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THEORETICAL AND EXPERIMENTAL ANALYSIS OF THE REDUCTION OF ROTOR BLADE VIBRATION IN TURBOMACHINERY THROUGH THE USE OF MODIFIED STATOR VANE SPACING

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SUMMARY

A theoretical analysis was made of the dynamic gas forces produced by stationary nozzle vanes and which cause some of the resonant vibrations of the rotating blading of turbomachinery. It was shown that substantial reductions of the excitation level could be obtained by altering the circumferential positions of the vanes by amounts less than ±10 percent of the normal spacing. Two of the configurations analyzed showed excitation level reductions of 68 percent of the excitation level produced by the standard equally spaced vane assembly.

Experimental verification of the analysis was obtained by using an air-interrupter disk and comparing the vibration amplitudes of turbine blades in a turbojet engine when two different nozzle vane configurations were used. One was an equally spaced vane configuration, and the other was one which the analysis had indicated would result in a reduction of the excitation level. The measured amplitudes of vibration of the turbine blades during engine operation compared favorably with those predicted by the analysis.

INTRODUCTION

Failure of one or more of the rotating blades of turbomachinery is often the cause for costly premature overhaul. The amount of internal damage in the unit usually depends upon the staging configuration and the location of the failure. In the case of a single-stage unit, or when a blade fails in the last stage of a multistage unit, the damage can be minor. However, when the failure occurs in any stage except the last of a multistage unit, the damage can be very extensive. Failure in the latter category in aircraft turbojets has occasionally been of catastrophic magnitude.

It has been recognized for many years that fatigue of the metal by resonant vibration has been one of the principal causes of rotor blade failure. The gas forces that the blades feel are not of a static character but vary considerably in magnitude during rotation and are periodic in that they are generally recurrent with each revolution of the wheel. When the frequency of the gas-force fluctuations coincides with the frequency of a natural mode of vibration of the blade, resonant vibration is produced. The vibratory stress level obtained depends on the magnitude of the gas-force fluctuations and the total damping of the system.

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The gas-force fluctuations arise mainly as a result of the channeling of the flow through various passages such as the inlet struts, combustion liners, transition pieces, and the stator vanes and also as a result of the nonuniform addition of energy in the individual combustion areas. Rotor blades are normally operated in close proximity to equally spaced stator vanes, which allows little opportunity for the wakes from the vanes to become diffused. The blades are therefore forced to pass through the wake of each vane and, hence, receive a series of equally spaced pulses. It is immediately obvious that, if it were possible to change the timing of the pulses with relation to each other, the effects of some pulses could be canceled by others, and a reduction of the excitation level would be effected.

A general method of analysis of the excitation forces resulting from the stator vanes was developed and is applicable to the blading of different types of turbomachinery such as axial-flow compressors, axial-flow pumps, and axial-flow turbines. The analytical method, involving the use of Fourier series, was applied to various vane spacing configurations. Configurations analyzed included those in which segments of vanes were shifted circumferentially with respect to each other (this type has been used in a military turbojet), those in which the vane spacing was changed in the various segments, and those in which random vane displacements were imposed on a basic configuration. In addition, various combinations of the above configurations were analyzed. Experimental verification of the theoretical results was obtained, first by using an air-interrupter disk with movable vanes to simulate the various configurations, and, second by determining the vibration response of the turbine blades in a turbojet to two different vane assemblies.

SYMBOLS

$A_n = \sqrt{a_n^2 + b_n^2}$ amplitude coefficient of harmonic order n

a_n amplitude coefficient of cosine term for order n

b_n amplitude coefficient of sine term for order n

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d	circumferential space between stator vanes
F(x)	total forcing function
$f_i(x)$	that portion of the total forcing function associated with the space between i^{th} and $i^{th} + 1$ stator vanes
K	total number of stator vanes
L	value of x at end of forcing cycle function
N	number of segments in vane assembly
n	order number
S	circumferential spacing change between vanes, percent of standard space
x	circumferential distance
δ	arbitrary vane circumferential displacement
φ	phase angle or relation between harmonics

Subscripts:

i,j,k,l	specific vanes in consecutive order
n	order number

ANALYTICAL PROCEDURE

General Approach

In one revolution of the wheel, each rotor blade is subjected to a complex variation of gas forces. This complex gas-force variation or fluctuation is, in general, repetitive with each successive revolution and may be considered as being composed of two parts. One part is an average static gas loading which produces a static bending of the airfoil. The other part is a dynamic gas loading superimposed on the static value and which produces the force variations responsible for the vibration of the blades. Reference 1 shows that the energy absorbed by a resonating system must be supplied in the form of a harmonic force having the same frequency as the natural frequency of the system and must act in phase with the harmonic velocity. In other words, if the dynamic force on the blades is produced by equal and evenly spaced gas pulses from the stator vanes and varies sinusoidally with time, there will be only one wheel speed at which any given mode of vibration of a blade can be excited. However, if the force variation is a complex periodic (nonharmonic) curve, such as would be obtained if the stator vanes were not equally spaced, then there will be more than one wheel speed at which any given mode can be excited. The speeds at which the resonances will occur and the relative amplitudes involved can be obtained by expressing the total forcing function in the form of a Fourier series. This procedure splits the complex forcing function into a series of harmonic curves, each having a specific frequency and amplitude relative to the fundamental of the total function. The over-all response of a blade can then be expressed in terms of its response to each individual harmonic component.

It is obvious that it might be possible to reduce the excitation forces resulting from the stator vanes by judiciously shifting the circumferential position of some or all of the vanes and, thus, in a broad sense, obtain a force cancellation effect. By this means, the harmonic content of the over-all forcing function is increased, but the magnitude of the over-all excitation level may be decreased and, hence, result in an improved environment for the rotor blades. The determination of the harmonic content of the various vane configurations considered was made through the use of trigonometric or Fourier series, and the magnitudes of the individual harmonic components were then evaluated in terms of a conventional equally spaced vane assembly, used hereafter as a standard for comparison.

Fourier Series Analysis

Assumptions and description of method. - Let it be assumed that each individual force pulse that the rotor blade is subjected to is of the same maximum value and that the force of this pulse varies sinusoidally with time. This forcing function, varying in amplitude from 0 to 1, can be seen in figure 1 where it is defined as

$$\left.\begin{aligned} f_i(x) &= 1 - \cos\frac{2\pi}{d_i}(x - x_i) && \text{for} \quad x_i \leq x \leq x_{i+1} \\ f_i(x) &= 0 && \text{for} \quad x_i > x > x_{i+1} \end{aligned}\right\} \qquad (1)$$

where d_i is the space between the i^{th} and $i^{th} + 1$ stator vanes or, symbolically, $d_i = x_{i+1} - x_i$.

One complete cycle of the forcing function occurs when the wheel has made one complete revolution. This complete force cycle is then made up of the sum of the fluctuations due to each stator vane and can be expressed as

$$F(x) = \sum_{i=1}^{K} f_i(x) = \sum_{i=1}^{K} \left[1 - \cos \frac{2\pi}{d_i} (x - x_i)\right] \tag{2}$$

where K is the total number of stator vanes. The value of x at the end of the cycle will be called L and is equal to x_{K+1} or $\sum_{j=1}^{K} d_j$.

An alternate way of describing the complete cyclic forcing function is in a Fourier series of the form (ref. 2):

$$F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{2\pi nx}{L} + b_n \sin \frac{2\pi nx}{L}\right) \tag{3}$$

where $a_0/2$ represents the average value of $F(x)$ over the cycle length L.

The coefficients of the series terms are given by

$$\left.\begin{aligned} a_n &= \frac{2}{L}\int_0^L F(x) \cos \frac{2\pi nx}{L}\, dx \\ b_n &= \frac{2}{L}\int_0^L F(x) \sin \frac{2\pi nx}{L}\, dx \end{aligned}\right\} \tag{4}$$

By substituting the definition of $F(x)$ from equation (2) into equations (4), the Fourier coefficients become

$$\left.\begin{aligned} a_n &= \sum_{i=1}^{K} \frac{2}{L}\int_{x_i}^{x_{i+1}} \cos \frac{2\pi nx}{L}\, dx - \sum_{i=1}^{K} \frac{2}{L}\int_{x_i}^{x_{i+1}} \cos \frac{2\pi}{d_i}(x - x_i) \cos \frac{2\pi nx}{L}\, dx \\ b_n &= \sum_{i=1}^{K} \frac{2}{L}\int_{x_i}^{x_{i+1}} \sin \frac{2\pi nx}{L}\, dx - \sum_{i=1}^{K} \frac{2}{L}\int_{x_i}^{x_{i+1}} \cos \frac{2\pi}{d_i}(x - x_i) \sin \frac{2\pi nx}{L}\, dx \end{aligned}\right\} \tag{5}$$

but

$$\sum_{i=1}^{K} \frac{2}{L} \int_{x_i}^{x_{i+1}} \cos \frac{2\pi nx}{L} \, dx = \frac{2}{L} \int_0^L \cos \frac{2\pi nx}{L} \, dx = 0$$

and

$$\sum_{i=1}^{K} \frac{2}{L} \int_{x_i}^{x_{i+1}} \sin \frac{2\pi nx}{L} \, dx = \frac{2}{L} \int_0^L \sin \frac{2\pi nx}{L} \, dx = 0$$

since x_1 is always taken to be zero and x_{K+1} is equal to L. Equations (5) can therefore be given by

$$\left.\begin{aligned} a_n &= -\sum_{i=1}^{K} \frac{2}{L} \int_{x_i}^{x_{i+1}} \cos \frac{2\pi}{d_i} (x - x_i) \cos \frac{2\pi nx}{L} \, dx = \sum_{i=1}^{K} a_i \\ b_n &= -\sum_{i=1}^{K} \frac{2}{L} \int_{x_i}^{x_{i+1}} \cos \frac{2\pi}{d_i} (x - x_i) \sin \frac{2\pi nx}{L} \, dx = \sum_{i=1}^{K} b_i \end{aligned}\right\} \quad (6)$$

where

$$a_i = -\frac{2}{L} \int_{x_i}^{x_{i+1}} \cos \frac{2\pi}{d_i} (x - x_i) \cos \frac{2\pi nx}{L} \, dx$$

$$b_i = -\frac{2}{L} \int_{x_i}^{x_{i+1}} \cos \frac{2\pi}{d_i} (x - x_i) \sin \frac{2\pi nx}{L} \, dx$$

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For $d_i \neq L/n$,

$$a_i = \frac{d_i}{2\pi}\left(\frac{1}{L - nd_i} - \frac{1}{L + nd_i}\right)\left(\sin \frac{2\pi n}{L} x_{i+1} - \sin \frac{2\pi n}{L} x_i\right)$$

$$b_i = \frac{d_i}{2\pi}\left(\frac{1}{L - nd_i} - \frac{1}{L + nd_i}\right)\left(\cos \frac{2\pi n}{L} x_{i+1} - \cos \frac{2\pi n}{L} x_i\right)$$

and for $d_i = L/n$, (7)

$$a_i = -\frac{1}{n} \cos \frac{2\pi}{d_i} x_i$$

$$b_i = -\frac{1}{n} \sin \frac{2\pi}{d_i} x_i$$

The coefficients of the Fourier series can now be combined as follows:

$$\sqrt{a_n^2 + b_n^2} = A_n \tag{8}$$

so that the Fourier series representation of the forcing function can be written as

$$F(x) = \frac{a_0}{2} + \sum_{i=1}^{\infty} A_n \sin\left(\frac{2\pi nx}{L} - \varphi_n\right) \tag{9}$$

where the A_n terms are the coefficients of the various harmonics that go into making up the forcing function and the φ_n terms are the phase relations for these harmonics.

Application of theory to a specific vane configuration. - If a particular vane configuration is to be analyzed, all the various spacings between stator vanes are, of course, known. The first step in the analysis is to calculate the length of the cycle L that is equal to the sum of all the spacings. This value of L along with the number of the harmonic n that is being calculated is substituted into equations (7). The value of the first stator spacing d_1 is then substituted into the proper equations (7), as determined by the value of L/n, and the a_1 and b_1 terms are then calculated. This procedure is followed for each successive value of stator spacing d until all the a_i and b_i terms have been calculated. The a_i and b_i terms are then summed up to give

a_n and b_n (eq. (6)), and, finally, the harmonic coefficient A_n for the n^{th} order is calculated by equation (8). If, now, the harmonic coefficient for some other order is desired, the value n of that harmonic is put into equations (7), and the entire procedure is repeated. This type of calculation lends itself very well to automatic computing where a configuration can be completely analyzed in a short time.

