NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 3810

CHARTS FOR ESTIMATING THE HOVERING ENDURANCE OF A HELICOPTER

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SUMMARY

As a means of comparing the performance capabilities of various helicopter propulsion systems, charts have been presented for estimating the hovering endurance of a helicopter in the form of a hovering-endurance parameter as a function of the ratio of total fuel load to helicopter initial weight for a range of initial thrust coefficients from 0.002 to 0.012 and for values of rotor mean lift coefficient from 0.24 to 0.72. The charts were prepared for a helicopter having rotor blades with ideal twist. However, corrections may be applied to the hovering endurance to account for other combinations of twist and taper. The effects of stall and compressibility have been neglected.

INTRODUCTION

The hovering endurance of a helicopter is a useful parameter in evaluating the performance capabilities of various helicopter propulsion systems; that is, piston, turbine, ram-jet, pulse-jet, and pressure-jet engines. For example, the helicopter having the best hovering endurance will usually prove to have the best range. The usual method of computing the hovering endurance of a helicopter is to make a time-consuming numerical analysis of the change in gross weight and power requirements as the fuel load is consumed. The purpose of this paper is to perform this analysis and to present the results in chart form so that the hovering endurance may be estimated with a minimum of calculation.

The charts are presented in the form of a hovering-endurance parameter as a function of the ratio of total fuel load to the initial gross weight of the helicopter for a range of initial thrust coefficients from 0.002 to 0.012 and for values of rotor mean lift coefficient from 0.24 to 0.72. The computations are based on a rotor with ideal twist (uniform inflow). Corrections that may be applied to the hovering-endurance parameter to account for various combinations of rotor-blade twist and taper are included. No allowance for the effects of stall or compressibility is made.

SYMBOLS

a	slope of curve of section lift coefficient plotted against section angle of attack (radian measure), assumed to be 5.73
B	tip-loss factor, assumed to be 0.97; blade elements outboard of radius BR are assumed to have profile drag but no lift
b	number of blades per rotor
$\bar{C}_L$	rotor mean lift coefficient, $6\frac{C_T}{\sigma}$
C_T	thrust coefficient; for hovering helicopter, $\frac{W}{\pi R^2 \rho (\Omega R)^2}$
C_Q	torque coefficient, $\frac{Q}{\pi R^2 \rho (\Omega R)^2 R}$
c	blade chord, ft
$c_{d,o}$	section profile-drag coefficient
c_e	equivalent blade chord, $\frac{\int_0^R cr^2 dr}{\int_0^R r^2 dr}$, ft
E	hovering endurance, hr
H	hovering-endurance parameter, $\frac{(\Omega R)(sfc)E}{550 \frac{W_{fl}}{W_i}}$
P	horsepower required for helicopter to hover, hp
Q	rotor torque, lb-ft
R	blade radius, ft
r	radial distance to a blade element, ft
sfc	specific fuel consumption, lb/(rotor hp)/hr

t	time, hr
W	gross weight of helicopter at any time in flight, lb
W_{fl}	total fuel load, lb
w	rate of fuel consumption, lb/hr
α_r	blade-section angle of attack measured from zero-lift line, radians
$\delta_0,\delta_1,\delta_2$	coefficients in power series expressing $c_{d,o}$ as a function of α_r; $c_{d,o} = \delta_0 + \delta_1\alpha_r + \delta_2\alpha_r^2$
ρ	mass density of air, slugs/cu ft
σ	rotor solidity based on equivalent blade chord, $\frac{bc_e}{\pi R}$
Ω	rotor angular velocity, radians/sec

Subscripts:

i	initial condition
f	final condition (after allotted fuel load is consumed)

ANALYSIS

Derivation of Basic Hovering-Endurance Equation

The hovering endurance of a helicopter is determined by the fully loaded weight of the aircraft, the total fuel load, and the fuel-consumption rate. The variation in the weight of a hovering helicopter may be expressed as

$$\frac{dW}{dt} = -w \tag{1}$$

where the rate of fuel consumption is given by

$$w = (sfc)P = (sfc)\frac{C_Q\pi R^2\rho(\Omega R)^3}{550} \tag{2}$$

At a constant tip speed and altitude, the combining of equations (1) and (2) and the use of the definition of C_T yields the relation

$$\frac{dC_T}{dt} = -\frac{(sfc)(\Omega R)C_Q}{550} \tag{3}$$

If the specific fuel consumption is assumed to be constant at some average value, equation (3) may be written

$$\int_{C_{T,f}}^{C_{T,i}} \frac{dC_T}{C_Q} = \int_0^E \frac{(sfc)(\Omega R)dt}{550} = \frac{(sfc)(\Omega R)E}{550} \tag{4}$$

Equation (4) relates the hovering endurance E to the integral of dC_T/C_Q between the initial and final values of C_T. A solution to equation (4) is possible for a known relation of C_T to C_Q.

