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RESEARCH MEMORANDUM

COOLING OF GAS TURBINES

II - EFFECTIVENESS OF RIM COOLING OF BLADES

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SUMMARY

An analysis is presented of rim cooling of gas-turbine blades; that is, reducing the temperature at the base of the blade (wheel rim), which cools the blade by conduction alone. Formulas for temperature and stress distributions along the blade are derived and, by the use of experimental stress-rupture data for a typical blade alloy, a relation is established between blade life (time for rupture), operating speed, and amount of rim cooling for several gas temperatures. The effect of a blade parameter combining the effects of blade dimensions, blade thermal conductivity, and heat-transfer coefficient is determined. The effect of radiation on the results is approximated. The gas temperatures ranged from 1300° to 1900° F and the rim temperatures, from 0° to 1000° F below the gas temperature. This report is concerned only with blades of uniform cross section, but the conclusions drawn are generally applicable to most modern turbine blades. For a typical rim-cooled blade, gas-temperature increases are limited to about 200° F for 500° F of cooling of the blade base below gas temperature, and additional cooling brings progressively smaller increases. In order to obtain increases in gas temperature of the order of 400° F either very large increases in thermal conductivity or very large decreases in heat-transfer coefficient or blade length are necessary, The increases in gas temperature allowable with rim cooling are particularly small for turbines of large dimensions and high specific mass flows. For a given effective gas temperature, substantial increases in blade life, however, are possible with relatively small amounts of rim cooling.

INTRODUCTION

An investigstion of the cooling of gas turbines is being conducted by the NACA that includes direct blade cooling by the passage of liquid or air through hollow blades and indirect blade

cooling by removal of heat from the blade root or tip. In part I (reference 1) of a series of reports written on this investigation, calculations of the radial temperature distribution through the rotor and blades of a gas turbine are made in which the blades are cooled indirectly by air blown over finned surfaces on the rotor and the blade tips. The calculations were made for assumed cooling-air and gas temperatures by means of an analysis developed in that report. It was found as a result of the calculations that the base of the blade was cooled about 500° F below the gas temperature.

The benefits of blade cooling may be measured by increases in allowable gas temperature, blade speed, or blade life, where life is the time to rupture due to centrifugal stress. Speed is ordinarily limited for aerodynamic reasons by the design Mach number and the purpose for which the turbine is used usually determines the desired life.

The purpose of the present report is to continue the analysis of indirect cooling of turbine blades, the difference between this report and part I being that the blade tips are assumed insulated in the present case and an analysis is developed for given rim temperatures rather than cooling-air temperatures. The benefits of blade cooling are determined in terms of turbine operating conditions by combining the temperature distribution in the blades with the mechanical limitations to blade operation. The report is limited to rim-cooled blades; that is, blades cooled only at the base. In this report the effectiveness of rim cooling is indicated by the increases in allowable gas temperature that can be obtained without changes in blade Mach number or blade life. The increase in blade life that could be obtained by a sacrifice of some of the increase in allowable gas temperature is indicated, and the effects on rim-cooling effectiveness of blade dimensions, blade thermal conductivity, and heat-transfer coefficient from the hot gases to the blade are evaluated. The elongation of the blade due to creep and its importance as a limitation on blade operation are determined. Calculations are made for gas temperatures of 1300° to 1900° F and for rim temperatures up to 1000° F below gas temperature.

SYMBOLS

a	velocity of sound at T_e, (ft/sec)
A	blade cross-sectional area, (sq ft)
B	number of blades

$\bar{D}$ mean diameter of turbine blades $(2r_t - L)$, (ft)

e percentage elongation at any blade point

e_m percentage elongation of blade metal

e_t total percentage elongation of blade

F fraction of blade surface area exposed to radiation from one side

g acceleration of gravity, 32.2 (ft/sec^2)

h heat-transfer coefficient based on T_e, Btu/(hr)(sq ft)($^{\circ}$F)

k blade thermal conductivity, Btu/(hr)(ft)($^{\circ}$F)

L blade length, (ft)

M tip Mach number index, V/a

M_{max} limiting tip Mach number index, V_{max}/a

p blade perimeter, (ft)

r radius of turbine at any point on blade, (ft)

r_t tip radius of turbine, (ft)

R gas constant, 53.5 (ft-lb)/(lb)($^{\circ}$R)

s actual stress at any point on blade, (lb/sq in.)

s_{max} stress for rupture for given life and temperature, (lb/sq in.)

T temperature of blade at any point, ($^{\circ}$R)

T_a allowable temperature at any blade point, ($^{\circ}$R)

T_e effective gas temperature for use in heat-transfer equations, ($^{\circ}$R)

ΔT_e permissible increase in effective gas temperature due to blade cooling for constant M_{max}, ($^{\circ}$F)

T_g total temperature of inlet gas, ($^{\circ}$R)

T_R average blade temperature used in radiation approximation, (°R)

T_0 temperature at blade root (wheel rim), (°R)

T_1 temperature of radiating surface on inlet side of blades, inlet nozzles, (°R)

T_2 temperature of radiating surface on exhaust side of blades, (°R)

V blade tip speed, (ft/sec)

V_m blade velocity at mean diameter, (ft/sec)

V_{max} limiting blade tip speed, (ft/sec)

V_w whirl component of inlet gas, (ft/sec)

x distance from blade root to any point on blade, (ft)

x_c critical blade point, (ft)

Y, Z parameters defined by equation (17) in appendix

α parameter equal to $\sqrt{hp/kA}$, $(ft)^{-1}$

γ ratio of specific heats

ϵ emissivity of metal surfaces

ρ density of blade metal, (slugs/cu ft)

σ Stefan-Boltzmann constant, 0.174×10^{-8} Btu/(hr)(sq ft)$(°R)^4$

ω angular velocity of turbine wheel, (radians/sec)

THEORETICAL ANALYSIS

Although a relation between the effective gas temperature T_e and the amount of cooling $T_e - T_0$ for a constant limiting Mach number index M_{max} is desired, the method of analysis makes it necessary to calculate first the values of the tip speed at rupture and the corresponding values of M_{max} for assumed values of T_e

and $T_e - T_0$. The method consists in combining calculated temperature and stress distributions in the blade with experimental stress-rupture data. The solution depends upon the choice of blade metal, the expected blade life, and the values of two dimensionless parameters αL and L/r_t. Means of accounting for radiation and of calculating the blade elongation due to creep are also presented.

Assumptions. - In the derivation of expressions for the temperature and stress distributions, the following simplifying assumptions are made:

1. The blade is of uniform cross-sectional area and perimeter.

2. The heat-transfer coefficient and the effective gas temperature are constant over the blade height. An average value for the thermal conductivity can be used over the blade height because the variation of conductivity with temperature is small for most turbine metals.

3. The blade tip is perfectly insulated. This assumption has little effect on the calculated temperature distribution except very close to the tip, because the temperature of the blade tip in the usual case differs only a small amount from the gas temperature and the tip area is usually small compared with the surface area of the blade. If cooling were applied at the tip, as in reference 1, the temperature distribution would be considerably affected, but the temperature at the critical point of the blade, which is nearly always within the inner half of the blade length, would remain essentially the same.

4. The temperature gradients in any cross section of the blade perpendicular to the radius of the turbine are negligible.

5. Bending stresses in the blade are negligible with the result that the calculated blade stress is uniform over any cross-sectional area.

Temperature distribution. - The blade temperature at any point is found from a heat balance of the blade. (See appendix.) If radiation is neglected,

$$T = T_e - \frac{\cosh\left[\alpha L\left(1 - \frac{x}{L}\right)\right]}{\cosh \alpha L}(T_e - T_0) \tag{1}$$

where

$$\alpha = \sqrt{hp/kA}$$

The inclusion of radiation, by means of an approximation suggested in a publication of the General Electric Company gives

$$T = \frac{Z}{Y} - \frac{\cosh\left[\sqrt{Y}\,L\left(1 - \frac{x}{L}\right)\right]}{\cosh\sqrt{Y}\,L}\left(\frac{Z}{Y} - T_0\right) \tag{2a}$$

where

$$\sqrt{Y}\,L = \alpha L\,\sqrt{1 + 8\sigma\epsilon F\,\frac{1}{h}\,T_R^{\,3}} \tag{2b}$$

and

$$\frac{Z}{Y} = \frac{T_e + \sigma\epsilon F\left(\frac{1}{h}\right)\left(T_1^{\,4} + T_2^{\,4} + 6T_R^{\,4}\right)}{1 + 8\sigma\epsilon F\left(\frac{1}{h}\right)T_R^{\,3}} \tag{2c}$$

For given values of gas temperature and rim temperature, the temperature distribution without radiation (equation (1)) as a function of x/L depends only on the dimensionless parameter αL, which is fixed by the blade dimensions, the blade conductivity, and the heat-transfer coefficient. This parameter may be considered as a measure of the ratio of the average temperature drop over the blade length to the average temperature drop across the film between the hot gases and the blade. Accordingly, as the value of αL is decreased, an increased percentage of the temperature drop between the gas and the blade root occurs through the film and all blade temperatures are reduced. The temperature distribution including radiation (equation (2a)) is identical to equation (1) with $\sqrt{Y}\,L$ substituted for αL and Z/Y for T_e.

Stress distribution. - The actual stress in the blade at any point x is found by integrating the centrifugal load over the portion of the blade from point x to the tip

$$s = \frac{\rho\omega^2\left(r_t^{\,2} - r^2\right)}{288} = \frac{\rho V^2}{288}\left\{1 - \left[1 - \frac{L}{r_t}\left(1 - \frac{x}{L}\right)\right]^2\right\} \tag{3}$$

For given values of blade metal density and of the ratio of blade height to turbine radius at the blade tip, the stress distribution as a function of x/L depends only on the blade tip speed.

