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TORSION TEST OF A MONOCOQUE BOX

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SUMMARY

A monocoque box of aluminum alloy was subjected to torques applied at the ends. The twist; the strain in the stringers, plate, and corner posts; and the buckling load were measured. The twist was found to be 20 to 50 percent less than that given by Bredt's theory for the torsion of a thin-walled box without reinforcements and the shear stress in the shear web, about 18 percent greater.

In order to obtain closer agreement between theoretical and experimental results, an analysis was developed for the twisting of a monocoque box reinforced by stringers, corner posts, and bulkheads. The measured twist agreed within 10 percent with this analysis. The measured and theoretical values for the strains and buckling load agreed within the error of observation.

INTRODUCTION

As a part of an investigation for the NACA on monocoque boxes, there was described in reference 1 results of compression tests. Torsion tests of the same specimen are reported in this paper. The torsion tests are designed to give information on the following points:

1. The agreement between the measured twist in a reinforced box and the theoretical twist for a simplified box without reinforcements given by R. Bredt (reference 2, p. 270)

2. The effect of buckling on the torsional stiffness

3. The magnitude of the shearing stresses and of the induced stresses due to bending

4. The variation in twist of the box with distance from the ends

5. The effect of the longitudinals and bulkheads on the stiffness of the box

SPECIMEN

The dimensions of the monocoque-box specimen are given in figure 1. The box was fabricated from 24S-T aluminum alloy; 0.075-inch sheet was used for the shear-web sides, and 0.026-inch sheet reinforced by Z-stringers, spaced 4 inches on centers, was used for the top and the bottom sides of the box. The stringers were fastened to the sheet by 1/8-inch brazier-head rivets, spaced 7/8 inch on centers. There were four intermediate bulkheads and antiroll members, spaced at 19 inches.

Particular care was taken in reinforcing the ends of the box to avoid concentration of the stresses on particular portions of the box. The reinforcements, consisting of steel angles and plates, are shown in figures 1, 2, and 3. Figure 3 also shows the construction of the bulkheads.

Tensile and compressive stress-strain curves of material from the corner posts, the stringers, and the sheet used in assembling the monocoque box are given in reference 1. Young's modulus and the yield strength, obtained from the stress-strain curves by the 0.002-offset method, are listed in table I.

TEST PROCEDURE

Figure 4 shows the monocoque-box specimen A mounted for a torsion test in a large lathe. The following procedure was used in mounting the specimen. First the specimen was centered in the lathe. Then one steel end plate was rigidly clamped on the face plate B of the lathe while the other was supported on the dead center in the tailstock by a ball bearing. This ball bearing was a single-row type that permitted slight rocking of the inner race. A sleeve was pressed on the dead center and ground to fit the inner race hole with a clearance of 0.005 inch in order to permit longitudinal motion.

Torque was applied to the specimen by the jack C acting on the lever D. The torque thus applied was resisted by a stop placed between a face-plate jaw and the lathe bed. The force exerted by the jack was measured with a weighing scale E of 2000-pound capacity. The force from the jack was balanced by the reaction of the ball bearing on the dead center of the lathe. Except for a negligibly small frictional torque, the specimen was therefore subjected to a pure torque equal to the product of the force applied by the jack and the distance between the point of application of this force and the center line of the lathe. A steel ball F was used to position accurately the point of application of the force exerted by the jack.

The twist in the specimen was measured by gages, one of which is identified as G in figure 4. A close-up of one of these gages is shown in figure 5. The twist was measured as the change in angle between two reflecting surfaces H attached to bars I by adjustable joints L. The bars I were clamped to the specimen at their centers by light clamps J. The area of contact with the specimen was a small ring 1/2 inch in diameter. This ring allowed attachment of a bar to the specimen over a rivet head. The pair of reflecting surfaces H consisted of a 20-millimeter 45° prism and a piece of plate glass selected for flatness. The plate glass was treated to eliminate the back-surface image. The twist was measured by reading the changes in angle between the two reflecting surfaces H with the Tuckerman autocollimator K. The least count of the autocollimator as used was 0.00001 radian.

The strains in the specimen were measured with Tuckerman optical strain gages used alone or with suitable adapters. The strains in the four corner posts were measured over 10-inch gage lengths with 2-inch strain gages having 8-inch extensions. Other strains on the outstanding portions of the box were measured by 2-inch strain gages mounted directly on the box, while the strains on the less accessible portions were measured by a special transfer of the lever type described in detail in reference 1. The strain on three gage lines 120° apart was measured by three 1-inch strain gages mounted on a rosette adapter.

The buckling of the sheet was observed visually by suitable illumination. Errors due to fluctuation of room temperature were minimized by conducting most of the experiments at night. This procedure was found to be necessary in order to measure the small strains in the stringers and corner posts more accurately.

RESULTS

The measured twists between bulkheads are plotted in figure 6. The twist was measured between all bulkheads by gages on the diagonally opposite lower east corner post and upper west corner post. The twist was also measured on the top center stringer between bulkheads 1 and 2. Figure 6 shows that the twist between a pair of bulkheads was practically independent of the location of the twist gages; this result indicates that the box twisted as a whole without appreciable distortion of the cross section. Figure 6 also shows that the twist was symmetrical with respect to a transverse plane through the center cross section of the box. It is interesting to note that the measured twist between bulkheads 1 and 2 and between bulkheads 3 and 4 was consistently higher than the measured twist at the center of the box between bulkheads 2 and 3.

The twisting of the box was not uniform along its length. Figure 7 shows the variation of twist with position along the box. The twist was measured over a 19-inch-gage length except at the ends of the box where the gage length was necessarily shorter. Figure 7 indicates that the twisting was fairly uniform between the three center bulkheads of the box, dropped off to a lower value in the unreinforced portions of the end bays, and increased considerably again between the ends of the box and points near the ends. The increase in twist at the ends of the box is somewhat surprising; the box is apparently less stiff in torsion at the reinforced ends than at the center. This condition indicates that the reinforcement, substantial though it seems (fig. 3), was unable to transfer the torque uniformly from the end plates to the sheet as well as to the longitudinal stringers and corner posts.

The maximum shearing stresses at four points in the center bay are given in figure 8. They were computed from the measured strains on three gage lines intersecting at angles of 120° using $E = 10.6 \times 10^6$ pounds per square inch and Poisson's ratio = 0.32. In no case did the direction of the maximum shearing stress computed from the measured strains differ from the perpendicular (or parallel) to the axis of the box by more than 1°. It is evident from figure 8 that the shearing stress in the shear web was nearly independent of the position between bulkheads. It is of interest to observe that the ratio of the shearing stress in the shear web to the shearing stress measured in the corner post, approximately 4.5, is very nearly equal to the inverse ratio of the thicknesses,

$$\frac{0.25 + 0.075}{0.075} = 4.3$$

The longitudinal strains induced in portions of the box by the twisting are given in figures 9 to 11. Figure 9 shows the strains in the corner posts and in three stringers midway between bulkheads 3 and 4, figure 10 shows the strains in the corner posts and in three stringers about 1 inch north of bulkhead 2, and figure 11 shows the variation of strain along one of the stringers and along an antiroll member. It is evident from these figures that the induced longitudinal strains were very small, the largest amount being less than 0.00004 corresponding to a stress of only 425 pounds per square inch; this value was about one-sixth the maximum shear stress in the shear web (fig. 8).

Buckling was observed in the top and bottom cover plates of the box near the center section at a torque of 40,800 inch-pounds. The change in load distribution due to the buckling was insufficient to show in the strain measurements; the only observed effect was a slight increase in twist per unit twisting moment (fig. 6).

SYMBOLS

The symbols used in the analysis are given in figure 12 and are defined as follows:

F shearing force applied to ends of side of box

M bending moment applied to ends of side of box

P_1 shearing forces applied by bulkheads 1 and 4

P_2 shearing forces applied by bulkheads 2 and 3

T torque applied to ends of box

s_1 shearing force per inch along corner of box between end of box and bulkhead 1 and between bulkhead 4 and other end of box

s_2 shearing force per inch along corner of box between bulkheads 1 and 2 and between bulkheads 3 and 4

s_3 shearing force per inch along corner of box between bulkheads 2 and 3

E Young's modulus of elasticity

G shear modulus of elasticity

A area of a side of box including reinforcements

k shear constant for side of box

I moment of inertia, including longitudinal reinforcement by stringers and corner posts, of a side of the box about an axis through center line and normal to plane of side

τ shearing stress

δ wall thickness

$$C = \frac{150E}{l^2 G}\left(a_t{}^2 I_h + a_h{}^2 I_t\right)\left(\frac{k_t}{a_h{}^2 A_t} \quad \frac{k_h}{a_t{}^2 A_h}\right)$$

2a width of side

l length of box

x distance from left end of box

y transverse displacement of side in its plane

θ twist per unit length of box

Subscripts:

t one pair of sides

h other pair of sides

b bending deformations

s shearing deformations

ANALYSIS

Bredt's Theory

Bredt, in 1896, developed a theory for torsion in thin tubes which is quoted by Timoshenko on page 270, of reference 2. This theory makes use of the membrane analogy to calculate the torque resisted by the tube. It assumes that the ends of the box are free to warp and does not take account of bending stresses or of reinforcements such as bulkheads. On the basis of Bredt's theory the twist per unit length in the box would be uniform and given by

$$\theta = \frac{T}{4A^2G} \int \frac{ds}{\delta}$$

where A is the area enclosed by the sides of the box

and $\int \frac{ds}{\delta}$ is the integral around the box of the reciprocal of the wall thickness. For this box (fig. 1)

$$\theta = 2.3 \times 10^{-9} T$$

Comparison of this formula with the observed twists given in figure 6 indicates that the box was twisted 20 to 50 percent less than would be indicated by the formula.

