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THE EROSION OF METEORS AND HIGH-SPEED VEHICLES

IN THE UPPER ATMOSPHERE

By C. Frederick Hansen

Ames Aeronautical Laboratory
Moffett Field, Calif.

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SUMMARY

A simple inelastic collision model of meteor-atmosphere interaction is used and analytic relations for velocity, deceleration, size, and relative luminous magnitude of meteors are derived and expressed in dimensionless parametric form. The analysis is compared with available quantitative observations of meteor behavior and it is indicated that a large fraction of the atmospheric bombardment energy is used in eroding meteor material. The erosion from large, high-speed vehicles as they traverse the high-altitude, free-molecule portion of the atmosphere is calculated, on the assumption that the vaporization process is similar to that which occurs for meteors. The maximum possible erosion does not create significant mass loss.

INTRODUCTION

The science of aerodynamics is constantly expanding into realms of higher speed flight. Already we are concerned with the problems associated with design and operation of ballistic and satellite vehicles which will traverse the atmosphere at velocities from 15,000 to 26,000 feet per second. In the foreseeable future, vehicles will be designed to enter gravitational-free space, and the problems which develop at speeds in excess of escape velocity, 37,000 feet per second, will need to be considered. It has become clear that some of the most serious problems of very high-speed flight will be due to the tremendous heating experienced by the vehicle as it traverses the atmosphere during the final stages of its flight. Unfortunately, the conditions experienced by such high-velocity vehicles have been difficult to reproduce in the laboratory and direct experiments in the atmosphere are costly. It is of interest, then, to examine a natural phenomenon from which some pertinent data may be deduced; namely, the travel of meteors through the earth's atmosphere.

The purpose of the present paper is: (1) to develop an analytical description of the physical behavior of meteors, (2) to use this analysis to calculate from observed meteor behavior the fraction of kinetic energy of atmospheric impact which is utilized in vaporizing meteor material, and (3) to deduce the amount of surface erosion which would occur on a vehicle traveling at high velocity through the upper atmosphere if the same surface processes occur as on meteors.

SYMBOLS

a	deceleration, ft/sec^2
A	frontal cross-section area, ft^2
C	heat capacity of meteor material, ft^2/sec^2 $^\circ$R
D	density of meteor, slugs/ft^3
$\overline{D}$	average density of nonhomogeneous body, slugs/ft^3
E	kinetic energy, ft-lb
$\overline{Ei}(u)$	exponential integral function of u, $\int^{u} \frac{e^x}{x}\,dx$
g	acceleration due to earth's gravity at the earth's surface, ft/sec^2
I	intensity of luminous radiation from meteors
m	mass of meteor, slugs
M	magnitude of luminous intensity $(-\frac{5}{2}\log I)$, also, molecular weight of meteor material
M'	reference magnitude (see eq. (23))
p	exponent on velocity in luminous intensity function (see eq. (18))
q	energy of vaporization per unit mass of meteor, ft^2/sec^2
r	effective radius of meteor, $\left(\frac{3V}{4\pi}\right)^{1/3}$, ft
R	radius of earth, ft
s	distance of meteor travel, ft
t	time, sec
u	dimensionless velocity parameter, $\frac{\zeta v^2}{12q}$
v	velocity of meteor, ft/sec
V	volume of meteor, ft^3
y	altitude above the earth's surface, ft
ln	logarithm to the base e
log	logarithm to the base 10

β atmospheric density scale factor, ft^{-1}

δ ratio of average density to surface density, $\frac{\overline{D}}{D}$

ζ fraction of kinetic energy change used to vaporize meteor material

θ angle of inclination to the vertical of the meteor path, radians

μ molecular weight of air particles

ρ density of atmosphere, lb/ft^3

σ Stefan-Boltzman radiation constant, lb/ft deg^4 sec

τ luminous efficiency factor (see eq. (17))

φ shape factor, $\left(\frac{4\pi}{3V}\right)^{2/3} \frac{A}{\pi}$

χ angle of inclination to the vertical of observer's line of sight, radians

ψ angle between meteor trail and observer's line of sight, radians

Subscripts

∞ conditions at infinite altitude

o conditions at the earth's surface

1 initial meteor conditions on entering the earth's atmosphere

e condition at the end point of meteor trail

m condition at point of maximum luminous intensity

THE PHYSICAL BEHAVIOR OF METEORS

Before proceeding with the analysis it will be helpful to review briefly the past work on meteors and also some of their salient characteristics which have been observed. This information will help to indicate what approximations can reasonably be made in setting up a model of the meteor-atmosphere interaction.

Much of the current interest in meteors is devoted to techniques of observing radio-wave reflections from meteor trails (ref. 1) and research in this field has not been particularly concerned with the physical behavior and properties of the meteors themselves. However, a small group of investigators has advanced the physical theory of meteors following the pioneer work of Lindemann and Dobson (ref. 2). The development of the theory up to 1937 is well summarized by Öpik (ref. 3) and Hoppe (ref. 4). Since that time much of the work on meteor theory has been due to Whipple (refs. 5 and 6). In particular, Whipple has been able to deduce, from observed meteor behavior, upper atmosphere densities that correlate well with the latest results from rocket research (ref. 7). For the purposes of the present paper, a solution in closed analytic form like that obtained by Hoppe (ref. 4) is the most convenient, though we shall find it desirable to use somewhat different approximations.

Meteors are apparently of two types, composed either of an igneous rock-like material or of a metallic nickeliferous iron (Grimminger, ref. 8). Judging from the meteor fall-out at the earth's surface, stone meteors outnumber the iron by a factor of about 10 (refs. 3 and 8). However, Öpik reports that among observed meteor radiation spectra both types seem to be equally prevalent and that perhaps the iron-type meteor is merely less likely to survive passage through the atmosphere (ref. 3). The estimated specific gravity and heat of vaporization for these two meteor materials are as follows:

	Specific gravity	Total heat of vaporization per unit mass from a cool state
Stonelike	3.4	77×10^6 ft^2/sec^2
Ironlike	7.8	77×10^6 ft^2/sec^2

The luminous intensity from meteors is a strongly increasing function of meteor size; whereas, the frequency of meteors decreases rapidly with size. Consequently, most of the meteors which are observed lie within a limited size range. According to the size-frequency distribution table cited in reference 8, visual meteors are generally from 0.01 to 1.0 centimeter (0.0003 to 0.03 ft) in diameter and 4×10^{-6} to 4 grams (10^{-8} to 10^{-2} lb) in mass. The luminous trails from meteors appear in the altitude range from about 40 to 150 kilometers (130,000 to 500,000 ft) (ref. 9) with initial velocities from 11 to 73 kilometers per second (36,000 to 240,000 ft/sec) (ref. 10).

When the size distribution of meteors is considered, it can be seen that most of them are in free-molecule flow at the altitudes where they appear luminous. Lindemann and Dobson (ref. 2) stated the fundamental processes that probably occur as the meteor streaks through the atmosphere: The impact of air molecules heats the meteor surface, vaporized meteor material collides with the atmosphere producing a trail of luminous radiation and ionization, while the body of the meteor is decelerated relatively slowly. The measurable line spectrum of this radiation consists mainly of line emission due to impact excitation of the meteor atoms (ref. 11), presumably because the ionization potentials for the meteor atoms are considerably lower than for the atmospheric constituents (ref. 12). Millman also reports no measurable evidence of nitrogen ionization spectra or afterglow in meteor trails (ref. 13). However, Millman later observed some spectral lines of oxygen and nitrogen in the infrared portion of a meteor spectrogram (ref. 14), and Cook and Millman (ref. 15) recently reported that bands of the neutral nitrogen molecule may account for much of the background continuum present in a spectrogram of a Perseid meteor. Thus, the atmospheric particles are probably excited to the level of visible radiation to some extent.

Finally, about 3 percent of the meteors observed split into two or more pieces in the upper atmosphere and nearly 10 percent show flares in brightness, apparently due to crumbling or breakage of the meteor (ref. 10). Most of the meteors, however, produce visible radiation that rises and then falls in a continuous manner (ref. 16). The point of maximum light moves toward the end of the trail as the initial velocity of the meteor increases, being about 66 percent along the trail at velocities near 30 km/sec (100,000 ft/sec) and about 81 percent along the trail at 72 km/sec (240,000 ft/sec)(ref. 5).

The detailed processes that occur between meteors and the atmosphere have not yet been deduced from the observable meteor phenomena. Therefore, we will consider what clues to the nature of these processes may be gleaned from laboratory experiments on sputtering and ion bombardment.

Meteor-Atmosphere Interaction

Recall that the typical meteor will be a small particle of iron or stone which vaporizes and becomes luminous at altitudes where the mean free path is large compared to the diameter of the meteor. Under these conditions, the atmospheric particles strike with the full kinetic energy due to the velocity of the meteor. Even at the minimum meteor velocity of 37,000 ft/sec (Appendix A), this energy is considerably greater than the binding energy of the meteor atoms. For example, at 37,000 ft/sec a molecule of nitrogen has 18.2 electron volts kinetic energy; whereas, the vaporization energy of iron is but 4.2 electron volts per atom. Consider for a moment a body-centered collision between an air particle and a meteor atom. If the collision is perfectly elastic, the atom kinetic energy will gain a fraction of the air particle's kinetic energy given by

$$\frac{\Delta E}{E} = \frac{4(M/\mu)}{[(M/\mu) + 1]^2} \tag{1}$$

where M is the atomic weight of the meteor material and μ the molecular weight of the air particle. For nitrogen molecule bombardment of iron M/μ is about 2, and thus the impact ablation of iron might be expected when the energy of the nitrogen molecules is greater than 9/8 of the atomic binding energy of iron, or 4.7 electron volts. Even if the collision were completely inelastic, it would deliver kinetic energy up to

$$\frac{\Delta E}{E} = \frac{M/\mu}{[(M/\mu) + 1]^2} \tag{2}$$

and the least kinetic energy of a nitrogen molecule required for ablation would then be 9/2 the heat of vaporization per iron atom, or 18.9 electron volts. It will be noted that impact energies of this order and greater are experienced by meteors. However, this simple energy balance concept apparently does not predict accurately the threshold of sputtering that results from gaseous bombardment of solid surfaces. Wehner (ref. 17) has measured the threshold of metal sputtering by mercury ion bombardment at normal incidence to the surface. The threshold energies were generally more than twice as large as needed to transfer an energy equal to the atomic heat of vaporization by a completely inelastic collision, though at grazing incidence, where the momentum gained by a metal atom from a collision is more likely to be directed away from the surface, lower threshold energies were detected. Wehner suggests that the elastic properties of the solid determine that fraction of kinetic energy transferred by a collision which is associated with momentum reflected outward from the surface. In turn, it is only this energy which is effective in sputtering the solid, while the remainder is dissipated as heat.

In addition to the kinetic energy of atmospheric bombardment, the heat of formation of oxygen and nitrogen compounds at the meteor surface is a possible source of energy for the ablation process. However, at present there is little evidence that nitrogen compounds will be formed, and the oxides have heats of formation which are generally small compared to the bombardment energy (heat of formation of the iron oxides is about 2.8 electron volts per oxygen atom, for example). Therefore, it will be assumed that the contribution of chemical energy to the vaporization of meteors is small.

In view of the above considerations, it seems likely that the process of meteor ablation changes with velocity as follows: Low velocity meteors are probably vaporized mainly by thermal heating while at higher velocities sputtering of meteor material would become the predominant ablation process. If one generalizes from Wehner's measurements, the threshold of sputtering of stone or iron by air molecules would be expected to occur at a velocity about 56,000 ft/sec. Wehner (ref. 18) finds that the sputtering yield is a linear function of the bombardment energy over a considerable range above the threshold energy value. Keywell (ref. 19) shows that the sputtering yield becomes nonlinear with bombardment energy above 1000 electron volts. However, meteors never suffer bombardment by particles of such high energy. For example, the 240,000 ft/sec maximum velocity meteor is hit by nitrogen molecules having a relative kinetic energy of 750 electron volts. Therefore, it will be assumed that meteors with velocities from 56,000 to 240,000 ft/sec are ablated by sputtering with a yield that is proportional to the bombardment energy.