Influence Coefficients

Objective. - It has been indicated in a previous section of this report that various selected vane configurations have been analyzed and their excitation levels compared with the standard equally spaced vane assembly. This procedure can be justly criticized on the basis that the configurations selected will not necessarily include the one having the lowest excitation level. For this reason, an attempt was made to find a method of arriving at the best possible configuration within the limits of practicality of aerodynamics and fabrication. Although a direct means of doing this was not found, a method involving influence coefficients was used in an attempt to obtain a partial solution to the problem. In this particular application, the influence coefficients were used to express the effect of circumferentially displacing a given vane by a small amount on the amplitude coefficients of the various harmonics. (Examples of the application of influence coefficients are presented in ref. 3.) The procedure involves arbitrarily changing the position of one or more vanes of a given configuration and calculating the harmonic and the influence coefficients. Vanes are then judiciously moved on the basis of the computed coefficients, and another calculation is made of the new coefficients. This procedure continues until it is no longer possible to reduce the over-all excitation level. It is obvious that it is necessary to decide in advance on the limits of the individual vane positions in order to prevent the procedure from converging on an absurd configuration.

Method of application. - Let the influence coefficients associated with the k^{th} stator vane be defined as the changes in the various harmonic coefficients due to a small positive displacement δ of the k^{th} stator vane. As can be seen from figure 2, a small displacement δ of the k^{th} blade changes the spacing on either side of this vane from d_j and d_k to $d_j + \delta$ and $d_k - \delta$, while all the other vane spacings and vane locations remain the same. Since this is the case, it can be seen from equations (7) that this small displacement δ of the k^{th} vane will change only the values of a_j, b_j, a_k, and b_k. The changes in a_n and b_n due to this displacement of the k^{th} vane are $\Delta a_{n,k}$ and $\Delta b_{n,k}$ and are given by

$$\begin{aligned}
\Delta a_{n,k} = {} & \frac{d_j + \delta}{2\pi}\left[\frac{1}{L - n(d_j + \delta)} - \frac{1}{L + n(d_j + \delta)}\right]\left[\sin\frac{2\pi n}{L}(x_k + \delta) - \sin\frac{2\pi n}{L}(x_j)\right] + \\
& \frac{d_k - \delta}{2\pi}\left[\frac{1}{L - n(d_k - \delta)} - \frac{1}{L + n(d_k - \delta)}\right]\left[\sin\frac{2\pi n}{L}(x_l) - \sin\frac{2\pi n}{L}(x_k + \delta)\right] - \\
& \frac{d_j}{2\pi}\left(\frac{1}{L - nd_j} - \frac{1}{L + nd_j}\right)\left[\sin\frac{2\pi n}{L}(x_k) - \sin\frac{2\pi n}{L}(x_j)\right] - \\
& \frac{d_k}{2\pi}\left(\frac{1}{L - nd_k} - \frac{1}{L + nd_k}\right)\left[\sin\frac{2\pi n}{L}(x_l) - \sin\frac{2\pi n}{L}(x_k)\right]
\end{aligned} \tag{10}$$

and a similar expression for $\Delta b_{n,k}$ except with the sine terms replaced by cosine terms.

If one of the four terms in $\Delta a_{n,k}$ and $\Delta b_{n,k}$ should become infinite, that one term should be replaced by an expression similar to the one given in the second part of equations (7). For example, if L were to be equal to $n(d_k - \delta)$, the second term of $\Delta a_{n,k}$ and $\Delta b_{n,k}$ in equations (10) would become infinite, and the entire second term in both $\Delta a_{n,k}$ and $\Delta b_{n,k}$ should be replaced by $-\frac{1}{n}\cos\left(\frac{2\pi}{d_k - \delta}x_j\right)$ and $-\frac{1}{n}\sin\left(\frac{2\pi}{d_k - \delta}x_j\right)$, respectively.

The influence coefficients associated with the k^{th} stator vane $\Delta A_{n,k}$ can now be computed for any particular stator vane configuration. These influence coefficients are given by

$$\Delta A_{n,k} = \sqrt{(a_n + \Delta a_{n,k})^2 + (b_n + \Delta b_{n,k})^2} - \sqrt{a_n^2 + b_n^2} \tag{11}$$

where a_n and b_n are calculated from equations (7), and $\Delta a_{n,k}$ and $\Delta b_{n,k}$ are calculated from equation (10). This procedure is repeated for all the different values of n that produce significant harmonic coefficients. It is obvious that, in order to make any calculations of influence coefficients, it is first necessary to choose some value for the displacement δ; in the calculations presented in this paper, a value of 1 percent of the standard spacing was chosen.

RESULTS OF THEORETICAL ANALYSIS

Vane Configurations Chosen for Analysis

An arbitrary selection of an equally spaced vane assembly having 48 vanes was made as the basis for the calculations. The specific number of vanes is unimportant to the analysis and does not affect the general conclusions that are drawn. In the experimental work discussed in later sections of the report, 36 and 64 vanes were used as a matter of convenience in assembling the necessary apparatus. As pointed out in the previous section, two assumptions were made: (1) the dynamic force on the rotor blades varies sinusoidally as the blades pass from the wake of one vane into the wake of the next, and (2) the amplitudes of all the sinusoidal pulses were assumed to be the same and equal to 1.00. Under the conditions of these assumptions, it can be readily shown that, for the 48 equally spaced vane assembly, the 48^{th} order is the prime forcing function with an amplitude coefficient of 1.00 and that the amplitude coefficients for all other harmonics are zero. In the analytical treatment of the other vane configurations the same two assumptions were made, which makes it possible to compare the amplitude coefficients of the various harmonics obtained with the amplitude coefficient of the 48^{th} order for the 48 equally spaced vane assembly. Thus, the excitation levels of any vane configuration can be expressed in percent of the excitation level of the equally spaced assembly. 4978

All the other vane configurations selected for analysis were obtained by modifying the positions of some or all of the vanes of the 48 equally spaced vane assembly. Four types of modifications were chosen for study:

(1) phasing (circumferential shifting) between N segments of the vane assembly with no vane spacing change within the segments

(2) changing the vane spacing in the N segments but with equal spacing within any one segment

(3) combinations of (1) and (2)

(4) random displacement of all vanes from a basic configuration

In the first type of modification, the equally spaced vane assembly is divided into N segments with the spacing between vanes in any one segment left unchanged. The individual segments are then displaced in the circumferential direction relative to each other. As the wheel rotates, the blades feel a series of equally spaced pulses as they pass before one segment and then feel another series of pulses that have a different phase relation to the first, and so on. The phase relation between the consecutive pulse series is defined in terms of the normal space between vanes being 360° in the unmodified equally spaced assembly. An example would be the case in which the normal vane assembly is divided into two segments and one segment rotated circumferentially with respect to the other by one-half of a normal vane space. This configuration would be termed a 180° phased assembly.

In the second type of modification, the equally spaced vane assembly is divided into N segments, and the spacing of the vanes within the various segments is changed. In any one segment, the spacing is constant. In the case of an assembly divided into two segments, the spacing in one segment is made $1 + \frac{S}{100}$, and, in the other, the spacing is made $1 - \frac{S}{100}$ where 1 is the spacing in the case of the standard equally spaced vane assembly. The circumferential spacing change S was allowed to vary from 0 to 15 percent of the standard spacing. It is recognized that there is a practical limit in the value of S from the aerodynamic design standpoint, and it was felt that a value of 15 percent would be large enough to cover the practical range. In the case of an assembly divided into three segments, the first segment is assumed to retain the original standard spacing, and the other two segments are assumed to have spacings of $1 + \frac{S}{100}$ and $1 - \frac{S}{100}$, respectively. In the case of an assembly divided into four segments, the spacing was assumed to be $1 + \frac{S}{100}$, $1 - \frac{S}{100}$, $1 + \frac{S}{100}$, and $1 - \frac{S}{100}$ for the four segments, respectively.

In the third type of modification, phasing between segments was combined with changing of the spacing within the individual segments to determine if any further reduction in the excitation levels could be obtained. In the fourth type of modification, each vane in a given basic vane configuration was moved circumferentially by a random amount. The amount of the movement and the sign (clockwise or counterclockwise) were obtained from tables of random numbers (ref. 4). The first basic vane configuration to which random movement was applied was the standard equally spaced 48 vane assembly. Limits were imposed on the maximum circumferential movement of any one vane to keep the resulting configuration within practical bounds. For four sets of random numbers, a maximum limit of ±5 percent of the basic spacing was imposed; in another trial of four different sets of random numbers, a maximum limit of ±10 percent was used. Random movement was also applied to two other basic vane configurations. One of these was a three-segment assembly with initial spacings in the segments of 1, 1.1, and 0.9, respectively. The other was a two-segment assembly with 112° phasing between the two segments, but with standard spacing within each segment.

A summary of all the various vane configurations studied is shown in table I.

Harmonic Coefficients for the Various Vane Configurations

Results of type I modification (phasing). - The excitation levels of vane configuration 2 (table I) are shown in figure 3. This figure and all succeeding figures that show the harmonic amplitude coefficient curves present in general those orders that result in maximum values of the harmonic coefficients for any given value of the variable involved. Presentation of all the orders computed would result in a needless confusion of curves. In general, the computations show that the harmonic coefficients decrease in magnitude as the order numbers either increase or decrease from a certain dominant range. Sufficient order numbers were therefore computed on either side of the dominant range to be reasonably certain that all significant coefficient values were obtained. In addition to order numbers in the vicinity of the dominant range, the coefficients for the order numbers 1, 2, 24, and 96 were also calculated. Configuration 2 is divided into two segments with standard spacing in each segment. The phasing between the segments is allowed to vary from 0° to 180°, and figure 3 shows the variation in the harmonic coefficients of the 47th, 48th, and 49th orders with phasing angle as the variable. With 0° phasing, the configuration reverts to number 1 in table I wherein the 48th order has a coefficient of 1.0 and all others are zero. As the phasing angle is increased from zero, the coefficient of the 48th order drops, while the other coefficients rise. At a phase angle of 180°, the 48th order decreases to a value of 0.05, while the 47th and 49th rise to values of 0.65 and 0.63, respectively. The best condition from the overall excitation level point of view occurs at a phase angle of approximately 112° where the coefficients of the 47th, 48th, and 49th are approximately equal at a value of 0.53.

In configuration number 3 of table I, the assembly is divided into three segments with standard spacing in each segment as illustrated in figure 4(a). The phasing between the three segments is obtained by changing the values of the spacings for the first, sixteenth, and thirty-second vanes but always keeping the sum of the three spacings constant. In order to calculate the effects of various combinations of phasings between the three segments, the following analysis procedure is used. A value of d_1 is chosen which fixes the position of the first segment with respect to the third segment, and a series of calculations are made for different positions of segment 2 with respect to the first and third segments. The value of d_1 is then changed, and the procedure is repeated. Figure 4(b) shows the results of a sample set of calculations where d_1 is taken as 0.9 (a 36° phase shift between segments 1 and 3), and the values of d_{16} and d_{32} are varied to rotate the second segment circumferentially. The results of these sets of calculations indicate that the best that can be achieved with phasing between the three groups is an excitation level of about 0.5, which may be obtained with several different combinations of phasings.

Results of type II modification (spacing change in segments). - In this type of modification the vane spacing in the segments was changed with the spacing in any one segment held constant. There was no phasing in this series. The first of this type is the two-segment assembly (number 4 in table I), and the harmonic coefficients are shown in figure 5 as a function of the spacing change S. The 48^{th} order drops very rapidly with an increase of S of only 3 percent. The other orders rise quite fast, however, leading to excitation levels in the range of 0.47 to 0.56 for values of S greater than 2.5 percent. The second of this type (number 5 in table I) contains three segments, and the harmonic coefficients plotted against spacing change S are shown in figure 6. It will be noted in this case that the excitation level at 6.5-percent S is down to 0.39, and at 10-percent S the excitation level is down to 0.32. The third configuration of this type (number 6 in table I), which is made up of four segments, is basically the same as a two-segment configuration and gives excitation levels that are the same as those of a two-segment configuration. All the configurations of type II revert to type I (table I) when S goes to zero.

Results of type III modification (phasing plus spacing change). - Two configurations of this type were analyzed and are noted as numbers 7 and 8 in table I. In number 7, the assembly was divided into two segments with 180° phasing between the segments. A spacing change in the two segments of $1 + \frac{S}{100}$ and $1 - \frac{S}{100}$ was incorporated as the variable. The results are plotted in figure 7 with the harmonic coefficients of the orders 42 to 48 shown as a function of S. Coefficients of the 49^{th} and higher orders were somewhat smaller than the corresponding orders below 48. This particular configuration displays no special merit since it will be noted that the excitation level varies from 0.48 to 0.57. In number 8, three segments were chosen with 120° phasing between segments. Again, the spacing was allowed to vary in the segments in accord with the relations $1 + 0$, $1 + \frac{S}{100}$, and $1 - \frac{S}{100}$. The results are plotted in figure 8 and indicate a considerable improvement over number 7. For example, at a value of S equal to approximately 8.5 percent, the excitation level is down to 0.32.

Results of type IV modification (random spacing). - Four configurations were chosen for studying the effect of moving the vanes in a random manner, and the results are presented in tabular form in table II. The first two of these (numbers 9 and 10 in table I) were based on the assembly in which all vanes were initially equally spaced. In number 9, four sets of random numbers were tried with limits of vane movement set at ±5 percent of the standard spacing. In number 10, four sets of random numbers were tried with the limits raised to ±10 percent. The harmonic coefficients of the orders 45 to 51 are compared in table II with coefficients for the standard equally spaced vane assembly. It will be noted that the 48th order was reduced from 1.00 to values ranging from 0.713 to 0.899. This reduction is not particularly good since several of the other configurations studied showed reductions of twice this amount.

