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TECHNICAL NOTE 4320

COMPRESSIBLE LAMINAR FLOW AND HEAT TRANSFER ABOUT A ROTATING ISOTHERMAL DISK

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SUMMARY

The flow and heat transfer about a rotating isothermal disk are reexamined to include the effects of compressibility and property variations. If viscous dissipation is neglected, the compressible problem is correlated to the incompressible problem by assuming linear variations of viscosity and thermal conductivity with temperature. Certain inaccuracies in several previous incompressible solutions are noted and corrected herein. The effect of compressibility appears as a distortion of the normal coordinate and normal velocity component and as a multiplicative factor in the heat-transfer coefficient, the Nusselt number, and in the expressions for the skin-friction components and torque required to rotate the disk.

INTRODUCTION

The steady laminar motion of an incompressible viscous fluid about an infinite rotating disk was considered by von Kármán (ref. 1). The Navier-Stokes equations were reduced to ordinary differential equations by separation of variables, and these were then solved by the Kármán-Pohlhausen integral method. Cochran (ref. 2) corrected several errors that he noted in von Kármán's solutions, and using the corrected integral solutions as a first approximation he obtained more accurate results by numerically integrating the differential equations. The flow was shown to be similar to that about a centrifugal fan: The fluid moves radially outward, especially near the disk, and to preserve continuity there is an axial flow toward the disk.

The heat transfer from a uniformly heated rotating disk was first considered by Wagner (ref. 3) who used von Kármán's uncorrected results to solve the energy equation neglecting dissipation and thereby derived a heat-transfer coefficient. In reference 4 the heat-transfer problem is also treated, but there Cochran's flow solutions are used and the energy equation including viscous dissipation is solved. However, the analyses (refs. 3 and 4) are restricted to apply to very small rotation and very small temperature differences. For large rotations or heating one might anticipate that the compressibility and property variations of the fluid would be important. Therefore, the problem is reexamined herein to include these effects.

BASIC EQUATIONS AND BOUNDARY CONDITIONS

The configuration to be studied is that of an infinite circular disk in the R,θ plane which is maintained at a uniform surface temperature and can rotate about the Z-axis (see fig. 1). The equations in cylindrical coordinates expressing conservation of mass, momentum, and energy for an axially symmetric compressible viscous flow are, respectively,

$$\frac{1}{R}\frac{\partial}{\partial R}(\rho RU) + \frac{\partial}{\partial Z}(\rho W) = 0 \tag{1a}$$

$$\rho\left(U\frac{\partial U}{\partial R} + W\frac{\partial U}{\partial Z} - \frac{V^2}{R}\right) = -\frac{\partial P}{\partial R} + \frac{\partial}{\partial R}\left(\mu\left\{2\frac{\partial U}{\partial R} - \frac{2}{3}\left[\frac{1}{R}\frac{\partial}{\partial R}(RU) + \frac{\partial W}{\partial Z}\right]\right\}\right) + \frac{\partial}{\partial Z}\left[\mu\left(\frac{\partial U}{\partial Z} + \frac{\partial W}{\partial R}\right)\right] + \frac{2\mu}{R}\left(\frac{\partial U}{\partial R} - \frac{U}{R}\right) \tag{1b}$$

$$\rho\left(U\frac{\partial V}{\partial R} + W\frac{\partial V}{\partial Z} + \frac{UV}{R}\right) = +\frac{\partial}{\partial R}\left[\mu\left(\frac{\partial V}{\partial R} - \frac{V}{R}\right)\right] + \frac{\partial}{\partial Z}\left[\mu\left(\frac{\partial V}{\partial Z}\right)\right] + \frac{2\mu}{R}\left(\frac{\partial V}{\partial R} - \frac{V}{R}\right) \tag{1c}$$

$$\rho\left(U\frac{\partial W}{\partial R} + W\frac{\partial W}{\partial Z}\right) = -\frac{\partial P}{\partial Z} + \frac{\partial}{\partial Z}\left(\mu\left\{2\frac{\partial W}{\partial Z} - \frac{2}{3}\left[\frac{1}{R}\frac{\partial}{\partial R}(RU) + \frac{\partial W}{\partial Z}\right]\right\}\right) + \frac{1}{R}\frac{\partial}{\partial R}\left[\mu R\left(\frac{\partial U}{\partial Z} + \frac{\partial W}{\partial R}\right)\right] \tag{1d}$$

$$\rho c_p\left(U\frac{\partial T}{\partial R} + W\frac{\partial T}{\partial Z}\right) - \left(U\frac{\partial P}{\partial R} + W\frac{\partial P}{\partial Z}\right) = \left[\frac{\partial}{\partial R}\left(k\frac{\partial T}{\partial R}\right) + \frac{k}{R}\frac{\partial T}{\partial R} + \frac{\partial}{\partial Z}\left(k\frac{\partial T}{\partial Z}\right)\right] + \Phi \tag{1e}$$

where the symbols are all defined in appendix A and where

$$\Phi = \mu\left[2\left(\frac{\partial U}{\partial R}\right)^2 + 2\left(\frac{U}{R}\right)^2 + 2\left(\frac{\partial W}{\partial Z}\right)^2 + \left(\frac{\partial V}{\partial Z}\right)^2 + \left(\frac{\partial U}{\partial Z}\right)^2 + \left(\frac{\partial V}{\partial R} - \frac{V}{R}\right)^2 - \frac{2}{3}\left(\frac{1}{R}\frac{\partial(RU)}{\partial R} + \frac{\partial W}{\partial Z}\right)^2\right]$$

