NACA

RESEARCH MEMORANDUM

A TORSIONAL STIFFNESS CRITERION FOR PREVENTING FLUTTER OF WINGS OF SUPERSONIC MISSILES

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A TORSIONAL STIFFNESS CRITERION FOR PREVENTING FLUTTER OF WINGS OF SUPERSONIC MISSILES

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SUMMARY

A formula, based on a semirational analysis, is presented for estimating the torsional stiffness necessary to prevent flutter of a sweptback or unswept uniform wing that attains supersonic speeds. Results of missile flights at speeds up to Mach number 1.4 demonstrate the usefulness of the formula.

INTRODUCTION

Failures probably due to flutter were encountered in NACA flight tests of several rocket-powered, drag-research missiles that were intended to attain Mach numbers of about 1.4. The wing failures of these missiles led to the development of a simple, semirational torsional stiffness criterion for preventing flutter of uniform, sweptback or unswept missile wings that attain supersonic speeds. Missiles that failed were redesigned in accordance with this stiffness criterion and proved to be safe in flight.

TORSIONAL STIFFNESS CRITERION

On the basis of the semirational analysis presented in appendix A, the following formula is proposed for estimating the torsional stiffness necessary to prevent flutter of a uniform sweptback or unswept wing that attains supersonic speeds:

$$GJ = 40\left(\frac{L^3c^2d}{L + 2c}\right) \tag{1}$$

where (fig. 1)

GJ torsional stiffness (ratio of torque to twist per unit length) of section normal to leading edge, pound-inches^2

L length, inches

c chord, normal to leading edge, inches

d $\dfrac{\text{Distance of center of gravity behind quarter-chord position}}{\text{Chord}}$

Equation (1) may be considered as probably most reliable for wings having the following characteristics:

(a) Low ratio of bending frequency to torsional frequency: $\dfrac{\omega_h}{\omega_\alpha} \ll 1$

(b) High relative density: $\dfrac{1}{\kappa} > 10$ (see appendix B)

(c) Center of gravity ahead of midchord position. However, for wings that do not quite satisfy those conditions, the criterion may be used as a design guide until more experimental and theoretical information becomes available.

The derivation of equation (1) was made for standard sea-level atmospheric conditions; application of the formula to high-altitude conditions is probably conservative.

Divergence of Unswept Wings

Failure by divergence rather than by flutter may occur in wings without sweepback. Let e be defined by:

$$\frac{\text{Distance of shear center behind quarter-chord position}}{\text{Chord}}$$

then the larger of d and e should be used in equation (1) in order to guard against the possibility of divergence as well as flutter of unswept wings.

FLIGHT TESTS

A number of rocket-powered missiles with uniform wooden wings, sweptback and unswept, have been flown by the NACA in the course of an investigation of drag at speeds up to Mach number 1.4. Some of the missiles lost their wings in flight; subsequent models of these missiles were flown successfully after the wings were reinforced with aluminum sheet bonded to the upper and lower surfaces in order to increase the torsional stiffness sufficiently to satisfy the criterion presented in the present paper.

A history of the flight experience with missiles is summarized in figure 2, which compares the actual wing stiffnesses (measured or calculated) with the stiffnesses required to prevent flutter according to equation (1). The data for figure 2 are shown in table I. It is to be noted that all missile wings with torsional stiffnesses that fall above the straight-line plot of the stiffness criterion did not fail in flight. The presence below the line of the two points representing missiles that did not fail indicates some conservatism of the criterion.

CONCLUDING REMARK

The usefulness of the torsional stiffness criterion presented has been demonstrated by the results of a limited number of flight tests of missiles with uniform, sweptback and unswept wings. However, the criterion should be regarded as subject to modification or replacement as experimental and theoretical data in greater quantity and at speeds higher than Mach number 1.4 become available.

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APPENDIX A

DERIVATION OF TORSIONAL STIFFNESS CRITERION

The following analysis refers throughout to unswept wings. However, considerable unpublished NACA data, as well as the data of reference 1, indicate that a wing of given L and c (fig. 1(a)) has a higher flutter speed if it is sweptback than if it is unswept. Hence, the stiffness criterion developed should be conservative when applied to sweptback wings.

Flutter at low subsonic speeds.- In reference 2, Theodorsen and Garrick present the following empirical formula for the flutter at low subsonic speed of a two-dimensional wing (fig. 1(c)):

$$\frac{v_f}{b\omega_\alpha} = \sqrt{\frac{r_\alpha^2}{\kappa} \frac{1/2}{\frac{1}{2} + a + x_\alpha}} \tag{A1}$$

The formula is stated to be reasonably good for wings having small ω_h/ω_α and small κ. The following theoretical formula for the divergence speed of a two-dimensional wing is also given in reference 2:

$$\frac{v_d}{b\omega_\alpha} = \sqrt{\frac{r_\alpha^2}{\kappa} \frac{1/2}{\frac{1}{2} + a}} \tag{A2}$$

Using strip theory, several authors (references 3, 4, and 5) have given as the divergence speed of an unswept uniform three-dimensional wing (fig. 1)

$$v_d = \frac{\pi}{cL} \sqrt{\frac{GJ}{2\rho\left(\frac{\partial C_L}{\partial \alpha}\right) e}} \tag{A3}$$

