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THE NEW INTERPRETATION OF THE LAWS OF AIR RESISTANCE.

By L. Prandtl, Göttingen.

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Researches on the laws of air resistance were for a long time neglected by physics, and up to nearly the end of the nineteenth century the data found by experimenters, many of them untrained, varied 50% or more. It was only when (through the development of aeronautics) better means were available for such investigations, that laboratories for the measurement of air resistance were established in the principal countries, some of these laboratories being well equipped for such work. The result has proved that, for a correct insight into the laws of air resistance, such laboratories were necessary, since the results obtained in small laboratories are not sufficiently accurate to enable us to draw correct conclusions regarding the conditions governing the use of aircraft. As a rule, these experimental establishments work with an artificial air stream, produced by propellers. The Göttingen Laboratory, established by the Kaiser Wilhelm Gesellschaft, has an air stream with a section of four square meters and a velocity which can be adjusted as required from zero to 55 m/sec (180.45 ft/sec).

According to a theory evolved by Newton and still generally held, especially by engineers, air resistance is due to the fact that the moving body collides with the individual particles of air in obedience to the law of elastic or non-elastic impact. This theory gives a result which is, in certain characteristics, approximately correct. It gives us a resistance proportional to

* Translated from the German.

the exposed surface, to the square of the velocity and to the density of the resisting medium. As regards the dependence of the resistance on the shape of the body, however, this theory leads to conclusions which are not borne out by experience. The fundamental error of Newton's formula is, that no account whatever is taken of the further fate of the impinging molecules.

A closer examination shows that it is well to consider the air as an ordinary fluid, and, indeed, for most of the velocities considered, as a non-compressible fluid, so long as the dimensions of the moving body are large in comparison with the mean free path of the particles of air. Certain definite, not minute, forces are required to compress appreciably a volume of air. If these forces do not arise, owing to the yielding of the air in front of the body, there is no appreciable change in volume. Changes in volume only become appreciable when the relative velocity of the body in the air is comparable with the velocity of sound. Tests with projectiles have demonstrated, in agreement with theoretical considerations, that the laws of flow and consequently the laws of air resistance only change greatly when the speed of the projectile exceeds the velocity of sound. This case is not considered in the following remarks. We shall only consider the laws of air resistance for medium velocities at which the volume of the air remains almost constant.

Fluids of constant volume have two characteristics properties: density, as a measure of the inertia of mass, and viscosity as a measure of the tension resulting from deformation of the fluid. Density is the mass per unit volume. Viscosity is the force exerted per unit area when deformation of the fluid is caused by the shearing motion of two parallel surfaces at unit distance from each other and moving at unit velocity with reference to each other. The form of flow in any fluid depends solely on the ratio of viscosity to density. This ratio is therefore called "kinematic viscosity." This quantity, which we will designate by ν, has, as can easily be shown, the dimension length^2/time. This dimension agrees with the product of length × velocity. If we divide the product of length and velocity by the kinematic viscosity, we get a dimensionless quantity, or pure coefficient. According to the known connection between the dimensions and the mechanical similarity of comparable processes, we find, when only the so-called physical constants come into play, that, with two geometrically similar bodies moving in fluids in a geometrically similar way, the forms of flow will also be geometrically similar, if the coefficient just mentioned has the same value in both cases. This coefficient is called the "Reynolds number" after the name of the discoverer of this law of similarity. It is to be noted that the Reynolds number applies only to motion within a fluid. In motions on the surface of a fluid, as, for instance, the motion of a ship on water, the force of gravity comes in as a third determining physical factor and interferes with geometric similarity. To such cases apply the laws discovered by Froude, into which we cannot go further here. In motions in air or entirely under water, gravity has no effect, because it is offset by the force of buoyancy which every particle of the fluid receives from its neighboring particles.

Turning now to the question of air resistance, it is evident that we again have Newton's Law for all geometrically similar motions, even though the mechanism be quite different. This is shown by the fact that, in similar motions, the numerical ratio of the forces of mass inertia and viscosity is the same at every point in the fluid, so that we must obtain a formula differing by only one numerical factor from that for mass inertia alone.