In configuration 11, the random movement was applied to a three-segment assembly with initial spacings in the segments of 1, 1.1, and 0.9, respectively. Three sets of random numbers were tried with limits of movement set at ±5 percent. The results are shown in table II, and it will be noted that there was no improvement in the over-all excitation level when compared with the basic configurations. In configuration 12, the random movement was applied to a two-segment assembly with 112° of phasing between the segments. Four sets of random numbers were tried and resulted in no marked improvement when the harmonic coefficients were compared with those of the basic design (table II).

Comparison of Excitation Levels of Various Vane Configurations and Effect of Modifications on Stage Efficiency

In the results that have been presented, it is immediately evident that only a very small proportion of the configurations analyzed leads to excitation level reductions greater than 60 percent. Those configurations involving the use of phasing alone provide at best a reduction of the order of 50 percent. When the assembly was divided into a number of segments having different spacings within the segments, a substantial improvement was obtained, and a reduction of 68 percent was noted for the three-segment arrangement having a 10-percent spacing change. A similar improvement was made by combining spacing change within the segments with segment phasing. This was obtained with an 8.5-percent spacing change in a three-segment configuration with 120° phasing between segments. When random positioning of the vanes was superimposed on any of the basic configurations (including the standard, equally spaced assembly), no substantial improvement could be obtained.

The effect of these vane assembly modifications on the aerodynamic performance was not studied in this investigation. For the small changes in spacing required (approx. 8 percent) it is likely that there would be little effect on the stage efficiency. If efficiency were adversely affected, however, it might be improved by either changing the vane setting angle to correct for velocity diagram alternations due to the solidity variation or to maintain constant solidity by altering the vane chord.

Example of Application of Influence Coefficients

A specific vane configuration of type 5 (table I) was arbitrarily selected to illustrate the application of influence coefficients for the purpose of obtaining further reductions in the over-all excitation level. The vane configurations chosen had three segments with 16 vanes in each segment. The spacing in the segments was 1, 1.1, and 0.9 for the three segments, respectively. The spacing was constant in any one segment, and no phasing was used between the segments. The computed harmonic amplitude coefficients for the particular configuration are listed as follows:

Order number, n	Harmonic amplitude coefficient, A_n
45	0.2720
46	.1642
47	.3257
48	.3178
49	.3315
50	.0944
51	.1455

As a first step, the influence coefficients were computed for a displacement δ of 1 percent of the standard spacing and for the vanes numbered 2 through 15 of the segment having an initial spacing of 1. The resulting values corresponding to the displacement of these vanes are shown in table III for order numbers 45 through 51. If a vane were to be displaced by some other value than 1 percent, for example, -3 percent, the influence coefficients for that vane would be multiplied by -3 in order to obtain the approximate changes in the harmonic coefficients. Note that the values in table III are very small.

For the vane configuration chosen for this illustration, it has already been shown that the harmonics having significant amplitude coefficients are the 47th, 48th, and 49th. With the aid of table III, new positions of vanes 2 through 15 can now be determined which will result in a reduction of the significant harmonics. One such new arrangement would be to displace vane 2 by +5 percent and vanes 13, 14, and 15 by -10 percent. The remaining vanes were not moved. The resulting harmonic amplitude coefficients are obtained as follows:

Order number, n	45	46	47	48	49	50	51
Original A_n	0.2720	0.1642	0.3275	0.3178	0.3315	0.0944	0.1455
ΔA_n due to +5 percent displacement of blade 2	.0045	-.0045	-.0125	.0020	.0050	.0040	.0010
ΔA_n due to -10 percent displacement of blade 13	-.0050	-.0110	.0030	-.0040	-.0060	.0090	-.0110
ΔA_n due to -10 percent displacement of blade 14	-.0090	-.0090	.0020	-.0040	-.0070	.0070	-.0120
ΔA_n due to -10 percent displacement of blade 15	-.0120	-.0060	-.0030	-.0040	-.0090	.0040	-.0120
New values of A_n	0.2505	0.1337	0.3170	0.3078	0.3145	0.1184	0.1115

If further reductions in harmonic coefficients are to be attempted, two series of calculations must next be made. More accurate values of the harmonic coefficients must be determined using the revised vane spacings. This is necessary because the influence coefficients are exact only for δ equal to 1 percent. For other values of δ, the coefficients become approximate because the relation between the influence coefficients and δ is not necessarily linear. In addition, new influence coefficients must be determined for all the vanes adjacent to vane spacings that were last altered. In the example presented, new influence coefficients would have to be determined for vanes 1, 2, 3, 12, 13, 14, 15, and 16. Once the new harmonic and influence coefficients are determined, the entire procedure for reducing the harmonic coefficients can be repeated. It should be noted that this procedure is very time-consuming even with present-day automatic computing machines and should be terminated as soon as possible when it becomes apparent that little improvement is being realized with additional computations. With the configuration presented, it is already apparent that further rearrangement of vanes 2 through 15 will probably result in only minor reductions in the over-all level of the harmonic coefficients.

EXPERIMENTAL CHECK OF ANALYTICAL ASSUMPTIONS

Objective and Equipment

In the theoretical analysis involving the Fourier series, it is assumed that each successive pulse felt by the turbine blades as they pass by the stator vanes will vary sinusoidally with time and that the amplitudes of all the pulses are equal. To obtain some indication concerning the general effect of these assumptions, the test rig shown in figure 9 was used.

The test rig consisted of an aluminum disk with 36 blades, capable of being rotated at a speed of 865 rpm by a constant-speed motor. An air nozzle was placed on one side of the blades, and a hot-wire anemometer was placed on the other side, directly opposite the air nozzle as shown in figure 9. The disk was constructed to permit changing the circumferential position of each of the 36 blades. By this means, various vane configurations could be simulated by accurately positioning each blade in accordance with the configuration desired. The rotating disk then simulates the vane assembly, and the stationary hot-wire probe provides a signal proportional to the dynamic force on the turbine blades.

Procedure and Results

The 36 blades on the disk were first set to provide equal spacing between all blades. The signal from the hot-wire was analyzed with an electronic wave analyzer and also was viewed on an oscilloscope. The wave analyzer showed that the 36^{th} order was highly dominant, and readings were obtained of the amplitude as a function of the airflow through the nozzle. The blades on the disk were then moved to simulate the three-segment configuration in which the vane spacings were 1, 1.04, and 0.96, respectively. The harmonic coefficients were measured with the wave analyzer using the same hot-wire and at the same airflows through the nozzle. This test was repeated for the three-segment configuration for additional values of S equal to 7 and 10 percent of the standard spacing.

The results of the tests are presented in table IV. The measured harmonic coefficients for the orders ranging from 31 to 40 inclusive are shown together with the coefficients calculated according to the analysis previously presented. It will be noted that the measured and calculated values of the coefficients agree well despite the obvious differences in amplitudes of the individual pulses and small departures from a sinusoidal shape as shown in figure 10. The wave form shown is typical of the signal obtained from the hot-wire. The good correlation is probably due, at least in part, to the fact that the wave analyzer tends to give an average value for each coefficient over a period of time determined by the time response of the meter. This is probably the same condition that the turbine blade finds as its environment in a turbojet engine. In other words, the amplitude of vibration of a turbine blade is probably determined by some sort of an average amplitude of the excitation pulses of a given harmonic and thus reponds in a delayed manner in accordance with the total damping present just the same as the wave analyzer responds in accordance with its damping. The results of the test therefore indicated that the basic theory used in the analysis could probably be applied to the case of the turbojet, and an attempt was then made to substantiate this conclusion. The results are discussed in the following section.

COMPARISON OF THEORETICAL ANALYSIS WITH BLADE VIBRATION DATA OBTAINED DURING ENGINE OPERATION

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Description of Engine and Modifications

The engine used in testing the validity of the theoretical analysis was an axial-flow turbojet in the 5000-pound thrust class. Two tests were made, one with a standard turbine stator vane assembly and one with a modified assembly having two segments phased 180°. Data were obtained from strain gages mounted on three turbine blades modified to conform to the test requirements.

Turbine stator vane assemblies. - The standard turbine stator vane assembly consisted of 64 vanes spaced at equal intervals. In the fabrication of the modified (phased) assembly, a standard unit was cut apart on a diameter. An arc segment containing one vane adjacent to a cut face was removed, and the remainder of that half of the assembly was revolved through an arc equal to one-half of a standard vane space. With the two parts fixed in this relation, material was added to the inner and outer annular rings where required to complete reassembly. Figure 11 shows the resulting vane spacing at the location of the phase change. There was a similar discontinuity nearly diametrically opposite. This configuration (equivalent to number 2 of table I with 180° phasing) was selected on the basis of relative ease of fabrication compared with other types included in the analytical investigation. It was not an optimum configuration insofar as reduction in excitation forces was concerned.

The dominant excitation force with the standard turbine stator vane assembly would occur at the 64th order of turbine speed. In the case of the 180° phased assembly, the 63rd and 65th would be expected to be the dominant orders of excitation, according to the analysis presented in this report.

Turbine blade modifications. - In the 75- to 100-percent speed range of the engine, the excitation frequencies to be expected from the stator vane assemblies were in the range of 6500 to 8500 cycles per second. Therefore, it was considered necessary to make the standard turbine blades more susceptible to vibration in the higher modes to permit reasonably accurate excitation level determinations when operating with the two different vane configurations. This was accomplished by machining the three test blades to decrease the rigidity in the tip region. The modification consisted of tapering the convex side from the base to a tip of 0.030 inch. There was no reduction in the base section.

Instrumentation and Operating Procedure

High-temperature strain gage. - The three modified turbine blades were installed 120° apart in the turbine wheel. Each blade was provided with a high-temperature strain gage mounted on the convex side of the airfoil at the base and near the trailing edge as shown in figure 12. This position has been shown in prior measurements to be sensitive in general to both low and high modes of vibration. The strain gages, fabricated by the pressure-stabilized grid method described in reference 5, were cemented to the blades with a material developed by the Protective Coatings Section of the Engineer Research and Development Laboratories at Fort Belvoir, Va. The military designation is MIL-P-14105A(CE). Procedure for the application of this material and a description of its characteristics are given in reference 6.

The strain-gage filaments were 0.001-inch Karma wire attached to tube-tipped leads of 0.012-inch Karma wire. The filaments were wound on a special jig to form the grid, and a press was used to produce 10- to 15-percent reduction in the wire diameter. The grid was 1/4 by 3/8 inch, with the strain-sensitive direction in the direction of the larger dimension. The pressure-stabilized grids were then transferred to cement-precoated surfaces on the turbine blades, and a cover coat of the cement was applied with an airbrush. The ceramic cement was dried at 150° F, and then the temperature was increased at a slow rate to approximately 850° F to transform the silicone resins present in the cement. The final step was to raise the temperature to 1550° F for 15 minutes. The finished coating was exceptionally hard, had good adhesion, and was resistant to erosion and abrasion. Total thickness of the strain-gage installation was 0.004 to 0.005 inch.

Signal transfer and recording equipment. - Secondary lead wires of 0.012-inch diameter were attached to the primary leads by spot-welding. These wires were encased in Inconel conduits swaged for firm support of the wire and the magnesium oxide insulation. The conduits led from the blade-base platforms to a terminal plate at the hub of the turbine wheel and were held against the face of the wheel by spot-welded Inconel ribbon. The shafts of the engine were bored axially to permit connection between the terminal plate and a slip-ring unit mounted at the front of the engine. Monel slip rings were used; the stationary brushes were 60 percent silver - 40 percent graphite, mounted at an angle of 8° to the ring radius at point of contact.

A separate and complete direct-current bridge was used in conjunction with each of the three gages with the three inactive arms of each bridge mounted on a dynamically unstrained portion of the rotating slip-ring body. The signals from the three gages were amplified and recorded on 1/2-inch magnetic tape at a tape speed of 30 inches per second. A standard frequency signal of 400 cycles per second and a signal from a tachometer generator on the engine were also recorded simultaneously on the tape.

Operating procedure. - The operating procedure consisted of starting the engine and taking it up to rated speed of 7950 rpm. At this speed the variable-area exhaust nozzle was set to give an exhaust gas temperature of 1260° F. The engine was then brought down to an idling speed of approximately 3000 rpm, and the strain-gage recording equipment was switched on. The speed was then slowly increased, covering the range of 3000 to 7953 rpm in approximately 15 minutes. The resulting acceleration was sufficiently low to permit all vibration resonances to build up to their maximum values. Deceleration runs were also made to check the possibility of nonlinear resonances.

After recording a complete run, the data were analyzed by playing back at a tape speed of 30 inches per second and observing the various signals with the aid of oscilloscopes, events-per-unit-time meters, and electronic voltmeters. Each resonance point was played back several times to confirm the exact frequency and amplitude of the signal. This was necessary to establish the exact order number of the excitation.

