NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 3639

APPROXIMATE INDICIAL LIFT FUNCTIONS FOR SEVERAL WINGS OF FINITE SPAN IN INCOMPRESSIBLE FLOW AS OBTAINED FROM OSCILLATORY LIFT COEFFICIENTS

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SUMMARY

The unsteady-lift functions for a wing undergoing a sudden change in sinking speed have been presented for delta wings having aspect ratios of 0, 2, and 4 and for rectangular and elliptical wings having aspect ratios of 0, 3, and 6. For the elliptical and rectangular wings the spanwise lift distributions were also presented. These functions were calculated from the lift coefficients associated with a wing oscillating harmonically in pure translational motion, as obtained from several sources.

The results of these calculations indicate that the normalized unsteady-lift functions are substantially independent of the shape of the plan form for elliptical, rectangular, or moderately tapered wings; however, for delta wings the increase of lift toward the steady-state value is much more rapid than that for the aforementioned wings of the same aspect ratio. These results also corroborate the results of other investigations in that the rate of growth of lift tends to increase with a decrease in aspect ratio. The shape of the spanwise distributions of the indicial lift seems to be, for all practical purposes, independent of time for rectangular and elliptical wings.

INTRODUCTION

The indicial lift functions due to a sudden change in sinking speed and due to the penetration of a sharp-edge normal gust (hereinafter designated as $k_1(s)$ and $k_2(s)$, respectively) are fundamental in the calculation of airplane transient motions and gust loads. These functions have been determined, however, for only a few wing configurations in incompressible flow and for only wings of infinite aspect ratio in compressible subsonic flow. Initial derivations of the functions $k_1(s)$ and $k_2(s)$ were made for two-dimensional incompressible flow by Wagner (ref. 1) and Küssner (ref. 2), respectively; in reference 3 Von Kármán and Sears pointed out and corrected an error in reference 2. Indicial lift functions for incompressible flow have been calculated by Robert T. Jones (ref. 4) for elliptical wings having aspect ratios of 3 and 6 and by W. Prichard Jones (ref. 5) for rectangular wings with aspect ratios of 4 and 6 and also for a moderately tapered wing having an aspect ratio of 5.84. For subsonic compressible flow, the indicial lift functions have been determined for infinite-aspect-ratio wings in references 6, 7, and 8. No information appears to be available, however, on these functions for wings of finite aspect ratio in subsonic compressible flow, although current work on oscillatory lift coefficients may in time lead to further information on the indicial lift functions for this condition.

The main purpose of the present report is to extend the knowledge of the incompressible-flow indicial function $k_1(s)$ to a plan form of current interest - namely, the delta wing - and to make a comparison of the k_1 functions for elliptical, rectangular, tapered, and delta wings in order to assess the effects of plan-form shapes and aspect ratios. An additional purpose is to present some information on the variation of the spanwise distribution of the indicial lift function $k_1(s)$ since no published information on this distribution has been found. Such information is of interest in the determination of transient stresses in wing structures.

The method used to obtain the k_1 function was similar to that used to obtain the functions for a wing in two-dimensional subsonic compressible flow, as reported in references 6 and 7. The indicial functions were obtained from their associated oscillatory lift coefficients (commonly used in flutter and frequency-response calculations) by means of Garrick's reciprocal relations (ref. 9). The oscillatory coefficients were determined by means of the methods presented by Lawrence and Gerber in reference 10 and by Reissner and Stevens in reference 11. (The method of ref. 11 also allows calculation of local oscillatory lift coefficients, leading to spanwise distribution of the indicial lift functions.)

The indicial lift functions $k_1(s)$ were derived for delta wings with aspect ratios of 2 and 4, and the spanwise variation of k_1 functions was determined for elliptical and rectangular wings with aspect ratios of 3 and 6. Also calculated were the unsteady-lift functions $k_1(s)$ and $k_2(s)$ for elliptical, rectangular, and delta wings of vanishingly small aspect ratio by a method based on an extension of the low-aspect-ratio theory of Robert T. Jones (ref. 12). The unsteady-lift functions previously obtained in references 1 to 5 are presented herein to show the effects of wing plan-form shape and aspect ratio. The limitations of the methods used in calculating the oscillatory coefficients precluded the calculation of the spanwise variation of the function $k_1(s)$ for the delta wing and, inasmuch as the proper oscillatory lift coefficients (for an oscillating airstream) were not available, the function $k_2(s)$ was not obtained either.

SYMBOLS

A aspect ratio

$b/2$ semispan of wing

$b(x)/2$ spanwise coordinate of the leading edge of wing, measured from root chord of wing

C_L steady-state lift coefficient

c section chord

$\bar{c}$ mean geometric chord

c_l section lift coefficient

c_r root chord of wing

$\bar{F}(k)$ real part of the total lift coefficient for a three-dimensional wing oscillating harmonically in pure translational motion in incompressible flow

$\bar{F}_l(k)$ real part of the local lift coefficient for a three-dimensional wing oscillating harmonically in pure translational motion in incompressible flow

$\dot{h}(t)$ vertical velocity of wing

$k = \omega c_r/2V$

$k_1(s)$ indicial lift functions for a wing experiencing a sudden change in sinking speed, normalized to unity by its steady-state value

$k_2(s)$ indicial lift function for a wing penetrating a sharp-edge gust, normalized to unity by its steady-state value

$l(x)$ lift per unit length

$q = \frac{1}{2} \rho V^2$

S	wing area
s	nondimensional distance traveled, root semichords
t	time
V	forward velocity
W_0	amplitude of vertical velocity
x	coordinate along root chord of wing, measured rearward from the leading edge
y	spanwise coordinate of wing
ρ	air density
ω	circular frequency

METHOD

The method used to obtain the k_1 function was similar to that used to obtain the functions for a wing in two-dimensional compressible flow, as reported in references 6 and 7. This method, which avoids some of the difficulties associated with the direct calculation of the k_1 function, makes use of Garrick's reciprocal relation (ref. 9) between the k_1 function and the oscillatory lift coefficients for a wing oscillating in pure translational motion.

A survey of some of the available methods for calculating oscillatory lift coefficients indicated that, as a result of limits in ranges of aspect ratio and wing geometry implied by various simplifying assumptions, no one method should be applied to all the wings considered herein. Consequently, two methods which appeared to provide a maximum of information for a reasonable effort were selected. These two methods, namely, that of Lawrence and Gerber (ref. 10) and that of Reissner and Stevens (ref. 11), do not directly apply, however, to all wings considered herein and, therefore, the consequences of extending the methods beyond their original limits were investigated. The results of this study together with a discussion of the characteristic features and limitations of each method are presented in the sections that follow.

Method of Reissner and Stevens

The method of Reissner and Stevens (ref. 11) for the calculation of oscillatory lift coefficients is characterized by simplifications which involve a basically two-dimensional analysis of strips taken along the span of the wing. The simplifications utilized by Reissner and Stevens reduce the surface integral equation to a single-variable integral equation in terms of the spanwise variable. The resulting equation is then solved by satisfying the boundary conditions along the span of the wing; the solution gives values of $\bar{F}_l(k)$, the real part of the local complex oscillatory lift coefficient. The simplifications used in the Reissner and Stevens method imply that the method is best suited for wings of high aspect ratio; in fact, the limiting two-dimensional case is contained directly.

The accuracy of the results obtained through the use of the Reissner and Stevens method for wings having aspect ratios as low as 3 was investigated for elliptical, rectangular, and delta plan forms. Estimates of the accuracy for elliptical and rectangular wings with aspect ratios of 3 were obtained by comparing k_1 functions based on the Reissner and Stevens method with a k_1 function obtained by Robert T. Jones in reference 4 for the elliptical wing and with a k_1 function based on the method of Lawrence and Gerber (ref. 10) for the rectangular wing. The results for the elliptical and rectangular wings are presented in figures 1 and 2, respectively. In both figures the agreement between the curves is considered satisfactory for values of s greater than 2. For s less than 2 the results obtained by use of the method of Reissner and Stevens and by that of Lawrence and Gerber are shown dashed to indicate a region where the functions were believed to be unreliable for reasons to be discussed in a section to follow. The accuracy of the results obtained by the Reissner and Stevens method for a delta wing having an aspect ratio of 4 was estimated by comparing the $\bar{F}_l(k)$ function obtained thereby with that obtained from the Lawrence and Gerber method. Appreciable differences in both shape and magnitude were observed and, inasmuch as the latter method was considered to be the more accurate, the Reissner and Stevens method was not used to determine either the local or the complete $k_1(s)$ function for the delta wing.

