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A PHENOMENOLOGICAL RELATION BETWEEN STRESS, STRAIN RATE, AND TEMPERATURE FOR METALS AT ELEVATED TEMPERATURES

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A PHENOMENOLOGICAL RELATION BETWEEN STRESS, STRAIN RATE, AND TEMPERATURE FOR METALS AT ELEVATED TEMPERATURES

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SUMMARY

A phenomenological relation between stress, strain rate, and temperature is suggested to account for the behavior of polycrystalline metals above the equicohesive temperature. The properties of the metal included in the relation are elasticity, linear thermal expansion, and viscosity. The relation may be integrated under various conditions to provide information on creep rates, creep rupture, stress-strain curves, and rapid-heating curves. It is shown that for one material - 7075-T6 aluminum-alloy sheet - the information yielded by the relation for these four applications agrees reasonably well with test data.

INTRODUCTION

The behavior of metals at elevated temperatures is constantly becoming a more important problem. Not only must information be at hand concerning creep at these temperatures, but information must also be available concerning when rupture might be expected if the creep were allowed to continue, what the stress-strain curve is likely to be at some particular temperature and strain rate, and how the material will behave when rapidly heated to a high temperature. It would be a useful achievement if some fundamental-law characteristic of the material could be found, such that these and other apparently distinct problems could be shown to be simply different aspects of the application of one law.

In studies of the creep of metals, it has become apparent that the creep behavior at elevated temperatures is different from that at low temperatures. For relatively pure metals, the line of demarcation for temperature occurs at about one-half the melting point and may be tentatively identified with the equicohesive temperature. In the high-temperature region, certain laws have been found and explicitly stated (ref. 1). These laws appear to give some promise that the behavior of metals in this region may be predicted for effects other than creep.

Creep is characterized by an initial transient or primary region which is followed by a steady or secondary region in which viscous flow occurs and finally by an accelerating or tertiary region which leads to rupture. A theory based on rate-process considerations is already available and accounts in a qualitative manner for the main effects shown by the steady creep in the high-temperature region (ref. 2). For stable metals, this theory requires a knowledge of only three constants to specify the creep behavior: one constant is roughly predictable from the melting point, and the other two must be determined from tests.

In this report, a phenomenological relation between stress, strain rate, and temperature is suggested for this region, and this relation utilizes the existing rate-process theory for steady creep. The relation is not an equation of state of the type proposed by Hollomon (ref. 3) but is an equation of "rate of change of state." Since different integrals of this relation exist under different conditions, there is no single "equation of state" in this case. These integrals are applied to creep, creep rupture, stress-strain curves, and rapid-heating curves, and comparisons of these numerical results are made with experimental results for 7075-T6 aluminum-alloy sheet.

SYMBOLS

ϵ	strain
t	time, hr
$\dot{\epsilon}$	strain rate, per hr
$\dot{\epsilon}_o$	strain rate used in obtaining stress-strain curves, per hr
σ	stress, ksi
σ_o	constant, ksi
σ_{lim}	limiting stress, ksi
E	Young's modulus, ksi
α	linear expansion coefficient, per ^{o}K
T	temperature, ^{o}K unless otherwise indicated
$\dot{T}_o$	rate of temperature change, ^{o}K per hr unless otherwise indicated

$\Delta T = \dot{T}_o t$

ΔH activation energy, cal per mole

R gas constant, taken as 2 cal per mole per ^oK

s constant, per hr per ^oK

$$\beta = \frac{\dot{\epsilon}_o}{2sT} e^{\frac{\Delta H}{RT}}$$

$$M = \frac{\sqrt{1 + \beta^2}}{2\beta} \frac{E\epsilon}{\sigma_o}$$

Subscript:

r conditions at rupture

STATEMENT OF PHENOMENOLOGICAL RELATION

A metal undoubtedly retains a certain elasticity at elevated temperatures, even though the elastic range is reduced from that prevailing at lower temperatures. The elasticity resides in the crystalline grains and is a measure of the resistance of the crystal lattice to distortion by stress. In addition to the elasticity, a polycrystalline metal also possesses a quasi-viscosity in the high-temperature region; in this region the application of a constant stress will result in a constant strain rate. Also, thermal expansion (or contraction) due to temperature change will create an apparent strain which can contribute to the total strain.

The suggested phenomenological relation which includes all these properties and which takes into account possible temperature changes may be stated as follows:

$$\frac{d\epsilon}{dt} = \dot{\epsilon} = \frac{d}{dt}\left(\frac{\sigma}{E}\right) + \alpha \frac{dT}{dt} + 2sTe^{-\frac{\Delta H}{RT}} \sinh \frac{\sigma}{\sigma_o} \tag{1}$$

In this relation the first term on the right supplies the contribution to the strain rate due to elasticity; the middle term, the contribution due to possible thermal expansion; and the final term, the contribution

due to viscosity. A relation somewhat similar to this but without the middle term has been proposed for plastics. (See ref. 4.)