After obtaining a complete resonance spectrum of the instrumented turbine blades with the standard stator vane configuration, the modified vane assembly was installed, and the engine runs were repeated. This was possible without disturbing the instrumented turbine blades by disconnecting the lead wires at the hub of the turbine wheel and withdrawing the wheel from the engine. The same gage mounts were therefore used in all engine runs, which eliminated such variables among different mounts as strain sensitivity factor and gage location.

Results of Engine Comparison

As mentioned previously, the theoretical analysis was made with the assumptions of equal pulse amplitudes and sinusoidal pulse wave form. It is obvious that neither of these assumptions is strictly correct in the case of a turbojet engine. It is therefore of considerable interest to determine the vibration response of turbine blades to different stator vane configurations and compare the results with the analysis. The vibration data from the three instrumented turbine blades are presented in the form of resonance spectrum diagrams. Figure 13 indicates the response of the three blades to the standard stator vane configuration, and figure 14 shows the response of the same blades to the modified vane assembly.

As would be expected, in the case of the standard configuration, the blades were excited at the 64^{th} order of the turbine speed. However, all the resonance points indicated on the 64^{th} order lines in figure 13 should not be interpreted as true vibrational modes of the individual instrumented blades. Most of these resonance points are of very small amplitude and are undoubtedly the result of energy transfer through the wheel from adjacent blades vibrating in their true modes. It is probable, however, that those few points which do have a relatively large amplitude when compared with the others are essentially true modes of the instrumented blades. It is these resonance points that are used in the correlation with the theoretical analysis.

In the case of the special vane configuration, it will be noted that the 64^{th} order is absent from the spectrum of all three blades, while the order numbers 61, 63, 65, and 67 are dominant. In the analysis section it was shown that these orders should be expected to be dominant with this type of vane spacing. The analysis also indicated that the vibration amplitudes as excited by the 63^{rd} and 65^{th} orders should be, respectively, 35 and 37 percent lower than the amplitude excited by the 64^{th} order of the standard equally spaced configuration. A comparison of the stresses at the strain-gage locations is shown in table V for both vane configurations together with the frequency of the vibrational modes. The vibration reductions in the 63^{rd} order were 22, 44, and 46 percent for the three blades, respectively, as compared with the predicted value of 35 percent. In the 65^{th} order, the vibration reductions were 26, 52, and 46 percent as compared with the predicted value of 37 percent. Although the procedure of averaging the results of the three blades may be open to question because of the limited sampling, the average measured reduction in the 63^{rd} order was 37 percent compared with the predicted 35 percent, and the average measured reduction in the 65^{th} order was 41 percent compared with the predicted 37 percent. As a result of this favorable agreement for the two particular vane configurations used in the engine, it is believed that the other vane configurations analyzed would also show correspondingly good agreement. This would indicate that the excitation level originating in the stator vanes could be reduced by approximately 68 percent relative to the equally spaced vane assembly through the use of a special vane arrangement. A spacing change of only 8.5 percent of the standard spacing is required.

SUMMARY OF RESULTS

Theoretical analysis of several modified stator vane configurations through the use of Fourier series indicates that the vibration excitation level experienced by the rotor blades can be considerably reduced relative to the standard equally spaced vane assembly. The modified configurations result in a forcing function having an increased harmonic content but with the amplitude of each harmonic well below the excitation amplitude for the standard assembly.

Of the configurations studied, two in particular showed very good reductions in over-all excitation level. One of these had three segments with spacings in the individual segments of $1 + 0$, $1 + \frac{S}{100}$, and $1 - \frac{S}{100}$, respectively, where S is the circumferential spacing change between vanes. At a value of S equal to 6.5 percent, the excitation level was down to 39 percent of the original, and at a value of S equal to 10 percent, the excitation level was decreased to 32 percent of the original. The other configuration had three segments with spacings in the segments of $1 + 0$, $1 + \frac{S}{100}$, and $1 - \frac{S}{100}$ in addition to 120° phasing between segments. At a value of S equal to 8.5 percent, the excitation level was down to 32 percent of the original.

Confirmation of the theoretical analysis was obtained by using an air-interrupter disk and measuring the amplitudes of vibration of three turbine blades in a turbojet engine when using two different vane configurations. One was an equally spaced vane assembly and the other was a two-segment, 180° phased arrangement with standard spacing within each segment. The analysis indicated that the phased assembly should have dominant 63^{rd} and 65^{th} orders with excitation level reductions of 35 and 37 percent, respectively, relative to the excitation level of the equally spaced assembly. For the three instrumented turbine blades, the average measured amplitude reductions in the 63^{rd} and 65^{th} orders were 37 and 41 percent, respectively, which compares favorably with the theoretical values.

On the basis of the good agreement obtained between the analysis and the engine results for the two different vane configurations used, it is believed that the other vane configurations analyzed would also show correspondingly good agreement. This would indicate that the excitation level originating in the stator vanes could be reduced by approximately 68 percent relative to the equally spaced vane assembly through the use of a special vane arrangement. A spacing change of only 8.5 percent of the standard spacing is required.

Lewis Flight Propulsion Laboratory
National Advisory Committee for Aeronautics
Cleveland, Ohio, July 14, 1958

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TABLE I. - VANE CONFIGURATIONS STUDIED

Modification type	Configuration	Number of segments	Number of blades per segment	Phasing between segments	Vane spacing within segments (in analytical analysis, S was varied from 0 to 15 percent)	Random vane spacing	Maximum limit for random movement, percent of basic spacing	Results presented in -
Standard	1	1	48	None	All vanes equally spaced, termed standard spacing	No	---	Text
I	2	2	24	0°-180°	Standard in both segments	No	---	Fig. 3
	3	3	16	0°-360°	Standard in all segments	No	---	Fig. 4(b)
II	4	2	24	None	$1 + \frac{S}{100}$, $1 - \frac{S}{100}$	No	---	Fig. 5
	5	3	16	None	$1 + 0$, $1 - \frac{S}{100}$, $1 + \frac{S}{100}$	No	---	Fig. 6
	6	4	12	None	$1 + \frac{S}{100}$, $1 - \frac{S}{100}$, $1 + \frac{S}{100}$, $1 - \frac{S}{100}$	No	---	Text
III	7	2	24	180°	$1 + \frac{S}{100}$, $1 - \frac{S}{100}$	No	---	Fig. 7
	8	3	16	120°	$1 + 0$, $1 + \frac{S}{100}$, $1 - \frac{S}{100}$	No	---	Table II
IV	9	1	48	None	Based on all vanes initially equally spaced	Yes	±5	Table II
	10	1	48	None	Based on all vanes initially equally spaced	Yes	±10	Table II
	11	3	16	None	Based on 1, 1.1, 0.9	Yes	±5	Table II
	12	2	24	112°	Based on standard spacing in both segments	Yes	±5	Table II

TABLE II. - HARMONIC COEFFICIENTS FOR CONFIGURATION INCORPORATING RANDOM SPACING

Order number, n	Standard equally spaced vane assembly as basic configuration									Three-segment vane assembly with spacings 1, 1.1, and 0.9 as basic configuration				Two-segment vane assembly with 112° phasing between segments as basic vane configuration				
	Basic vane configuration	±5-Percent random movement set number				±10-Percent random movement set number				Basic vane configuration	±5-Percent random movement set number			Basic vane configuration	±5-Percent random movement set number			
		1	2	3	4	1	2	3	4		1	2	3		1	2	3	4
45	0	0.015	0.043	0.069	0.035	0.109	0.037	0.164	0.174	0.272	0.260	0.276	0.238	0.170	0.135	0.199	0.175	0.136
46	0	.090	.048	.091	.137	.125	.079	.022	.175	.164	.157	.144	.177	.036	.190	.210	.261	.165
47	0	.199	.249	.390	.295	.463	.189	.196	.374	.326	.315	.310	.348	.517	.502	.467	.649	.504
48	1	.802	.899	.750	.792	.733	.775	.889	.713	.318	.353	.335	.312	.556	.425	.431	.105	.377
49	0	.198	.216	.376	.346	.317	.204	.209	.439	.332	.297	.371	.320	.510	.595	.492	.546	.403
50	0	.052	.087	.126	.095	.095	.051	.084	.035	.094	.150	.089	.103	.035	.094	.147	.248	.232
51	0	.079	.068	.028	.026	.182	.083	.135	.118	.146	.167	.119	.154	.164	.175	.142	.200	.130

TABLE III. - COMPUTED INFLUENCE COEFFICIENTS FOR A DISPLACEMENT OF 1 PERCENT OF THE VANES OF ONE SEGMENT OF A THREE-SEGMENT ASSEMBLY

Vane number	Influence coefficient $\Delta A_{n,k}$ for various order numbers						
	45	46	47	48	49	50	51
2	0.0009	-0.0009	-0.0025	0.0004	0.0010	0.0008	0.0002
3	.0005	-.0006	-.0025	.0004	.0010	.0004	-.0003
4	.0000	-.0004	-.0024	.0004	.0009	.0001	-.0008
5	-.0005	-.0001	-.0023	.0004	.0009	-.0003	-.0012
6	-.0009	.0003	-.0020	.0004	.0007	-.0005	-.0013
7	-.0012	.0006	-.0019	.0004	.0007	-.0009	-.0013
8	-.0013	.0009	-.0017	.0004	.0007	-.0010	-.0011
9	-.0012	.0011	-.0016	.0004	.0007	-.0012	-.0007
10	-.0009	.0012	-.0013	.0004	.0006	-.0013	-.0002
11	-.0005	.0013	-.0009	.0004	.0006	-.0012	.0002
12	.0000	.0013	-.0006	.0004	.0006	-.0011	.0007
13	.0005	.0011	-.0003	.0004	.0006	-.0009	.0011
14	.0009	.0009	-.0002	.0004	.0007	-.0007	.0012
15	.0012	.0006	.0003	.0004	.0009	-.0004	.0012

TABLE IV. - COMPARISON OF CALCULATED HARMONIC COEFFICIENTS WITH THOSE OBTAINED FROM AIR-INTERRUPTER RIG

[36 Blades in disk set to simulate three-segment vane assembly having vane spacings of 1, $1 + \frac{S}{100}$, and $1 - \frac{S}{100}$ with values of S equal to 4, 7, and 10 percent.]

Order number, n	Configuration having S = 4 percent		Configuration having S = 7 percent		Configuration having S = 10 percent	
	Calculated	Measured	Calculated	Measured	Calculated	Measured
31					0.12	0.17
32			0.11	0.15	.29	.30
33	0.12	0.13	.29	.33	.38	.36
34	.21	.23	.37	.45	.38	.34
35	.53	.49	.51	.55	.24	.20
36	.57	.54	.23	.23	.42	.34
37	.51	.45	.45	.50	.21	.16
38	.21	.17	.35	.34	.29	.20
39			.29	.30	.30	.18
40			.17	.18		

TABLE V. - REDUCTION IN TURBINE BLADE STRESSES THROUGH USE OF A MODIFIED VANE ASSEMBLY

Blade number	Frequency of vibrational mode, cps	Standard, equally spaced vane assembly having 64 vanes	Modified assembly having two segments with 180° phasing between segments			
			63^{rd} Order excitation		65^{th} Order excitation	
		Stress at gage location resulting from 64^{th} order excitation, psi	Stress at gage location, psi	Reduction in comparison with standard assembly, percent	Stress at gage location, psi	Reduction in comparison with standard assembly, percent
1	6603	4290	3350	22	3190	26
2	7275	3940	2210	44	1890	52
3	6600	4490	2430	46	2430	46
Average measured reduction				37		41
Calculated theoretical reduction				35		37

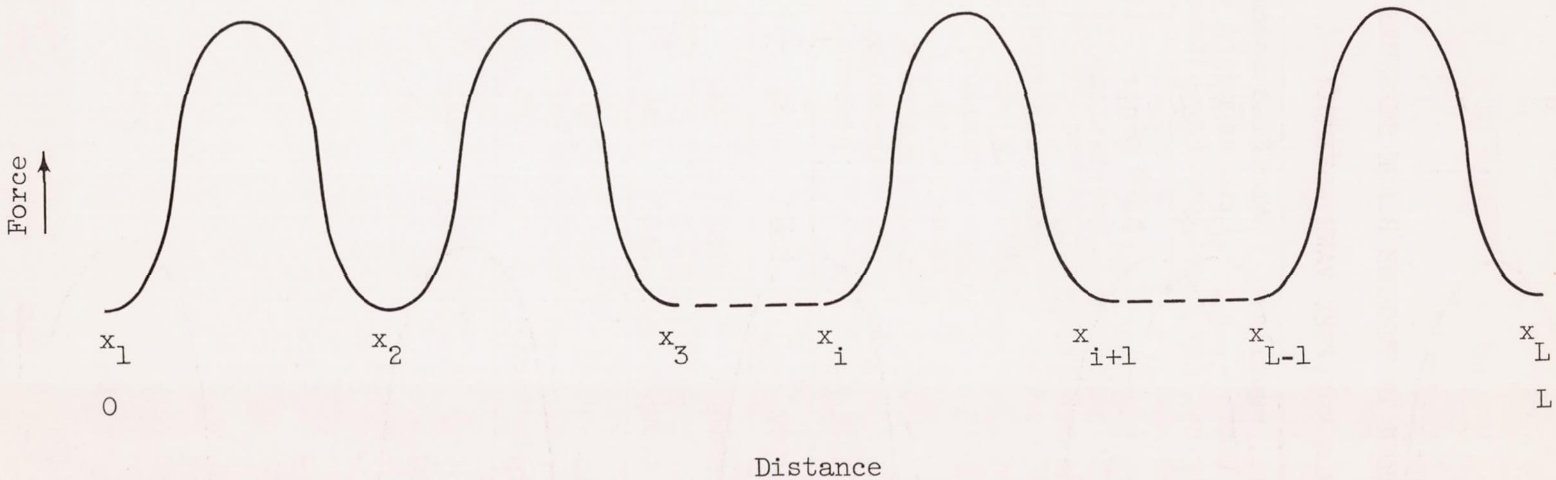


Figure 1. - Representation of assumed forcing function produced by turbine stator vanes.
$f_i(x) = 1 - \cos \frac{2\pi}{d_i} (x - x_i)$ for $x_i \leq x \leq x_{i+1}$.