Method of Lawrence and Gerber

The method of Lawrence and Gerber (ref. 10) for the calculation of oscillatory lift coefficients is characterized by the reduction of the surface integral equation to a single-variable integral equation in terms of a chordwise variable by the choice of a spanwise weighting function. The lifting equation is then solved by satisfying the boundary conditions at various stations along the chord. The method carries the restrictions that the plan form have a straight trailing edge normal to the stream. The simplifying assumptions in the method of Lawrence and Gerber imply that the method is best suited for wings of low aspect ratio.

The accuracy of results obtained through use of the Lawrence and Gerber method for wings having aspect ratios as high as 4 was estimated by comparing the k_1 function for a rectangular wing with an aspect ratio of 6 based on coefficients calculated by the Lawrence and Gerber method with the k_1 function obtained by W. Prichard Jones (ref. 5) for the same wing. As is indicated in figure 3, in which the two k_1 functions are presented, there is only a difference of approximately 3 percent between the two functions for values of s greater than 2. As in figure 2, the function based on coefficients obtained by the Lawrence and Gerber method is shown dashed in a region where the function was believed to be unreliable.

Application of Methods

In view of the results presented in figures 1 to 3, the k_1 function for the delta wing was calculated from oscillatory lift coefficients obtained by use of the method of Lawrence and Gerber; consequently, the spanwise variation of local k_1 functions was not determined for the delta wing. Local k_1 functions at various spanwise stations were obtained by use of the Reissner and Stevens method, however, for rectangular and elliptical wings.

Lack of information precluded the calculation of the indicial lift function $k_2(s)$ for gust penetration of wings of finite aspect ratio. Although it is shown in reference 9 that the function $k_2(s)$ can be obtained directly from the k_1 function for wings in two-dimensional flow, the relation used is one derived from closed-form expressions for the lift response to sinusoidal sinking oscillations and to sinusoidal gusts; this relation is not valid for wings of finite aspect ratio. The determination of the k_2 function requires a knowledge of the lift due to sinusoidal gusts or due to harmonically oscillating flaps for all values of flap-chord ratio; this information is not available for wings of finite span. The k_2 and k_1 functions for several wings of vanishingly small aspect ratio were determined, however, to aid in the illustration of trends of these functions with aspect ratio. A derivation of expressions for these functions for vanishingly small aspect ratio is given in the appendix, and the indicial lift functions for rectangular-, elliptical-, and delta-wing plan forms are presented in table I.

Method of Calculation

The indicial lift function for a wing undergoing a sudden change in sinking speed can be calculated from the real part of the complex lift coefficients for a wing oscillating in pure translational motion by means of the reciprocal relation (ref. 9)

$$k_1(s) = \frac{2}{\pi}\int_0^\infty \frac{\bar{F}(k)\ \sin ks}{k}\,dk \tag{1}$$

The indicial lift functions $k_1(s)$ for delta and rectangular wings were calculated from equation (1) by using the $\bar{F}(k)$ coefficients presented in reference 10 together with additional coefficients which were calculated in increments of $k = 0.2$ by the method of reference 10 for reduced frequencies $k \leqq 2$. The validity of the method of reference 10 for very high reduced frequencies has been investigated and the method has been found to be inapplicable directly for high frequencies. Consequently, the $\bar{F}(k)$ coefficients were faired for $2 < k < \infty$ to a value of $\bar{F}(\infty) = 1$. This value has been chosen because the method of reference 10 is primarily a low-aspect-ratio theory, and for vanishingly small aspect ratios the starting lift $k_1(0)$, which has been shown in reference 7 to be equal to $\bar{F}(\infty)$, is presented in the appendix and is equal to 1.0. The fairing of $\bar{F}(k)$ was in the form of a part of a hyperbola, as used in a similar problem reported in reference 6. Reference 6 provides a means of estimating the error due to fairing as it affects the k_1 function, and for the present calculations it is indicated that the error is significant only for wing travel s (semichords) less than 2; hence, in this region the curves presented are shown by dashed lines. Equation (1) was evaluated by numerical integration for $k \leqq 2$ and by analytical integration for $k > 2$. The k_1 functions for rectangular wings with aspect ratios of 3 and 6 (the former being presented in fig. 2) are in agreement with those derived from reference 11, and it is concluded therefore that the $\bar{F}(k)$ coefficients as calculated from reference 10 are accurate for values of k at least as large as 2.0.

The procedure followed in the calculation of the "local" k_1 functions for elliptical and rectangular wings was the same as that for $k_1(s)$ for the delta wing except that the oscillatory coefficients used in equation (1) were local values $\bar{F}_l(k)$ as calculated by the method of reference 11, and the $\bar{F}_l(k)$ curves were faired to the two-dimensional values of $\bar{F}_l(\infty) = \pi/C_L$, which was consistent with the method of reference 11.

The k_1 functions for the complete elliptical and rectangular wings were obtained by spanwise graphical integration of the local k_1 functions at six equally spaced stations along the wing semispan. Six stations were used rather than the four used in reference 11 because the lift distributions obtained from four stations were believed to be insufficiently defined for the purpose of this report. One of the six stations chosen was at the wing tip, where the lift was assumed to be zero.

RESULTS AND DISCUSSION

Delta-Plan-Form k_1 Function

The indicial lift functions for a sudden change in sinking speed $k_1(s)$ were determined for delta wings of aspect ratios 0, 2, and 4 and are presented in figure 4. It might be noted that, for the delta wing as in the cases of wings of other plan forms, a decrease in aspect ratio results in a more rapid growth of lift to the steady-state value.

Indicial Spanwise Loading Distributions

The local indicial lift functions for a sudden change in sinking speed were calculated for six stations along the wing semispan on both elliptical and rectangular wings of aspect ratios 3 and 6. These data for rectangular wings are presented in figure 5 in the form of spanwise local lift coefficients per unit wing lift coefficient for several values of wing distance traveled. As indicated by figure 5, the spanwise distributions are substantially independent of time, with the possible exception of the very beginning of motion. No indication of the initial spanwise distribution can be given, however, since the theory is not valid in this region. The data for the elliptical wings exhibited characteristics similar to those for the rectangular wings in that the spanwise distributions were, for all practical purposes, equal to the steady-state lift distribution, which in this case is elliptical.

Effect of Plan-Form Shape on the k_1 Function

The effects of plan-form shape on the k_1 functions for elliptical, rectangular, moderately tapered, and delta wings are shown in figure 6. The k_1 functions for the first three wings of aspect ratio 6 (aspect ratio of 5.84 for tapered wing) are grouped together in figure 6(a) wherein it is indicated that the k_1 functions are almost identical. The results for elliptical and rectangular wings of aspect ratio 3, although not plotted, show similar characteristics.

The k_1 functions for rectangular and delta wings of aspect ratio 4 are plotted in figure 6(b). Inasmuch as it has been established that the function $k_1(s)$ for a rectangular wing is substantially the same as those for elliptical and moderately tapered wings of the same aspect ratio, it may be concluded from figure 6(b) that the delta wing exhibits a more rapid growth of lift to the steady-state value than do the other three plan forms considered.

The functions $k_1(s)$ presented in figure 6 were obtained from the following sources: The indicial functions for the elliptical wing were reproduced from reference 4, whereas those for the rectangular and tapered wings were reproduced from reference 5. The delta-wing indicial function was reproduced from figure 4.