The collision process could excite numerous energy modes besides the dissolution of chemical bonds of meteor material, of course. The internal energy of the air particles might be excited, for example. However, the rotational and vibrational energy levels are small compared to the collision energy and will be neglected. On the other hand, the ionization potentials of the air are the same order of magnitude as the collision energy, but here the low intensity level of air ionization spectra observed in meteor trails indicates that this is not a major energy sink for the collision process. Next the meteor itself may be heated to a high temperature and radiate into space. It should be noted that the meteor need not necessarily be heated for the sputtering process to occur, and indeed there is evidence that some meteorites have been stopped by the atmosphere without being heated above their melting points, according to reference 6. Obviously heating is required, though, before the thermal vaporization of low-velocity meteors can occur. But with either process, the surface temperature will be limited to the vaporization temperature of the meteor material. It is shown in Appendix B that heating to this temperature can occur largely at altitudes above those where meteors are observed, and that the possible radiation losses, in the interval where meteor ablation is predominant, are small compared to the energy flux from the atmosphere. Therefore it will be assumed that the kinetic energy lost in the collision with the atmosphere is principally absorbed in the vaporization of meteor material.

Now, the chemical potential being neglected, the maximum heat energy available equals the relative kinetic energy of the atmosphere which impacts on the meteor. This maximum is realized if the bombardment is completely inelastic, in which case the vaporized meteor material and the impinging air particles are emitted from the meteor surface with zero average velocity. In the analysis which follows, the bombardment will be assumed inelastic, so that we may deduce from meteor observations the fraction of maximum possible energy which is utilized in vaporizing the meteor. Moreover, it will be assumed that this fraction ζ is a constant. This is consistent with the observation that the yield is directly proportional to bombardment energy for the sputtering process that is expected to occur for high-velocity meteors. It is also consistent with the thermal vaporization process expected for low-velocity meteors if the heat of vaporization is considered constant, since then the mass vaporized is proportional to the energy input. Of course, this fraction ζ may be different for the two processes.

Meteor Mechanics

With the above meteor-atmosphere collision model in mind, consider a meteor of volume V and velocity v consisting of a homogeneous material of density D. For convenience, we define an effective radius $r = (3V/4\pi)^{1/3}$. The meteor will intercept a mass of air per unit time $\varphi\pi r^2\rho v$, where ρ is the atmospheric density and φ is the shape factor, $(4\pi/3V)^{2/3}(A/\pi)$, which accounts for any nonsphericity of the meteor; A is the frontal cross-section area of the meteor. Since no external forces act on the meteor-atmosphere system, the total rate of momentum change is zero.

$$\frac{4}{3}\pi r^3 D \frac{dv}{dt} + \varphi\pi r^2 \rho v^2 = 0 \tag{3}$$

It will be noted that the rate of mass loss is not involved in this expression since, by assumption, the mass vaporized leaves, on the average, with zero velocity relative to the meteor. The vaporized material then suffers no further momentum change until its next collision with an atmospheric particle.

The rate of change of total kinetic energy of the meteor-atmosphere system is

$$\frac{dE}{dt} = \frac{\varphi\pi r^2 \rho v^3}{2} + \frac{4}{3}\pi r^3 D v \frac{dv}{dt} \tag{4}$$

If the potential energy change due to the earth's gravitational field were included, this would contribute a kinetic energy term $\frac{4}{3}\pi r^3 D v g \cos\theta$ to the right side of equation (4). It will be shown later, however, that this term can be safely neglected because, when the meteor becomes visible, its deceleration, dv/dt, is two magnitudes larger than g.

Note that from equations (3) and (4) the rate of change in the system's kinetic energy is just

$$\frac{dE}{dt} = -\frac{\varphi\pi r^2 \rho v^3}{2} \tag{5}$$

that is, the negative of the flux of atmospheric bombardment energy into the meteor surface. By definition, a fraction ζ of this energy is utilized to vaporize meteor material. Let q be the heat of vaporization per unit mass; then

$$\zeta \frac{dE}{dt} = 4\pi D q r^2 \frac{dr}{dt} \tag{6}$$

and by combination of equations (3), (4), and (6) there results

$$v \frac{dv}{dt} = \frac{6q}{\zeta r} \frac{dr}{dt} \tag{7}$$

The variable u will be defined

$$u = \frac{\zeta v^2}{12q} \tag{8}$$

Then, if it be assumed that q and ζ are constants, equation (7) is integrable to

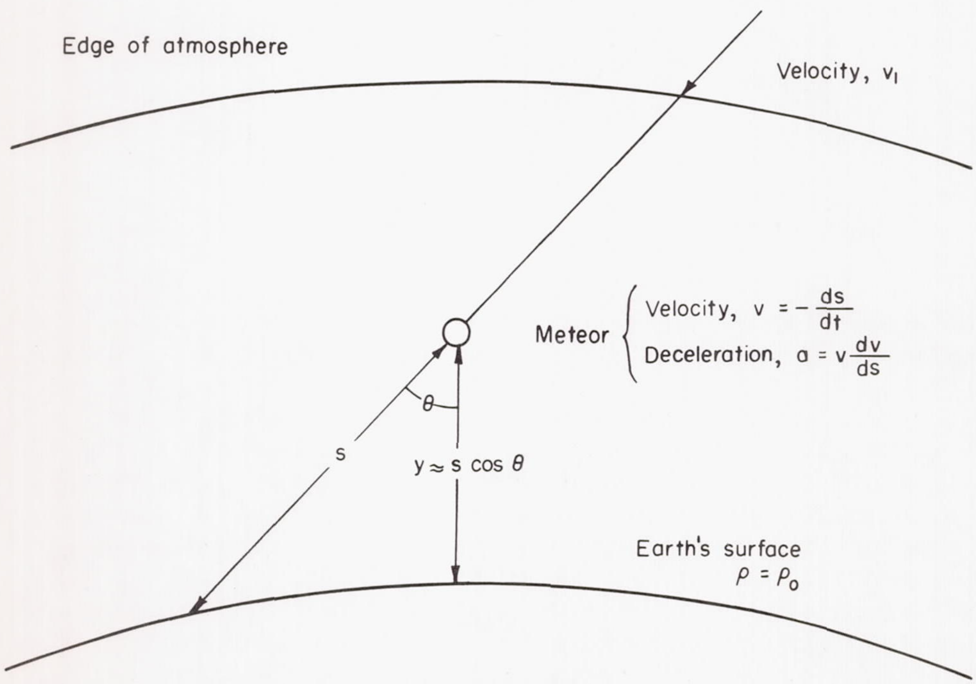
$$\ln \frac{r}{r_1} = -(u_1 - u) \tag{9}$$

Now it will be assumed that the atmospheric density varies exponentially

$$\frac{\rho}{\rho_o} = \exp(-\beta y) \tag{10}$$

where ρ_o is the density at the earth's surface and β is the scale factor chosen to fit the actual density over the range of altitude which is of interest. The meteor will be taken to follow a straight line path at an angle of inclination, θ, from the vertical which is not too large. Then if s is the distance from the point of impact on the earth's surface (see sketch (a)), the altitude y is approximately

$$y = s \cos \theta \tag{11}$$



Sketch (a)

For nearly horizontal incidence, corrections for curvature of the atmosphere and of the meteor path will, of course, need to be included.

From equations (3), (9), (10), and (11) there develops

$$\frac{3\varphi\rho_o}{2r_1 D} \int_s^\infty \exp(-\beta s \cos\theta) ds = \int_u^{u_1} \exp(u - u_1) \frac{du}{u} \tag{12}$$

which integrates to

$$\beta y + \ln \frac{2r_1 \beta D \cos\theta}{3\varphi\rho_o} = u_1 - \ln\left[\overline{Ei}(u_1) - \overline{Ei}(u)\right] \tag{13}$$

The deceleration of the meteor is (sketch (a))

$$a = v \frac{dv}{ds} = v \cos\theta \left(\frac{dy}{dv}\right)^{-1} \tag{14}$$

where from equations (8) and (13)

$$\frac{dy}{dv} = \frac{\zeta v}{6q\beta} \frac{d\overline{Ei}(u)}{du} \left[\overline{Ei}(u_1) - \overline{Ei}(u)\right]^{-1} \tag{15}$$

whence

$$\frac{a\varphi\zeta}{6q\beta \cos\theta} = ue^{-u}\left[\overline{Ei}(u_1) - \overline{Ei}(u)\right] \tag{16}$$

The shape factor, φ, has been carried along for generality so that the equations may be used for bodies of arbitrary shape. It is exactly unity, of course, for a sphere, and it is close to unity for any body which is approximately equally thick in all dimensions. Moreover, the time-averaged value of φ is also near unity for elongated or flattened bodies that are tumbling. For these reasons φ will be presumed very close to unity for the typical meteor.

The analytic description of meteor mechanics is concluded in equations (8), (9), (13), and (16). These equations uniquely prescribe the size, velocity, and deceleration of a meteor as a function of altitude in terms of the initial size, the initial velocity, the density, the specific vaporization energy of the meteor material, and the energy fraction ζ. It will be noted that the meteor properties enter the equations in dimensionless parameters, so that a series of universal curves will describe the behavior of all meteors. Figure 1 presents the size parameter $\ln(r/r_1)$, which equals the velocity parameter $u - u_1$, as a function of the altitude parameter $\beta y + \ln \dfrac{2r_1\beta D \cos\theta}{3\varphi\rho_o}$. Figure 2 presents the deceleration parameter $\dfrac{a\zeta}{6q\beta \cos\theta}$ as a function of this same altitude parameter.

It will be noted that the solution for meteor size (eq. (9)) is identical with that obtained by Hoppe (ref. 4) if the atmosphere particle collisions with the meteor are perfectly inelastic and ζ is taken as unity. The altitude-velocity relation (eq. (13)) is also similar to Hoppe's solution, but the velocity parameter is introduced by the exponential integral function $\overline{Ei}(u)$ rather than by the function $Ei(-u)$ (i.e., $-\int_u^\infty e^{-x}x^{-1}dx$) as given by Hoppe. As a result of this difference, Hoppe predicts that the meteor is eroded to a limiting size with about one half the initial mass, whereas equation (13) predicts almost complete vaporization of the meteor during its deceleration in the atmosphere (see fig. 1). (The $\overline{Ei}(u_1) - \overline{Ei}(u)$ function does have an inflection point near $u = 1$ which limits the erosion, but normally before this point is reached the meteor has been reduced to less than one molecule and the solution loses physical significance.)

Luminous Radiation From Meteors

A description of meteor behavior is hardly adequate without some account of the luminous radiation produced by the meteors. The processes involved are complex and probably cannot be described accurately by a simple functional relation. However, it is still desirable to find a simple relation that will describe the essential gross features of the phenomenon. For this purpose it has usually been assumed (refs. 3 and 5) that the intensity of luminous radiation, I, produced is proportional to the flux of kinetic energy into the atmosphere due to the vaporized meteor material.

$$I = -\frac{\tau}{2}\frac{dm}{dt}V^2 \tag{17}$$

However, it is clear that luminous intensity cannot be strictly proportional to this kinetic energy flux, since it is observed that most of the radiation comes from de-excitation of meteor atoms. This means that the first collision or two which excite the vaporized meteor atom are the important ones, and the excess kinetic energy carried away from these collisions by the atmospheric particles is relatively ineffective for producing luminous radiation. As expected, then, the proportionality factor τ is not a constant. Öpik (ref. 3), concludes that it will vary approximately linearly with velocity.

For the present paper it will be assumed that the luminous intensity is proportional to the rate of mass loss and to a function of velocity that may be approximated by v^{2p-1}, where p is a constant. Then,

$$I \sim \frac{dm}{ds} v^{2p} \tag{18}$$

The mass loss per unit path length can be expressed

$$\frac{dm}{ds} = -\frac{4\pi r^2 D}{v} \frac{dr}{dt} \tag{19}$$

while from equations (3) and (7)

$$\frac{dr}{dt} = -\frac{\zeta \varphi \rho v^3}{8qD} \tag{20}$$

Combining equations (18), (19), and (20) and transforming to the variable u, one obtains

$$I \sim \varphi \left(\frac{q}{\zeta}\right)^p \rho r^2 u^{p+1} \tag{21}$$

The definition of the magnitude of luminous intensity is (ref. 20)

$$M = -\frac{5}{2} \log I \tag{22}$$

so that

$$M = M' - \frac{5}{2 \ln 10}\left[\ln \varphi r_1^2 + p \ln \frac{q}{\zeta} - \beta y - 2(u_1 - u) + (p + 1)\ln u\right] \tag{23}$$

where the constant M' absorbs all other constants and sets the reference level of the magnitude. In parametric form equation (23) becomes

$$M + \frac{5}{2} \log\left[\frac{(r_1\beta)^3 D \cos \theta}{\rho_o} \left(\frac{q\beta}{\zeta g}\right)^p\right]$$

$$= -\frac{5}{2 \ln 10}\left\{\ln \left[\overline{Ei}(u_1) - \overline{Ei}(u)\right] - u_1 - 2(u_1 - u) + (p + 1)\ln u\right\} \tag{24}$$

where the reference magnitude has been assigned zero value. This magnitude parameter is shown as a function of the altitude parameter in figure 3.