The form of the viscous term is taken from reference 2 and is now well established. The exponential term describes the principal effect of temperature. The hyperbolic sine term describes the effect of stress. Kauzmann in reference 2 gave this term as $\sinh\left(m \frac{\sigma}{T}\right)$ where m is a constant, but it has been demonstrated repeatedly by tests that the argument of the term is independent of temperature and is of the form $\frac{\sigma}{\sigma_o}$, where σ_o is a constant. The remainder of the expression, which consists of a number of individual constants multiplied by the temperature T as given in reference 2, is replaced here by $2sT$; the factor 2 is retained for convenience.

The quantities E and α are values which are usually known as functions of the temperature. The quantities s, $\Delta H/R$, and σ_o are found from measurements of the steady creep of the metal under constant stress and temperature. A range of temperature in the high-temperature region and also a range of stress are required to establish $\Delta H/R$ and σ_o, respectively.

APPLICATIONS

With different conditions imposed upon the metal, the suggested relation should generate in turn the laws governing such phenomena as steady creep, creep rupture, stress-strain characteristics, and behavior under rapid-heating conditions. These laws are derived from equation (1).

Steady Creep

If stress σ and temperature T are constant, equation (1) reduces to

$$\dot{\epsilon} = 2sTe^{-\frac{\Delta H}{RT}} \sinh \frac{\sigma}{\sigma_o} \tag{2}$$

or

$$\dot{\epsilon} = sTe^{-\frac{\Delta H}{RT}} e^{\frac{\sigma}{\sigma_o}} \qquad (\sigma \gg \sigma_o) \tag{2a}$$

which indicates steady creep. Equation (2a) may also be written as

$$\frac{\sigma}{\sigma_o} = \frac{\Delta H}{RT} + \log_e \dot{\epsilon} - \log_e sT \tag{3}$$

in which $\log_e sT$ is substantially constant over a limited temperature range. Stress plotted against $\log_e \dot{\epsilon}$ should, therefore, yield straight lines with slope σ_o which are displaced from one another at different temperatures by amounts depending mainly on the value of $\frac{\Delta H}{RT}$. Stress is sometimes plotted against the parameter $\frac{\Delta H}{RT} + \log_e \dot{\epsilon}$ (ref. 5) to demonstrate that data taken at different temperatures will fall on a single curve.

Creep Rupture

At constant stress and temperature, the strain at a time t after the application of stress is

$$\epsilon = \dot{\epsilon}t$$

since the creep is steady under these conditions.

Reference 6 discloses as an experimental fact that if t_r is the time to rupture under conditions of creep, then

$$\dot{\epsilon}t_r \approx \text{Constant} = \epsilon_r$$

Thus,

$$\dot{\epsilon} = \frac{\epsilon_r}{t_r}$$

and equation (3) may be rewritten as

$$\frac{\sigma_r}{\sigma_o} = \frac{\Delta H}{RT} - \log_e t_r + \log_e \frac{\epsilon_r}{sT} \tag{3a}$$

Since the last term is substantially constant, the rupture stress σ_r plotted against the parameter $\frac{\Delta H}{RT} - \log_e t_r$ should yield a single

straight line with slope σ_o. Such a parameter has been employed in the correlation of creep-rupture tests (ref. 7).

Stress-Strain Curves

At constant temperature, equation (1) becomes

$$\dot{\epsilon} = \frac{1}{E}\frac{d\sigma}{dt} + 2sTe^{-\frac{\Delta H}{RT}} \sinh \frac{\sigma}{\sigma_o} \tag{4}$$

Obviously, if the stress is applied very rapidly, the effect of the final viscous term may be negligible compared with the term $\frac{1}{E}\frac{d\sigma}{dt}$; thus, the material may appear to be elastic.

If an attempt is made to hold the strain rate constant, as is sometimes done with conventional testing machines, by setting the strain rate equal to the testing rate $\dot{\epsilon}_o$, a differential equation in σ results, and the solution which is a family of theoretical stress-strain curves is given as follows:

$$\frac{\sigma}{\sigma_o} = \log_e \frac{\sqrt{1 + \beta^2} + (1 + \beta)\tanh M}{\sqrt{1 + \beta^2} + (1 - \beta)\tanh M} \tag{5}$$

where

$$\beta = \frac{\dot{\epsilon}_o}{2sT} e^{\frac{\Delta H}{RT}}$$

$$M = \frac{\sqrt{1 + \beta^2}}{2\beta} \frac{E\epsilon}{\sigma_o}$$

$$\epsilon = \dot{\epsilon}_o t$$

Replacement of $\tanh M$ by exponentials gives

$$\frac{\sigma}{\sigma_o} = \log_e \frac{\left(\sqrt{1 + \beta^2} + 1 + \beta\right)e^{2M} + \sqrt{1 + \beta^2} - 1 - \beta}{\left(\sqrt{1 + \beta^2} + 1 - \beta\right)e^{2M} + \sqrt{1 + \beta^2} - 1 + \beta}$$

and since ordinarily $\beta \gg 1$, the expression for the stress-strain curves becomes

$$\frac{\sigma}{\sigma_o} = \log_e \frac{(2\beta + 1)e^{2M} - 1}{e^{2M} + 2\beta - 1} \tag{6}$$

At the lower end of the curves where the strain is small, M is small, $e^{2M} \approx 1 + 2M$, and

$$\frac{\sigma}{\sigma_o} = \log_e \frac{2\beta + 2M + 4M\beta}{2\beta + 2M} = \log_e \left(1 + \frac{2M\beta}{M + \beta}\right) \approx \frac{2M\beta}{M + \beta} \approx 2M = \frac{E\epsilon}{\sigma_o}$$

which shows that the initial portion of the curves is elastic. At the upper end of the curves where the strains are large, M is large, and from equation (6),

$$\frac{\sigma_{lim}}{\sigma_o} = \log_e(2\beta + 1) \approx \log_e 2\beta \tag{7}$$

where σ_{lim} is the limiting stress which can be developed at that temperature and strain rate.

