NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 3176

WALL INTERFERENCE IN WIND TUNNELS WITH SLOTTED AND POROUS BOUNDARIES AT SUBSONIC SPEEDS

By Barrett S. Baldwin, Jr., John B. Turner,
and Earl D. Knechtel

Ames Aeronautical Laboratory
Moffett Field, Calif.

WASHINGTON
May 1954

V

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 3176

WALL INTERFERENCE IN WIND TUNNELS WITH SLOTTED AND POROUS BOUNDARIES AT SUBSONIC SPEEDS

By Barrett S. Baldwin, Jr., John B. Turner, and Earl D. Knechtel

SUMMARY

Linearized compressible-flow analysis is applied to the study of wind-tunnel-wall interference for subsonic flow in either two-dimensional or circular test sections having slotted or porous walls. Expressions are developed for evaluating blockage and lift interference.

INTRODUCTION

In solid-wall wind tunnels the effects of blockage severely limit model sizes that can be tested at high subsonic speeds; in fact, the model must become vanishingly small as sonic speed is approached. It has been demonstrated that if the walls are ventilated (e.g., slotted or porous) then blockage is reduced and much larger models can be tested. However, wall-interference effects, although reduced, still exist and must be evaluated in order to correct the wind-tunnel data to free-air conditions.

It is the objective of the present investigation to analyze two of the principal wall-interference effects, blockage and lift interference, for two- and three-dimensional subsonic flows in ventilated test sections, where blockage refers to the mean incremental velocity induced in the vicinity of the model by wall interference and lift interference is the mean upwash so induced. In the three-dimensional case it is convenient to perform the analysis for a circular test section. The results obtained for the circular test section may be applied to a square test section of equal cross-sectional area since the wall interference at the center of the tunnel should be relatively insensitive to such a change in the shape of the wall.

SYMBOLS

A	factor in Fourier integral transform of φ^*
a	slot width of slotted wall (see fig. 1)
b	wing span of model wing
c	constant factor in nonlinear term of boundary equation
G	Fourier integral transform with respect to x of φ
g	dummy variable of Fourier transform
h	half tunnel height
I_0	modified Bessel function of the first kind and order zero
I_1	modified Bessel function of the first kind and order one
K_0	modified Bessel function of the second kind and order zero
K_1	modified Bessel function of the second kind and order one
K	slot constant, $-\frac{l}{\pi} \ln\left[\sin\left(\frac{\pi a}{2l}\right)\right]$
l	slot separation of slotted wall (see fig. 1)
L	lift on the model
M	free-stream Mach number
m_e	parameter proportional to size of two-dimensional model, $\frac{\tau_e U}{\beta}$
m_r	parameter proportional to size of three-dimensional model, $\tau_r U$
n	coordinate in the direction of the outward normal to the wall
q	dummy variable of integration
R	porosity parameter
r,θ,x	cylindrical coordinates
U	free-stream velocity

u,v,w	perturbation velocity components in the x, y, z directions, respectively
u^*,v^*,w^*	additional velocity components due to the presence of the walls
$\tilde{u},\tilde{v},\tilde{w}$	additional velocity components having rapid spacewise variation near the walls
$\Delta u,\Delta v,\Delta w$	additional velocity components at the position of the model due to the walls
W	complex velocity in the y, z plane
X	complex variable equal to $z + iy$ (physical plane)
x,y,z	Cartesian coordinates
α	dummy constant in limiting process
β	$\sqrt{1 - M^2}$
Γ	circulation
Φ	complex velocity potential
φ	total perturbation velocity potential, $\varphi_1 + \varphi^*$
φ_1	approximate perturbation potential due to model in free air
φ^*	additional perturbation potential due to tunnel walls
$\delta\varphi^*$	additional wall-interference potential arising from non-linear term in boundary equation
ρ	free-stream density
τ_e	cross-sectional area of two-dimensional model
τ_r	volume of three-dimensional model
ξ	function of X, equal to $\zeta + i\eta$ (transformed plane)
ζ,η	Cartesian coordinates in transformed plane

ANALYSIS

General Statement of the Problem

The effect of the tunnel walls on the flow around a model, in the case of ventilated walls, can be calculated using the same basic method as that used in reference 1 for the closed-wall case. As in reference 1 the analysis is based on the linearized equation of subsonic compressible flow

$$\beta^2 \frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} + \frac{\partial^2 \varphi}{\partial z^2} = 0 \tag{1}$$

where φ is the perturbation velocity potential of the flow in the tunnel.

Let $\varphi = \varphi_1 + \varphi^*$, where φ_1 is the potential of the flow about the model in free air and φ^* is the potential of the additional flow due to the presence of the walls.

If φ_1 is taken to be a known solution of equation (1) which approximates the true free-air potential at points far from the model, φ^* can be calculated from the fact that the sum $\varphi_1 + \varphi^*$ satisfies a known boundary condition at the wall. Since the values of φ_1 at the wall only are used, any inaccuracy in the value of φ_1 near the model should not affect the calculation of φ^* appreciably.

The primary objective in this procedure is to estimate the change in stream conditions caused by the walls at the position of the model. It is assumed that the velocity components derived from φ^* are constants near the model which can be subtracted from the stream velocity to obtain the equivalent free air stream velocity. Thus,

$$\Delta u = \frac{\partial \varphi^*}{\partial x} \qquad \text{at} \quad x=y=z=0$$

is the blockage correction, and

$$\Delta w = \frac{\partial \varphi^*}{\partial z} \qquad \text{at} \quad x=y=z=0$$

is the upwash correction in the three-dimensional case.

Boundary Conditions

In this section a single expression approximately representing the boundary conditions of solid, porous, and slotted walls and an open jet will be developed.

Let x be the coordinate in the direction of the free stream and n the coordinate in a direction perpendicular to the x direction. Consider a wall which is perpendicular to the n direction (i.e., parallel to the free stream). If the wall is solid, the condition of no flow through the wall can be expressed as

$$\frac{\partial \varphi}{\partial n} = 0 \qquad \text{at the wall}$$

In the case of an open jet there is no pressure drop across the jet boundary so that there is zero perturbation pressure at the boundary. With a disturbance in the stream this boundary does not remain parallel to the free stream. However, for convenience, the condition of zero perturbation pressure is imposed at a surface parallel to the free stream and coinciding with the jet boundary far upstream of the disturbance (see ref. 2). Also, for convenience, this surface can be called an open wall and the boundary condition can be expressed as

$$\frac{\partial \varphi}{\partial x} = 0 \qquad \text{at the wall}$$

In reference 3 an average boundary condition for a porous wall is derived. The average velocity normal to the wall is assumed to be proportional to the pressure drop through the wall, a linearized approximation to viscous flow through a porous medium, and the pressure outside the wall is assumed equal to the free-stream pressure. This leads to the boundary equation

$$\frac{\partial \varphi}{\partial x} + \frac{1}{R}\frac{\partial \varphi}{\partial n} = 0 \qquad \text{at the wall} \tag{2}$$

The quantity R is a porosity parameter defined by

$$\Delta p = \frac{\rho U}{R}\frac{\partial \varphi}{\partial n} \tag{3}$$

where

Δp pressure drop through the wall

ρ stream density

U stream velocity

The quantity $\rho U/R$ can be determined experimentally by measuring the mass flow and pressure drop through a sample of the wall under conditions corresponding to zero stream velocity.

Porous walls to which equations (2) and (3) are applicable will henceforth be referred to in this report as ideal porous walls.

An approximate boundary equation for a slotted wall is derived in Appendix A. The pressure at the slots is assumed constant and equal to the free-stream pressure. The resulting uniform boundary condition is

$$\frac{\partial\varphi}{\partial x} + K\frac{\partial^2\varphi}{\partial x\partial n} = 0 \qquad \text{at the wall} \tag{4}$$

where K is related to the slot geometry by

$$K = -\frac{l}{\pi}\,ln\left[\sin\left(\frac{\pi a}{2l}\right)\right] \tag{5}$$

Slotted walls to which equation (4) is applicable will henceforth be referred to in this report as ideal slotted walls.

Solutions for wall interference based on equations (2) and (4) can be obtained in one calculation by combining them in the form

$$\frac{\partial\varphi}{\partial x} + K\frac{\partial^2\varphi}{\partial x\partial n} + \frac{1}{R}\frac{\partial\varphi}{\partial n} = 0 \qquad \text{everywhere at the wall} \tag{6}$$

Thus, wall-interference solutions based on equation (6) contain, as special cases, those of the closed wall ($K \rightarrow \infty$ or $1/R \rightarrow \infty$), ideal porous wall ($K = 0$), ideal slotted wall ($1/R = 0$), and open jet ($K = 0$ and $1/R = 0$). Furthermore, equation (6) can be assumed to describe a slotted wall having mixed potential and viscous flows in the slots. In that case the porosity parameter R can be determined experimentally, as it is in the case of a porous wall, by measuring the mass flow for a given pressure drop through a sample of the wall.

If it is found that a nonlinear relationship between pressure drop and mass flow exists, it may be necessary to add a term of the form $f(\partial\varphi/\partial n)$ to equation (6). This case is discussed in Appendix B.

In addition to the foregoing interpretations of equation (6), an interpretation identifying it with slotted walls with tapered slots (l and a functions of x) and potential flow in the slots is possible. This case is discussed in Appendix C where it is found that instead of representing viscous effects, the parameter R is related to the taper by

$$\frac{1}{R} = \left.\frac{dK}{dx}\right|_{x=0} \tag{7}$$

where $K(x)$ is the same as in equation (5).

Blockage in a Two-Dimensional-Flow Tunnel

Under the assumption of infinitesimal model size, the blockage correction will be calculated using equation (6) as the boundary equation.

Let x be the coordinate in the free-stream direction and y the coordinate in the direction perpendicular to the walls. Let h be the half tunnel height so that the walls are at $y = -h$ and at $y = +h$. In these coordinates equation (1) becomes

$$\beta^2 \frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} = 0 \tag{8a}$$

and equation (6) becomes

$$\left(\frac{\partial \varphi}{\partial x} \pm K \frac{\partial^2 \varphi}{\partial x \partial y} \pm \frac{1}{R} \frac{\partial \varphi}{\partial y} \right)_{y = \pm h} = 0 \tag{8b}$$

The $\pm$ signs on the second and third terms are required because at the upper wall, $n = +y$ and at the lower wall, $n = -y$, n being the coordinate in the direction of the outward normal to the wall.

If φ is replaced by $\varphi_1 + \varphi^*$, equations (8a) and (8b) yield

$$\beta^2 \frac{\partial^2 \varphi^*}{\partial x^2} + \frac{\partial^2 \varphi^*}{\partial y^2} = 0 \tag{9a}$$

$$\left(\frac{\partial \varphi^*}{\partial x} \pm K \frac{\partial^2 \varphi^*}{\partial x \partial y} \pm \frac{1}{R} \frac{\partial \varphi^*}{\partial y} \right)_{y = \pm h} = -\left(\frac{\partial \varphi_1}{\partial x} \pm K \frac{\partial^2 \varphi_1}{\partial x \partial y} \pm \frac{1}{R} \frac{\partial \varphi_1}{\partial y} \right)_{y = \pm h} \tag{9b}$$

These two equations are sufficient to determine φ^* when φ_1, the disturbance due to the model in free air, is known.