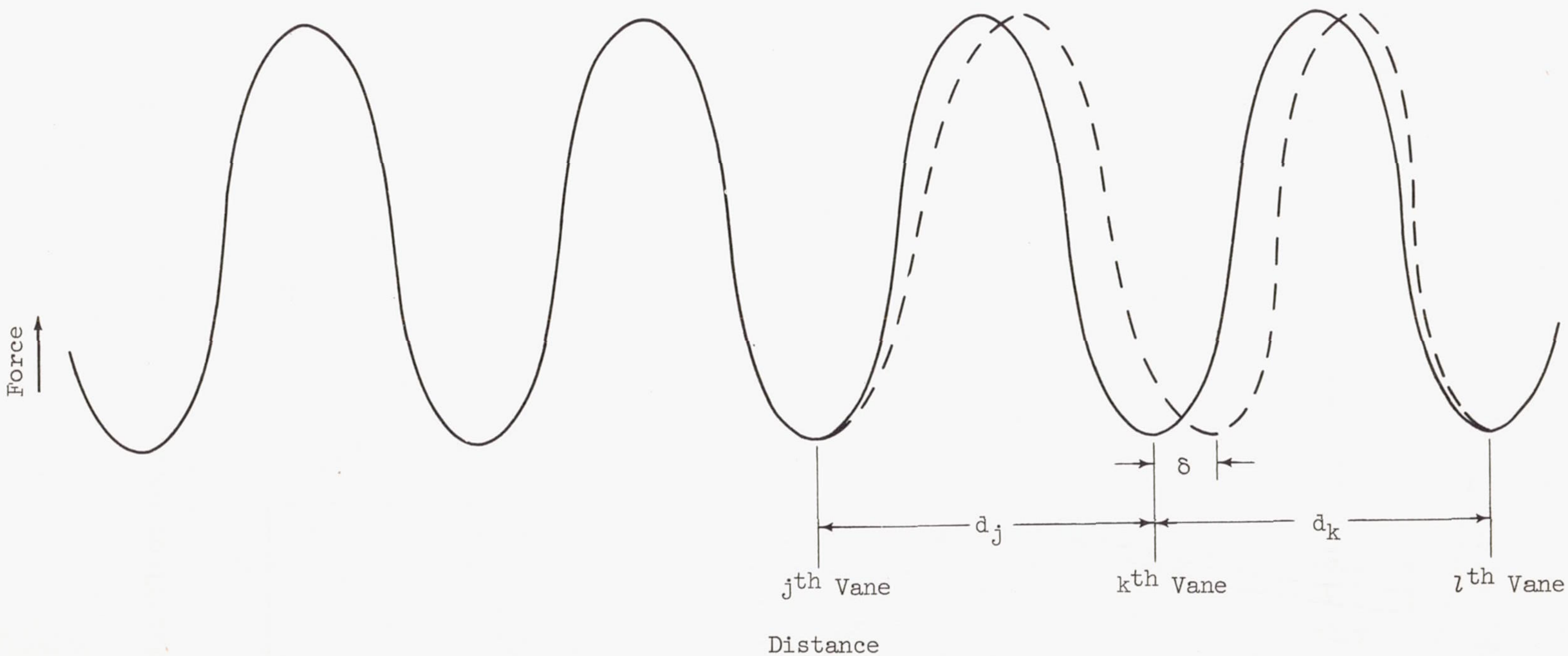


Figure 2. - Notation for vane displacement in influence coefficient method.

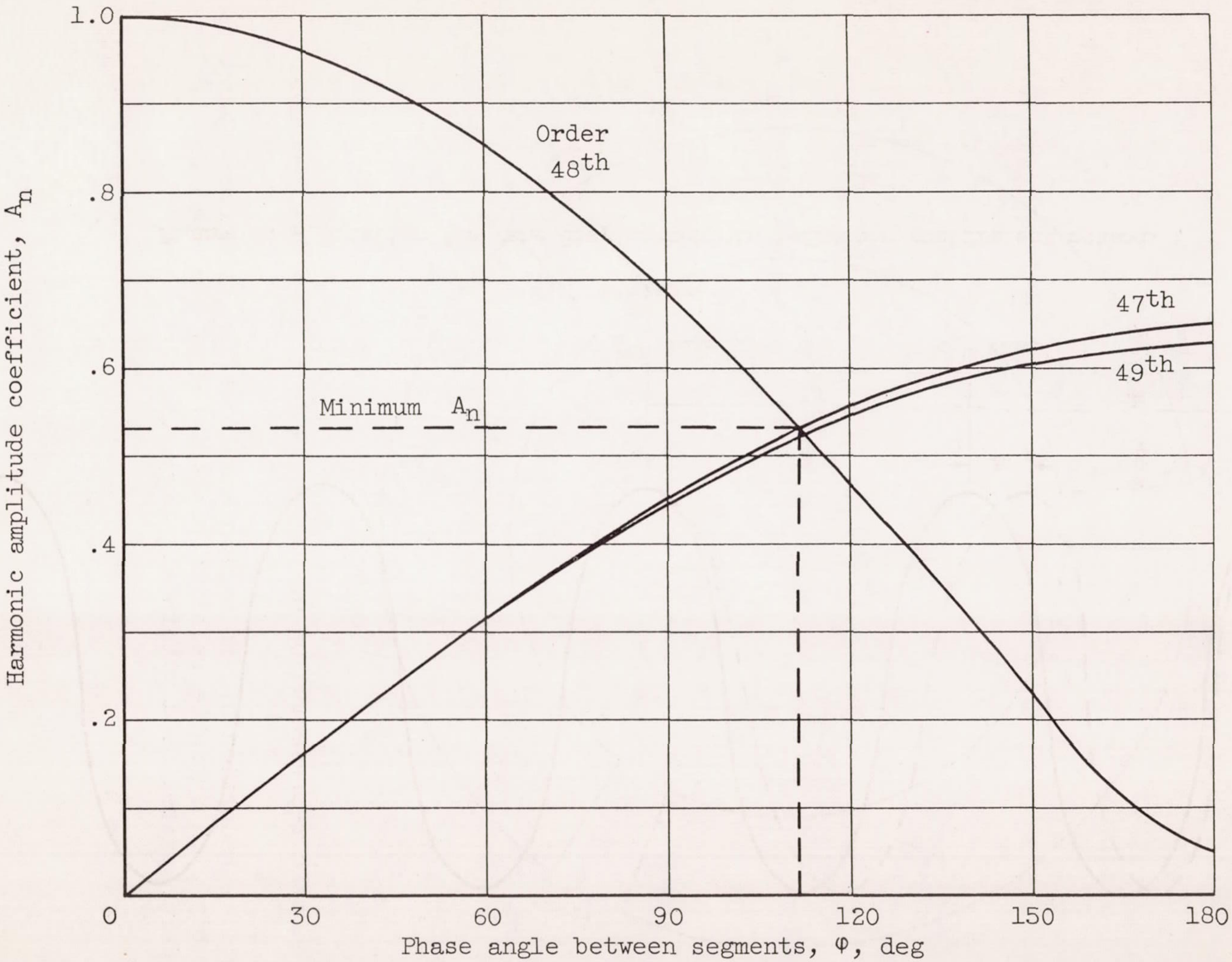
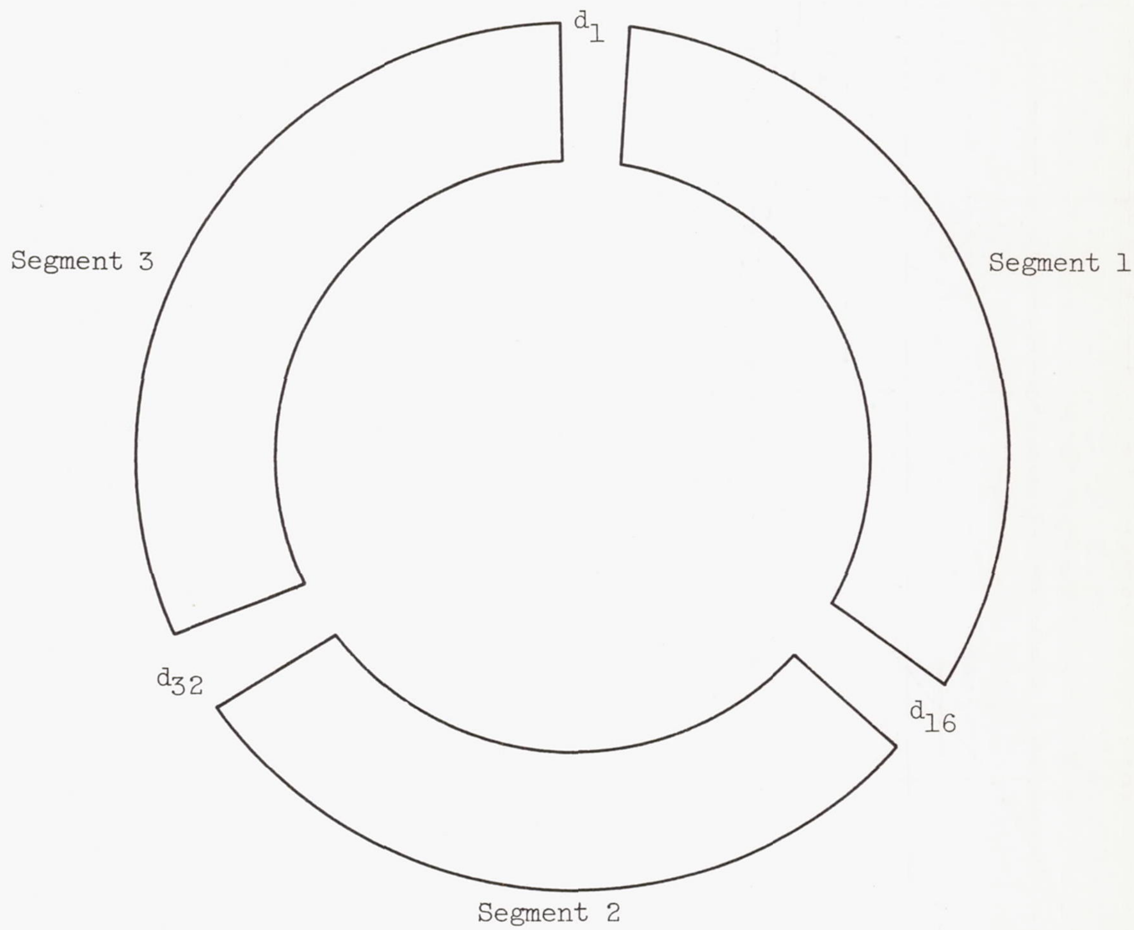


Figure 3. - Harmonic coefficients for two-segment assemblies with variation in phasing between segments. Standard spacing used in both segments.



(a) Schematic representation of phasing. $d_1 + d_{16} + d_{32}$ = constant.

Figure 4. - Reduction of excitation made possible by three-segment assembly with variable relative positioning of the segments. Standard spacing used within all segments.