Limiting tip speed. - Experimental stress-rupture data for blade metals give the maximum stress the metal can withstand for a given temperature and life. If the blade life is specified, the maximum stress is a function of only the temperature

$$s_{max} = s_{max}\ (T) \tag{4}$$

Equations (3) and (4) may be combined to find the allowable temperature at any blade point and for any blade speed for a given blade metal and constant values of blade life, gas temperature T_e, amount of rim cooling $(T_e - T_0)$, and the parameters αL and L/r_t:

$$T_a = T_a\left(\frac{x}{L},\ V\right) \tag{5}$$

A family of curves of T_a as a function of x/L with V as a parameter may be drawn.

Failure occurs when the actual blade temperature at one blade point is greater than the allowable temperature. The limiting tip speed is that for which the allowable temperature is just equal to the actual temperature at one point on the blade, which is called the critical blade point because the blade would fracture at this point if the tip speed were increased. Mathematically, the limiting tip speed and the critical blade point are defined by two simultaneous equations

$$T\left(\frac{x_c}{L}\right) = T_a\left(\frac{x_c}{L},\ V_{max}\right) \tag{6a}$$

$$\frac{d\left[T\left(\frac{x_c}{L}\right)\right]}{d\left(\frac{x}{L}\right)} = \frac{\partial\left[T_a\left(\frac{x_c}{L},\ V_{max}\right)\right]}{\partial\left(\frac{x}{L}\right)} \tag{6b}$$

where $T(x/L)$ is the temperature distribution given by equation (1) or (2a). If the solution gives a critical point outside the blade length, equation (6b) is invalid and the critical point is at the root; in this case V_{max} is found from equation (6a) with $x_c/L = 0$. Because of the empirical origin of the function T_a, these equations are conveniently solved by graphical methods, as illustrated in figure 1.

The tip Mach number index is defined as the tip speed of the blade divided by the velocity of sound in the gas and is a measure of the actual Mach number of the gas flowing past the blade for a given velocity diagram. If the velocity of sound is calculated from the effective gas temperature, the limiting tip Mach number index is expressed

$$M_{max} = \frac{V_{max}}{a} = \frac{V_{max}}{\sqrt{\gamma g R T_e}} \tag{7}$$

Blade creep. - Experimental data on the creep properties of blade metals give the percentage elongation for a given life as a function of metal temperature and stress

$$e_m = e_m\ (T,\ s) \tag{8}$$

When equation (8) is combined with the actual stress-distribution equation (3) and the temperature-distribution equation (1) or (2), the actual percentage elongation of the blade at any point and for any tip speed may be found

$$e = e\left(\frac{x}{L},\ V\right) \tag{9}$$

The total percentage elongation of the blade is found by integration over the blade height for any blade speed,

$$e_t = \int_0^1 e\left(\frac{x}{L},\ V\right) d\left(\frac{x}{L}\right) \tag{10}$$

This equation is useful only for values of $V \leqq V_{max}$ because, for greater values, the elongation of the blade becomes infinite, corresponding to blade fracture. The evaluation of equation (10) may be carried out graphically.

APPLICATION OF ANALYSIS

The foregoing analysis is general and applies to any blade of uniform cross section. A blade section was assumed and the corresponding values of the dimensionless parameter αL and L/r_t were used to obtain quantitative results. These values are referred to as the "basic" values. In order to show the effect of heat-transfer coefficient, blade conductivity, and blade dimensions on the results, calculations were made for values of αL equal to 1/3, 1/2, 1, 2, and 3 times the basic value. A single value of L/r_t was used throughout because the effectiveness of cooling depends very little on L/r_t for the usual range of values for constant αL. The quantitative results are also dependent on the assumed mechanical properties of the blade metal. Values of the effective gas temperature from 1300° to 1900° F and of amounts of rim cooling from 0° to 1000° F were assumed. The value of the limiting Mach number index was calculated for each pair of values of T_e and $T_e - T_0$ for a value of γ of 1.31.

Blade dimensions, conductivity, and heat-transfer coefficient. - The basic values of αL and L/r_t were determined from values of the blade dimensions, the blade-metal conductivity, and the heat-transfer coefficient representative of those found in modern high-temperature gas turbines:

Area of blade cross section A, sq ft (or 0.1047 sq in.)	0.000728
Perimeter of blade cross section p, ft (or 2.40 in.)	0.20
Blade length L, ft (or 1.75 in.)	0.146
Radius at tip r_t, ft (or 6.84 in.)	0.570
Thermal conductivity of the blade k, Btu/(hr)(ft)(°F)	12
Heat-transfer coefficient h, Btu/(hr)(sq ft)(°F)	40
Basic value of αL	4.42
Basic value of L/r_t	0.26

The value of h of 40 Btu/(hr)(sq ft)(°F) corresponds to relatively low mass flows; because of this relatively low value of h and the choice of a fairly short blade, the basic value of αL probably is as low as is obtainable at present.

Mechanical properties of blade metal. - The assumed mechanical properties of the blade metal are shown in figures 2 and 3. These properties are based on available data (references 2 and 3) for S497 alloy, which is a forged ferrous alloy, high in nickel, chromium, and cobalt content with a density of 0.31 pound per cubic inch, and are extrapolated to include a wide range of temperatures. The stress-rupture properties (fig. 2) are typical of the stronger alloys now available for turbine blades but the creep properties (fig. 3) are relatively poor. S497 was selected in order to accentuate any possible creep limitations in blade design.

Effective gas temperature. - The effective gas temperature T_e rather than the total temperature at the inlet to the turbine T_g is used in the analysis because it is the temperature that enters into the heat-transfer equation. The power and the efficiency of a compressor-turbine unit, however, depend on T_g, and the relation between these two temperatures should be known.

The effective temperature is approximately equal to the total temperature relative to the moving blades; the ratio of the total inlet temperature to the effective temperature calculated at the mean blade diameter is therefore

$$\frac{T_g}{T_e} = 1 + \frac{\gamma - 1}{2}\left(\frac{V_m}{a}\right)^2\left(2\frac{V_w}{V_m} - 1\right)$$

where

$$\frac{V_m}{a} = \frac{V}{a}\frac{D}{2r_t}$$

For a constant blade Mach number and a given velocity diagram, which determines V_w/V_m, the inlet total temperature is directly proportional to the effective temperature. For example, if the Mach number index is set at 0.5 (and $D/2r_t = 0.87$), $T_g/T_e = 1.041$ for $V_w/V_m = 1.2$ (typical of reaction blades) and $T_g/T_e = 1.088$ for $V_w/V_m = 2.0$ (typical of impulse blades).

Radiation. - The effect of radiation on the temperature distribution depends to a great extent upon the particular installation of the turbine, which determines the values of the temperatures T_1 and T_2 as well as the validity of the approximations used in treating radiation. (See appendix.) The results were calculated without radiation, but the manner in which the various radiation assumptions modified these results was investigated.

In the application of the radiation equation (2a), the following assumptions were made:

1. The average blade temperature T_R used in the approximation

$$T^4 = 4T_R^3\,T - 3T_R^4$$

given in the appendix as equation (15), was set equal to $T_e - (1/4)\,(T_e - T_0)$; this value was determined by comparing the temperature distribution given by equation (2) for several values of T_R with one found by numerical integration of the exact differential equation for a typical case. For extremely high conductivities or low heat-transfer coefficients, this value of T_R gives blade temperatures that are slightly low. A somewhat lower value for T_R would be more accurate for this case.

2. Values of T_2/T_1 of 1.0, 0.9, 0.75, and 0 with T_1 equal to T_e were assumed; more generally, if T_1 does not equal T_e, these assumed values correspond to values of $T_1^4 + T_2^4$ of 2.0, 1.656, 1.316, and 1.0 times the value of T_e^4, respectively. The values of T_1 for a given T_e depends on the difference between the inlet total temperature and the effective gas temperature and on the extent to which the nozzles are cooled. The temperature T_2 on the exit side of the blades is particularly dependent on the installation because the value of T_2 depends on whether the blades exhaust to the stators of another stage, to an exhaust hood, or directly to the atmosphere.

3. The factor ϵF, which enters equations (2b) and (2c), was set equal to 0.312. This value was obtained by assuming closely spaced blades, for which the total blade-surface area exposed to radiation from either side is approximately equal to the annulus area $\pi \bar{D} L$. Consequently, F was set equal to $\pi \bar{D} L/BpL$ which was evaluated as 0.312 using the basic blade dimensions and a value of B of 50. The value of ϵ was considered equal to 1, which is close to the value for oxidized metal surfaces.

RESULTS AND DISCUSSION

Effectiveness of rim cooling for basic blade. - The over-all effect of rim cooling for the basic blade ($\alpha L = 4.42$) is shown in figure 4; radiation effects and blade elongation are not included. Because the basic value of αL is lower than that for most modern turbines, these results for the basic blade may be considered representative of the turbines for which rim cooling is most effective. Blade life is plotted on a logarithmic scale against the limiting Mach number index; families of curves, each for a constant effective gas temperature and several amounts of rim cooling are shown. Large increases in blade life can be obtained with small increases in the amount of cooling at a constant Mach number index. For example, increasing the amount of cooling from 200° to 400° F at a gas temperature of 1500° F and a constant limiting Mach number index of 0.5 results in increasing the allowable life eightfold. Because the curves of figure 4 are approximately parallel, results for a single value of blade life are fairly typical of all values.

A cross plot of curves of the type in figure 4 at 1000 hours (fig. 5) shows the effective gas temperature as a function of the amount of rim cooling for constant values of the limiting Mach number index. Figure 5 indicates the effect of different amounts of rim cooling on the allowable gas temperature for a given design because the limiting Mach number index at which a turbine is to be operated is determined by the design.

The data of figure 5 for a constant limiting Mach number index of 0.5 have been replotted in figure 6 as the increase in gas temperature ΔT_e greater than that allowable without cooling. The corresponding increase in the inlet total temperature is from 2 to 10 percent greater than the value of ΔT_e, depending on the velocity diagram of the turbine. Lines of constant limiting Mach number index of 0.4 and 0.6 give values of ΔT_e that vary less than 30° F from those in figure 6. This small effect of Mach number index can be explained by the fact that the lines of constant limiting Mach number index are approximately parallel in figure 5. At lower limiting Mach ..

numbers, the values of ΔT_e are slightly less; therefore, rim cooling is somewhat less effective at high gas temperatures.

