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A THEORETICAL INVESTIGATION OF THE SHORT-PERIOD DYNAMIC LONGITUDINAL STABILITY OF AIRPLANE CONFIGURATIONS HAVING ELASTIC WINGS OF 0° TO 60° SWEEPBACK

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SUMMARY

A theoretical investigation has been made to determine the effects of an elastic wing on the dynamic longitudinal stability of thin-wing airplane configurations. In order to investigate the effects of various important parameters, the configurations were assumed to vary in wing sweep angle from 0° to 60°, in center-of-gravity location from 25 percent mean aerodynamic chord to 45 percent mean aerodynamic chord, and in ratio of wing mass to airplane mass from 0.15 to 0.50.

Three degrees of freedom were assumed - freedom in vertical translation of the rigid airplane, pitching rotation of the rigid airplane, and displacement of the wing tip due to bending of the elastic wing. The elastic wing mode was determined from consideration both of the deflection under static loading and the deflection in the primary ground-vibration mode and was represented by a combination of bending in the primary mode and the associated torsion. Lagrange's method was used to obtain an equation of motion for each degree of freedom. The characteristic equation of the system was solved for the period and damping of the airplane mode and of the wing mode. Solutions were also obtained for three simplified airplane and wing systems: (1) the wing mode alone, (2) the airplane under quasi-static conditions, and (3) the rigid airplane.

An analysis of the solutions showed that, for configurations having 40° to 60° sweepback, no dynamic instability due to wing flexibility was indicated; however, the loss in static stability due to wing flexibility was found to be a fairly serious problem. For configurations having no sweepback, the wing was subject to a decrease in oscillatory stability for the large ratio of wing mass to airplane mass accompanied by forward center-of-gravity locations. The quasi-static method gave results comparable to those of the semirigid method for sweptback wings; however, for straight wings the quasi-static method gave poor results.

INTRODUCTION

As a result of the trend toward the use of thin sweptback wings on large high-speed aircraft, the effect of structural flexibility on dynamic longitudinal stability must be considered. The inclusion of flexibility modes as degrees of freedom in addition to those of the rigid airplane results in higher order characteristic equations. As the order of the characteristic equation increases, the amount and difficulty of work involved in obtaining the solution increases. The additional work is, of course, unjustified if the results do not differ appreciably from the results obtained by using less rigorous methods.

The present paper presents a theoretical investigation of the effects of wing flexibility on the dynamic longitudinal stability of airplanes by including a degree of freedom for the elastic wing. The results of the higher order equations are compared with the results of less rigorous methods. In order to investigate the effects of various important parameters, the configurations are assumed to vary in wing sweep angle from 0° to 60°, in center-of-gravity locations from 25 percent mean aerodynamic chord to 45 percent mean aerodynamic chord, and in ratio of wing mass to airplane mass from 0.15 to 0.50. A single value of wing bending and of torsional stiffness typical of those possessed by an actual airplane is used in the analysis.

The equations of motion are derived in the appendix by using Lagrange's method. The resulting semirigid three-degree-of-freedom equations of motion are solved at various flight conditions for the period and damping of the two modes of oscillation, the first involving primarily wing motion and the second, airplane motion. These solutions are compared with solutions obtained for three simplified airplane and wing systems: (1) the wing mode alone, (2) the airplane under quasi-static conditions (flexible inertia and damping terms assumed to be zero, and (3) the rigid airplane.

SYMBOLS

A diagram showing the system of axes and positive directions of forces and moments on the airplane is presented in figure 1.

A aspect ratio, $\dfrac{(b \cos \Lambda)^2}{S}$

A_{Zh} generalized nondimensional mass coupling term between Z and h degrees of freedom, $a_{Zh}/\rho S\bar{c}$

A_{hh} generalized nondimensional mass term of flexible wing mode between elastic wing and h degree of freedom, $a_{hh}/\rho S\bar{c}$

$A_{\theta h}$ generalized nondimensional mass coupling term between θ and h degrees of freedom, $a_{\theta h}/\rho S\bar{c}^2$

a_{Zh} generalized mass coupling term between Z and h degrees of freedom, $2\int_0^{b_o/2}\left[m_w' f_z(y) - S_w' f_\phi(y)\right]dy_o$, slugs

a_{hh} generalized mass term of flexible wing mode between elastic wing and h degree of freedom,

$$2\int_0^{b_o/2}\left\{m_w'\left[f_z(y)\right]^2 - 2S_w' f_z(y) f_\phi(y) + I_w'\left[f_\phi(y)\right]^2\right\}dy_o,$$

slugs

$a_{\theta h}$ generalized mass coupling term between θ and h degrees of freedom,

$$2\int_0^{b_o/2}\left[-S_w' f_z(y) + I_w' f_\phi(y) - m_w' e f_z(y) + S_w' e f_\phi(y)\right]dy_o,$$

slug-ft

b span (along elastic axis), ft

C_F force coefficient due to elastic wing deflection, F_h/qS

C_N normal-force coefficient, N/qS

C_{N_o} trim normal-force coefficient, W/qS

C_m pitching-moment coefficient about Y-axis, $M/qS\bar{c}$

$\bar{c}$ mean aerodynamic chord, ft

c chord, ft

c_m' section pitching-moment coefficient about wing elastic axis, $m'/qc\bar{c}$

c_n section normal-force coefficient, n/qc

$$D = \frac{d}{dt\,\frac{V}{\bar{c}}}$$

EI	bending stiffness, lb-in.^{2}
E_k	kinetic energy, ft-lb
E_p	potential energy, ft-lb
e	longitudinal distance from airplane center of gravity to wing elastic axis (function of spanwise location), positive forward, ft
F	force, lb
f	section force, lb/ft
$f_z(y)$	spanwise bending mode shape along wing elastic axis
$f_\phi(y)$	spanwise twisting mode shape about wing elastic axis per unit tip bending deflection, radians/ft
GJ	torsional stiffness, lb-in.^{2}
g	acceleration due to gravity, ft/sec^2
H	wing-tip deflection, $h/\bar{c}$, chords
h	wing-tip deflection of elastic axis due to bending, positive downward, ft
I	moment of inertia about Y-axis, $m_A x^2$, $m_f x^2$, or $m_w x^2$, slug-ft^2
I'	section moment of inertia, $m_w' x^2$, slug-ft^2/ft
K_Y	radius of gyration about Y-axis, chords
k	reduced angular frequency, $\omega\bar{c}/V$
l_t	longitudinal distance from quarter chord of wing mean aerodynamic chord to quarter chord of tail mean aerodynamic chord, ft
M	pitching moment about Y-axis, ft-lb
m	section pitching moment about Y-axis, ft-lb/ft
m_A, m_f, m_w	mass, slugs
m'	section pitching moment about elastic axis, ft-lb/ft

m_w'	section mass, slugs/ft
N	normal force, positive downward, lb
n	section normal force, lb/ft
Q	generalized coordinate
q	dynamic pressure, lb/sq ft; also, pitching angular velocity, radians/sec
r_c	ratio of local chord to root chord, c/c_r
S	wing plan-form area, sq ft
S'	section mass moment about elastic axis, $m_w'x$, slug-ft/ft
S_A,S_f,S_w	mass moment, m_Ax, m_fx, or m_wx, slug-ft/ft
$T_{0.1}$	time to damp to 0.1 amplitude, sec
t	time, sec
V	velocity, fps
W	weight, lb
X	longitudinal axis of displacement fixed at airplane center of gravity
x	longitudinal displacement, positive forward, ft
Y	lateral axis of reference fixed at airplane center of gravity
y	lateral or spanwise displacement, ft
Z	vertical displacement of airplane center of gravity, positive downward, ft
z	vertical wing deflection of elastic axis due to wing bending, positive downward, ft
α	angle of attack, positive wing leading edge up, radians
δ	elevator deflection, positive trailing edge down, radians
η	dimensionless spanwise coordinate, $\frac{y}{b/2}$, fraction of semispan

$\bar{\eta}_o$	dimensionless spanwise coordinate, $\frac{y_o}{b_o/2}$, fraction of exposed semispan
θ	angle of pitch about airplane center of gravity, positive airplane nose up, radians
Λ	sweep angle of elastic axis of wing, deg
μ	nondimensional airplane mass, $m_A/\rho S\bar{c}$
ρ	mass density of air, slugs/cu ft
ϕ	angle of twist of airfoil in plane perpendicular to elastic axis, positive wing leading edge up, radians
ω	angular frequency, radians/sec

Subscripts:

A	airplane
av	average
cg	center of gravity
f	fuselage
h	flexible-wing degree of freedom
i	intersection of elastic axis with fuselage
max	maximum
o	exposed wing
Q	generalized coordinate
r	wing root
t	tail
w	wing
Z	vertical degree of freedom
θ	pitching degree of freedom

Dots are used to indicate differentiation with respect to time; for example, $\dot{\theta} = \frac{d\theta}{dt}$.

The subscripts α, $\dot{\theta}$, $\dot{h}$, h, q, H, and δ indicate differentiation with respect to the subscript; for example, $C_{N_\alpha} = \frac{dC_N}{d\alpha}$.

BASIS OF ANALYSIS

Semirigid Method

A flexible structure may be considered to have an infinite number of degrees of freedom because it can deflect an infinite number of ways depending on the loading. However, in order to make the solution for the motion of the flexible airplane as simple as possible, the number of degrees of freedom and consequently the number of equations of motion should be kept to a minimum. This method of approach is presented in reference 1 and is called the semirigid concept. In this approach the mode of flexure under load is always the same regardless of the loading.

Three degrees of freedom were assumed for the configuration studied herein - freedom in vertical translation of the rigid airplane Z, pitching rotation of the rigid airplane θ, and displacement of the wing tip due to bending of the elastic wing h - and an equation of motion must be derived for each of these three degrees of freedom. A simplified way of deriving these equations is by Lagrange's method which is described in reference 2. Lagrange's method consists in writing an expression for the total kinetic energy and the total potential energy of the system. The operations indicated by the general Lagrangian equation are then performed in the expression for total energy, and an equation of motion for each degree of freedom is obtained. The application of this method to a configuration with a flexible wing is presented in reference 3. For the convenience of those who are not familiar with Lagrange's method, a derivation of the equations of motion used in this paper is presented in the appendix.

The flexible wing mode shape was assumed to consist of bending $f_z(y)$, combined with twisting per unit bending deflection at the wing tip $f_\phi(y)$. When the wing is flexed in the degree of freedom characterized by the wing-tip bending deflection h, the bending deflection at station y is z and the torsional deflection is ϕ. No motion of the fuselage was considered to be associated with the wing deflection mode. In practice, some fuselage rotation or bending would occur in conjunction with the wing mode and should be considered when making a specific analysis.

In order to use the equations of motion, the characteristics of a wing mode which satisfactorily represents the flexing configuration must be determined, and the conditions under which the wing flexes must be specified. The wing was assumed to be a flexible cantilever beam which was anchored to the fuselage in a plane through the point of intersection of the elastic axis and the fuselage and perpendicular to the elastic axis. The basic mode of oscillation was considered to be a combination of primary bending in a vertical plane containing the elastic axis and the associated twisting about the elastic axis. The shape of the bending and twisting modes could be expected to vary somewhat with the span load distribution on the wing. At zero frequency the loading is static, the result of steady additional lift forces. As the frequency increases toward the wing natural frequency, the wing mode shape would be expected to approach the mode shape corresponding to an elastic vibration of the wing on the ground. Simple beam theory was used to obtain the static bending mode shape and the twisting mode shape under steady load. The moment of the additional lift forces about the wing elastic axis was used in determining the twisting mode shape and the amount of twist per unit tip deflection. The twisting mode was determined solely for static loading inasmuch as the twist occurring in an elastic vibration of the wing appeared to be negligible. This observation resulted from the fact that, for the assumed wing mass distribution, the wing was almost perfectly mass balanced about the elastic axis.

The bending mode shape of the wing oscillating at the ground natural frequency was obtained with the aid of reference 4. The two calculated bending modes and the calculated twisting mode are presented in figure 2. The calculated bending modes closely approximate each other despite the large variation in distribution between the static and dynamic loading. A parabolic variation of spanwise bending $f_z(y) = \bar{\eta}_o^2$ and a linear variation of spanwise twist $f_\phi(y) = \frac{\partial \phi}{\partial h} \bar{\eta}_o$ are also presented in figure 2. The agreement between the calculated mode shapes and these simplified approximating curves is seen to be very good; therefore, the simplified curves were used to represent the mode shapes.