To determine the minimum magnitude (maximum luminous intensity), equation (23) is differentiated with respect to u

$$\frac{2}{5} \ln 10 \frac{dM}{du} = \left\{u \exp(-u)\left[\overline{Ei}(u_1) - \overline{Ei}(u)\right]\right\}^{-1} - 2 - \frac{p+1}{u} \tag{25}$$

For large velocities, the term $(p+1)/u$ will be small. Then dM/du will be approximately zero when

$$\exp(-u_1)\left[\overline{Ei}(u_1) - \overline{Ei}(u_m)\right] = \frac{1}{2u_m} \exp(u_m - u_1) \tag{26}$$

The function $\exp(-u)\overline{Ei}(u)$ approaches u^{-1} for large u so that a further approximation is

$$\exp(u_1 - u_m) = \frac{3}{2} \frac{u_1}{u_m} \tag{27}$$

It can be seen from the solution of equation (27) that u_m will be close to u_1, so that

$$u_1 - u_m \simeq \ln \frac{3}{2} \tag{28}$$

Equations (24), (26), and (28) can be used to arrive at

$$M_m + \frac{5}{2} \log\left[\left(\frac{r_1 \beta D}{\rho_o}\right)^3 \cos \theta\right] = M' - \frac{5}{2} p \log \frac{u_1}{\zeta} \tag{29}$$

where again all constants not shown are collected in the reference magnitude M'.

Reduction of Experimental Data

The following properties of meteor behavior have been measured: velocity v and deceleration a at altitude y, the end point altitude y_e of the meteor trail, the altitude y_m at which the intensity of luminous radiation is a maximum, the magnitude of maximum luminous intensity M_m, and the angle of inclination θ of the meteor path. The end point altitude will prove useful because it is essentially dependent only on the initial velocity and size of the meteor (see figs. 1 and 3). For large velocities where u_1 is large compared to unity $\overline{Ei}(u_e) \ll \overline{Ei}(u_1)$, and a good approximation is, according to equation (13)

$$\exp(-\beta y_e) = \frac{2\beta r_1 D \cos\theta}{3\rho_o} \exp(-u_1)\overline{Ei}(u_1) \tag{30}$$

Then from equations (13), (16), and (30)

$$\overline{Ei}(u) = \overline{Ei}(u_1)[1 - \exp(\beta y_e - \beta y)] \tag{31}$$

and

$$\exp(-u)\overline{Ei}(u) = \frac{2a}{\beta \cos\theta v^2}[1 - \exp(\beta y - \beta y_e)] \tag{32}$$

Equation (32) may be solved for u by graphical or numerical means; the kinetic energy fraction ζ is determined from the definition, equation (8); and u_1 is calculated from equation (31). The unknowns r_1 and D cannot be separated, but an estimate of D can hardly be off more than a factor of 2 or 3 so the initial size r_1 is determinable within the same factor. The product r_1D is calculated from equation (30).

At very high velocities where both u_1 and u are large compared to unity, the data may be reduced analytically from the approximations

$$u \approx \frac{\beta \cos\theta v^2}{2a}[1 - \exp(\beta y - \beta y_e)]^{-1} \tag{33}$$

$$u_1 - u \approx \ln[1 - \exp(\beta y_e - \beta y)] \tag{34}$$

$$r_1 D \approx \frac{3\rho_o u_1}{2\beta \cos\theta} \exp(-\beta y_e) \tag{35}$$

With r_1D and u_1 determined from the above equations, the exponent p of the luminous intensity velocity function (eq. (18)) may be determined. The sum $M_m + \frac{5}{2}\log\left(\frac{\beta r_1 D}{\rho_o}\right)^3 \cos\theta$ is plotted as a function of $\log u_1/\zeta$, and according to equation (29) the slope will be -2.5p. It may be noted that the intensity seen by the observer on the earth's surface will be inversely proportional to the square of the distance from the meteor to the observer, $y^2/\cos^2\psi$, where ψ is the angle of inclination of the observer's line of sight. In addition, the meteor trail is a line source with apparent brightness which varies inversely with $\cos\chi$, where χ is the angle between the meteor trail and the observer's line of sight. Thus, the minimum magnitude will be

$$M_m = -\frac{5}{2}\log\left[I_m\cos\chi_m\left(\frac{y_m}{\cos\psi_m}\right)^2\right] \tag{36}$$

and it is this quantity which should be used to test the relation given by equation (29).

EXPERIMENTAL DATA AND DISCUSSION OF RESULTS

It is disappointing that most of the observed physical behavior of meteors which is reported in the literature is merely descriptive or very incomplete. The one exception is a table of quantitative measurements of height, velocity, deceleration, and luminous magnitudes for a group of about 20 meteors which is presented by Whipple (ref. 5). The pertinent data from this group of measurements are abstracted in table I, except for meteor 505 data which have been omitted because this meteor broke into distinct fragments and is atypical. Also shown in table I are the values calculated for ζ, u_1/ζ, $r_1\beta D/\rho_0$, and $M_m + \frac{5}{2}\log\left(\frac{r_1\beta D}{\rho_0}\right)^3\cos\theta$. The two separate sets of observations for meteor 663 are inconsistent, and only the calculations based on the first set of observations are included. In these calculations the following constants were used: $\beta^{-1} = 8.02$ km (26,300 ft), $q = 7.2$ km^2/sec^2(77×10^6 ft^2/sec^2), and $\rho_0 = 1.29\times10^{-3}$ gm/cm^3 (0.00238 slug/ft^3). The mean value and root mean square deviation for ζ which fits these data is 0.89 ± 0.67. The values of $M_m + \frac{5}{2}\log\left(\frac{r_1\beta D}{\rho_0}\right)^3\cos\theta$ are plotted as a function of $\log u_1/\zeta$ in figure 4. The linear regression giving the least mean squares fit to these data is also shown in figure 4 and has a slope -5.77, whence p takes the value 2.30 according to equation (29).

The luminous efficiency factor τ (eq. (17)) varies as the (2p-3) power of velocity according to the notation of equation (18). Thus the value of 2.30 for p corresponds to τ proportional to $v^{1.6}$. This variation is somewhat stronger than the linear function proposed by Öpik. However, it will be noted from table I that almost all of the data available have a strong central tendency around $v_1 = 35$ km/sec (115,000 ft/sec) and therefore these data are not really suitable for accurately determining the slope. A group of observations on some slower meteors and on some very high-speed meteors is needed to anchor the end points of the best linear fit. Such data could very well yield a value for p of 2, which would give the linear relation of τ with velocity proposed by Öpik. The point of maximum light intensity moves nearer the end of the meteor trail as velocity increases according to the relationship shown in figure 3. This position was determined for a heat of vaporization per unit mass of 77×10^6 ft^2/sec^2, assuming that meteor light is detectable over a range of 6 magnitudes. The relation with velocity is shown in figure 5 along with the observations of J. F. Foster as reported in reference 5. The choice of maximum detectable magnitude is somewhat arbitrary, of course, as it depends on the sensitivity of the detector. The maximum intensity position also depends on meteor size and path inclination as well as on velocity. The curve on figure 5 would be displaced upward for a more sensitive detector, and downward for larger meteor size and smaller path inclination angle.

The calculated meteor performance characteristics agree, at least in a qualitative sense, with observed meteor behavior. For example: (1) the appearance of meteors is predicted in the proper range of altitudes; (2) the luminous intensity curves rise and then fall, as observed, with the maximum progressively nearer the end of the trail as velocity increases; and (3) the meteors exhibit nearly constant velocity throughout most of their path until the end where they are highly decelerated and rapidly vaporized in a short interval of altitude and time. Whipple (ref. 7) was able to integrate the meteor equations numerically for the determination of atmospheric density by taking advantage of this characteristic that velocity is nearly constant over a wide range of altitude.

Since the absolute values of the meteor characteristics are not immediately perceptible from the parametric relations shown in figures 1, 2, and 3, the performance of a typical meteor was calculated. An iron sphere, 0.01 foot in radius, entering the atmosphere at zero inclination was chosen to represent this typical meteor and a value of 0.9 was used for ζ. The resulting size, velocity, and deceleration are shown as functions of altitude in figures 6, 7, and 8.

It is obvious from table I and figures 4 and 5 that the experimental data are not sufficiently refined to provide a good quantitative check on the analysis. This scatter in results is typical of meteor data taken at the present state of the art. Table I shows, for example, that independent observations of the same meteor sometimes result in rather different decelerations and end-point altitudes. The velocity measurements are probably subject to the least error, but it has been recognized that decelerations are particularly difficult to measure (ref. 21). This is because the decelerations are very large and change rapidly as shown in figures 2 and 8. Figure 8 also shows that the gravitational acceleration is negligible compared to the atmospheric deceleration over most of the meteor path, as was assumed in the analysis.

Very likely more precise data would fit the analysis with considerably less scatter. However, some of the variance is probably due to differences between individual meteors which have not been taken into account. For example, the values of q and ζ are probably somewhat different for the stonelike and ironlike meteor material. It would be expected, then, that precise data might group into two categories, one for each type of material. (The predicted performance characteristics of stonelike meteors duplicate those shown for iron meteors in figures 6, 7, and 8 except that all events occur at slightly higher altitudes due to the decrease in meteor density (see eq. (13)).) Also, the values deduced for ζ would likely shift in the velocity range where the erosion process changes from vaporization to sputtering. In addition to the above differences, some meteors apparently crumble or spall rather than vaporize evenly (ref. 10) and this will result in a high value for ζ, which can be greater than unity. This is indeed the case for some of the meteors listed in table I. For the purpose of this paper, the total mass loss is the significant factor, and it does not matter whether this lost material is in a completely vaporized state or in molecular clusters. From this viewpoint, the most important quantitative result derived from the fit of data to the analysis is that ζ is the order of unity. It is concluded then that atmospheric bombardment erodes material from the surface of meteors with relatively high efficiency.

EROSION OF LARGE BODIES IN THE UPPER ATMOSPHERE

Vehicles made for travel through the upper atmosphere will generally move at considerably lower velocity than meteors. For example, the velocity of a satellite vehicle in altitude equilibrium is given by

$$v^2 = \frac{gR}{1 + (y/R)} \tag{37}$$

where g is the gravitational acceleration at the earth's surface and R the radius of the earth. Thus, the satellite will travel slower than the minimum velocity meteor by a factor of about $\sqrt{2}$. Ballistic type vehicles will be designed to travel even slower. Therefore, it cannot be concluded that these large, man-made vehicles will be eroded by atmospheric bombardment with the same efficiency as meteors. However, it seems likely that the meteor theory will set an upper limit on the erosion rate, and it will therefore be of interest to calculate this limit.

