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EXPERIMENTAL INVESTIGATION OF THE TURBULENT-BOUNDARY-
LAYER TEMPERATURE-RECOVERY FACTOR ON BODIES OF
REVOLUTION AT MACH NUMBERS FROM 2.0 TO 3.8

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SUMMARY

The local temperature-recovery factor of a turbulent boundary layer produced by natural transition on a thin-walled, metal, 10° cone was measured at Mach numbers of 1.97 and 3.77 and at length Reynolds numbers, based on the surface kinematic viscosity, from $4 \times 10^5$ to $4 \times 10^6$. The recovery factor in the fully developed turbulent zone was found to have a value of 0.862 ± 0.008 which was essentially independent of both Mach number and Reynolds number. The recovery factor was somewhat greater toward the end of the region of boundary-layer transition but did not exceed 0.892.

The recovery factor was also measured on a 40° cone-cylinder combination at Mach numbers of 3.10 and 3.77 and at length Reynolds numbers from $3 \times 10^5$ to $1 \times 10^6$. An increase in local turbulent recovery factor above that on the 10° cone of less than 2 percent was observed; the maximum value was 0.896. A recovery factor in the turbulent boundary layer of 0.885 ± 0.011 is considered to be adequately representative of the values obtained with both bodies in the present investigation. Similar results have been found by previous investigators at lower Mach numbers.

INTRODUCTION

The temperature which occurs at the insulated surface of a vehicle in supersonic flight may be thought to result from two superimposed effects. The first effect, which determines the static temperature just outside the boundary layer, is due to the shape of the body; the second is brought about by the frictional dissipation of kinetic energy in the boundary layer. In most cases the static temperature can be calculated with good accuracy, and for Mach numbers up to 2.5 the temperature rise through turbulent boundary layers can be determined by recourse to information such as that given in references 1 through 3.
For Reynolds numbers of about one million and for Mach numbers less than 2.5 the information contained in references 1 through 3 indicates that when the boundary layer is turbulent, about 89 percent of the available kinetic energy can be expected to appear as heat at the surface of a body. However, for Mach numbers greater than about 2.5 the available theory and experiments (references 4 and 5) are not in agreement. The data of reference 5 indicate that very large values of the turbulent-boundary-layer recovery factor (0.92 to 0.97) are to be expected at Mach numbers of 2.87 and 4.25, while the theory of reference 4 indicates that the recovery factor for a 1/7-power turbulent-boundary-layer velocity profile should decrease to 0.863 at a local Mach number of 4.25.

The purpose of the present wind-tunnel experiments was to obtain additional values of the turbulent-boundary-layer recovery factor in the Mach number range from 2 to 4 and to compare these values with the predictions of reference 4 and the data of reference 5.

**NOTATION**

- \( a \) speed of sound, feet per second
- \( C_r \) recovery factor \( \left( \frac{T_p-T_1}{T_0-T_1} \right) \), dimensionless
- \( c_p \) constant-pressure specific heat, Btu per pound, \(^\circ\)F
- \( c_v \) constant volume specific heat, Btu per pound, \(^\circ\)F
- \( g \) acceleration of gravity, feet per second squared
- \( k \) thermal conductivity coefficient, Btu per second, square foot, \(^\circ\)F per foot
- \( M \) Mach number \( \left( \frac{V}{a} \right) \), dimensionless
- \( N \) reciprocal of exponent defining boundary-layer velocity profile, dimensionless
- \( P \) pressure, pounds per square foot
- \( Pr \) Prandtl number \( \left( \frac{c_p \mu}{k} \right) \), dimensionless
R  gas constant, feet per °F
Re  Reynolds number \( \left( \frac{Vx}{V} \right) \), dimensionless
T  absolute temperature, °F
t  time, seconds
V  velocity, feet per second
x  distance from nose along body generator, feet
y  coordinate normal to body surface, feet
γ  ratio of specific heats \( \left[ \frac{c_p}{c_v} (= 1.4) \right] \), dimensionless
μ  absolute viscosity, pound-seconds per square foot
ρ  mass density, slugs per cubic foot
ν  kinematic viscosity \( \left( \frac{μ}{ρ} \right) \), square feet per second