It should be emphasized that figures 4 and 5 give values at rupture, whereas in actual operation a turbine would be run with some factor of safety. The actual operating point could be determined either by expressing the desired life as some fraction of that for rupture or by using an effective gas temperature somewhat lower than that for rupture. If these actual operating conditions were shown in figures 4 and 5, the only significant change would be a general displacement of the curves relative to the axes. If the factor of safety is introduced only by means of a specified reduction in the effective gas temperature, the curves of figure 6 are valid for the actual operating conditions as well as for those at rupture.

For the basic blade ($\alpha L = 4.42$), increases in effective gas temperature from 140° to 200° F are made available by the first 250° to 500° F of rim cooling (fig. 6) but 1000° F of cooling gives an additional increase of only 60° F. This effect can be explained to some extent by the fact that, for the greater amounts of cooling, the critical blade point is farther from the root (fig. 7) and therefore more difficult to cool. Actually, the first amounts of cooling and the corresponding large increases in allowable gas temperature may not be available in most turbines because the rim may be 100° to 300° F below the gas temperature as a result of the cooling of the wheel and the shaft by lubricating oil and radiation.

Effect of blade dimensions, conductivity, and heat-transfer coefficient. - The value of the dimensionless parameter αL completely determines the effect of blade dimensions, conductivity, and heat-transfer coefficients on the results when radiation is neglected and the ratio L/r_t is constant.

The temperature distribution in the blade is shown in figure 8 for an effective gas temperature of 1500° F and a value of $(T_e - T_0)$ of 700° F for seven values of αL including the limiting values of ∞ and 0; these curves are superimposed on a family of allowable-temperature curves for a 1000-hour blade life to determine the critical blade points. For values of αL greater than 4.42, the blade temperatures are very close to the gas temperature over half the length of the blade, whereas for smaller values of αL all blade temperatures are considerably lower than that of the gas. The critical blade point is close to the root for small values of αL and approaches the blade tip as αL is increased to a value of about 4, but for larger values of αL the critical point again becomes closer to the root. At low values of αL, the critical blade point actually

remains at the root for the first several hundred degrees of cooling but then moves quite rapidly toward the tip as cooling is increased (fig. 7).

The variation in the effectiveness of rim cooling with changes in αL is shown in figure 6 for a blade life of 1000 hours. The increase in allowable effective gas temperature is much greater for the lower values of αL; thus, for a value of $T_e - T_0$ of 600° F, the increase is 440° F for αL of 1.47, 215° F for 4.42, and only 90° F for 13.26. At values of αL greater than 4.42 there is little increase in ΔT_e with increasing amounts of cooling above 300° to 400° F but, for the very low values of αL, ΔT_e continues to increase almost proportionally with $T_e - T_0$ over the entire range considered. This cooling can be used, of course, to increase blade life beyond 1000 hours by sacrificing some of the increase in gas temperature; for example, a sacrifice of about 60° F in gas temperature for αL of 4.42 makes possible a tenfold increase in life.

The low values of αL necessary for rim cooling to be very effective could possibly be obtained by increasing the value of conductivity or decreasing the value of the heat-transfer coefficient from the basic values. For example, the value αL is halved if k is increased from 12 to 48 Btu/(hr)(ft)(°F) or if h is decreased from 40 to 10 Btu/(hr)(sq ft)(°F). Unfortunately, none of the alloys now available for use at high temperatures and stresses has a conductivity much greater than 12. In a given turbine the increased cooling effectiveness obtained by increasing the conductivity might be offset by the fact that additional heat must be removed from the blade to maintain the same rim temperature.

A relatively low value of h might be obtained by a proper adjustment of the design velocities over the blades, but these velocities are usually restricted by aerodynamic considerations. The use of insulating coatings on the blade results in small reductions in the over-all heat-transfer coefficient from the hot gases to the metal blade from an initially low value of h, such as 40 Btu/(hr)(sq ft)(°F), although the coating can produce relatively large reductions from a high value. For example, if h is 40, a coating 0.010 inch thick with a conductivity of 0.5 Btu/(hr)(ft)(°F) will reduce the over-all coefficient only 6 percent; if h is 250, the same coating will reduce the over-all coefficient 30 percent. A coating, however, may weaken the blade or increase the ratio of blade perimeter to area p/A and thus αL, which might completely counteract the favorable effect of the reduction in the over-all film coefficient.

A low value of αL can also be obtained by the use of a short blade or a blade with a small p/A ratio. In order to obtain the

same mass flow when the blade length is decreased, the tip radius must be increased, which means that the increased cooling effectiveness is gained at the price of increasing the size of the turbine. The value of L/r_t is also decreased in this case with the result that the calculations are not strictly applicable; it may be shown, however, that although a decrease in L/r_t causes a considerable increase in the limiting Mach number index the decrease has little effect on the curves of ΔT_e against $T_e - T_0$ (fig. 6). The ratio of blade perimeter to area, which depends upon the shape and the size of the blade, is limited by aerodynamic considerations and no significant change in αL can usually be obtained by a variation in p/A.

On the other hand, large high-power turbines are likely to have values of αL considerably greater than 4.42 for two reasons: (a) these turbines have large specific mass flows and consequently the value of h is greater than 40; and (b) these turbines have dimensions greater than the basic values assumed. For example, if the heat-transfer coefficient is increased from 40 to 120 and all the dimensions are multiplied by 3 (thus multiplying $\sqrt{p/A}\,L$ by $\sqrt{3}$), αL is increased from 4.42 to 13.26.

Blade creep. - The distribution of local elongation due to creep over the blade length and the corresponding stress and temperature distributions are shown in figure 9 for a particular set of conditions. The total blade elongation obtained by integration of the distribution of local elongation is also given in the figure for each tip speed. The percentage elongation of the blade at rupture depends primarily on the temperature at the critical point where most of the elongation occurs. The temperature at the critical point for effective gas temperatures from 1400° to 1500° F is in the neighborhood of 1350° F (fig. 8), at which temperature the elongation of the blade is a maximum (fig. 3); accordingly, the maximum blade elongation for these gas temperatures should be the maximum elongation for any gas temperature.

The elongation of the basic blade for a gas temperature of 1500° F is plotted against Mach number index for several amounts of cooling in figure 10. The maximum elongation indicated is only about one-sixteenth inch, which is less than most blade-tip clearances. If a shorter blade life were assumed, the maximum elongation at rupture would be expected to be somewhat larger and might be excessive. For longer blades the maximum percentage elongation is expected to be about the same; for such blades, therefore, the total elongation at rupture may be considerably greater than allowed by the tip clearances.

Actual blade elongations, however, will be much smaller than the maximum values indicated because the turbine is actually operated with a factor of safety and the elongation is greatly reduced by a small decrease in Mach number index or a small increase in cooling. An actual turbine would never be operated at a speed corresponding to the vertical portion of the curves of figure 10, for very little overspeed would result in rupture. Inasmuch as the creep properties assumed for the blade metal were relatively poor, actual turbine operation will not be limited by blade creep.

Effect of radiation. - The effect of radiation on these results varies greatly with the assumed values of the radiation temperatures T_1 and T_2. If these temperatures are equal to the gas temperature, the effect of radiation is approximately the same as the effect of an increased value of the heat-transfer coefficient or of αL. The influence of radiation on the effectiveness of cooling can then be found by using values of $\sqrt{Y}\,L$ as effective values of αL in figure 6. The magnitude of the increase depends upon h and T_R alone for a constant value of ϵF (equation (2b)). For small values of h and large values of T_R (corresponding to high gas temperatures), the value of $\sqrt{Y}\,L$ may be as much as two or three times the value of αL; however, for the basic value of h (40 Btu/(hr)(sq ft)(°F)), $\sqrt{Y}\,L$ is only about 1.3 times the value of αL.

If T_1 or T_2 is assumed less than T_e, as is likely for most turbines, the effective increase in αL will be counteracted by a decrease in Z/Y, which has the same effect on the temperature distribution as decreasing T_e. In figure 11, the temperature distribution for the basic value of αL of 4.42 and no radiation is compared with that for four radiation approximations. As the value of T_2/T_1, and correspondingly Z/Y, is decreased, all blade temperatures are decreased and the limiting Mach number indices increase; the limiting Mach number index for no radiation is approximately the mean of the values for different radiation assumptions and the critical blade point is somewhat closer to the tip than for the examples that include radiation. The increase in allowable gas temperature with radiation is compared in the following table with that for no radiation for equal amounts of cooling, $T_1 = T_e = 1960^\circ$ R, and a 1000-hour blade life:

T_2/T_1	h (Btu/(hr)(sq ft)(°F))	Z/Y (°R)	Z/Y - T_0 (°F)	$\sqrt{Y}$ L	αL	ΔT_e (no radiation) (°F)	ΔT_e (radiation) / ΔT_e (no radiation)
0.9	40	1925	665	5.62	4.42	224	1.0
.9	40	1920	360	5.76	4.42	173	1.0
.9	360	1954	694	13.69	13.26	95	1.0
.9	40	1925	665	1.87	1.47	480	1.0
.9	4.44	1884	624	3.78	1.47	455	.7
.75	4.44	1791	531	3.78	1.47	400	.9
.75	40	1883	623	5.62	4.42	218	1.0
.75	360	1947	687	13.69	13.26	95	1.0

Results are shown for the two most probable radiation approximations, T_2/T_1 equal to 0.9 and 0.75 and for values of αL of 1.47, 4.42, and 13.26. The cases in which the values of αL of 1.47 and 13.26 are obtained by varying h from its basic value are considered separately from those in which k or the blade dimensions are varied. When radiation is considered, the blade temperature with no cooling is approximately Z/Y; the amount of cooling is thus Z/Y - T_0. Values of ΔT_e (no radiation) were obtained from figure 6 at points where $T_e - T_0 = Z/Y - T_0$.