If we now consider the cases in which the Reynolds number has different values for otherwise constant geometrical conditions, we shall not be surprised to find that the numerical factor of the law of resistance, which in Newton's law is a constant, always assuming different values. If we write for the law of resistance

$$D = c\, S\, \rho\, \frac{v^2}{2}$$

(in which S = area, ρ = density, and v = velocity), then the drag coefficient c is not a constant, as stated by Newton, but a function of the Reynolds number. It is probable, moreover, that c is constant over quite a long reach, especially for bodies having edges.

In order to elucidate the actually existing ratios, we will take a series of tests recently made at Göttingen on the resistance of cylinders exposed to an air stream perpendicular to their axes. The tests were begun on wires no larger than a hair and were extended to cylinders as large as a tree trunk. Since the kinematic viscosity of the air is about 0.14 cm²/sec, we get Reynolds numbers, $v\,d/\nu$, from 4.2 to 800000 (d = diameter). The drag coefficients found for these values are shown by curves in Fig. 1. The Reynolds number, $v\,d/\nu$, as also the drag coefficient c, is plotted in a logarithmic scale, the only way in which quantities of such various orders can be simultaneously represented on paper. The law of similarity is shown by the fact that, when plotted according to Reynolds numbers, the results of the whole series of tests fall naturally on a single curve. The accuracy with which they fit the curve is a measure of the accuracy of the tests, which, moreover, are verified by Relf's tests* (Fig. 1).

The course of the curve is surprisingly complicated and undergoes many remarkable changes. The Reynolds number gives essentially the ratio between the forces due to viscosity and the forces due to inertia. We might, therefore, assume, by analogy with experiments in similar physical phenomena, that, with sufficiently high Reynolds numbers (say 100 or 1000), the influence of viscosity would disappear. Were this the case, the curve would be horizontal from that point. In reality, the effect of viscosity is still apparent even with $v\,d/\nu = 1000000$, which is about as far as we can go in the Göttingen laboratory. Furthermore, the sum total of all observations, even on other bodies, admits of the conclusion that, with perfectly smooth surfaces, aside from the observed variations in the law, nothing further can be expected. For bodies with rough surfaces (and therefore to be regarded as totally dissimilar geometrically from bodies with smooth surfaces),

* Relf. Report 1913-14, Brit. Adv. Comm. for Aeron., p. 47.

there is a great increase in resistance, as shown by a test with a thin, rough, wooden strut, which was afterwards ground smooth and tested again (Fig. 2).

The strong influence of viscosity is due to the fact that it causes vortices to form in a thin film of an even slightly viscous fluid flowing about the object and that these vortices are propagated into the fluid, thereby considerably increasing the resistance.* The study of the processes in this film furnishes the key to the explanation of the phenomena.

Beginning with the smallest value of the Reynolds number, we find first a region in which the resistance is mainly due to the effects of viscosity and can be expressed by approximate mathematical formulas deduced from theory. In this region the Reynolds numbers are small compared to unity. The way the curve rises to the left gives a natural connection with this region. To the right the curve gradually assumes an almost horizontal course and then bends downward (at about $v\, d/\nu = 130$). This is due to the fact that the form of flow changes here. While below this value the form of flow remains for a time unaltered and the motion is therefore, "stationary," while above it the flow begins to oscillate, thus leading to the formation of more or less regular vortices. The connection of such a series of vortices with the resistance has been very clearly explained by von Karman.** He shows that only a vortex system consisting of vortices rotating alternately to the right and left can have a stable configuration

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* L. Prandtl. Verhandl. d. III internat. Math.-Kongr. zu Heidelberg, 1904. Leipzig, 1905, p.484, or "Flüssigkeitsbewegung" in Handwörterbuch der Naturw., 1913.
** Physikalische Zeitschrift, 1912, p.49.

and that this is borne out by observation. The further course of the curve up to $v\,d/\nu = 180000$ must be explained by the gradual modification of the vortex formation. The details have not yet been sufficiently investigated.

