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INHERENT STABILITY OF HELICOPTERS.

By G. Arturo Crocco.

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INHERENT STABILITY OF HELICOPTERS.*

By G. Arturo Crocco.

The equilibrium, in still air, of a "stationary" helicopter (i.e., of one having neither vertical nor translational velocity, but a tendency to remain practically motionless within restricted limits of space) presents some difficulty in practice and justifies a theoretical investigation of its "inherent stability," i.e., independent of the pilot.

Let us imagine, therefore, a helicopter reduced to its simplest form, namely, a pair of propellers revolving in opposite directions about the same axis of symmetry and driven by means of gearing inclosed in a sphere beneath.

Without presuming to know the best practical form it may be given by inventive genius, such a type is theoretically possible and may therefore be adopted as the ideal type.

Let us assume also that the resulting lift is centralized** and passes exactly through the axis of symmetry; that the latter passes through the center of gravity; and that there is a state of equilibrium between the lift and the force of gravity at the altitude where the helicopter is desired to remain stationary.

* From "Rendiconti della R. Accademia Nazionale dei Lincei," August, 1923, pp. 47-52.

** The lift may be decentralized by cyclic variations of incidence or velocity, spontaneous or voluntary, which occur during a gyration.

What would then happen, if some disturbing cause should incline the axis of symmetry z from its vertical position by a small angle γ?

Let us observe that there is no stabilizing action of position, such as maintains the equilibrium of a balloon and tends to restore the original vertical position of the axis, but that there is only a lateral thrust, due to the composition of the lifting force (acting at an oblique angle of γ to the vertical) and the weight mg.

This thrust, approximately $mg\gamma$, tends to produce lateral motion ("drift"), from which there can arise only an indirect straightening effect, capable of producing a rolling motion. "Drift" and "roll" are thus combined in a motion which, by extension, will be called "stable", when it consists of more or less decreasing oscillations about a mean position; and "unstable", when it consists of oscillations tending to increase.*

It is easily demonstrated that such a resultant motion, when composed of the lateral and vertical forces indicated, is essentially unstable.

We will designate by u the velocity of drift arising from the lateral thrust $mg\gamma$ toward the axis x, perpendicular to z; by u' its derivative or the baricentric acceleration of the moving body; with γ' and γ'' the successive derivatives of γ, or the

* We arrived at similar conclusions in the "Lateral Stability of Airplanes" (Rendiconti d'Istituto Sperimentale Aeronautico, May-July, 1912), in which the resultant motion is more complex, on account of the dissymmetry in the direction of drift.

angular velocity and acceleration of the rotation γ about the axis y perpendicular to x and z; by j the moment of inertia of the helicopter about the axis y.

As we shall demonstrate farther along, in propellers of special shape, there is generated in the drift a straightening moment $h\,u$, directly proportional to the velocity. Hence, for small angles, the equations of motion in this case may be written:

$$\begin{aligned} \gamma'' + h\,u &= 0 \text{ (roll)} \\ u' - g\gamma &= 0 \text{ (drift)} \end{aligned} \qquad (1)$$

in which the coefficients of u and γ are divided respectively by the mass and moment of inertia of the moving body.

By combining the above two equations, we obtain the single equation:

$$\gamma''' + hg\,\gamma = 0 \qquad (2)$$

which is satisfied in general by values of γ of exponential form, with exponents which are roots of the cubical equation:

$$x^3 + hg = 0 \qquad (3)$$

which, for $h > 0$, has only one real negative root, $-x_1$; and a pair of complex roots $\alpha \pm \beta i$, with a real positive part $\alpha = \frac{x_1}{2}$; and coefficient of the imaginary part $\beta = \alpha\sqrt{3}$.

There is generated therefore, at least for an exponential term rapidly tending toward zero, an oscillatory motion of semiperiod $T = \frac{\pi}{\alpha\sqrt{3}}$ and of increasing amplitude according to the logarithmic increment $\alpha T = \frac{\pi}{\sqrt{3}}$.

It is noteworthy that this increment is a fixed number, independent of the conditions of the problem and especially of the straightening moment. Such motion is therefore irreducibly unstable.