When T_2/T_1 is 0.9, the difference between the effectiveness of rim cooling with or without radiation is slight. The only significant variation is for the extreme case in which h has been decreased to one-ninth its basic value. Radiation then becomes a large percentage of the total heat input and the cooling effectiveness drops with the allowable increase in effective gas temperature with radiation only 70 percent of that without radiation. If T_2/T_1 is assumed equal to 0.75, Z/Y becomes relatively low and tends to counteract the effective increase in the value of αL. For example, for the extreme case when h is 4.44, the ratio of ΔT_e with radiation to ΔT_e without radiation is increased from 0.7 to 0.9 when T_2/T_1 is decreased from 0.9 to 0.75.

CONCLUSIONS

From an analysis of the rim cooling of a typical gas-turbine blade and also variation of a blade parameter, the following conclusions were drawn:

1. The benefits of rim cooling are limited to increases in allowable gas temperature of the order of 200° F for most blades.

2. For a given effective gas temperature, the addition of small amounts of rim cooling allows extremely large increases in blade life.

3. In order to obtain gas temperature increases of the order of 400° F, extreme increases in thermal conductivity or decreases in heat-transfer coefficient or blade length are necessary; consequently for turbines of large dimensions and high specific mass flows, the increases in gas temperature possible with rim cooling are particularly small.

4. Rim cooling is most effective for the first 250° to 500° F of cooling of the rim (below gas temperature); for the typical blade, 500° F of cooling allows a gas temperature increase of 200° F but additional cooling yields only relatively small gains in allowable gas temperature.

5. In order to obtain large increases in gas temperature, some direct method of cooling would probably have to be used, in which a considerable portion of the blade length is cooled by contact with a coolant fluid.

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APPENDIX - DERIVATION OF TEMPERATURE-DISTRIBUTION EQUATIONS

A heat balance for the steady state for a differential element dx between two planes perpendicular to the blade length and at distances x and $x + dx$ from the blade root is considered. The difference between the amount of heat leaving the element by conduction dQ_x and the amount entering by conduction $dQ(x + dx)$ is equal to the sum of the heat entering by convection dQ_c and by radiation dQ_R:

$$dQ_x - dQ_{(x + dx)} = dQ_c + dQ_R \tag{11}$$

$$\left(kA \frac{dT}{dx}\right) - \left(kA \frac{dT}{dx} + kA \frac{d^2T}{dx^2} dx\right) = hp\ (T_e - T)\ dx + dQ_R \tag{12}$$

If dQ_R is set equal to zero and $hp/kA = \alpha^2$

$$\frac{d^2T}{dx^2} = -\alpha^2 (T_e - T)$$

A solution of this equation is

$$T = T_e - C_1 \cosh \alpha\ (x - C_2) \tag{13}$$

where C_1 and C_2 are arbitrary constants Because the tip is assumed to be insulated $\frac{dT}{dx} = 0$ at $x = L$. At $x = 0$, $T = T_0$.

Substitution of these boundary conditions in equation (13) gives the final equation for the temperature distribution when radiation is neglected

$$T = T_e - \frac{\cosh \left[\alpha L \left(1 - \frac{x}{L}\right)\right]}{\cosh \alpha L} (T_e - T_0) \tag{1}$$

Radiation may be approximately accounted for by assuming the blades are faced on the inlet side by nozzles at an average temperature T_1 and on the exhaust side by exit stators or an exhaust hood at an average temperature T_2. If the fraction of the blade-surface area exposed to radiation from either side is F and if contributions to dQ_R from radiation between the blades are neglected,

$$dQ_R = \epsilon\sigma\left[\left(T_1^4 - T^4\right) + \left(T_2^4 - T^4\right)\right]Fpdx \tag{14}$$

As an approximation T^4 is expanded linearly about an average blade temperature T_R

$$T^4 = 4T_R^{\,3}\,T - 3T_R^{\,4} \tag{15}$$

When equations (14) and (15) are combined and the result is substituted in equation (12)

$$\frac{d^2T}{dx^2} - YT + Z = 0 \tag{16}$$

where

$$\left.\begin{aligned} Y &= \frac{hpL + 8\epsilon\sigma FpL\,T_R^{\,3}}{kAL} \\ Z &= \frac{hpL\,T_e + \epsilon\sigma FpL\left(T_1^{\,4} + T_2^{\,4} + 6T_R^{\,4}\right)}{kAL} \end{aligned}\right\} \tag{17}$$

If Y and Z are assumed constant over the blade height, the general solution of equation (16) is

$$T = \frac{C_1 \cosh\left[\sqrt{Y}\,(x + C_2)\right] + Z}{Y} \tag{18}$$

The arbitrary constants C_1 and C_2 are determined by applying the same boundary conditions as before, and the resulting equation for the temperature distribution when radiation is included is

$$T = \frac{Z}{Y} - \frac{\left(\frac{Z}{Y} - T_0\right)\cosh\left[\sqrt{Y}\,L\left(1 - \frac{x}{L}\right)\right]}{\cosh\sqrt{Y}\,L} \tag{2a}$$

Equations (17) may also be written

$$\sqrt{Y}\,L = \alpha L\sqrt{1 + 8\sigma\epsilon F\,\frac{1}{h}\,T_R^{\,3}} \tag{2b}$$

$$\frac{Z}{Y} = \frac{T_e + \sigma\epsilon F\,\frac{1}{h}\left(T_1^{\,4} + T_2^{\,4} + 6T_R^{\,4}\right)}{1 + 8\sigma\epsilon F\,\frac{1}{h}\,T_R^{\,3}} \tag{2c}$$

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3. Cross, Howard C., and Simmons, Ward F.: Progress Report on Heat-Resisting Metals for Gas Turbine Parts (N-102). OSRD No. 3651, Ser. No. M-280, NDRC, OSRD, War Metallurgy Div., May 10, 1944.

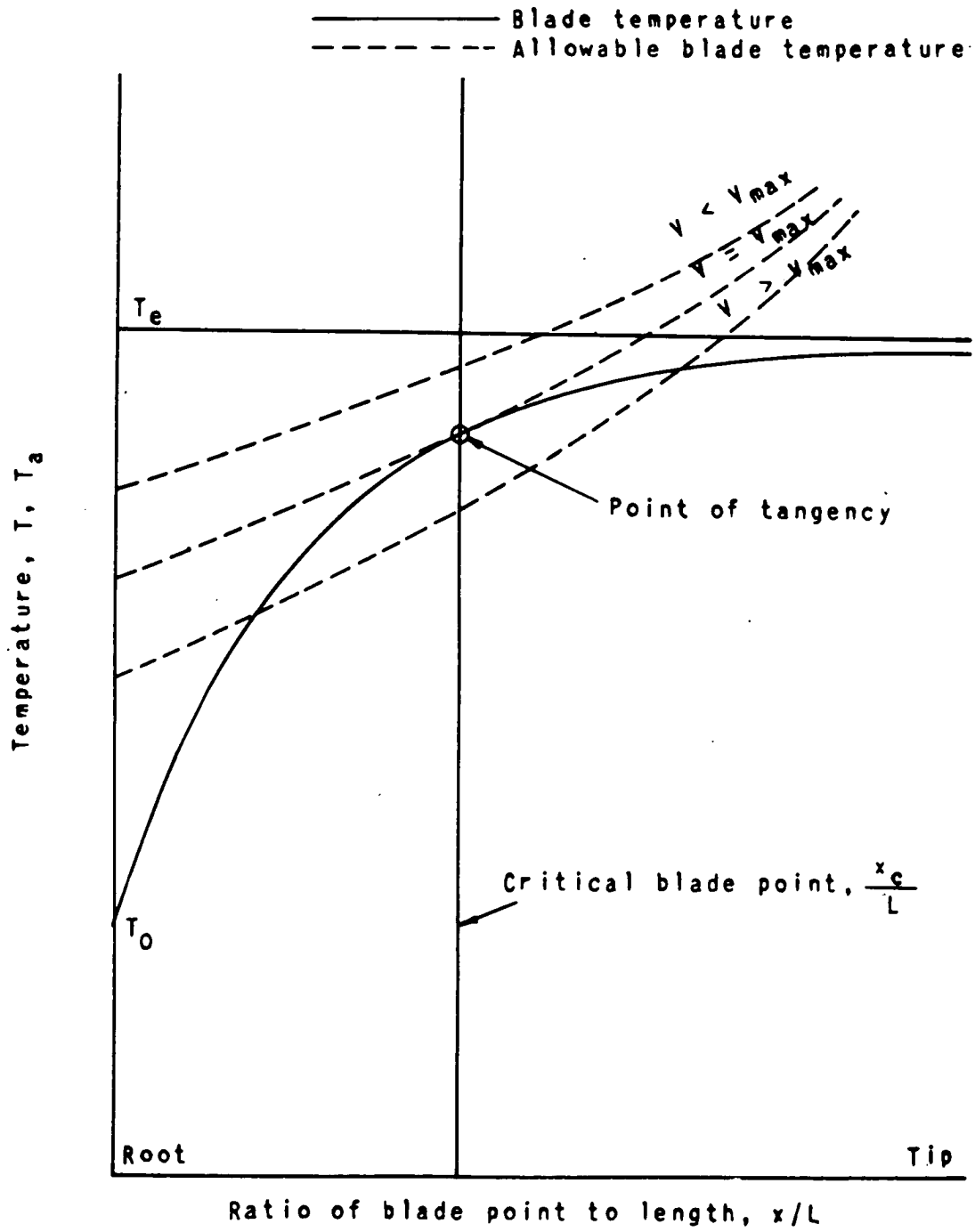

Figure 1. - Method of determining limiting speed and critical blade point from curves of temperature distribution and allowable temperature.