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For a disk rotating with an angular velocity Ω, the no-slip condition for viscous fluids requires that

$$U(R,0) = W(R,0) = 0; \qquad V(R,0) = \Omega R \tag{2}$$

At large distances from the disk it is specified that

$$U(R,\infty) = V(R,\infty) = 0 \tag{3}$$

The thermal boundary conditions for an isothermal disk are

$$T(R,0) = T_w, \qquad T(R,\infty) = T_\infty \tag{4}$$

In order to nondimensionalize the basic equations and reduce them to ordinary differential equations, let

$$\left.\begin{aligned} R = (\upsilon_\infty/\Omega)^{1/2}\, r, \qquad Z = (\upsilon_\infty/\Omega)^{1/2}\, z \\ U = (\Omega\upsilon_\infty)^{1/2}\, r\, f(z),\ V = (\Omega\upsilon_\infty)^{1/2}\, r\, g(z),\ W = (\Omega\upsilon_\infty)^{1/2}\, h(z) \\ P = \rho_\infty\upsilon_\infty\Omega\pi(z);\ \rho = \rho_\infty\bar{\rho};\ \tau(z) = (T - T_\infty)/(T_w - T_\infty) \end{aligned}\right\} \tag{5}$$

and assume the viscosity and conductivity laws

$$\mu = C(z)\mu_\infty/\bar{\rho};\ k = \frac{\mu c_p}{Pr} = \frac{c_p}{Pr}\frac{\mu_\infty}{\bar{\rho}}\, C(z) \tag{6}$$

Equations (1) then become

$$\frac{\partial}{\partial r}(\bar{\rho} r^2 f) + \frac{\partial}{\partial z}(\bar{\rho} r h) = 0 \tag{7a}$$

$$\bar{\rho}(f^2 + hf' - g^2) = \frac{\partial}{\partial z}\left[\frac{C(z)}{\bar{\rho}}\, f'\right] \tag{7b}$$

$$\bar{\rho}(2fg + hg') = \frac{\partial}{\partial z}\left[\frac{C(z)}{\bar{\rho}}\, g'\right] \tag{7c}$$

$$\bar{\rho}hh' = -\pi' + \frac{4}{3}\frac{\partial}{\partial z}\left[\frac{C(z)}{\bar{\rho}}\,(h' - f)\right] + \frac{1}{r}\frac{\partial}{\partial r}\left[\frac{C(z)}{\bar{\rho}}\, r^2 f'\right] \tag{7d}$$

$$\begin{aligned} Pr\,\bar{\rho}\,h\tau' - \frac{Pr\ \upsilon_\infty\Omega}{c_p(T_w - T_\infty)}\, h\pi' = \frac{\partial}{\partial z}\left[\frac{C(z)}{\bar{\rho}}\,\tau'\right] + \frac{Pr\ \mu\Omega}{\rho_\infty c_p(T_w - T_\infty)}\,\frac{4}{3}\,(f - h')^2 \\ + \frac{Pr\ \mu\Omega}{\rho_\infty c_p(T_w - T_\infty)}\, r^2(g'^2 + f'^2) \end{aligned} \tag{7e}$$

where the primes denote differentiations with respect to z and where c_p/Pr is assumed to be constant.

The boundary conditions (eqs. (2) to (4)) become

$$f(0) = h(0) = 0; \qquad g(0) = 1 \tag{8}$$

$$f(\infty) = g(\infty) = 0 \tag{9}$$

$$\tau(0) = 1; \qquad \tau(\infty) = 0 \tag{10}$$

If equations (7a) to (7c) were written in terms of the velocities and their derivatives, they would be recognized as the three-dimensional boundary-layer equations for compressible axisymmetric flow. The boundary-layer nature of the flow is also pointed out in references 1 and 4. Equation (7d) is essentially an equation for the normal pressure gradient that is negligible within the boundary-layer approximation. The pressure in this problem is therefore constant.

In order that equations (7a) to (7c) and (7e) be a consistent set, it is necessary that the last term of (7e) be neglected because of its r-variation. To find the conditions under which this term can be omitted, it is noted that the coefficient appearing in equation (7e) is essentially given by $(\gamma - 1)Pr_\infty M^2/Re\left[(T_W/T_\infty) - 1\right]$. Since the r is not of unit order but actually equal to the square root of the Reynolds number Re, the last term of equation (7e) can be neglected if

$$\frac{(\gamma - 1)Pr_\infty M^2}{\dfrac{T_W}{T_\infty} - 1} \ll 1 \tag{11}$$

This condition is obviously satisfied for relatively moderate rotations and large temperature ratios. The second-last term of equation (7e) is even smaller than the last terms and therefore is also negligible if inequality (11) is satisfied. These terms are neglected in the remainder of the report.

Equations (7) can be reduced to the incompressible equations for this problem as given essentially in reference 4 by taking $C(z)$ to be a constant, C (implying a linear viscosity-temperature law) and letting

$$\left.\begin{aligned}
\zeta &= \frac{1}{\sqrt{C}}\int_0^z \bar{\rho}\, dz \\
f = F,\ g &= G,\ h = \frac{\sqrt{C}}{\bar{\rho}} H \\
\tau = S,\ \pi &= \Pi = \text{constant}
\end{aligned}\right\} \tag{12}$$

Equations (7) then become

$$2F + H' = 0 \tag{13a}$$

$$F^2 + HF' - G^2 = F'' \tag{13b}$$

$$2FG + HG' = G'' \tag{13c}$$

$$\Pi' = 0 \tag{13d}$$

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$$PrHS' = S'' \tag{13e}$$

where here the primes denote differentiation with respect to ζ. The boundary conditions are

$$F(0) = H(0) = 0;\ G(0) = 1 \tag{14}$$

$$F(\infty) = G(\infty) = 0 \tag{15}$$

$$S(0) = 1;\ S(\infty) = 0 \tag{16}$$

Equations (13a) to (13c) with the boundary conditions (eqs. (14) and (15)) were solved in references 1 and 2. Equation (13e) was solved in reference 4 with the boundary conditions (eq. (16)), where $S = Q_1$ and $Pr = \gamma\sigma$. Thus, once the solutions of equations (13a) to (13e) are known, the solutions can be transformed back to the compressible solution by means of equations (5), (6), and (12). The effect of compressibility, therefore, is to distort the normal coordinate and normal velocity.