The assumption of a constant specific fuel consumption is not truly representative of an actual helicopter. However, since the rotor power required can be obtained as a function of rotor thrust coefficient and since the variation in specific fuel consumption can be obtained from the manufacturer's engine specifications, a series of constant specific fuel consumptions may then be chosen so as to approximate a curve of specific fuel consumption as a function of thrust coefficient. However, if it becomes necessary to use a great number of constant specific fuel consumptions in order to approximate the curve, the use of the charts developed in this paper may provide little or no reduction in the time or the calculations necessary to compute the hovering endurance.

Solutions to the Hovering-Endurance Equations

An expression relating C_T to C_Q will be dependent upon the airfoil characteristics and rotor configuration of the particular helicopter being investigated. It has been shown in reference 1 that the relation between C_T and C_Q for a typical helicopter rotor blade can be approximated by the corresponding relation for an ideally twisted blade. The ideally twisted blade is defined as a blade having a twist which will produce uniform inflow over the disk and, consequently, have minimum induced losses. The relation of C_T to C_Q for the ideally twisted blade in the absence of ground effect is given by the expression

$$C_Q = \frac{C_T^{3/2}}{\sqrt{2}\,B} + \frac{\sigma}{8}\delta_0 + \frac{2}{3}\frac{\delta_1}{a}\frac{C_T}{B^2} + \frac{4\delta_2}{\sigma a^2}\left(\frac{C_T}{B^2}\right)^2 \qquad (5)$$

In this analysis, the constants are assumed to be as follows:

$$B = 0.97$$
$$a = 5.73$$
$$\delta_0 = 0.0087$$
$$\delta_1 = -0.0216$$
$$\delta_2 = 0.400$$

These values are representative of a well-constructed blade. The effects of stall and compressibility on the rotor power requirements are not included in equation (5). A plot of equation (5) for a range of rotor solidities is given in figure 1. The relation between C_T and C_Q for the ideally twisted rotor blade, as given in equation (5), may be substituted into equation (4) and integrated in closed form. However, the resultant equation is extremely cumbersome and difficult to use and, therefore, the hovering endurance of a helicopter with ideally twisted blades was determined by a graphical integration of equation (4) for a range of parameters, and the results are presented in chart form.

PRESENTATION OF RESULTS

Description of Charts

The charts for estimating the hovering endurance of a helicopter with ideally twisted blades are presented in figure 2 in terms of the hovering-endurance parameter

$$H = \frac{(\Omega R)(sfc)E}{550\,\frac{W_{fl}}{W_i}}$$

which is a function of the ratio of total fuel load to the initial weight of the helicopter for values of $\overline{C}_L$ from 0.24 to 0.72 and for a range of initial thrust coefficients from 0.002 to 0.012. The range of solidities considered is from 0.02 to 0.14. The parameters used in figure 2 were chosen as best suited for the reading of values of the hovering endurance in the low range of W_{fl}/W_i. The value of the hovering-endurance parameter when the fuel load is zero may be shown to be equal to $C_{T,i}/C_{Q,i}$.

The selection of an ideally twisted blade is purely arbitrary and only serves to provide a basis from which the relation between C_T and C_Q for a blade with a particular taper and twist can be determined. The increase in the power required by a rotor having various combinations of taper and twist over the power required by a rotor having ideally twisted blades is given in reference 1 and is, for convenience, repeated in the following table:

Blade twist, deg	Blade taper (ratio of root chord to tip chord)	Percent increase in power (percent decrease in hovering endurance)
0	1	5 to 6
-8	1	3
-12	1	2
0	3	3
-8	3	0
-12	3	0

Although the values in the preceding table were computed for solidities of 0.060 and 0.042, a reasonable degree of accuracy may be expected when these values are used for the range of solidities utilized in the computation of the charts.

The hovering endurance of a rotor with twist and taper will be reduced from the value of the hovering endurance of a rotor with ideally twisted blades by the same percentage as the power required for hovering is increased over that required by a rotor with ideal twist. For example, a rotor with a twist of 12° and no taper will require 2 percent more power to hover than a rotor with ideal twist and will, therefore, have a hovering endurance of 2 percent less than the hovering endurance of a rotor with ideal twist.

Application of Hovering-Endurance Charts

The application of the hovering-endurance charts may best be illustrated by an example. Assume that a helicopter has an initial weight of 5,000 pounds, a total fuel load of 1,000 pounds, a disk loading of 3 lb/sq ft, a tip speed of 700 ft/sec, and an initial value of $\bar{C}_L = 0.36$. Also, assume that the helicopter is powered by a turbine engine for which the curve of specific fuel consumption (including gear-box losses) as a function of rotor horsepower required is given in figure 3(a). From this curve and through the use of figure 1 or a similar curve, the variation of specific fuel consumption with thrust coefficient or, as given in figure 3(b), with the amount of fuel consumed may be obtained. As indicated in figure 3(b), the curve of specific fuel consumption can be approximated by a specific fuel consumption of 1.03 lb/(rotor hp)/hr for the first 45 percent of the fuel consumed and by a specific fuel consumption of 1.08 lb/(rotor hp)/hr for the remainder of the fuel consumed.