As a corollary, note that the following equation results when the value of 2β is substituted into equation (7):

$$\dot{\epsilon}_o = sTe^{-\frac{\Delta H}{RT}} e^{\frac{\sigma_{lim}}{\sigma_o}}$$

which is of the same form as equation (2a) for steady creep. Thus, the limiting stress σ_{lim} obtainable bears the same relation to the testing rate $\dot{\epsilon}_o$ that any creep stress σ bears to its corresponding creep

rate $\dot{\epsilon}$. This correlation has been noted experimentally by Sherby and Dorn in reference 5 where the ultimate strength in tension is used for σ_{lim}.

Rapid-Heating Curves

The material composing the skin of a high-speed airplane or missile may be subjected to rapid increases in temperature as a result of aerodynamic heating during flight. It is important to know what to expect from the material under such conditions.

In equation (1), let $\frac{dT}{dt} = \dot{T}_o$ in which $\dot{T}_o$ is a constant. Then, for a constant stress $\sigma \gg \sigma_o$,

$$\epsilon = \int \dot{\epsilon}\, dt = \sigma \int d\left(\frac{1}{E}\right) + \alpha \dot{T}_o t + s e^{\frac{\sigma}{\sigma_o}} \int T e^{-\frac{\Delta H}{RT}}\, dt \tag{8}$$

in which allowance should be made for the variation of E and α with temperature. Equation (8) may be written in terms of finite increments as follows:

$$\epsilon = \frac{\sigma}{E} + \alpha\, \Delta T + s e^{\frac{\sigma}{\sigma_o}} \sum T e^{-\frac{\Delta H}{RT}} \Delta t \tag{9}$$

where

$$\Delta T = \dot{T}_o t$$

The initial term $\frac{\sigma}{E}$ gives the contribution to the total strain of the application of stress, the second term gives the contribution due to thermal expansion, and the last term gives the contribution due to viscosity.

COMPARISON OF THEORY AND EXPERIMENT

A comparison of the laws generated by the assumed relation of equation (1) with experiments performed on one material, 7075-T6 aluminum-alloy sheet, is given in this section.

The alloy is in the solution heat treated and aged condition. Further exposure to elevated temperature results in overaging. The material becomes unstable and suffers a marked reduction in strength with time when subsequently heated in the range from 300° to 600° F. In addition, different batches of the material may vary somewhat among themselves in properties. In view of this variability in the material, the constants in the viscous term were adjusted in each case to give a reasonable fit to the data. Table I shows the constants used in the calculations.

The temperature which separates the high and the low regions is about 300° F for this alloy. Tests performed at temperatures below 300° F should not be expected to show any agreement with calculations. In most cases, the results of such tests have been included to show the extent of the disagreement.

Steady Creep

Figure 1(a) shows steady-creep data for 7075-T6 aluminum-alloy sheet. These data are obtained from reference 8 (solid curves) and from preliminary unpublished NACA data (points). The dashed lines were computed from equation (3) by using the appropriate constants in table I. The dashed lines show a good fit to the data from reference 8 at 300° F and 375° F; the disagreement at 211° F is plainly evident. The dashed lines computed for 450° F and 500° F go through the points obtained at the higher stresses in each case; whereas, the lower stresses give somewhat less creep than would be expected from equation (3). This peculiarity at low stresses has been observed previously by Dorn and Shepard (ref. 1).

Figure 1(b) shows the same data plotted against the parameter $\frac{\Delta H}{RT} + \log_e \dot{\epsilon}$. All the data fit reasonably well with the calculated curve (eq. (3)) with the exception of the points obtained at 211° F.

Creep Rupture

Figure 2 shows the creep-rupture data, taken from reference 8 and from preliminary unpublished NACA data, plotted against the parameter $\frac{\Delta H}{RT} - \log_e t_r$, where t_r is the rupture time at the stress σ_r and the temperature T. Data show that a reasonable value for the constant ϵ_r is 0.0077; the straight line computed from equation (3a), which uses this value and the values given in table I, goes through the line of points, with the exception as noted before of the very lowest stresses and of the points at 212° F.

Stress-Strain Curves

Several investigators in the past have pointed out relations between creep and stress-strain data; for example, Hollomon (ref. 3) suggested a graphical procedure for passing from one to the other. In the present paper, the passage is accomplished analytically through equation (6).

