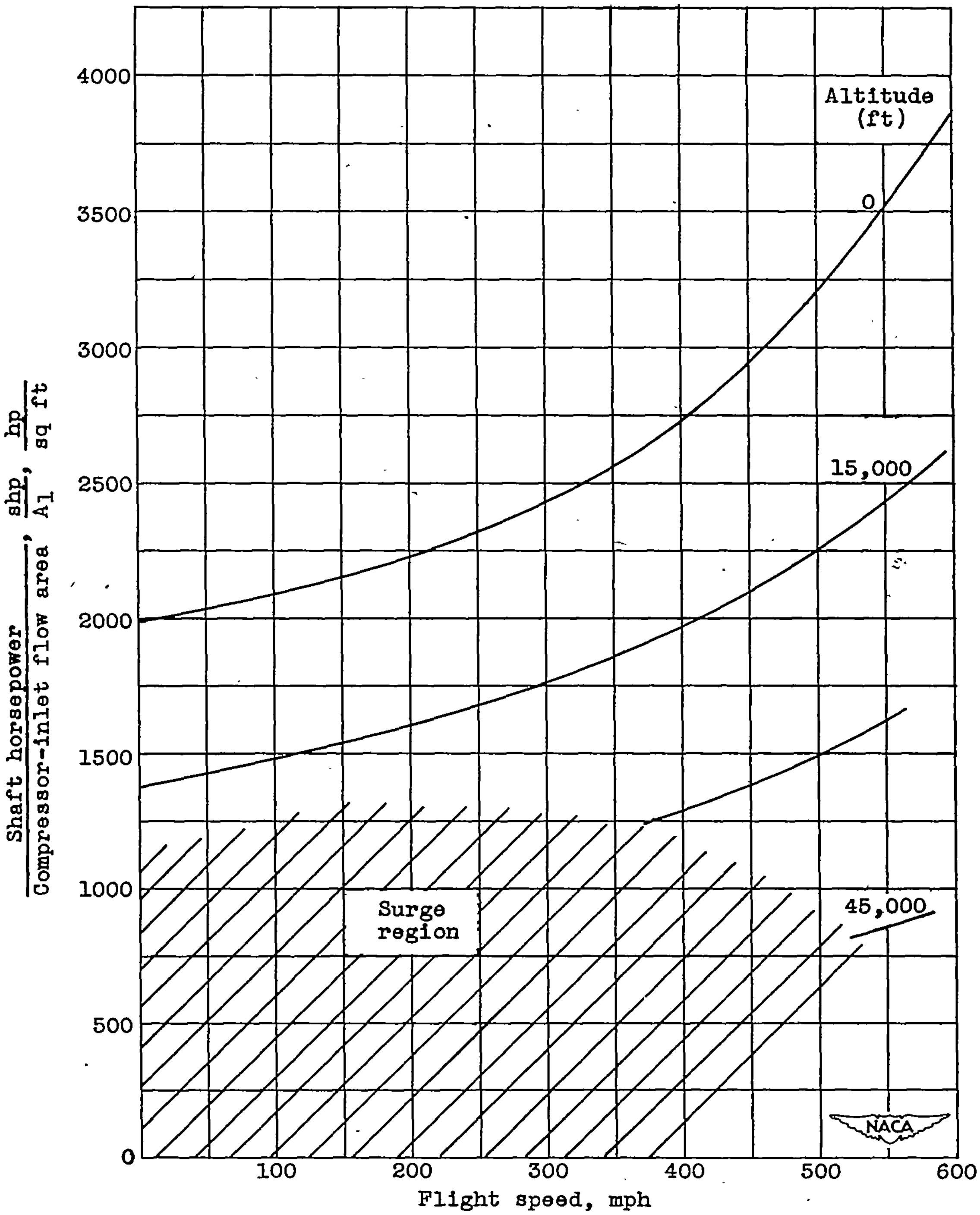