As in reference 1, the disturbance due to the model at zero angle of attack in free air is approximated by a two-dimensional doublet which can be expressed as

$$\varphi_1 = \frac{m_e}{2\pi} \left(\frac{x}{x^2 + \beta^2 y^2} \right) \tag{10}$$

The reasoning behind the choice of the doublet is as follows: The source-sink distribution representing a nonlifting model contains the same total sink strength as total source strength, so that the distant flow field would not resemble that of a single source or sink. The center of gravity of the source distribution would lie forward on the model compared to the center of the sink distribution which would be aft. Hence, there would be a dipole or doublet effect of the model at a distance. Other higher-order effects would be much less than that of the doublet at large distances. The constant m_e is related to the size of the model with sufficient accuracy for present purposes by

$$m_e = \frac{\tau_e U}{\beta} \tag{11}$$

where τ_e is the area of the model in the xy plane.

It is convenient to solve equations (9) by the Fourier transform method. The Fourier transform of φ_1 is defined by the relations

$$G_1(g,y) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \varphi_1(x,y)e^{igx}\, dx \tag{12a}$$

$$\varphi_1(x,y) = \int_{-\infty}^{\infty} G_1(g,y)e^{-igx}\, dg \tag{12b}$$

Analogous expressions defining the Fourier transforms of φ^* are

$$G^*(g,y) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \varphi^*(x,y)e^{igx}\, dx \tag{13a}$$

$$\varphi^*(x,y) = \int_{-\infty}^{\infty} G^*(g,y)e^{-igx}\, dg \tag{13b}$$

With the substitution of equation (13b) into (9a) and with an interchange in the order of differentiation and integration there results

$$\int_{-\infty}^{\infty} \beta^2 \frac{\partial^2}{\partial x^2} G^*(g,y)e^{-igx}\, dg + \int_{-\infty}^{\infty} \frac{\partial^2}{\partial y^2} G^*(g,y)e^{-igx}\, dg = 0$$

Performing the indicated x differentiation and collecting the two terms under the common integral yields

$$\int_{-\infty}^{\infty} \left(-\beta^2 g^2 G^* + \frac{\partial^2 G^*}{\partial y^2}\right)e^{-igx}\, dg = 0$$

This integral will be zero if

$$-\beta^2 g^2 G^* + \frac{\partial^2 G^*}{\partial y^2} = 0 \tag{14a}$$

Substituting equations (12b) and (13b) into equation (9b) and proceeding as before yields

$$\left(-ig\, G^* \mp ig\, K \frac{\partial G^*}{\partial y} \pm \frac{1}{R} \frac{\partial G^*}{\partial y}\right)_{y = \pm h} = -\left(-ig\, G_1 \mp ig\, K \frac{\partial G_1}{\partial y} \pm \frac{1}{R} \frac{\partial G_1}{\partial y}\right)_{y = \pm h} \tag{14b}$$

Equations (14a) and (14b), involving the transforms G_1 and G^* of φ_1 and φ^*, can now be solved in the place of solving equations (9a) and (9b). The general solution of equation (14a) is

$$G^* = A(g) \cosh (\beta g y) + B(g) \sinh (\beta g y)$$

Substituting the second term of this expression into equation (13b) yields a term of φ^* having an odd y dependence. From symmetry considerations it can be seen that φ^* should have even y dependence. Therefore, $B(g)$ must be zero and, hence,

$$G^* = A(g) \cosh (\beta g y) \tag{15}$$

The unknown factor $A(g)$ will be determined by substituting equation (15) into equation (14b), but G_1 must be known for this purpose.

In order to find G_1, equation (10) is substituted into equation (12a) and there results

$$G_1 (g,y) = \frac{m_e}{2\pi} \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{x}{x^2 + \beta^2 y^2} e^{igx}\, dx \tag{16}$$

From reference 4, it is found that[1]

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{1}{x^2 + \beta^2 y^2} e^{igx}\, dx = \frac{1}{2\beta |y|} e^{-|g| \beta |y|}$$

---

[1]In reference 4, $G(g)$ is defined as $G(g) = \int_{-\infty}^{\infty} F(f) e^{i2\pi fg}\, df$. In the tables of $F(f)$ versus $G(g)$, F is usually given as a function of p where $p = i2\pi f$. If $P(p)$ is the function of p given in the table, it follows that $F(f) = P(p) = P(i2\pi f)$. With the substitution, $x = 2\pi f$, it is found that $G(g) = \frac{1}{2\pi} \int_{-\infty}^{\infty} P(ix) e^{ixg}\, dx$. Thus, $G(g)$ in the tables is the Fourier transform, as defined in equations (12) of the function of x which results from replacing p by ix in $P(p)$.

where the symbol $|\ |$ denotes absolute value. Differentiating both sides of this with respect to g yields

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{ix}{x^2 + \beta^2 y^2} e^{igx}\, dx = -\frac{1}{2}\frac{g}{|g|} e^{-|g|\beta|y|}$$

so that

$$G_1 = \frac{m_e}{4\pi} i \frac{g}{|g|} e^{-|g|\beta|y|} \tag{17}$$

Substituting equations (15) and (17) into (14b) yields

$$A(g)\left[-ig\cosh(\beta gy) \mp igK\beta g\sinh(\beta gy) \pm \frac{1}{R}\beta g\sinh(\beta gy)\right]_{y=\pm h} =$$

$$-\frac{m_e}{4\pi} i \frac{g}{|g|}\left[-ig\, e^{-|g|\beta|y|} \mp igK\left(-|g|\beta\frac{y}{|y|}\right)e^{-|g|\beta|y|} \pm\right.$$

$$\left.\frac{1}{R}\left(-|g|\beta\frac{y}{|y|}\right)e^{-|g|\beta|y|}\right]_{y=\pm h}$$

Making the indicated substitutions $(y = \pm h)$ and solving for $A(g)$ yields the single complex equation

$$A(g) = -\frac{m_e}{4\pi}\frac{\left(i\frac{g}{|g|} - igK\beta + \frac{\beta}{R}\right)e^{-|g|\beta h}}{\left[\cosh(\beta gh) + K\beta g\sinh(\beta gh) + i\frac{\beta}{R}\sinh(\beta gh)\right]} \tag{18}$$

Substituting equations (15) and (18) into equation (13b) yields

$$\varphi^* = -\frac{m_e}{4\pi}\int_{-\infty}^{\infty}\frac{\left(i\frac{g}{|g|} - igK\beta + \frac{\beta}{R}\right)e^{-|g|\beta h}\cosh(\beta gy)\, e^{-igx}}{\left[\cosh(\beta gh) + K\beta g\sinh(\beta gh) + i\frac{\beta}{R}\sinh(\beta gh)\right]}\, dg$$

Upon separation into real and imaginary parts and making use of the fact that an integral from $-\infty$ to $+\infty$ is twice the value of the integral from 0 to ∞ if the integrand is an even function and zero if it is an odd function, this becomes

$$\varphi^* = -\frac{m_e}{2\pi}\left[\frac{\beta}{R}\int_0^\infty \frac{\cosh(\beta gy)\cos(gx)}{\left[\cosh(\beta gh) + K\beta g\sinh(\beta gh)\right]^2 + \left[\frac{\beta}{R}\sinh(\beta gh)\right]^2}\,dg\right. +$$

$$\frac{1}{2}\int_0^\infty \left\{\frac{\left[1 - (K\beta g)^2 - \left(\frac{\beta}{R}\right)^2\right] + \left[\left(1 - K\beta g\right)^2 + \left(\frac{\beta}{R}\right)^2\right]e^{-2\beta gh}}{\left[\cosh(\beta gh) + K\beta g\sinh(\beta gh)\right]^2 + \left[\frac{\beta}{R}\sinh(\beta gh)\right]^2}\right\}$$

$$\left.\cosh(\beta gy)\sin(gx)\,dg\right]$$

With the substitution, $q = \beta gh$, this becomes

$$\varphi^* = -\frac{m_e}{2\pi\beta h}\left[\frac{\beta}{R}\int_0^\infty \frac{\cosh\left(\frac{qy}{h}\right)\cos\left(\frac{qx}{\beta h}\right)}{\left[\cosh(q) + \frac{K}{h}q\sinh q\right]^2 + \left[\frac{\beta}{R}\sinh(q)\right]^2}\,dq\right. +$$

$$\frac{1}{2}\int_0^\infty \left\{\frac{\left[1 - \left(\frac{K}{h}q\right)^2 - \left(\frac{\beta}{R}\right)^2\right] + \left[\left(1 - \frac{K}{h}q\right)^2 + \left(\frac{\beta}{R}\right)^2\right]e^{-2q}}{\left[\cosh(q) + \frac{K}{h}q\sinh(q)\right]^2 + \left[\frac{\beta}{R}\sinh(q)\right]^2}\right\}$$

$$\left.\cosh\left(\frac{qy}{h}\right)\sin\left(\frac{qx}{\beta h}\right)dq\right] \tag{19}$$

The integrals indicated in equation (19) have not been found in closed form. However, if the derivatives of φ^* are desired at a small number of points for a small number of values of K/h and β/R, the derivatives can be taken in the integrand and the resulting integrals evaluated numerically or graphically.

The quantity of primary interest is Δu, the value of $\partial\varphi^*/\partial x$ evaluated at $x = y = 0$. Performing the indicated differentiation on equation (19) and setting $x = y = 0$, the expression

$$\Delta u = -\frac{m_e}{4\pi\beta^2h^2}\int_0^\infty \left\{\frac{\left[1-\left(\frac{K}{h}q\right)^2-\left(\frac{\beta}{R}\right)^2\right]+\left[\left(1-\frac{K}{h}q\right)^2+\left(\frac{\beta}{R}\right)^2\right]e^{-2q}}{\left[\cosh(q)+\frac{K}{h}q\sinh(q)\right]^2+\left[\frac{\beta}{R}\sinh q\right]^2}\right\}q\,dq \tag{20}$$

is obtained for the additional stream velocity at the position of the model due to the walls.