The values of wing bending and torsional stiffness and the wing structural weight distribution for the assumed airplane configurations were based on those possessed by an actual swept-wing bomber airplane and are presented in figure 3. The bending and torsional stiffnesses vary along the span in accordance with r_c^4, the fourth power of the ratio of the local chord to the root chord, as suggested in reference 5. This assumption is in good agreement with the structural characteristics of the actual airplane. The actual airplane had a ratio of wing structural mass to airplane mass of 0.15. The higher values of the ratio of wing mass to airplane mass of 0.33 and 0.50 as presented in figure 3 approximate some typical wing-airplane mass ratios for wings having additional masses attached to them such as nacelles and external or internal stores. However, in these cases it should be noted that the assumed mass distribution does not necessarily correspond to an airplane having such additional masses attached to it.

The bending mode shape was used to calculate the ground natural frequency of the wing. In order to obtain a range of natural frequencies of the wing, the wing stiffness was assumed to remain constant and the mass of the wing was assumed to increase so that the airplane mass also increased. Frequencies were calculated for ratios of wing mass to airplane mass of 0.15, 0.33, and 0.50 and are presented in table I.

Additional configurations used in the analysis were obtained by modifying a basic configuration. The variations were obtained by holding the wing area and the span along the elastic axis constant as the wing panels were swept about a vertical axis which was located at the juncture of the wing elastic axis and the airplane center line. As the wing was varied in sweep, the wing root was moved forward or rearward so as to keep the tail length l_t constant. Configurations having sweep angles of 0°, 40°, and 60° measured with respect to the elastic axis were assumed and are shown in figure 4. Because of the large aspect ratio, only a small difference in angle of sweep between the elastic axis and the quarter-chord line exists. In view of this small difference and for purposes of convenience, the angles of sweepback used herein refer to the sweep of the elastic axis which is the 38-percent-chord line. The fuselage and tail dimensions were held constant. The airplane center of gravity was assumed to be located at 25, 35, and 45 percent of the wing mean aerodynamic chord.

The inertia terms and the aerodynamic damping and restoring-spring parameters were calculated for the configurations and are presented in table II. The airplane lift and pitching-moment characteristics were calculated by using references 6 and 7. Strip theory uncorrected for compressibility effects was used in calculating the force parameters. In the computations of the generalized mass terms, some small terms were neglected. The equations of motion were solved for period and damping, and solutions for each configuration were obtained at three or more dynamic pressures. Also, transient solutions for a configuration at two altitudes were presented in order to show the relation of the wing oscillation to the airplane oscillation. Because of the variations with altitude of the relative density of the airplane and the generalized masses, the solutions of the characteristic equation for values of dynamic pressure at one altitude are not applicable for the same values of dynamic pressure at other altitudes.

Other Methods

In order to determine whether more simple methods could be used to predict the dynamic characteristics of the flexible airplane and wing, results were obtained by the quasi-static method and the wing-mode-alone method. The quasi-static method consisted in eliminating all the inertia and damping terms pertaining to the flexible mode (terms containing D^2H and DH) from the equations of motion and solving the resulting second-order characteristic equation for period and damping. In effect, the results of this method are comparable to those of the rigid-airplane method except that the effects of flexibility on the aerodynamic parameters are accounted for at any given dynamic pressure. It should be noted that the rigid-airplane solutions are the same as the solutions of the semirigid airplane at zero dynamic pressure. The simple wing-mode-alone method consists in neglecting the first two equations of motion and solving the third equation of motion after neglecting the coupling terms (terms containing $D\alpha$, α, $D^2\theta$, and $D\theta$) that appear in the third equation. It should be noted at this point that the conditions for the wing-mode-alone method do not simulate the conditions used in flutter work. The consideration of a separate torsional mode (as in flutter work) is beyond the scope of the problem dealt with in this paper. The frequencies of oscillation of the modes are low enough so that consideration of unsteady-lift effects is unwarranted. The wing-mode-alone method was evolved in order to understand better the effects of short-period coupling in the wing mode. This method, therefore, represents the wing motion that would occur if the airplane center of gravity was constrained to move in a straight line at all times without pitching.

RESULTS AND DISCUSSION

The equations of motion were solved for the damping and frequency of the flexible wing mode and of the airplane mode for various dynamic pressures at a standard altitude of 8,000 feet. The solutions for ratios of wing mass to airplane mass of 0.15, 0.33, and 0.50 and for center-of-gravity locations of 25, 35, and 45 percent of the wing mean aerodynamic chord at zero sweep angle are presented in figure 5. For sweep angles of 40° and 60° the solutions did not change appreciably with increasing mass ratio. Therefore the results for the mass ratio of 0.33 only and with center-of-gravity locations of 25, 35, and 45 percent of the mean aerodynamic chord are presented in figure 6. The wing characteristics covered in parts (a) of figures 5 and 6 are expressed in terms of actual time; whereas the airplane characteristics covered in parts (b) of these figures are expressed in terms of nondimensional time. The results are presented in different forms inasmuch as each form is believed to enable the best interpretation of the data. The wing ground natural frequency is shown in the plots of wing frequency against dynamic pressure in figures 5(a) and 6(a). The wing-mode-alone solutions and the solutions for the quasi-static conditions of the airplane mode are also shown in these figures.

The results for the airplane mode are presented as a function of the period and of the reciprocal of the time to damp to 0.1 amplitude where time is expressed nondimensionally as units of distance traveled measured in fuselage lengths. Fuselage length, rather than the usual unit of wing chord, was used because the fuselage length remained constant for all configurations whereas the wing mean aerodynamic chord varied with the sweepback angle. Solutions for the rigid airplane configuration presented in this form are independent of forward speed. At zero dynamic pressure flexibility effects must vanish, and the solution for the flexible airplane mode is identical to that of the rigid configuration. Therefore, the deviation of the solutions from the values for the configurations at zero dynamic pressure represents the effect of flexibility at any particular dynamic pressure at an altitude of 8,000 feet.

Before examining the data, it is well to note a relation between the wing mode and the airplane mode. By expanding the nondimensional characteristic equation, the sum of the damping of the airplane mode and the wing mode for a particular configuration can be shown to remain constant with variation in dynamic pressure. Any changes in wing damping, therefore, result in opposite changes in airplane damping.

Semirigid Method

The solutions for the configurations having zero sweep obtained by the semirigid method are discussed first. The effects of variation of wing mass and center-of-gravity location are shown for zero sweep in figure 5. In the lower half of figure 5(a), the damping curves do not extend to zero dynamic pressure. In the region of zero dynamic pressure, the damping curves would approach infinity because of the assumed absence of structural damping.

The effective damping of the wing oscillation decreases as the mass of the wing increases. This effect is shown in the damping curves by an increase in time to damp to 0.1 amplitude and also in the frequency curves by a tendency for the frequency to become constant at higher dynamic pressures. The damping for the wings of greater mass also shows an appreciable variation with center-of-gravity location at the highest dynamic pressures. For the center of gravity at the most forward location ($0.25\bar{c}$), the oscillation tends to become less stable with increase in dynamic pressure. This effect is indicated by an increase in time to damp to 0.1 amplitude while the frequency becomes approximately constant. Solutions are presented at dynamic pressures which would be above the critical Mach number of a configuration in order to gain a general idea of the stability trends. Although the solutions at those dynamic pressures are meaningless for the present configuration, they would apply at lower dynamic pressures for a configuration with a wing of greater flexibility.

Some effects of the airplane mode of oscillation on the wing mode may be seen from a comparison of the wing-alone solutions with the semirigid solutions for the wing. To analyze these effects better the frequencies of the airplane mode have been put in dimensional form and are plotted in figure 5(a) together with the wing frequencies. The airplane motion generally decreases the damping of the wing oscillation especially for the $m_W/m_A = 0.50$ configuration at the higher dynamic pressures. The frequency of the airplane oscillation approaches the natural frequency of the wing oscillation at these dynamic pressures. The temporary increase in wing frequency above the ground natural frequency of the wing at low dynamic pressures also may be attributed to the effect of the airplane oscillation on the wing oscillation. This conclusion is substantiated by the lack of increase in frequency for the wing-mode-alone method. As is well-known, damping in a single-degree-of-freedom system, whether positive or negative, will decrease the frequency from that of the system with no damping.

For the airplane mode (fig. 5(b)), the damping shows a slight general decrease with increase in wing mass and, hence, airplane mass. For the light configuration $m_W/m_A = 0.15$ with the center of gravity located at 45 percent mean aerodynamic chord, the damping is sufficient to cause the airplane oscillation to become critically over damped. The damping of the airplane oscillation increases slightly with increase in dynamic pressure. Generally, the damping of the airplane mode for configurations having zero sweep angle is good.

At zero sweep angle the effects on the local incidence angle of wing flexibility are due to the wing twisting - the bending deflections have no effect. As the sweep angle of a wing increases, the effect of twist on the local incidence angle gradually decreases and the bending component of the flexible mode begins to exert a powerful effect. At 3° of sweep for the assumed basic wing, the effect of twist on the local wing incidence is neutralized by the effect of bending; therefore, the wing is aeroisoclinic. Above 3° of sweep the bending effects are predominant.

An examination of figure 6(a) shows that the wing frequency increases with increasing dynamic pressure for wings of 40° and 60° sweepback. The trend of the increase in wing frequency above the wing ground natural frequency at low dynamic pressures is the same as for the zero sweep angles; for the 40° and 60° swept wings, however, the wing frequency continues to increase apparently as a result of increased aerodynamic restoring moments which add to the elastic restoring moments of the wing. In general, the data show that the wing mode is always satisfactorily damped for the expected flight range of dynamic pressures. The damping appears to become poorer at low dynamic pressures, but these results are pessimistic because of the assumption of zero structural damping. It is also apparent that the center-of-gravity position has no appreciable effect on the characteristics of the wing mode. The damping is slightly less for the 60° sweptback configuration than for the 40° sweptback configuration because the lift-curve slope is less for the 60° configuration.

The characteristics of the airplane mode for the swept-wing configurations (fig. 6(b)) indicate that the static-stability considerations completely overshadow the dynamic stability characteristics in importance. At the lowest dynamic pressures for the most rearward center-of-gravity location the semirigid configurations possess an oscillation similar to that found in the rigid airplanes. With increased dynamic pressures, this oscillation changes into two convergences as the maneuvering neutral point approaches the center-of-gravity position being considered. With still further increase in dynamic pressure, the maneuver point becomes coincident with the center of gravity being considered, and one of the convergences changes to a divergence. Any further increase in dynamic pressure causes the divergence to be more severe as the maneuvering neutral point moves farther ahead of the center of gravity. For more forward locations of the center of gravity the same sequence occurs except at higher dynamic pressures. This analysis is confirmed by the data of figure 7, which shows the location of the maneuvering neutral point with dynamic pressure obtained by the quasi-static method for 0°, 40°, and 60° sweepback angles.

The top part of figure 6(b) shows that when an oscillation does exist the period increases with increasing dynamic pressure as would be expected from the decrease in static stability with increase in dynamic pressure. A comparison between the variation of wing frequency and airplane frequency with dynamic pressure shows that these variations are in opposite directions so that interaction between these two modes appears to be impossible. The loss in static stability with increasing dynamic pressure is caused by a forward shift in wing aerodynamic center. There is, of course, a compensating factor in the form of decreased wing-lift-curve slope due to bending, but this factor is less important than the forward shift in wing aerodynamic center.

Evaluation of Simplified Methods

Results obtained from the rigid-airplane solutions have already been covered in the previous discussion inasmuch as the results are the same as for the flexible airplane at zero dynamic pressure. In order to determine whether more simple methods could be used to predict the dynamic characteristics of the flexible airplane and wing, results were obtained by the quasi-static method and the wing-mode-alone method, and these results are shown in figures 5 and 6.

An inspection of figure 5(b) shows that, for the unswept airplane configuration at the highest dynamic pressures, the period and damping can be determined with only a fair degree of accuracy by the quasi-static method. It is noteworthy that the results of the quasi-static method do not differ appreciably from the results obtained with the rigid-airplane equations of motion. For swept-wing configurations (fig. 6(b)), however, the period and damping are seen to be in somewhat better agreement with those determined by the semirigid method. The major effects of static stability are shown by the quasi-static method. The variations in damping with dynamic pressure are almost exactly the same as those given by the semirigid method. It therefore appears that the quasi-static method can be used to give a good first approximation of the dynamic characteristics of a swept-wing airplane having a flexible wing.