The curve now undergoes a transition, assuming a steep slope downward and finally leads to a drag coefficient which is only a third of the value at the beginning of the slope. This phenomenon has been verified by tests on spheres, made almost simultaneously by Eiffel in air* and Costanzi in water.** The theoretical explanation is furnished by work done at Göttingen.*** It has been shown that this sudden fall of the drag coefficient is due to the fact that the layer of the fluid on the surface of the body, in which viscosity works, passes from a parallel or smooth to a turbulent or eddying flow. From this point of view, we can comprehend the fact observed that, with intensified vorticity of the air stream, the critical velocity is lowered. As a crucial test for this conception, we may mention that, if a one-millimeter wire be placed on a sphere having a diameter of 28 cm (71.1 in) the position of the wire being slightly in front of the spot where parallel flow leaves the surface (thereby creating small local vortices), the condition of small resistance can be obtained even at much smaller velocities.

The reduction of the resistance depends on such a change in

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* Comptes Rendus 155, p. 1597 (1912).

** Rendiconti delle esper. aeronaut, del genio Vol. II, No. 4.

*** L. Prandtl. Nachrichten der Gesellsch. d. Wiss., Göttingen; Math. Phys. Klasse 1914, p. 177;
C. Wieselsberger. Zeitschr. f. Flugt. u. Motorl., 1914, p. 140.

the flow that the point of separation which lay about 85° from the most forward point, is shifted backwards 30° or more, thus considerably reducing the whole vortex formation. The flow (and with it, the distribution of pressure) thus very nearly approximates that calculated on the theory of the frictionless fluid, the characteristic of which is the fact that the stream combines behind the body just as it separated in front and that therefore no resistance is created. The distribution of pressure about a cylinder in the turbulent condition has been measured by G. I. Taylor,* who obtained a maximum negative pressure at ±82° amounting to 245% of the positive pressure at the leading "edge" while the theory of the ideal fluid gives 300% negative pressure at ±90°.

The turbulent state is of the highest technical importance, on account of its small resistance. Since resistance must be reduced as far as possible on aircraft, such forms should be chosen as will create the turbulent condition. This is facilitated by the fact that the critical Reynolds number lies much lower for streamline profiles than for cylinders. By utilizing suitable forms, the resistance can be reduced until it is no greater than the frictional resistance of a thin, smooth plate. For such bodies, the practical and theoretical conceptions of flow and pressure distribution are in almost perfect agreement. Practical people were formerly rather disdainful of the theory of an ideal fluid, but during the last ten years it has achieved very important results. Fig. 3 gives an example of this.** The plain lines

* Report 1915-16, British Advisory Committee for Aeronautics, p. 30.
** G. Fuhrmann. Jahrb. d. Motorluftschiff-Studienges, 1911-12, p. 63.

give the calculated pressure distribution for three airship models, while the tiny circles indicate the result of pressure tests in a wind tunnel. The agreement between the two is surprisingly good until the rear end is reached, where there is a systematic deviation owing to the frictional drag of the air.

Lack of space prevents our going into further details concerning the results which might be obtained with airplane wings by the help of the theory of the ideal fluid.* Lift could be shown to depend directly on the vortex system. That part of the resistance which is directly connected with the lift and which has its equivalent in the kinetic energy left in the air, could be calculated by purely theoretical formulas. The rest of the resistance arises under favorable conditions, only from friction, for which we have empirical laws. In principle, therefore, we are now in a position to calculate theoretically, for a wing of given section and shape, both lift and drag, as well as pressure distribution, with sufficient practical approximation. The only problem which we cannot yet solve mathematically is whether a given profile is really good, or whether it gives rise to detrimental vortices, since this depends directly on the behavior of the boundary layer retarded by viscosity.