SOLUTIONS

After F was eliminated by means of equation (13a), the dependent variables G and H were determined by numerical integration of equations (13b) and (13c) along with the boundary conditions (14) and (15) on an IBM 650 high-speed computing machine. The problem was treated as an initial-value problem, and a forward integration technique similar to that described in appendix A of reference 5 was employed. Five-point integration formulas were used throughout. The recalculation of these functions was made here to extend them to large values of the independent variable. This extension is necessary for an accurate calculation of the temperature distribution and hence the heat transfer.

The dependent variable S was then calculated from equation (13e) on a desk computer for $Pr = 0.72$ using the results for H as calculated herein.

RESULTS

Velocities

From equations (5) and (12) it can be seen that the velocities are given essentially by the functions F, G, and H. The functions H, H', H'', G, and G' are listed in table I. H and G differ slightly from Cochran's results (ref. 2) with $H(\infty)$ tending to the value -0.8845 as compared to -0.886 for Cochran (ref. 2). The velocities, which are plotted in figure 2 against ζ, tend to their limits very rapidly, as they should for a boundary-layer flow.

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Temperatures

From equations (5) and (12) it can be seen that the temperature is essentially equal to S. The function S is plotted in figure 3 against ζ along with the function Q_1 for $\sigma = 0.514$ (Pr = 0.72 for air) from reference 4. The function S, as calculated in the present report, is more accurate because the H function is known to four decimal places for all ζ, while in reference 4 the values of H were taken from reference 2, where H was tabulated for values of ζ to 4.4.

Heat Transfer

The average heat-transfer coefficient is defined as

$$\bar{h} = \frac{q}{(T_w - T_\infty)A} = -\frac{\int_A k_w \left(\frac{\partial T}{\partial Z}\right)_{Z=0} dA}{A(T_w - T_\infty)} = -\frac{k_w}{\sqrt{C}} \frac{\rho_w}{\rho_\infty} \left(\frac{\partial S}{\partial \zeta}\right)_{\zeta=0} \left(\frac{\Omega}{\upsilon_\infty}\right)^{1/2}$$

However, because of equation (6),

$$\bar{h} = -\sqrt{C}\, k_\infty \left(\frac{\partial S}{\partial \zeta}\right)_{\zeta=0} \left(\frac{\Omega}{\upsilon_\infty}\right)^{1/2} \tag{17}$$

The constant C (from the viscosity and conductivity laws) can be chosen for best correlation.

To compare with the previous work in which only small temperature differences were considered, C can be taken to be unity. Therefore, in the present case

$$\bar{h} = 0.329\, k_\infty \left(\frac{\Omega}{\upsilon_\infty}\right)^{1/2} \tag{18}$$

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Wagner in reference 3, defining $\bar{h}$ as in equation (18), obtained

$$\bar{h} = 0.339\ k_\infty \left(\frac{\Omega}{\upsilon_\infty}\right)^{1/2} \tag{19}$$

By means of an enthalpy balance, Wagner also found

$$\bar{h} = 0.335\ k_\infty \left(\frac{\Omega}{\upsilon_\infty}\right)^{1/2} \tag{20}$$

where in equations (19) and (20) he assumed $Pr = 0.74$. The difference between the present results and Wagner's (ref. 3) may be due to two things: (1) Wagner used von Kármán's (ref. 1) uncorrected velocity profiles in his calculations, and (2) Wagner used a larger value of Pr than is used herein; this would tend to give a larger initial slope for the temperature function according to figure 7 of reference 4. If the results of reference 4 were used to determine $\bar{h}$, the coefficient would be 0.286. The difference occurs because an incorrect energy equation was used in reference 4, which led the parameter Pr/γ (taken as 0.514 there) to appear rather than the Prandtl number itself (see eq. (13e)). This point was first noted in reference 6 and is discussed in detail in appendix B.

The heat-transfer data of references 6 and 7, along with the analyses of references 3 and 4 and the present report, can all be expressed in terms of an average Nusselt number defined as

$$Nu = \frac{\bar{h}R_0}{k_\infty} \tag{21a}$$

where

$$Nu = -\sqrt{C}\left(\frac{\partial s}{\partial \zeta}\right)_{\zeta=0}\left(\frac{\Omega R_0^2}{\upsilon_\infty}\right)^{1/2} = -\sqrt{C}\left(\frac{\partial s}{\partial \zeta}\right)_{\zeta=0}(Re_0)^{1/2} \tag{21b}$$

for the present report. In figure 4, the ratio of the Nusselt number to the square root of the Reynolds number from various analyses and experimental studies is plotted against the angular velocity, where, for the present analysis, it is assumed that $C = 1$. If the mean of the heat-transfer data as correlated in reference 6 is used, only the experimental results of reference 7 vary with Ω. The data in reference 7 give results that are much higher than any others, while the mean of the data for reference 6 are in good agreement with the present analysis. Reference 6 points out that the results of reference 7 are too high because of heat losses through the insulation and that the results are also affected at low rotational speeds by free convection (due to gravitational force). The latter effect is clearly shown in figure 4. The value of

$\bar{h}$, as given by equation (20), is used in equation (21a) to plot the analysis of reference 3 in figure 4, while the value of $\bar{h}$ corresponding to $Pr/\gamma = 0.514$ is used for reference 4. The latter is used in order to illustrate the importance of using the correct form of the energy equation. If the results for $Pr/\gamma = 0.72$ from reference 4 are used, the value of $Nu/\sqrt{Re}$ is 0.35.