Subscripts

ō  stagnation condition
{l}  local condition just outside the boundary layer
r  insulated surface condition
s  measured surface condition
∞  free-stream condition

THEORY

The theory for turbulent boundary layers is incomplete because the mechanism of turbulence is not well understood. With the exception of the theory of Tucker and Maslen (reference 4), the existing theories which lead to a prediction of the turbulent recovery factor are summarized in reference 3. The theory of Tucker and Maslen extends the
incompressible analysis of Squire for a flat plate to include the effects of Mach number. The result of their analysis is the following approximation formula:

\[
\ln(C_T) = \left( \frac{N + 1 + 0.528 M_1^2}{3N + 1 + M_1^2} \right) \ln(\text{Pr})
\]  

(1)

wherein \(\text{Pr}\) is the Prandtl number and \(N\) is the reciprocal of the exponent of the boundary-layer velocity profile as approximated by the power law. Since the parameter \(N\) can be shown to increase with the Reynolds number, it is apparent from equation (1) that the recovery factor predicted by the theory of reference 4 increases with increasing Reynolds number and decreases with increasing Mach number. Squire's result for the incompressible case is obtained from equation (1) when the Mach number is zero, and the approximation \(C_T = \text{Pr}^{1/3}\) is obtained as the limit when the parameter \(N\) is increased indefinitely.

The recovery factor at any point on an insulated body can be found by measuring its surface temperature, \(T_r\), the total temperature of the air stream, \(T_0\), and the local Mach number, \(M_1\), just outside the boundary layer (which defines the local static temperature). In the present investigation these quantities were measured by well-known techniques and were then combined to form the recovery factor according to the following equation:

\[
C_T = \frac{T_r}{T_0} \left( 1 + \frac{\gamma-1}{2} M_1^2 \right) - 1
\]

(2)

Equation (2) can be derived by combining the definition of the recovery factor (see notation) with the adiabatic energy equation.

**APPARATUS**

Wind Tunnels

The present investigation was conducted in the Ames 1- by 3-foot supersonic wind tunnels No. 1 and No. 2. Wind tunnel No. 1 is of the closed-circuit, continuous-operation, variable-pressure type and is equipped with a flexible-plate nozzle that provides a range of Mach numbers from 1.2 to 2.4. The absolute pressure in the tunnel settling
chamber can be varied from one-fifth of an atmosphere to three atmospheres to provide changes in the test Reynolds number. The absolute humidity of the air is maintained at less than 0.0001 pound of water per pound of dry air so that the effects of water content on the supersonic flow are negligible. The No. 2 wind tunnel is of the intermittent-operation, nonreturn, variable-pressure type and uses the dry air at high pressure (six atmospheres absolute) from the Ames 12-foot wind tunnel. The air is expanded to atmospheric pressure through the 1- by 3-foot test section, which is structurally identical to that of the No. 1 wind tunnel. The Mach number can be varied from about 1.2 to 3.8. The steady running time available for each test depends largely on the test Mach number and varies from about 18 minutes at a Mach number of 2.0 to 5 minutes at a Mach number of 3.8. The total pressure in the wind-tunnel settling chamber is controlled by means of a butterfly throttling valve in the supply pipe. Because the air in the supply system expands during each test, the stagnation temperature decreases with time; the maximum rate of decrease is about 40°F per minute.

Test Bodies

In order to obtain measurements for comparison with existing theory, a body with uniform surface pressure and temperature was desired. In supersonic flow this consideration required, for instance, a cone or a flat plate. Since the turbulent boundary layer on a conical body has been shown by the theory presented in reference 6 to be related to that on a flat plate, and because bodies of revolution are more convenient to test than flat plates, a 10° cone was employed to obtain data for comparison with the theory of reference 4. Because a 40° cone cylinder was used to obtain the data reported in reference 5, a body with almost identical external shape was made and tested to obtain comparable data.