The shearing stress τ in the shear web is, according to Bredt's theory,

$$\tau = \frac{T}{2(2a_t 2a_h)\delta_h} = 0.0278T$$

This value is about 18 percent less than the measured values plotted in figure 8.

It is apparent that Bredt's theory is inadequate in describing the torsion of the monocoque box. A more accurate description was obtained by the analysis given in the following section.

Torsion of Monocoque Box with Bulkheads and Corner Posts

An analysis of the torsion in a monocoque box with bulkheads and corner posts was derived by treating the box as an assembly of four beams with wide webs which are joined at the edges and to which transverse forces are applied at the bulkheads.

This analysis differs from similar analyses of box beams in torsion by Reissner (reference 3), Ebner (reference 4), Williams (references 5 and 6), and Payne (reference 7) in taking account of the effect of the individual bulkheads instead of assuming the section of the box beam to remain rectangular at all points, as would be done for an infinite number of bulkheads. Consideration of the individual bulkheads seemed advisable in the present case of bulkhead spacings which are comparable with the transverse dimensions of the box, in order to compute the forces acting on the bulkheads.

It follows from the equilibrium of forces and moments acting on a portion of the side between adjacent bulkheads (fig. 12) that the longitudinal shear s per unit length at the extreme fiber must be constant. At the section x = constant, where $4l/5 < x < l$, according to the simple beam theory:

$$EI\frac{d^2y_b}{dx^2} = -M + F(l - x) - 2as_1(l - x) \qquad (1)$$

By integration:

$$EI\,\frac{dy_b}{dx} = -\,Mx + (F - 2as_1)\,\left(lx - \frac{x^2}{2}\right) + \text{constant} \qquad (2)$$

The integration constant may be determined by assuming rigid clamping of the sides at the end of the box:

$$x = l, \quad \frac{dy_b}{dx} = 0$$

therefore

$$EI\,\frac{dy_b}{dx} = M(l - x) - (1/2)\,(F - 2as_1)(l - x)^2 \qquad (3)$$

According to the simple beam theory this slope produces a longitudinal displacement $a(dy_b/dx)$ toward the ends of the box at one extreme fiber and an equal displacement away from the end of the box at the opposite extreme fiber. The condition of continuity of the box requires these displacements to be equal at adjoining extreme fibers:

$$a_t\,\frac{dy_{tb}}{dx} = -\,a_h\,\frac{dy_{hb}}{dx} \qquad (4)$$

If equation (3) is substituted in both sides of equation (4),

$$\frac{a_t}{I_t}\left[M_t\,(l - x) - (1/2)\,(F_t - 2a_ts_1)(l - x)^2\right]$$
$$= -\,\frac{a_h}{I_h}\left[M_h\,(l - x) - (1/2)(F_h - 2a_hs_1)(l - x)^2\right] \qquad (5)$$

In order for equation (5) to be true for all values of x, the coefficients of $(l - x)$ and $(l - x)^2$ must equal zero.

$$0 = \frac{a_tM_t}{I_t} + \frac{a_hM_h}{I_h} \qquad (6)$$

$$s_1 = \left(\frac{a_t F_t}{I_t} + \frac{a_h F_h}{I_h}\right) \Big/ \left(\frac{2a_t^2}{I_t} + \frac{2a_h^2}{I_h}\right) \tag{7}$$

Between $x = \frac{3l}{5}$ and $x = \frac{4l}{5}$

$$EI \frac{d^2 y_b}{dx^2} = -2as_2\left(\frac{4l}{5} - x\right) + F(l - x) + P_1\left(\frac{4l}{5} - x\right) - \frac{2as_1 l}{5} - M \tag{8}$$

Integrating gives

$$EI \frac{dy_b}{dx} = \left(-\frac{8as_2 l}{5} - \frac{2as_1 l}{5} + \frac{4P_1 l}{5} - M + Fl\right) x + \left(as_2 - \frac{F}{2} - \frac{P_1}{2}\right) x^2 + \text{constant} \tag{9}$$

At $x = \frac{4l}{5}$ the slope given by equations (3) and (9) is the same. This value determines the constant in equation (9). Substituting for the constant in equation (9) its value gives

$$EI \frac{dy_b}{dx} = \left(as_2 - \frac{F}{2} - \frac{P_1}{2}\right)x^2 + \left(Fl + \frac{4P_1 l}{5} - M - \frac{2as_1 l}{5} - \frac{8as_2 l}{5}\right)x - \frac{Fl^2}{2} - \frac{8P_1 l^2}{25} + Ml + \frac{9as_1 l^2}{25} + \frac{16as_2 l^2}{25} \tag{10}$$

Substituting equation (10) in equation (4) and equating the coefficient of x^2 to zero gives

$$s_2 = \left[\frac{a_t\left(F_t + P_{1_t}\right)}{I_t} + \frac{a_h\left(F_h + P_{1_h}\right)}{I_h}\right] \Bigg/ \left(\frac{2a_t^{\,2}}{I_t} + \frac{2a_h^{\,2}}{I_h}\right) \quad (11)$$

Equating the coefficient of x and the constant term to zero leads again to equations (6) and (7).

Similarly, between $x = \frac{2l}{5}$ and $x = \frac{3l}{5}$

$$EI\frac{d^2y_b}{dx^2} = -2as_3\left(\frac{3l}{5} - x\right) + F(l - x) + P_1\left(\frac{4l}{5} - x\right)$$

$$+ P_2\left(\frac{3l}{5} - x\right) - \frac{2as_1 l}{5} - \frac{2as_2 l}{5} - M \quad (12)$$

Integrating and determining the constant so that the slope at $x = \frac{3l}{5}$ equals the value given by equation (10) results in

$$EI\frac{dy_b}{dx} = \left(-\frac{6as_3 l}{5} - \frac{2as_2 l}{5} - \frac{2as_1 l}{5} - M + Fl + \frac{4P_1 l}{5} + \frac{3P_2 l}{5}\right)x$$

$$+ \left(as_3 - \frac{F}{2} - \frac{P_1}{2} - \frac{P_2}{2}\right)x^2 - \frac{Fl^2}{2} - \frac{16P_1 l^2}{50} - \frac{9P_2 l^2}{50} + Ml$$

$$+ \frac{9as_1 l^2}{25} + \frac{7as_2 l^2}{25} + \frac{9as_3 l^2}{25} \quad (13)$$

Substituting (13) in (4) and equating the coefficient of x^2 to zero gives

$$s_3 = \left[\frac{a_t\left(F_t + P_{1_t} + P_{2_t}\right)}{I_t} + \frac{a_h\left(F_h + P_{1_h} + P_{2_h}\right)}{I_h}\right] \Bigg/ \left(\frac{2a_t^{\,2}}{I_t} + \frac{2a_h^{\,2}}{I_h}\right) \quad (14)$$

The curvature $\frac{d^2y_b}{dx^2}$ is zero by symmetry when $x = \frac{l}{2}$. From equation (12) this value gives

$$M = \frac{Fl}{2} + \frac{3P_1 l}{10} + \frac{P_2 l}{10} - \frac{2as_1 l}{5} - \frac{2as_2 l}{5} - \frac{as_3 l}{5} \tag{15}$$

In addition, there is a deflection of the side due to shear in its plane by the transverse forces F, P_1, and P_2. The slope due to shear between $x = \frac{4l}{5}$ and $x = l$ is:

$$\frac{dy_s}{dx} = k\frac{F}{GA} \tag{16}$$

where k is a constant depending on the stress distribution (reference 8). For example, a rectangular section that is free at the ends has a value of $k = 1.5$, while for the same section when clamped at the ends, $k = 1.2$. Between $x = 3l/5$ and $x = 4l/5$,

$$\frac{dy_s}{dx} = k\frac{F + P_1}{GA} \tag{17}$$

and between $x = 2l/5$ and $x = 3l/5$,

$$\frac{dy_s}{dx} = k\frac{F + P_1 + P_2}{GA} \tag{18}$$

The slope of a side of the box between $x = 4l/5$ and $x = l$ is, from equations (3) and (16),

$$\frac{dy}{dx} = \frac{dy_b}{dx} + \frac{dy_s}{dx} = \frac{1}{EI}\left[M(l - x) - \frac{1}{2}(F - 2as_1)(l - x)^2\right] + \frac{kF}{GA} \tag{19}$$