Application of the wing-mode-alone method to the prediction of the period and damping of the flexible wing mode (figs. 5(a) and 6(a)) shows that for unswept wings the period and damping can be predicted with a fair degree of accuracy except in the case of large wing mass ratios with far-forward center-of-gravity positions. For sweptback wings within the ranges of parameters covered, the wing-mode-alone method seems to give good results.

Transient Solution

The period and damping for two modes of oscillation of a series of flexible-wing aircraft have been presented and discussed. In order to give a better idea of the relative importance of the wing oscillation in the total airplane motion, solutions have been obtained on the Reeves Electronic Analog Computer for the transient motions following a disturbance. The longitudinal transient responses of a 35° swept-wing aircraft configuration to an elevator step input at altitudes of 8,000 feet and 30,000 feet are presented in figure 8. The configuration closely resembles the 40° swept aircraft for flight configuration of $m_W/m_A = 0.33$ in mass, structural, and aerodynamic parameters. The plot of wing-tip deflection h shows only a very small excitation of the wing mode which quickly disappears. It is possible that the wing oscillation would be excited to a greater degree by a gust or some other form of disturbance; however, these records indicate that the wing oscillation is not easily excited by use of the elevator. The airplane oscillation is heavily damped at both altitudes.

CONCLUSIONS

On the basis of an analysis of the effects of wing flexibility on configurations varying in sweep angle, center-of-gravity location, and ratio of wing mass to airplane mass, the following conclusions are indicated:

1. For configurations having 40° to 60° sweepback, no dynamic instability due to wing flexibility was indicated; however, the loss in static stability due to wing flexibility was found to be a fairly serious problem.

2. For configurations having no sweepback, the effects of wing flexibility were found to be serious only in the case of configurations having a large ratio of wing mass to airplane mass and forward center-of-gravity locations; for these cases, the analysis indicated that the wing was subject to a decrease in oscillatory stability.

3. The quasi-static method appears to be fairly realistic as a means of obtaining a good approximation to the dynamic characteristics of a swept-wing configuration. However, the method does not appear to be accurate for unswept configurations at high dynamic pressures.

4. A simplified wing-mode-alone method was found to give good results in predicting the characteristics of the flexible wing mode of motion for swept-wing configurations. However, this method did not appear to be reliable for use with unswept configurations.

Langley Aeronautical Laboratory,
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APPENDIX

DERIVATION OF EQUATIONS OF MOTION

The Lagrangian equation is

$$\frac{d}{dt}\left(\frac{\partial E_k}{\partial \dot{Q}}\right) - \frac{\partial E_k}{\partial Q} + \frac{\partial E_p}{\partial Q} = F_Q \tag{A1}$$

where E_k and E_p are the kinetic and potential energies of the dynamic system. The kinetic energy of the system is equal to the sum of the contributions of the fuselage and of the elements of the wing which are external to the wing-fuselage intersection. In this analysis the fuselage is assumed to be rigid, and no motion of the fuselage is considered to be associated with the wing deflection mode. The motion of the fuselage is therefore due to motion of the rigid airplane, and the motion of elements of the wing is due to motion of the rigid airplane and motion of the flexible wing. The tail assembly is considered part of the fuselage. The velocities and the geometric relations of the fuselage and wing elements to the airplane center of gravity are presented in figure 9. From figure 9(a) the velocity at the center of gravity of the fuselage $\dot{Z}_f$ is seen to be

$$\dot{Z}_f = \dot{Z} - x\dot{\theta} \tag{A2}$$

and the kinetic energy of the fuselage unit is

$$E_{k_f} = \frac{1}{2} m_f(\dot{Z} - x\dot{\theta})^2 + \frac{1}{2} I_{cg_f}\dot{\theta}^2 \tag{A3}$$

By expanding equation (A3), the kinetic energy of the fuselage becomes

$$E_{k_f} = \frac{1}{2} m_f\dot{Z}^2 - S_f\dot{Z}\dot{\theta} + \frac{1}{2} I_f\dot{\theta}^2 \tag{A4}$$

where

$$I_f\dot{\theta}^2 = m_f(x\dot{\theta})^2 + I_{cg_f}\dot{\theta}^2$$

For the wing, the velocity at the center of gravity of a small element of wing $\dot{Z}_W$ may be obtained with the aid of figure 9(b) as

$$\dot{Z}_W = \dot{Z} - (e + x)\dot{\theta} + f_Z(y)\dot{h} - f_\phi(y)\dot{h}x \tag{A5}$$

The kinetic energy of a small element of wing is

$$dE_{k_W} = \frac{1}{2} m_W'\left[\dot{Z} - (e + x)\dot{\theta} + f_Z(y)\dot{h} - f_\phi(y)\dot{h}x\right]^2 dy + \frac{1}{2} I_{cg_W}'\left[\dot{\theta} + f_\phi(y)\right]^2 dy \tag{A6}$$

In the present analysis the term $\frac{1}{2} I_{cg_W}\left[\dot{\theta} + f_\phi(y)\right]^2 dy$ has been considered to be small for the wings of high aspect ratio which have been considered herein and, therefore, has not been carried further in the derivation. Equation (A6) is integrated over the exposed semispan $b_o/2$. To put the equation in proper form for integration, let the variable over the exposed semispan be $y_o = y - y_i$. The equation is then integrated from 0 at the wing-fuselage intersection to $b_o/2$ at the wing tip. Expanding equation (A6) results in

$$\begin{aligned} E_{k_W} = 2\int_0^{b_o/2}\Biggl(&\frac{1}{2} m_W'\left\{\dot{Z}^2 - 2\dot{Z}e\dot{\theta} + 2\dot{Z}f_Z(y)\dot{h} + (e\dot{\theta})^2 - 2e\dot{\theta}f_Z(y)\dot{h} + \right. \\ &\left.\left[f_Z(y)\dot{h}\right]^2\right\} - S_W'\left[\dot{Z}\dot{\theta} + \dot{Z}f_\phi(y)\dot{h} - e\dot{\theta}^2 - e\dot{\theta}f_\phi(y)\dot{h} + \dot{\theta}f_Z(y)\dot{h} + \right. \\ &\left. f_Z(y)f_\phi(y)\dot{h}^2\right] + \frac{1}{2} I_W'\left\{\dot{\theta}^2 + 2f_\phi(y)\dot{\theta}\dot{h} + \left[f_\phi(y)\dot{h}\right]^2\right\}\Biggr)dy_o \end{aligned} \tag{A7}$$

Equation (A7) contains mass terms pertaining to the rigid and flexible degrees of freedom. These relative mass terms can be grouped and assigned definitions. The terms of equation (A7) pertaining to the rigid degrees of freedom are combined with equation (A4) to obtain the kinetic-energy equation of the rigid airplane. The mass terms in equation (A7) pertaining to the rigid degrees of freedom are obtained by removing the mass terms containing the flexible degree of freedom $\dot{h}$ from the equation. Then the kinetic energy of the rigid wing is obtained as

$$E_{k_w} = 2\int_0^{b_o/2}\left\{\frac{1}{2} m_w'\left[\dot{Z}^2 - 2e\dot{\theta}\dot{Z} + (e\dot{\theta})^2\right] - S_w'(\dot{Z}\dot{\theta} - e\dot{\theta}^2) + \frac{1}{2} I_w'\dot{\theta}^2\right\}dy_o \tag{A8}$$

If the $\dot{Z}^2$ terms of equations (A4) and (A8) are combined, the kinetic energy of the rigid airplane in vertical translation becomes

$$\frac{1}{2} m_A\dot{Z}^2 = \frac{1}{2} m_f\dot{Z}^2 + 2\int_0^{b_o/2} \frac{1}{2} m_w'Z^2 dy_o \tag{A9}$$

Similarly, if the $\dot{\theta}^2$ terms of equations (A4) and (A8) are combined, the kinetic energy of the rigid airplane in pitch becomes

$$\frac{1}{2} I_A\dot{\theta}^2 = \frac{1}{2} I_f\dot{\theta}^2 + 2\int_0^{b_o/2}\left[\frac{1}{2} m_w'(e\dot{\theta})^2 + S_w'e\dot{\theta}^2 + \frac{1}{2} I_w'\dot{\theta}^2\right]dy_o \tag{A10}$$

The terms remaining in equations (A4) and (A8) can be recognized as static unbalance terms. Because equations (A4) and (A8) are derived about the airplane center of gravity, the sum of the static unbalance terms must be equal to zero or

$$-S_f\dot{Z}\dot{\theta} - 2\int_0^{b_o/2}\left(m_w'e\dot{\theta}\dot{Z} + S_w'\dot{Z}\dot{\theta}\right)dy_o = 0$$

Combining equations (A9) and (A10) yields the total kinetic energy of the rigid airplane in vertical translation and pitch

$$E_k = \frac{1}{2} m_A\dot{Z}^2 + \frac{1}{2} I_A\dot{\theta}^2 \tag{A11}$$

In a similar manner, some new terms representing the inertia effects of the flexible airplane may be defined. After the terms in equations (A7) containing $\dot{h}^2$ are collected, the generalized mass pertaining to the flexible degree of freedom may be defined as

$$a_{hh}\dot{h}^2 = 2\dot{h}^2\int_0^{b_o/2}\left\{m_w'\left[f_z(y)\right]^2 - 2S_w'f_z(y)f_\phi(y) + I_w'\left[f_\phi(y)\right]^2\right\}dy_o \tag{A12}$$

By collecting the mass terms from equation (A7) containing $\dot{Z}\dot{h}$ and $\dot{\theta}\dot{h}$, the generalized masses pertaining to the coupling terms may likewise be defined as

$$2a_{Zh}\dot{Z}\dot{h} = 2\dot{Z}\dot{h}\left\{2\int_0^{b_o/2}\left[m_w'f_z(y) - S_w'f_\phi(y)\right]dy_o\right\} \tag{A13}$$

$$2a_{\theta h}\dot{\theta}\dot{h} = 2\dot{\theta}\dot{h}\left\{2\int_0^{b_o/2}\left[-S_w'f_z(y) + I_w'f_\phi(y)\right]dy_o - 2\int_0^{b_o/2}\left[m_w'ef_z(y) - S_w'ef_\phi(y)\right]dy_o\right\} \tag{A14}$$

The abbreviated equation for kinetic energy is then

$$2E_k = m_A\dot{Z}^2 + I_A\dot{\theta}^2 + a_{hh}\dot{h}^2 + 2a_{Zh}\dot{Z}\dot{h} + 2a_{\theta h}\dot{\theta}\dot{h} \tag{A15}$$

The potential energy of the system is composed of a contribution from the airplane and a contribution from the flexible wing. The potential energy of the airplane due to its vertical position is given by $-ZW_A$. The potential energy of the flexible wing is expressed in terms of the frequency of the wing oscillating in the assumed deflection mode. If the wing is performing a sinusoidal oscillation, then the potential energy at its point of maximum deflection is equal to the kinetic energy of the wing as it passes through the point of zero deflection. For a small strip of wing vibrating sinusoidally in the deflection mode, the maximum velocity is $\omega(z_{max} - \phi_{max}x)$ and the corresponding potential energy expressed in terms of kinetic energy is

$$\Delta E_{p_w} = \frac{1}{2} m_w'\left[\omega(z_{max} - \phi_{max}x)\right]^2 dy_o$$

For any spanwise location, $z_{max} = f_z(y)h$ and $\phi_{max} = f_\phi(y)h$; substituting these values and integrating yields

$$E_{p_w} = \frac{1}{2}\omega^2h^2\left(2\int_0^{b_o/2}\left\{m_w'\left[f_z(y)\right]^2 - 2S_w'f_z(y)f_\phi(y) + I_w'\left[f_\phi(y)\right]^2\right\}dy_o\right)$$

$$= \frac{1}{2}\omega^2h^2a_{hh}$$

If the potential energy of the airplane is combined with the potential energy of the elastic wing, the total potential energy of the system can be expressed as

$$E_p = -m_A g Z + \frac{1}{2} \omega^2 h^2 a_{hh} \tag{A16}$$

where $W_A = m_A g$.