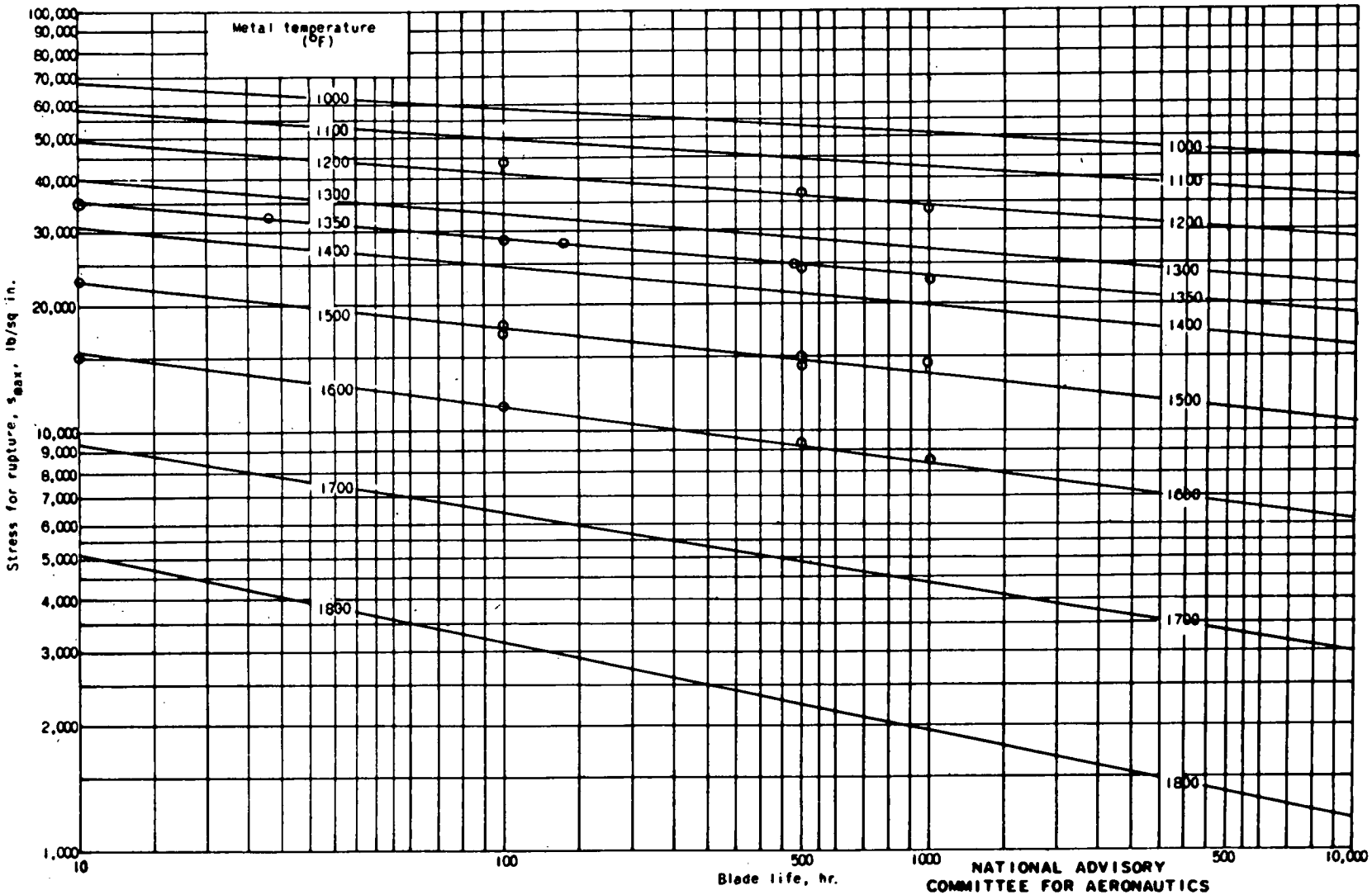

Figure 2. - Stress-rupture characteristics for high-temperature alloy S497. (Points from experimental data in references 2 and 3.)

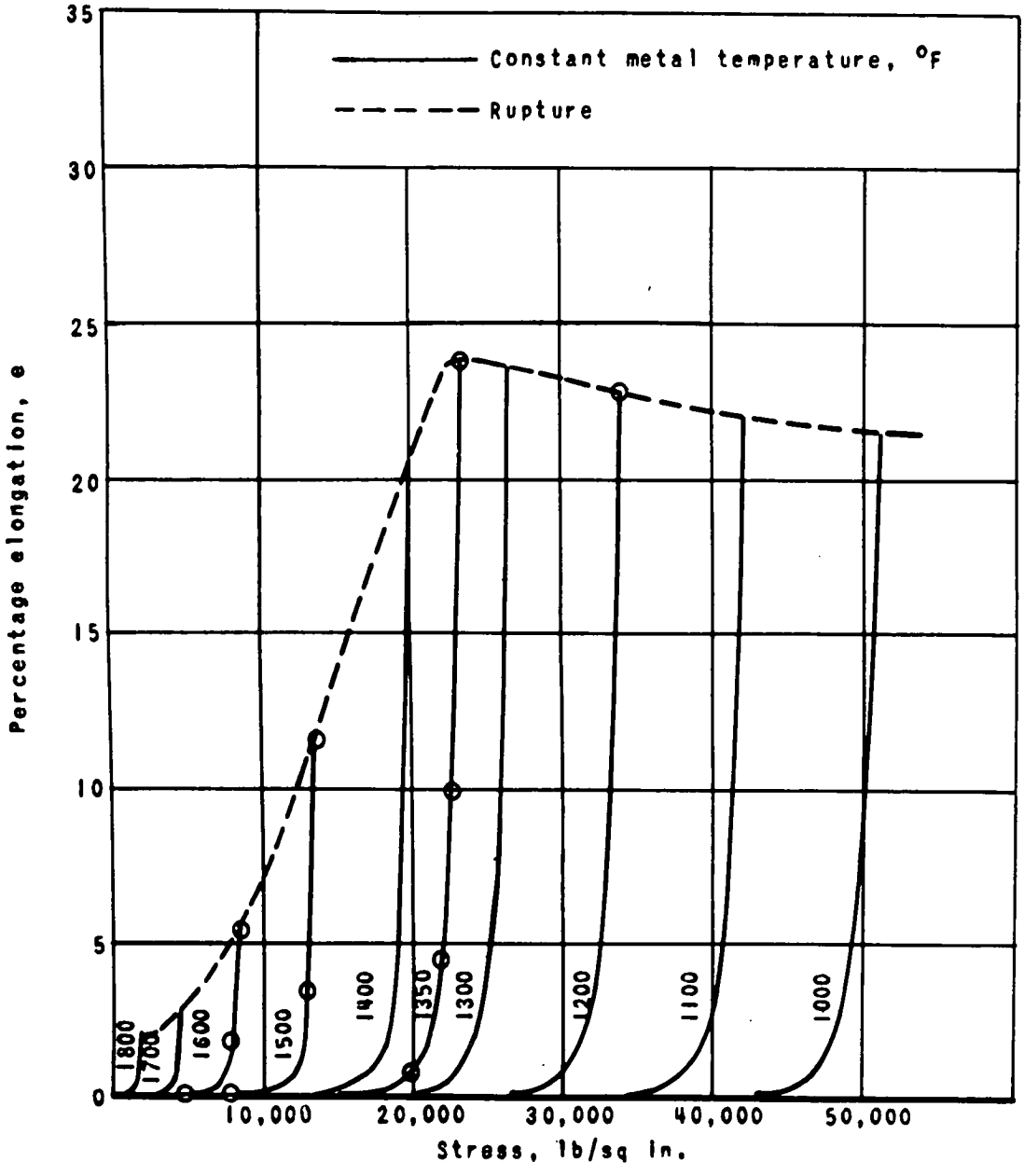

Figure 3. - Effect of stress on elongation due to creep for S497 at constant metal temperatures and blade life of 1000 hours. (Points from experimental data in references 2 and 3.)