Solid wall.- Letting $K \to \infty$ or $1/R \to \infty$ in equation (20) yields

$$\Delta u\Big|_{\frac{K}{h}\to\infty} = \frac{m_e}{2\pi\beta^2h^2}\int_0^\infty \frac{e^{-q}q}{\sinh(q)}\,dq = \frac{\pi}{24}\frac{m_e}{\beta^2h^2} \tag{21}$$

Ideal porous wall.- At $K = 0$ equation (20) becomes

$$\Delta u\Big|_{\frac{K}{h}=0} = -\frac{m_e}{2\pi\beta^2h^2}\int_0^\infty \frac{\left[\cosh(q)-\left(\frac{\beta}{R}\right)^2\sinh q\right]}{\left[\cosh(q)\right]^2+\left[\left(\frac{\beta}{R}\right)\sinh(q)\right]^2}\,e^{-q}q\,dq \tag{22}$$

Ideal slotted wall.- With $1/R = 0$ we have

$$\Delta u\Big|_{\frac{\beta}{R}=0} = -\frac{m_e}{\pi\beta^2h^2}\int_0^\infty \frac{\left(1-\frac{K}{h}q\right)e^{-2q}q\,dq}{\left(1+\frac{K}{h}q\right)+\left(1-\frac{K}{h}q\right)e^{-2q}} \tag{23}$$

Open jet.- For both $K = 0$ and $1/R = 0$

$$\Delta u\Big|_{\substack{\frac{K}{h}=0\\ \frac{\beta}{R}=0}} = -\frac{m_e}{\pi\beta^2h^2}\int_0^\infty \frac{e^{-2q}q\,dq}{1+e^{-2q}} = -\frac{\pi}{48}\frac{m_e}{\beta^2h^2} \tag{24}$$

In the general case (eq. 20), the value of Δu lies between the values for the solid wall and the open jet, and Δu can be made arbitrarily small by an appropriate choice of K/h and β/R. Figure 2 is a plot of the values of K/h and β/R at which $\Delta u = 0$. These values were computed numerically near the ends of the curve and interpolated in the middle. Equations (21), (22), and (24) are in agreement with the results of reference 3. Figure 3 is a plot of equation (23), showing the variation of blockage factor with slot parameter for the two-dimensional-flow, ideal, slotted tunnel.

Since the effect on the blockage correction of letting β approach zero is the same as letting $1/R$ approach zero, it can be concluded that as the stream Mach number approaches unity, the blockage correction factor for an ideal porous wall approaches that of an open jet. Similarly any effect of viscosity or taper of a slotted wall described by equation (6) would be suppressed at near sonic speed so that the blockage correction factor would approach that of an ideal slotted wall.

Blockage in a Circular Tunnel

Again, the blockage due to the wall interference of a very small model will be calculated using equation (6) as the boundary equation.

Let x be the coordinate in the free-stream direction and r the cylindrical coordinate perpendicular to the x direction. Let r_o be the tunnel radius so that the wall is at $r = r_o$. Using these coordinates, equation (1) becomes, in the case of rotational invariance,

$$\beta^2 \frac{\partial^2 \varphi}{\partial x^2} + \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \varphi}{\partial r}\right) = 0 \tag{25}$$

and equation (6) becomes

$$\left(\frac{\partial \varphi}{\partial x} + K \frac{\partial^2 \varphi}{\partial x \partial r} + \frac{1}{R} \frac{\partial \varphi}{\partial r}\right)_{r = r_o} = 0 \tag{26}$$

Then φ^* must satisfy the equations

$$\beta^2 \frac{\partial^2 \varphi^*}{\partial x^2} + \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \varphi^*}{\partial r}\right) = 0 \tag{27a}$$

$$\left(\frac{\partial \varphi^*}{\partial x} + K \frac{\partial^2 \varphi^*}{\partial x \partial r} + \frac{1}{R} \frac{\partial \varphi^*}{\partial r}\right)_{r = r_o} = -\left(\frac{\partial \varphi_1}{\partial x} + K \frac{\partial^2 \varphi_1}{\partial x \partial r} + \frac{1}{R} \frac{\partial \varphi_1}{\partial r}\right)_{r = r_o} \tag{27b}$$

(See fig. 1.)

Again, from reference 1 the free-air solution is approximated by

$$\varphi_1 = \frac{m_r}{4\pi} \frac{x}{(x^2 + \beta^2 r^2)^{3/2}} \tag{28}$$

The constant m_r is related to the size of the model by

$$m_r = U \tau_r \tag{29}$$

where τ_r is the volume of the model. Correction factors which take into account the shape of the model and the presence of shock waves are discussed in reference 5.

Substituting, as before, the Fourier integral expressions for φ^* and φ_1 into equations (27) yields

$$-\beta^2 g^2 G^* + \frac{1}{r} \frac{\partial}{\partial r}\left(r \frac{\partial G^*}{\partial r}\right) = 0 \tag{30a}$$

$$\left(-ig\ G^* - K\ ig \frac{\partial G^*}{\partial r} + \frac{1}{R} \frac{\partial G^*}{\partial r}\right)_{r = r_o} = -\left(-ig\ G_1 - K\ ig \frac{\partial G_1}{\partial r} + \frac{1}{R} \frac{\partial G_1}{\partial r}\right)_{r = r_o} \tag{30b}$$

The general solution of the first of equations (30) is

$$G^* = A(g)\ I_o\ (\beta g r) + B(g)\ K_o\ (\beta g r)$$

where I_o and K_o are the modified Bessel functions of zero order of the first and second kinds, respectively (see ref. 6). Since $K_o\ (\beta g r)$ has a singularity at $r = 0$ which would lead to a singularity in φ^* at $r = 0$, $B(g)$ must be zero and, hence,

$$G^* = A(g)\ I_o\ (\beta g r) \tag{31}$$

Substituting equation (28) into the Fourier integral expression of φ_1 yields

$$G_1 = \frac{m_r}{4\pi} \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{x}{(x^2 + \beta^2 r^2)^{3/2}} e^{igx}\ dx \tag{32}$$

From reference 4 it is found that

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{1}{\sqrt{x^2 + \beta^2 r^2}} e^{igx}\ dx = \frac{1}{\pi} K_o\ (\beta r |g|)$$

According to equations (12), the inverse relation

$$\int_{-\infty}^{\infty} \frac{1}{\pi} K_o\ (\beta r|g|)\ e^{-igx}\ dg = \frac{1}{\sqrt{x^2 + \beta^2 r^2}}$$

follows. Taking the derivative with respect to x of both sides of this equation yields

$$\int_{-\infty}^{\infty} (-ig)\ \frac{1}{\pi} K_o\ (\beta r|g|) e^{-igx}\ dg = -\frac{x}{(x^2 + \beta^2 r^2)^{3/2}}$$

From this, according to equations (12),

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{x}{(x^2 + \beta^2 r^2)^{3/2}}\ e^{igx}\ dx = ig\ \frac{1}{\pi} K_o\ (\beta r|g|)$$

so that

$$G_1 = \frac{m_r}{4\pi^2}\ ig\ K_o\ (\beta r|g|) \tag{33}$$

Substituting equations (31) and (33) into equation (30b) yields

$$A(g)\left[-ig\ I_o\ (\beta g r_o)\ -ig\ K\beta g\ I_1\ (\beta g r_o) + \frac{1}{R}\ \beta g\ I_1\ (\beta g r_o)\right] =$$

$$-\frac{m_r}{4\pi^2}\ ig\left[-ig\ K_o\ (\beta r_o|g|)\ -ig\ K(-\beta|g|)\ K_1\ (\beta r_o|g|)\ +\right.$$

$$\left.\frac{1}{R}\ (-\beta|g|)\ K_1\ (\beta r_o|g|)\right]$$

the derivatives of I_o and K_o being found in reference 6. Upon solving for $A(g)$, substituting the result into equation (13), and substituting q for $\beta r_o g$ there results

$$\varphi^* =$$

$$-\frac{m_r}{4\pi\beta^2 r_o^2}\left\{\frac{\beta}{R}\,\frac{2}{\pi}\int_0^\infty \frac{\left[K_1(q)I_o(q) + K_o(q)I_1(q)\right] I_o\left(\frac{qr}{r_o}\right)\cos\left(\frac{qx}{\beta r_o}\right) q\,dq}{\left[I_o(q) + \frac{K}{r_o}\,qI_1(q)\right]^2 + \left[\frac{\beta}{R}\,I_1(q)\right]^2} + \right.$$

$$\left.\frac{2}{\pi}\int_0^\infty \frac{\left[-q\frac{K}{r_o}\,K_1(q)I_o(q) + K_o(q)I_o(q) + q\,\frac{K}{r_o}\,K_o(q)I_1(q) - \left(q^2\frac{K^2}{r_o^2} + \frac{\beta^2}{R^2}\right)K_1(q)I_1(q)\right] I_o\left(\frac{qr}{r_o}\right)\sin\left(\frac{qx}{\beta r_o}\right) q\,dq}{\left[I_o(q) + q\,\frac{K}{r_o}\,I_1(q)\right]^2 + \left[\frac{\beta}{R}\,I_1(q)\right]^2}\right\} \tag{34}$$

Differentiating with respect to x and setting $x = r = 0$ yields

$$\Delta u = -\frac{m_r}{2\pi^2\beta^3 r_o^3}\int_0^\infty \frac{\left\{K_o(q)I_o(q) + q\,\frac{K}{r_o}\left[-K_1(q)I_o(q) + K_o(q)I_1(q)\right] - \left(q^2\,\frac{K^2}{r_o^2} + \frac{\beta^2}{R^2}\right)K_1(q)I_1(q)\right\} q^2 dq}{\left[I_o(q) + q\,\frac{K}{r_o}\,I_1(q)\right]^2 + \left[\frac{\beta}{R}\,I_1\,(q)\right]^2} \tag{35}$$

<u>Closed wall</u>.- Setting $1/R \rightarrow \infty$ in equation (35) yields

$$\Delta u\,\bigg|_{\frac{\beta}{R}\rightarrow\infty} = \frac{m_r}{4\pi\beta^3 r_o^3}\,\frac{2}{\pi}\int_0^\infty \frac{K_1(q)}{I_1(q)}\,q^2 dq = \frac{2.4m_r}{2\pi^2 r_o^3\beta^3} \tag{36}$$

<u>Ideal porous wall</u>.- At $K = 0$ equation (35) becomes

$$\Delta u \Bigg|_{\frac{K}{r_o}=0} = -\frac{m_r}{4\pi\beta^3 r_o^3}\,\frac{2}{\pi}\int_0^\infty \frac{\left[K_o(q)I_o(q) - \frac{\beta^2}{R^2}K_1(q)I_1(q)\right]q^2 dq}{\left[I_o(q)\right]^2 + \left[\frac{\beta}{R}I_1(q)\right]^2} \tag{37}$$

Ideal slotted wall.- With $(1/R) = 0$ we have

$$\Delta u \Bigg|_{\frac{\beta}{R}=0} = -\frac{m_r}{4\pi\beta^3 r_o^3}\,\frac{2}{\pi}\int_0^\infty \frac{\left[K_o(q) - q\frac{K}{r_o}K_1(q)\right]q^2 dq}{\left[I_o(q) + q\frac{K}{r_o}I_1(q)\right]} \tag{38}$$

Open jet.- For both $(1/R) = 0$ and $K = 0$

$$\Delta u \Bigg|_{\substack{\frac{\beta}{R}=0 \\ \frac{K}{r_o}=0}} = -\frac{m_r}{4\pi\beta^3 r_o^3}\,\frac{2}{\pi}\int_0^\infty \frac{K_o(q)}{I_o(q)}q^2 dq = -\frac{0.63\,m_r}{2\pi^2 r_o^3\beta^3} \tag{39}$$

The values of Δu in equation (35) lie between the values for the closed wall and the open jet. Figure 4 is a plot of values of K/r_o versus β/R at which $\Delta u = 0$; the shape of the curve was calculated near the ends and interpolated in the middle. A graph of equation (38) appears in figure 5 showing the variation of blockage factor with slot parameter for the cylindrical, ideal slotted tunnel.