Skin Friction and Torque

Because of angular symmetry and the form of the normal velocity component (eq. (5)), the expressions for the components of tangential and radial skin friction are

$$\text{Radial skin friction} = \int (\tau_{ZR})_{Z=0}\, dA = \int \mu_w \left(\frac{\partial U}{\partial Z}\right)_{Z=0} dA \tag{22a}$$

$$\text{Tangential skin friction} = \int (\tau_{Z\theta})_{Z=0}\, dA = \int \mu_w \left(\frac{\partial V}{\partial Z}\right)_{Z=0} dA \tag{22b}$$

where A equals the area of a finite disk of radius R_0. Thus, if equations (5), (6), and (12) are used, these expressions can be written as

$$\text{Radial skin friction} = \frac{2}{3}\pi\rho_\infty\Omega\sqrt{C\upsilon_\infty\Omega}\, R_0^3F'(0) = 1.069\ \rho_\infty\Omega\sqrt{C\upsilon_\infty\Omega}\, R_0^3 \tag{23a}$$

$$\text{Tangential skin friction} = \frac{2}{3}\pi\rho_\infty\Omega\sqrt{C\upsilon_\infty\Omega}\, R_0^3G'(0) = -1.289\ \rho_\infty\Omega\sqrt{C\upsilon_\infty\Omega}\, R_0^3 \tag{23b}$$

The torque, or the rotational moment necessary to turn the disk, is

$$\text{Torque} = \int (\tau_{Z\theta})_{Z=0}\, R\,dA = 2\pi \int_0^{R_0} (\tau_{Z\theta})_{Z=0}\, R^2 dR \tag{24a}$$

Again, if equations (5), (6), and (12) are used, this equation becomes

$$\text{Torque} = \frac{\pi}{2}\, \rho_\infty\Omega\sqrt{C\upsilon_\infty\Omega}\, R_0^4G'(0) = -0.98\ \rho_\infty\Omega\sqrt{C\upsilon_\infty\Omega}\, R_0^4 \tag{24b}$$

Von Kármán (ref. 1) derived a similar expression using the angular momentum leaving the cylindrical surface formed by the disk and the gaseous boundary layer. The coefficient in reference 1 was 0.92. Again, the results are affected by the compressibility as evidenced by the $\sqrt{C}$ which appears in the equations.

SUMMARY OF RESULTS

The flow and heat transfer about a rotating isothermal disk have been reexamined to include the effects of compressibility and property variations. For flows in which a relatively highly heated or cooled disk is rotating with a moderate velocity so that viscous dissipation is negligible, the compressible problem is correlated to the incompressible problem by assuming linear variations of viscosity and thermal conductivity with temperature. Certain inaccuracies in several previous incompressible solutions have been noted and corrected herein. The effect of compressibility appears as a distortion of the normal coordinate and normal velocity component and as a multiplicative factor in the heat-transfer coefficient, the Nusselt number, and in the expressions for the skin-friction components and torque required to rotate the disk.

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APPENDIX A

SYMBOLS

A	surface area of disk
$C(z)$	function in viscosity and conductivity laws, eq. (6)
c_p	specific heat at constant pressure
c_v	specific heat at constant volume
$F(\zeta),G(\zeta),H(\zeta)$	"incompressible" dimensionless velocity functions
$f(z),g(z),h(z)$	compressible dimensionless velocity functions
$\bar{h}$	average heat-transfer coefficient, $q/A(T_w - T_\infty)$
k	coefficient of thermal conductivity
M	Mach number, $\Omega R/\sqrt{\gamma \bar{R} T_\infty}$
Nu	Nusselt number, $\bar{h}R_0/k_\infty$
P	pressure
Pr	Prandtl number, $c_p\mu/k$
Q_1	dimensionless temperature function defined in ref. 4 equal to $S(\zeta)$
q	total heat flux
R,θ,Z	cylindrical coordinates, Z normal to disk surface
$\bar{R}$	gas constant, $P = \rho \bar{R} T$
Re	Reynolds number, $\Omega R^2/\nu_\infty$
r,z	dimensionless cylindrical coordinates
$S(\zeta)$	"incompressible" dimensionless temperature function
T	temperature
U,V,W	radial, tangential, and normal velocities

γ	ratio of specific heats, c_p/c_v
ζ	dimensionless axial coordinate
μ	absolute-viscosity coefficient
υ	kinematic-viscosity coefficient
π,Π	dimensionless pressure functions
ρ	density
σ	Pr/γ
τ	dimensionless compressible temperature function, $\tau(z) = (T - T_\infty)/(T_w - T_\infty)$
τ_{ZR}	radial shear per unit area on planes parallel to the disk
$\tau_{Z\theta}$	angular shear per unit area on planes parallel to the disk
Φ	dissipation function
Ω	angular disk velocity

Subscripts:

w	disk surface
0	disk edge
∞	undisturbed region

Superscript:

'	denotes differentiation with respect to z in eqs. (7) and with respect to ζ in eqs. (13)

APPENDIX B

COMPARISON OF ENERGY EQUATIONS

To understand the difference between the energy equations used in reference 4 and herein, the two forms of the energy equation will be considered neglecting dissipation for axially symmetric boundary layers. These are

$$\rho c_v\left(U\frac{\partial T}{\partial R}+W\frac{\partial T}{\partial Z}\right)=\frac{\partial}{\partial Z}\left(k\frac{\partial T}{\partial Z}\right)-P\left(\frac{\partial U}{\partial R}+\frac{U}{R}+\frac{\partial W}{\partial Z}\right) \tag{B1}$$

$$\rho c_p\left(U\frac{\partial T}{\partial R}+W\frac{\partial T}{\partial Z}\right)=\frac{\partial}{\partial Z}\left(k\frac{\partial T}{\partial Z}\right)+\left(U\frac{\partial P}{\partial R}+W\frac{\partial P}{\partial Z}\right) \tag{B2}$$

Equation (B1) was used in reference 4 and (B2) was used herein. The last terms on the right were omitted in both cases so that the remaining equations are identical except for the different specific heats. Clearly, for gases the two equations give different results (as has already been pointed out). The question then arises whether the omission of these terms is really justified in both cases. If the state equation

$$P=\rho\bar{R}T$$

is applied to equation (B2), there results

$$\rho(c_p-\bar{R})\left(U\frac{\partial T}{\partial R}+W\frac{\partial T}{\partial Z}\right)=\frac{\partial}{\partial Z}\left(k\frac{\partial T}{\partial Z}\right)+\bar{R}T\left(U\frac{\partial\rho}{\partial R}+W\frac{\partial\rho}{\partial Z}\right)$$

or

$$\rho c_v\left(U\frac{\partial T}{\partial R}+W\frac{\partial T}{\partial Z}\right)=\frac{\partial}{\partial Z}\left(k\frac{\partial T}{\partial Z}\right)+\bar{R}T\left(U\frac{\partial\rho}{\partial R}+W\frac{\partial\rho}{\partial Z}\right) \tag{B3}$$

Equation (B3) is identical with (B1) except that the last terms are in a different form. From equation (B3) it is evident that the last term is of the same order of magnitude as the convection (left side) term. Therefore, with the c_v form of the energy equation it is not correct to omit the last terms of equation (B1) for gases as was done in reference 4. Looking at (B2), however, it can be seen that within the boundary-layer assumptions the last term can be neglected relative to the convection terms as was done herein.