The equations of motion for each degree of freedom determined by substituting equations (A15) and (A16) into the Lagrangian equation (A1) and using the generalized coordinates Z, θ, and h are

$$m_A\ddot{Z} + a_{Zh}\ddot{h} - m_A g = F_Z \tag{A17}$$

$$I_A\ddot{\theta} + a_{\theta h}\ddot{h} = F_\theta \tag{A18}$$

$$a_{hh}\ddot{h} + a_{Zh}\ddot{Z} + a_{\theta h}\ddot{\theta} + a_{hh}\omega^2 h = F_h \tag{A19}$$

The generalized forces F_Z, F_θ, and F_h account for the forces not included in the potential-energy term E_p. In order to obtain the components of the generalized force term, the airplane is assumed to be composed of the fuselage unit and the wing unit as in the development of the inertia parameters. On a unit there are acting a normal force $n\,dy$ due to the variation in coordinate Z, a moment $m\,dy$ about the airplane center of gravity due to the variation in coordinate θ, and a force $F\,dy$ due to the variation in coordinate h. The h coordinate is composed of bending and twisting of the exposed wing. Therefore, the force $F\,dy$ is composed of force $n\,dy$ and moment $m'dy$ about the local elastic axis. The total section work done by the forces on these units corresponding to a virtual displacement ΔQ is

$$f_Q\,\Delta Q\,dy = \Delta Q\left[n\frac{\partial Z}{\partial Q} + m\frac{\partial \theta}{\partial Q} + \left(n\frac{\partial Z}{\partial h} + m'\frac{\partial \phi}{\partial h}\right)\frac{\partial h}{\partial Q}\right]dy \tag{A20}$$

where

$$n = \frac{\dot{Z}}{V} \frac{\partial n}{\partial(\dot{Z}/V)} + \theta \frac{\partial n}{\partial \theta} + \dot{\theta} \frac{\partial n}{\partial \dot{\theta}} + h \frac{\partial n}{\partial h} + \dot{h} \frac{\partial n}{\partial \dot{h}} \tag{A21}$$

$$m = \frac{\dot{Z}}{V} \frac{\partial m}{\partial(\dot{Z}/V)} + \theta \frac{\partial m}{\partial \theta} + \dot{\theta} \frac{\partial m}{\partial \dot{\theta}} + h \frac{\partial m}{\partial h} + \dot{h} \frac{\partial m}{\partial \dot{h}} \tag{A22}$$

$$m' = \frac{\dot{Z}}{V} \frac{\partial m'}{\partial(\dot{Z}/V)} + \theta \frac{\partial m'}{\partial \theta} + \dot{\theta} \frac{\partial m'}{\partial \dot{\theta}} + h \frac{\partial m'}{\partial h} + \dot{h} \frac{\partial m'}{\partial \dot{h}} \tag{A23}$$

By definition

$$\alpha = \frac{\dot{Z}}{V} + \theta \tag{A24}$$

Substituting the partial derivatives of equation (A24) into equations (A21), (A22), and (A23) yields

$$n = \alpha \frac{\partial n}{\partial \alpha} + \dot{\theta} \frac{\partial n}{\partial \dot{\theta}} + h \frac{\partial n}{\partial h} + \dot{h} \frac{\partial n}{\partial \dot{h}} \tag{A25}$$

$$m = \alpha \frac{\partial m}{\partial \alpha} + \dot{\theta} \frac{\partial m}{\partial \dot{\theta}} + h \frac{\partial m}{\partial h} + \dot{h} \frac{\partial m}{\partial \dot{h}} \tag{A26}$$

$$m' = \alpha \frac{\partial m'}{\partial \alpha} + \dot{\theta} \frac{\partial m'}{\partial \dot{\theta}} + h \frac{\partial m'}{\partial h} + \dot{h} \frac{\partial m'}{\partial \dot{h}} \tag{A27}$$

Upon integrating over the wing, equation (A20) becomes

$$F_Q = N \frac{\partial Z}{\partial Q} + M \frac{\partial \theta}{\partial Q} + 2 \int_0^{b_o/2} \left(n \frac{\partial Z}{\partial h} + m' \frac{\partial \phi}{\partial h} \right) \frac{\partial h}{\partial Q} \, dy_o \tag{A28}$$

where N and M include the normal-force contributions of the fuselage and tail. The integral pertaining to the elastic degree of freedom h is limited to the exposed wing span because no component of the elastic motion has been assumed for the fuselage. Partial differentiation of equation (A28) with respect to Z, θ, and h yields

$$F_Z = N = \alpha N_\alpha + \dot{\theta} N_{\dot{\theta}} + h N_h + \dot{h} N_{\dot{h}} \tag{A29}$$

$$F_\theta = M = \alpha M_\alpha + \dot{\theta} M_{\dot{\theta}} + h M_h + \dot{h} M_{\dot{h}} \tag{A30}$$

$$F_h = 2\int_0^{b_o/2} \left(n \frac{\partial Z}{\partial h} + m' \frac{\partial \phi}{\partial h}\right) dy_o \tag{A31}$$

The generalized force term F_h is probably the least familiar; therefore, the components of the force term are obtained in coefficient form. Dividing equation (A31) by qS and multiplying by $\frac{b_o/2}{b_o/2}$ yields

$$\frac{F_h}{qS} = \frac{b_o/2}{qc_{av}\frac{b}{2}} \int_0^1 \left(n \frac{\partial Z}{\partial h} + m' \frac{\partial \phi}{\partial h}\right) \frac{dy_o}{b_o/2} \tag{A32}$$

Let $C_F = F_h/qS$, $n = c_n qc$, $m' = c_m' qc\bar{c}$, and $\bar{\eta}_o = \frac{y_o}{b_o/2}$. Then equation (A32) becomes

$$C_F = \frac{b_o}{b} \int_0^1 \left[c_n \frac{c}{c_{av}} \frac{\partial Z}{\partial h} + c_m' \frac{c}{c_{av}} \bar{c} \frac{\partial \phi}{\partial h}\right] d\bar{\eta}_o \tag{A33}$$

and equations (A25) and (A26) become

$$c_n = \alpha c_{n_\alpha} + q \frac{1}{2} c_{n_q} + H c_{n_H} + \dot{H} c_{n_{\dot{H}}} \tag{A34}$$

$$c_m' = \alpha c_{m_\alpha}' + q \frac{1}{2} c_{m_q}' + H c_{m_H}' + \dot{H} c_{m_{\dot{H}}}' \tag{A35}$$

where by convention, $c_{n_{\dot{\theta}}} = \frac{1}{2} c_{n_q}$, $c_{m_{\dot{\theta}}}' = \frac{1}{2} c_{m_q}'$, and $H = \frac{h}{\bar{c}}$. Substituting equations (A34) and (A35) into (A33) yields

$$C_F = \frac{b_o}{b} \int_0^1 \left[\frac{c}{c_{av}} \frac{\partial Z}{\partial h} \left(\alpha c_{n_\alpha} + q \frac{1}{2} c_{n_q} + H c_{n_H} + \dot{H} c_{n_{\dot{H}}} \right) + \right.$$

$$\left. \frac{c\bar{c}}{c_{av}} \frac{\partial \phi}{\partial h} \left(\alpha c_{m_\alpha}' + q \frac{1}{2} c_{m_q}' + H c_{m_H}' + \dot{H} c_{m_{\dot{H}}}' \right) \right] d\bar{\eta}_o \qquad (A36)$$

Partial differentiation of equation (A36) with respect to the various generalized force components α, q, H, and DH, with $\dot{H} = \frac{V}{\bar{c}} DH$, yields

$$C_{F_\alpha} = \frac{b_o}{b} \int_0^1 \left(c_{n_\alpha} \frac{c}{c_{av}} \frac{\partial Z}{\partial h} + c_{m_\alpha}' \frac{c}{c_{av}} \bar{c} \frac{\partial \phi}{\partial h} \right) d\bar{\eta}_o \qquad (A37)$$

$$\frac{1}{2} C_{F_q} = \frac{b_o}{b} \int_0^1 \left(\frac{1}{2} c_{n_q} \frac{c}{c_{av}} \frac{\partial Z}{\partial h} + \frac{1}{2} c_{m_q}' \frac{c}{c_{av}} \bar{c} \frac{\partial \phi}{\partial h} \right) d\bar{\eta}_o \qquad (A38)$$

$$C_{F_H} = \frac{b_o}{b} \int_0^1 \left(c_{n_H} \frac{c}{c_{av}} \frac{\partial Z}{\partial h} + c_{m_H}' \frac{c}{c_{av}} \bar{c} \frac{\partial \phi}{\partial h} \right) d\bar{\eta}_o \qquad (A39)$$

$$C_{F_{DH}} = \frac{b_o}{b} \int_0^1 \left(c_{n_{DH}} \frac{c}{c_{av}} \frac{\partial Z}{\partial h} + c_{m_{DH}}' \frac{c}{c_{av}} \bar{c} \frac{\partial \phi}{\partial h} \right) d\bar{\eta}_o \qquad (A40)$$

The equations of motion (eqs. (A17) to (A19) and (A29) to (A31)) are now put in nondimensional form. Time is expressed as $S = t \frac{V}{\bar{c}}$, the distance traveled in chord lengths. Nondimensional derivatives are obtained as

$$\dot{\theta} = \frac{d\theta}{dt} = \frac{V}{c}\frac{d\theta}{dS} = \frac{V}{c}D\theta$$

and

$$\ddot{h} = \frac{\ddot{h}\bar{c}}{\bar{c}} = \bar{c}\ddot{H} = \frac{\bar{c}d^2H}{dt^2} = \bar{c}\left(\frac{V}{\bar{c}}\right)^2\frac{d^2H}{dS^2} = \bar{c}\left(\frac{V}{\bar{c}}\right)^2 D^2H$$

Linear quantities are expressed in chord lengths $\bar{c}$, and frequency is expressed as $\omega = k\frac{V}{\bar{c}}$. Force equations are divided by $\frac{\rho}{2}V^2S$ and moment equations by $\frac{\rho}{2}V^2S\bar{c}$. The resulting three equations of motion in nondimensional form are

$$2\mu D(\alpha - \theta) + 2A_{Zh}D^2H - C_{N_o} - \alpha\left(C_{N_\alpha}\right) - D\alpha\left(\frac{1}{2}C_{N_{D\alpha}}\right) - D\theta\left(\frac{1}{2}C_{N_q}\right) -$$

$$H\left(C_{N_H}\right) - DH\left(C_{N_{DH}}\right) = C_{N_\delta}\delta \tag{A41}$$

$$2\mu K_Y{}^2D^2\theta + 2A_{\theta h}D^2H - \alpha\left(C_{m_\alpha}\right) - D\alpha\left(\frac{1}{2}C_{m_{D\alpha}}\right) - D\theta\left(\frac{1}{2}C_{m_q}\right) -$$

$$H\left(C_{m_H}\right) - DH\left(C_{m_{DH}}\right) = C_{m_\delta}\delta \tag{A42}$$

$$2A_{hh}D^2H + 2A_{Zh}D(\alpha - \theta) + 2A_{\theta h}D^2\theta + 2A_{hh}k^2H - \alpha\left(C_{F_\alpha}\right) - D\theta\left(\frac{1}{2}C_{F_q}\right) -$$

$$H\left(C_{F_H}\right) - DH\left(C_{F_{DH}}\right) = C_{F_\delta}\delta \tag{A43}$$

The elastic properties of the flexible-wing mode are manifested primarily in the k^2 term of equation (A43). The elastic mode shape of the flexible wing is employed implicitly in the generalized mass and force terms pertaining to the flexible mode.

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TABLE I.- CONFIGURATIONS AND PERTINENT CHARACTERISTICS

[S = 1,460 sq ft; l_t = 46.5 ft; e.a. location, 38 percent wing chord; wing section c.g. location, 38 percent wing chord (approx.)]