Again, letting $\beta \rightarrow 0$ has the same effect on the blockage correction as letting $(1/R) \rightarrow 0$, so that near sonic speed the ideal porous wall should act like an open jet and the slotted wall should act like an ideal slotted wall.

Lift Interference in a Circular Tunnel

The upwash correction will be calculated using the infinitesimal model size approximation and the approximate boundary condition of equation (6).

Let x be the coordinate in the free-stream direction, z the coordinate in the direction of lift on the model, and y the remaining

Cartesian coordinate. The cylindrical coordinates r and θ are related to the Cartesian coordinates by $r = \sqrt{y^2 + z^2}$ and $z = r \sin \theta$. It can be seen from reference 1 that the appropriate free air solution is

$$\varphi_1 = \frac{\Gamma b}{4\pi} \left(1 + \frac{x}{\sqrt{x^2 + \beta^2 r^2}}\right) \frac{\sin \theta}{r} \tag{40}$$

which is the potential of a horseshoe vortex having infinitesimal span. The fact that the actual span is finite introduces higher-order terms which are negligible at distances large compared to the size of the model. Here Γb is related to the lift on the model by

$$L = \rho U \Gamma b \tag{41}$$

The Fourier transform with respect to x of φ_1 cannot be found. An arbitrary parameter α will be introduced into the potential so that the Fourier transform of a related function can be found. Let this function be

$$\varphi_1' = \frac{\Gamma b}{4\pi} \left(e^{-\alpha\sqrt{x^2 + \beta^2 r^2}} - \frac{1}{\alpha} \frac{\partial}{\partial x} e^{-\alpha\sqrt{x^2 + \beta^2 r^2}}\right) \frac{\sin \theta}{r} \tag{42}$$

so that

$$\lim_{\alpha \to 0} \varphi_1' = \varphi_1$$

Then α will be eliminated from the resulting $\varphi^{*\prime}$ by taking the limit with α at zero.

From reference 4 it is found that the Fourier transform of

$e^{-\alpha\sqrt{x^2 + \beta^2 r^2}}$ is $\dfrac{\alpha\beta r\, K_1\left(\beta r\sqrt{g^2 + \alpha^2}\right)}{\pi\sqrt{g^2 + \alpha^2}}$. By the use of an inversion,

differentiation, and reinversion, as in the derivation of equation (33),

it is found that the transform of $\dfrac{\partial}{\partial x} e^{-\alpha\sqrt{x^2 + \beta^2 r^2}}$ is

$$- \frac{ig\alpha\beta r K_1(\beta r\sqrt{g^2+\alpha^2})}{\pi\sqrt{g^2+\alpha^2}}$$ so that the transform of φ_1' is

$$G_1' = \frac{\Gamma b\beta}{4\pi^2}(\alpha+ig)\frac{K_1(\beta r\sqrt{g^2+\alpha^2})\sin\theta}{\sqrt{g^2+\alpha^2}} \tag{43}$$

From physical reasoning, it can be seen that the $\sin\theta$ factor of φ_1 will result in the same angle dependence for φ^*, so that $\sin\theta$ will be a factor common to φ_1', $\varphi^{*'}$ and their Fourier transforms G_1' and $G^{*'}$. With cylindrical coordinates, equation (1) is

$$\beta^2\frac{\partial^2\varphi}{\partial x^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\varphi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2\varphi}{\partial\theta^2} = 0$$

In the case of $\sin\theta$ dependence, this becomes

$$\beta^2\frac{\partial^2\varphi}{\partial x^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\varphi}{\partial r}\right) - \frac{1}{r^2}\varphi = 0$$

and the boundary condition remains the same as in equation (26). Substituting the Fourier integral expressions for φ_1' and $\varphi^{*'}$ in these equations, as before, yields

$$-\beta^2g^2G^{*'} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial G^{*'}}{\partial r}\right) - \frac{1}{r^2}G^{*'} = 0 \tag{44a}$$

$$\left(-igG^{*'} - Kig\frac{\partial G^{*'}}{\partial r} + \frac{1}{R}\frac{\partial G^{*'}}{\partial r}\right)_{r=r_o} = -\left(-igG_1' - Kig\frac{\partial G_1'}{\partial r} + \frac{1}{R}\frac{\partial G_1'}{\partial r}\right)_{r=r_o} \tag{44b}$$

The solution of equation (44a) which has no singularity at $r = 0$ is

$$G^{*'} = A(g)\, I_1(\beta rg)\sin\theta \tag{45}$$

Substituting this and equation (43) into equation (44b) yields

$$A(g)\left\{-ig\,I_1(\beta r_o g) + ig\,K\,\frac{I_1(\beta r_o g)}{r_o} - ig\,K\,\beta g I_o(\beta r_o g) - \frac{1}{R}\,\frac{I_1(\beta r_o g)}{r_o} + \frac{\beta g}{R}\,I_o(\beta r_o g)\right\}\sin\theta =$$

$$-\frac{\Gamma b\beta}{4\pi^2}(\alpha+ig)\left\{-ig\,\frac{K_1(\beta r_o\sqrt{g^2+\alpha^2})}{\sqrt{g^2+\alpha^2}} + ig\,K\,\frac{K_1(\beta r_o\sqrt{g^2+\alpha^2})}{r_o\sqrt{g^2+\alpha^2}} + ig\,K\beta K_o(\beta r_o\sqrt{g^2+\alpha^2}) - \frac{1}{R}\,\frac{K_1(\beta r_o\sqrt{g^2+\alpha^2})}{r_o\sqrt{g^2+\alpha^2}} -\right.$$

$$\left.\frac{\beta}{R}\,K_o(\beta r_o\sqrt{g^2+\alpha^2})\right\}\sin\theta$$

Solving for A, substituting in the integral expression for $\varphi^{*\prime}$, and taking the limit of $\varphi^{*\prime}$ as $\alpha \longrightarrow 0$ yields

$$\varphi^* = \frac{\Gamma b}{4\pi r_o}\left[\frac{r}{r_o}\sin\theta - \frac{\beta}{R}\,\frac{2}{\pi}\int_0^\infty \frac{\left[K_1(q)I_o(q) + K_o(q)I_1(q)\right] q^2 I_1\left(\frac{qr}{r_o}\right)\sin\theta\cos\left(\frac{qx}{\beta r_o}\right)dq}{q^2\left[\left(1-\frac{K}{r_o}\right)I_1(q) + q\left(\frac{K}{r_o}\right)I_o(q)\right]^2 + \left(\frac{\beta}{R}\right)^2\left[I_1(q) - qI_o(q)\right]^2} - \frac{2}{\pi}\right.$$

$$\int_0^\infty\left\{\frac{q^2\left[\left(1-\frac{K}{r_o}\right)K_1(q) - q\,\frac{K}{r_o}\,K_o(q)\right]\left[\left(1-\frac{K}{r_o}\right)I_1(q) + q\,\frac{K}{r_o}\,I_o(q)\right] + \left(\frac{\beta}{R}\right)^2\left[K_1(q)+qK_o(q)\right]\left[I_1(q)-qI_o(q)\right]}{q^2\left[\left(1-\frac{K}{r_o}\right)I_1(q) + q\,\frac{K}{r_o}\,I_o(q)\right]^2 + \left(\frac{\beta}{R}\right)^2\left[I_1(q) - q\,I_o(q)\right]^2}\right\}$$

$$\left.I_1\left(\frac{qr}{r_o}\right)\sin\theta\sin\left(\frac{qx}{\beta r_o}\right)dq\right] \qquad (46)$$

Replacing $I_1\ (qr/r_0)$ by its power series expansion, differentiating with respect to z, which is equal to $r \sin\theta$, and setting $r = 0$ yields

$$\left.\frac{\partial\varphi^*}{\partial z}\right|_{r=0} = \frac{\Gamma_b}{4\pi r_0^2}\left[1-\frac{\beta}{R}\frac{1}{\pi}\int_0^\infty \frac{\left[K_1(q)\,I_0(q)+K_0(q)\,I_1(q)\right]q^3\cos\left(\frac{qx}{\beta r_0}\right)dq}{q^2\left[\left(1-\frac{K}{r_0}\right)I_1(q)+q\frac{K}{r_0}I_0(q)\right]^2+\left(\frac{\beta}{R}\right)^2\left[I_1(q)-qI_0(q)\right]^2} - \frac{1}{\pi}\right.$$

$$\int_0^\infty\left\{\frac{q^2\left[\left(1-\frac{K}{r_0}\right)K_1(q)-q\frac{K}{r_0}K_0(q)\right]\left[\left(1-\frac{K}{r_0}\right)I_1(q)+q\frac{K}{r_0}I_0(q)\right]+\left(\frac{\beta}{R}\right)^2\left[K_1(q)+qK_0(q)\right]\left[I_1(q)-q\,I_0(q)\right]}{q^2\left[\left(1-\frac{K}{r_0}\right)I_1(q)+q\frac{K}{r_0}I_0(q)\right]^2+\left(\frac{\beta}{R}\right)^2\left[I_1(q)-q\,I_0(q)\right]^2}\right\}$$

$$\left.q\sin\left(\frac{qx}{\beta r_0}\right)dq\right] \tag{47}$$

for the upwash at $r = 0$.

When $x = 0$ we have

$$\Delta w = \frac{\Gamma_b}{4\pi r_0^2}\left\{1-\frac{\beta}{R}\frac{1}{\pi}\int_0^\infty\frac{\left[K_1(q)\,I_0(q)+K_0(q)\,I_1(q)\right]q^3dq}{q^2\left[\left(1-\frac{K}{r_0}\right)I_1(q)+q\frac{K}{r_0}I_0(q)\right]^2+\left(\frac{\beta}{R}\right)^2\left[I_1(q)-q\,I_0(q)\right]^2}\right\} \tag{48}$$

for the value of upwash at the position of the model due to the walls.

<u>Solid wall</u>.- Letting $1/R \rightarrow \infty$ yields

$$\left.\Delta w\right|_{\frac{\beta}{R}\to\infty} = \frac{\Gamma_b}{4\pi r_0^2}$$

Ideal porous wall.- At $K = 0$ equation (48) becomes

$$\Delta w\Big|_{\frac{K}{r_o}=0} = -\frac{\Gamma b}{4\pi r_o^2}\left\{1 - \frac{\beta}{R}\frac{1}{\pi}\int_0^\infty \frac{[K_1(q)I_o(q) + K_o(q)I_1(q)]\, q^3 dq}{q^2 I_1^2(q) + \left(\frac{\beta}{R}\right)^2 [I_1(q) - qI_o(q)]^2}\right\} \tag{49}$$

Ideal slotted wall.- When $1/R$ is set equal to zero in equation (48), a limiting process is required to obtain the correct result at $(\beta/R) = 0$. The result of this process is

$$\Delta w\Big|_{\frac{\beta}{R}=0} = \frac{\Gamma b}{4\pi r_o^2}\left(\frac{\frac{K}{r_o} - 1}{\frac{K}{r_o} + 1}\right) \tag{50}$$

Open jet.- Letting $K = 0$ in equation (50) yields

$$\Delta w\Big|_{\substack{\frac{\beta}{r}=0 \\ \frac{K}{r_o}=0}} = -\frac{\Gamma b}{4\pi r_o^2}$$

The value of Δw in the general case of equation (48) lies between the values for the closed wall and the open jet.