The initial thrust coefficient for operation at sea level ($\rho = 0.002378$ slug/cu ft) is $C_{T,i} = 0.00258$. After the first 45 percent of the fuel is consumed, the thrust coefficient is $C_T = 0.00234$. The final thrust coefficient is $C_{T,f} = 0.00206$.

The hovering endurance parameter for the first 45 percent of the fuel load consumed may be obtained from figure 2(b) for a ratio of fuel load to initial weight of 0.09. The value of H is 18.5. The hovering endurance is 1.27 hours. The ratio of fuel load to initial weight for the remainder of the flight is 0.121. The initial thrust coefficient is 0.00234, and the value of $\bar{C}_{L,i}$ is 0.328. The hovering-endurance parameter is about 18.5, and the hovering endurance is about 1.63. Therefore, the hovering endurance of the assumed helicopter is the sum of the hovering endurances calculated, or 2.90 hours. If the rotor blades had no taper or twist, the hovering endurance would be about 5 percent less, or about 2.75 hours.

CONCLUDING REMARKS

As a means of comparing the performance capabilities of various helicopter propulsion systems, charts have been presented for estimating the hovering endurance of a helicopter. These charts are in the form of a hovering-endurance parameter as a function of the ratio of total fuel load to helicopter initial weight for a range of initial thrust coefficients from 0.002 to 0.012 and for values of rotor mean lift coefficient from 0.24 to 0.72. The charts are presented for a helicopter with rotor blades having ideal twist. The effects of twist and taper are accounted for by the use of corrections to the charts from data showing the difference between a blade with ideal twist and a blade with conventional combinations of twist and taper. The effects of stall and compressibility have been neglected.

The method of using the charts for estimating the hovering endurance of a helicopter has been illustrated through the computation of a sample problem.

Langley Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., June 18, 1956.

REFERENCE

1. Gessow, Alfred: Effect of Rotor-Blade Twist and Plan-Form Taper on Helicopter Hovering Performance. NACA TN 1542, 1948.