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* L. Prandtl. Nachr. d. Ges. d. Wiss. zu Göttingen, Math-Phys. Kl. 1918, p.451, and 1919, p.107; also Jahrb. d. Wiss. Ges. f. Luftfahrt V (1920) p.37; A. Betz, Naturwissenschaften, 1918, p.557; Beiheft (supplement) 2 to Zeitschr. f. Flugtechn. u. Motorluftsch., p.1, (1920).

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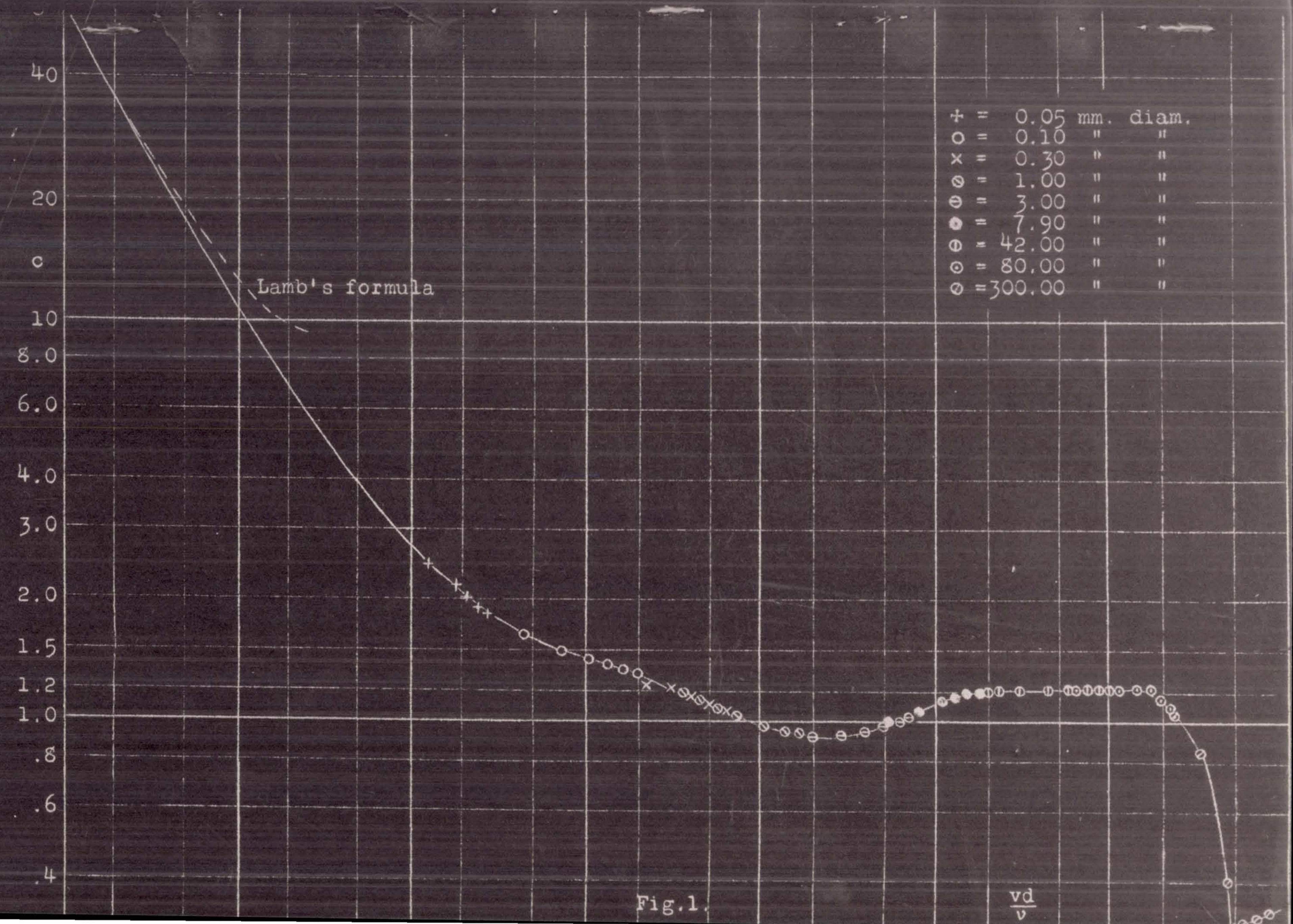