CONCLUDING REMARKS

A method of evaluating wall interference of partly open walls involving mixed potential and viscous flows has been presented. Expressions for blockage and lift interference for both slotted and porous walls have been derived. Some new details of the method may prove useful in other theoretical treatments of this type of problem.

The results of the analysis indicate that near sonic speed, the blockage correction for an ideal porous wall approaches that of an open jet. Similarly, any linear viscous or taper effect of a slotted wall is suppressed near sonic speed, so that the blockage correction approaches that of an ideal slotted wall.

Ames Aeronautical Laboratory
National Advisory Committee for Aeronautics
Moffett Field, Calif., May 29, 1953

APPENDIX A

DERIVATION OF THE BOUNDARY EQUATION

In this appendix an approximate smoothed or average boundary equation for a slotted wall will be derived.

An ideal slotted wall has zero perturbation pressure at the slots and zero normal flow at the strips. These conditions can be expressed as

$$\left.\begin{aligned} \frac{\partial\varphi}{\partial x} &= 0 \qquad \text{at the slots} \\ \frac{\partial\varphi}{\partial n} &= 0 \qquad \text{at the strips} \end{aligned}\right\} \tag{A1}$$

When the slot spacing and model dimensions are small compared to tunnel dimensions, the perturbation flow can be separated into a rapidly varying and a relatively uniform part so that the two parts can be investigated separately. It will be shown that the effect of the rapidly varying part can be replaced by a condition on the relatively uniform part.

Let $\tilde{\varphi}$, $\tilde{u}$, $\tilde{v}$, and $\tilde{w}$ represent the rapidly varying part of the flow field and φ, u, v, and w the remaining part of the perturbation flow. For a plane wall at $z = h$ equations (A1) require that

$$\left.\begin{aligned} \tilde{u} + u &= 0 \qquad \text{at the slots} \\ \tilde{w} + w &= 0 \qquad \text{at the strips} \end{aligned}\right\} \tag{A2}$$

In addition

$$\tilde{u} = \tilde{v} = \tilde{w} = 0 \qquad \text{far from the wall} \tag{A3}$$

In order to solve for $\tilde{\varphi}$, use can be made of the fact that u, v, and w are nearly constant at the wall compared to $\tilde{u}$, $\tilde{v}$, and $\tilde{w}$, so that u and w can be considered constant in equations (A2).

Since the slots lie along the x direction, $\tilde{u}$ is nearly constant in the x direction, so that $\partial\tilde{u}/\partial x$ can be neglected compared to $\partial\tilde{v}/\partial y$ and $\partial\tilde{w}/\partial z$. As in slender airplane theory, this leads to a two-dimensional crossflow for which

$$\frac{\partial^2 \tilde{\varphi}}{\partial y^2} + \frac{\partial^2 \tilde{\varphi}}{\partial z^2} = 0 \tag{A4}$$

Then $\tilde{\varphi}$ can be of the form $Q(x)\ f(y,z)$ where $Q(x)$ is a slowly varying function of x, and $f(y,z)$ satisfies the two-dimensional Laplace equation. Then,

$$\tilde{u} = \frac{\partial \tilde{\varphi}}{\partial x} = \frac{\partial Q}{\partial x} f(y,z)$$

and since $\tilde{u}$ is equal to $-u$, a constant, at the slots, $f(y,z)$ must be constant at the slots or

$$\tilde{v} = \frac{\partial \tilde{\varphi}}{\partial y} = Q \frac{\partial f}{\partial y} = 0 \qquad \text{at the slots}$$

This equation can be used to replace the first of equations (A2), and altogether there results

$$\left.\begin{aligned} \tilde{v} &= 0 \qquad \text{at the slots} \\ \tilde{w} = -w &= \text{constant at the strips} \\ \tilde{\varphi} = \tilde{v} = \tilde{w} &= 0 \qquad \text{far from the wall} \end{aligned}\right\} \tag{A5}$$

assuming that $\partial Q/\partial x$ is not absolutely zero.

Equations (A4) and (A5) can be solved using the conformal transformation technique.

Let the wall at $z = h$ be slotted periodically from $y = -\infty$ to $y = +\infty$ with the center of a slot at $y = 0$. Let a be the slot width and l the slot separation (see fig. 1). It is sufficient to consider only one period of the periodic flow configuration so that solid boundaries can be placed at $y = \pm(l/2)$ and attention confined to the region between them.

Let $X = z + iy$ be the complex physical plane and $\xi = \zeta + i\eta$ the transformed plane. Let Φ be the complex velocity potential such that $W = \tilde{w} - i\tilde{v}$, the complex velocity, is equal to $d\Phi/dX$, and $\tilde{\varphi}$ = real part Φ. Then equation (A4) is satisfied if Φ is any analytic function of X. The boundary conditions (A5) are satisfied by finding the analytic function $\xi(X)$ which transforms the boundaries in the X plane into a configuration for which the flow field with the desired flow at the boundaries can be found.

In the X plane the region under consideration is from $z = -\infty$ to $z = h$ and from $y = -(l/2)$ to $y = +(l/2)$. The slot lies on the line $X = h + iy$ from $y = -(a/2)$ to $+(a/2)$. (See fig. 6.) The two half strips lie on the same line from $y = -(l/2)$ to $y = -(a/2)$ and from $y = (a/2)$ to $y = (l/2)$. The remainder of the solid boundary lies along the line $X = z - i(l/2)$ from $z = -\infty$ to $z = h$ and along the line $X = z + i(l/2)$ from $z = -\infty$ to $z = h$.

The transformation which will place the origin at the wall, and the domain under consideration in the right half-plane is

$$\xi_1 = h - X \tag{A6}$$

The effect of the transformation on the positions of the boundaries is obtained by substituting the equations representing the boundaries into equation (A6). Thus, the line $X = h + iy$ becomes $\xi_1 = \zeta_1 + i\eta_1 = -iy$ in the ξ_1 plane and the slot lies on this line from $\eta_1 = -(a/2)$ to $\eta_1 = +(a/2)$. The two half strips lie on the same line from $\eta_1 = -(l/2)$ to $-(a/2)$ and from $a/2$ to $l/2$. Similarly, the remainder of the solid boundary in the ξ_1 plane lies on the line $\xi_1 = h - z - i(l/2)$ from $\zeta_1 = 0$ to $\zeta_1 = \infty$ and on the line $\xi_1 = h - z + i(l/2)$ from $\zeta_1 = 0$ to $\zeta_1 = \infty$.

To satisfy the boundary condition at the strip, a term $w\xi_1 = w(h-X)$ is added to the potential in the transformed plane.

The remaining boundary-value problem is solved with the aid of two further transformations.

The transformation

$$\xi_2 = \sinh\left(\frac{\pi}{l}\,\xi_1\right) \tag{A7}$$

transforms the region under consideration in the ξ_1 plane to the entire right half of the ξ_2 plane, as can be seen by following the procedure outlined for the first transformation. It is found that in the ξ_2 plane, the slot lies along the imaginary axis from $-i \sin(\pi a/2l)$ to $+i \sin(\pi a/2l)$ and the solid boundary including the two half strips lies along the imaginary axis outside of $\pm i \sin(\pi a/2l)$.

The transformation

$$\xi = \xi_2 + \sqrt{\xi_2^{\,2} + \sin^2\left(\frac{\pi a}{2l}\right)} \tag{A8}$$

transforms the region under consideration to the positive real half of the ξ plane, excluding the circle of radius $\sin(\pi a/2l)$ centered at the origin. The slot lies along the half of this circle in the right half plane, and the complete solid boundary lies along the imaginary axis outside the circle. Thus, a source at the origin in the ξ plane will satisfy the condition of no flow through the solid boundary and no flow across the slot. Hence, the desired potential is

$$\Phi = A\,ln\,(\xi) + w\xi_1 \tag{A9}$$

Substituting equations (A6), (A7), and (A8) in (A9) yields

$$\Phi = A\,ln\left\{\sinh\left[\frac{\pi}{l}(h-X)\right] + \sqrt{\sinh^2\left[\frac{\pi}{l}(h-X)\right] + \sin^2\left(\frac{\pi a}{2l}\right)}\right\} + w(h-X)$$

The constant A is evaluated by the last of equations (A5), $\Phi\Big|_{X\to-\infty} = 0$ being the equivalent of that equation.

As $X \to -\infty$, the potential simplifies to

$$\begin{aligned}\Phi\Big|_{X\to-\infty} &\cong A\,ln\left\{2\sinh\left[\frac{\pi}{l}(h-X)\right]\right\} + w(h-X)\\ &\cong A\,ln\left[e^{+\frac{\pi}{l}(h-X)}\right] + w(h-X)\\ &\cong A\frac{\pi}{l}(h-X) + w(h-X)\end{aligned}$$

The neglected terms are of order smaller than $1/h-X$. Thus, Φ will be zero at infinity if $A = -(l/\pi)w$ so that

$$\Phi = -w\frac{l}{\pi}\,ln\left\{\sinh\left[\frac{\pi}{l}(h-X)\right] + \sqrt{\sinh^2\left[\frac{\pi}{l}(h-X)\right] + \sin^2\left(\frac{\pi a}{2l}\right)}\right\} + w(h-X) \tag{A10}$$

Then

$$\Phi\Big|_{y=0} = -w\frac{l}{\pi}\,ln\left\{\sinh\left[\frac{\pi}{l}(h-z)\right] + \sqrt{\sinh^2\left[\frac{\pi}{l}(h-z)\right] + \sin^2\left(\frac{\pi a}{2l}\right)}\right\} + w(h-z)$$

It was assumed that the velocity potential is constant over the slot. The value of this constant is

$$\Phi \Big|_{\substack{y=0\\z=h}} = -w \frac{l}{\pi} \ln\left[\sin\left(\frac{\pi a}{2l}\right)\right] = \tilde{\varphi} \qquad \text{at the slot}$$

and

$$\frac{\partial\tilde{\varphi}}{\partial x} = \frac{\partial w}{\partial x}\left\{-\frac{l}{\pi} \ln\left[\sin\left(\frac{\pi a}{2l}\right)\right]\right\} = \tilde{u} \qquad \text{at the slot}$$

Substituting this in the first of equations (A2) yields

$$u + \left\{-\frac{l}{\pi} \ln\left[\sin\left(\frac{\pi a}{2l}\right)\right]\right\} \frac{\partial w}{\partial x} = 0 \qquad \text{at the slot}$$

Since it was assumed that u and w do not vary appreciably from slot to strip, this equation applies everywhere at the wall and yields

$$\left.\begin{aligned} &\frac{\partial\varphi}{\partial x} + K \frac{\partial^2\varphi}{\partial x \partial n} = 0 \\ &K = -\frac{l}{\pi} \ln\left[\sin\left(\frac{\pi a}{2l}\right)\right] \end{aligned}\right\} \text{at the wall} \qquad \text{(A11)}$$

for a plane wall.