Configuration			$\bar{c}$, ft	A	Wing ground natural frequency, ω, radians/sec
Sweep angle, Λ, deg	m_W/m_A	c.g. location, percent $\bar{c}$			
0	0.15	25	11	13.4	9.87
		35	11	13.4	9.87
		45	11	13.4	9.87
	.33	25	11	13.4	5.98
		35	11	13.4	5.98
		45	11	13.4	5.98
	.50	25	11	13.4	4.21
		35	11	13.4	4.21
		45	11	13.4	4.21
40	.33	25	14.38	7.9	5.98
		35	14.38	7.9	5.98
		45	14.38	7.9	5.98
60	.33	25	22	3.4	5.98
		35	22	3.4	5.98
		45	22	3.4	5.98

TABLE II.- INERTIA TERMS AND DAMPING AND RESTORING-SPRING PARAMETERS

Sweep angle, Λ, deg	m_W/m_A	c.g. location, percent $\bar{c}$	Parameters																			
			μ	A_{hh}	$A_{\alpha h}$	$A_{\theta h}$	K_Y^2	C_{N_α}	C_{m_α}	$\frac{1}{2} C_{N_{D\alpha}}$	$\frac{1}{2} C_{m_{D\alpha}}$	$\frac{1}{2} C_{N_q}$	$\frac{1}{2} C_{m_q}$	C_{N_H}	C_{m_H}	$C_{N_{DH}}$	$C_{m_{DH}}$	C_{F_α}	$C_{F_{D\alpha}}$	$\frac{1}{2} C_{F_q}$	C_{F_H}	$C_{F_{DH}}$
0	0.15	25	129.5	2.36	4.6	0.6	3.32	-5.66	-1.25	-1.07	-4.52	-3.96	-11.69	0.042	0	-1.39	0	-1.39	0	-0.224	0.019	-0.77
		35	129.5	2.36	4.6	.13	3.32	-5.66	-.68	-1.04	-4.3	-3.44	-11.17	.042	0	-1.39	0	-1.39	0	-.134	.019	-.77
		45	129.5	2.36	4.6	-.32	3.32	-5.66	-.11	-1.02	-4.1	-2.93	-10.65	.042	0	-1.39	0	-1.39	0	-.045	.019	-.77
	.33	25	164	6.45	12.55	1.63	2.64	-5.66	-1.25	-1.07	-4.52	-3.96	-11.69	.042	0	-1.39	0	-1.39	0	-.224	.019	-.77
		35	164	6.45	12.55	.38	2.64	-5.66	-.68	-1.04	-4.3	-3.44	-11.17	.042	0	-1.39	0	-1.39	0	-.134	.019	-.77
		45	164	6.45	12.55	-.88	2.64	-5.66	-.11	-1.02	-4.1	-2.93	-10.65	.042	0	-1.39	0	-1.39	0	-.045	.019	-.77
	.50	25	219	12.9	25.2	3.27	1.98	-5.66	-1.25	-1.07	-4.52	-3.96	-11.69	.042	0	-1.39	0	-1.39	0	-.224	.019	-.77
		35	219	12.9	25.2	.75	1.98	-5.66	-.68	-1.04	-4.3	-3.44	-11.17	.042	0	-1.39	0	-1.39	0	-.134	.019	-.77
		45	219	12.9	25.2	-1.76	1.98	-5.66	-.11	-1.02	-4.1	-2.93	-10.65	.042	0	-1.39	0	-1.39	0	-.045	.019	-.77
40	.33	25	125.8	4.93	9.61	9.61	1.75	-4.24	-.93	-.82	-2.65	-2.98	-9.25	-.40	-.268	-1.00	-.95	-1.00	0	-.848	-.177	-.55
		35	125.8	4.93	9.61	8.65	1.75	-4.24	-.51	-.79	-2.49	-2.61	-8.85	-.40	-.268	-1.00	-.95	-1.00	0	-.783	-.177	-.55
		45	125.8	4.93	9.61	7.68	1.75	-4.24	-.08	-.77	-2.33	-2.24	-8.46	-.40	-.268	-1.00	-.95	-1.00	0	-.719	-.177	-.55
60	.33	25	82.1	3.22	6.28	6.07	.83	-2.98	-.65	-.53	-1.13	-1.91	-4.01	-.50	-.319	-.62	-.564	-.62	0	-.416	-.222	-.34
		35	82.1	3.22	6.28	5.44	.83	-2.98	-.36	-.51	-1.02	-1.68	-3.75	-.50	-.319	-.62	-.564	-.62	0	-.378	-.222	-.34
		45	82.1	3.22	6.28	4.82	.83	-2.98	-.06	-.48	-.92	-1.45	-3.50	-.50	-.319	-.62	-.564	-.62	0	-.342	-.222	-.34

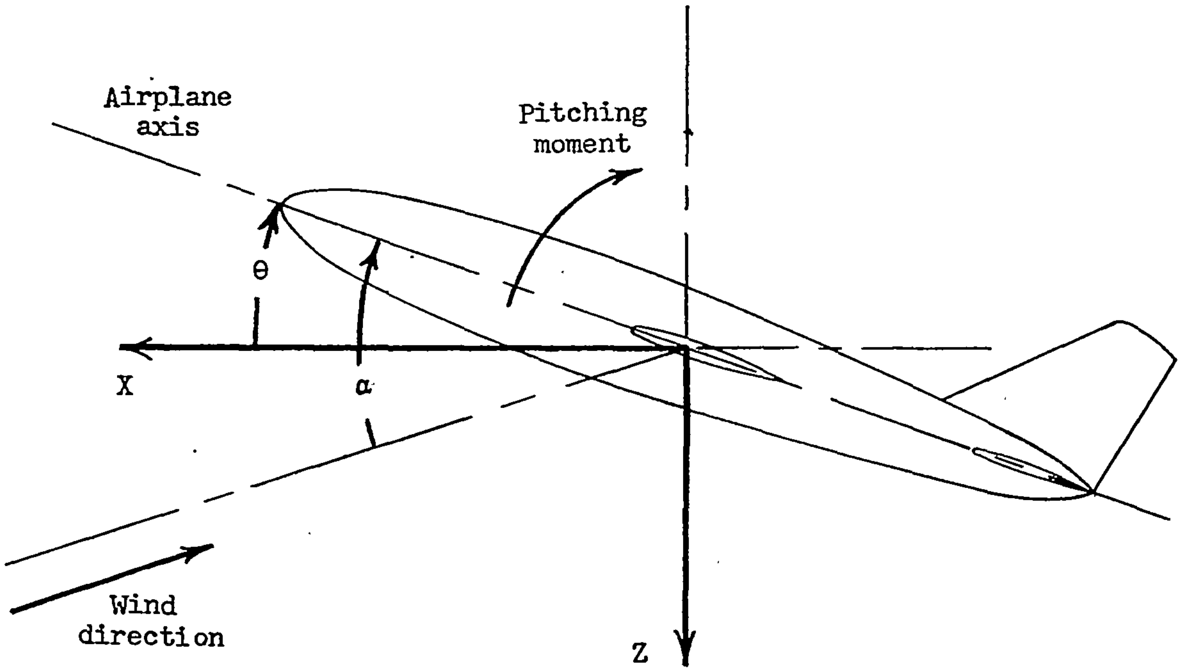


Figure 1.- A diagram showing the stability axes and positive directions of forces and moments.

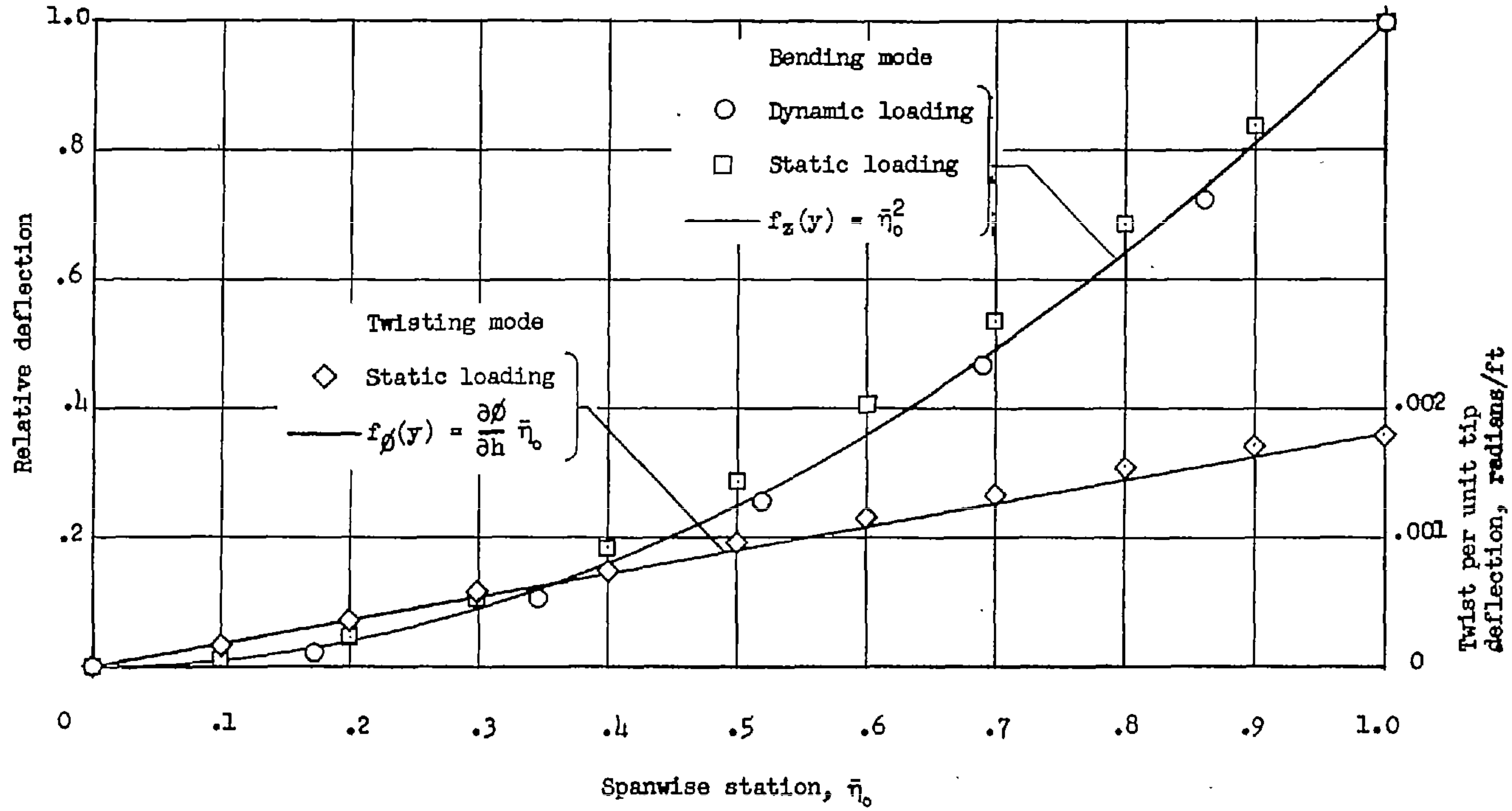


Figure 2.- Bending and twisting modes of the flexible wing.