Figure 3 shows stress-strain curves for the 7075-T6 aluminum-alloy sheet, as given in reference 9, for three temperatures. The strain rate was 0.12 per hr. The theoretical curves obtained by computation from equation (6) (see table I) fit the data well at the higher temperatures but overshoot the 200° F curve by a considerable amount.

Rapid-Heating Curves

Figure 4 shows the rapid-heating curves for the 7075-T6 aluminum alloy, taken from reference 9, for three stresses and for widely different temperature rates. The calculated curves represent a severe test of the assumed relation given in equation (1) not only because the cooperation of all four terms is required in equation (9), which is used for computation, but also because the temperatures reached may be considerably higher and the strain rates considerably larger than those encountered in other types of tests. Near the upper ends of the curves, calculations were made at sufficiently close intervals of temperature, usually 5° to 10° K, to establish better the points. The shape and the separation of the curves with heating rate are given reasonably well by the theoretical curves except near 300° F, and then the predicted final rapid increases in strain occur at a somewhat higher temperature than that actually observed. The agreement at the lower temperature rate on the curve for 17.2 ksi is not as good as the others.

DISCUSSION

It would appear from the tests reported here that the validity of the relation suggested in equation (1) is borne out for the 7075-T6 aluminum alloy. The relation has shown itself capable of accounting for the steady creep, the creep-rupture data, the stress-strain curves, and the rapid-heating curves for this alloy.

A more satisfactory test of the validity of the relation would be with a more stable material such as pure aluminum. The alloy selected in this paper is not such a material but was used because of the existence of the different types of data available. The elevated-temperature phenomena are reasonably well accounted for, however, in spite of this handicap.

The constants in the viscous term vary to different degrees. The activation energy ΔH of the three constants is most nearly constant and is structurally insensitive, being independent of stress, temperature, grain size, and minor alloy additions, and has been tentatively identified with the activation energy for self-diffusion (ref. 1). The magnitude of ΔH is roughly proportional to the melting point of the metal. The quantity σ_o, on the other hand, is structurally sensitive; for pure aluminum, it has a magnitude of about 0.2 ksi, whereas for the 7075-T6 alloy it has a magnitude of 4.3 ksi. For stable materials, it is independent of stress, temperature, and grain size. The quantity s was found to be a constant that is independent of stress and temperature and is subject to the same degree of uncertainty as the strain rate. Repeated steady-creep tests on the same material under identical conditions will yield creep rates within a range of perhaps 2:1. This uncertainty in the creep rate is reflected in a similar uncertainty in the quantity s. It is, therefore, not surprising to find a similar variation in s in the different applications. (See table I.)

For materials which are unstable in the sense that the constants in the viscous term are mainly functions of the temperature, one might expect equation (1) to be still usable if such variations in the constants are allowed for in the computations. If the materials are unstable in the sense that the constants in the viscous term are mainly functions of the time, however, then equation (1) is probably not usable.

Equation (1) is evidently valid for values of stress and temperature which are changing smoothly in one direction, even though these changes may be fairly rapid. For sudden or discontinuous changes in stress or temperature, transients are known to occur which will not be given by equation (1). Some modification of this relation, perhaps by an additional term, will be required to account properly for such transients. Such a modification would have to account for transient creep as well as for other conditions; the nature of the modification is obscure at the present time.

Certain parameters have appeared as a result of this relation and may prove useful in comparing data at different temperatures above the equicohesive temperature. For example, if steady-creep data are available at different stresses and temperatures, stress plotted against the parameter $\frac{\Delta H}{RT} + \log_e \dot{\epsilon}$ should yield a straight line with slope equal to σ_o. For the stress-strain curves, the parameter which determines the separation of the curves with temperature is $2\beta = \frac{\dot{\epsilon}_o}{sT} e^{\frac{\Delta H}{RT}}$, since the maximum height of the curves is $\sigma_o \log_e 2\beta$. For rupture resulting

from creep, rupture stress plotted against $\frac{\Delta H}{RT} - \log_e t_r$ should yield a single straight line with slope equal to σ_o. If the rupture stress were plotted against the parameter $\frac{\Delta H}{RT} + \log_e \frac{\dot{\epsilon}_r}{sT} - \log_e t_r$, the straight line should pass through the origin.

CONCLUDING REMARKS

The application of the suggested phenomenological rate relation to the determination of steady creep rates, creep-rupture time, stress-strain curves, and rapid-heating curves for 7075-T6 aluminum-alloy sheet has shown that such a relation can be used to cover a wide variety of applications. The possibility of predicting the behavior of materials under widely different load and temperature conditions is indicated for temperatures above the equicohesive temperature for the material. Additional verification of the validity of the relation would be desirable as suitable data over a sufficiently wide range of test conditions become available.

Langley Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., February 18, 1957.