Integrating this equation gives

$$y = \frac{1}{EI}\left[-\frac{M}{2}(l - x)^2 + \frac{1}{6}(F - 2as_1)(l - x)^3\right] + \frac{kFx}{GA} + \text{constant} \tag{20}$$

The increase in y between $x = \frac{4l}{5}$ and $x = l$ is

$$\Delta y_1 = \frac{1}{750EI}\left(15Ml^2 - Fl^3 + 2as_1 l^3\right) + \frac{kF}{GA}\frac{l}{5} \tag{21}$$

The box must maintain its rectangular section at the reinforced ends and at the bulkheads. The increase in transverse displacement of the sides between end sections and bulkheads must, therefore, be such as to produce rotation of all four sides through the same angle:

$$\frac{\Delta y_t}{a_h} = \frac{\Delta y_h}{a_t} \tag{22}$$

Substituting from equation (21) in equation (22) yields

$$\frac{1}{750EI_t a_h}\left(15M_t l^2 - F_t l^3 + 2a_t s_1 l^3\right) + \frac{k_t F_t}{GA_t a_h}\frac{l}{5}$$

$$= \frac{1}{750EI_h a_t}\left(15M_h l^2 - F_h l^3 + 2a_h s_1 l^3\right) + \frac{k_h F_h}{GA_h a_t}\frac{l}{5} \tag{23}$$

The increase in y between $x = \frac{3l}{5}$ and $\frac{4l}{5}$ is, from equations (10) and (17),

$$\Delta y_2 = \frac{1}{750EI}\left[45Ml^2 - 7Fl^3 - P_1 l^3 + 12as_1 l^3 + 2as_2 l^3\right] + k\,\frac{F + P_1}{GA}\frac{l}{5} \tag{24}$$

From equation (22),

$$\frac{1}{750EI_t a_h}\left(45M_t l^2 - 7F_t l^3 - P_{1_t} l^3 + 12a_t s_1 l^3 + 2a_t s_2 l^3\right) + \frac{F_t + P_{1_t}}{GA_t a_h}\frac{lk_t}{5}$$

$$= \frac{1}{750EI_h a_t}\left(45M_h l^2 - 7F_h l^3 - P_{1_h} l^3 + 12a_h s_1 l^3 + 2a_h s_2 l^3\right) + \frac{F_h + P_{1_h}}{GA_h a_t}\frac{lk_h}{5} \qquad (25)$$

The increase in y between $x = \frac{2l}{5}$ and $\frac{3l}{5}$ is, from equations (13) and (18),

$$\Delta y_3 = \frac{1}{750EI}\left(75Ml^2 - 19Fl^3 - 7P_1 l^3 - P_2 l^3 + 24as_1 l^3\right.$$

$$\left. + 12as_2 l^3 + 2as_3 l^3\right) + k\frac{F + P_1 + P_2}{GA}\frac{l}{5} \qquad (26)$$

When equation (22) is applied,

$$\frac{1}{750EI_t a_h}\left(75M_t l^2 - 19F_t l^3 - 7P_{1_t} l^3 - P_{2_t} l^3 + 24a_t s_1 l^3\right.$$

$$\left. + 12a_t s_2 l^3 + 2a_t s_3 l^3\right)$$

$$+ \frac{F_t + P_{1_t} + P_{2_t}}{GA_t a_h}\frac{lk_t}{5} = \frac{1}{750EI_h a_t}\left(75M_h l^2\right.$$

$$- 19F_h l^3 - 7P_{1_h} l^3 - P_{2_h} l^3 + 24a_h s_1 l^3$$

$$\left. + 12a_h s_2 l^3 + 2a_h s_3 l^3\right) + \frac{F_h + P_{1_h} + P_{2_h}}{GA_h a_t}\frac{lk_h}{5} \qquad (27)$$

Simplifying equations (23), (25), and (27) by substituting for s_1, s_2, s_3, and M their values as given by equations (7), (11), (14), and (15), respectively, gives

$$0 = 13a_hF_t - 13a_tF_h + 9a_hP_{1_t} - 9a_tP_{1_h} + 3a_hP_{2_t} - 3a_tP_{2_h}$$

$$+ \frac{150E}{l^2G}\left[(k_tF_t)/(A_ta_h) - (k_hF_h)/(A_ha_t)\right](a_t^2I_h + a_h^2I_t) \qquad (28)$$

$$0 = 31a_hF_t - 31a_tF_h + 25a_hP_{1_t} - 25a_tP_{1_h} + 9a_hP_{2_t} - 9a_tP_{2_h}$$

$$+ \frac{150E}{l^2G}\left[\frac{k_t\left(F_t + P_{1_t}\right)}{A_ta_h} - \frac{k_h\left(F_h + P_{1_h}\right)}{A_ha_t}\right]\left(a_t^2I_h + a_h^2I_t\right) \qquad (29)$$

$$0 = 37a_hF_t - 37a_tF_h + 31a_hP_{1_t} - 31a_tP_{1_h} + 13a_hP_{2_t} - 13a_tP_{2_h}$$

$$+ \frac{150E}{l^2G}\left[\frac{k_t\left(F_t + P_{1_t} + P_{2_t}\right)}{A_ta_h} - \frac{k_h\left(F_h + P_{1_h} + P_{2_h}\right)}{A_ha_t}\right]\left(a_t^2I_h + a_h^2I_t\right) \qquad (30)$$

It will be immaterial whether the corner posts are apportioned to the moment of inertia I_t or I_h since, in equations (28), (29), and (30), the moment of inertia enters only in the expression $(a_t^2I_h + a_h^2I_t)$, which does not vary with this apportionment.

The torque applied to the ends of the box is given by

$$T = 2a_hF_t + 2a_tF_h \qquad (31)$$

Since the torque applied at the bulkheads is zero,

$$0 = 2a_hP_{1_t} + 2a_tP_{1_h} \qquad (32)$$

$$0 = 2a_hP_{2_t} + 2a_tP_{2_h} \qquad (33)$$

The six unknown forces F_t, F_h, P_{1_t}, P_{1_h}, P_{2_t}, and P_{2_h} can now be determined by solving simultaneously equations (28), (29), (30), (31), (32), and (33) with the result,

$$F_t = \frac{T/2a_h}{1 + \dfrac{\dfrac{304 + 160C + 13C^2}{112 + 40C + C^2} + \dfrac{150Ek_t}{l^2GA_ta_h{}^2}\left(a_t{}^2I_h + a_h{}^2I_t\right)}{\dfrac{304 + 160C + 13C^2}{112 + 40C + C^2} + \dfrac{150Ek_h}{l^2GA_ha_t{}^2}\left(a_h{}^2I_t + a_t{}^2I_h\right)}} \tag{34}$$

$$F_h = \frac{T/2a_t}{1 + \dfrac{\dfrac{304 + 160C + 13C^2}{112 + 40C + C^2} + \dfrac{150Ek_h}{l^2GA_ha_t{}^2}\left(a_t{}^2I_h + a_h{}^2I_t\right)}{\dfrac{304 + 160C + 13C^2}{112 + 40C + C^2} + \dfrac{150Ek_t}{l^2GA_ta_h{}^2}\left(a_t{}^2I_h + a_h{}^2I_t\right)}} \tag{35}$$

$$P_{1_t} = \frac{72 + 18C}{112 + 40C + C^2}\left(\frac{a_t}{a_h}F_h - F_t\right) \tag{36}$$

$$P_{1_h} = \frac{72 + 18C}{112 + 40C + C^2}\left(\frac{a_h}{a_t}F_t - F_h\right) \tag{37}$$

$$P_{2_t} = \frac{-24 + 6C}{112 + 40C + C^2}\left(\frac{a_t}{a_h}F_h - F_t\right) \tag{38}$$

$$P_{2h} = \frac{-24 + 6C}{112 + 40C + C^2}\left(\frac{a_h}{a_t}F_t - F_h\right) \tag{39}$$

The average twist per unit length in the end bays between $x = 4l/5$ and $x = l$ is given by

$$\theta_1 = \frac{(\Delta y_t)_1}{a_h}\frac{5}{l} \tag{40}$$

Substituting from equation (21) gives

$$\theta_1 = \frac{1}{150 l a_h E I_t}\left(15 M_t l^2 - F_t l^3 + 2 a_t s_1 l^3\right) + \frac{k_t F_t}{G A_t a_h} \tag{41}$$