REFERENCES

1. von Kármán, Th.: Uber laminare und turbulent Reibung. Z.a.M.M., Bd. 1, Heft 4, Aug. 1921, pp. 233-252.

2. Cochran, W. G.: The Flow Due to a Rotating Disc. Proc. Cambridge Phil. Soc., vol. 30, July 30, 1934, pp. 365-375.

3. Wagner, C.: Heat Transfer from a Heated Disk to Ambient Air. Jour. Appl. Phys., vol. 19, no. 9, Sept. 1948, pp. 837-839.

4. Millsaps, Knox, and Pohlhausen, Karl: Heat Transfer by Laminar Flow from a Rotating Plate. Jour. Aero. Sci., vol. 19, no. 2, Feb. 1952, pp. 120-126.

5. Ostrach, Simon: An Analysis of Laminar Free-Convection Flow and Heat Transfer About a Flat Plate Parallel to the Direction of the Generating Body Force. NACA Rep. 1111, 1953. (Supersedes NACA TN 2635.)

6. Cobb, E. C., and Saunders, O. A.: Heat Transfer from a Rotating Disk. Proc. Roy. Soc. (London), ser. A, vol. 236, no. 1206, Aug. 2, 1956, pp. 343-351.

7. Young, R. L.: Heat Transfer from a Rotating Plate. Trans. ASME, vol. 78, no. 6, Aug. 1956, pp. 1163-1168.

TABLE I. - DIMENSIONLESS SOLUTIONS

ζ	H''	H'	H	G'	G	S'	S
0	-1.0205	0	0	-0.6159	1.0000	-0.3286	1.0000
.2	-.6676	-.1673	-.0179	-.5987	.8780	-.3276	.9343
.4	-.3989	-.2727	-.0628	-.5577	.7621	-.3265	.8588
.6	-.2031	-.3320	-.1239	-.5048	.6557	-.3222	.8039
.8	-.0635	-.3578	-.1934	-.4476	.5605	-.3149	.7401
1.0	.0314	-.3603	-.2855	-.3911	.4766	-.3047	.6781
1.2	.0922	-.3475	-.3565	-.3381	.4058	-.2917	.6184
1.4	.1279	-.3251	-.4058	-.2898	.3411	-.2766	.5616
1.6	.1456	-.2975	-.4661	-.2470	.2875	-.2598	.5079
1.8	.1508	-.2677	-.5227	-.2095	.2419	-.2419	.4577
2.075	.1452	-.2287	-.5906	-.1662	.1905	-.2166	.3955
2.275	.1357	-.1988	-.6331	-.1401	.1599	-.1984	.3558
2.475	.1239	-.1726	-.6702	-.1179	.1342	-.1807	.3157
2.675	.1113	-.1491	-.7025	-.0991	.1125	-.1639	.2813
2.875	.0986	-.1281	-.7300	-.0832	.0944	-.1477	.2502
3.075	.0865	-.1096	-.7537	-.0698	.0791	-.1327	.2222
3.275	.0752	-.0934	-.7740	-.0585	.0663	-.1189	.1970
3.475	.0650	-.0794	-.7912	-.0491	.0556	-.1062	.1746
3.675	.0559	-.0674	-.8059	-.0412	.0466	-.0947	.1545
3.875	.0478	-.0570	-.8183	-.0345	.0390	-.0843	.1366
4.075	.0408	-.0482	-.8288	-.0289	.0327	-.0748	.1207
4.275	.0346	-.0407	-.8377	-.0242	.0274	-.0664	.1066
4.475	.0294	-.0343	-.8451	-.0203	.0230	-.0587	.0939
4.675	.0249	-.0289	-.8514	-.0170	.0192	-.0520	.0828
4.875	.0210	-.0243	-.8567	-.0143	.0161	-.0459	.0728
5.075	.0177	-.0204	-.8612	-.0119	.0135	-.0406	.0644
5.275	.0149	-.0172	-.8649	-.0100	.0113	-.0358	.0568
5.475	.0126	-.0144	-.8681	-.0084	.0095	-.0316	.0501
5.675	.0106	-.0121	-.8707	-.0070	.0079	-.0279	.0441
5.875	.0089	-.0102	-.8730	-.0059	.0067	-.0246	.0389
6.075	.0075	-.0085	-.8748	-.0049	.0056	-.0220	.0342
6.275	.0063	-.0071	-.8764	-.0041	.0047	-.0191	.0302
6.475	.0053	-.0060	-.8777	-.0035	.0039	-.0169	.0266
6.675	.0044	-.0050	-.8788	-.0029	.0033	-.0149	.0234
6.875	.0037	-.0042	-.8797	-.0024	.0027	-.0127	.0206
7.075	.0031	-.0035	-.8805	-.0020	.0023	-.0115	.0182
7.275	.0026	-.0030	-.8811	-.0017	.0019	-.0102	.0160
7.475	.0022	-.0025	-.8817	-.0014	.0016	-.0090	.0141
7.675	.0018	-.0021	-.8821	-.0012	.0013	-.0079	.0124
7.875	.0015	-.0017	-.8825	-.0010	.0011	-.0070	.0109
8.075	.0013	-.0015	-.8828	-.0008	.0009	-.0061	.0096
8.2	.0012	-.0013	-.8830	-.0008	.0008	-.0056	.0089
8.4	.0010	-.0011	-.8832	-.0006	.0007	-.0050	.0078
8.6	.0008	-.0009	-.8834	-.0005	.0006	-.0044	.0069
8.8	.0007	-.0008	-.8836	-.0004	.0005	-.0039	.0061
9.0	.0006	-.0006	-.8837	-.0004	.0004	-.0034	.0053
9.2	.0005	-.0005	-.8839	-.0003	.0003	-.0030	.0047
9.4	.0004	-.0005	-.8840	-.0003	.0003	-.0026	.0041
9.6	.0003	-.0004	-.8840	-.0002	.0002	-.0023	.0036
9.8	.0003	-.0003	-.8841	-.0002	.0002	-.0020	.0032
10.0	.0002	-.0003	-.8842	-.0002	.0002	-.0018	.0028
10.2	.0002	-.0002	-.8842	-.0001	.0001	-.0016	.0025
10.4	.0002	-.0002	-.8843	-.0001	.0001	-.0014	.0022
10.6	.0001	-.0002	-.8843	-.0001	.0001	-.0012	.0019
10.8	.0001	-.0001	-.8843	-.0001	.0001	-.0011	.0017