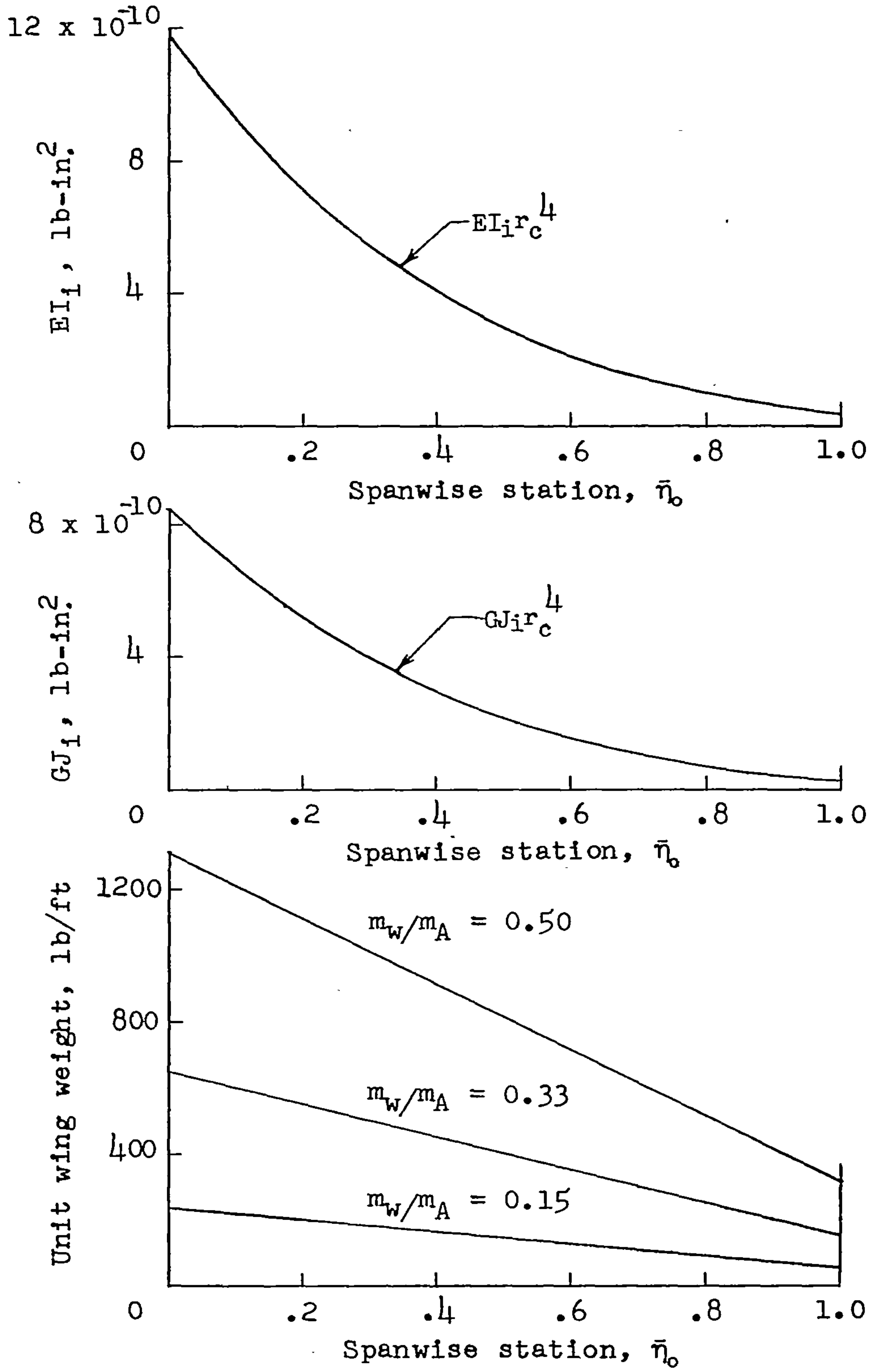


Figure 3.- Wing spanwise elastic and weight distributions.

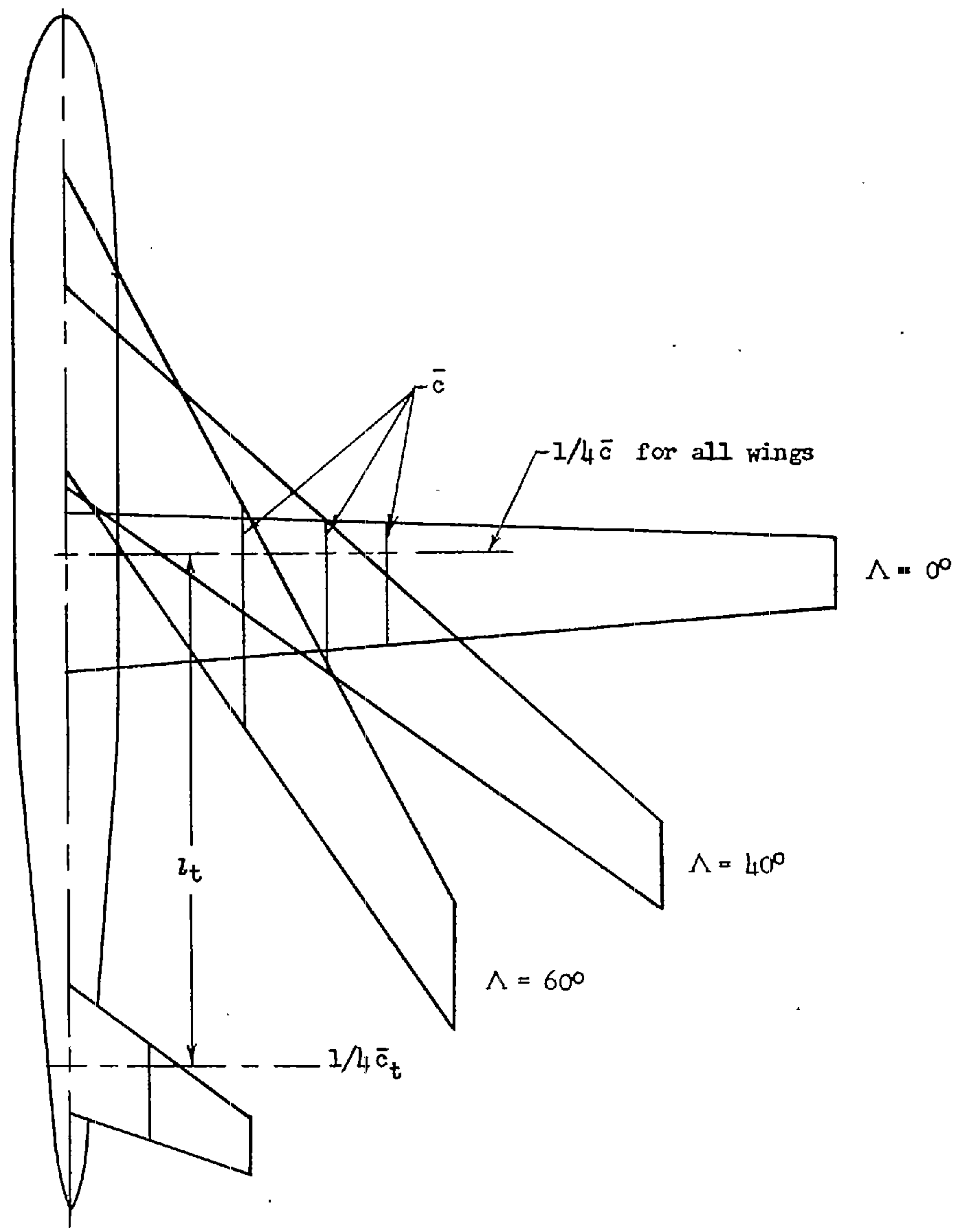
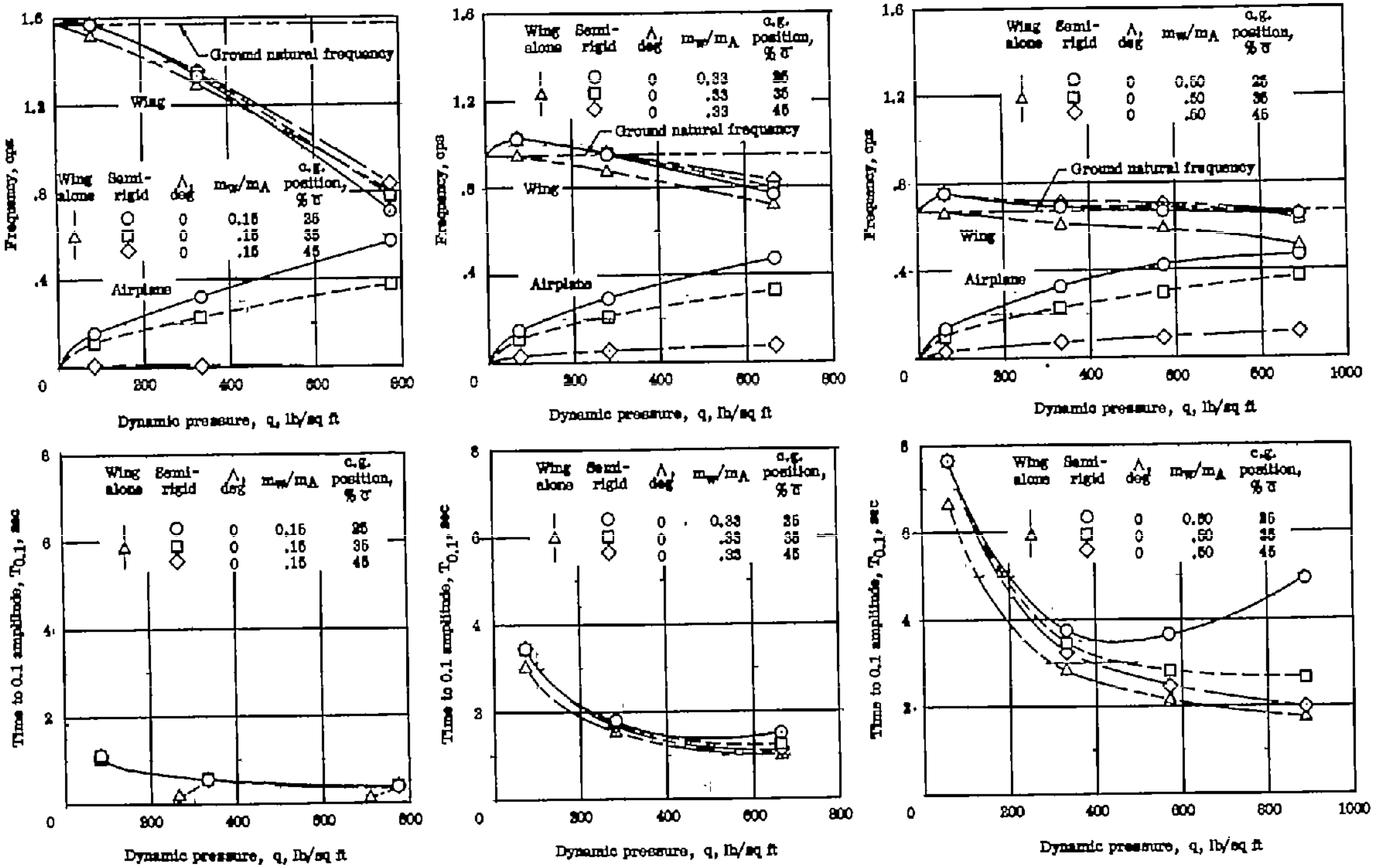
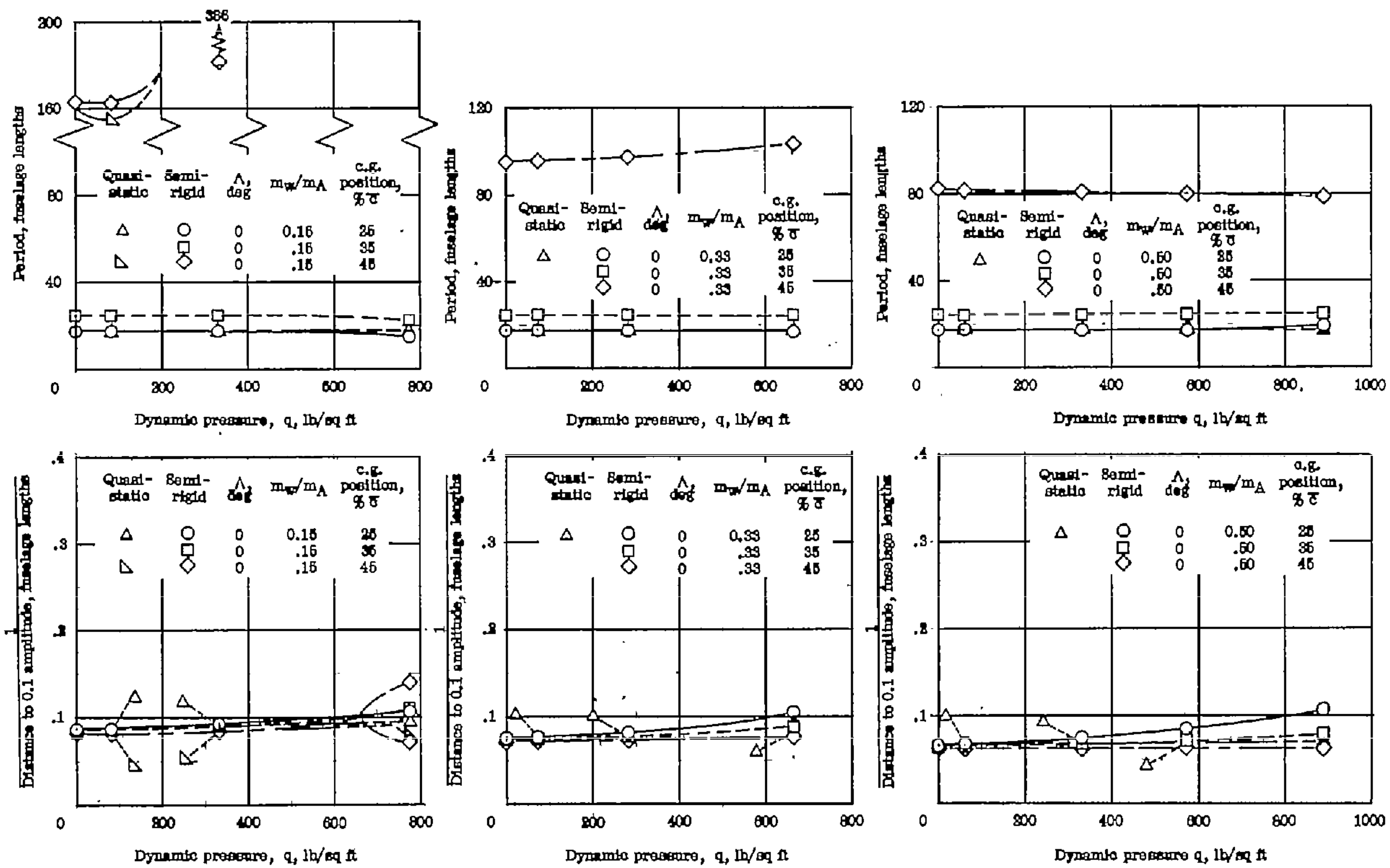


Figure 4.- Plan forms of 0°, 40°, and 60° configurations. l_t = Constant.