Substituting for M_t and s_1 their values as given by equations (15) and (7), respectively, yields

$$\theta_1 = \frac{l^2}{300E\left(a_t{}^2 I_h + a_h{}^2 I_t\right)}\left(13 a_h F_t - 13 a_t F_h + 9 a_h P_{1_t} - 9 a_t P_{1_h} + 3 a_h P_{2_t} - 3 a_t P_{2_h}\right) + \frac{k_t F_t}{G A_t a_h} \tag{42}$$

Similarly, the average twist per unit length in the second bay between $x = 3l/5$ and $x = 4l/5$ is, from equation (24),

$$\theta_2 = \frac{l^2}{300E\left(a_t{}^2 I_h + a_h{}^2 I_t\right)}\left(31 a_h F_t - 31 a_t F_h + 25 a_h P_{1_t} - 25 a_t P_{1_h} + 9 a_h P_{2_t} - 9 a_t P_{2_h}\right) + k_t \frac{F_t + P_{1_t}}{G A_t a_h} \tag{43}$$

and the average twist per unit length in the center bay between $x = 2l/5$ and $x = 3l/5$ is, from equation (26),

$$\theta_3 = \frac{l^2}{300E\left(a_t{}^2 I_h + a_h{}^2 I_t\right)}\left(37 a_h F_t - 37 a_t F_h + 31 a_h P_{1_t} - 31 a_t P_{1_h} + 13 a_h P_{2_t} - 13 a_t P_{2_h}\right) + k_t \frac{F_t + P_{1_t} + P_{2_t}}{G A_t a_h} \tag{44}$$

COMPARISON BETWEEN THEORY AND EXPERIMENT

The side t of the box (fig. 12) was taken as the cover plate and included the stringers; the side h was taken as the shear web and included the corner posts. From figure 1,

$$a_t = 12 \text{ inches}$$

$$a_h = 5 \text{ inches}$$

$$l = 95 \text{ inches}$$

$$I_t = 46.0 \text{ inches}^4$$

$$I_h = 64.3 \text{ inches}^4$$

$$A_t = 1.290 \text{ inches}^2$$

$$A_h = 2.94 \text{ inches}^2$$

$$a_t^2 I_h + a_h^2 I_t = 10{,}410 \text{ inches}^6$$

The shear constant k_t was determined on the basis that all the cover plate was effective in transmitting shear and that the stringer areas contributed nothing to the shear resistance:

$$k_t = \frac{1.290}{0.026 \times 24} = 2.063$$

The shear constant k_h was determined on the basis that those parts of the shear web and the $\frac{1}{4}$-inch reinforcing plate lying between the cover plates (less than 5 in. from the center line) were effective in transmitting shear and that the rest of the corner post area contributed nothing to the shear resistance:

$$k_h = \frac{2.94}{0.075 \times 10 + 2 \times 0.25 \times 1.688} = 1.845$$

The elastic properties are, from table I:

$E = 10.6 \times 10^6$ pounds per square inch

$\mu = 0.32$ = Poisson's ratio

$$G = \frac{E}{2(1 + \mu)} = 4.01 \times 10^6 \text{ pounds per square inch}$$

The computed constants are

$$C = \frac{150E}{l^2G}\left(a_t{}^2I_h + a_h{}^2I_t\right)\left(\frac{k_t}{a_h{}^2A_t} + \frac{k_h}{a_t{}^2A_h}\right) = 31.25$$

$$\frac{150Ek_t}{l^2Ga_h{}^2A_t}\left(a_t{}^2I_h + a_h{}^2I_t\right) = 29.26$$

$$\frac{150Ek_h}{l^2Ga_t{}^2A_h}\left(a_t{}^2I_h + a_h{}^2I_t\right) = 1.99$$

The forces at the ends and at the bulkheads are, from equations (34) to (39),

$F_t = 0.02080$ T pounds

$F_h = 0.03300$ T pounds

$P_{1_t} = 0.01582$ T pounds

$P_{1_h} = -0.00660$ T pounds

$P_{2_t} = 0.00408$ T pounds

$P_{2_h} = -0.00170$ T pounds

It is interesting to note the rapid decrease in the force applied by the bulkheads in passing from the ends toward the center of the box.

The average twists per unit length between bulkheads are, from equations (42) to (44),

$10^6\theta_1 = 0.00105$ T radian per inch (end bay between $x = \frac{4l}{5}$ and $x = l$)

$10^6\theta_2 = 0.00164$ T radian per inch (second bay between $x = \frac{3l}{5}$ and $x = \frac{4l}{5}$)

$10^6\theta_3 = 0.00179$ T radian per inch (center bay between $x = \frac{2l}{5}$ and $x = \frac{3l}{5}$)

These twists are compared with the measured twists in figure 6. The theory gives a consistent increase in twist per unit length in passing from the ends toward the center, while the measurements show about 10 percent less twist for the center bay than for the two adjacent bays. (See also fig. 7.)

The average shearing force per unit depth of center-bay shear web is

$$\frac{F_h + P_{1_h} + P_{2_h}}{2a_h} = T\,\frac{0.03300 - 0.00660 - 0.00170}{2 \times 5} = 0.00247 \text{ T pounds per inch}$$

Since the forces resisting bending are mostly in the corner posts, the shearing force should be nearly constant, except at the corner posts, and the shear stress may be obtained by dividing the shearing force per inch by the thickness of the wall. On this basis, the stress at the center of the 0.075-inch shear web is

$$\frac{0.00247\ T}{0.075} = 0.0329 \text{ T pounds per square inch}$$

and for the stress at the inner side of the 0.25-inch reinforcing plate

$$\frac{0.00247\ T}{0.250 + 0.075} = 0.0076 \text{ T pounds per square inch}$$

These stresses are compared with the measured stresses in figure 8. The theoretical values agree closely with the measured values.

The strain due to bending in the upper corner post of the t side or the lower corner post of the h side (fig. 12) is

$$a_t \left(\frac{d^2y_b}{dx^2}\right)_t = - a_h \left(\frac{d^2y_b}{dx^2}\right)_h$$

From equations (8), (15), (11), and (14) with $x = \frac{7l}{10}$ this expression reduces to

$$\frac{a_t a_h l}{5E\left(a_t^2 I_h + a_h^2 I_t\right)}\left(- a_h F_t + a_t F_h - a_h P_{1_t} + a_t P_{1_h} - \frac{a_h P_{2_t}}{2} + \frac{a_t P_{2_h}}{2}\right) = 11.71\,T \times 10^{-10}$$

and, with $x = \frac{39l}{95}$ by use of equations (12), (15), (7), (11), and (14) the expression reduces to

$$\frac{17 a_t a_h l}{190E\left(a_t^2 I_h + a_h^2 I_t\right)}\left(a_h F_t - a_t F_h + a_h P_{1_t} - a_t P_{1_h} + a_h P_{2_t} - a_t P_{2_h}\right) = - 4.30\,T\,10^{-10}$$

These theoretical values are shown in figures 9 and 10. Theoretical values for the strains in the stringers, derived on the assumption that the strain due to bending varies linearly across the sides of the box, are also given in figures 9 and 10. The theoretical and observed values are in agreement.

The buckling load of the cover plate was computed from values for the critical buckling stress given in reference 9. The cover plate was divided by the stringers and bulkheads into panels having a width to length ratio of $\frac{4}{19} = 0.21$. The thickness of the plate was 0.026 inch and the width 4 inches. If it is assumed that the stringers give simple support, the critical buckling stress is

$$\tau_{cr} \text{ (simple support)} = 2260 \text{ pounds per square inch}$$

If it is assumed that the stringers give clamped support and that the ratio of critical loads for clamped and simply supported plates having a width to length ratio of 0.21 is the same as the ratio for infinitely long plates,

$$\tau_{cr} \text{ (clamped)} = \frac{8.98}{5.35}\, 2260 = 3790 \text{ pounds per square inch}$$

The shearing stress at the center of the cover plate (t side) is

$$\frac{k_t}{A_t}\left(F_t + P_{1_t} + P_{2_t}\right) = \frac{2.063}{1.290}\left(0.02080 + 0.01582 + 0.00408\right)T$$

$$= 0.0651\ T \text{ pounds per square inch}$$

The critical torques are therefore

$$T \text{ (simple support)} = \frac{2260}{0.0651} = 34{,}700 \text{ inch-pounds}$$

$$T \text{ (clamped)} = \frac{3790}{0.0651} = 58{,}200 \text{ inch-pounds}$$

The measured value of the critical torque was 40,800 inch-pounds, a value between the theoretical values corresponding to clamped and simple support at the stringers.

CONCLUSIONS

The measured twist in the monocoque box between bulkheads 1 and 2 and between bulkheads 3 and 4 was consistently higher than the measured twist at the center of the box between bulkheads 2 and 3. The reinforcement at the ends of the box, substantial though it seems, was unable to transfer the torque uniformly from the end plates to the sheet as well as to the stringers and corner posts.