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Lamb's formula
+ = 0.05 mm. diam.
○ = 0.10 " "
× = 0.30 " "
⊗ = 1.00 " "
⊖ = 3.00 " "
● = 7.90 " "
⦶ = 42.00 " "
⊙ = 80.00 " "
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Fig.1.

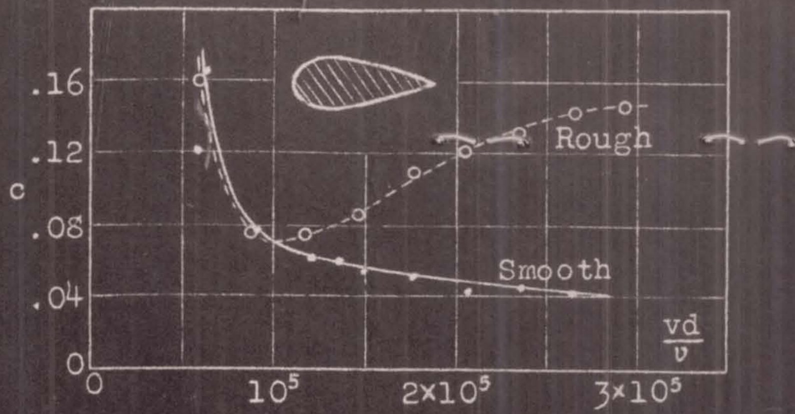

Fig.2

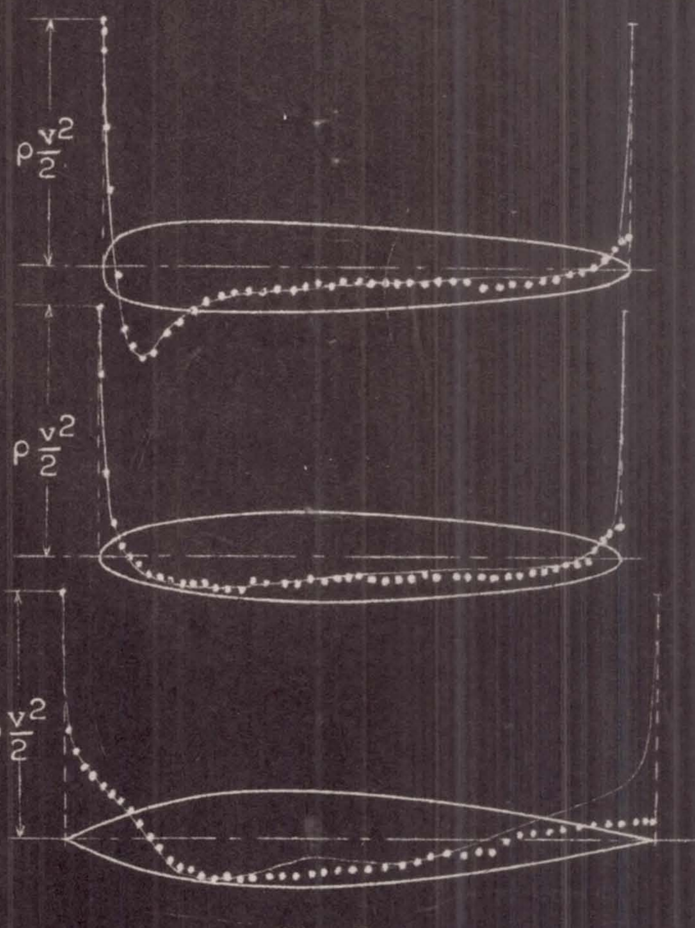

Fig.3