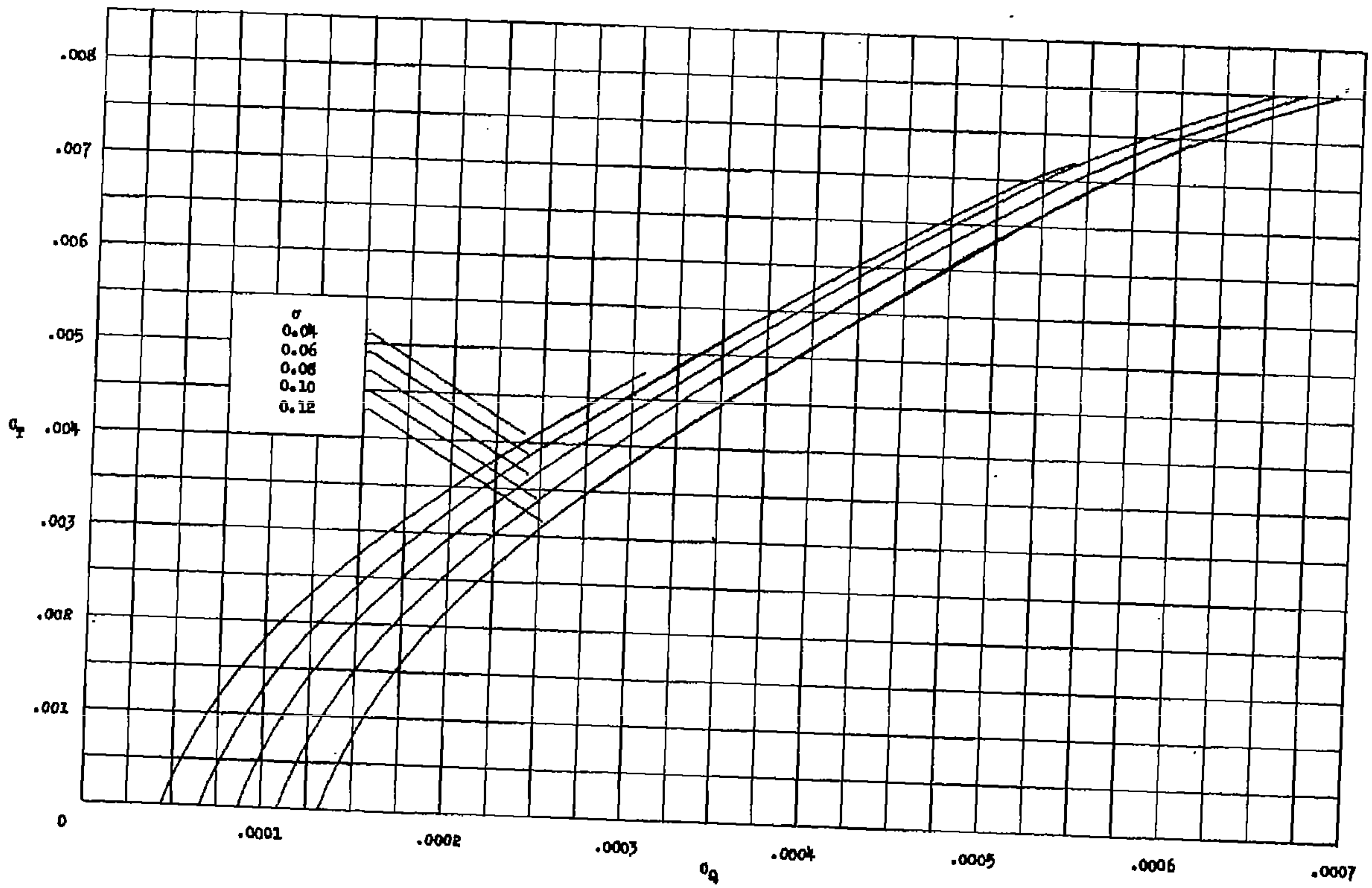
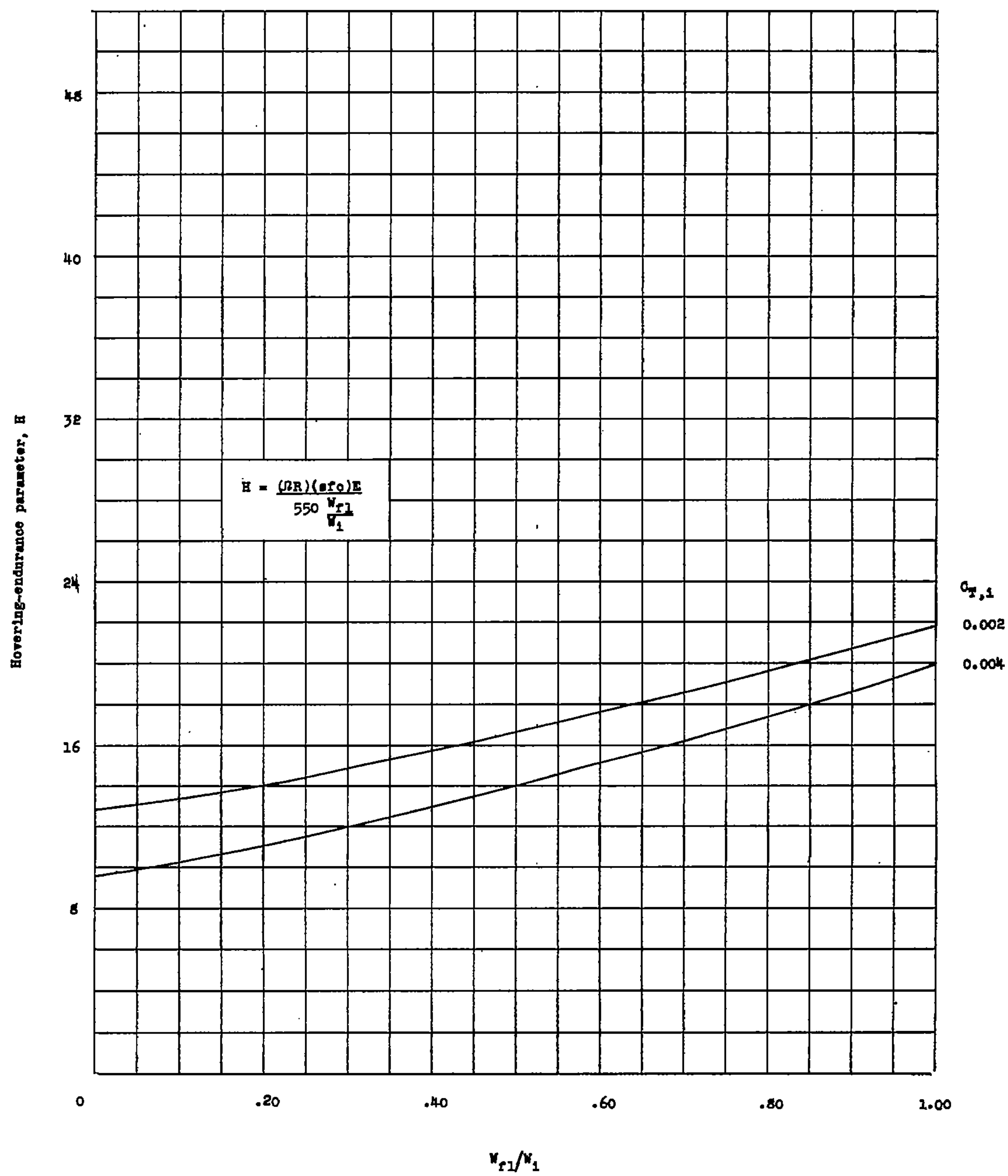