The shear stress in the shear web was independent of the position between bulkheads within the error of measurement. The ratio of measured shearing stress in the shear web and in the corner post was inversely proportional to the wall thickness at the points of measurement.

The measured longitudinal strains were very small, the largest being less than 0.00004 corresponding to a stress of only 425 pounds per square inch; this value was about one-sixth the maximum measured shearing stress.

Buckling was observed in the cover plates near the center section of the box at a torque of 40,800 inch-pounds. This buckling caused only a slight decrease in the stiffness of the box for higher moments.

Comparison of the measured twists with Bredt's theory indicates that the box was twisted 20 to 50 percent less than this theory would indicate. Comparison of the measured shearing stress with Bredt's theory indicates that his theory gives values about 18 percent less than the measured values.

Using an analysis of the torsion in a monocoque box with bulkheads and corner posts derived by treating the box as an assembly of four beams with wide webs joined at the edges and subjected to transverse forces at the bulkheads, a check of the measured twist within 10 percent was obtained. This analysis also gave theoretical values of the shearing stresses and longitudinal stresses which agreed closely with the measured values.

The observed buckling load of the cover sheet was between the computed values for simple support and clamped support at the edges.

National Bureau of Standards,
Washington, D. C., April 2, 1942

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TABLE I.- MECHANICAL PROPERTIES OF MATERIAL

Sample	Young's modulus (lb/sq in.)		Yield strength offset = 0.2 percent (lb/sq in.)		Tensile strength (lb/sq in.)	Elongation in 2 in. (percent)
	Tension	Compression	Tension	Compression		
Corner angle	10.4×10^6	10.8×10^6	48,000	42,000	61,600	21
Stringer 2	10.4	10.8	48,300	40,700	63,110	25
Stringer 1	10.4	10.8	48,700	40,500	63,100	25
0.075-in. shear web (longitudinal)	10.5	10.7	53,700	44,000	70,020	20
0.026-in. top and bottom plating (longitudinal)	10.5	10.8	57,100	46,800	73,500	18

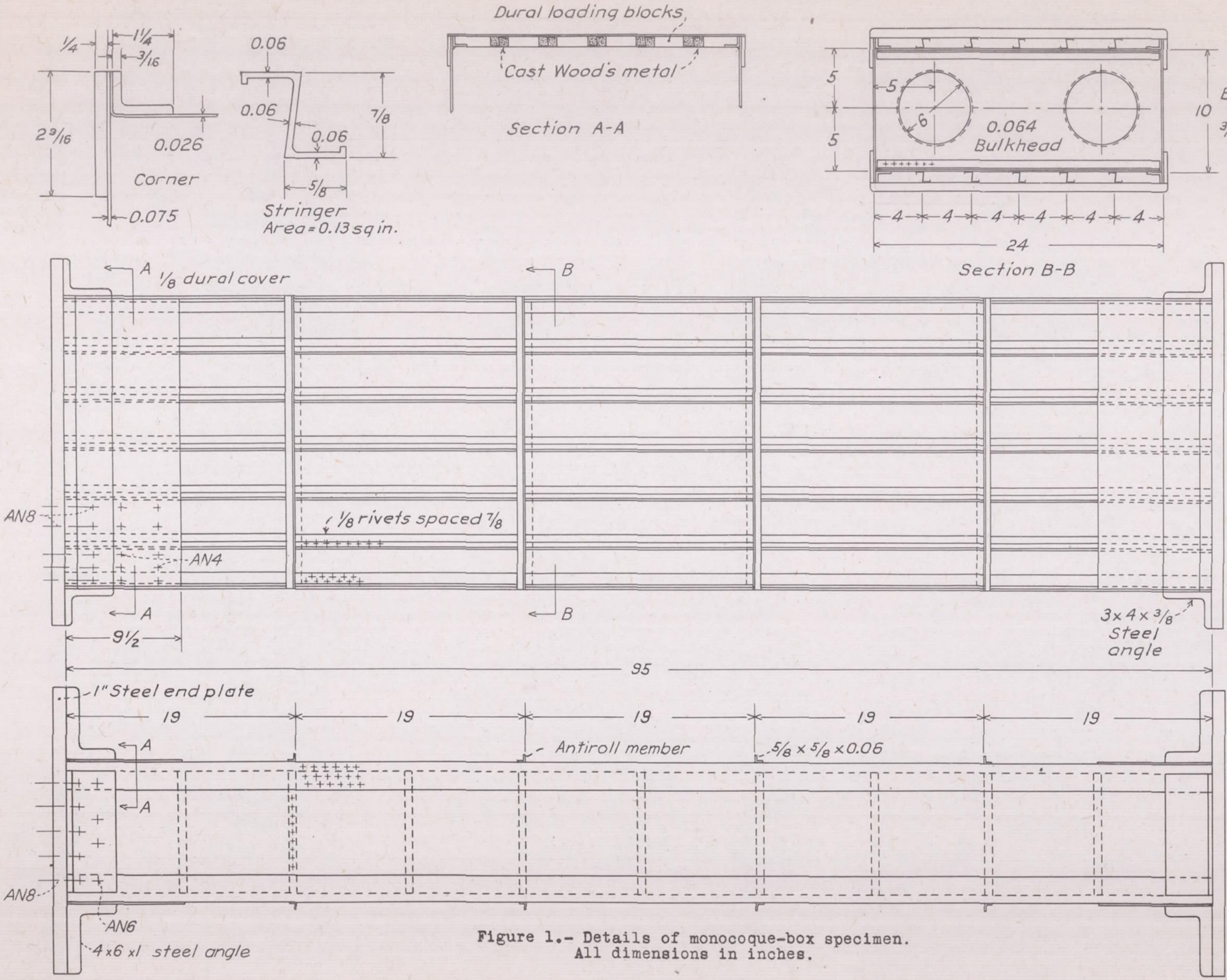

Figure 1.- Details of monocoque-box specimen.
All dimensions in inches.

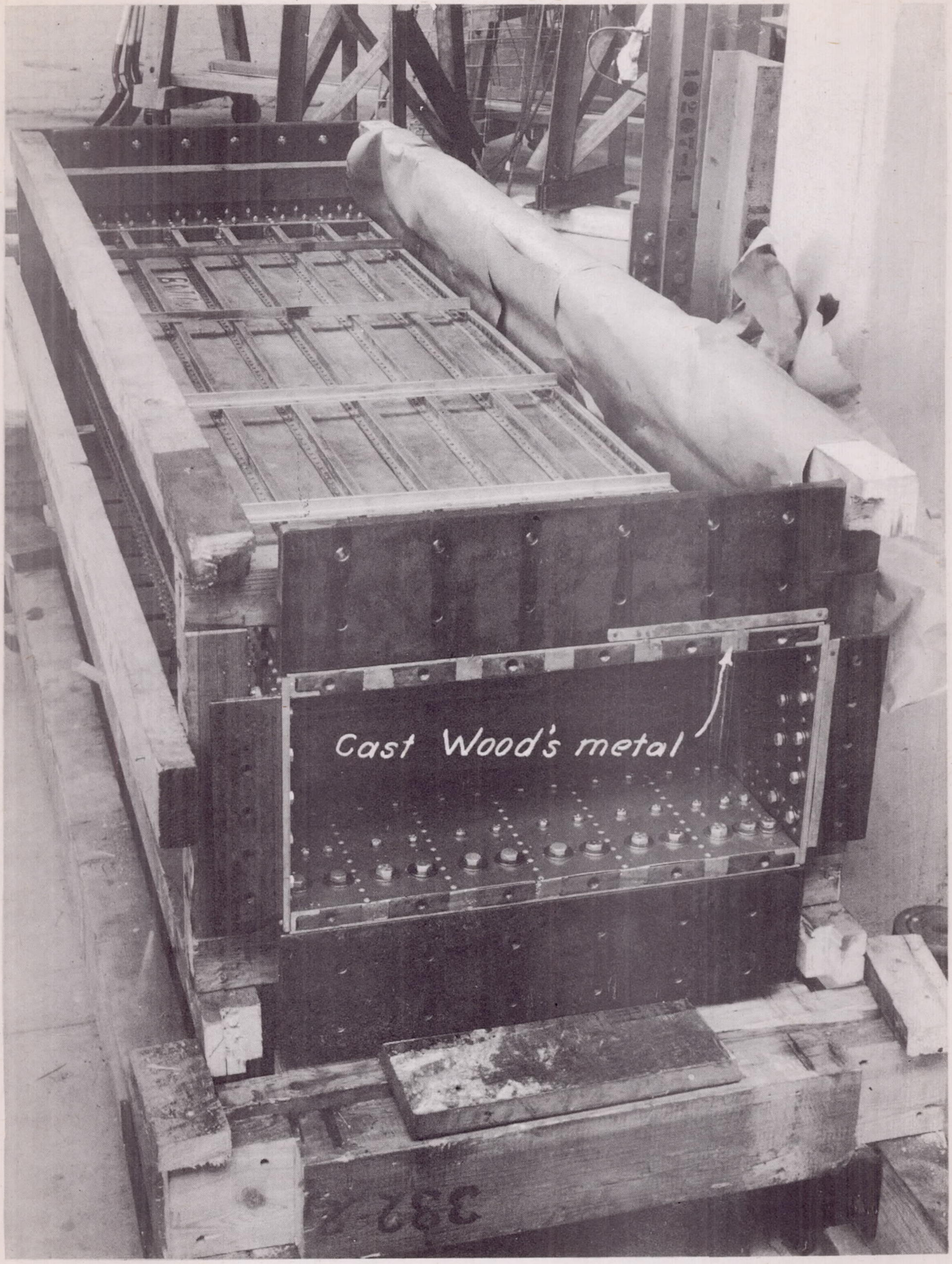

Figure 2.- Over-all view of monocoque box (end plate removed).