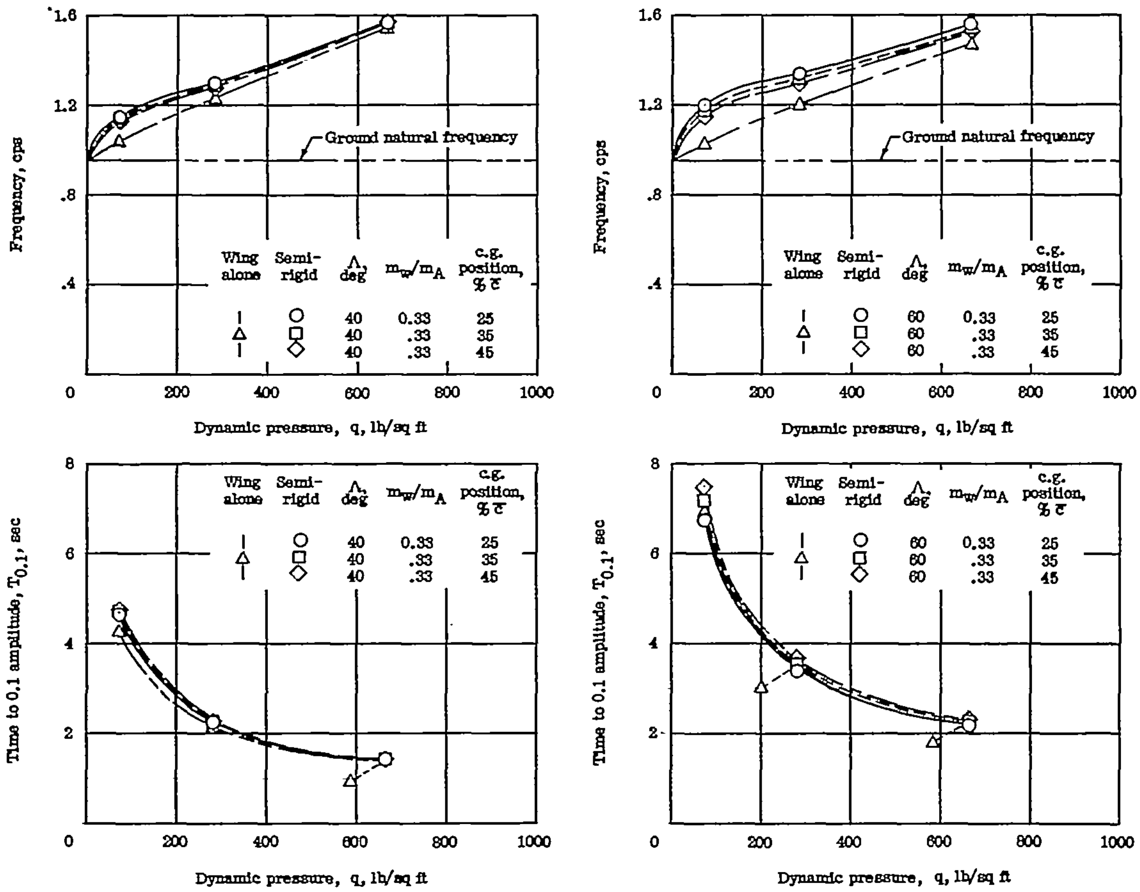
(a) Frequency and damping of elastic wing mode and frequency of airplane short-period longitudinal stability mode in dimensional terms as a function of dynamic pressure.

Figure 5.- Dynamic stability of airplane longitudinal mode and flexible wing mode obtained by the semirigid method and less rigorous methods as a function of dynamic pressure. Altitude, 8,000 feet; $\Lambda = 0^{\circ}$.



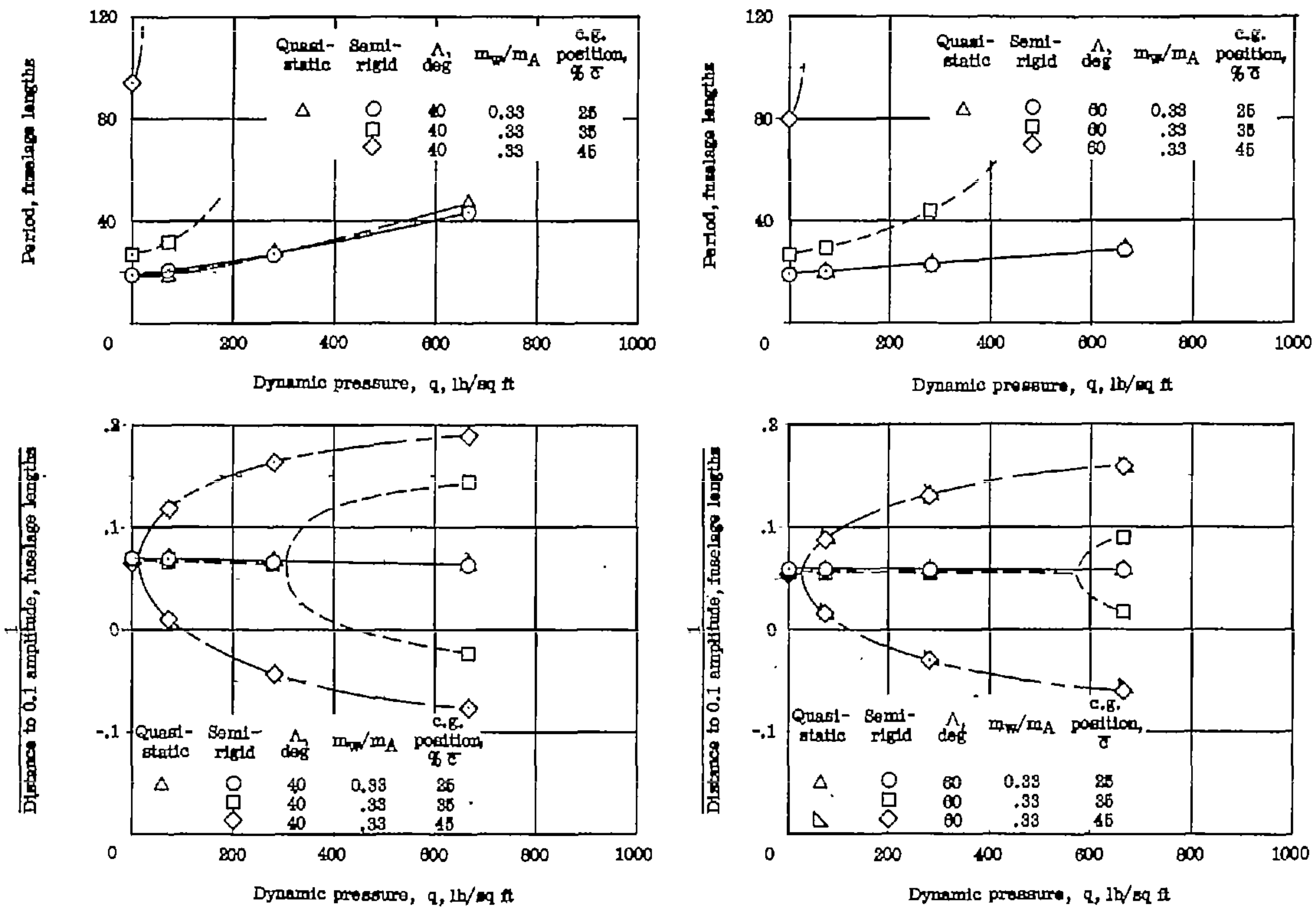
(b) Period and damping of airplane short-period longitudinal stability mode in nondimensional terms as a function of dynamic pressure.

Figure 5.- Concluded.



(a) Frequency and damping of elastic wing mode in dimensional terms as a function of dynamic pressure.

Figure 6.- Dynamic stability of airplane longitudinal mode and flexible wing mode obtained by the semirigid method and less rigorous methods as a function of dynamic pressure. Altitude, 8,000 feet; $\Lambda = 40^{\circ}$ and 60°.



(b) Period and damping of airplane short-period longitudinal stability mode in nondimensional terms as a function of dynamic pressure.

Figure 6.- Concluded.