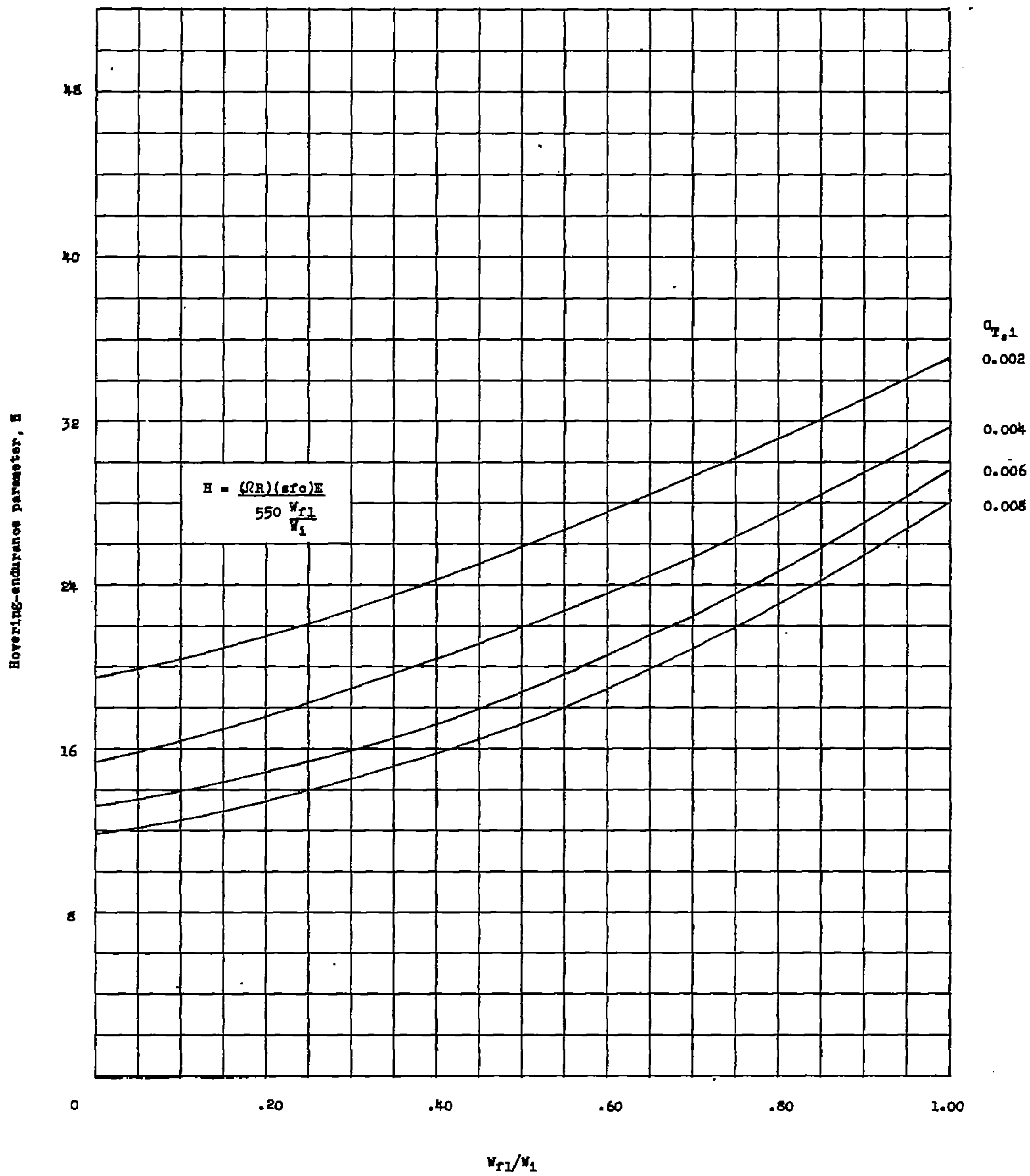