The vehicles being considered will be much larger than a representative meteor and continuum flow conditions will exist up to somewhat over 300,000 feet altitude. Therefore, the meteor erosion model will only apply to these vehicles at altitudes in excess of this, and the vaporization which may occur at lower altitudes should be calculated from the heat transfer to the vehicle which is predicted by continuum aerodynamic theory. In addition, the vehicle will generally not be homogeneous. The inhomogeneity of the structure can be accounted for by defining an average density $\overline{D} = \delta D$ where D is the density of the surface material. Then $\overline{D}$ replaces D in equations (3) and (4) but not in equation (6). The solution then proceeds in exactly the same form as before except that the parameter u now becomes $\delta\zeta v^2/12q$. However, for bodies with mass-to-surface ratio as great as that of a probable man-made vehicle, this solution is more complex than necessary. Such bodies are not decelerated appreciably in the upper atmosphere and terms with the factor dv/dt may be neglected. In other words, the kinetic energy change of the body is negligible compared to the change in kinetic energy of the atmosphere. Then

$$\frac{dE}{dt} = -\frac{\varphi\pi r^2 \rho v^3}{2} \tag{38}$$

where, as before, r is defined as $(3V/4\pi)^{1/3}$ and φ is the shape factor. Consistent with the meteor analysis, a fraction ζ of this energy is assumed to vaporize surface material.

$$\zeta \frac{dE}{dt} = 4\pi Dqr^2 \frac{dr}{dt} \tag{39}$$

For an exponentially scaled atmosphere, equations (38) and (39) lead to

$$\frac{dr}{dt} = -\frac{\varphi\zeta v^3 \rho_o}{8qD} \exp(-\beta y) \tag{40}$$

Equation (40) can be integrated after expressing the trajectory and velocity as functions of time. A reasonably good estimate of erosion rate is obtained by considering a vehicle entering the atmosphere at constant velocity and angle of incidence, since the change in both velocity and incidence due to gravitational acceleration is small and the drag forces are negligible within that portion of the atmosphere with which we are concerned. Then noting that

$$dt = \frac{dy}{v \cos \theta} \tag{41}$$

equation (40) integrates to

$$r_1 - r = \frac{\varphi\zeta v^2 \rho_o}{8\beta qD \cos \theta} \exp(-\beta y) \tag{42}$$

The amount of erosion which will occur on vehicles which have penetrated the atmosphere to 360,000 feet altitude is shown in figure 9 as a function of velocity for a variety of surface materials. It can be seen from figure 9 that this erosion will be small and will probably not be serious in terms of strength or mass loss. The principal effect might be to create sufficient roughness so that turbulent-flow heat transfer would be experienced by the vehicle during its descent through the lower, continuum atmosphere. The heat transfer during this portion of the trajectory could thus be increased by an order of magnitude. In addition, high surface polishes for the purpose of reflecting radiation could be rendered ineffective by the high-altitude, free-molecule bombardment ablation. The answers to these conjectures are among the important problems which need to be solved by further research if we are to appreciate the practical limitations of high-altitude flight. In any event, the choice of a surface material with a large qD product should minimize the erosion, according to equation (42).

Substituting equation (37) in (40) gives the rate of surface loss from a satellite

$$\frac{dr}{dt} = -\frac{\varphi\zeta\rho_o}{8qD}\left[\frac{gR}{1 + (y/R)}\right]^{3/2}\exp(-\beta y) \tag{43}$$

The surface loss rate of a satellite is plotted in figure 10 as a function of altitude, again for various surface materials. If the shape factor φ and energy fraction ζ are considered near unity, it can be seen that a satellite which is to persist for about one year with less than 0.01-inch surface erosion should orbit at altitudes greater than 800,000 feet. Satellites will, in general, have to travel at altitudes much higher than this anyway so that aerodynamic drag will not influence their orbit. Therefore, it is concluded that ablation of the satellite will probably not be detrimental to its function, but might influence the chance of successful recovery of the vehicle, due to the effect of surface conditions on heat transfer during entry into the atmosphere.

CONCLUSIONS

1. An analysis based on the assumptions that the atmospheric bombardment of meteors is inelastic, and that a constant fraction ζ of the bombardment energy is used in vaporizing meteor material, qualitatively predicts the size, velocity, and deceleration of meteors as a function of altitude in terms of the initial size, the initial velocity, the density, the specific vaporization energy of the meteor, and the energy fraction ζ. The relative magnitude of luminous radiation from meteors is also in qualitative agreement with observation.

2. The behavior of meteors can be correlated conveniently in terms of dimensionless parameters. The size, velocity, deceleration, and the luminous magnitude parameters are functions of the altitude parameter which are uniquely determined by the initial velocity parameter.

3. The quantitative agreement between meteor theory and observation is erratic due mainly to the scatter in the observed data, particularly in the deceleration and end-point altitude measurements. However, the data do indicate that meteors are efficiently eroded by the atmosphere bombardment; that is to say, the energy fraction ζ is the order of unity.

4. Observations of meteor behavior which are more self-consistent are needed in order to detect effects of different meteor materials and of different erosion processes that may occur for high and low velocity meteors. In addition, more observations on very high velocity meteors and on very low velocity meteors are required to determine the dependence of luminous intensity on velocity.

5. Vehicles traversing the upper atmosphere in free-molecule flow conditions at meteor velocities will not suffer noticeable loss of mass or strength by erosion. Further research is needed to determine whether such erosion might create sufficient surface roughness so that the vehicle would not efficiently reflect radiant energy and would experience turbulent-flow heat transfer during its descent through the lower, continuum atmosphere.

Ames Aeronautical Laboratory
National Advisory Committee for Aeronautics
Moffett Field, Calif., Jan. 2, 1957

APPENDIX A

MINIMUM AND MAXIMUM VELOCITY OF METEORS

Consider first a meteor accelerated from infinity by the earth's gravitational field in the absence of any perturbing bodies. Then the change in kinetic energy of the meteor equated to the change in potential energy is

$$\frac{V^2 - V_\infty^2}{2} = \int_y^\infty \frac{gR^2}{(y + R)^2}\, dy = \frac{gR}{1 + (y/R)} \tag{A1}$$

where g is the gravitational acceleration at $y = 0$ and R is the radius of the earth. The atmosphere is exceedingly thin compared to the radius of the earth, so the entrance velocity of the meteor into the atmosphere is given approximately by neglecting y/R compared to unity in equation (A1). All meteors at infinity which are initially moving away from the earth will never be collected and the minimum velocity meteor will be that which falls through the gravitational potential from an initial velocity relative to the earth which is close to zero.

Now the meteor can be slowed down in its fall from infinite altitude by a collision with another body (collision includes, of course, a change in orbit due to attractions by other bodies). However, it can be seen from equation (A1) that the effect on the minimum velocity will be negligible unless the collision occurs within the range y/R less than about 20. The chance of a collision is small within this range and will be neglected. Thus the minimum velocity with which meteors will enter the atmosphere is normally just the escape velocity from the earth's surface.

$$(V_1)_{min} = \sqrt{2gR} = 37{,}000 \text{ ft/sec} \tag{A2}$$

If meteors should originate outside the solar universe it is expected that they would have a statistical distribution of velocities from the minimum velocity up, due to their orbiting collisions with other mass concentrations throughout space. If, however, the meteors originate within the solar universe or from a near portion of this galaxy which is traveling about the same speed as our solar system, then the maximum velocity meteor would normally be one that is intercepted by earth as it falls into the sun from rest at infinity and from a direction opposite to the earth's velocity vector. The escape velocity from the sun at the earth's orbit is 138,000 ft/sec and the earth's velocity in this orbit is 100,000 ft/sec. The fall through the earth's potential adds about 3,000 ft/sec to the velocity, so meteors of solar origin could enter the atmosphere at velocities up to 240,000 ft/sec. Meteor velocities do, in fact, cut off at about 73 km/sec (240,000 ft/sec) (ref. 10).

APPENDIX B

HEATING OF METEORS AND RADIATION LOSSES

Meteors which enter the atmosphere with a low velocity (from 36,000 to 56,000 ft/sec) are probably vaporized by thermal heating rather than by sputtering. In this case some of the energy flux received from the atmosphere will be required to heat the meteor up to vaporization temperature. Neglecting radiation losses and assuming that initially all of the incident energy flux is transformed into heat (see ref. 22) which diffuses uniformly throughout the meteor, we can express the energy equation:

$$\frac{\varphi\pi r^2\rho v^3}{2} + \frac{4}{3}\pi r^3 D v \frac{dv}{dt} = -\frac{4}{3}\pi r^3 DC \frac{dT}{dt} \qquad (B1)$$

where C is the heat capacity of the meteor material. The momentum equation remains the same

$$\frac{4}{3}\pi r^3 D \frac{dv}{dt} + \varphi\pi r^2\rho v^2 = 0 \qquad (B2)$$

and with equation (B1) leads to

$$v \frac{dv}{dt} = -2C \frac{dT}{dt} \qquad (B3)$$

whence

$$\frac{v^2{}_1 - v^2}{2} = 2C(T - T_1) \qquad (B4)$$

For an exponentially scaled atmosphere, equation (B2) becomes

$$\frac{4rD}{3v}\frac{dv}{ds} = \varphi\rho_o \exp(-\beta s \cos\theta) \qquad (B5)$$

and, for the initial portion of the trajectory where the meteor is not yet vaporizing, r is constant so that equation (B5) may be integrated to

$$\ln\frac{v}{v_1} = \frac{3\varphi\rho_o}{4\beta r D \cos\theta}\exp(-\beta y) \qquad (B6)$$

Equations (B4) and (B6) lead to

$$\frac{4C(T - T_1)}{v_1^2} = 1 - \exp\left(- \frac{3\varphi\rho_o}{2\beta rD \cos \theta} e^{-\beta y}\right) \tag{B7}$$

The exponential argument is very small for the values of βy being considered, so that equation (B7) is approximately equivalent to

$$\frac{4C(T - T_1)}{v_1^2} = \frac{3\varphi\rho_o}{2\beta rD \cos \theta} \exp(-\beta y) \tag{B8}$$

and the altitude at which a meteor of radius r will reach a temperature T is given by

$$\beta y + \ln rD \cos \theta = \ln\left[\frac{3\varphi\rho_o}{8\beta} \frac{v_1^2}{C(T - T_1)}\right] \tag{B9}$$

It is found that the size and velocity of the meteor predicted by the analysis have suffered very little change from their initial values at the altitude where T reaches vaporization temperature, and therefore only a small lag in time and altitude should be needed for heating the meteor. This can be seen from equation (13), where for large values of u the approximation $\overline{Ei}(u) \approx u^{-1} \exp u$ is used

$$\beta y + \ln rD \cos \theta \approx - \ln \frac{2\beta}{3\varphi\rho_o u_1}\left[1 - \exp(u - u_1)\right] \tag{B10}$$

It follows from equations (9), (B9), and (B10) that the radius which is predicted for an eroding meteor corresponding to the altitude where the meteor should be heated to a temperature T, is

$$1 - \frac{r}{r_1} = \frac{\varphi\zeta C(T - T_1)}{3q} \tag{B11}$$

For φ and ζ near unity, $C(T - T_1) = 16 \times 10^6$ ft^2/sec^2, and $q = 77 \times 10^6$ ft^2/sec^2, values which are appropriate to the vaporization of iron, for example, this change is less than 7 percent. It is expected, therefore, that the correction to the analysis needed to account for the heating lag will be small.

When the meteor heats up it also suffers losses by radiation which have been neglected. The rate of loss by radiation will be

$$\frac{dR}{dt} = 4\pi r^2 \epsilon \sigma T^4 \tag{B12}$$

where ϵ is the emissivity of the meteor surface and σ the Stefan-Boltzmann constant. The energy flux received from the atmosphere is

$$\frac{dE}{dt} = \frac{\varphi \pi r^2 \rho v^3}{2} \tag{B13}$$

and the ratio of radiation loss to incident energy flux is

$$\frac{dR}{dE} = \frac{8 \epsilon \sigma T^4}{\varphi \rho_o v^3 \exp(-\beta y)} \tag{B14}$$

For ϵ and φ about unity and T about 5400° R, this ratio is very small compared to unity at the altitudes where significant erosion occurs. Even for the very lowest velocity meteors, $v_1 = 37{,}000$ ft/sec, this ratio is less than 6 percent at altitudes of meteor activity. Therefore it is concluded that the correction to the energy equation needed to account for radiation losses is also small.