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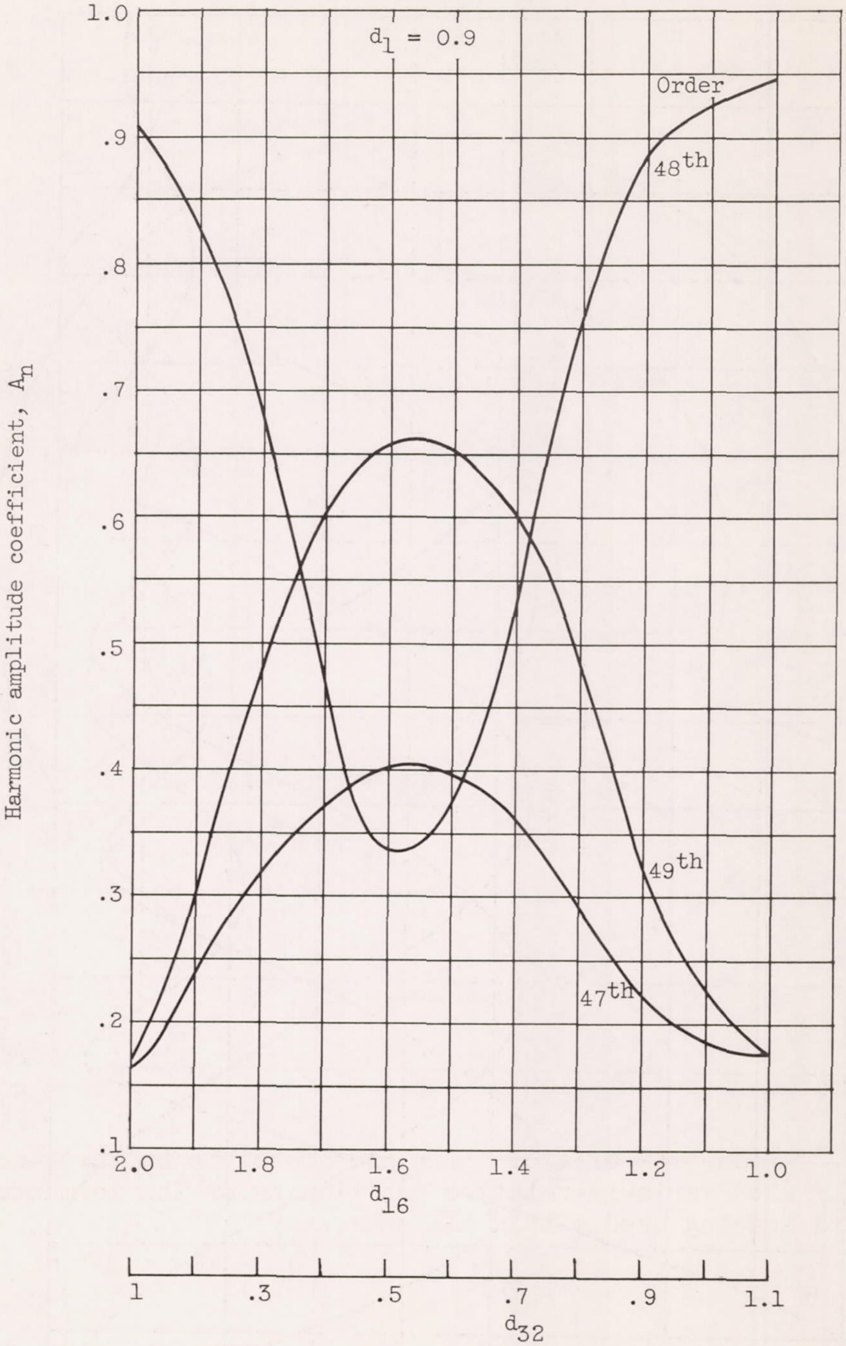

(b) Harmonic coefficients.

Figure 4. - Concluded. Reduction of excitation made possible by three-segment assembly with variable relative positioning of the segments. Standard spacing used within all segments.

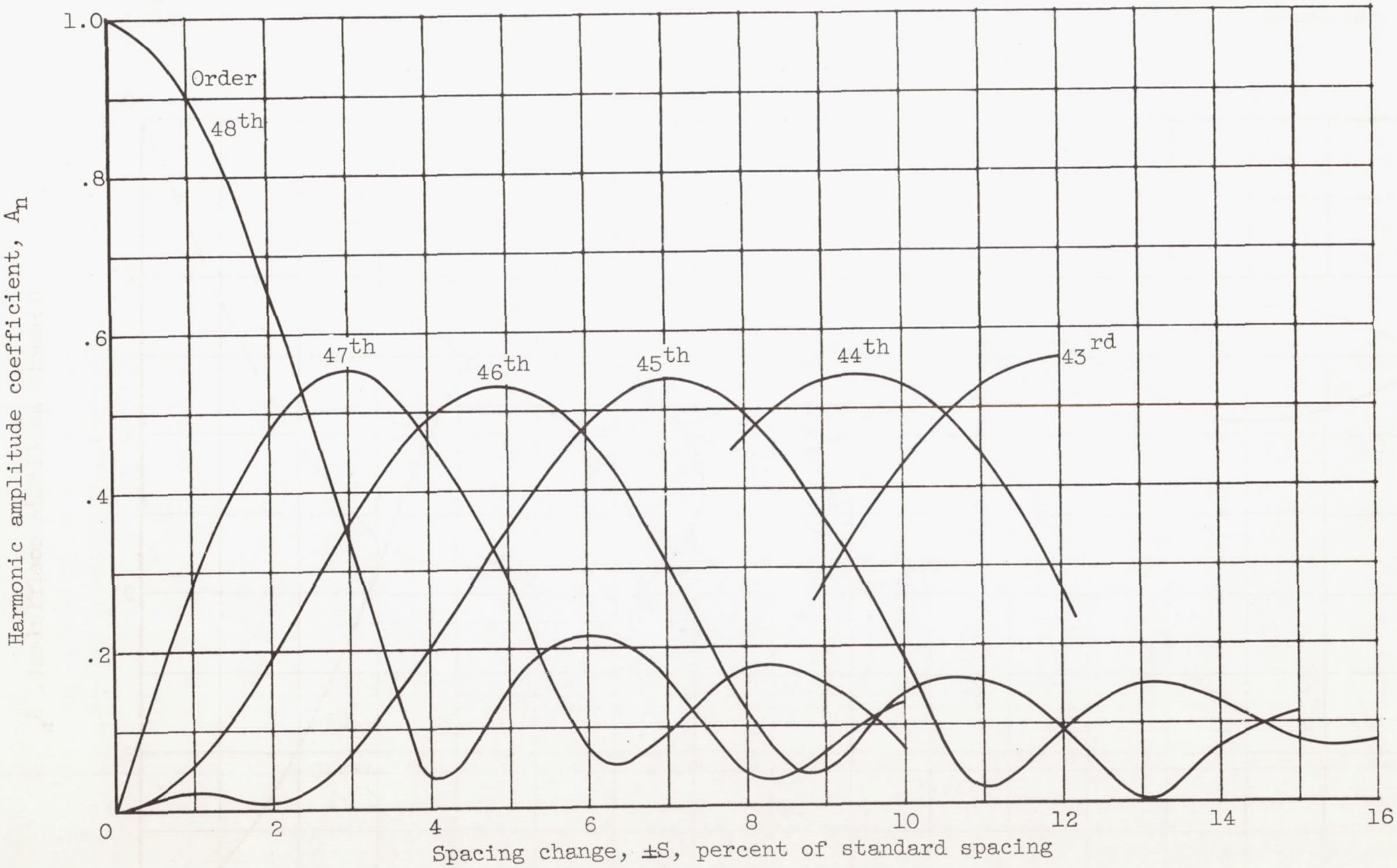

Figure 5. - Harmonic coefficients for two-segment assembly with variable spacing in segments. No phasing between segments.

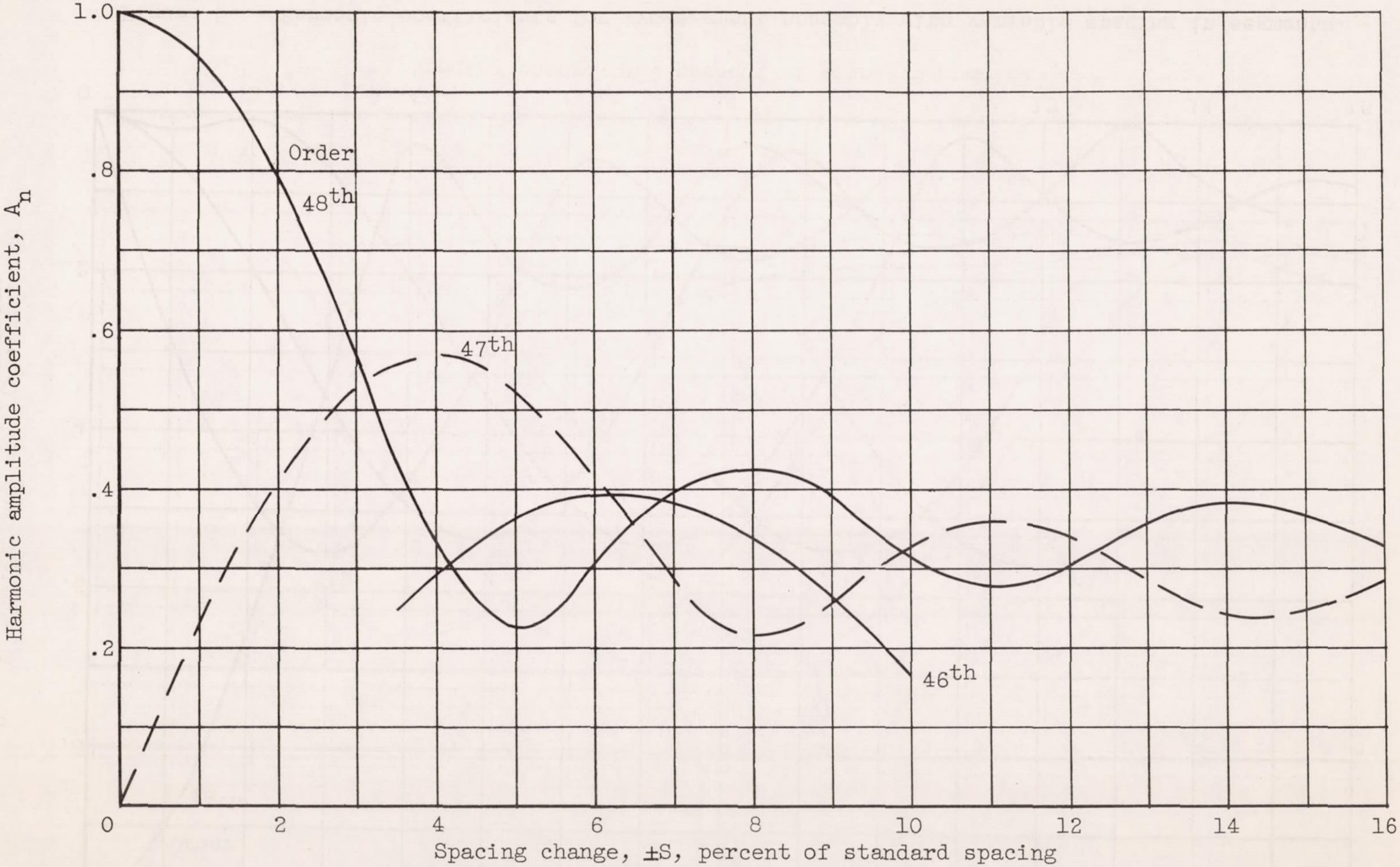

Figure 6. - Harmonic coefficients for three-segment assembly with variable spacing in segments. No phasing between segments.

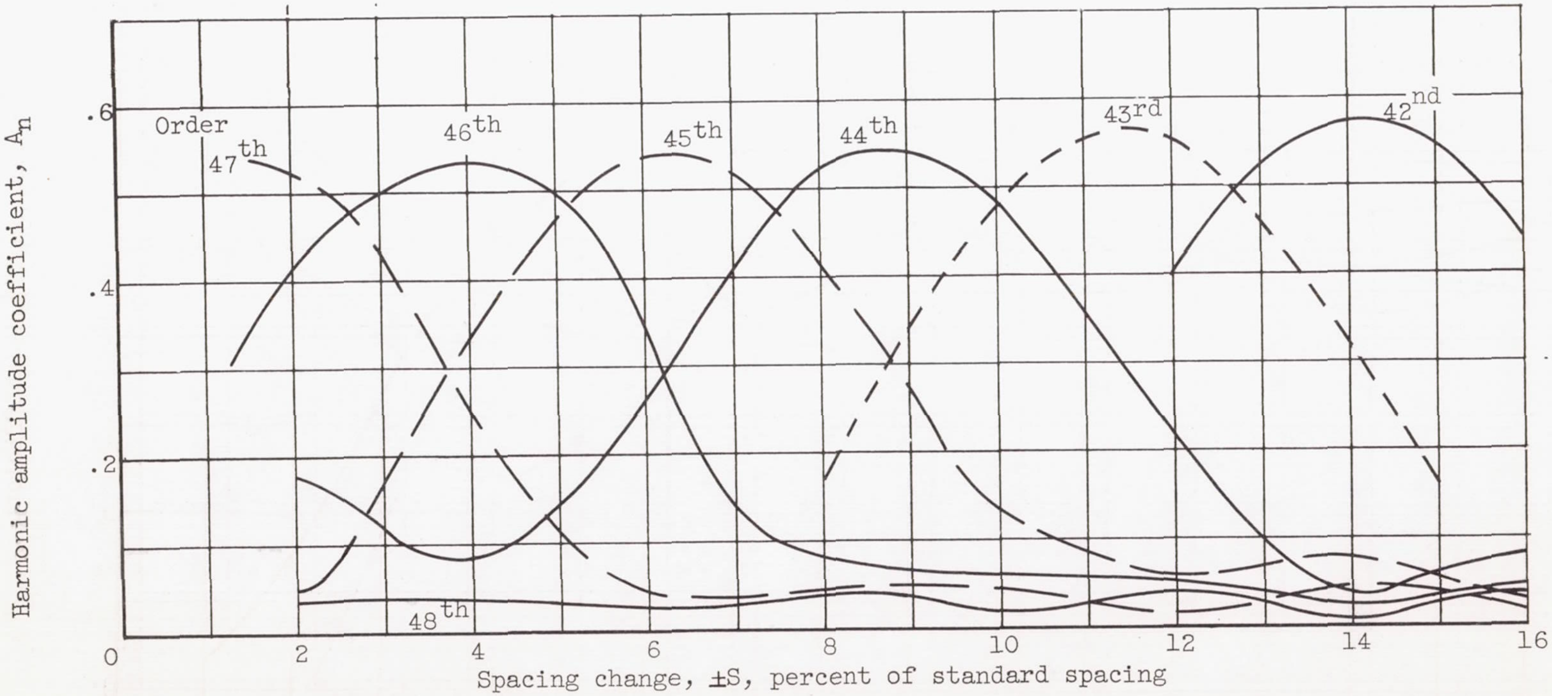

Figure 7. - Harmonic coefficient for two-segment assembly with variable spacing in segments. 180° phasing between segments.