Effect of Aspect Ratio on the k_1 Function

The effect of aspect ratio on the k_1 function for delta wings has already been indicated in figure 4 and for elliptical, rectangular, and moderately tapered wings this effect is indicated in figure 7, in which k_1 functions are plotted for rectangular wings having aspect ratios of 0, 3, 4, 6, and ∞. The curves in figure 7 were obtained from the following sources: The function for zero aspect ratio was determined by the method presented in the appendix of the present report. The functions for aspect ratios of 3 and 6 were calculated by means of the method of reference 10. The functions for aspect ratios of 4 and ∞ were obtained from references 5 and 1, respectively. As shown previously, the results for the rectangular wings are representative of those for elliptical and moderately tapered wings. The general effect of decrease in aspect ratio for the plan forms considered is a more rapid growth of lift toward the steady-state value, as indicated by figures 4 and 7.

Inasmuch as the indicial functions presented herein are normalized to unity at the steady-state condition ($s = \infty$ or $k = 0$), the application of these functions requires a knowledge of the steady-state lift-curve slope. Although the normalized indicial lift functions $k_1(s)$ have been shown to be substantially independent of method of calculation (at least for elliptical, rectangular, and moderately tapered wings), the associated steady-state lift-curve slopes can be shown to vary appreciably with method of calculation. Because a choice of a range of values of lift-curve slope for a given normalized function must be made, the choice might as well be from a source one considers most reliable.

Effect of Aspect Ratio on the k_2 Function

The effect of aspect ratio and shape of plan form on the k_2 function is indicated in figure 8. In figure 8(a) is plotted the k_2 function for rectangular wings of aspect ratios 0, 4, and 6, and in figure 8(b) is plotted the k_2 function for elliptical wings of aspect ratios 0, 3, and 6. The zero-aspect-ratio functions for both wing shapes are those calculated by means of the method given in the appendix. The remaining k_2 functions for the rectangular and elliptical wings were reproduced from references 5 and 4, respectively. The effect of aspect ratio is generally the same as it was for the k_1 functions. Although not shown here, it might be interesting to note that the k_2 functions for the elliptical and rectangular wings of aspect ratio 6 are almost the same, as is the case for the k_1 functions. However, for aspect ratios of 3 or less the k_2 functions differ significantly, so that for low aspect ratios the effect of plan-form shape appears to be more significant for the k_2 function than it is for the k_1 function.

CONCLUSIONS

The indicial lift functions for a wing undergoing a change in sinking speed in an incompressible medium have been calculated from the oscillatory lift coefficients for a wing oscillating harmonically in pure translational motion. These functions are presented for delta wings having aspect ratios of 0, 2, and 4, and the local as well as the total functions are presented for elliptical and rectangular wings having aspect ratios of 0, 3, and 6.

The present investigation indicates the following results for the cases considered herein:

1. The growth of lift for delta wings is much more rapid than that for elliptical and rectangular wings of the same aspect ratio.

2. For elliptical or rectangular wings, the spanwise distributions of the lift are substantially independent of time.

3. The normalized unsteady-lift functions are relatively independent of plan-form shape for elliptical, rectangular, or moderately tapered wings.

4. The growth of lift, for all wings investigated, is more rapid as the aspect ratio is decreased.

Langley Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., January 11, 1956.

APPENDIX

INDICIAL LIFT OF WINGS OF ZERO ASPECT RATIO

The lift per unit length along a slender body or a wing of very low aspect ratio undergoing pure translational motion is shown in reference 13 to be

$$l(x) = \left(\frac{\partial}{\partial t} + V \frac{\partial}{\partial x}\right)\left[\pi\rho \frac{b^2}{4}(x)\dot{h}\right] \tag{A1}$$

where $\frac{b(x)}{2}$ is designated as $R(s)$ in reference 13.

For pure sinking motion

$$\dot{h}(t) = W_0 \mathbf{1}(t) \tag{A2}$$

where $\mathbf{1}(t)$ is the unit jump function. The total lift coefficient can then be written as

$$qSC_L \frac{W_0}{V} k_1(s) = \int_0^{c_r} l(x)dx$$

$$k_1(s) = \mathbf{1}(s) + \delta(s) \int_0^2 \left[\frac{b(x^*)}{b}\right]^2 dx^* \tag{A3}$$

where c_r is root chord, $x^* = 2x/c_r$, $s = 2Vt/c_r$, and $\delta(s)$ is the unit impulse function.

For a wing penetrating a sharp-edge gust

$$\dot{h}(t) = W_0 \mathbf{1}\left(t - \frac{x}{V}\right) \tag{A4}$$

so that

$$\left.\begin{aligned} k_2(s) &= \left[\frac{b(s)}{b}\right]^2 & \left(0 \leqq s \leqq 2x_0/c_r\right) \\ k_2(s) &= 1 & \left(s > 2x_0/c_r\right) \end{aligned}\right\} \quad (A5)$$

where x_0 is defined by $b\left(x_0\right) = b$.

These expressions (eqs. (A3) and (A5)) may be evaluated for any specific plan form. For instance, the results for rectangular, elliptical, and delta wings are given in table I.

REFERENCES

1. Wagner, Herbert: Über die Entstehung des dynamischen Auftriebes von Tragflügeln. Z.a.M.M., Bd. 5, Heft 1, Feb. 1925, pp. 17-35.

2. Küssner, H. G.: Zusammenfassender Bericht über den instationären Auftrieb von Flügeln. Luftfahrtforschung, Bd. 13, Nr. 12, Dec. 20, 1936, pp. 410-424.

3. Von Kármán, Th., and Sears, W. R.: Airfoil Theory for Non-Uniform Motion. Jour. Aero. Sci., vol. 5, no. 10, Aug. 1938, pp. 379-390.

4. Jones, Robert T.: The Unsteady Lift of a Wing of Finite Aspect Ratio. NACA Rep. 681, 1940.

5. Jones, W. Prichard: Aerodynamic Forces on Wings in Non-Uniform Motion. R. & M. No. 2117, British A.R.C., Aug. 1945.

6. Mazelsky, Bernard, and Drischler, Joseph A.: Numerical Determination of Indicial Lift and Moment Functions for a Two-Dimensional Sinking and Pitching Airfoil at Mach Numbers 0.5 and 0.6. NACA TN 2739, 1952.

7. Mazelsky, Bernard: Numerical Determination of Indicial Lift of a Two-Dimensional Sinking Airfoil at Subsonic Mach Numbers From Oscillatory Lift Coefficients With Calculations for Mach Number 0.7. NACA TN 2562, 1951.

8. Lomax, Harvard, Heaslet, Max. A., Fuller, Franklyn B., and Sluder, Loma: Two- and Three-Dimensional Unsteady Lift Problems in High-Speed Flight. NACA Rep. 1077, 1952. (Supersedes NACA TN 2403 by Lomax, Heaslet, Sluder, TN 2387 by Lomax, Heaslet, Fuller; contains material from TN 2256 by Lomax, Heaslet, Fuller.)

9. Garrick, I. E.: On Some Reciprocal Relations in the Theory of Nonstationary Flows. NACA Rep. 629, 1938.

10. Lawrence, H. R., and Gerber, E. H.: The Aerodynamic Forces on Low Aspect Ratio Wings Oscillating in an Incompressible Flow. Jour. Aero. Sci., vol. 19, no. 11, Nov. 1952, pp. 769-781.

11. Reissner, Eric, and Stevens, John E.: Effect of Finite Span on the Airload Distributions for Oscillating Wings. II - Methods of Calculation and Examples of Application. NACA TN 1195, 1947.

12. Jones, Robert T.: Properties of Low-Aspect-Ratio Pointed Wings at Speeds Below and Above the Speed of Sound. NACA Rep. 835, 1946. (Supersedes NACA TN 1032.)

13. Clevenson, S. A., Widmayer, E., Jr., and Diederich, Franklin W.: An Exploratory Investigation of Some Types of Aeroelastic Instability of Open and Closed Bodies of Revolution Mounted on Slender Struts. NACA TN 3308, 1954. (Supersedes NACA RM L53E07.)