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TABLE I.- OBSERVED AND CALCULATED METEOR CHARACTERISTICS

Observed data from reference 5							Calculated data			
Meteor number	y, km	y_e, km	u, km/sec	a, km/sec^2	$\cos\theta$	$-M_m$	[1]ζ	u_1/ζ	$r_1\beta D/\rho_o$, $\times 10^{-3}$	$-\left[M_m + \frac{5}{2}\log\left(\frac{r_1\beta D}{\rho}\right)^3\cos\theta\right]$
642	82.4	73.8	30.65	2.28	0.815	2.6	1.00	11.3	2.19	22.84
642	77.4	73.8	29.79	8.40	.815	2.6	.92	11.4	2.03	23.02
651	84.3	75.1	36.56	1.09	.978	2.8	2.25	15.6	4.83	20.20
660	58.9	48.1	13.26	.492	.562	3.2	2.16	2.2	32.48	14.98
660	62.9	47.1	15.47	.57	.682	3.5	1.04	2.9	19.62	16.72
663	114.7	109.4	79.69	.34	.123	4.0	2.07	74.2	2.18	26.23
663	99.5	96.8	68.43	.30	.074	3.7	---	---	---	---
670	74.4	71.0	23.88	2.07	.465	4.8	2.29	7.1	7.78	20.47
689	91.8	83.4	61.19	10.4	.734	4.6	.21	44.3	6.05	22.47
642	82.4	73.8	30.65	2.42	.815	2.6	.94	11.4	2.07	22.95
642	77.4	73.8	29.79	8.90	.815	2.6	.87	11.5	1.93	23.18
697	80.5	68.4	32.09	3.82	.711	2.8	.28	12.9	1.58	24.17
697	75.0	68.4	33.9	10.0	.711	2.8	.30	15.3	1.99	23.44
705	86.7	55.9	30.65	.60	.660	3.8	.13	10.8	3.21	22.96
705	78.5	55.9	30.20	1.56	.660	3.8	.14	11.4	3.54	22.63
705	69.7	55.9	28.47	7.11	.660	3.8	.11	10.9	2.72	23.50
705	61.7	55.9	23.65	19.1	.660	3.8	.18	10.0	4.01	22.23
710	80.6	67.6	30.17	1.75	.891	1.9	.68	10.7	2.81	20.16
710	74.7	67.6	28.44	7.6	.891	1.9	.44	10.7	1.79	22.63
712	79.4	63.5	29.20	1.14	.897	4.6	.68	10.0	4.33	22.45
712	70.2	63.5	28.10	4.5	.897	4.6	.82	10.0	5.13	21.90
727	82.7	74.3	35.31	1.6	.891	1.6	1.62	14.7	3.97	19.74
730	73.4	58.1	35.96	2.4	.968	4.3	.38	15.3	6.73	20.72
733	93.5	81.7	38.32	.73	.768	4.2	1.69	17.2	2.22	24.39
736	81.5	70.5	36.05	1.52	.880	1.7	1.06	15.3	4.39	19.53
736	81.5	70.5	36.05	1.78	.880	1.7	.90	15.5	3.76	20.03
778	77.0	64.6	33.25	2.62	.795	3.8	.44	13.4	3.70	22.30
778	77.0	64.6	33.25	2.26	.795	3.8	.51	13.3	4.27	21.82

[1](0.89 ± 0.67) mean value of ζ

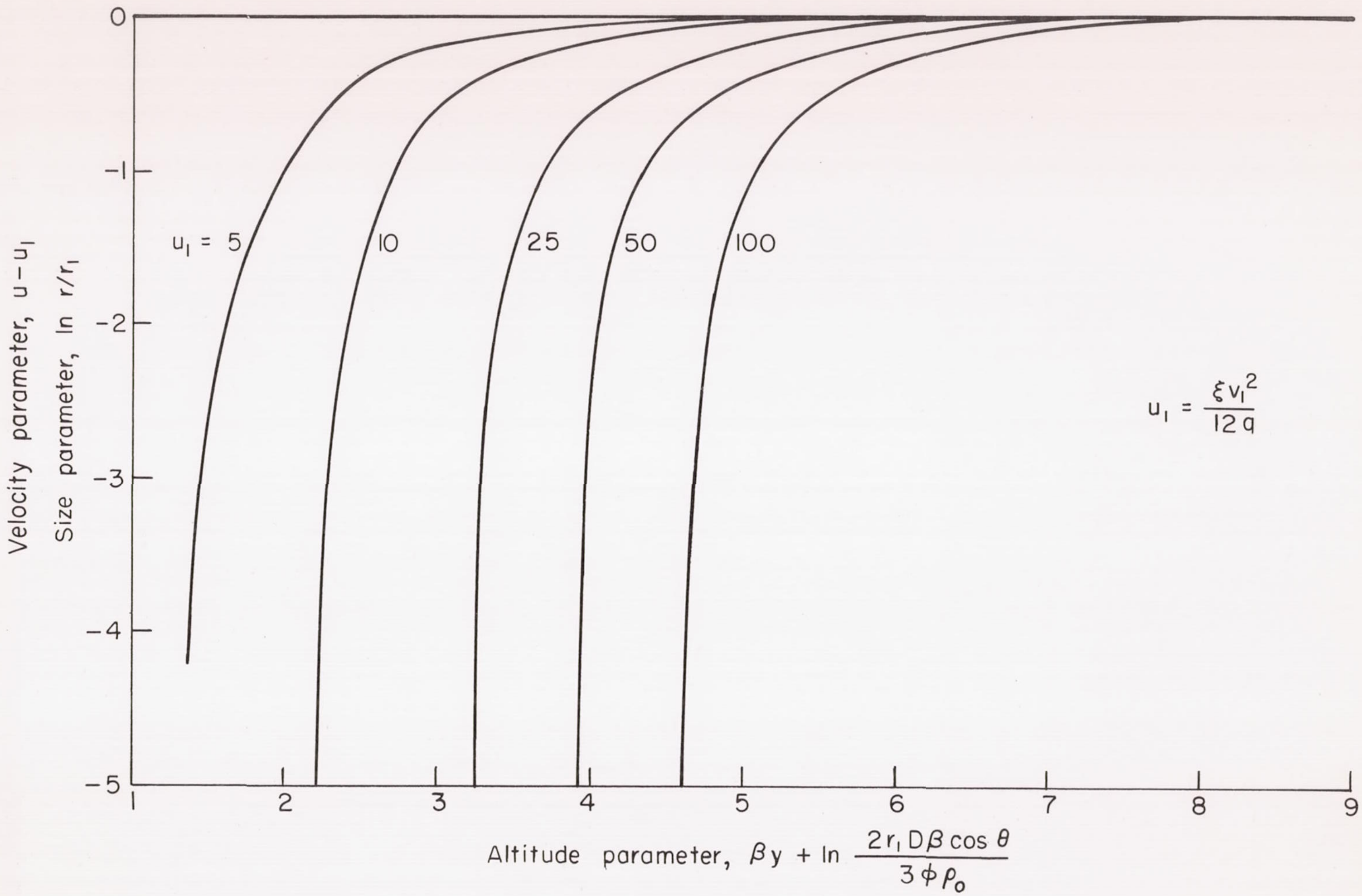

Figure 1.- Calculated velocity and size parameters for meteors as a function of altitude parameter.

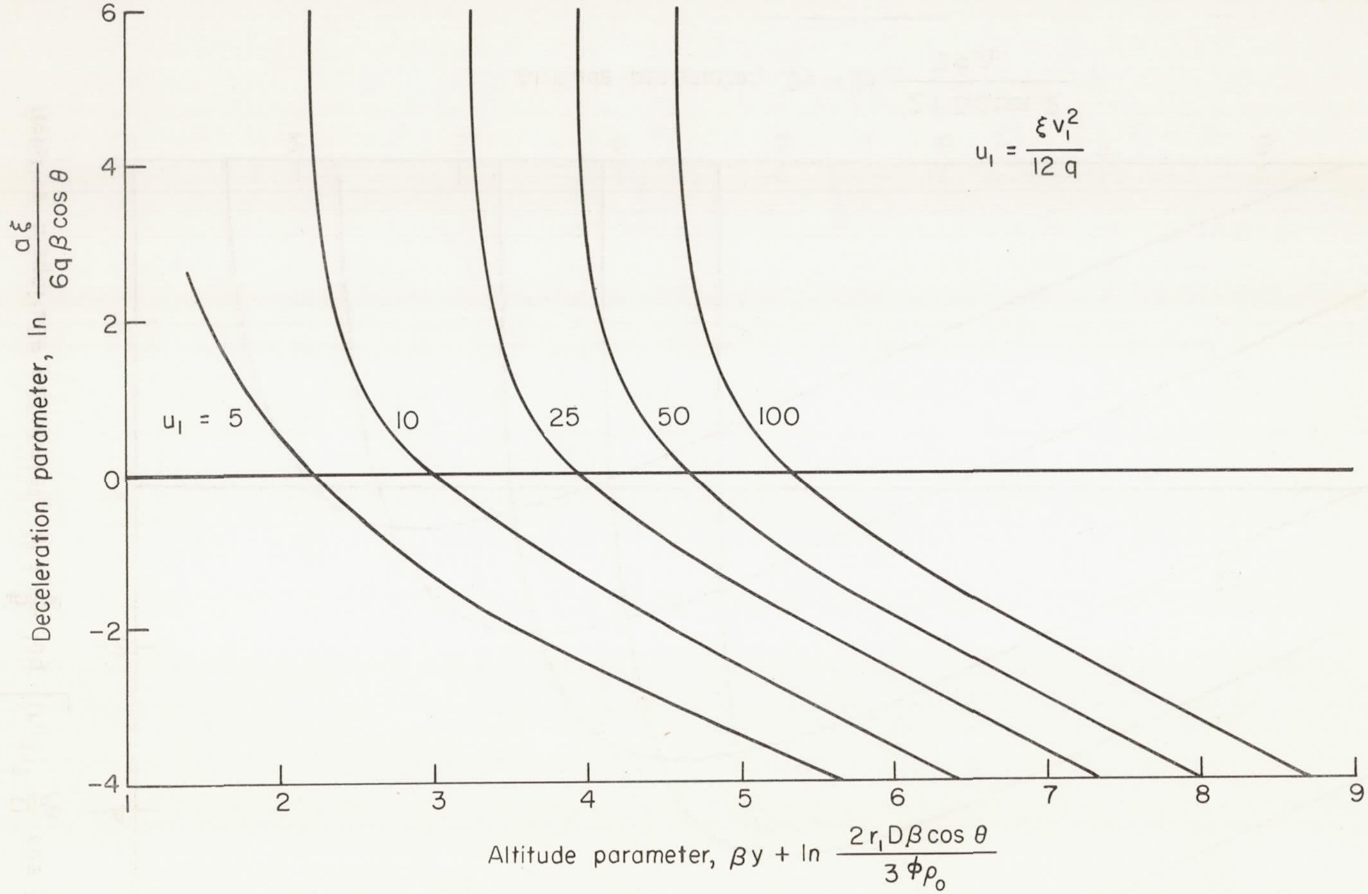

Figure 2.- Calculated deceleration parameter for meteors as a function of altitude parameter.

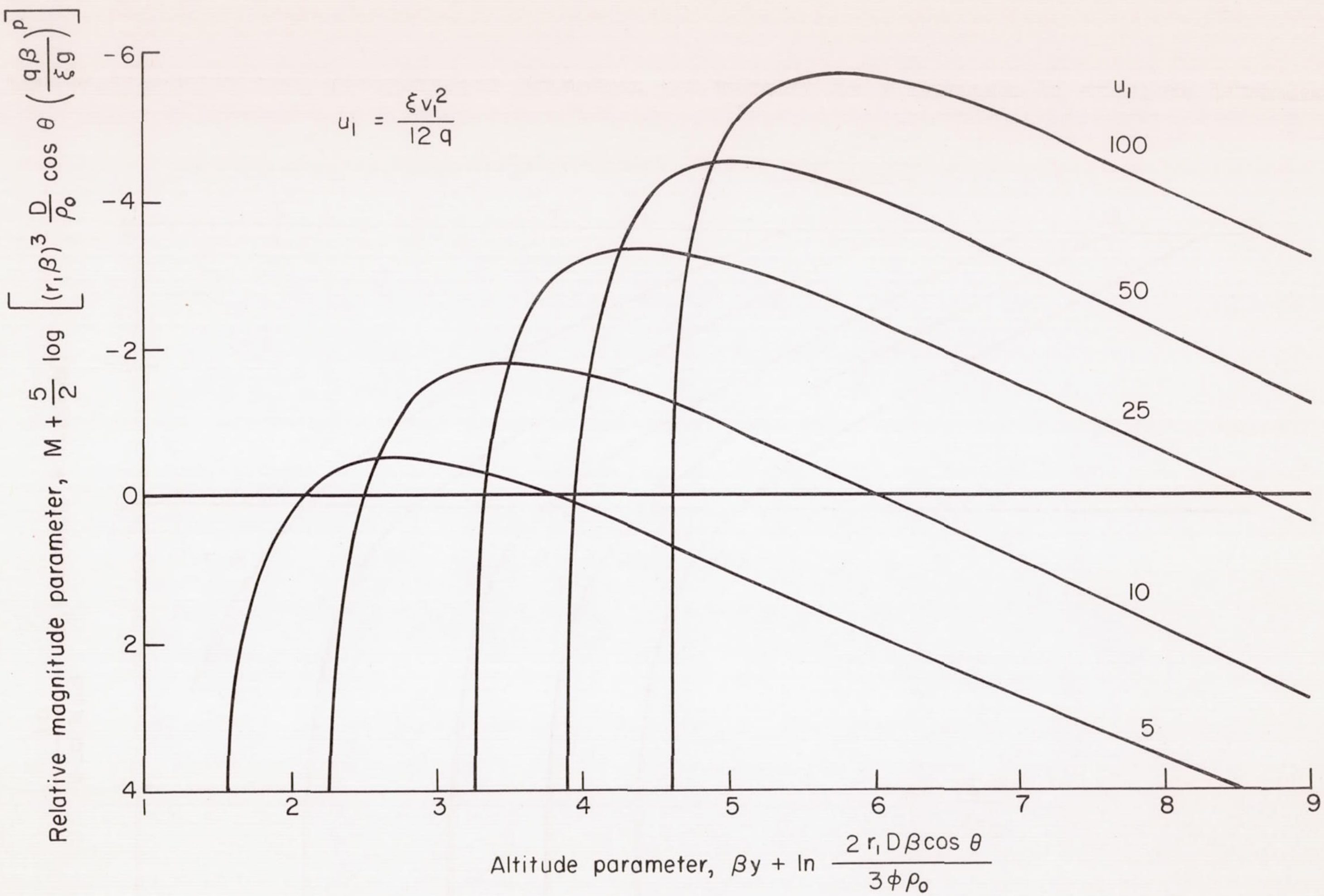

Figure 3.- Calculated magnitude of luminous intensity for meteors as a function of altitude parameter.