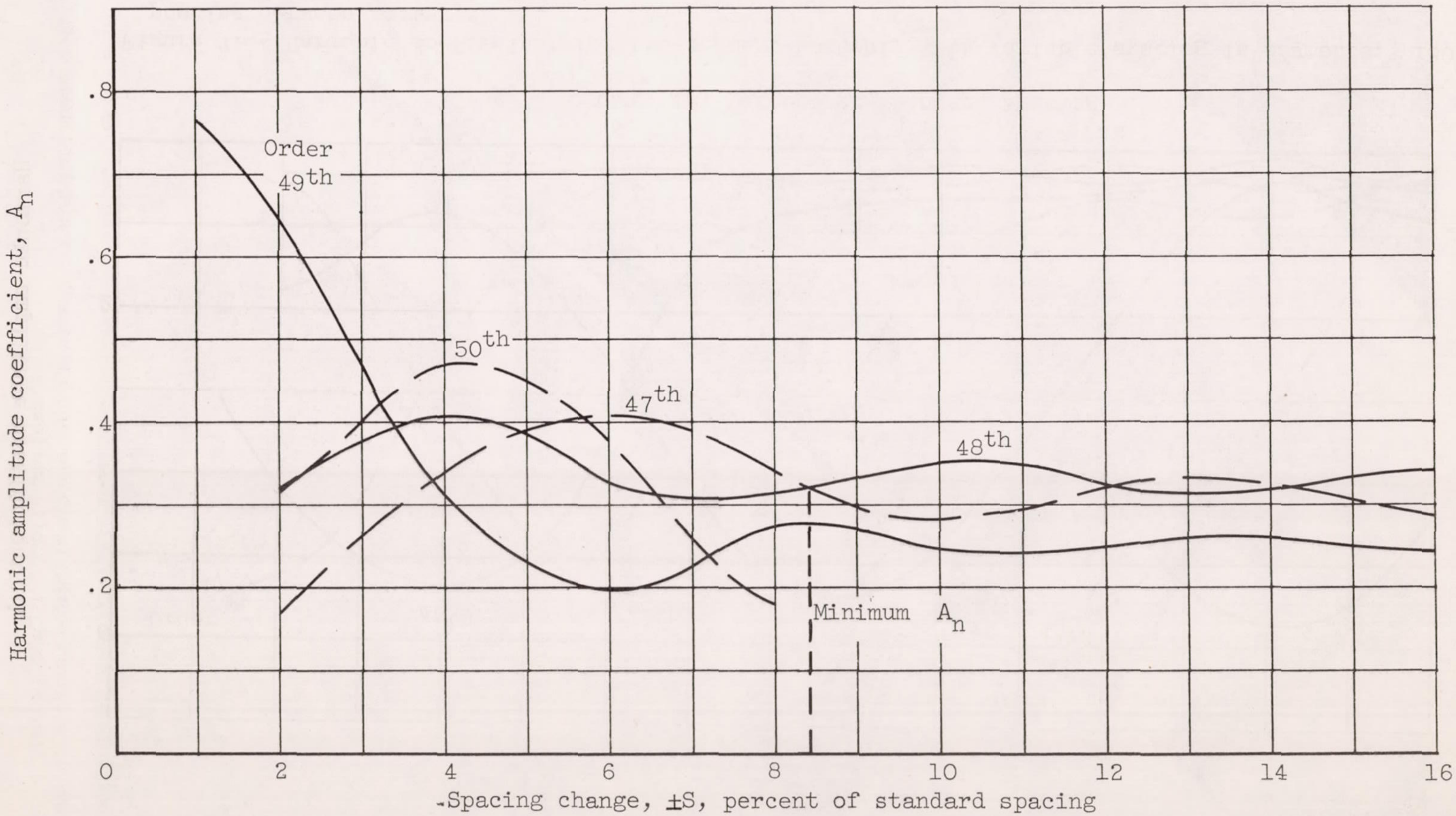

Figure 8. - Harmonic coefficients for three-segment assembly with variable spacing between segments. 120° phasing between segments.

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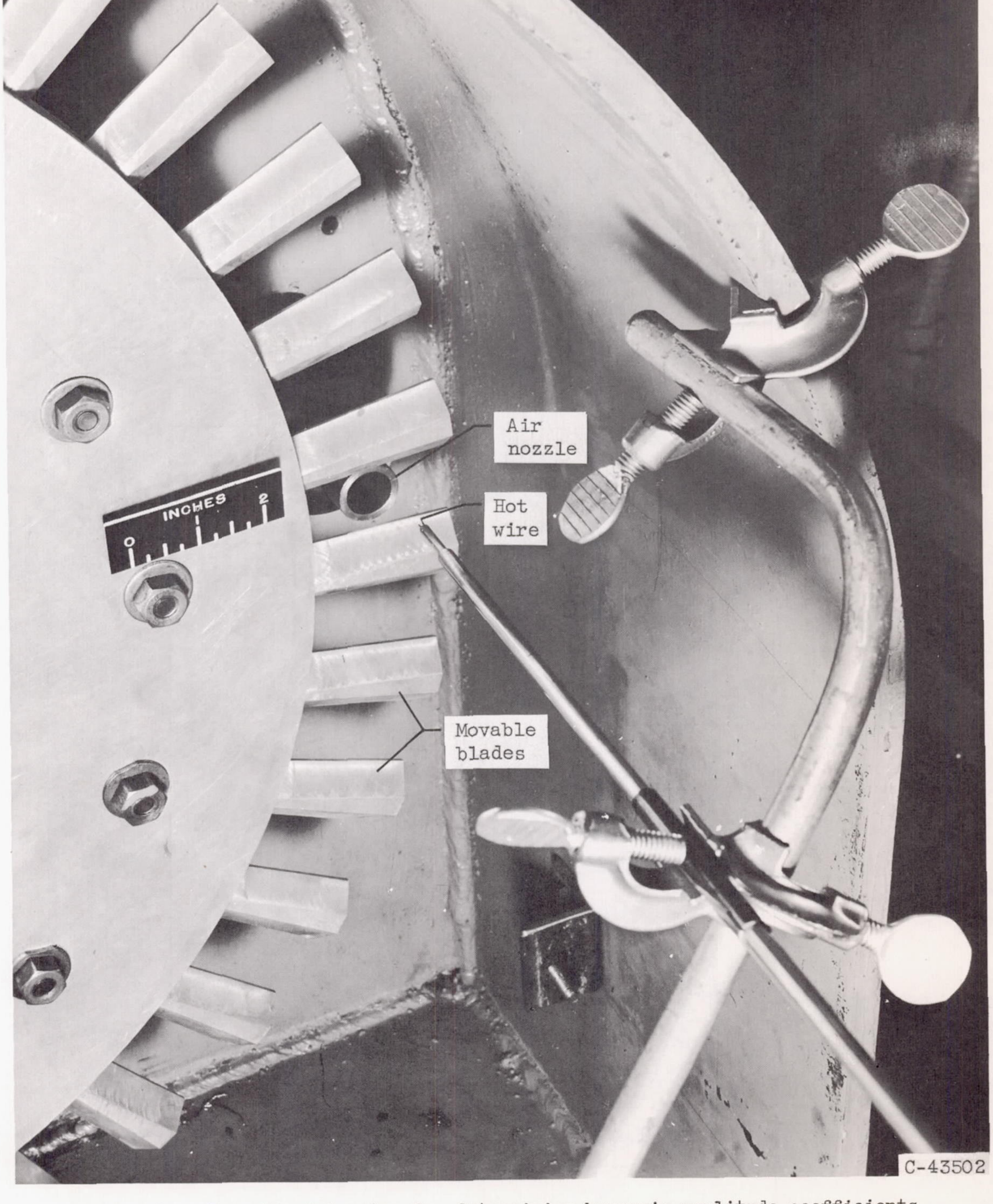

Figure 9. - Experimental setup for determining harmonic amplitude coefficients for different spacing configurations.

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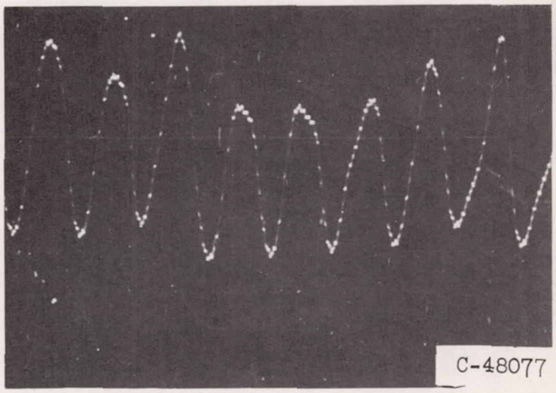


Figure 10. - Typical hot-wire signal from vane configuration simulator.

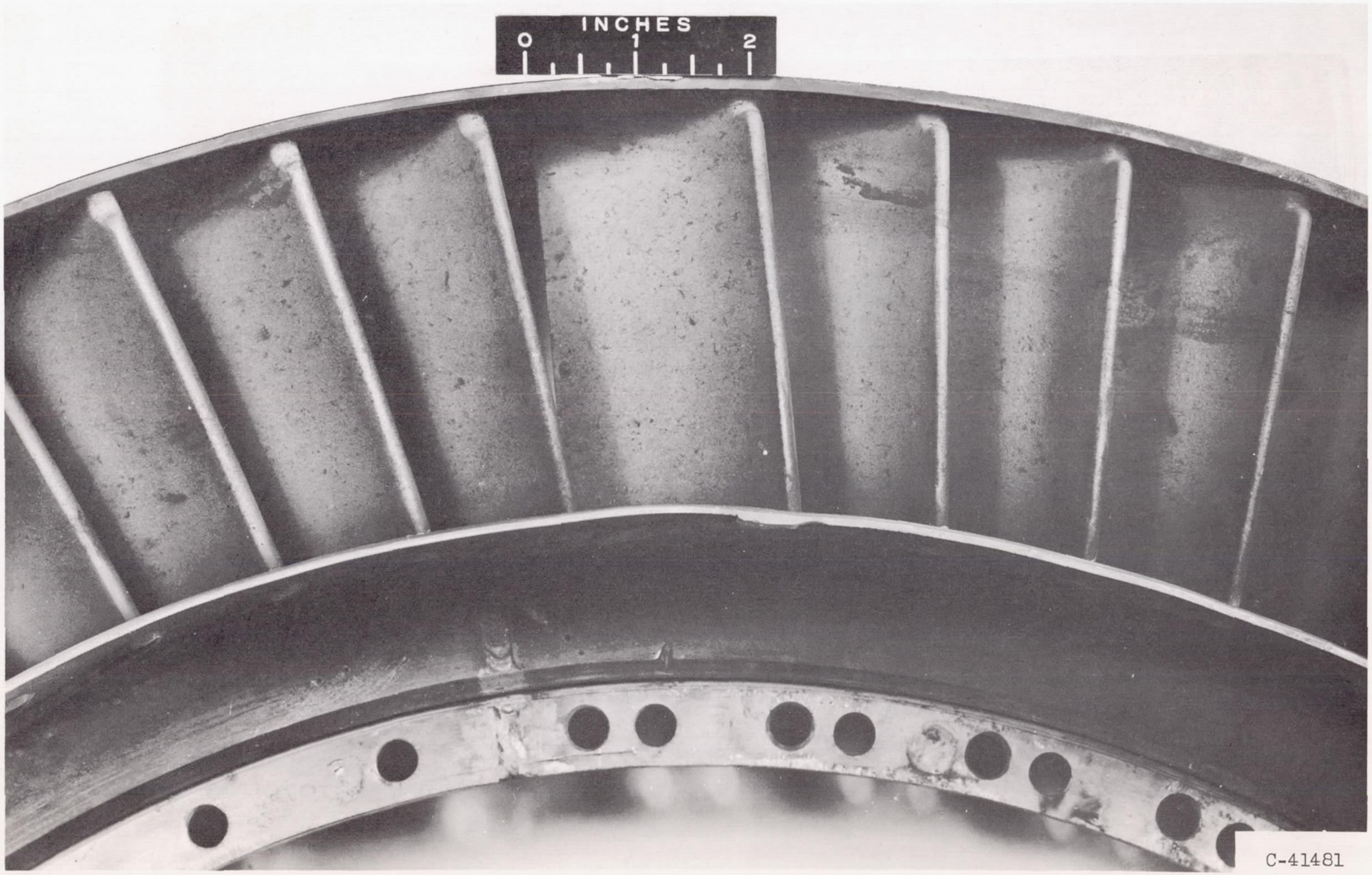


Figure 11. - Modified vane configuration used in turbojet tests showing transition region between segments.