TABLE I

INDICIAL LIFT FUNCTIONS FOR RECTANGULAR, ELLIPTICAL, AND DELTA WINGS OF VANISHINGLY SMALL ASPECT RATIO

Plan form	$\left[\frac{b(s)}{b}\right]^2 = \left[\frac{b(x^*)}{b}\right]^2$	$k_1(s)$	$k_2(s)$
Rectangular	$\mathit{1}$	$\mathit{1}(s) + 2\delta(s)$	$\mathit{1}(s)$
Elliptical	$1 - (s - 1)^2$	$\mathit{1}(s) + \frac{4}{3}\delta(s)$	$s(2 - s) \quad 0 \leqq s \leqq 1$ $\mathit{1}(s) \quad s > 1$
Delta	$s^2/4$	$\mathit{1}(s) + \frac{2}{3}\delta(s)$	$s^2/4 \quad 0 \leqq s \leqq 2$ $\mathit{1}(s) \quad s > 2$

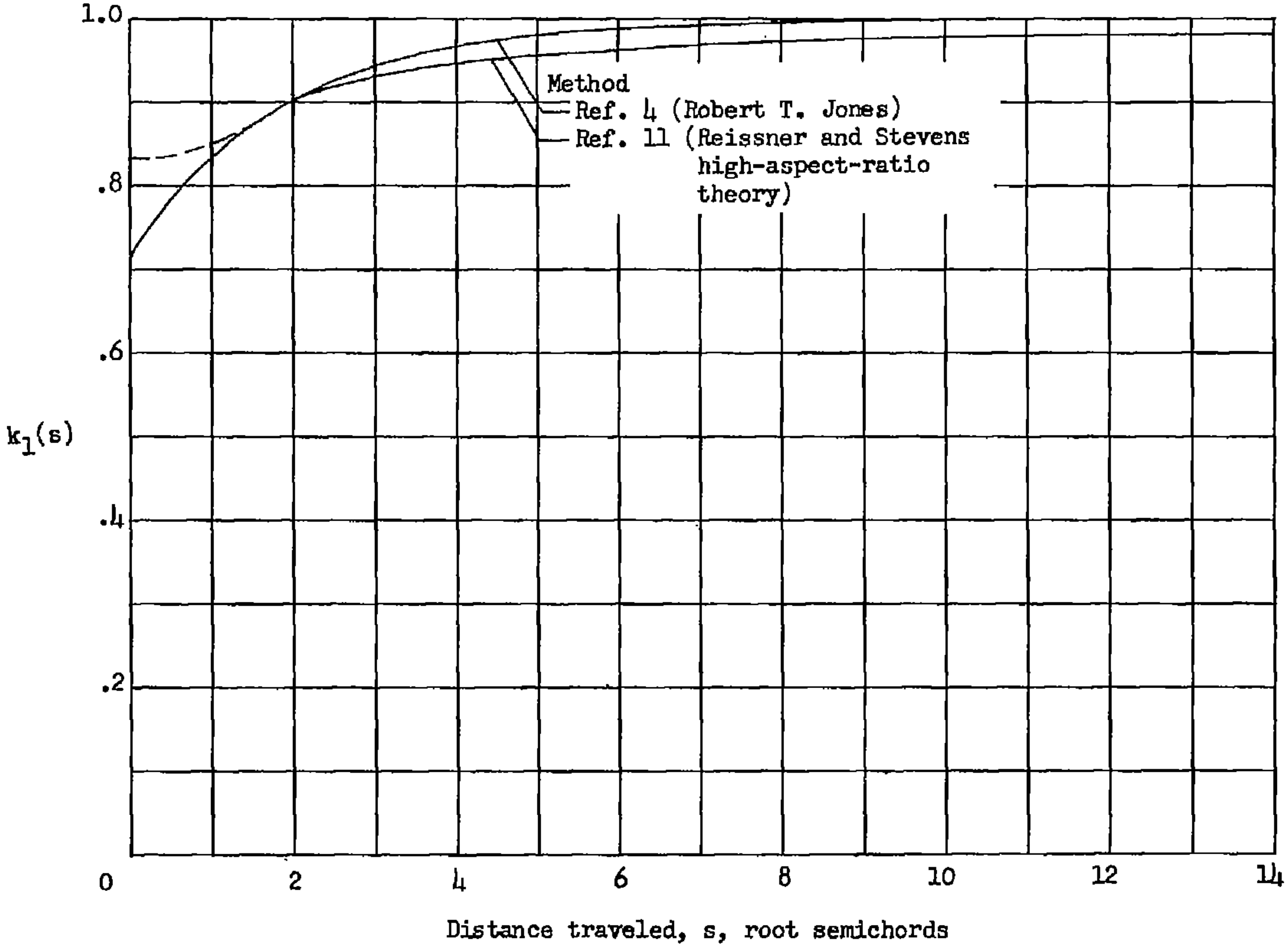


Figure 1.- Indicial lift functions $k_1(s)$ for an elliptical wing of aspect ratio 3 as calculated by method of reference 11 and as reproduced from reference 4.

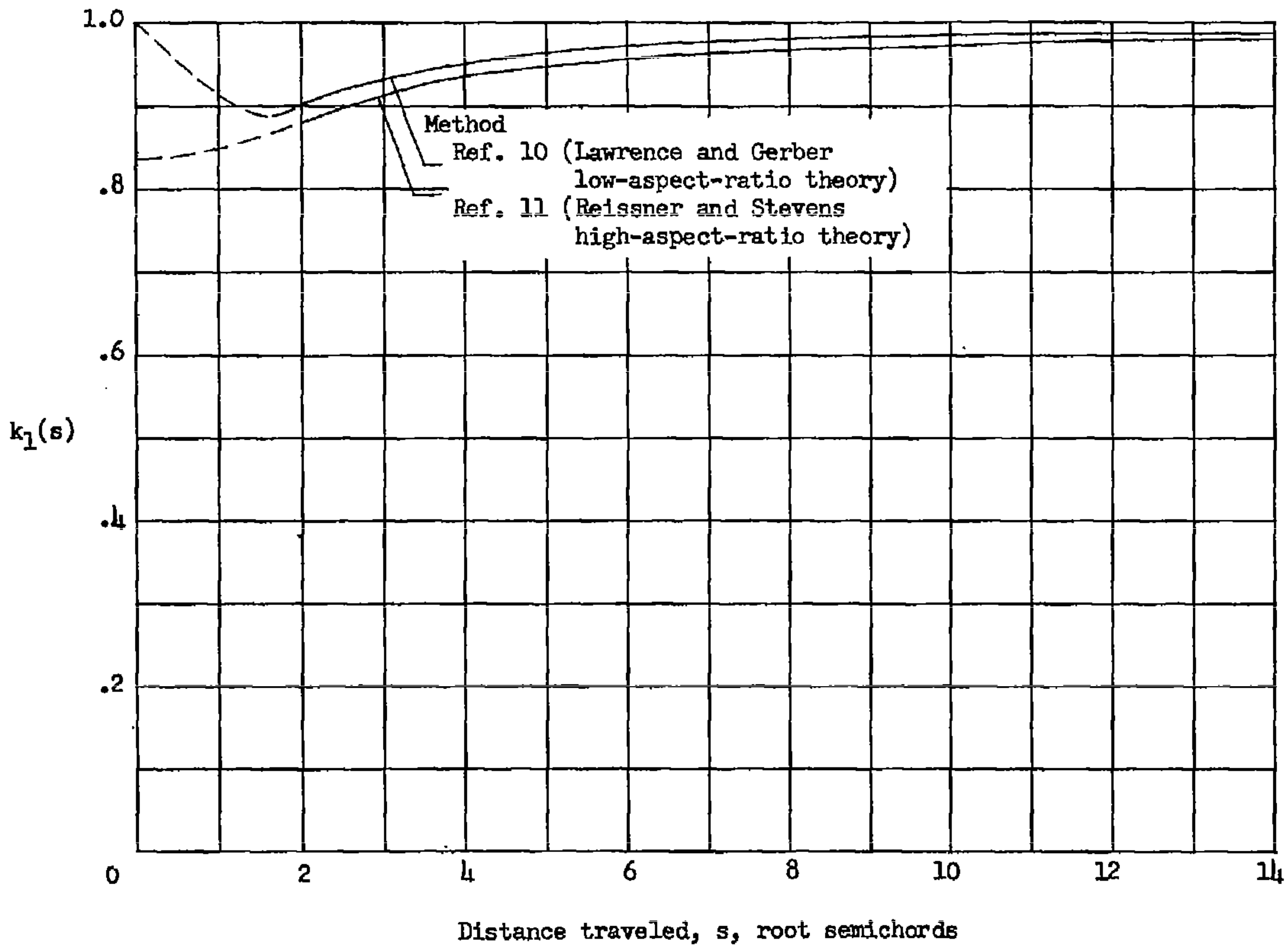


Figure 2.- Indicial lift functions $k_1(s)$ for a rectangular wing of aspect ratio 3 as calculated by methods of references 10 and 11.