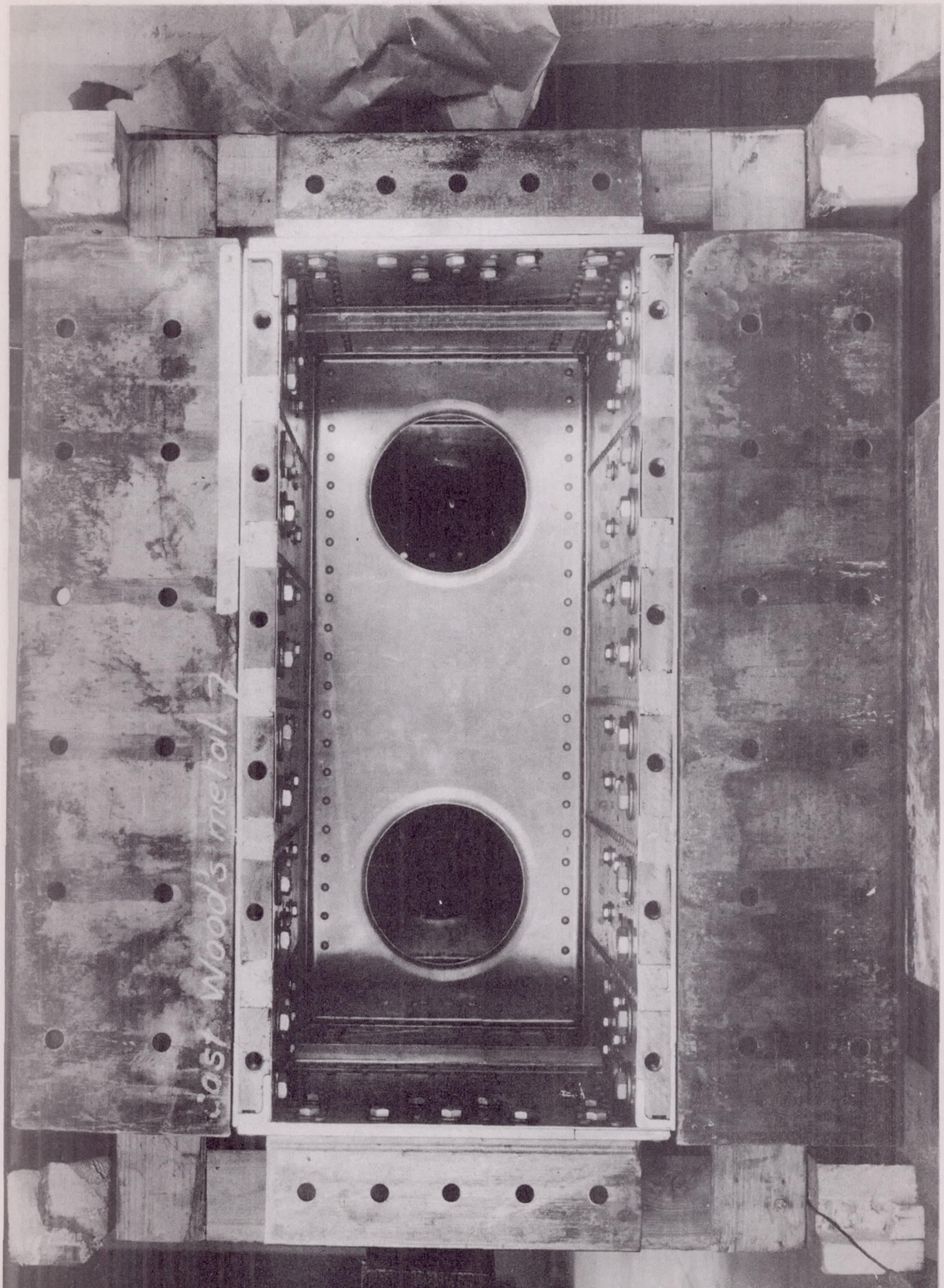

Figure 3.- End view of monocoque box (end plate removed).

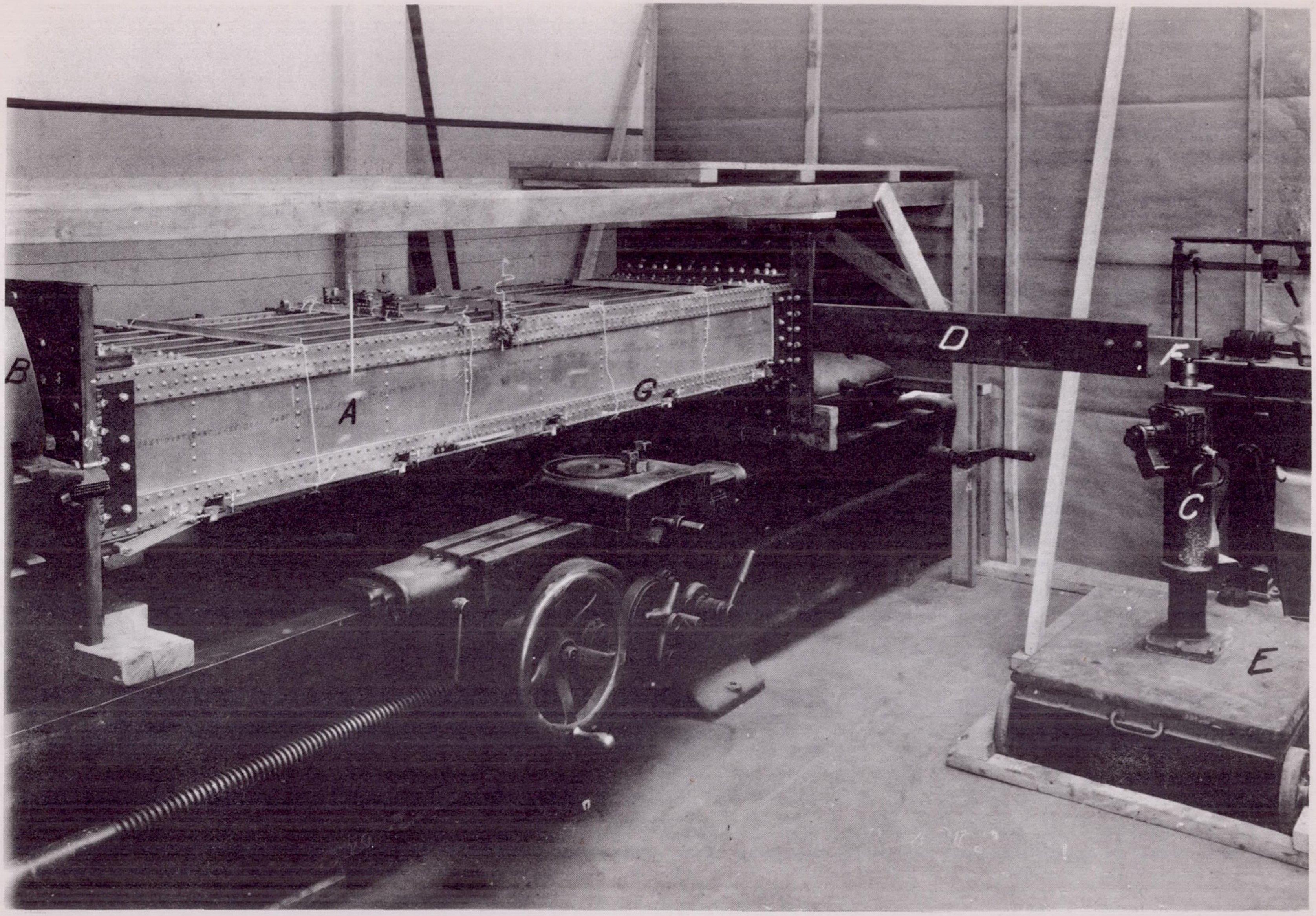

Figure 4.- Monocoque box mounted for torsion test with strain gages and twist gages attached.