ζ	H''	H'	H	G'	G	S'	S
11.0	0.0001	-0.0001	-0.8844	-0.0001	-0.0001	-0.0010	0.0015
11.2	.0001	-.0001		-.0001	.0001	-.0008	.0013
11.4	.0001	-.0001		-.0000	.0000	-.0007	.0012
11.6	.0001	-.0001				-.0006	.0010
11.8	.0000	-.0001				-.0006	.0010
12.0		-.0000				-.0005	.0009
12.2						-.0004	.0007
12.4						-.0004	.0006
12.6			-.8845			-.0003	.0005
12.8						-.0003	.0005
13.0						-.0003	.0004
13.2						-.0002	.0004
13.4						-.0002	.0003
13.6						-.0002	.0003
13.8						-.0002	.0003
14.0						-.0001	.0002
14.2						-.0001	.0002
14.4						-.0001	.0002
14.6						-.0001	.0002
14.8						-.0001	.0001
15.0						-.0001	.0001
15.2						-.0001	.0001
15.4						-.0001	.0001
15.6						-.0001	.0001
15.8						-.0000	.0001
16.0							.0001
16.2							.0001
16.4							.0001
16.6							.0001
16.8							.0000
17.0							
17.2							
17.4							
17.6							
17.8							
18.0							
18.2							
18.4							
18.6							
18.8							
19.0							
19.2							
19.4							
19.6							
19.8							
20.0							
20.2							
20.4							
20.6							

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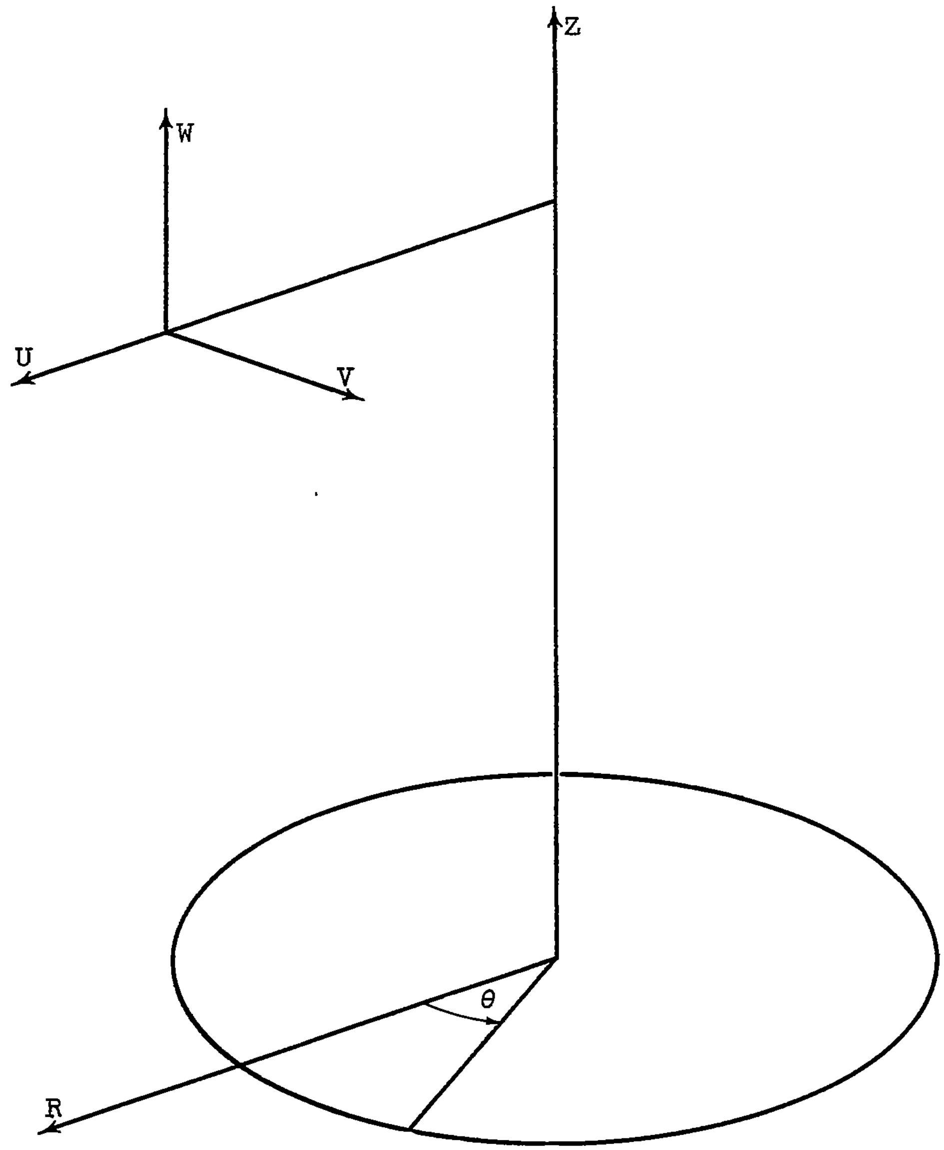

Figure 1. - Schematic sketch of configuration considered.