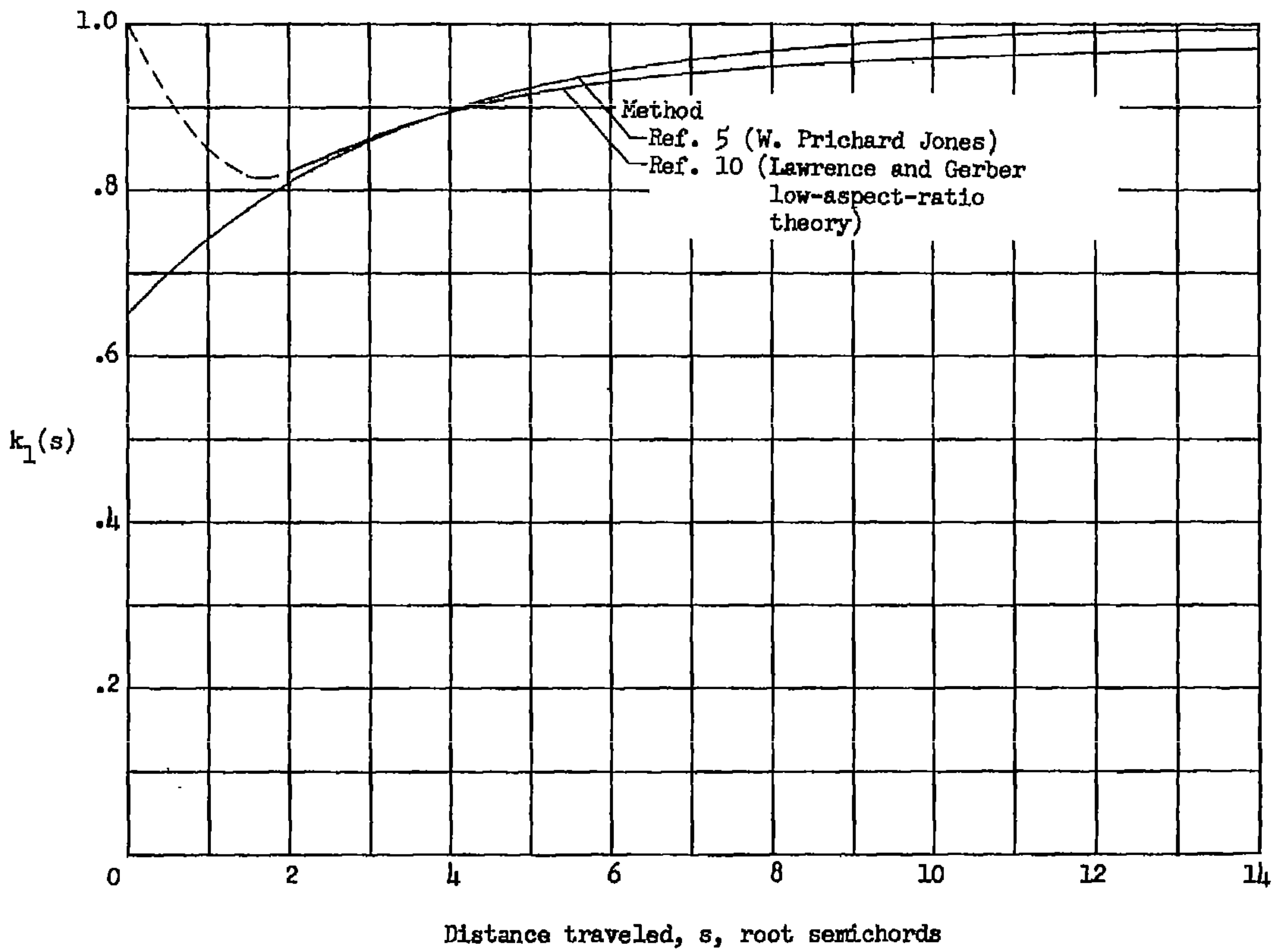


Figure 3.- Indicial lift functions $k_1(s)$ for a rectangular wing of aspect ratio 6 as calculated by method of reference 10 and as reproduced from reference 5.