In considering a curved cylindrical wall, it appears that the above results are not altered appreciably if the radius of curvature of the wall is everywhere large compared to the slot spacing.[2] Hence, it can be assumed that equations (A11) are applicable to any slotted wall.

[2]For a circular cylindrical slotted wall, solution of the boundary value problem

$$\frac{\partial\tilde{\varphi}}{\partial\theta} = 0 \qquad \text{at slots}$$

$$\frac{\partial\tilde{\varphi}}{\partial r} = -w \qquad \text{at strips}$$

$$\tilde{\varphi} = 0 \qquad \text{at } r = 0$$

yields a value for K identical with that obtained from equation (A11).

In an attempt to take viscous effects in the slots into account, it can be assumed that as in the case of the "ideal" porous wall there is a pressure drop through the wall which is proportional to the normal velocity at the wall. In that case the first of equations (A2) is replaced by $(u + \tilde{u}) + (1/R)w = 0$ at the slots which leads to

$$\frac{\partial \varphi}{\partial x} + K \frac{\partial^2 \varphi}{\partial x \partial n} + \frac{1}{R} \frac{\partial \varphi}{\partial n} = 0 \quad \text{everywhere at the wall} \tag{A12}$$

where K remains the same as in equation (A11) and R is to be determined experimentally.

Equation (A10) can be used to calculate the neglected variations in flow quantities near the wall and also at the model if the variations are not negligible there.

DISCUSSION OF A CRITICAL ASSUMPTION IN THE DERIVATION OF THE BOUNDARY EQUATION

It is assumed that the perturbation pressure at the wall is proportional to $u + \tilde{u}$. This is a good approximation only if

$$(v + \tilde{v})^2 + (w + \tilde{w})^2 \ll 2\,(u + \tilde{u})\,U$$

in addition to the usual requirements for linearization. This additional requirement can be reasonably relaxed to

$$(w + \tilde{w})^2 \ll 2uU \quad \text{at the wall} \tag{A13}$$

Equation (A10) indicates singularities in $\tilde{w}$ at the edges of the slots. Experience with wing leading edges indicates that this discrepancy can be reasonably ignored. However, equation (A13) should at least be satisfied at the center of the slots. The effect of this condition on u and v will now be determined. Differentiating equation (A10) yields

$$W = \frac{d\Phi}{dX} = w \frac{\cosh\left[\frac{\pi}{l}(h-X)\right]}{\sqrt{\sinh^2\left[\frac{\pi}{l}(h-X)\right] + \sin^2\left(\frac{\pi a}{2l}\right)}} - w$$

Then

$$W\Big|_{z=h} = w\frac{\cos\left(\frac{\pi y}{l}\right)}{\sqrt{\sin^2\left(\frac{\pi a}{2l}\right) - \sin^2\left(\frac{\pi y}{l}\right)}} - w = \tilde{w}\Big|_{z=h}$$

and

$$(w + \tilde{w})\Big|_{\substack{z=h\\ y=0}} = \frac{w}{\sin\left(\frac{\pi a}{2l}\right)}$$

so that equation (A13) becomes

$$\frac{w^2}{\sin^2\left(\frac{\pi a}{2l}\right)} \ll 2Uu$$

or

$$w^2 \ll 2\sin^2\left(\frac{\pi a}{2l}\right)Uu \quad \text{at the wall} \tag{A14}$$

This result places a lower limit on the ratio of open to total area (a/l) for which the results of this analysis can be expected to apply to slotted sections.

APPENDIX B

BLOCKAGE IN A CIRCULAR TUNNEL WITH NONLINEAR VISCOUS EFFECT

If, in the experimental determination of the porosity parameter R, it is found that an additional term of the form $f\ (\partial\varphi/\partial n)$ is needed in the boundary equation, the result is

$$\frac{\partial\varphi}{\partial x} + K\frac{\partial^2\varphi}{\partial x\partial n} + \frac{1}{R}\frac{\partial\varphi}{\partial n} + f\left(\frac{\partial\varphi}{\partial n}\right) = 0 \quad \text{at the wall} \tag{B1}$$

or

$$\frac{\partial\varphi^*}{\partial x} + K\frac{\partial^2\varphi^*}{\partial x\partial n} + \frac{1}{R}\frac{\partial\varphi^*}{\partial n} = -\frac{\partial\varphi_1}{\partial x} - K\frac{\partial^2\varphi_1}{\partial x\partial n} - \frac{1}{R}\frac{\partial\varphi_1}{\partial n} - f\left(\frac{\partial\varphi_1}{\partial n} + \frac{\partial\varphi^*}{\partial n}\right) \tag{B2}$$

This type of equation cannot be solved exactly by any presently known method because of the nonlinearity. However, the equation becomes linear if the φ^* in the nonlinear term is neglected under the assumption that

$$f\left(\frac{\partial\varphi_1}{\partial n} + \frac{\partial\varphi^*}{\partial n}\right) - f\left(\frac{\partial\varphi_1}{\partial n}\right) << f\left(\frac{\partial\varphi_1}{\partial n}\right) \quad \text{at the wall} \tag{B3}$$

Hence, the equation

$$\frac{\partial\varphi^*}{\partial x} + K\frac{\partial^2\varphi^*}{\partial x\partial n} + \frac{1}{R}\frac{\partial\varphi^*}{\partial n} = -\frac{\partial\varphi_1}{\partial x} - K\frac{\partial^2\varphi_1}{\partial x\partial n} - \frac{1}{R}\frac{\partial\varphi_1}{\partial n} - f\left(\frac{\partial\varphi_1}{\partial n}\right) \quad \text{at the wall} \tag{B4}$$

will be discussed.

The transforms G^* and G_1 of φ^* and φ_1 remain the same as before, with the exception that the factor $A(g)$ must be evaluated from the transform of equation (B4). For this purpose, the transform of $f(\partial\varphi_1/\partial r)$ is needed and can be found by evaluating

$$G_f = \frac{1}{2\pi}\int_{-\infty}^{\infty} f\left(\frac{\partial\varphi_1}{\partial r}\right)\bigg|_{r=r_o} e^{igx}\,dx \tag{B5}$$

The transform of equation (B4) is found to be

$$A(g)\left[-igI_o\ (\beta gr_o) - igK\beta gI_1 + \frac{1}{R}\ \beta gI_1\right] =$$

$$-\frac{m_r}{4\pi^2}\ ig\ \left(-igK_o + ig\ \beta\left|g\right|KK_1 - \frac{1}{R}\ \beta\left|g\right|K_1\right) - G_f$$

Upon solving for $A(g)$ and substituting in the inverse transform expression for φ^* equation (34) is obtained plus the additional term

$$\delta\varphi^* = -\int_{-\infty}^{\infty}\left\{\frac{1}{\frac{1}{R}\ \beta gI_1(\beta r_o g) - ig\left[I_o(\beta r_o g) + K\beta gI_1(\beta r_o g)\right]}\right\} G_f I_o(\beta rg)\, e^{-igx} dg$$

With the substitution $q = \beta r_o g$ this becomes

$$\delta\varphi^* = -\int_{-\infty}^{\infty}\left\{\frac{1}{\frac{\beta}{R}\ I_1(q) - i\left[I_o(q) + \frac{K}{r_o}\ qI_1(q)\right]}\right\} G_f e^{-i\ \frac{qx}{\beta r_o}}\ I_o\left(\frac{qr}{r_o}\right)\ \frac{dq}{q} \qquad (B6)$$

As an example of the use of equation (B6) let

$$f\left(\frac{\partial\varphi}{\partial n}\right) = \frac{c}{U^2}\left(\frac{\partial\varphi}{\partial n}\right)^3 \qquad (B7)$$

This function is chosen rather than one proportional to $(\partial\varphi/\partial n)^2$ because the pressure drop should be an odd function of the normal velocity.

Hence,

$$G_f = -\frac{c}{U^2}\left(\frac{m_r}{4\pi}\right)^3 \left(3\beta^2 r_o\right)^3\ \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{x^3}{\left[x^2 + \beta^2 r_o^{\,2}\right]^{15/2}}\ e^{\,igx} dx$$

From reference 4 it is found that

$$\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{1}{(x^2+\beta^2 r_o{}^2)^{15/2}}\,e^{igx}\,dx=\frac{|g|^7\,K_7\,(\beta r_o|g|)}{\sqrt{\pi}\,\Gamma\left(\frac{15}{2}\right)(2\beta r_o)^7}$$

Differentiating three times with respect to g yields

$$\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{-ix^3}{(x^2+\beta^2 r_o{}^2)^{15/2}}\,e^{igx}\,dx=\frac{1}{\sqrt{\pi}\,\Gamma\left(\frac{15}{2}\right)(2\beta r_o)^7}\,\frac{\partial^3}{\partial g^3}\,|g|^7 K_7(\beta r_o|g|)$$

so that

$$G_f=-i\,\frac{c}{U^2}\left(\frac{m_r}{4\pi\beta^3 r_o{}^3}\right)^3\frac{3^3}{\sqrt{\pi}\,\Gamma\left(\frac{15}{2}\right)2^7}\,\beta^8 r_o{}^5\,\frac{\partial^3}{\partial g^3}\,|g|^7\,K_7\,(\beta r_o|g|)$$

or

$$G_f=-i\beta^3\,\frac{c}{U^2}\left(\frac{m_r}{4\pi\beta^3 r_o{}^3}\right)^3\frac{3^3}{2^7\sqrt{\pi}\,\Gamma\left(\frac{15}{2}\right)}\,\beta r_o\,\frac{\partial^3}{\partial q^3}\left[|q|^7\,K_7(|q|)\right] \tag{B8}$$

Substituting this in equation (B6) and separating real and imaginary parts yields

$$\delta\varphi^*=-\beta^3\,\frac{c}{U^2}\left(\frac{m_r}{4\pi\beta^3 r_o{}^3}\right)^3\frac{3^3}{2^6\sqrt{\pi}\,\Gamma\left(\frac{15}{2}\right)}$$

$$\left\{\int_0^{\infty}\frac{\left[I_o\,(q)+\frac{K}{r_o}\,qI_1(q)\right]\frac{\beta r_o}{q}\,\frac{d^3}{dq^3}\left[q^7K_7(q)\right]\cos\left(\frac{qx}{\beta r_o}\right)I_o\left(\frac{qr}{r_o}\right)dq}{\left[\frac{\beta}{R}\,I_1(q)\right]^2+\left[I_o(q)+\frac{K}{r_o}\,qI_1(q)\right]^2}-\right.$$

$$\left.\frac{\beta}{R}\int_0^{\infty}\frac{I_1(q)\,\frac{\beta r_o}{q}\,\frac{d^3}{dq^3}\left[q^7\,K_7\,(q)\right]\sin\left(\frac{qx}{\beta r_o}\right)I_o\left(\frac{qr}{r_o}\right)dq}{\left[\frac{\beta}{R}\,I_1\,(q)\right]^2+\left[I_o\,(q)+\frac{K}{r_o}\,qI_1\,(q)\right]^2}\right\} \tag{B9}$$