In the case of the helicopter, this motion of "drift" generates damping and resisting forces which modify the result and the consideration of which forms the principal object of this article.*

In order to find these forces, together with the straightening action already mentioned, let us imagine the two helicopter propellers, with four blades each and so regulated that each propeller furnishes half of the lift and absorbs about half of the power. We can then examine one alone and the results will apply approximately to the other.

Let F represent the lifting force of each blade in still air, with zero "drift" and "roll" and Q the resistance encountered by each blade during rotation. If U is a suitably selected mean tangential velocity, we can write:

$$\begin{aligned} F &= f(\varphi)\,U^2 \\ Q &= q(\varphi)\,U^2 \end{aligned} \qquad (4)$$

f and q being experimental functions of the mean angle of attack φ.

Let us now consider the helicopter in the motion of "drift" and "roll". There arise variations, $\Delta\varphi$ and ΔU, in both factors of equations 4 and consequently there are increments in the lift and in the resistance or drag, which, by indicating with the

* The effect of the damping forces on stability was discussed by me in 1904 in a paper on the critical speed of airships; then in 1909, in setting forth the conditions of stability of airplanes (this publication, June 5, 1909); and again in 1912.

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apexes the first derivatives of f and q, we can write with sufficient approximation:

$$\Delta F = f'(\varphi)U^2 \Delta\varphi + f(\varphi)2U\,\Delta U$$
$$\Delta Q = q'(\varphi)U^2 \Delta\varphi + q(\varphi)2U\,\Delta U \qquad (5)$$

Let us now calculate the increments ΔU and $\Delta\varphi$, due to the "drift", for a blade, whose axis of rotation makes an angle of α with the axis y. We immediately obtain $\Delta U = u \cos\alpha$, but, as to $\Delta\varphi$, nothing can be affirmed, unless we imagine the propeller to be "bell-shaped," i.e., with the blades inclined to the plane perpendicular to the axis of rotation, like an umbrella turned inside out. It is the usual shape of propellers, when the thrust is offset by the centrifugal force.

If ω is the angle of the "bell", i.e., of the axis of the blades with the above-mentioned plane, it is easily demonstrated that, for the blade thus defined and with close approximation, we obtain

$$U\,\Delta\varphi = u \sin\alpha \tan\omega^*$$

Moreover, it is easy to calculate the increments due to the "roll". With its angular velocity γ', defined above, this appreciably affects only φ, but it is necessary to know the distance ρ of the mean center of pressure of the blade from the axis z. In this case we have:

$$U\,\Delta_1\varphi = \rho \sin\alpha\,\gamma'$$

* The existence and the calculation of the effects of the transverse dihedral on the "drift" of airplanes, with analogous formulas, were discussed by me in 1912, in this publication.

By substituting all these values in equation 5, we obtain the increments ΔF and ΔQ for the blade under consideration. From similar considerations, there were also found the corresponding values for the other three blades at the same instant.

In fact, the problem is to determine the three projections of these increments ΔF and ΔQ, on the three axes, x, y, z; and the three moments of the same with respect to the three said axes, knowing the mean distance ρ from the center of pressure; and, lastly, to add the simultaneous values thus obtained for all four blades.

We thus obtain the following results relative to "bell" propellers with four blades, in motion of "drift" and "roll".

The resultant projection of the ΔF on z and the resultant moment of the ΔQ with respect to z are practically zero in every instance.

The resultant projection of the ΔQ on y and the resultant moment of ΔF with respect to x differ from zero, but they are offset by those of the lower propeller, so that they do not affect the stability.

On the other hand, a result differing from zero is obtained by adding the resultant projection of the ΔQ on the axis of "drift" to that of the lower propeller. It is a true resisting force of constant instantaneous value $4\,q(\varphi)U\,u$, so that, for both propellers, we have approximately:

$$\text{Resisting force} = \frac{75\ HP}{U^2}\,u \qquad (6)$$

in which HP represents the horsepower absorbed by the helicopter.

It then differs from zero and is also added to that of the lower propeller, the resultant moment of the ΔF with respect to the axis y caused by the "drift". D being their diameter, we have, for both propellers, approximately:

$$\text{Straightening moment} = \frac{mg}{8} \times \frac{f'(\varphi)}{f\ (\varphi)} \ \frac{D \tan \omega}{U} u \qquad (7)$$

This also differs from zero and is added to that of the lower propeller, the resultant moment of the ΔF, with reference to the same axis y, caused by the "roll", giving, for both propellers, approximately:

$$\text{Damping moment} = \frac{mg}{24} \ \frac{f'(\varphi)}{f\ (\varphi)} \ \frac{D^2}{U} \gamma' \qquad (8)$$

All these instantaneous values are practically constant for a four-bladed propeller, while there would be oscillations during a revolution, i.e., dependent on the position α, for two, three, or five-bladed propellers.