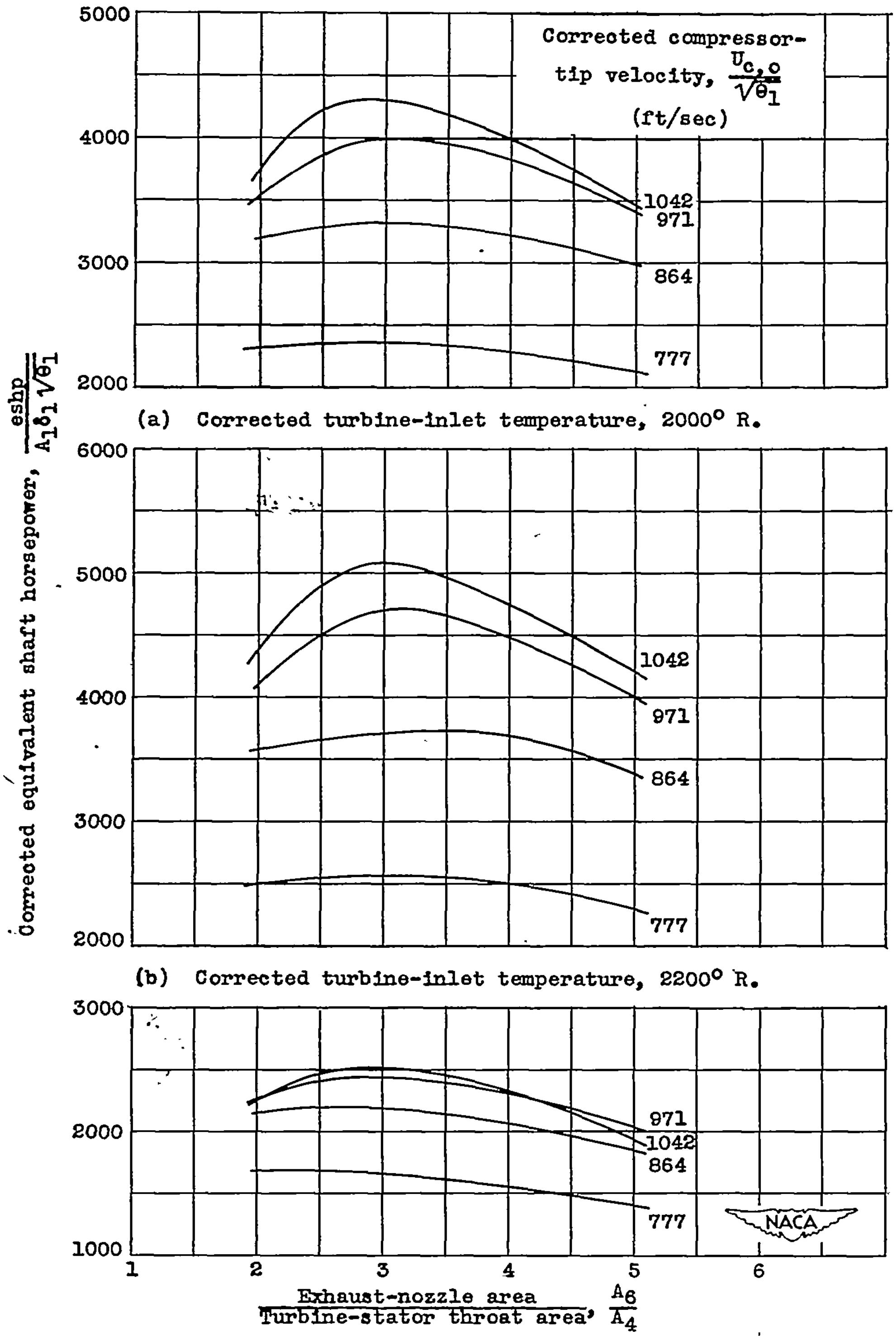