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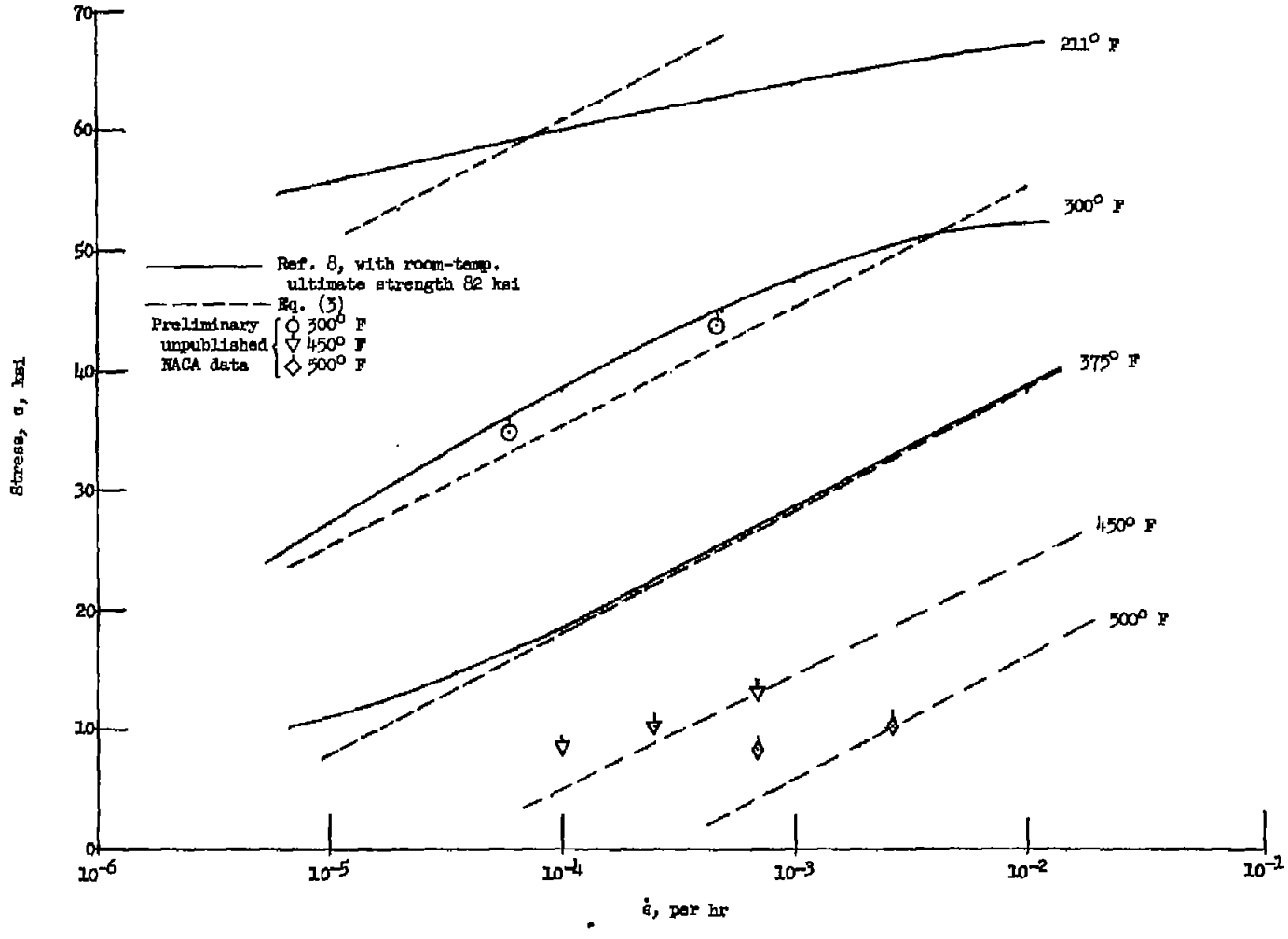
TABLE I