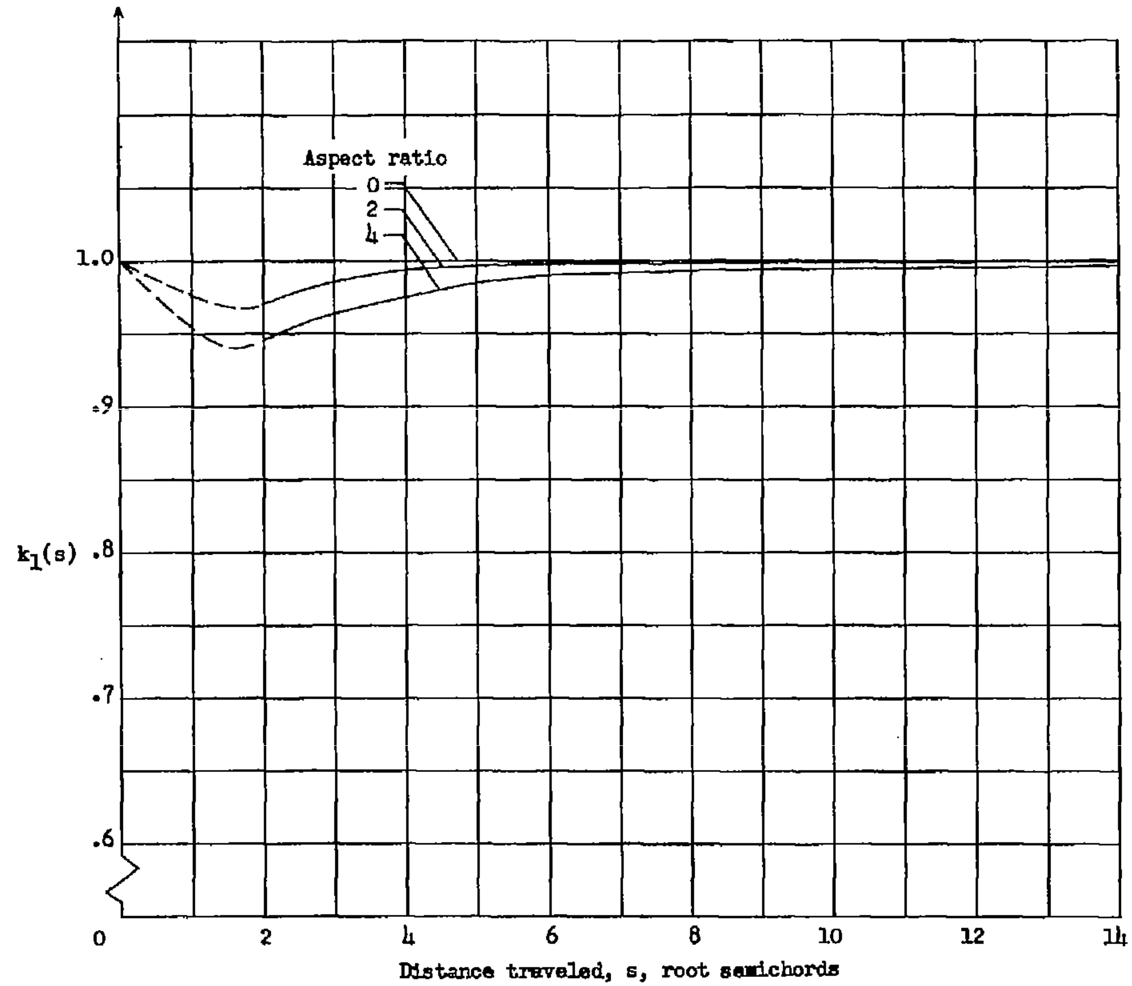
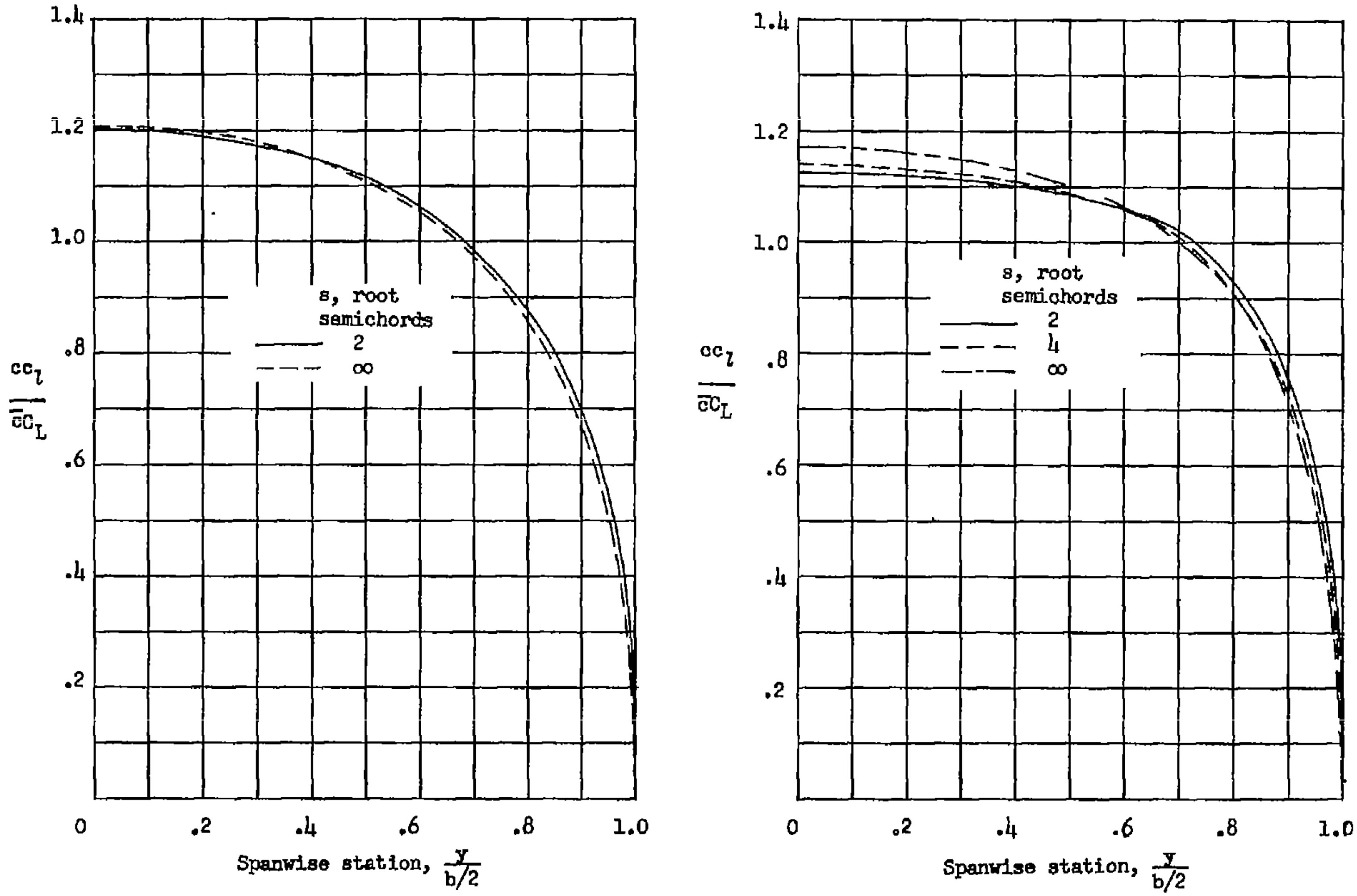


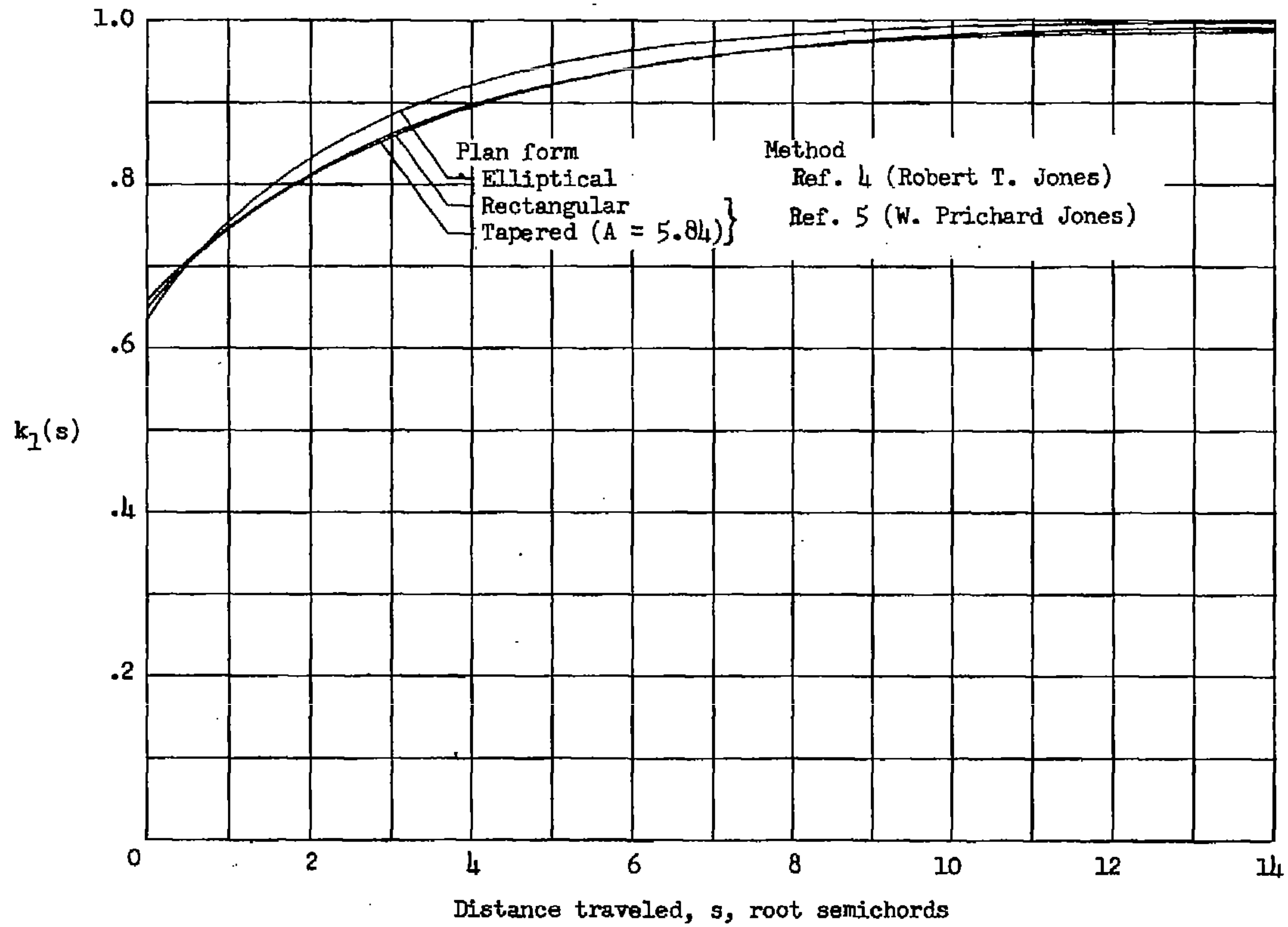
Figure 4.- Indicial lift functions for delta wings having aspect ratios of 0, 2, and 4 based on method of Lawrence and Gerber (ref. 10).