Figure 15. - Available shaft horsepower per unit compressor flow area for various flight speeds and altitudes. Engine of illustrative example; turbine-inlet temperature, 2000° R; compressor-tip velocity, 971 feet per second; constant exhaust-nozzle area.



(c) Corrected turbine-inlet temperature, 1600° R.

Figure 16. - Effect of variable-area exhaust nozzle on corrected equivalent shaft horsepower. Engine of illustrative example; ram pressure ratio, 1.5.

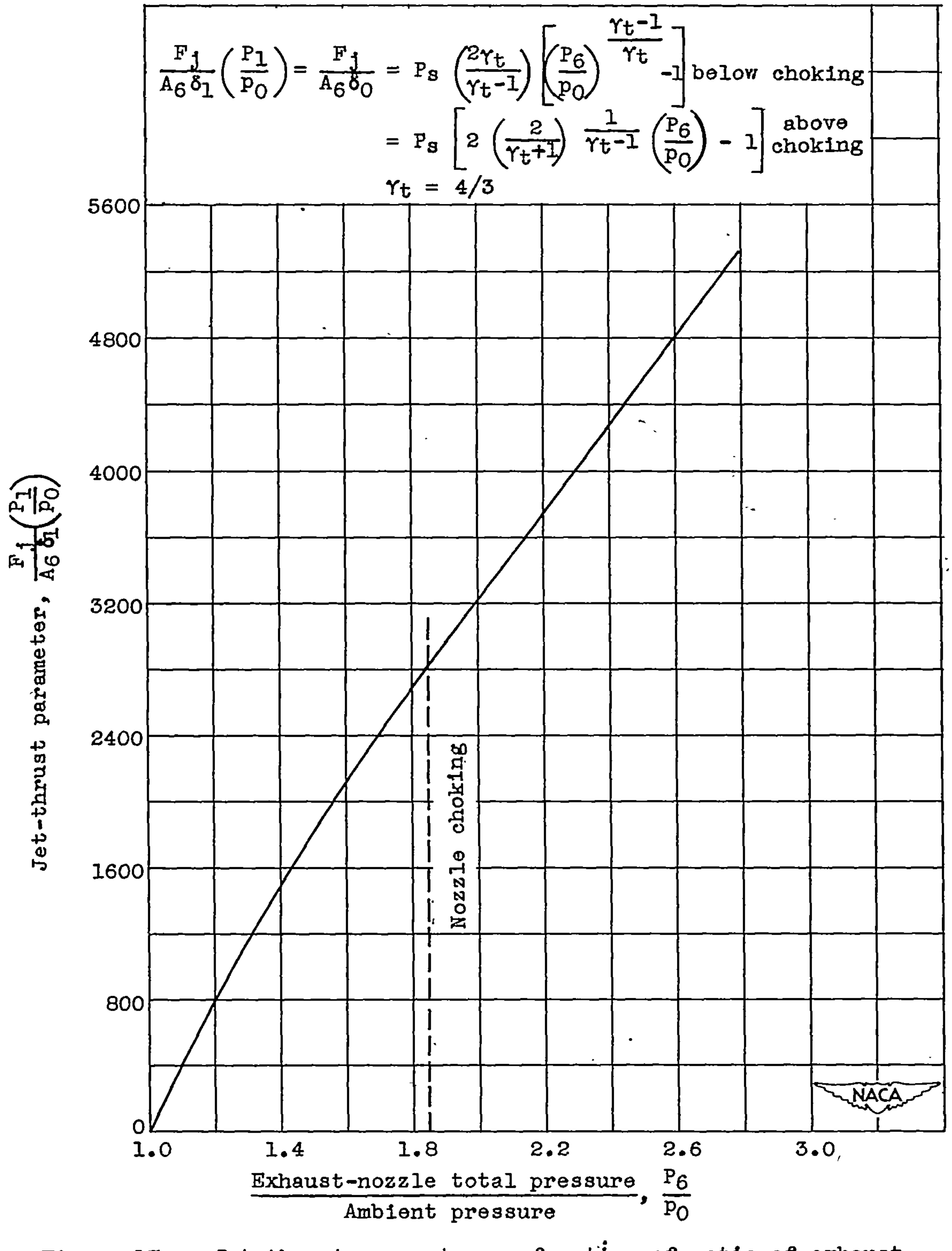


Figure 17. - Jet-thrust parameter as function of ratio of exhaust-nozzle total pressure to ambient pressure. Convergent exhaust nozzle.