Upon differentiation with respect to x and setting $x=r=0$, there results

$$\delta\Delta u = \beta^3 \frac{c}{U^2} \left(\frac{m_r}{4\pi\beta^3 r_o^3}\right)^3 \frac{3^3}{2^6\sqrt{\pi}\,\Gamma\left(\frac{15}{2}\right)} \frac{\beta}{R} \int_0^\infty \frac{I_1(q) \frac{d^3}{dq^3}\left[q^7 K_7(q)\right] dq}{\left[\frac{\beta}{R} I_1(q)\right]^2 + \left[I_o(q) + \frac{K}{r_o} I_1(q)\right]^2}$$

From reference 6 it is found that

$$\frac{d^3}{dq^3}\left[q^7 K_7(q)\right] = (1152 + 168q^2 - 5q^4)q^2 K_1(q) + (576 + 12q^2 - q^4)q^3 K_o(q)$$

and

$$\frac{3^3}{2^6\sqrt{\pi}\,\Gamma\left(\frac{15}{2}\right)} = \frac{1}{5005}\,\frac{2}{\pi}$$

so that

$$\delta\Delta u = \left(\frac{m_r}{4\pi\beta^3 r_o^3}\right)^3 \beta^3 \frac{c}{U^2} \frac{\beta}{R} \frac{1}{5005} \frac{2}{\pi} \left\{ \int_0^\infty \frac{(1152 + 168q^2 - 5q^4)q^2 I_1(q) K_1(q)\, dq}{\left[\frac{\beta}{R} I_1(q)\right]^2 + \left[I_o(q) + \frac{K}{r_o} I_1(q)\right]^2} + \int_0^\infty \frac{(576 + 12q^2 - q^4)\, q^3 I_1(q) K_o(q)}{\left[\frac{\beta}{R} I_1(q)\right]^2 + \left[I_o(q) + \frac{K}{r_o} I_1\,(q)\right]^2} dq \right\} \tag{B10}$$

Setting K/r_o equal to zero in (B10) and recombining it with equation (37) yields

$$\Delta u = -\frac{m_r}{2\pi^2\beta^3 r_o^3}\left\{\int_0^\infty \frac{\left[I_o(q)K_o(q) - \frac{\beta^2}{R^2}K_1(q)I_1(q)\right]q^2\,dq}{\left[I_o(q)\right]^2 + \left[\frac{\beta}{R}I_1(q)\right]^2} - \right.$$

$$\left.\left(\frac{\tau_r}{4\pi r_o^3}\right)^2 \frac{c}{\beta^3}\frac{\beta}{R}\frac{1}{5005}\int_0^\infty \frac{[(1152 + 168q^2 - 5q^4)I_1(q)K_1(q) + (576 + 12q^2 - q^4)qI_1(q)K_o(q)]\,q^2\,dq}{\left[I_o(q)\right]^2 + \left[\frac{\beta}{R}I_1(q)\right]^2}\right\} \tag{B11}$$

as the blockage in a porous tunnel for which the boundary equation is

$$\frac{\partial\varphi}{\partial x} + \frac{1}{R}\frac{\partial\varphi}{\partial r} + \frac{c}{U^2}\left(\frac{\partial\varphi}{\partial r}\right)^3 = 0 \qquad \text{at the wall}$$

The linear-viscous-effect term is zero when equation (B11) is evaluated numerically at $\beta/R = 1.21$, and

$$\Delta u = \frac{m_r}{2\pi^2\beta^3 r_o^3}\left[\left(\frac{\tau_r}{4\pi r_o^3}\right)^2 \frac{c}{\beta^3}(0.173)\right]$$

compared with the value of

$$\Delta u = \frac{m_r}{2\pi^2\beta^3 r_o^3}\;[2.4]$$

for a solid wall.

Since the factor $\tau_r/4\pi r_0^3$ is very small, the nonlinear contribution to the blockage correction should be negligible at subsonic speeds where β is not small. But as contrasted to the linear viscous effect which becomes small as M approaches 1, this nonlinear viscous effect may become large.

APPENDIX C

DERIVATION OF THE BOUNDARY EQUATION

FOR A TAPERED SLOTTED WALL

In this appendix an approximate smoothed boundary equation for a slotted wall with tapered slots will be derived.

The development in this case is identical with that in Appendix A up to the point where it is found that

$$\tilde{\varphi} = w\left\{-\frac{l}{\pi}\ln\left[\sin\left(\frac{\pi a}{2l}\right)\right]\right\} \qquad \text{at the slots}$$

or

$$\tilde{\varphi} = w\,K \qquad \text{at the slot} \tag{C1}$$

When the slots are tapered, the slot parameter K is a slowly varying function of x which can be expanded in a power series about $x = 0$ so that neglecting the higher order terms

$$K(x) \cong K\Big|_{x=0} + \frac{dK}{dx}\Big|_{x=0} x$$

Then

$$\tilde{\varphi} = K\Big|_{x=0} w + \frac{dK}{dx}\Big|_{x=0} xw \qquad \text{at the slots}$$

and

$$\tilde{u} = \frac{\partial\tilde{\varphi}}{\partial x} = K\Big|_{x=0}\frac{\partial w}{\partial x} + \frac{dK}{dx}\Big|_{x=0} w + \frac{dK}{dx}\Big|_{x=0}\frac{\partial w}{\partial x} x \qquad \text{at the slot}$$

Assuming that the last term will be negligible compared to the others, we have

$$\tilde{u} = K\Big|_{x=0}\frac{\partial w}{\partial x} + \frac{dK}{dx}\Big|_{x=0} w \qquad \text{at the slots} \tag{C2}$$

Substituting this equation in equation (A2) of Appendix A yields

$$u + K\Big|_{x=0} \frac{\partial w}{\partial x} + \frac{dK}{dx}\Big|_{x=0} w = 0 \qquad \text{at the slots}$$

Then, as in Appendix A, since u and w do not vary appreciably from slot to strip

$$\left.\begin{aligned} &\frac{\partial \varphi}{\partial x} + K\Big|_{x=0} \frac{\partial^2 \varphi}{\partial x \partial n} + \frac{dK}{dx}\Big|_{x=0} \frac{\partial \varphi}{\partial n} = 0 \qquad \text{everywhere at the wall} \\ &K(x) = -\frac{l(x)}{\pi} \ln\left[\sin \frac{\pi a(x)}{2l(x)}\right] \end{aligned}\right\} \tag{C3}$$

Hence, the parameter $1/R$ in equation (6) of the text can be interpreted as the quantity $dK/dx\Big|_{x=0}$ which is related to the taper of the slots in a slotted wall having purely potential flow in the slots.

REFERENCES

1. Baranoff, A. v.: Tunnel Correction for Compressible Subsonic Flow. NACA TM 1162, 1947.

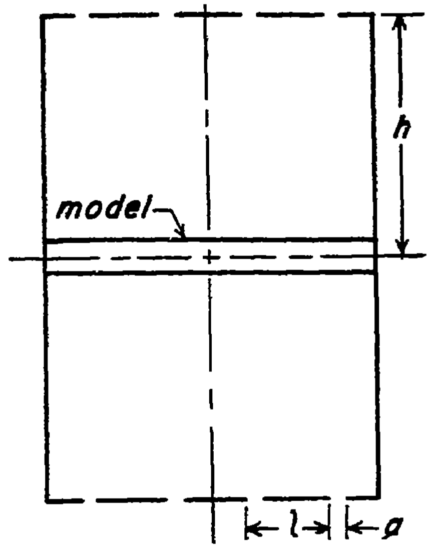
2. Glauert, H.: Wind Tunnel Interference on Wings, Bodies and Airscrews. R & M No. 1566, British A.R.C., 1933.

3. Goodman, Theodore R.: The Porous Wall Wind Tunnel. Part II - Interference Effect on a Cylindrical Body in a Two-Dimensional Tunnel at Subsonic Speed. Cornell Aeronautical Laboratory, Inc., Rep. No. AD-594-A-3, 1950.

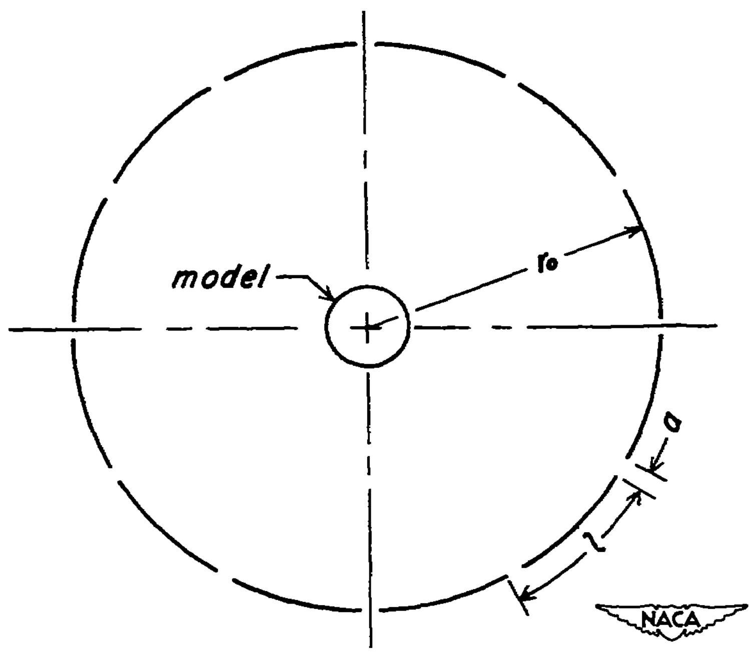
4. Campbell, George A., and Foster, Ronald M.: Fourier Integrals for Practical Applications. D. Van Nostrand Company, New York, 1948.

5. Göthert, B.: Wind-Tunnel Corrections at High Subsonic Speeds Particularly For an Enclosed Circular Tunnel. NACA TM 1300, 1952.

6. McLachlan, N.W.: Bessel Functions for Engineers, Oxford University Press, London, 1946.



(a) Two-dimensional-flow slotted test section.



(b) Circular slotted test section.

Figure 1.- Cross-sectional diagrams of the two slotted test sections in a plane perpendicular to the free stream.