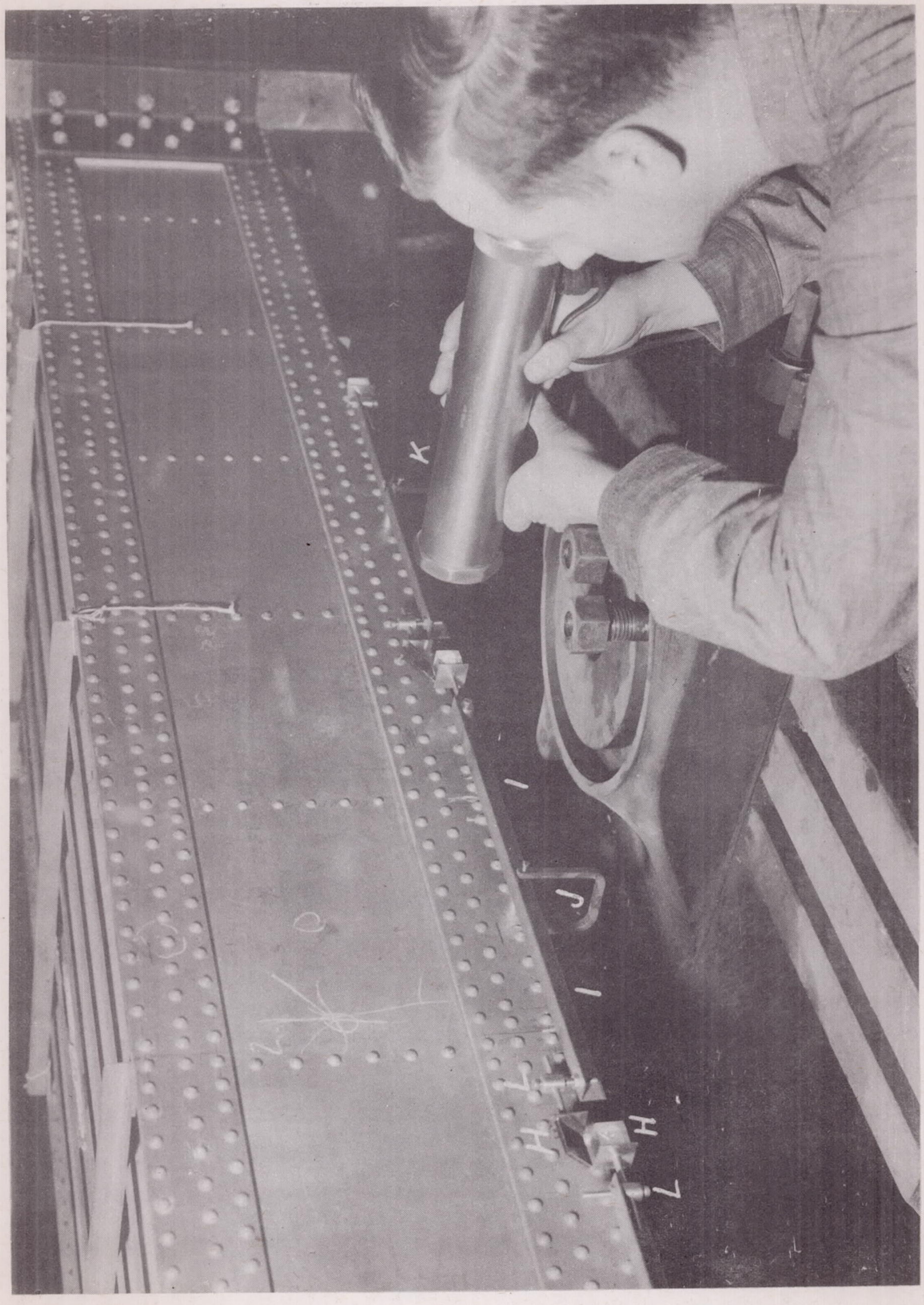

Figure 5.- Twist-measuring gages on monocoque box.

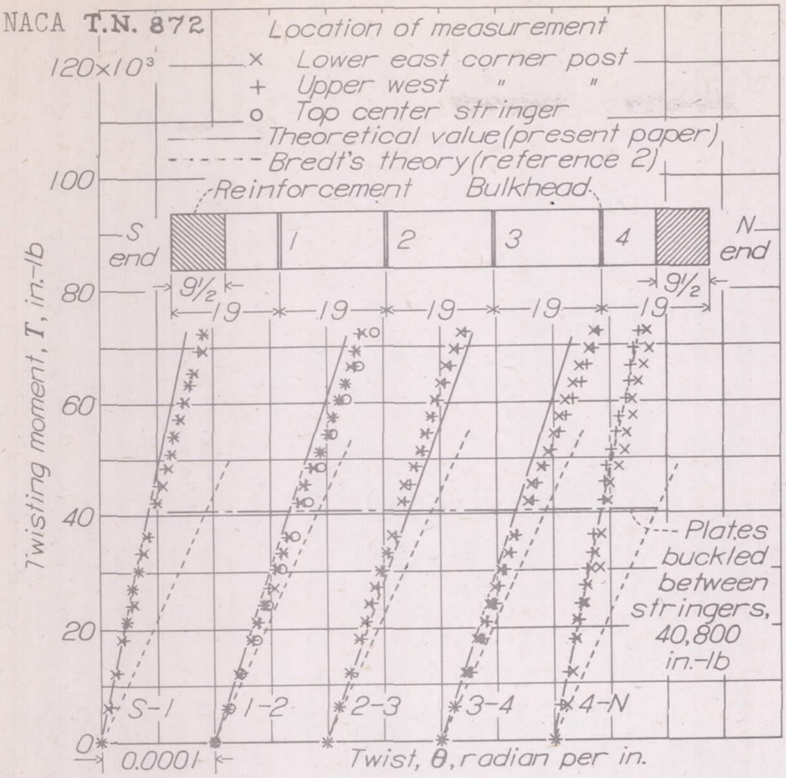

Figure 6.- Average twist of monocoque box between bulkheads.

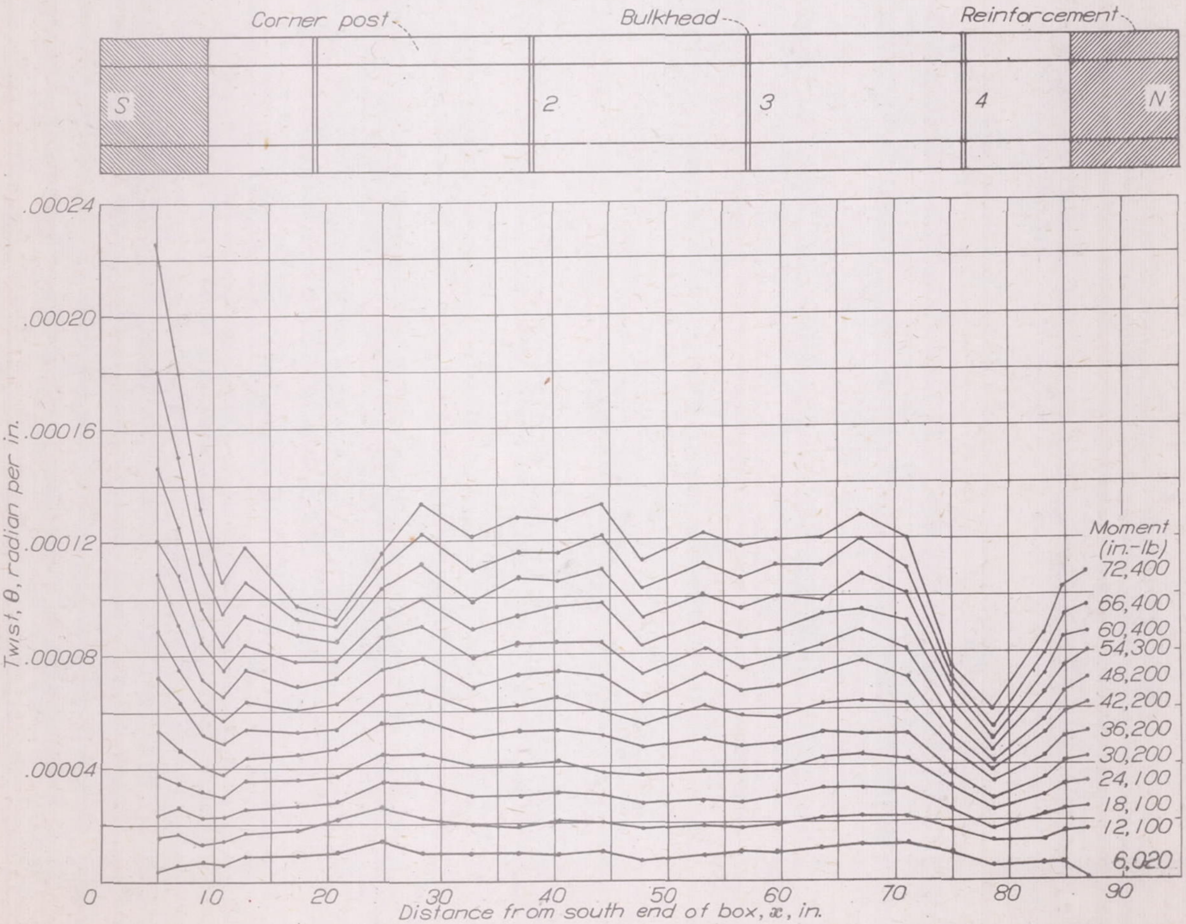

Figure 7.- Variation of average twist with position along box. (Gage length of 19 in. except at ends where gage length is twice the distance from the point in question to the end of the box.)