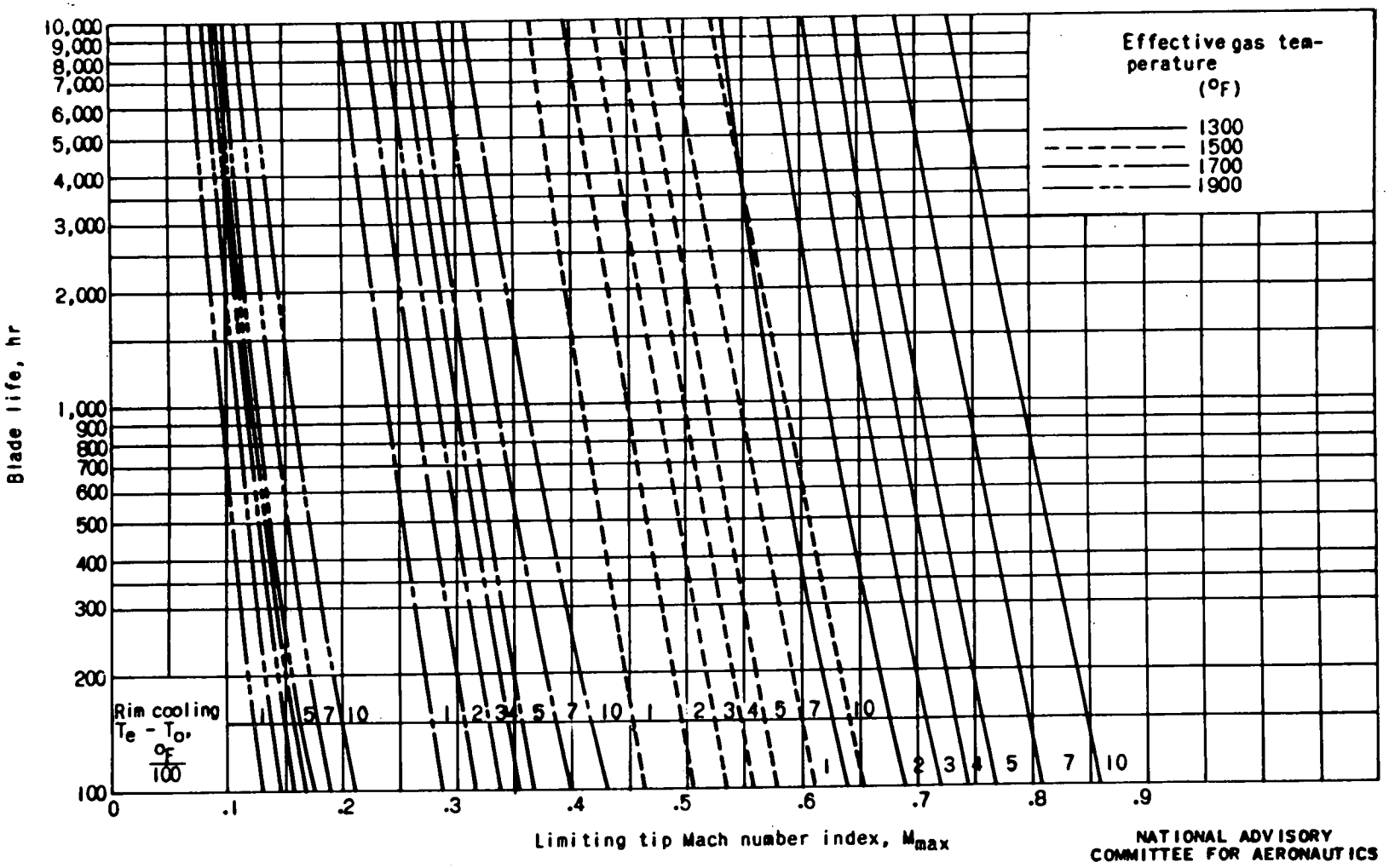

Figure 4. - Effect of limiting tip Mach number index, rim cooling, and exhaust-gas temperature on expected life of turbine blade when α_L is 4.42.

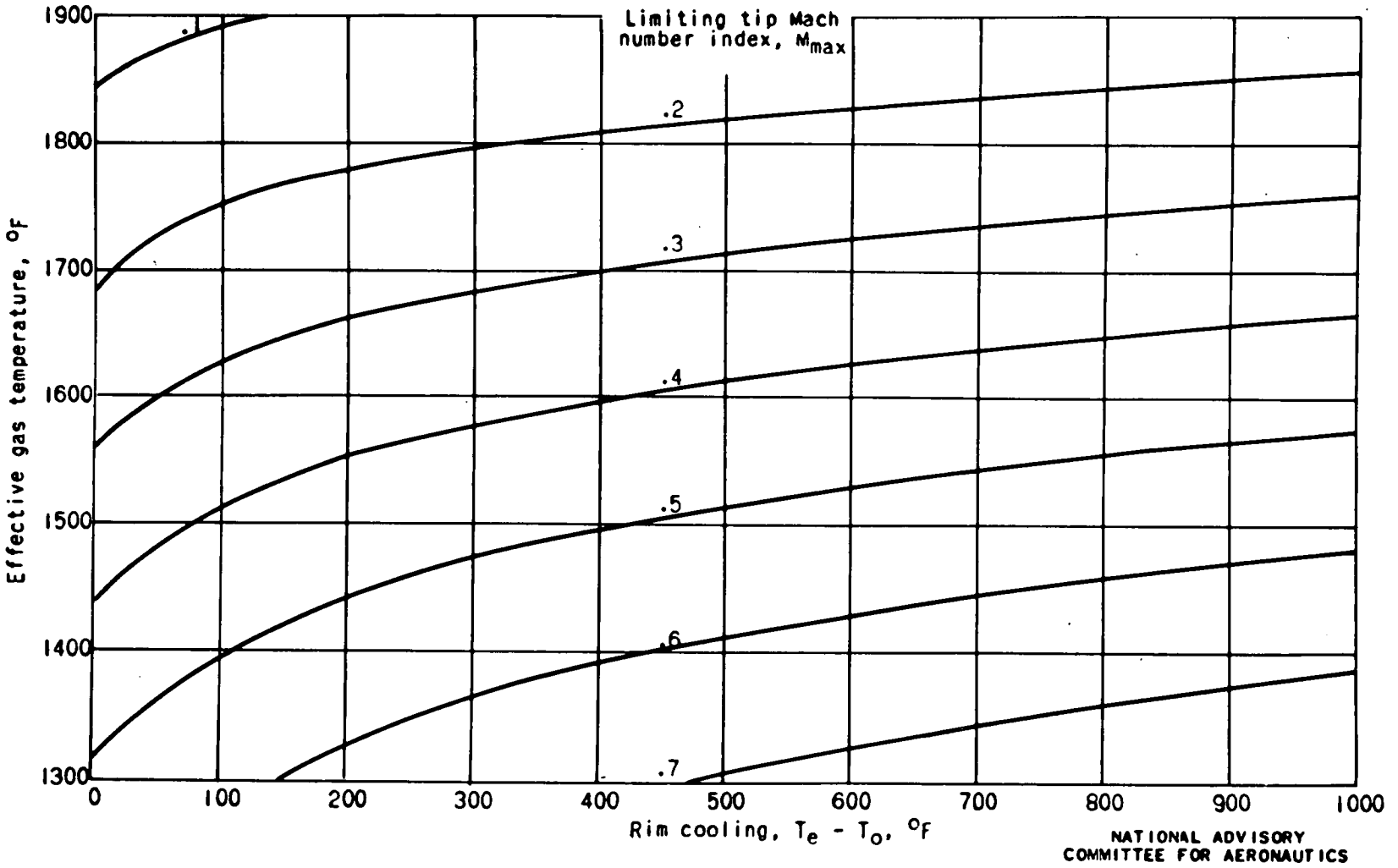

Figure 5. - Effect of rim cooling on allowable effective gas temperature for various limiting Mach number indices, αL of 4.42, and blade life of 1000 hours.

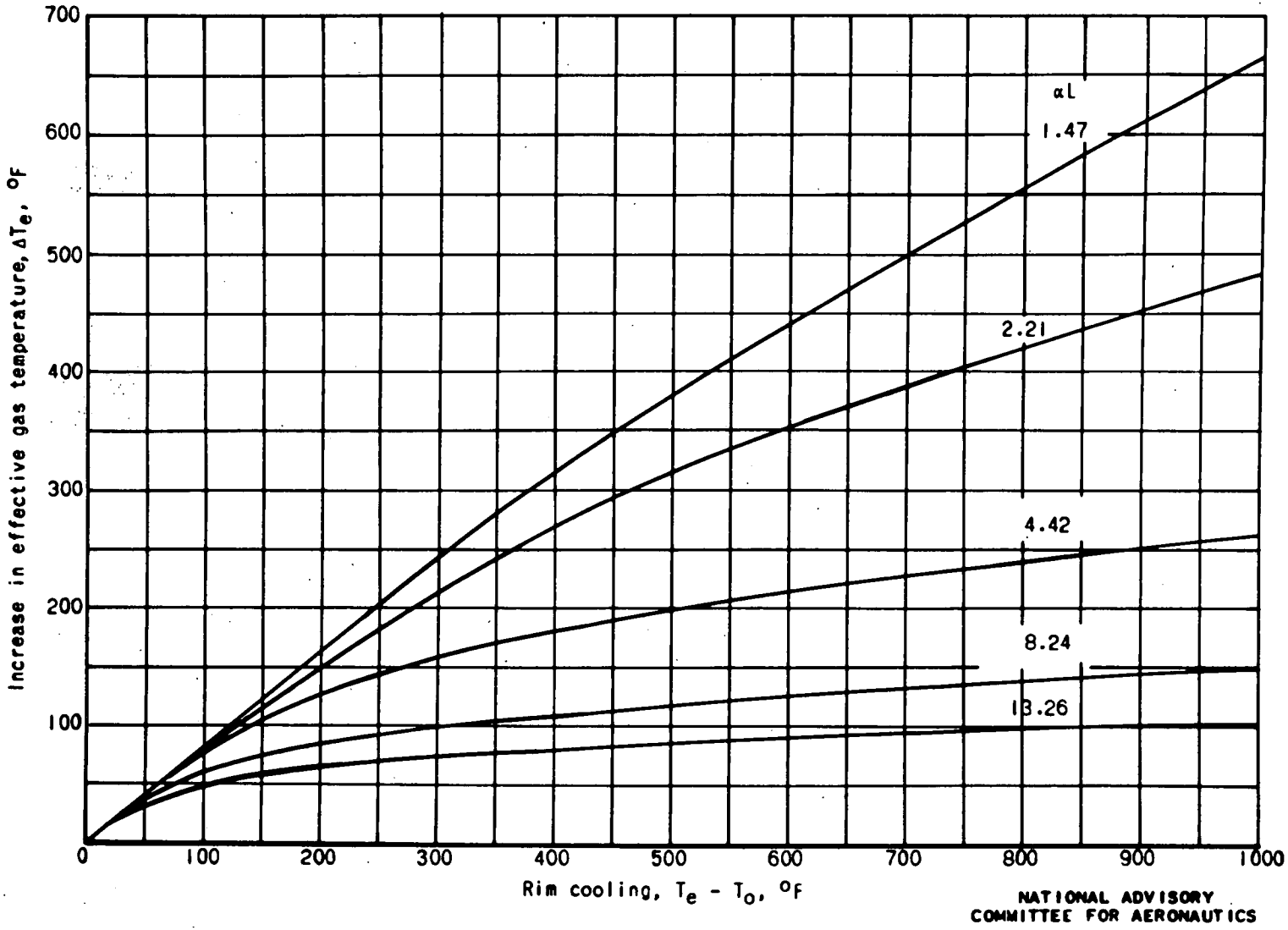

Figure 6. - Allowable increase in effective gas temperature for various amounts of rim cooling, several values of αL, limiting Mach number index of 0.5, and blade life of 1000 hours.