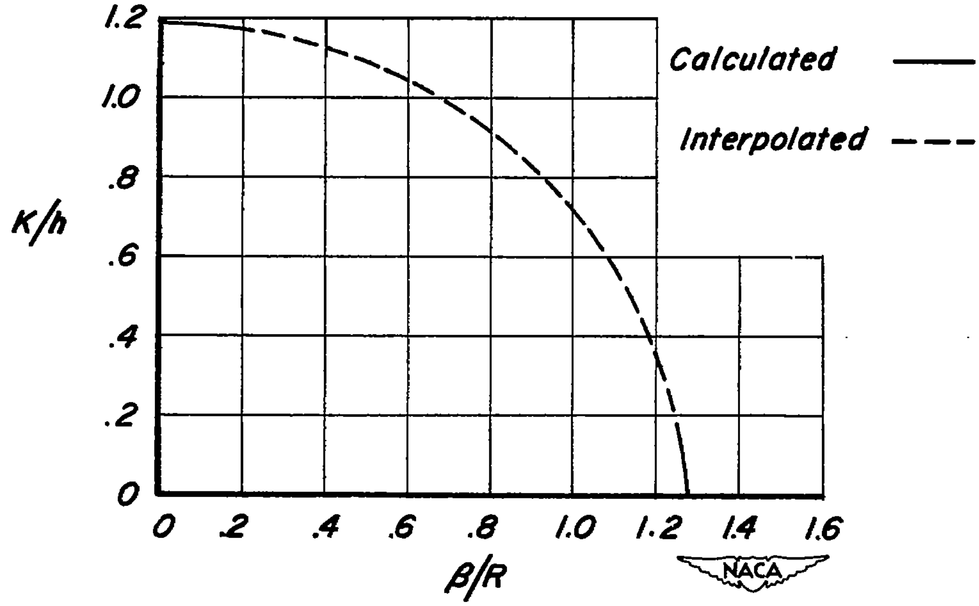


Figure 2.- Simultaneous values of slot parameter and porosity parameter for zero blockage in a two-dimensional-flow tunnel.

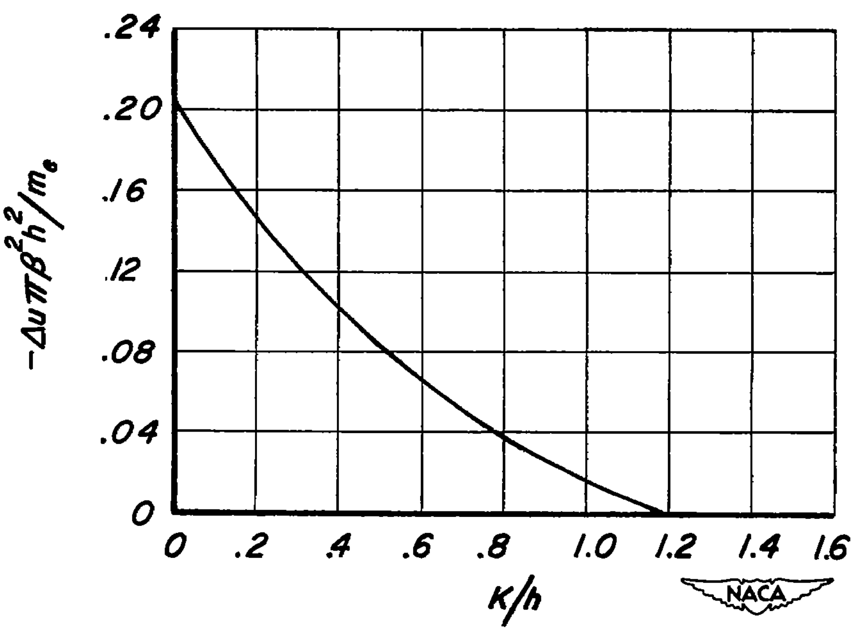


Figure 3.- Blockage factor as a function of slot parameter in a two-dimensional-flow, ideal, slotted tunnel.

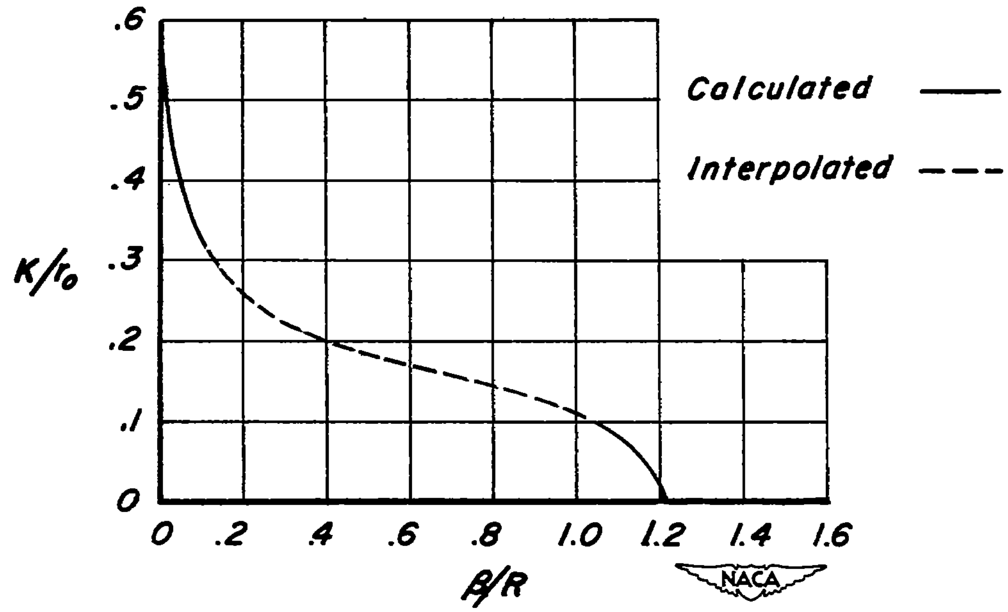


Figure 4.- Simultaneous values of slot parameter and porosity parameter for zero blockage in a circular cylindrical tunnel.

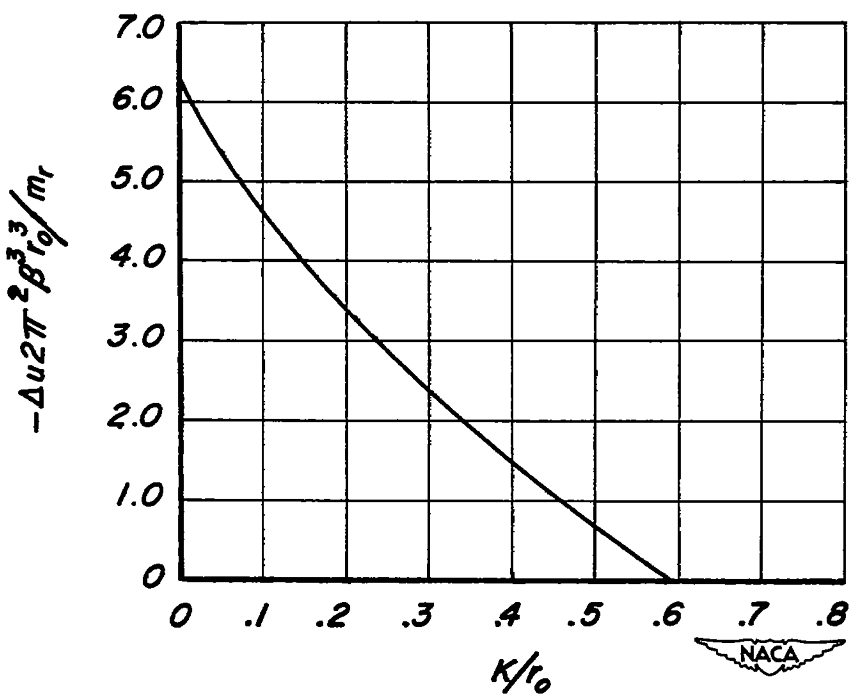
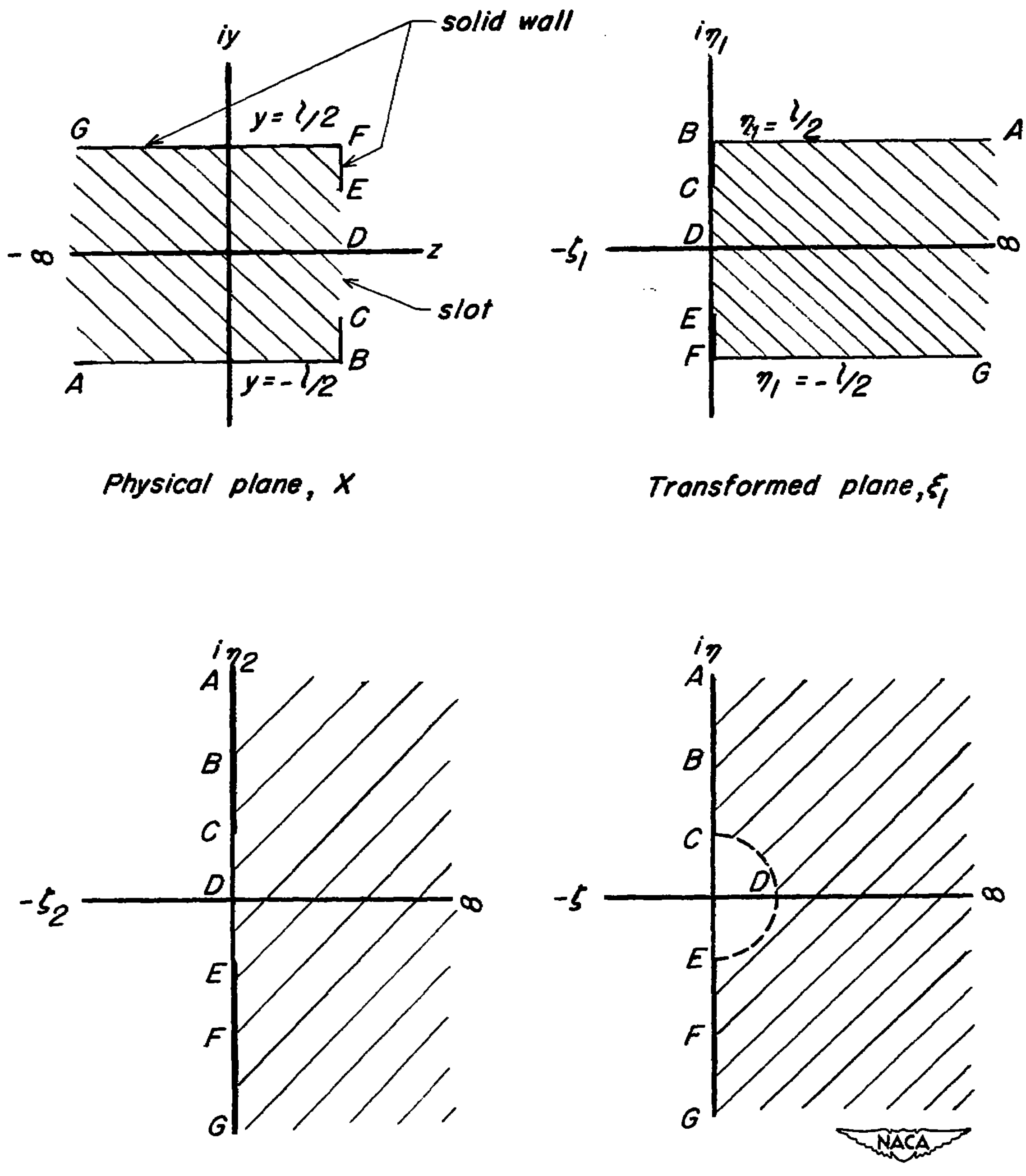


Figure 5.- Blockage factor as a function of slot parameter in a circular cylindrical, ideal, slotted tunnel.



Physical plane, X

Transformed plane, ξ_1

Transformed plane, ξ_2

Transformed plane, ξ

Figure 6.- Sketch of complex planes (Appendix A).

NACA-Langley - 5-26-54 - 1000