(a) Rectangular plan form; A = 3.

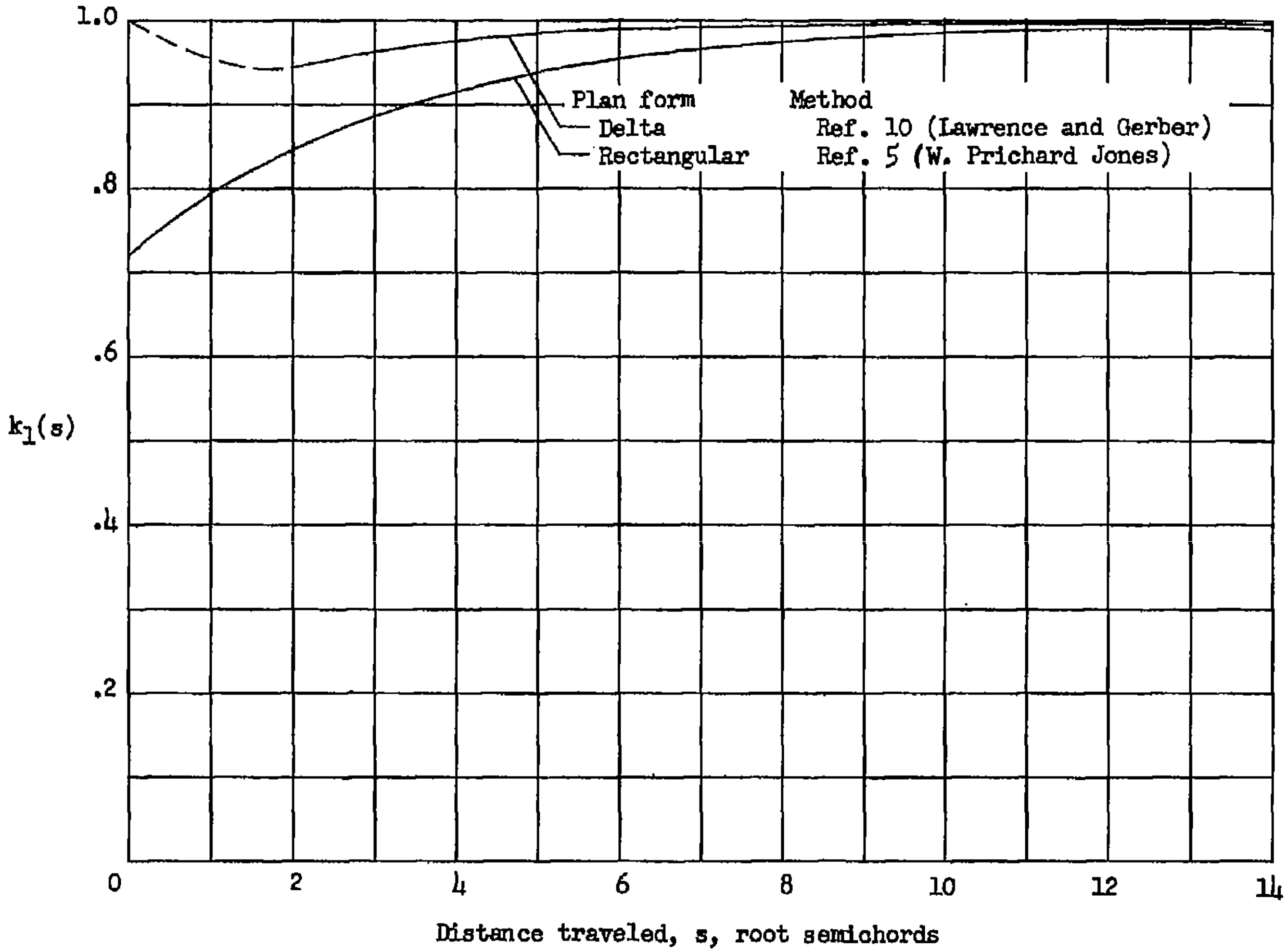
(b) Rectangular plan form; A = 6.

Figure 5.- Instantaneous spanwise loading coefficients for several values of distance traveled s.



(a) Elliptical, rectangular, and tapered wings of aspect ratio 6.

Figure 6.- Indicial lift functions $k_1(s)$ for wings of various aspect ratios and plan-form shapes.



(b) Rectangular and delta wings of aspect ratio 4.

Figure 6.- Concluded.