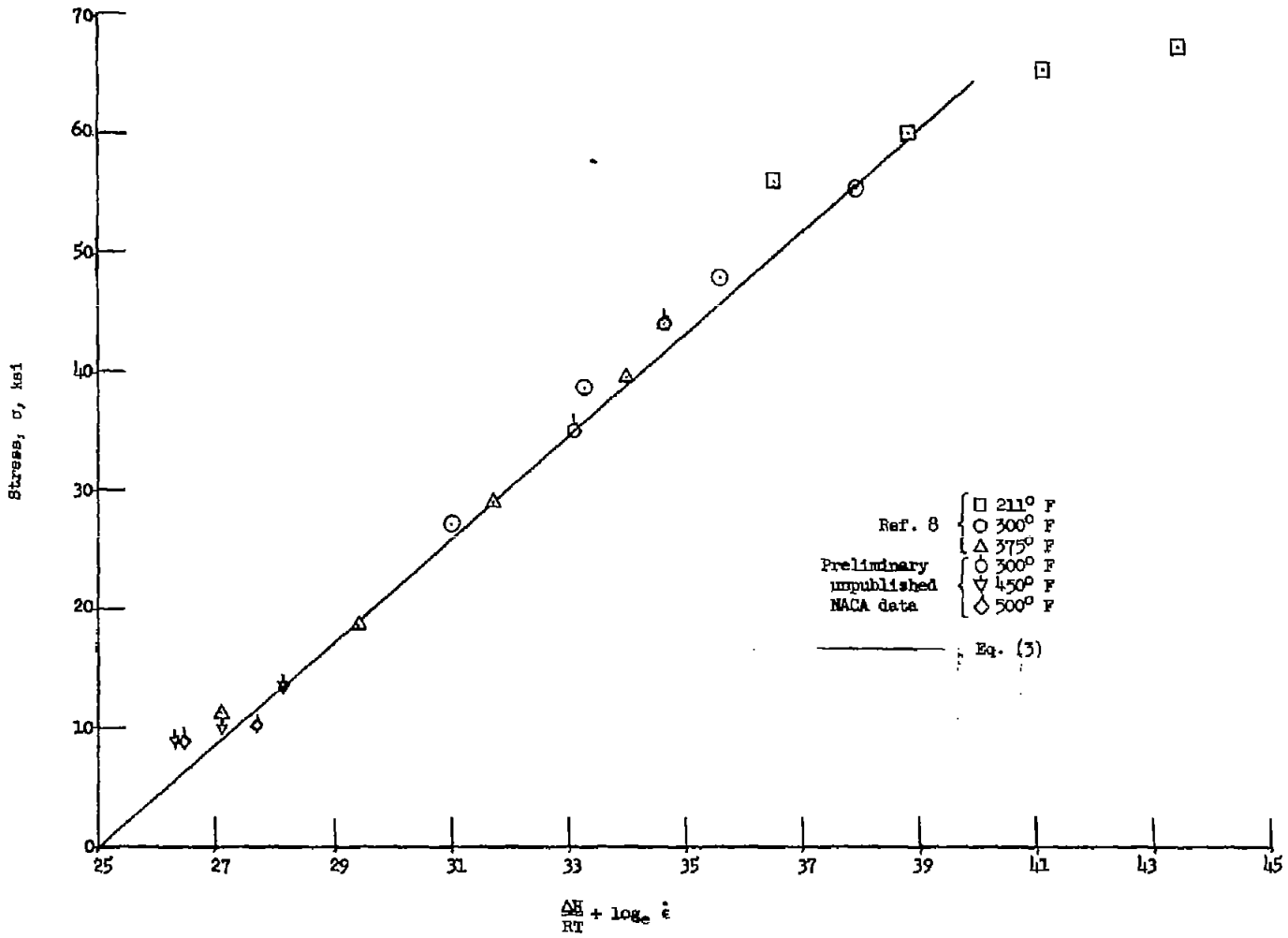
VALUES OF THE CONSTANTS USED IN THE APPLICATIONS

Figure	Application	Reference for experimental data	Equations used in calculations	Constants used in viscous term		
				ΔH, cal per mole	σ_o, ksi	s, per hr per ^{o}K
1(a)	Steady creep	8	(3)	35,800	4.3	1.62×10^{8}
1(b)	Steady creep	8	(3)	35,800	4.3	1.62
2	Creep rupture	8	(3a)	35,800	4.3	1.62
3	Stress-strain curves	9	(6)	34,700	4.3	1.50
4	Rapid-heating curves	9	(9)	34,700	4.3	3.60



(a) Creep rates at different temperatures.

Figure 1.- Steady creep rates for 7075-T6 aluminum-alloy sheet.



(b) Correlation of creep rates with parameter $\frac{\Delta H}{RT} + \log_e \dot{\epsilon}$.

Figure 1.- Concluded.