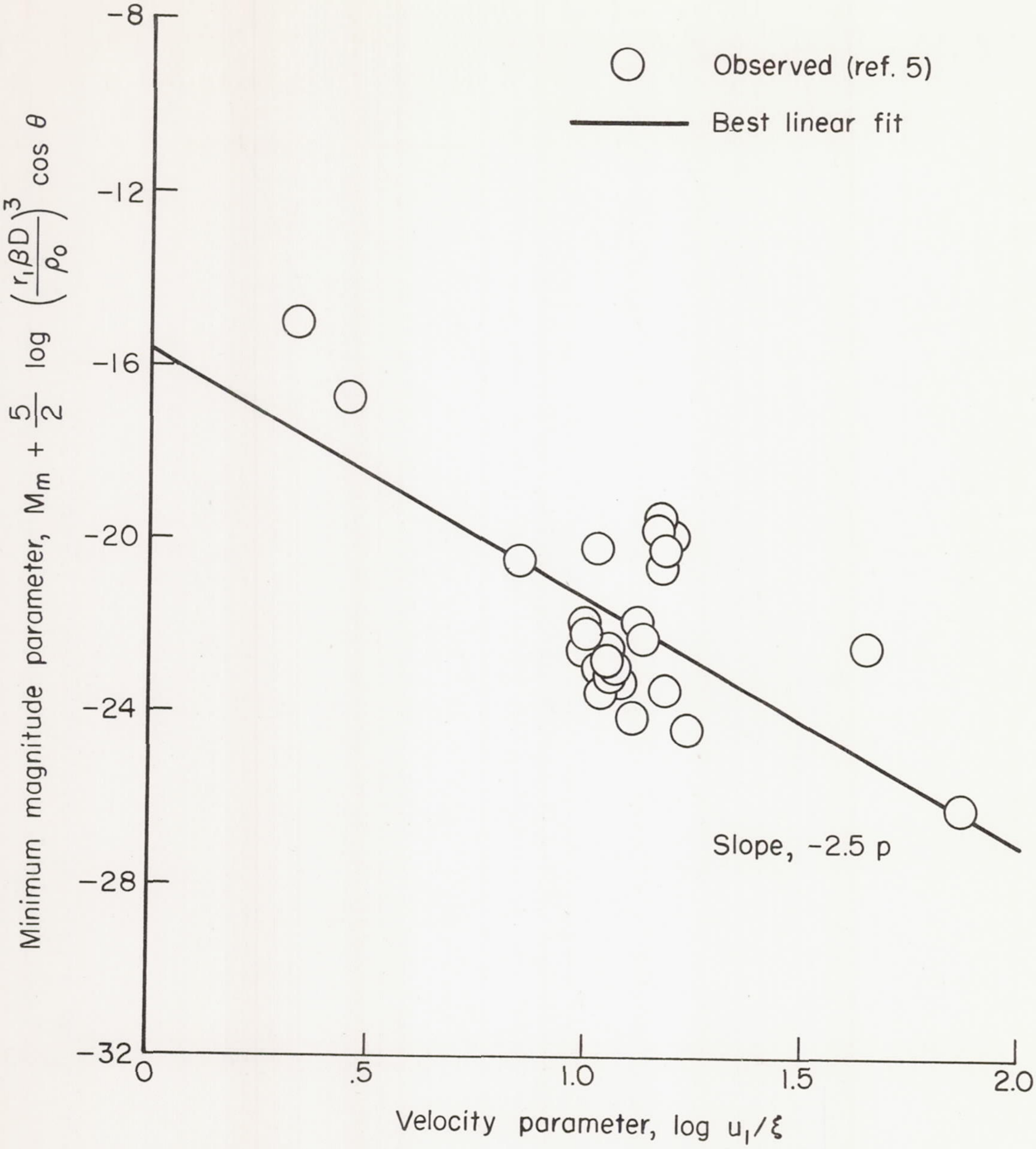

Figure 4.- Relation between the observed minimum magnitudes and velocities of meteors.

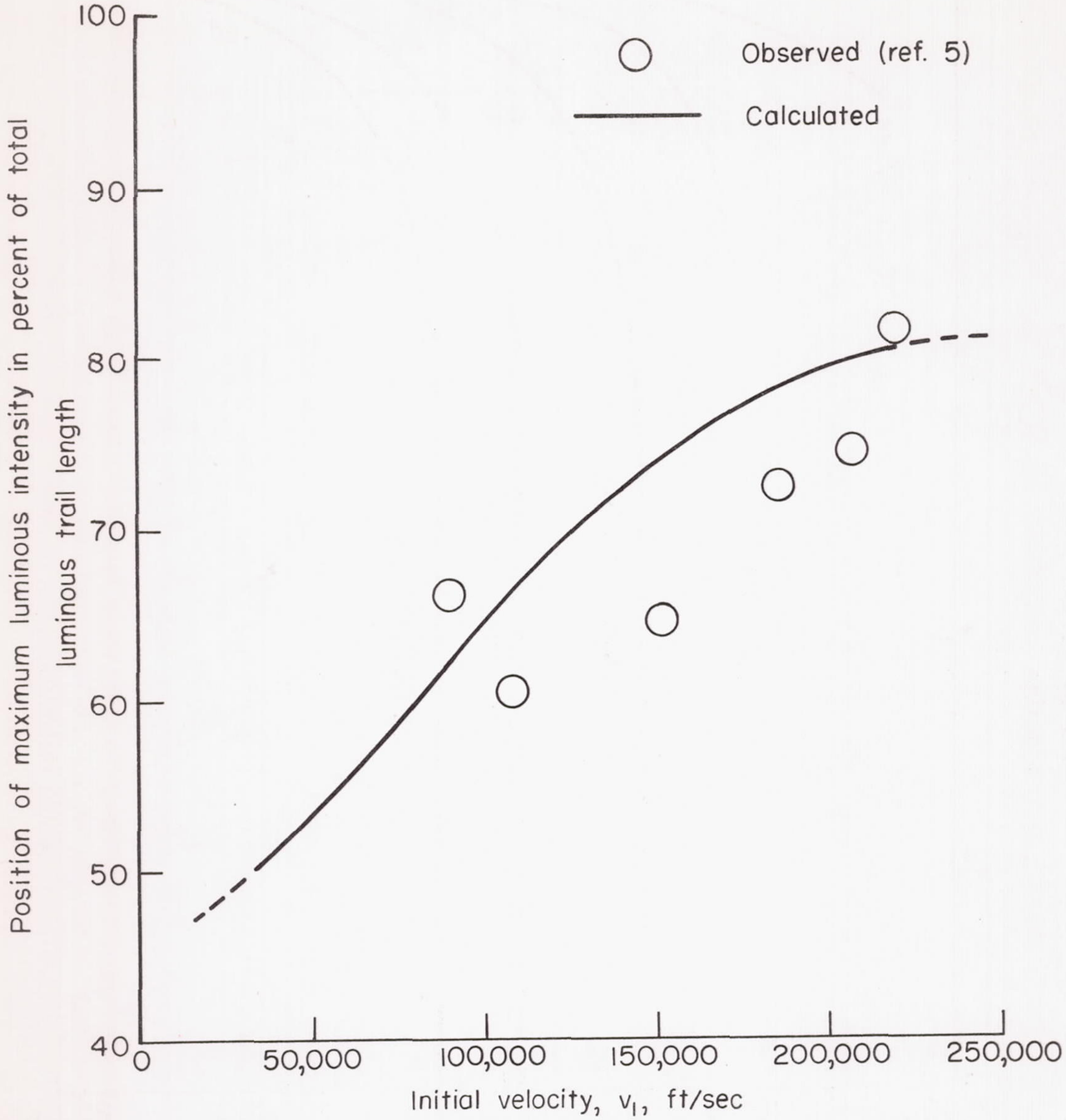

Figure 5.- Position of maximum luminous intensity of a meteor trail as a function of initial velocity.

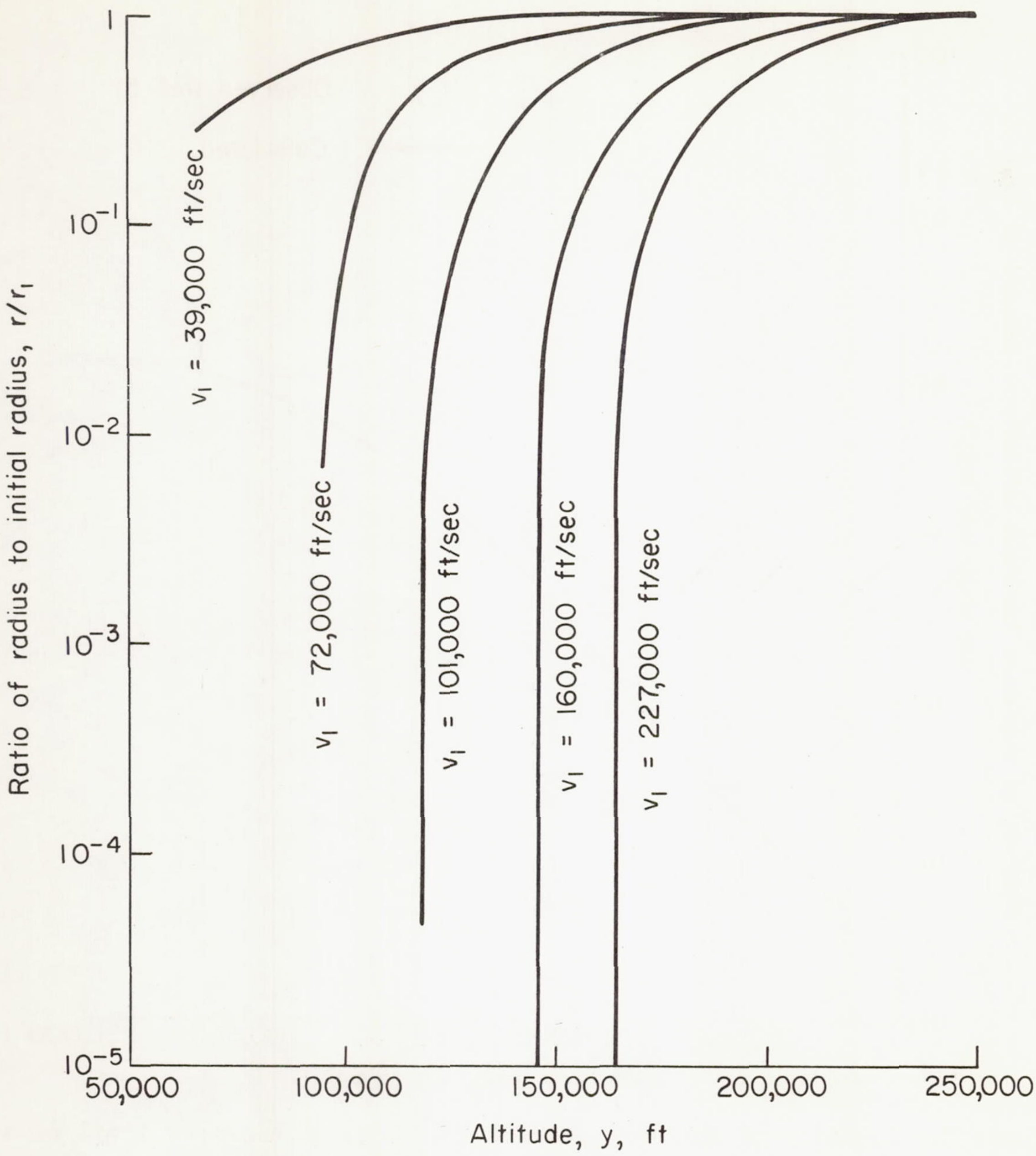

Figure 6.- Calculated size as a function of altitude for a 0.01 foot iron meteor entering the atmosphere at zero inclination.