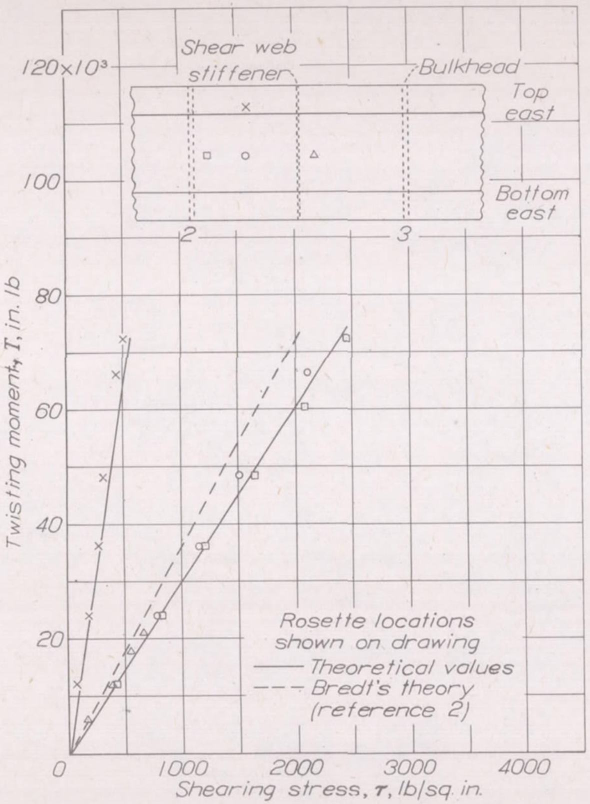

Figure 8.- Shearing stresses in shear web, center bay between bulkheads 2 and 3.

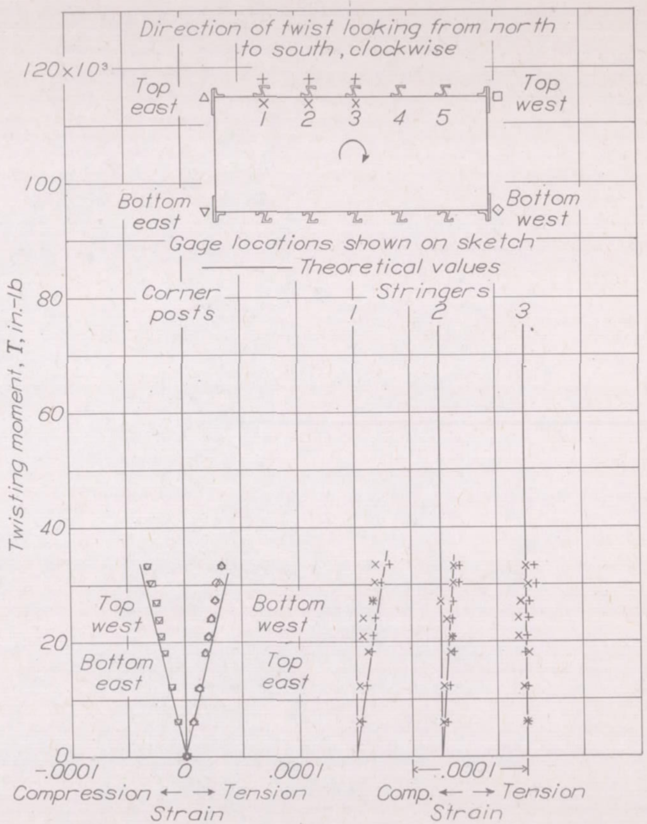

Figure 9.- Strains midway between bulkheads 3 and 4.

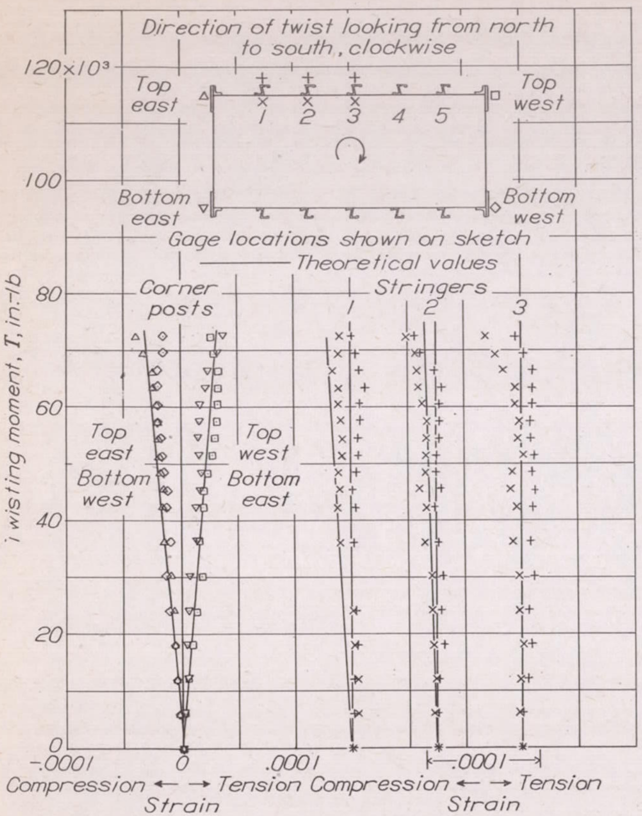

Figure 10.- Strains 1-inch north of bulkhead 2.

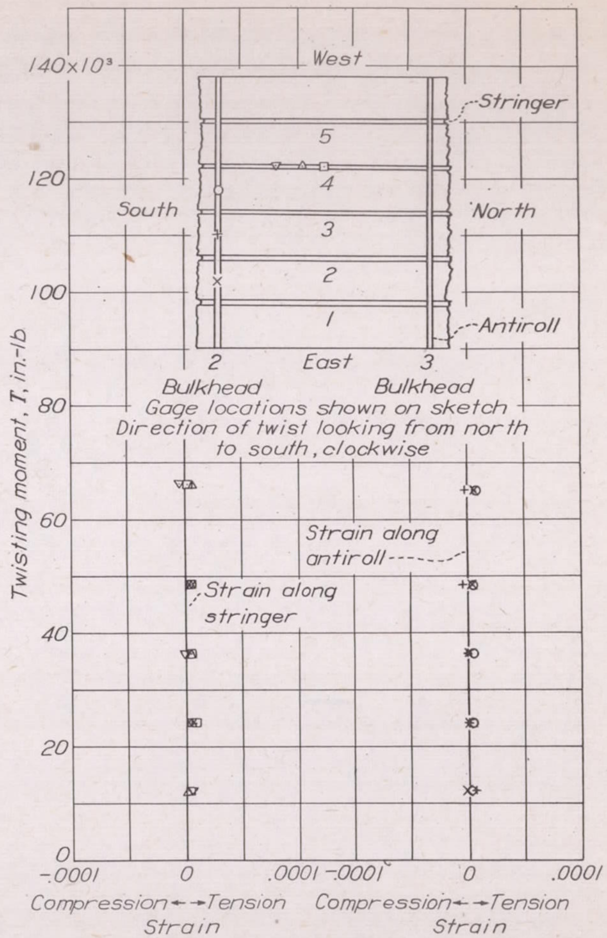

Figure 11.- Strain on top of box along top of stringer 4 between bulkheads 2 and 3 and along antiroll member on top of bulkhead 2.

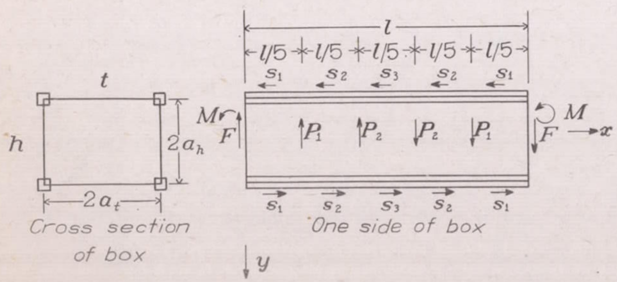

Figure 12.- Sketch showing symbols used in theory.