Figure 1.- Relation between thrust coefficient and torque coefficient for ideally twisted rotor blades.



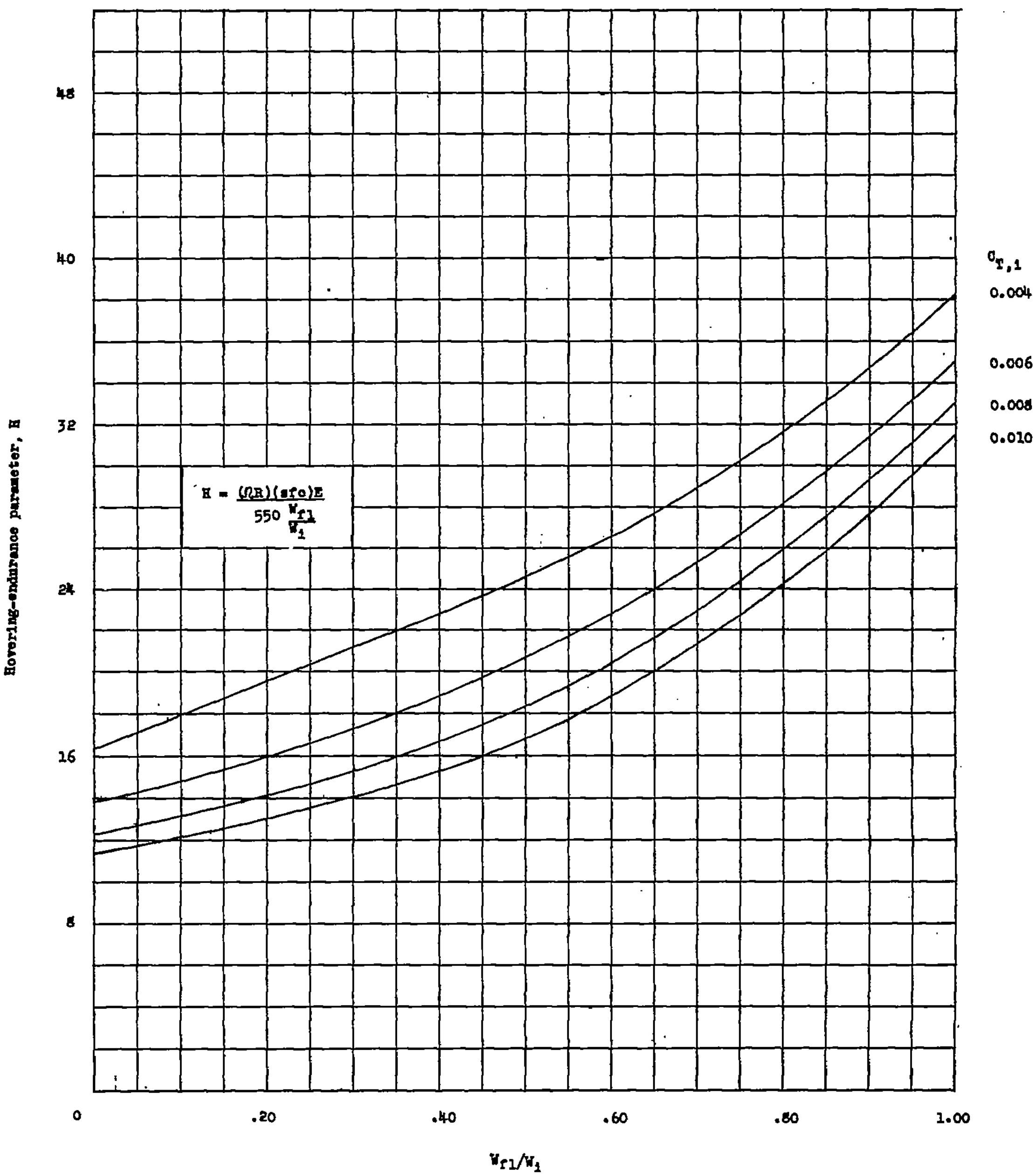
(a) $\bar{C}_{L,i} = 0.24$.

Figure 2.- Charts for estimating the hovering-endurance parameter as a function of the ratio of fuel load to initial helicopter weight for a rotor with ideally twisted blades.



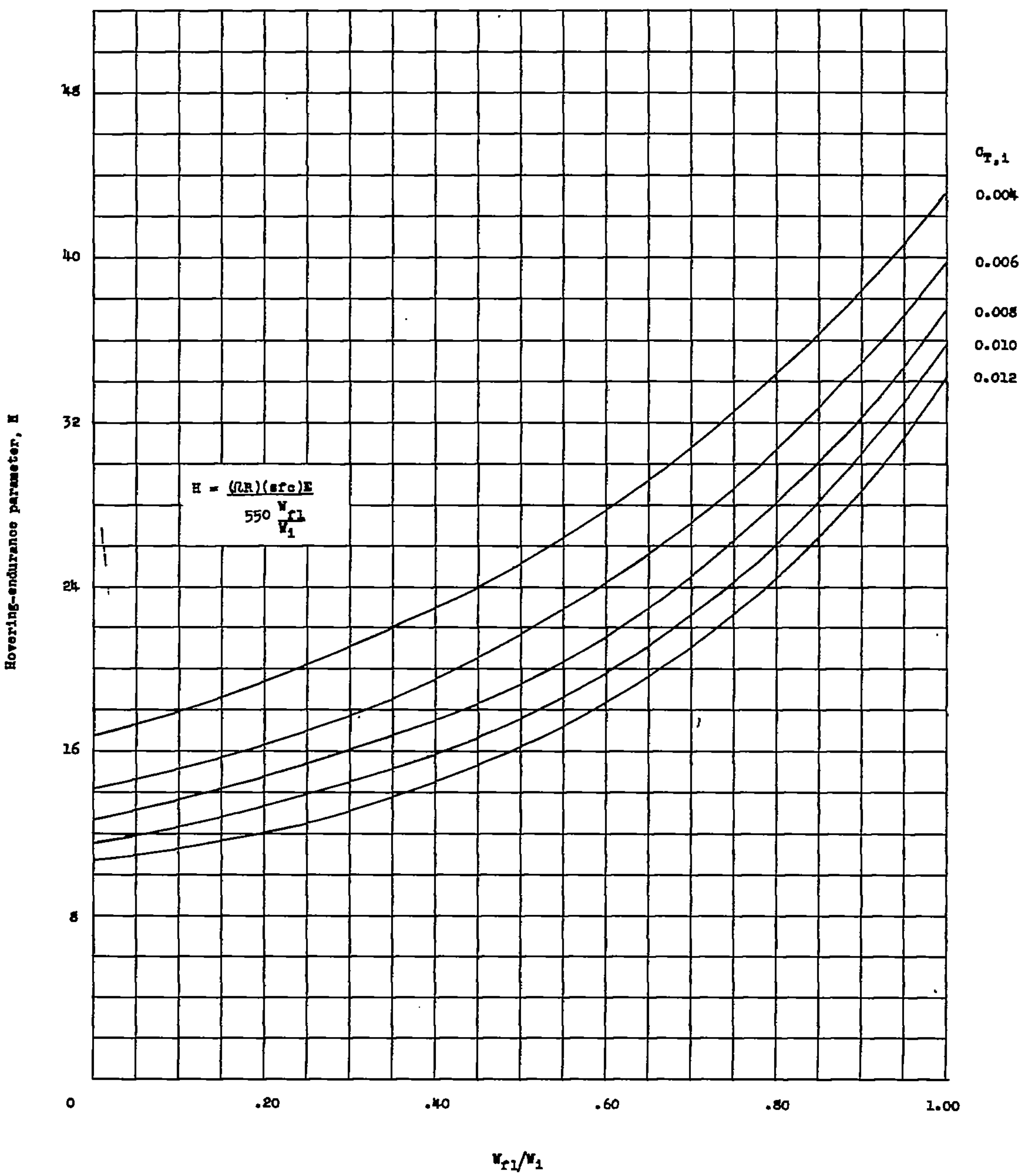
(b) $\bar{C}_{L,i} = 0.36$.