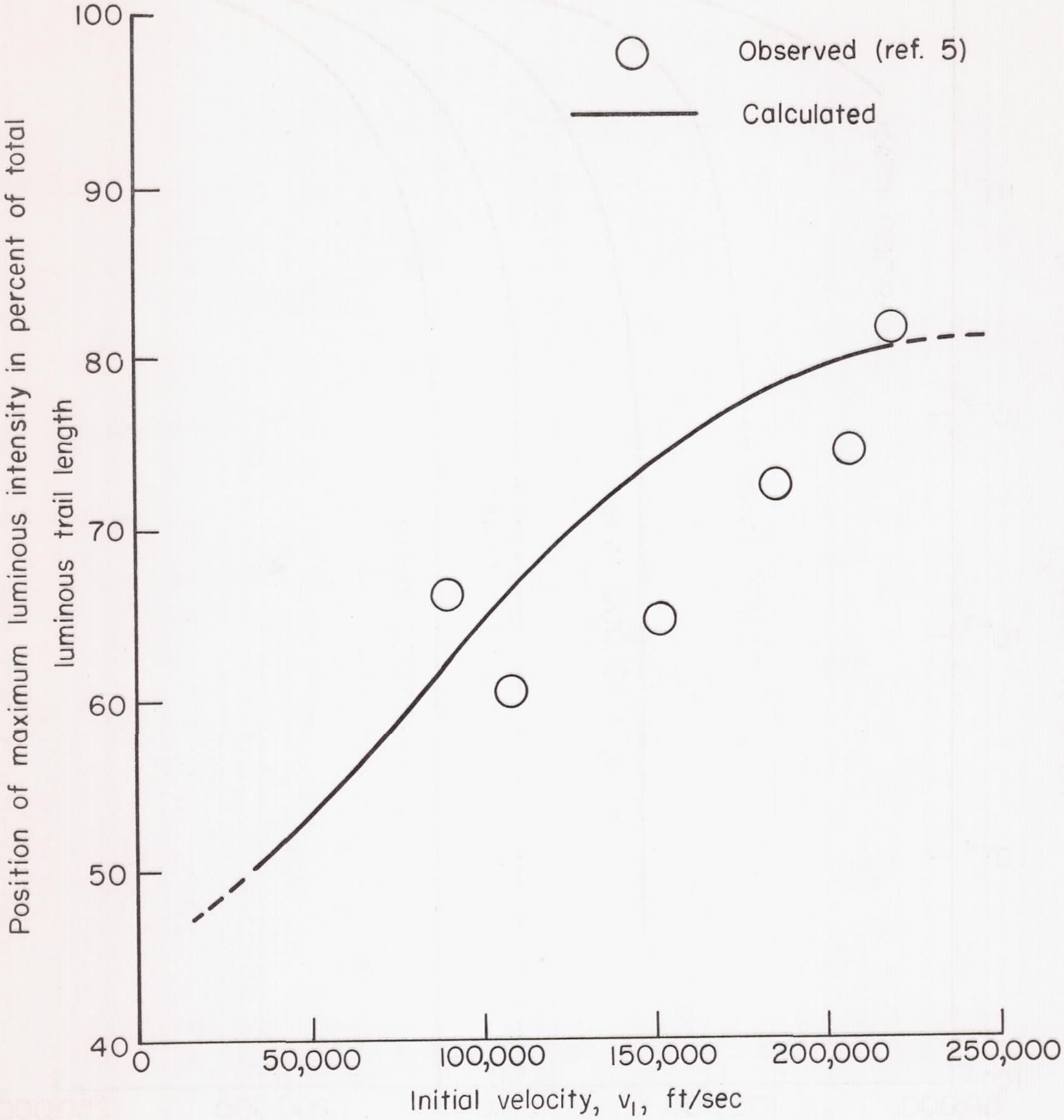

Figure 5.- Position of maximum luminous intensity of a meteor trail as a function of initial velocity.

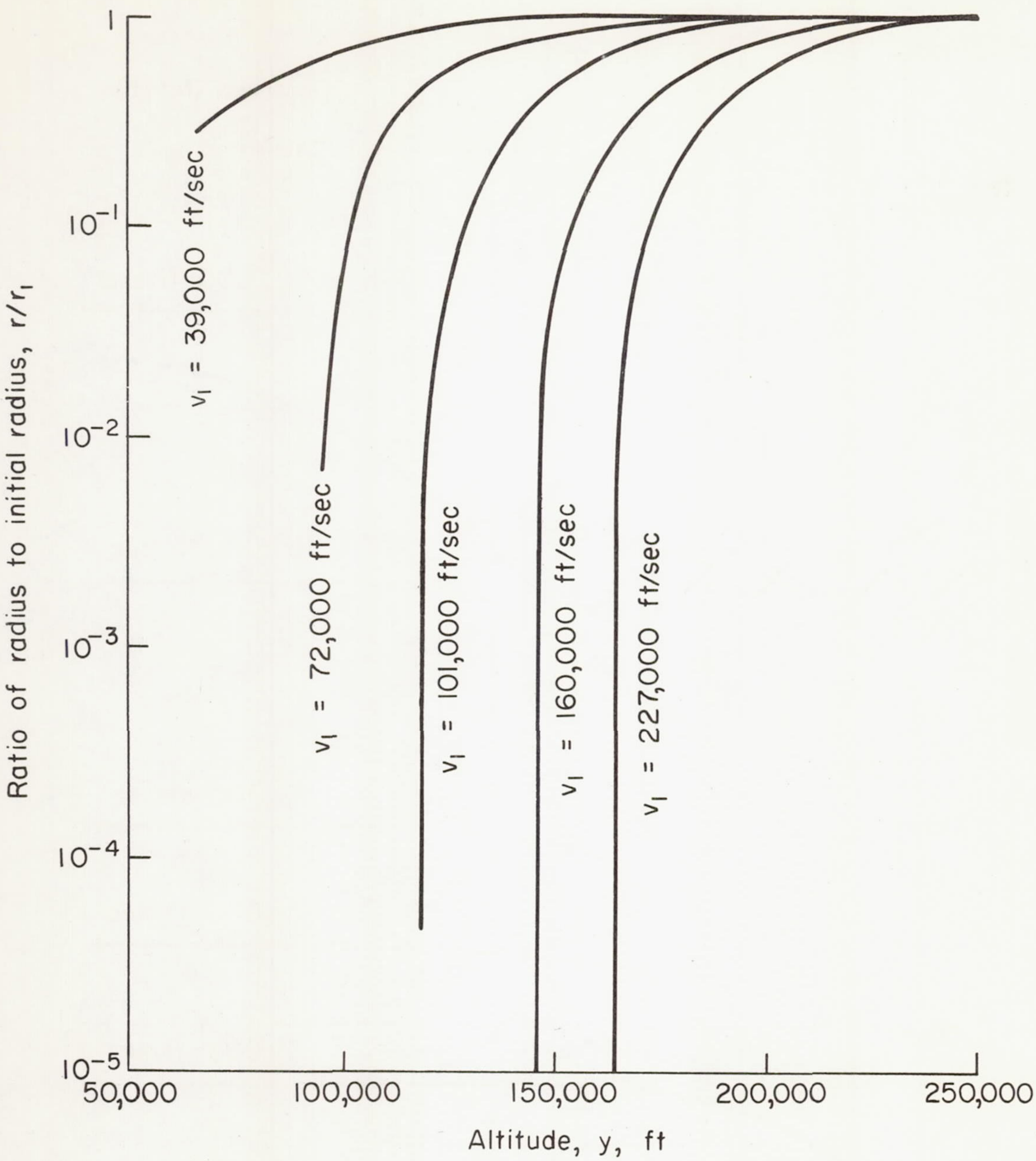

Figure 6.- Calculated size as a function of altitude for a 0.01 foot iron meteor entering the atmosphere at zero inclination.

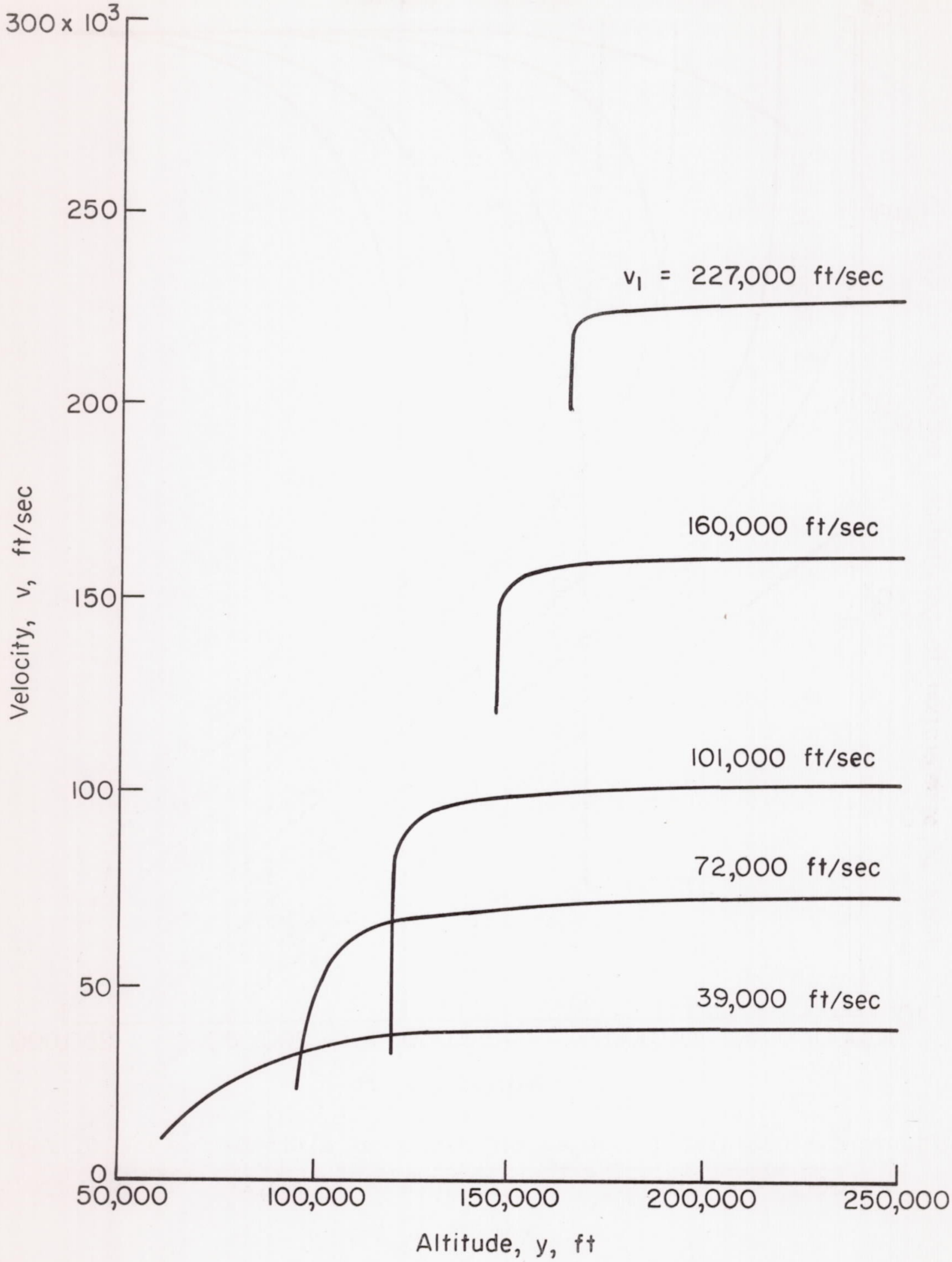

Figure 7.- Calculated velocity as a function of altitude for a 0.01 foot radius iron meteor entering the atmosphere at zero inclination.