10° cone. - The 10° included-angle cone (fig. 1(a)) was made of stainless steel and, with the exception of an inaccessible region at the tip and a threaded section at the base, the wall thickness was 0.032 inch. The thin wall served to minimize both the heat capacity of the model and the longitudinal heat conduction within the shell. Stainless steel was used to further reduce longitudinal conduction because of its low thermal conductivity relative to other metals. Twenty constantan thermocouple wires were soldered into holes in the shell spaced along a ray of the cone as shown in figure 1(a). Four additional constantan wires were installed along the opposite ray of the cone to provide a check on the uniformity of the circumferential surface-temperature distributions. A single stainless steel wire, connected to the base of the cone, completed the return circuit for the 24 stainless-steel-constantan thermocouples. The exterior surface
of the cone was ground and then polished until the maximum roughness was reduced to less than 15 microinches. Careful inspection of light reflections from the cone surface revealed that the surface waviness was small.

40° cone cylinder.—The 40° cone-cylinder model which was included in the present investigation to obtain data for comparison with that of reference 5 is shown in figure 1(b). The cone-cylinder models of the two investigations were made of different materials and with different fineness ratios. The model of reference 5 was made in segments of a paper-base plastic. The afterbody could be varied in length up to a maximum of about 3 feet. The model of the present investigation was made with a thin (0.063 inch thick) brass shell. Brass was used to facilitate construction. The afterbody length was selected to provide over-all length Reynolds numbers comparable to those of reference 5. As shown in figure 1(b), six iron-constantan thermocouples were soldered into the shell, one being placed on the conical portion of the body. In addition, six static-pressure orifices were located along the opposite side of the body to provide data from which the local Mach number could be calculated. The maximum surface roughness of the 40° cone-cylinder body was 20 microinches.

Instrumentation and Accuracy

The thermocouple voltages on the bodies, as well as those from the thermocouples used to measure the total temperature in the wind tunnel, were read on either indicating or recording self-balancing potentiometers that were calibrated and were accurate to ±0.25°F. Although the instrument accuracies were ±0.25°F, the repeatability of the temperature measurements during a test was ±0.5°F because of minor variations in the stagnation temperature. Several iron-constantan thermocouples located on the downstream screen in each of the wind-tunnel settling chambers were used to measure the total temperatures of the air streams. In the No. 2 wind tunnel, the stagnation-temperature distribution across the settling chamber was uniform within ±0.5°F. In the No. 1 wind tunnel, because of the cooling system, there was a temperature variation of 3°F across the settling chamber at the higher pressure levels. The average of the temperature readings in the settling chambers was used in the reduction of the test data. As a result, temperature data obtained in the No. 2 wind tunnel can have a maximum error of ±0.5°F, while the maximum error of those obtained in the No. 1 wind tunnel can be approximately ±1.5°F.

The local Mach number just outside the boundary layer on the 10° cone was computed from the known Mach number distribution in the wind-tunnel test section and the charts of reference 7. In the case of the cone cylinder, the local Mach number was computed from the data of
references 7 and 8 and the static pressures observed during the tests. The values of local Mach number were estimated to be accurate to within ±0.8 percent at the lower Mach numbers, but the accuracy of the static pressure measurements on the cone-cylinder model at a Mach number of 3.77 provided values of the local Mach number that were only accurate to ±1.2 percent. However, the effect of the decrease in accuracy of the Mach number is compensated by a reduction in the effect of the Mach number error on the over-all recovery-factor accuracy as the Mach number increases (see equation 2).

The maximum probable error in the local recovery factor, based on the individual accuracies of the Mach numbers and temperatures, is approximately ±1 percent for values from both wind tunnels at all the test Mach numbers. The Reynolds numbers were determined with similar accuracy.