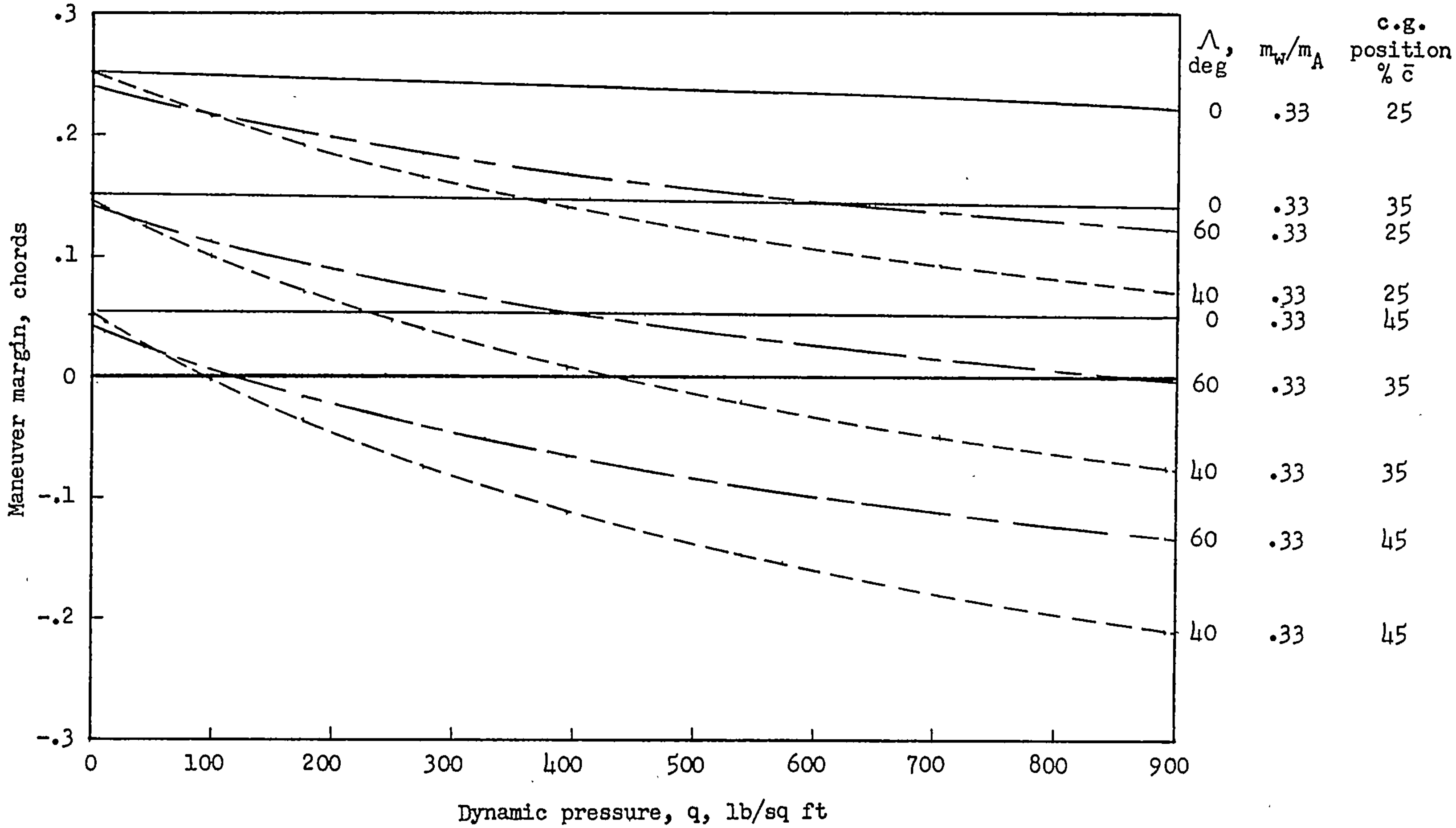
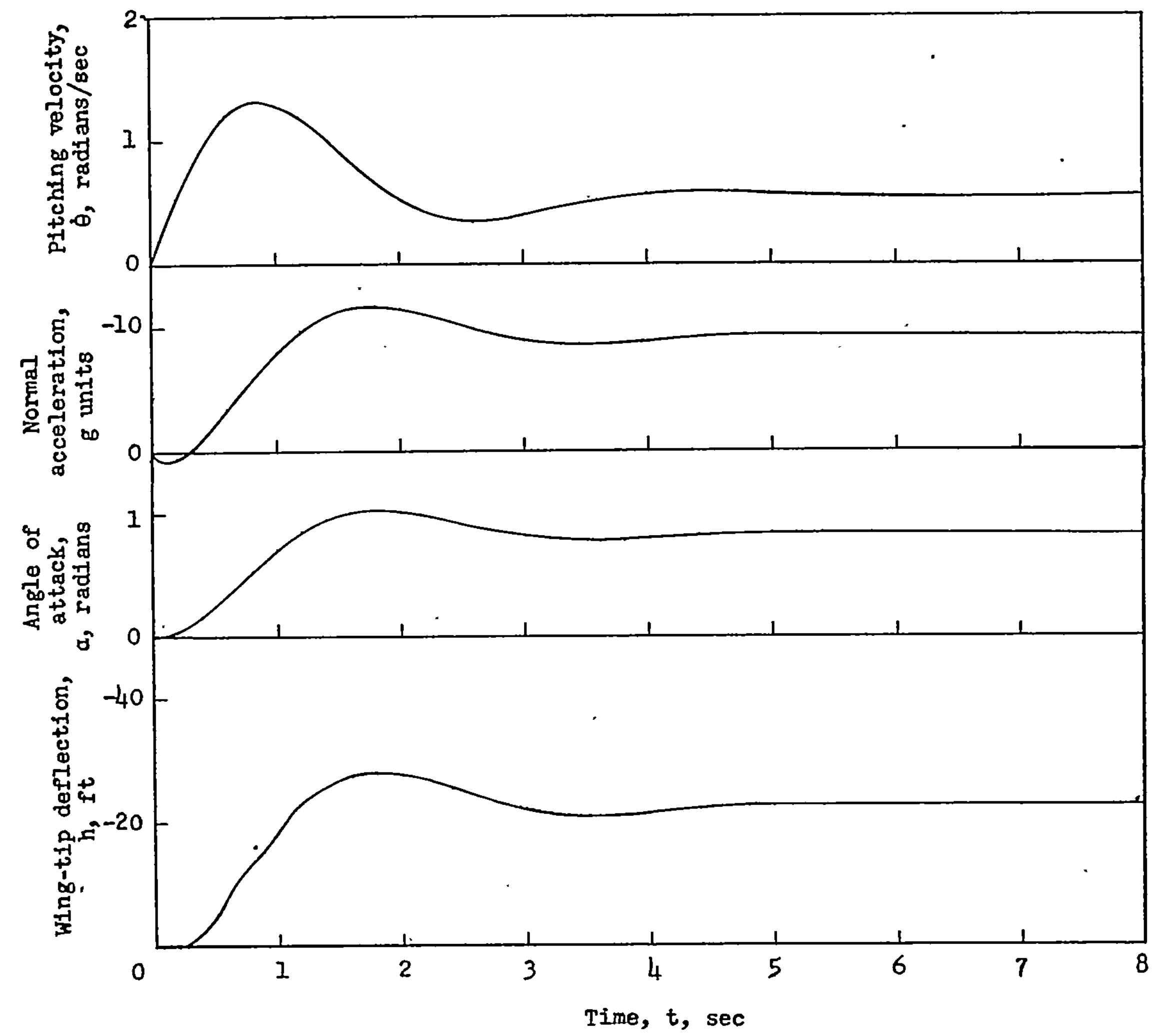
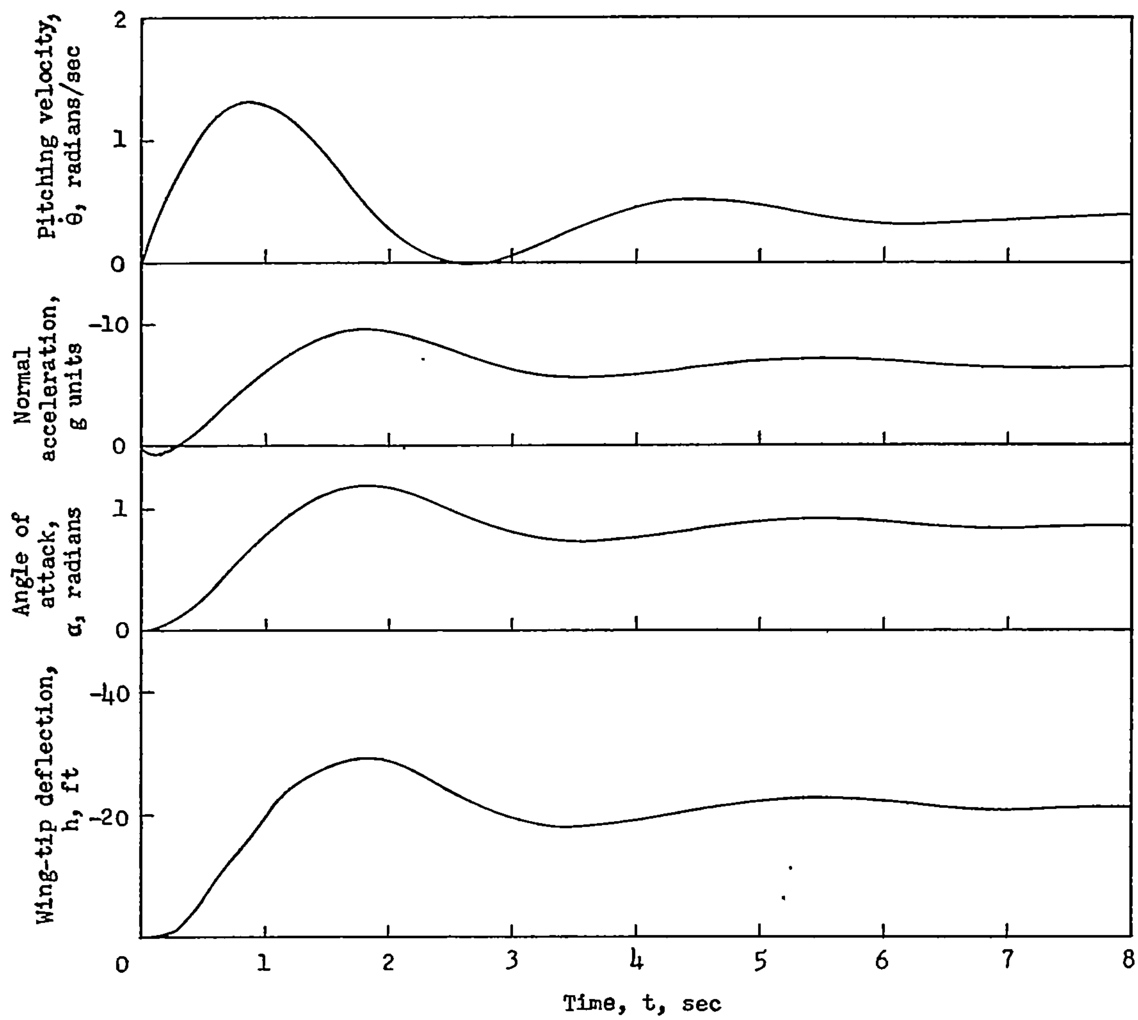


Figure 7.- Variation of the maneuver margin with dynamic pressure for configurations with flexible wings. Quasi-static method. Altitude, 8,000 feet.



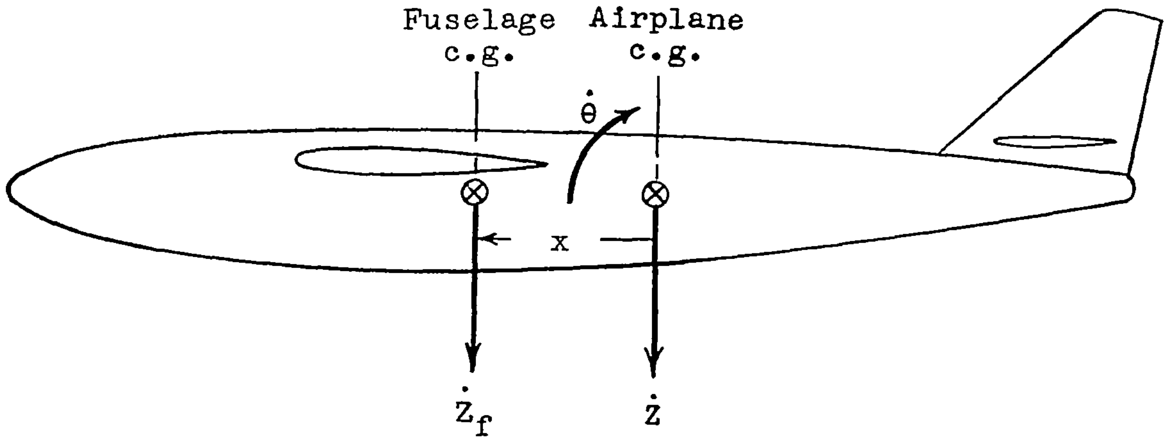
(a) Altitude, 8,000 feet; Mach number, 0.4; q = 176 lb/sq ft.

Figure 8.- Longitudinal transient responses of typical configuration with flexible wing to 1-radian step inputs of the elevator at altitudes of 8,000 feet and 30,000 feet. Center-of-gravity location, 20 percent mean aerodynamic chord; $\Lambda = 35^{\circ}$.

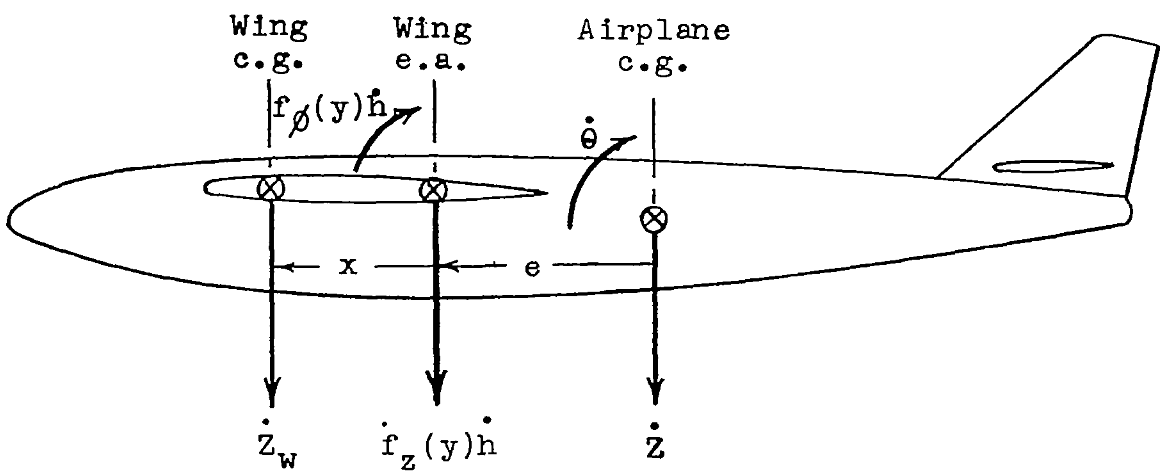


(b) Altitude, 30,000 feet; Mach number, 0.6; q = 158 lb/sq ft.

Figure 8.- Concluded.



(a) Geometric relation of fuselage center of gravity to airplane center of gravity.



(b) Geometric relation of local wing center of gravity to airplane center of gravity.

Figure 9.- Linear and angular velocities and geometric relations of the fuselage and wing centers of gravity to the airplane center of gravity.

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