We are now in a position to lay down the equations of motion in a more complete manner, i.e., by indicating, with r, h, and s, the coefficients now determined by equations 6, 7 and 8, respectively divided by m, j and j:

$$\gamma'' + s\gamma' + h\,u = 0 \ (\text{"roll"}) \qquad (9)$$

$$u' + r\,u - g\gamma = 0 \ (\text{"drift"})$$

and, by combining,

$$\gamma''' + (r + s)\,\gamma'' + r\,s\,\gamma' + hg = 0$$

satisfied by the exponential form with exponents x which are roots of the cubical equation:

$$x^3 + (r + s)\,x^2 + r\,s\,x + hg = 0 \qquad (10)$$

It is here possible to determine the conditions necessary and sufficient for stability, i.e., the conditions for which the real roots and the real parts of the complex radicals of equation 10 are negative, which cannot happen in equation 3.

It is well known that it is only necessary for the three coefficients r, s, h and the expression $rs\,(r + s) - hg$ to be positive. This gives us the key to the problem of stability, since, between the values assumable as constructive data it allows us to vary the bell shape at will and to give it the value:

$$\mathrm{tg}\,\omega < \frac{f'(\varphi)}{f\,(\varphi)}\;\frac{D^3}{U^3}\;\frac{HP}{j} \qquad (11)$$

which gives us <u>an intrinsically stable helicopter</u>.

In this particular case, we can write:

$$(r + s)\,rs = hg \qquad (12)$$

In this case, the three roots of equation 10 are: one real negative root $-(r + s)$ and two imaginary roots with the coefficient

$$\beta = \sqrt{\frac{hg}{r + s}}$$

so that, at least for one initial term which contains a rapidly diminishing exponent, the amplitude of the oscillation of the "roll" is in definite harmony with the semiperiod:

$$T = \pi\sqrt{\frac{r + s}{hg}}$$

It is readily seen that, since the ratio between r and s is very small in comparison with unity, this semiperiod corresponds approximately to that of a simple pendulum of the length:

$$\lambda = \frac{s}{h} = \frac{D}{3 \tan \omega}$$

which can be determined on the axis, by cutting it off with a perpendicular to the "bell" traced at a distance of $\frac{D}{3}$ from the center.

It is, moreover, noteworthy that, by calculating the "excursion" (semiamplitude) σ, of the "drift" by means of equation 9 and introducing the above-mentioned approximation, we obtain the maximum amplitude of the excursion

$$\sigma_o = \gamma_o \lambda$$

which is that of the simple pendulum, oscillating with an angular amplitude γ_o equal to the maximum angular amplitude of the "roll".

From this particular case, it is readily deduced, by turning to the general case, that if we depart from the conditions verifying equation 12, in the sense of diminishing the straightening moment as indicated in equation 11, there will arise a damping factor $e^{\alpha t}$ with $\alpha = 0$, which will diminish the amplitude of the oscillations and will restore the helicopter, more or less rapidly, to its original vertical position. This is the case of stability.

If, on the contrary, we depart from equation 12 in the sense of increasing the straightening moment, there will arise an exponential factor with the exponent $\alpha > 0$ and the amplitude of oscillation will increase. This is the case of instability, tending toward the limit furnished by the example in which we assumed $r = s = 0$, with $h > 0$.

It is also easy to consider the opposite limit, in which r and s are large in comparison with h. In this case, by making $h = 0$ in the first of the equations 9 and assuming that there is a permanent original decentration of the lift, we find it impossible to satisfy the equation with a final value of γ' zero, as was the case for $h > 0$. This indicates instability.

We conclude that the ideal helicopter taken for illustration, can remain "inherently stable," provided there exist, in addition to the straightening moment, damping forces of the kind described; and provided the straightening moment be, in opposition to this, sufficiently moderate to satisfy the determined condition of stability.

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