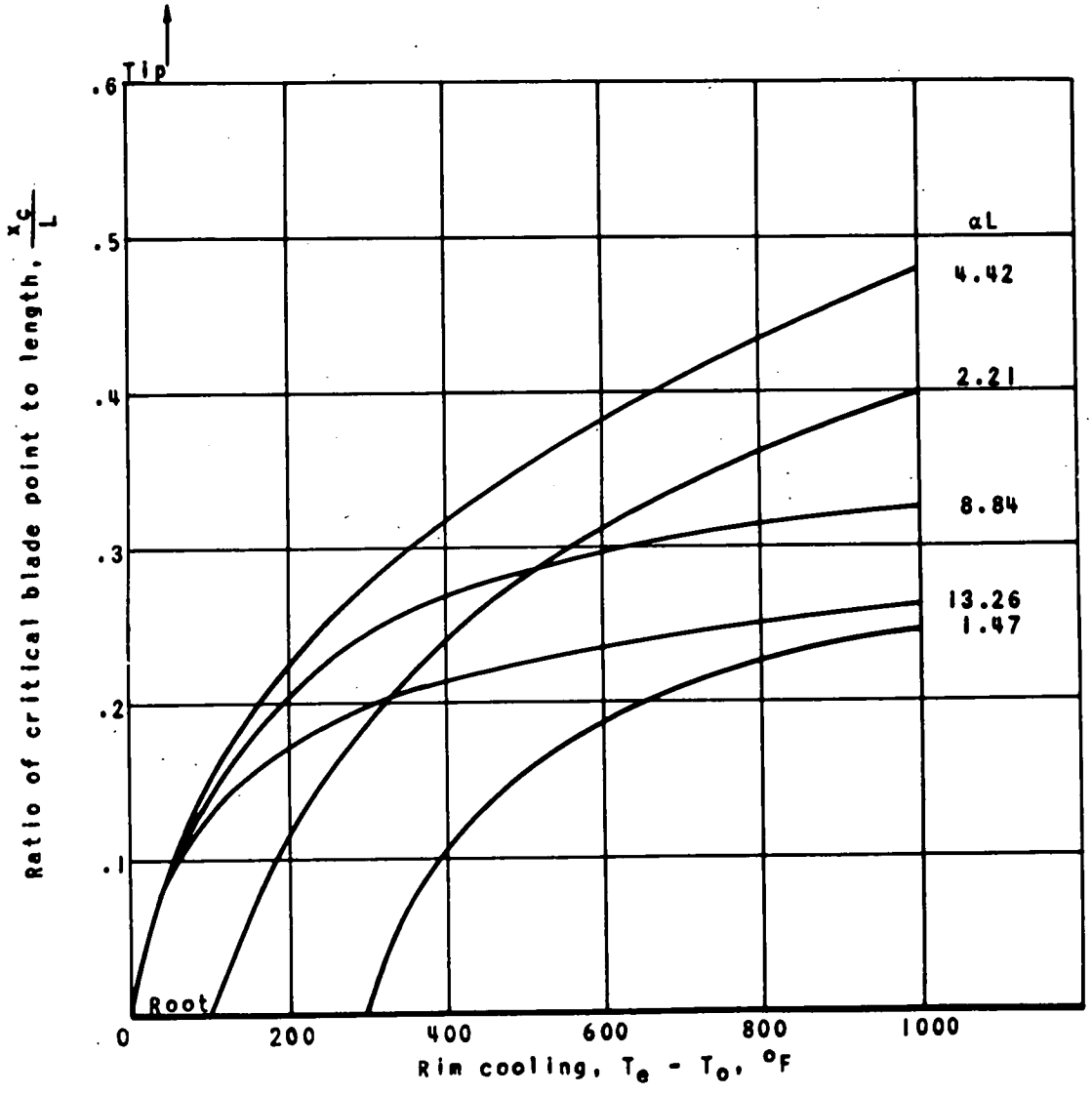

Figure 7. - Effect of rim cooling on location of critical blade point for various values of αL, effective gas temperature of 1500° F, and blade life of 1000 hours.

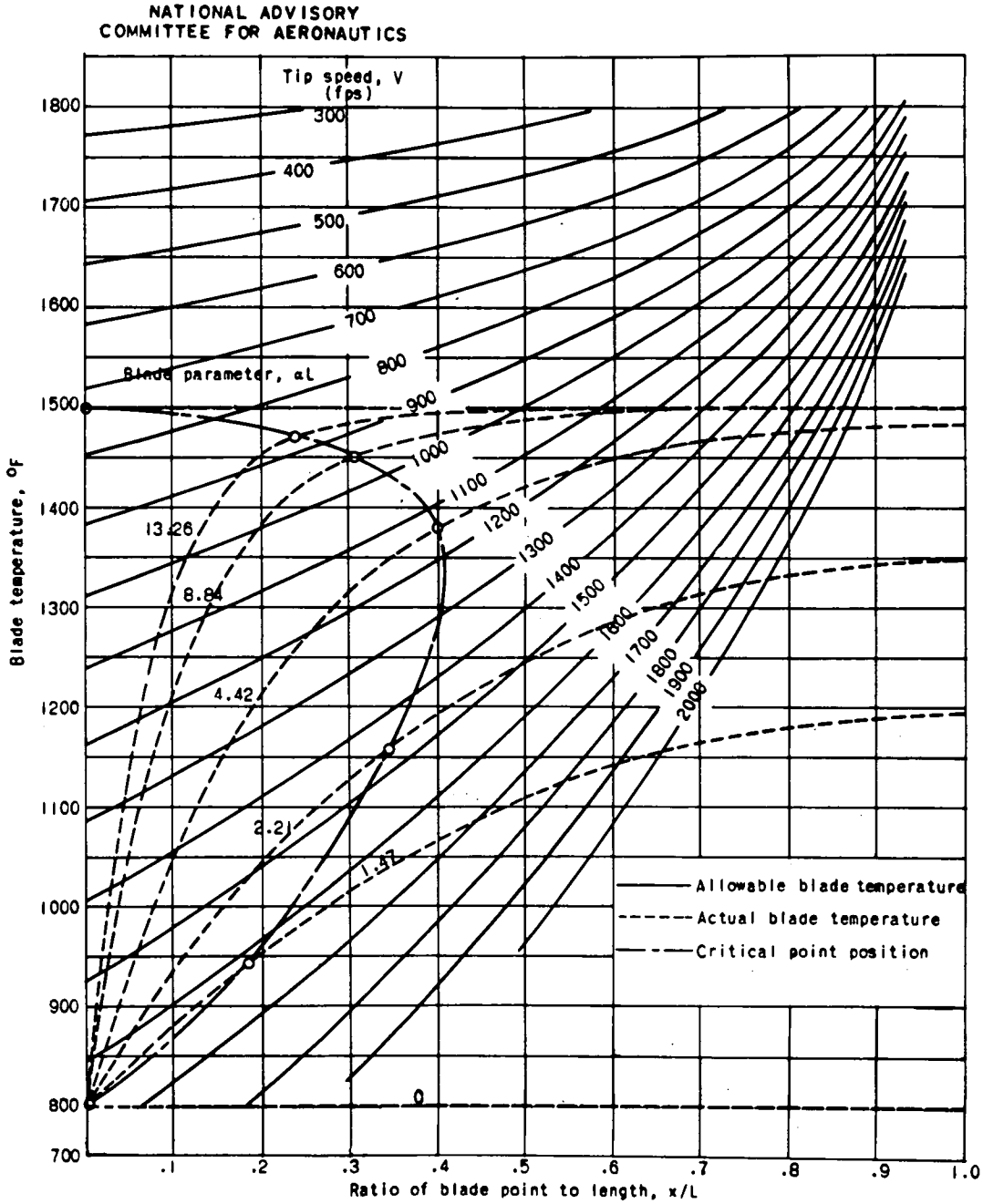

Figure 8. - Effect of αL on temperature distribution and location of blade critical point for effective gas temperature of 1500° F, $T_e - T_o$ of 700° F, and blade life of 1000 hours.