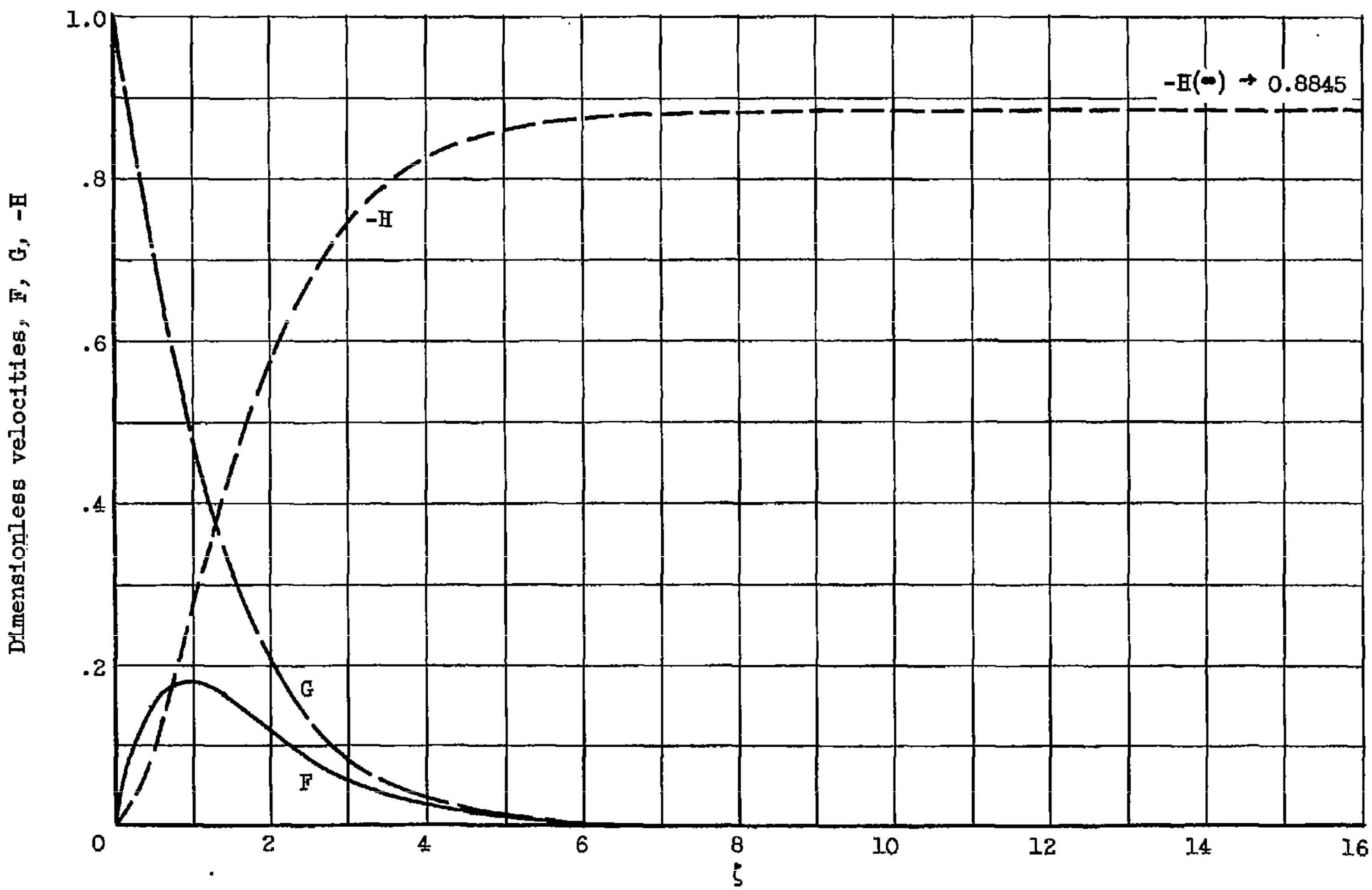

Figure 2. - Dimensionless velocity functions, $F = U/\Omega R$; $G = V/\Omega R$; $H = \bar{\rho}W/C^{1/2}(\Omega\nu_{\infty})^{1/2}$.