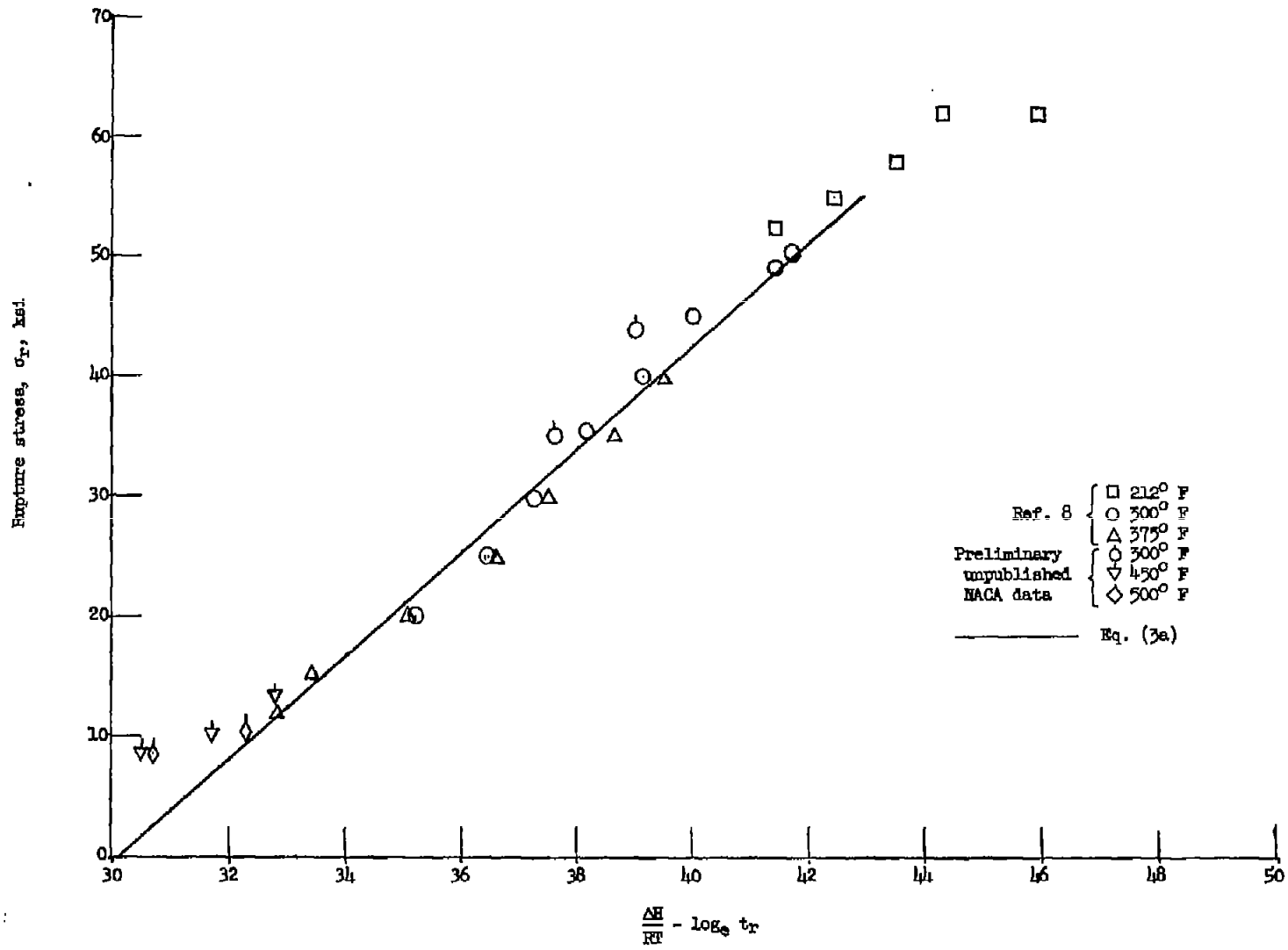


Figure 2.- Correlation of stress with rupture time for creep rupture of 7075-T6 aluminum-alloy sheet.