PROCEDURE AND TESTS

A preliminary series of tests with the 10° cone were conducted in the Ames 1- by 3-foot supersonic wind tunnel No. 1 to obtain the steady-state value of the turbulent-boundary-layer recovery factor at a nominal Mach number of 2 and an over-all length Reynolds number based on free-stream conditions in the wind-tunnel test section of 7.7 x 10⁶. The data from these tests were obtained for comparison with data obtained under transient temperature conditions in the No. 2 wind tunnel but at the same Mach and Reynolds numbers. The length Reynolds number was held constant in the intermittent-operation wind tunnel by gradually decreasing the total pressure as a function of the decreasing total temperature. This comparison served two purposes. First, a check was obtained on the rate at which the tunnel-model combination reached equilibrium temperature. Equilibrium was assumed to exist when the surface-to-stagnation temperature ratio, Tₛ/T₀, at each point on the cone became constant as a function of the elapsed testing time. Second, by comparing these constant values of the temperature ratios with those obtained under steady-state conditions in the continuous-operation wind tunnel, the possible errors due to the small but finite heat transfer which accompanied the stagnation-temperature drift could be evaluated.

After the accuracy of the testing technique had been evaluated at a Mach number of 2.0, tests were conducted with the 10° cone in the No. 2 wind tunnel at a nominal Mach number of 3.8. The test conditions were selected so that the stagnation-temperature drift and the over-all heat-transfer rate corresponded to those of the tests at a Mach number of 2.0. The test duration at the higher Mach number was less than at a Mach number of 2.0 because of the fixed maximum pressure and the greater
pressure ratio required to maintain supersonic flow; however, as will be discussed later, sufficient time was available for conditions to reach equilibrium with this model.

After the tests with the 10° cone, the 40° cone cylinder was tested in the No. 2 wind tunnel at nominal Mach numbers of 3.1 and 3.8. At the higher Mach number the wall thickness of this model (0.063 inch) prevented the temperature ratio $T_s/T_0$ from reaching equilibrium in the available testing time. The data were plotted as a function of $e^{-t}$, where $t$ is the elapsed testing time in seconds, and the ordinates at infinite time ($e^{-t} = 0$) from the extrapolated data were taken as the experimental values of the temperature ratios.

RESULTS AND DISCUSSION

10° Cone

The results of the preliminary tests in the No. 1 and No. 2 wind tunnels to determine the suitability of the thin-walled, metal 10° cone for tests under transient temperature conditions are shown in figure 2. These data were obtained at a Mach number of 1.97 in both wind tunnels and indicate that the value of the temperature ratio for turbulent flow ($T_s/T_0 = 0.950$) in the boundary layer is independent of the test facility. The locations of the transition regions are quite different, however.

From the data shown in figure 2 it can be concluded that the thin-walled, metal cone is as satisfactory for recovery-factor measurements in the No. 2 wind tunnel as in the No. 1 wind tunnel. Additional evidence of this is shown in figure 3 by the values of the temperature ratio $T_s/T_0$ at two different elapsed times during a test at a Mach number of 3.77. As shown in figure 3, the values of the temperature ratio in the first 8 inches of model length changed somewhat with increasing time, but in the 8- to 14-inch region the change was negligible. Since the total and surface temperatures were recorded continuously with time during the tests in the No. 2 wind tunnel, it was possible to plot values of the temperature ratio at several points on the cone as a function of $e^{-t}$ and determine the value of the ratio for infinite time ($e^{-t} = 0$). These values were found to correspond to those shown in figure 3 for an elapsed time of 4.5 minutes, indicating that within the limits of experimental accuracy the results shown were obtained at the condition of thermal equilibrium.

The computed local Mach number distributions along the 10° cone at free-stream Mach numbers of 1.97 and 3.77 are shown in figure 4. The local recovery factors were calculated using the local Mach number, $M_1$. 
and the temperature ratio \( \frac{T_s}{T_o} \approx \frac{T_r}{T_o} \) in equation (2) and are shown as a function of local Reynolds number, \( Re_s \), in figure 5. For convenience, the definition of Reynolds number employed in reference 5 has been used:

\[
Re_s = \frac{V_1 x}{v_s}
\]  

(3)

The viscosity, \( \mu_s \), was calculated from Sutherland's equation as a function of the measured surface temperature, \( T_s \), and the static pressure was assumed constant across the boundary layer. Two theoretical values of the recovery factor, the square root, and the cube root of the Prandtl number (\( Pr = 0.715 \)) have been included in figure 5 for comparison with the experimental data. In figure 6 the curve of Tucker and Maslen (equation (1)) for a Prandtl number of 0.715 and a 1/7-power velocity profile is shown along with experimental points taken from figure 5 and the data of reference 3. These points were selected so that, on the basis of free-stream properties and when account is taken of geometric differences (reference 6), the Reynolds number is approximately constant.