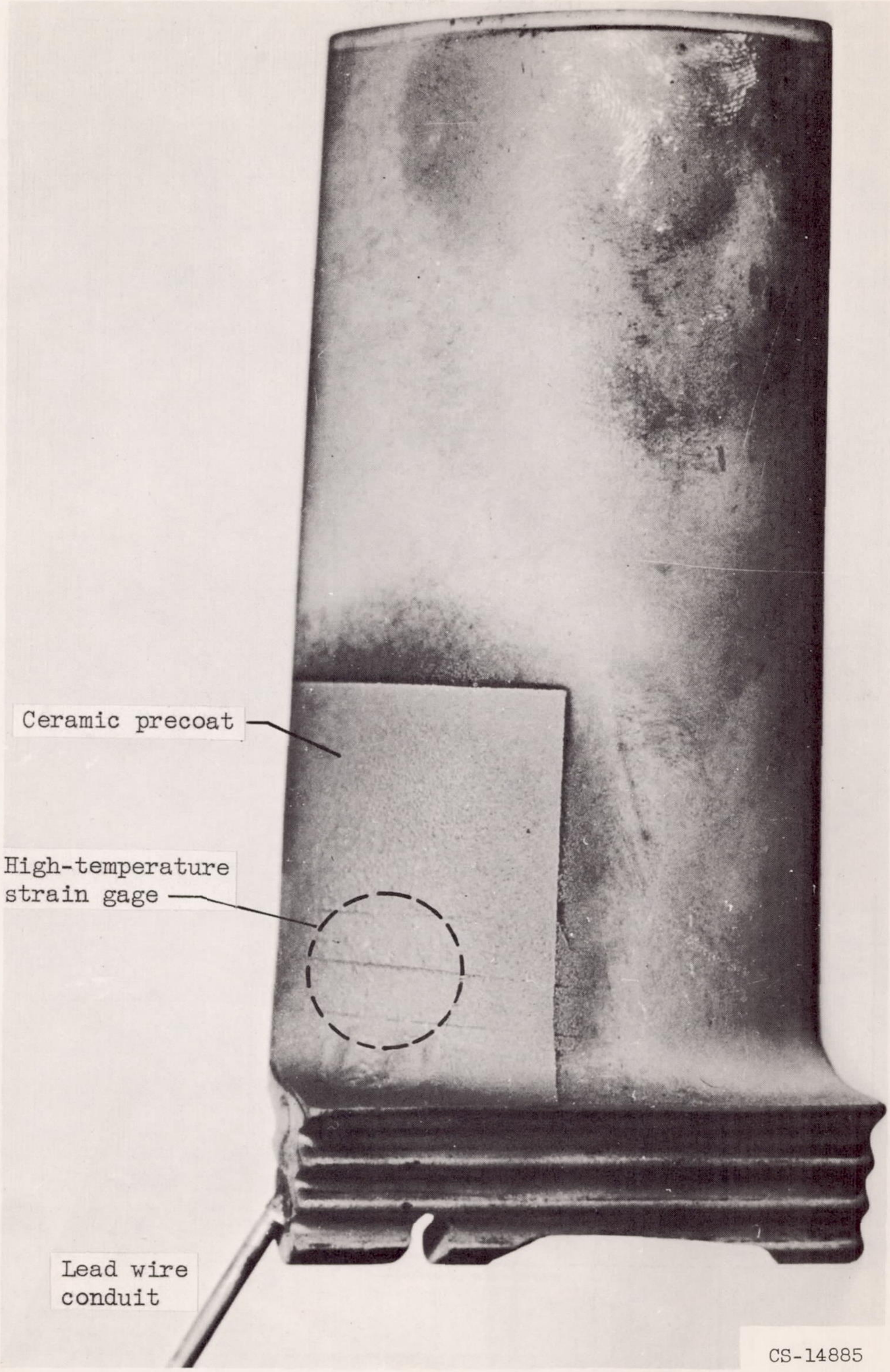


Figure 12. - Turbine blade showing installation of high-temperature strain gage.

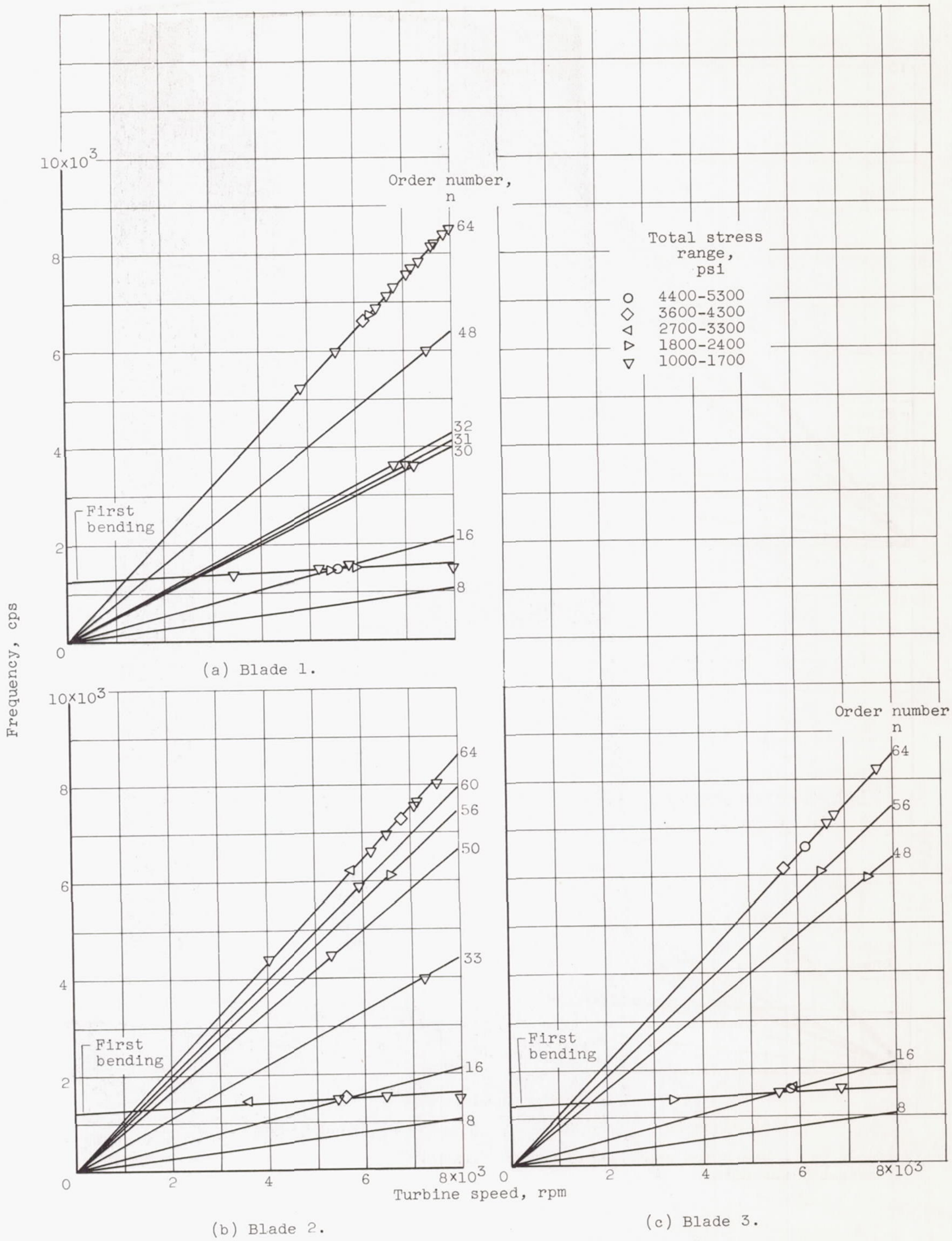
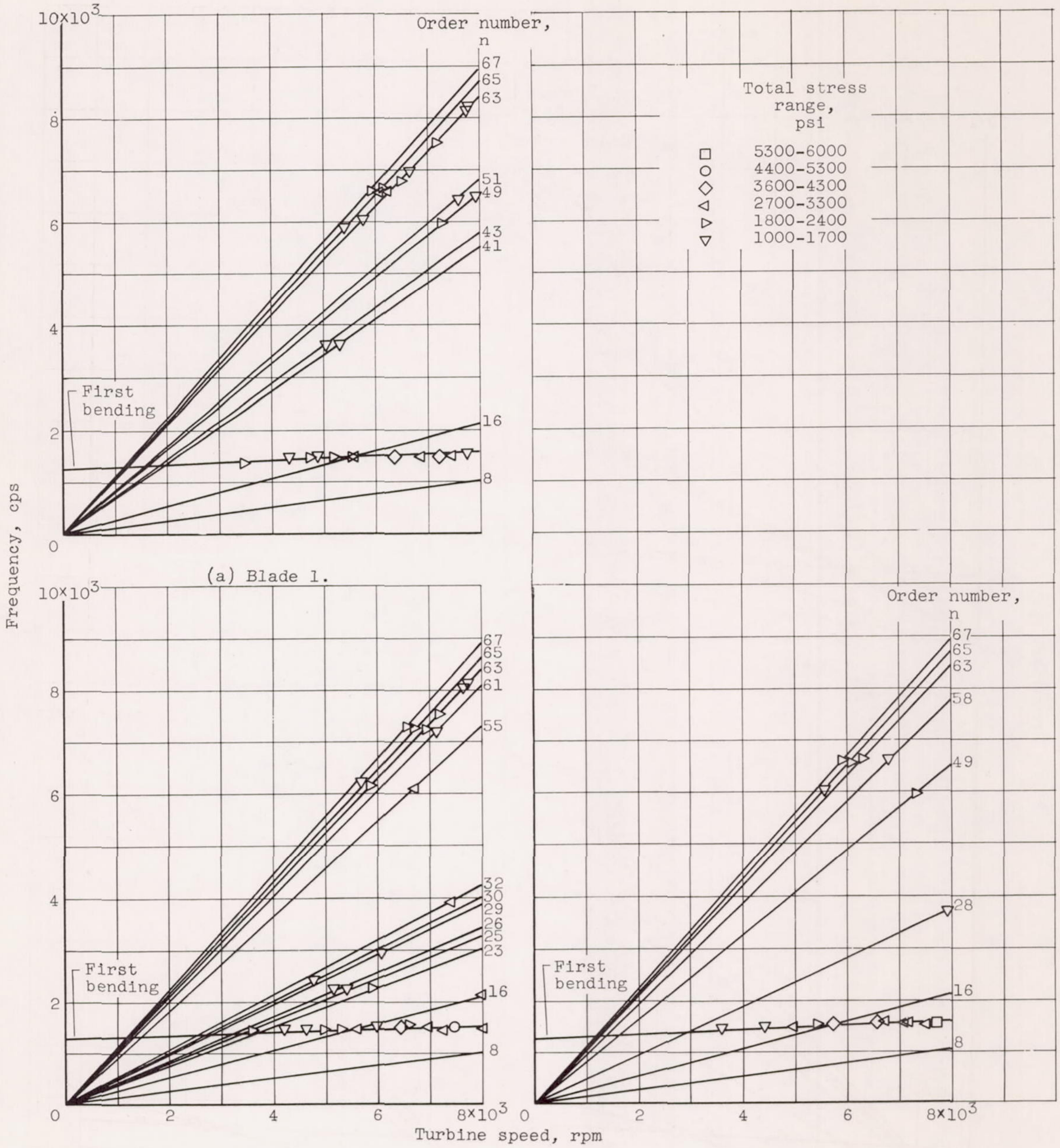


Figure 13. - Vibration spectra for turbine blades when standard equally spaced vane assembly was used.



(b) Blade 2.

(c) Blade 3.

Figure 14. - Vibration spectra for turbine blades when 180° phased, two-segment vane assembly was used.