Figure 2.- Continued.



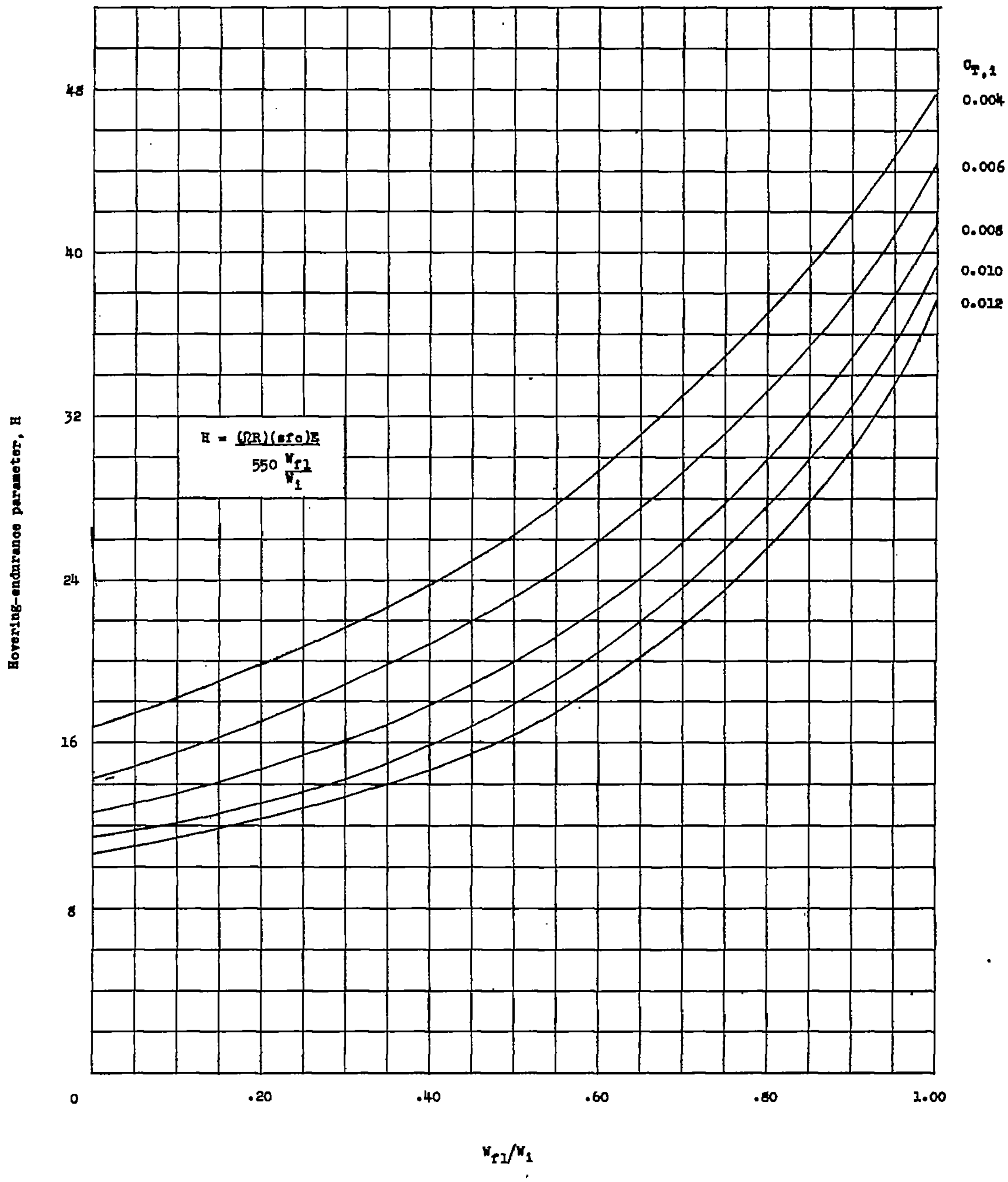
(c) $\bar{C}_{L,i} = 0.48$.