It is apparent from figure 6 that there is little variation of the experimental recovery factor with Mach number, and that for the theory of reference 4 to be consistent with the data the parameter \( N \) and the local Mach number \( M_1 \) must be related in such a manner that the entire exponent of the Prandtl number in equation (1) is almost constant and at a value near Squire's exact value for \( N = 7 \) and \( M_1 = 0 \). (An empirical modification of the theory of reference 4 which predicts a recovery factor consistent with the data of the present investigation has been proposed by Mr. Maurice Tucker of the NACA Lewis Flight Propulsion Laboratory.)

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1. Reynolds number based on wall properties can be converted to Reynolds number based on local free-stream properties for the present test conditions by the approximate relation:

\[
\frac{V_1}{v_s} = \frac{Re_s}{Re_1} = \left( \frac{T_s}{T_o} \times \frac{1}{1 + \frac{7-1}{2} M_1^2} \right)^{1.9}
\]

2. In NACA TN 2337, 1951, the arithmetic mean temperature of the boundary layer is proposed as a suitable reference temperature for evaluating the state properties of compressible turbulent boundary layers. This hypothesis gives a reasonable agreement between experimental and predicted drag coefficients in supersonic flow. If the same hypothesis is used to obtain a Prandtl number representative of the turbulent boundary layer, the recovery factors predicted for local Mach numbers of 1.93 and 3.61 are 0.885 and 0.883, respectively, for \( N = 7 \). These values are in close agreement with the data shown in figure 6.
The local large values of recovery factor at the completion of transition (fig. 5) could result from the fact that $N$, in effect, is greater than 7 at the beginning of the turbulent-boundary-layer region. If this is true, the greatest possible recovery factor according to equation (1) could not exceed the cube root of the Prandtl number ($N \rightarrow \infty$).

The experimental turbulent-boundary-layer recovery factor some distance after transition is independent of Mach number and Reynolds number within the limits of experimental accuracy for the range of test conditions covered in the present investigation. The experimental values lie between 0.879 and 0.885, and these recovery factors are in agreement with those reported in references 1, 2, and 3.

40° Cone Cylinder

In the tests of the 40° cone cylinder at the maximum Mach number (3.77), it was found that the surface temperature did not reach equilibrium in the 4.5-minute test period. The values of the temperature ratio $T_e/T_o$ from each thermocouple for this test are shown in figure 7(a) plotted as functions of $e^{-t/60}$. The values of the temperature ratio along the body for infinite time ($e^{-t} = 0$) are shown in figure 7(b). (Symbols correspond to thermocouple locations tabulated in fig. 7(a).) The data for a free-stream Mach number of 3.10 are also shown in figure 7(b). It was not necessary to extrapolate to infinite time to obtain these latter data. The thermocouple at $x = 1.63$ inches was located on the 40° cone and the temperature obtained at this point is shown connected to the other points by a dashed line indicating an unknown variation because the change from a cone to a cylinder in this region is accompanied by both an abrupt change in local Mach number and the beginning of transition.

The Mach number distributions along the cone cylinder at Mach numbers of 3.10 and 3.77 are shown in figure 8. The local recovery factors calculated by use of equation (2) are plotted as a function of local Reynolds number in figure 9. (For clarity, the data for a Mach number of 3.10 are shown with flagged symbols.) It can be observed that the values of the local recovery factor measured on the conical nose are, within the limits of experimental accuracy, the same as those measured near the tip of the 10° cone despite the differences in cone angle and model material. The value of the turbulent-boundary-layer recovery factor at a Mach number of 3.10 is 0.890. At a Mach number of 3.77 the recovery factor is slightly greater. No local increase in the recovery factor at the completion of transition is apparent in the data for the cone cylinder. It is possible that this small characteristic local increase could have been eliminated by longitudinal heat conduction.
in the rather thick brass shell or masked by the effect of change in body shape in the transition zone. It is also possible that any local increase could have occurred between the rather widely spaced thermo-couples and therefore remained undetected.