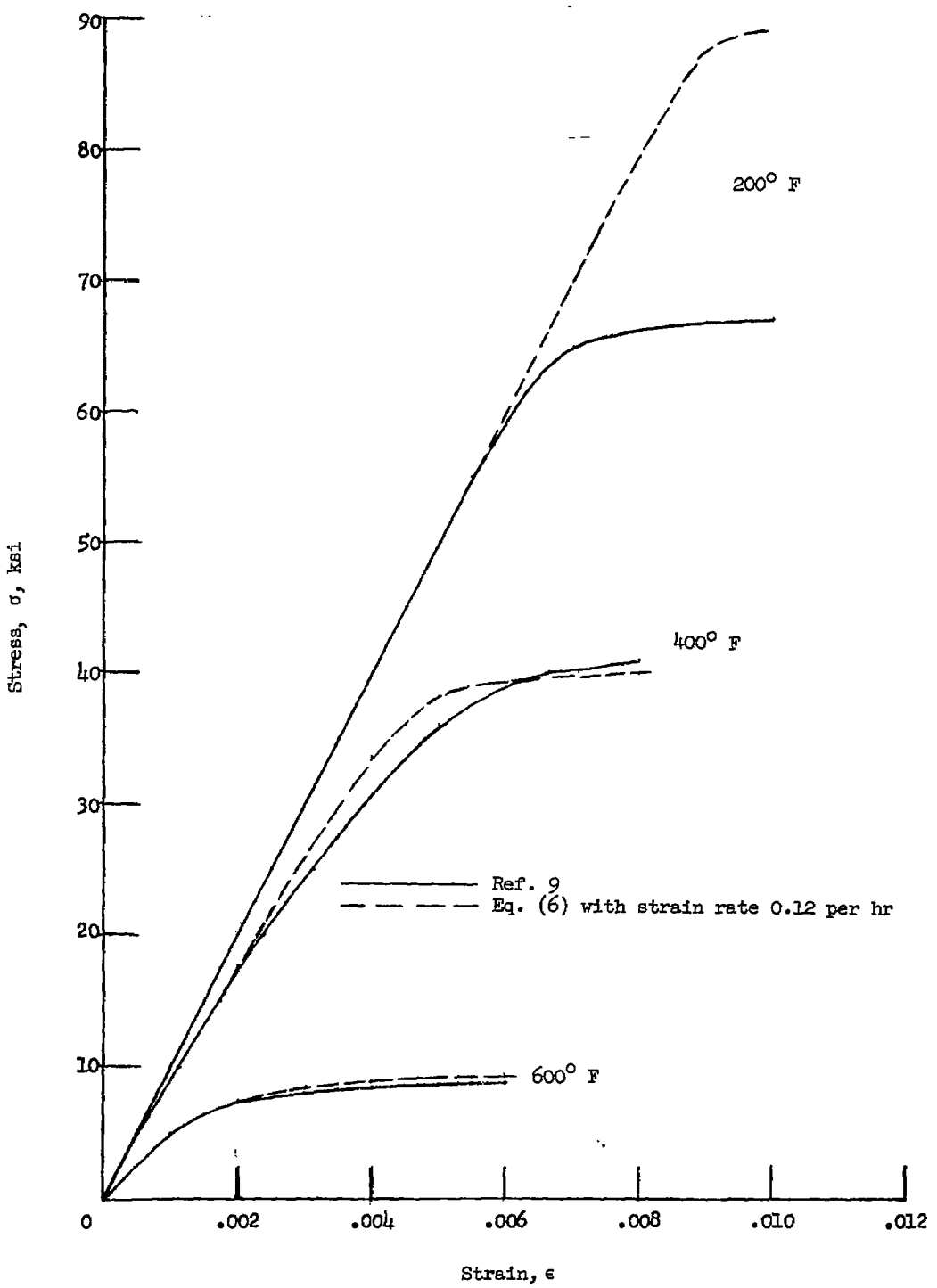


Figure 3.- Tensile stress-strain curves for 7075-T6 aluminum-alloy sheet.

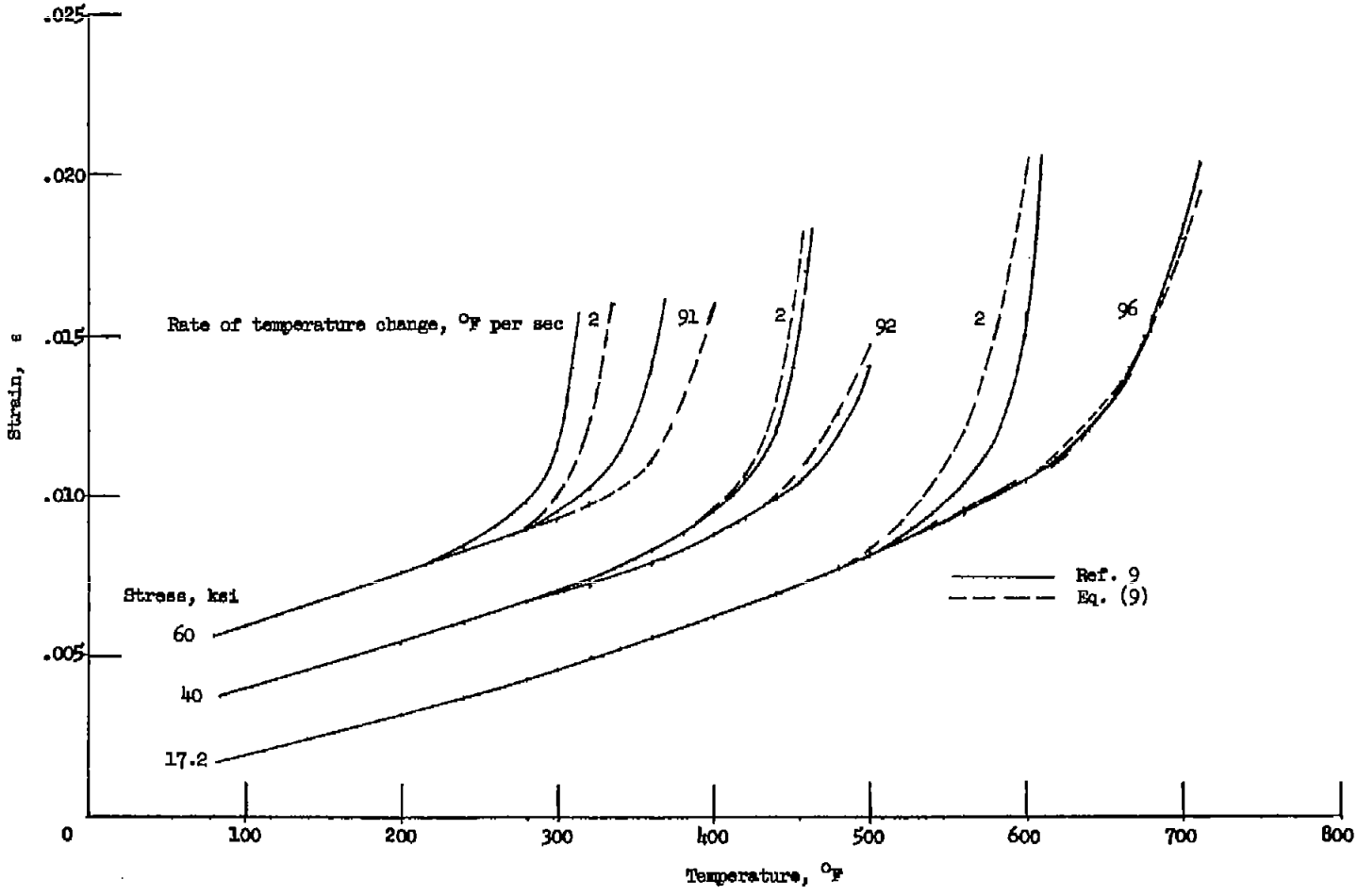


Figure 4.- Rapid-heating curves for 7075-T6 aluminum-alloy sheet for three stresses.

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