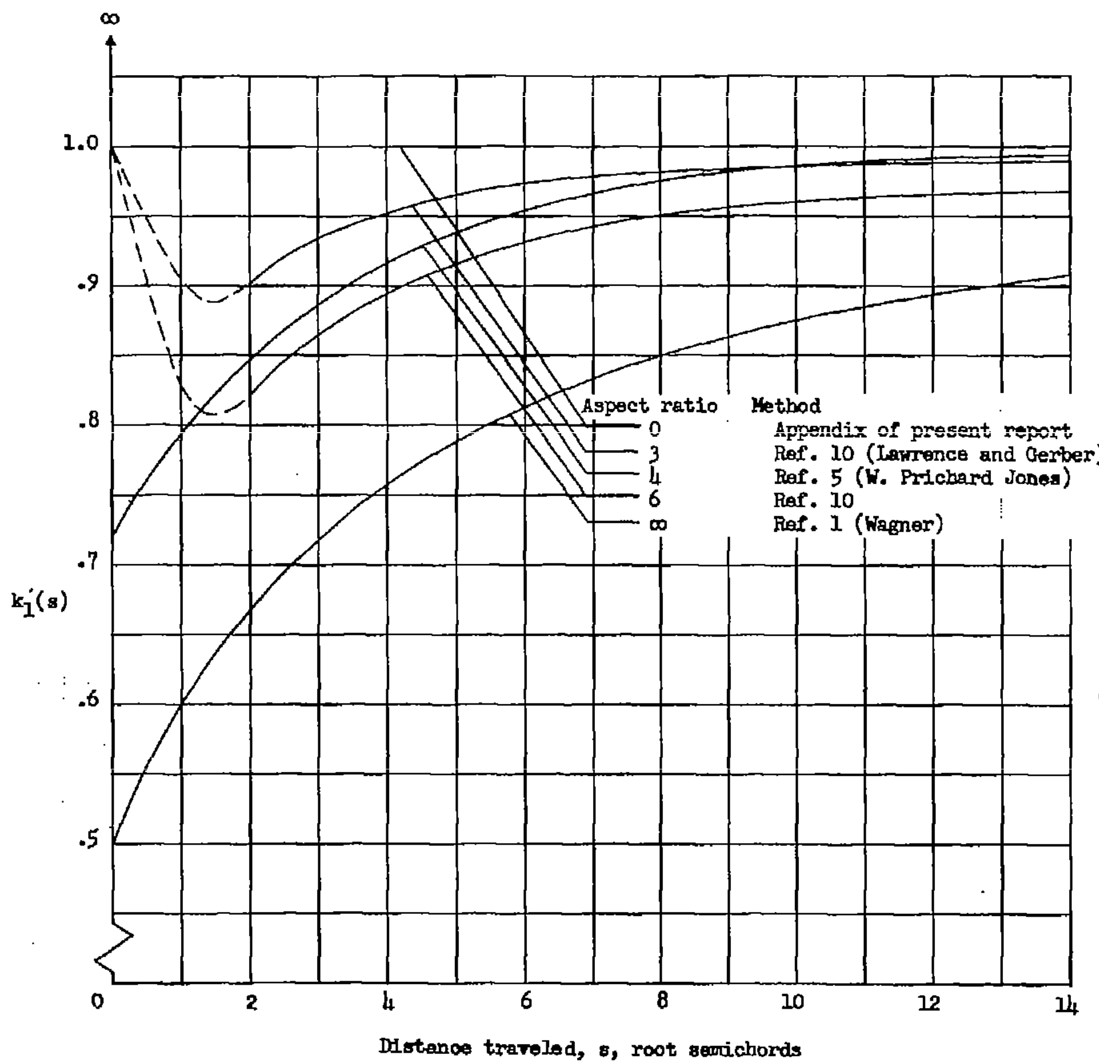
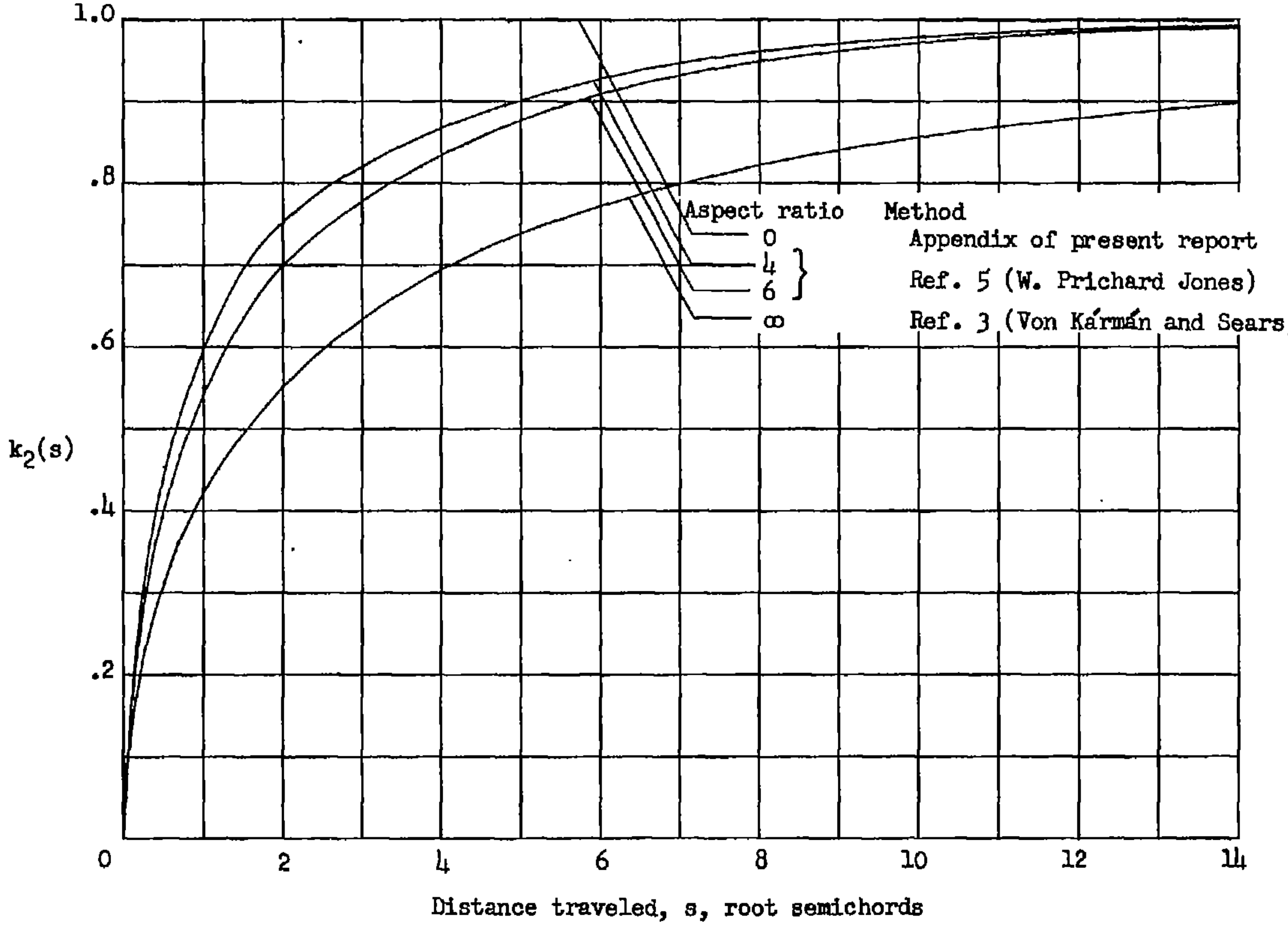
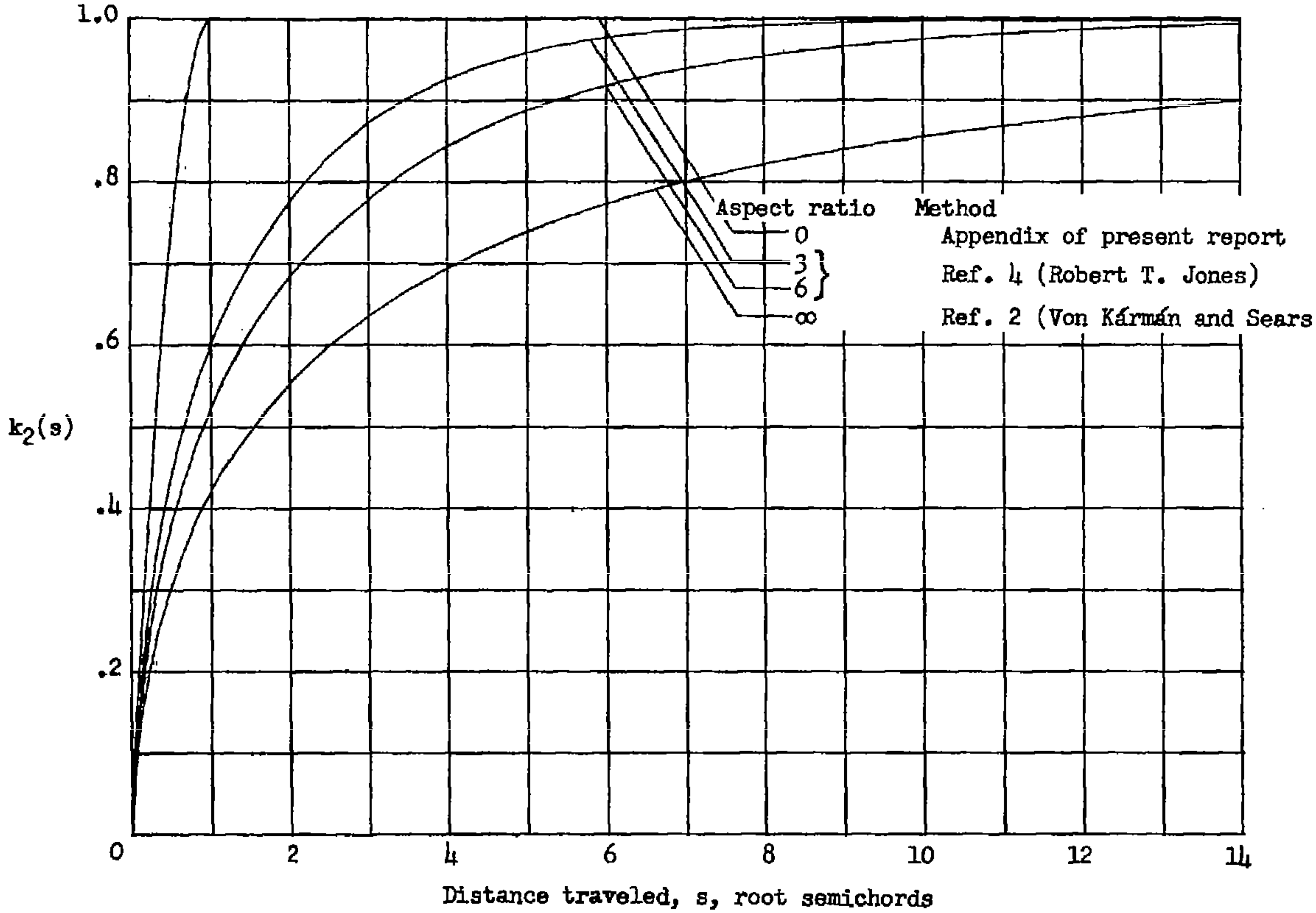


Figure 7.- Indicial lift functions $k_1(s)$ for rectangular wings of various aspect ratios.



(a) Rectangular wings.

Figure 8.- Indicial lift functions $k_2(s)$ for wings of various aspect ratios, penetrating a sharp-edge gust.



(b) Elliptical wings.

Figure 8.- Concluded.