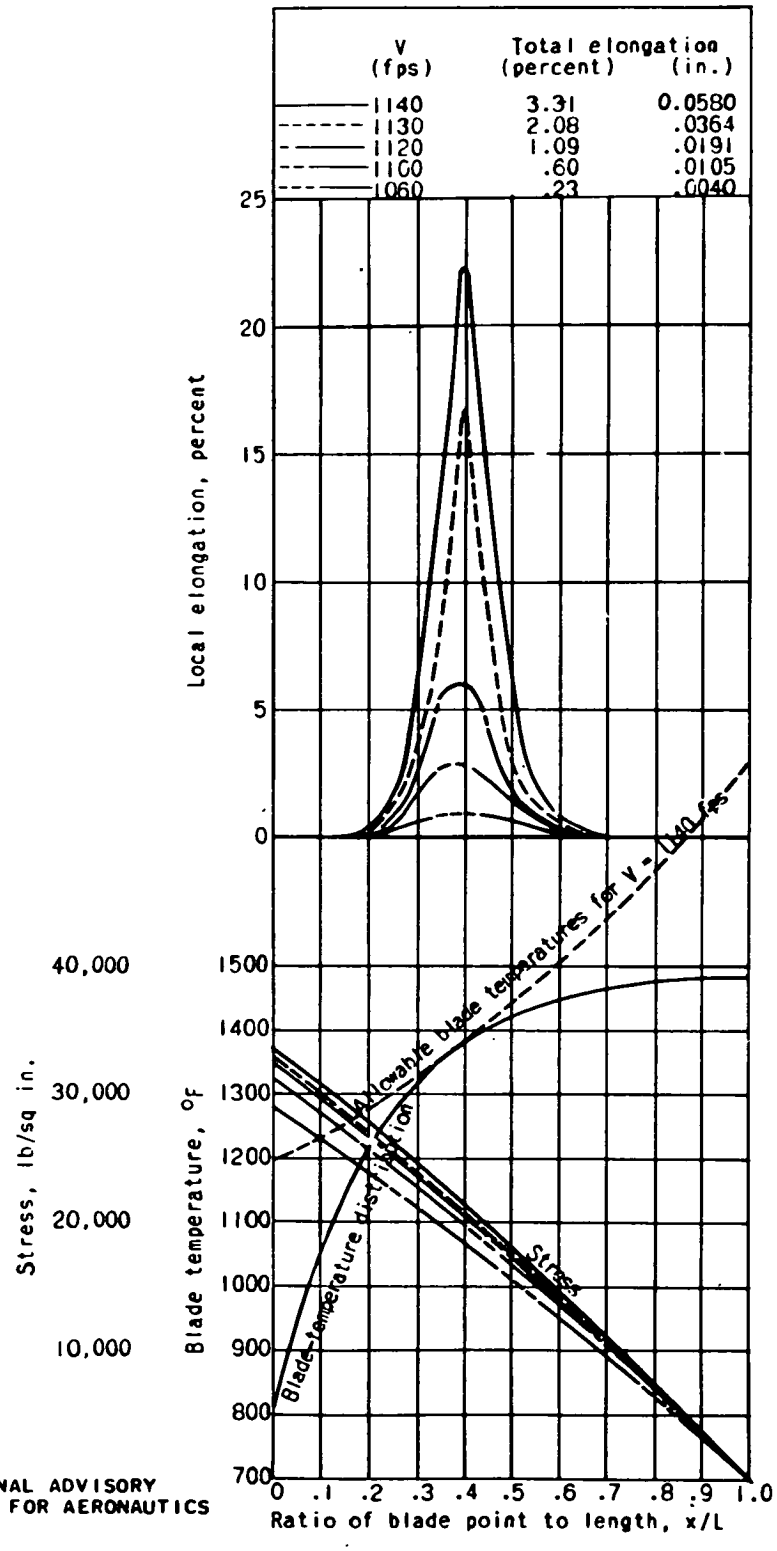

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Figure 9. - Distribution of temperature, stress, and elongation due to creep for αL of 4.42, effective gas temperature of 1500° F, $T_e - T_o$ of 700° F, and blade life of 1000 hours.

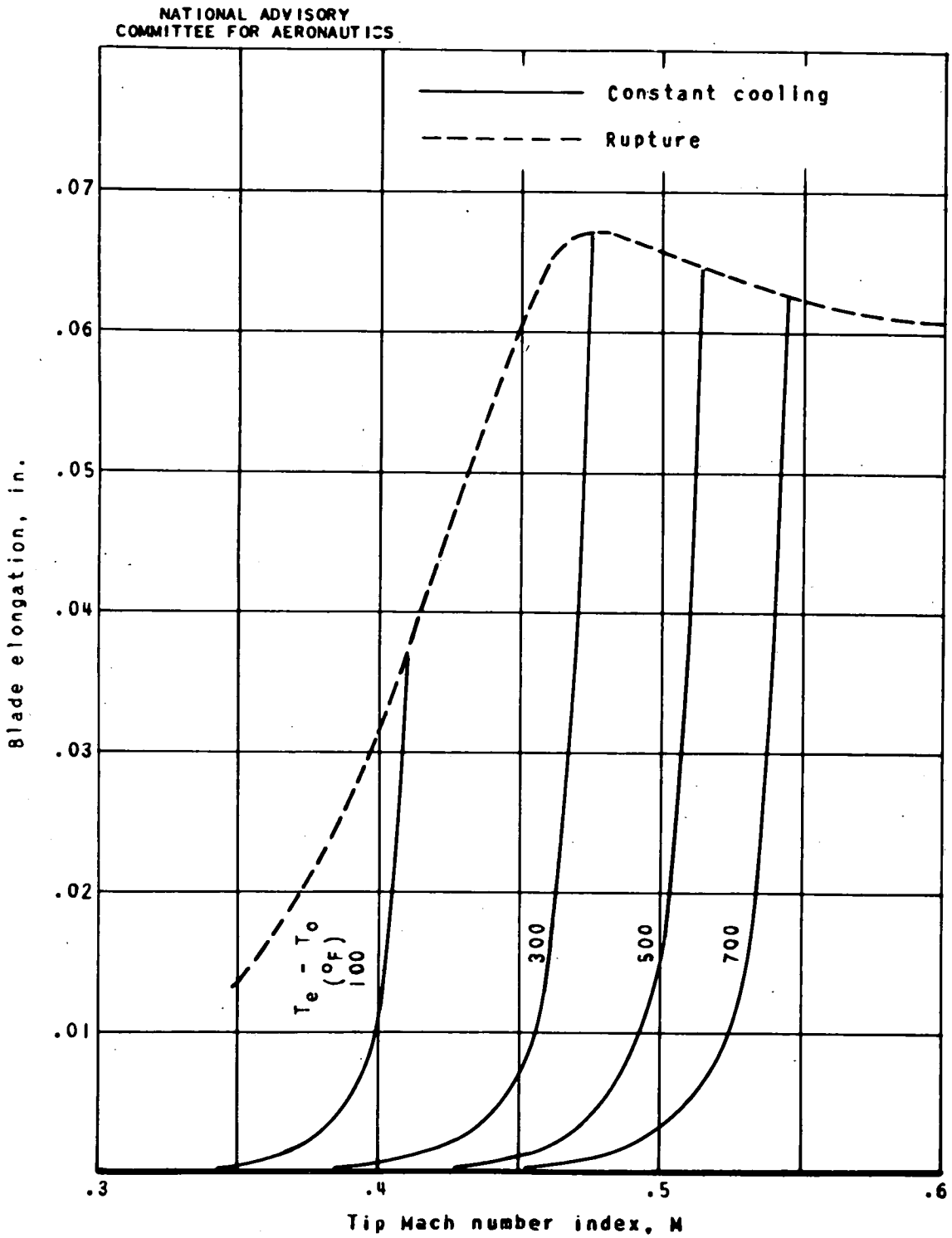

Figure 10. - Effect of tip Mach number index on blade elongation due to creep for constant rim cooling, αL of 4.42, effective gas temperature of 1500° F, and blade life of 1000 hours.

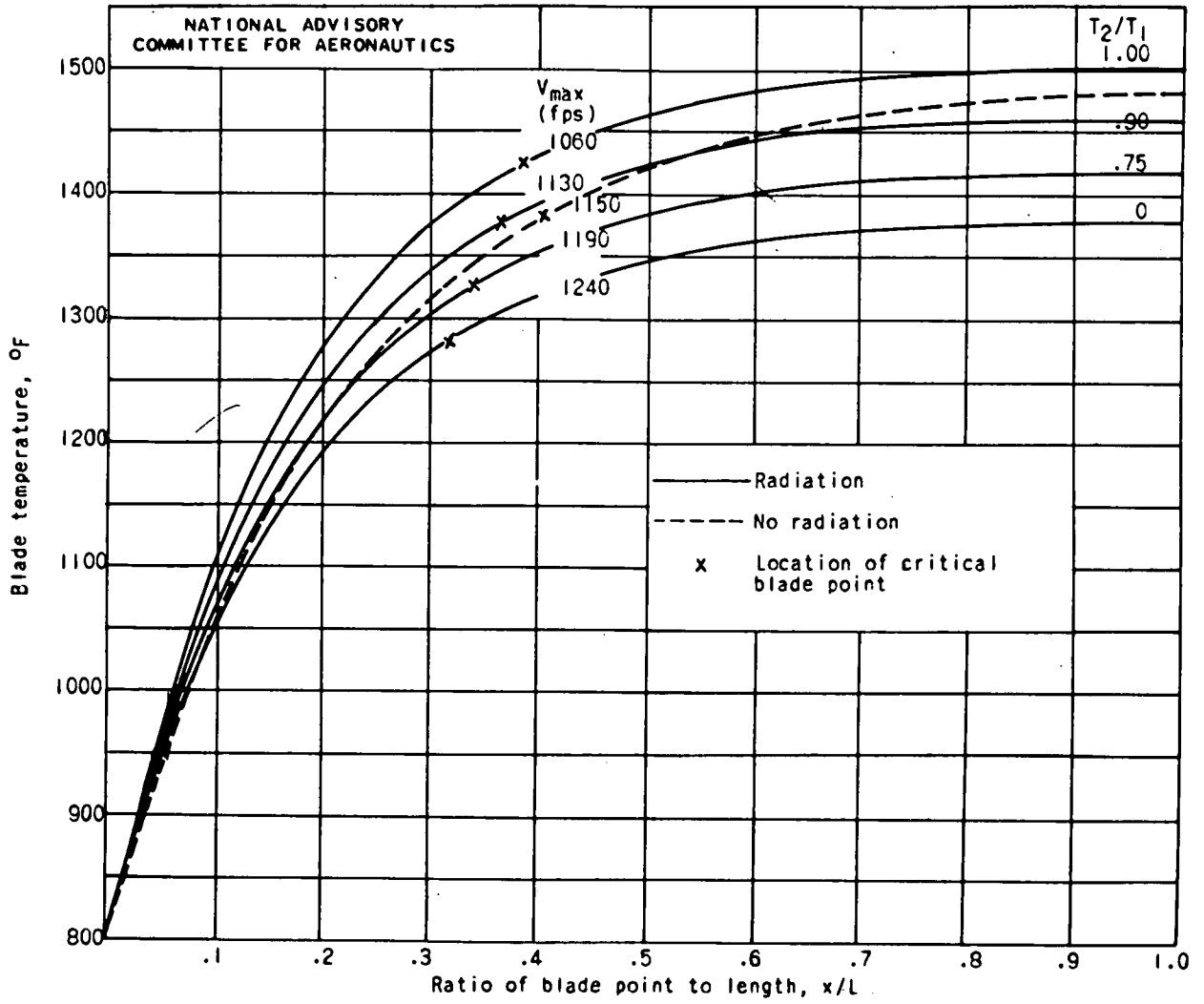

Figure 11. - Comparison of temperature distribution, limiting tip speed, and critical blade position without radiation and for various radiation assumptions, αL of 4.42, effective gas temperature of 1500° F, $T_e - T_o$ of 700° F, and blade life of 1000 hours.

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