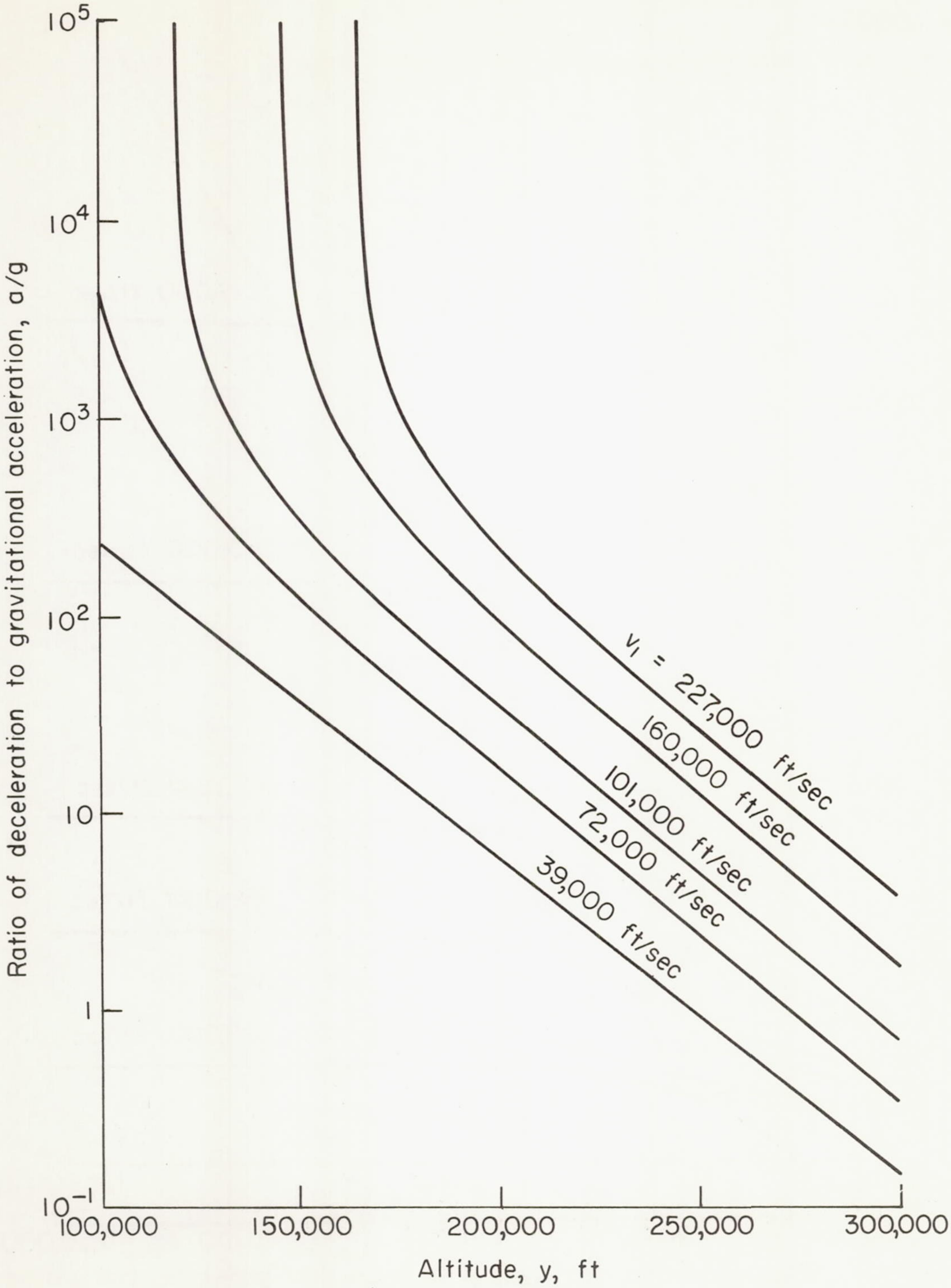

Figure 8.- Calculated deceleration as a function of altitude for a 0.01 foot radius iron meteor entering the atmosphere at zero inclination.

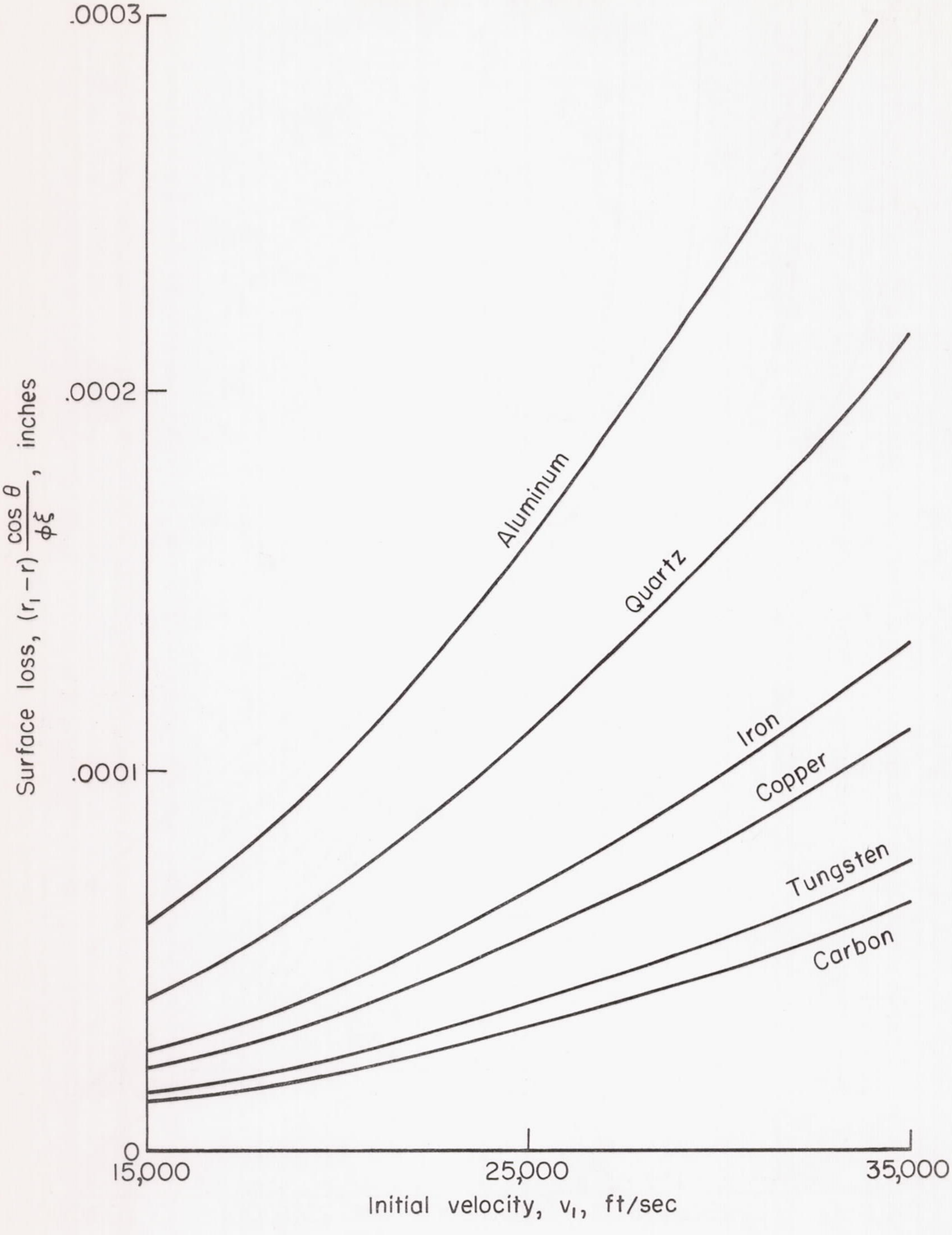

Figure 9.- Surface erosion on a vehicle, which has penetrated the atmosphere to an altitude of 360,000 feet as a function of initial velocity, for various surface materials.

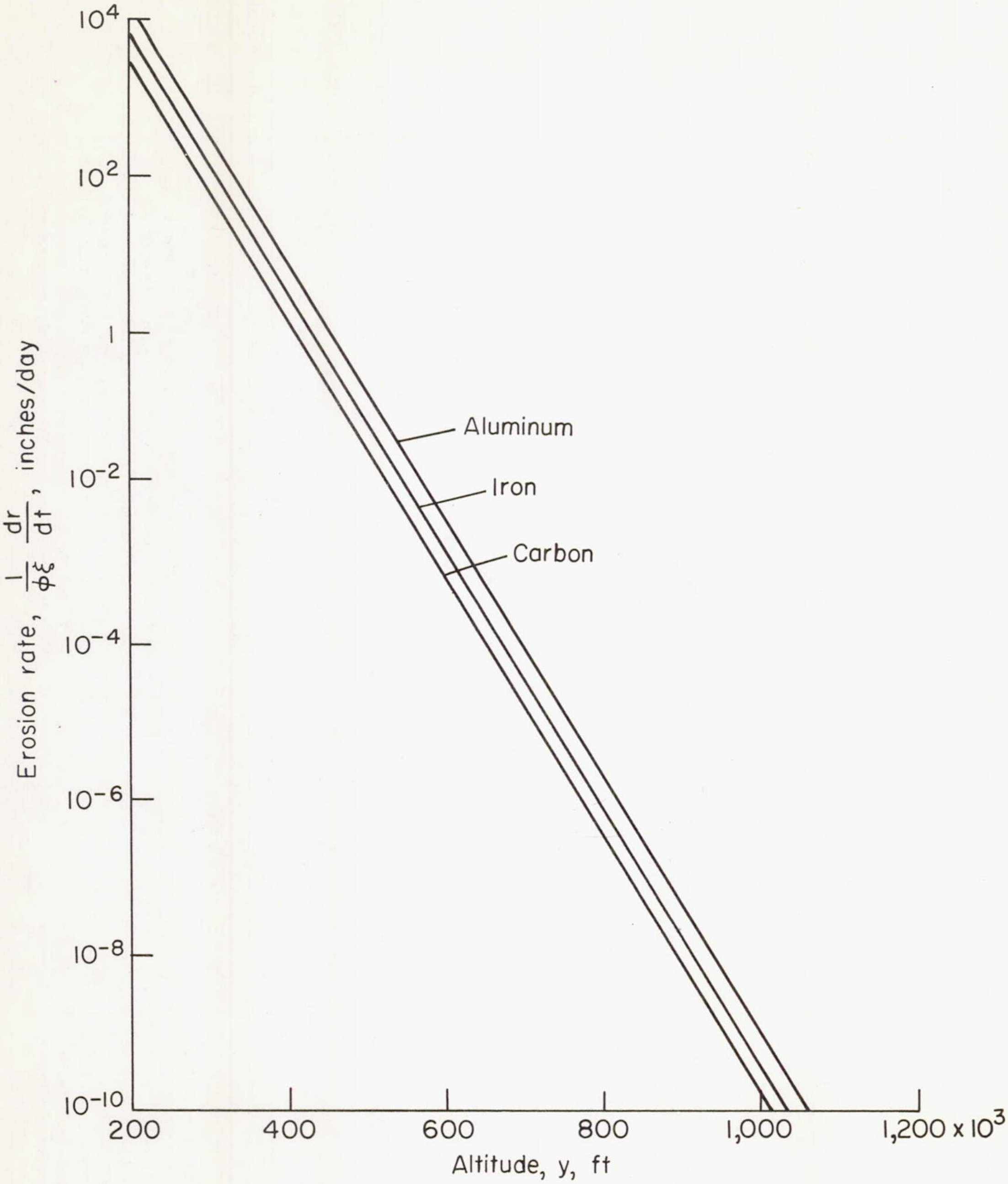

Figure 10.- Erosion rate of a satellite as a function of altitude for various surface materials.