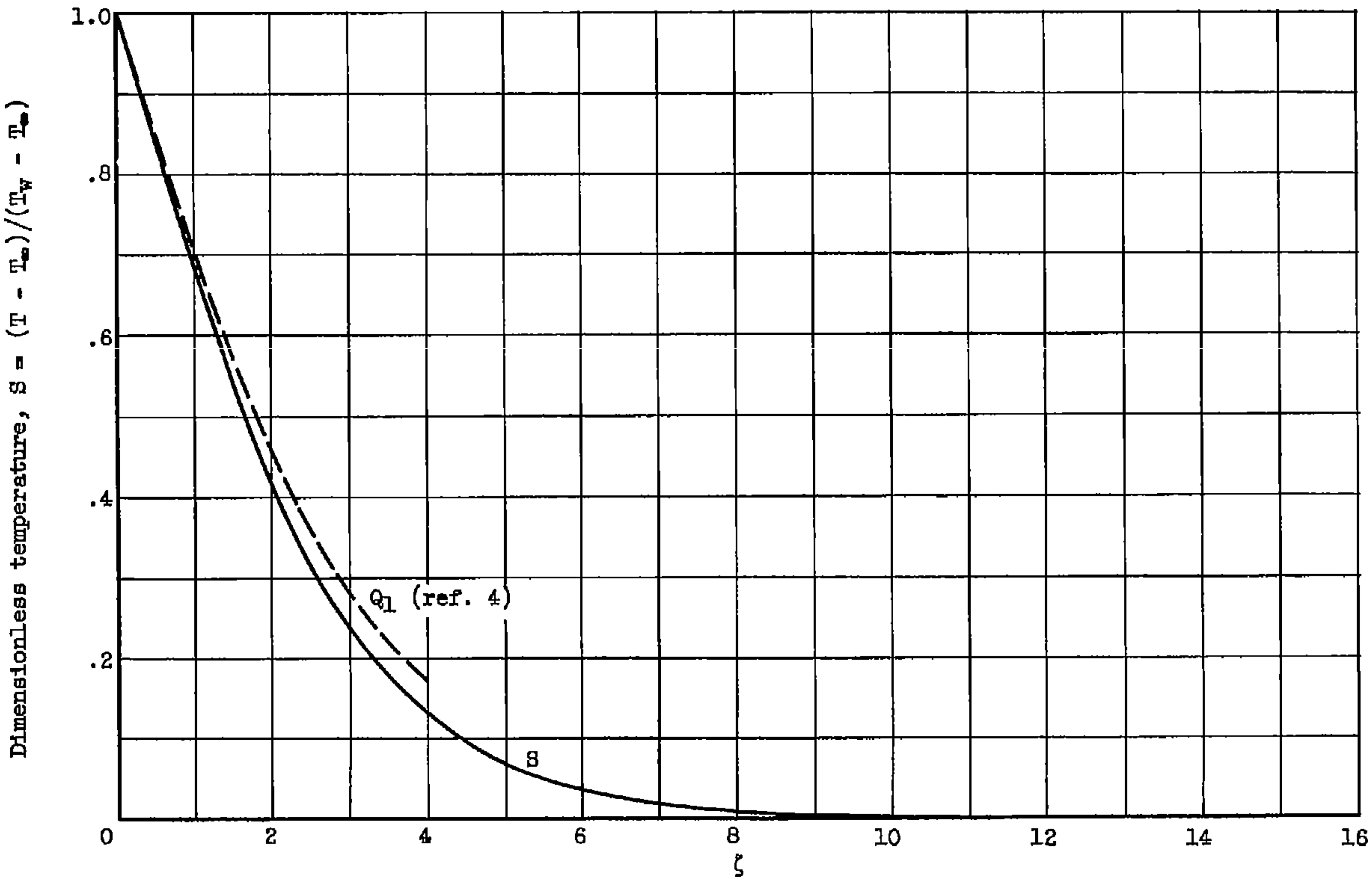

Figure 3. - Dimensionless temperature distribution.

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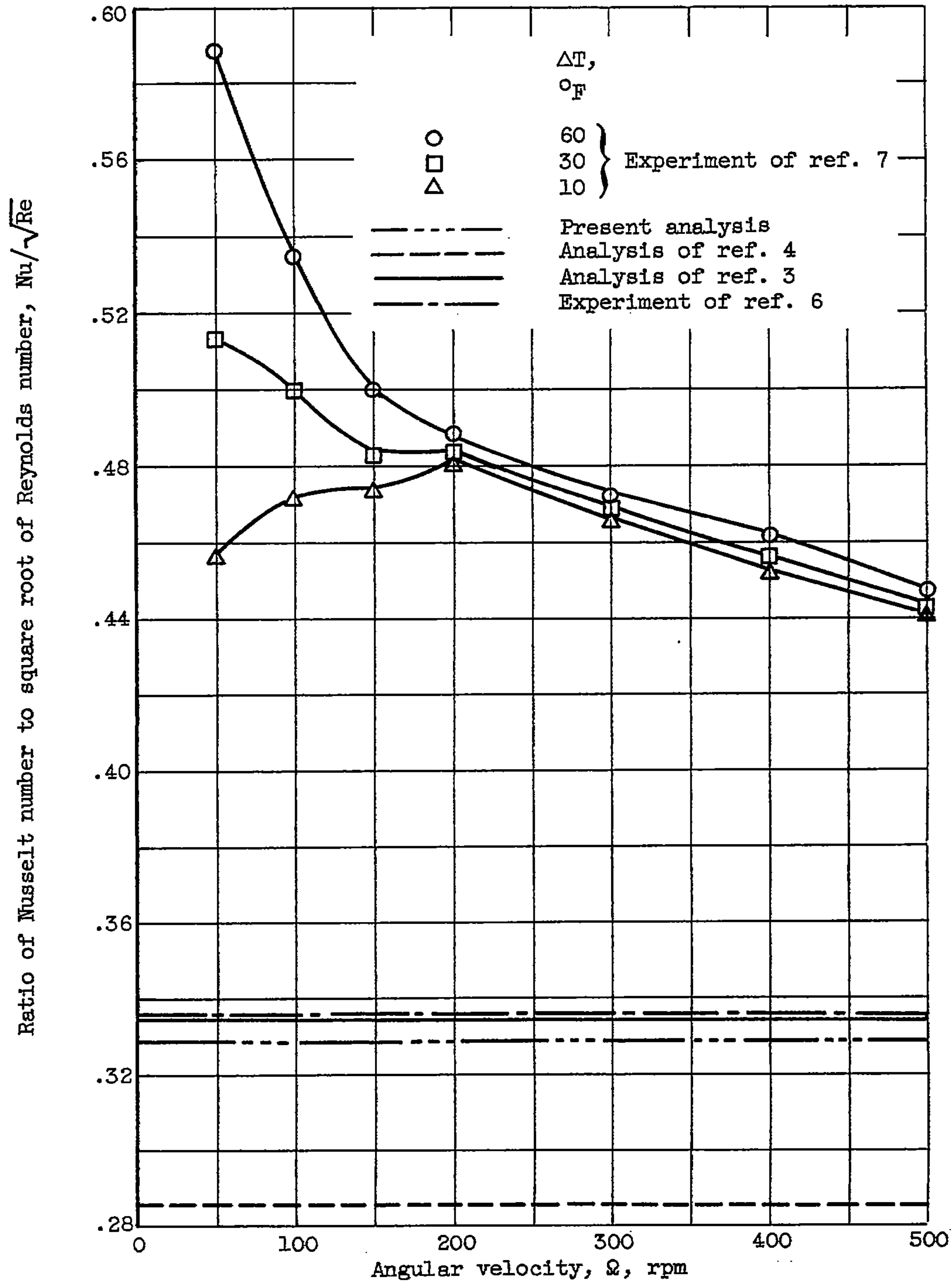

Figure 4. - Ratio of Nusselt number to square root of Reynolds number as function of disk angular velocity.