Figure 2.- Continued.



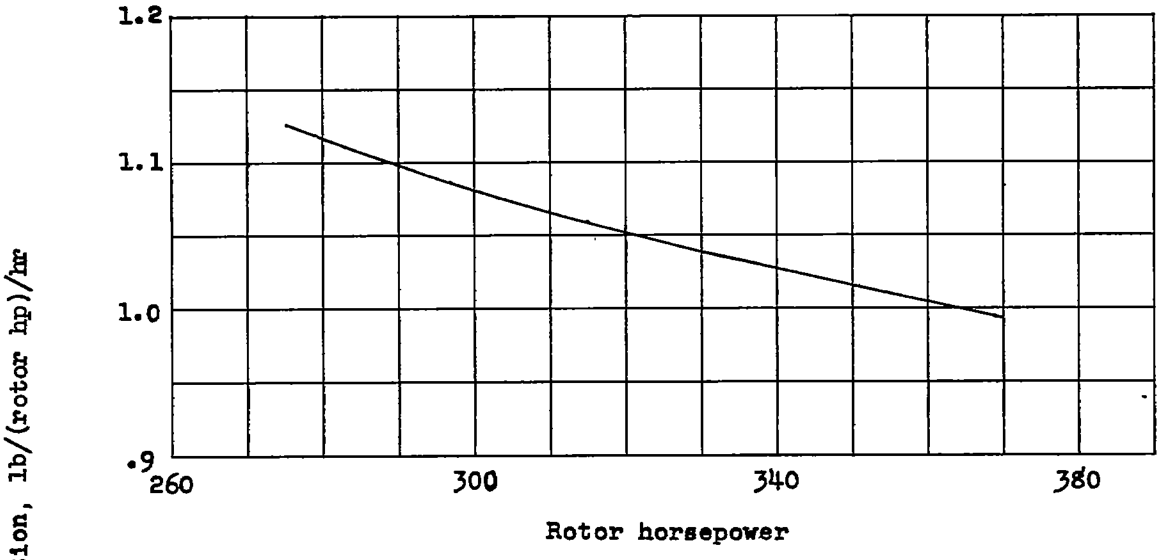
(d) $\bar{C}_{L,i} = 0.60$.

Figure 2.- Continued.

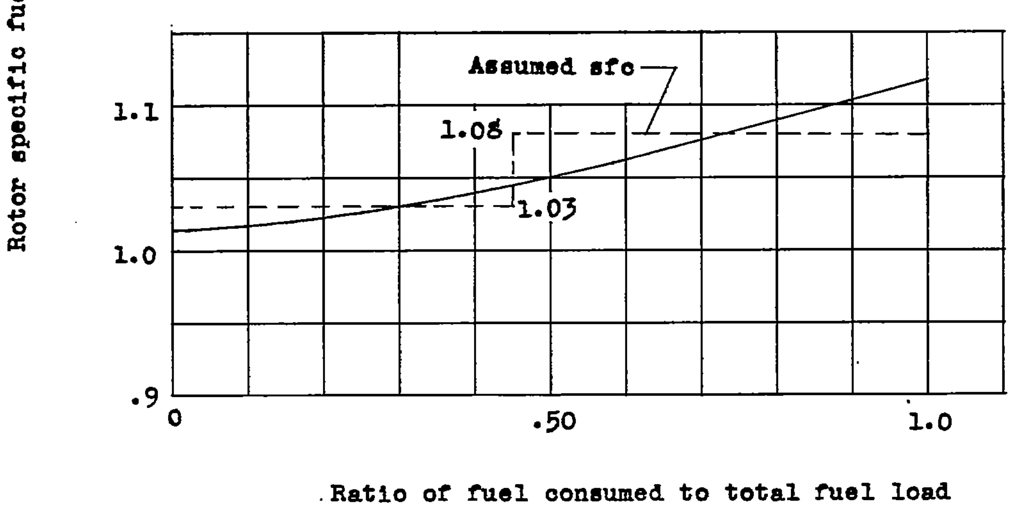


(e) $\bar{C}_{L,i} = 0.72$.

Figure 2.- Concluded.



(a) Rotor horsepower.



(b) Weight of fuel consumed.

Figure 3.- Variation of specific fuel consumption with rotor horsepower and weight of fuel consumed.

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