It should be noted in figure 9 that the decrease in length Reynolds number in going from the cone to the cylinder results from the fact that the magnitude of the local Reynolds number per foot on the cylindrical portion is considerably less than on the conical nose.

The recovery-factor data from reference 5 obtained at Mach numbers of 2.87 and 4.25 are shown in figure 9 for comparison with the present data. All the data from reference 5 were obtained on the cylindrical portion of the body. Major differences are apparent between these data and those of the present investigation. Although the reasons for the differences between the general levels of the recovery-factor data are not known, it should be noted that two differences between the respective test conditions are the Reynolds number per foot (1 x 10^5 in the tests of reference 5 and 7 x 10^5 in the present tests) and the transition Reynolds number of the wind tunnels. This suggests that wind-tunnel turbulence level or approach toward free molecule flow may be affecting the results.9

The results of the present tests of the 40° cone cylinder indicate that the turbulent-boundary-layer recovery factor for this configuration is substantially the same as that for the 10° cone. The greatest value obtained with the cone cylinder (0.896 at M = 3.77) is about 2 percent greater than the minimum value obtained with the 10° cone (0.879 at M = 1.97). Thus, for presumably completely developed turbulent boundary layers on both bodies, the recovery factors are in agreement within the limits of experimental accuracy (±1 percent). A value of 0.890 is probably adequately representative of the experimental results for Reynolds numbers per foot (Reₙₘ) greater than about 2 x 10^5.

9Recent exploratory tests with the 10° cone in the Ames 10- by 14-inch hypersonic wind tunnel (Mₓ = 3 to 7, continuous operation at Pₒ = 75 psig maximum) have indicated that the turbulent-boundary-layer recovery factors at length Reynolds numbers equal to those in the 1- by 3-foot wind tunnel No. 2 were almost identical to those of the present investigation. However, at Reynolds numbers less than 2 x 10^5 per foot the recovery factor exceeds those reported herein and the value appears to increase with decreasing Reynolds numbers per foot. Values as high as 0.92 have been measured at a Reynolds number of 1.5 x 10^5 per foot and a Mach number of 4.
CONCLUDING REMARKS

Turbulent-boundary-layer recovery factors have been measured on two bodies of revolution, a 10° cone and a 40° cone cylinder. The results for the 10° cone have been compared with the theory of Tucker and Maslen and the effect of Mach number predicted by this theory was not detected. However, the velocity profile was not measured in the present experiments and such measurements are necessary before the validity of the theory can be determined. The data obtained with the 40° cone cylinder were compared with data obtained by Eber for a model of identical shape and markedly different results were obtained. The difference is believed to be caused by possible differences in the conditions which caused transition. For Reynolds numbers \((R_e)\) greater than about \(2 \times 10^5\) per foot, and for Mach numbers up to 3.77, a value of \(0.885 \pm 0.011\) is adequately representative of the results obtained with both bodies in the present investigation. Similar results have been found by previous investigators at lower Mach numbers.

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REFERENCES


(a) 10° cone.

All dimensions in inches.

(b) 40° cone-cylinder.

Figure 1. Model details.
Figure 2.—Longitudinal distributions of surface-to-total temperature ratio obtained on the 10° cone in the No. 1 and No. 2 wind tunnels, $M_{\infty} = 1.97$.

Figure 3.—Effect of test duration in the No. 2 wind tunnel on the surface-to-total temperature ratio for the 10° cone at $M_{\infty} = 3.77$. 
Figure 4.—Variation of local Mach number along the length of the $10^\circ$ cone.

Figure 5.—Variation of the local recovery factor with local Reynolds number for the $10^\circ$ cone.
Figure 6 - Variation of local recovery factor with local Mach number.
Effect of duration of test on temperature ratio.

Effect of Mach number on temperature ratio.

Figure 7—Variation of the local surface-to-total temperature ratio along the 40° cone cylinder.
Figure 8.—Variation of local Mach number along the 40° cone cylinder.

Figure 9.—Variation of the local recovery factor with local Reynolds number for the 40° cone cylinder.