Equations (A1) and (A2) differ only in that the term $\frac{1}{2} + a$ in the divergence equation is replaced by $\frac{1}{2} + a + x_\alpha$ in the flutter equation. Then, by analogy with equations (A1) and (A2), equation (A3) can be modified to give the flutter speed of a uniform three-dimensional wing by replacing e by d; thus,

$$v_f = \frac{\pi}{cL}\sqrt{\frac{GJ}{2\rho\left(\frac{\partial C_L}{\partial\alpha}\right)d}} \tag{A4}$$

The value of $\partial C_L/\partial\alpha$ to be used in equation (A4) is the two-dimensional value 2π multiplied by an aspect-ratio correction. In calculating divergence speeds by strip theory, Shornick (reference 6) makes the approximation

$$\frac{\partial C_L}{\partial\alpha} = \left(\frac{\partial C_L}{\partial\alpha}\right)_\infty\left(\frac{L/c}{\frac{L}{c} + 2}\right)$$

or

$$\frac{\partial C_L}{\partial\alpha} = \left(\frac{\partial C_L}{\partial\alpha}\right)_\infty\left(\frac{L}{L + 2c}\right) \tag{A5}$$

(For the case of a uniform wing with $L/c = 3.14$, this assumption gives for the divergence speed computed by strip theory, equation (A3), a value that differs by less than 1 percent from the exact result calculated by lifting-line theory by Hildebrand and Reissner (reference 5).) Use of equation (A5) in the flutter equation (A4) gives for the required torsional stiffness to prevent flutter at low subsonic speeds

$$GJ = \frac{2\rho v_f^2}{\pi^2}\left(\frac{L^3c^2d}{L + 2c}\right)\left(\frac{\partial C_L}{\partial\alpha}\right)_\infty \tag{A6}$$

<u>Flutter at high subsonic speeds</u>.- Equation (A6) may be extended to speeds up to about $M = 0.75$ by using the Glauert-Prandtl compressibility correction on $\left(\frac{\partial C_L}{\partial\alpha}\right)_\infty$; thus, in equation (A6) let

$$\left(\frac{\partial C_L}{\partial\alpha}\right)_\infty = \frac{2\pi}{\sqrt{1 - M^2}} \tag{A7}$$

This use of the Glauert-Prandtl correction is suggested by Garrick (reference 7) for wings with small ω_h/ω_α and small κ. From Garrick's study of numerical flutter calculations in reference 7, it may be concluded that for other types of wings this correction is conservative.

Flutter at transonic and supersonic speeds.- From the studies of Garrick and Rubinow (reference 8) on supersonic flutter, the following conclusion may be drawn: For wings having low ω_h/ω_α and low κ, and having the center of gravity ahead of the midchord position, the transonic range appears critical for bending-torsion flutter. That is, if the wing passes through the transonic range without fluttering, it will probably not flutter at higher speeds. Furthermore, even if the conditions specified on ω_h/ω_α, κ, and the position of the center of gravity are not wholly fulfilled, it is probable that if the wing passes safely through transonic speeds, it will not flutter until a Mach number considerably higher than 1 is attained.

With these considerations in mind, it appears that a procedure to prevent flutter of a large class of supersonic missiles is to design against transonic flutter. Since there is very little transonic aerodynamic information available, the method to be used is to extend the form of equation (A6). For the purpose of the present analysis it will be assumed that equation (A7) holds up to $M = 0.75$, and that between $M = 0.75$ and $M = 1$ (fig. 3),

$$\left(\frac{\partial C_L}{\partial \alpha}\right)_\infty = \frac{2\pi}{\sqrt{1 - (0.75)^2}}$$

$$\doteq 2\pi(1.51)$$

The design value of v_f will be taken as the velocity of sound. Then, substituting in the design formula (A6) the values

$v_f = 1120$ ft/sec (Velocity of sound at sea level)

$\rho = 0.002378\ \text{lb-sec}^2/\text{ft}^4$ (Density at sea level)

$$\left(\frac{\partial C_L}{\partial \alpha}\right)_\infty = 2\pi(1.51)$$

gives

$$GJ = 39.8\left(\frac{L^3c^2d}{L + 2c}\right)$$

where L and c are in inches, and GJ in pound-inches^2. Rounding off the value of the constant gives as the final design formula

$$GJ = 40\left(\frac{L^3c^2d}{L + 2c}\right) \tag{A8}$$

APPENDIX B

SYMBOLS

c	chord normal to leading edge (See fig. 1.)
d	$\dfrac{\text{Distance of center of gravity behind quarter-chord position}}{\text{Chord}}$
e	$\dfrac{\text{Distance of shear center behind quarter-chord position}}{\text{Chord}}$
L	length along leading edge (See fig. 1.)
v_d	divergence speed
v_f	flutter speed
GJ	torsional stiffness, ratio of torque to twist per unit length
M	Mach number
ρ	air density
$\dfrac{\partial C_L}{\partial \alpha}$	lift-curve slope, finite wing
$\left(\dfrac{\partial C_L}{\partial \alpha}\right)_\infty$	lift-curve slope, infinite wing
Λ	angle of sweepback

The following symbols and their definitions are essentially those of Theodorsen and Garrick, reference 2:

b	half chord, used as reference unit length
$\frac{1}{2} + a$	$\dfrac{\text{Distance of shear center behind quarter-chord position}}{\text{Half-chord}}$ (See fig. 1.); $\frac{1}{2} + a = 2e$
$\frac{1}{2} + a + x_\alpha$	$\dfrac{\text{Distance of center of gravity behind quarter-chord position}}{\text{Half-chord}}$ (See fig. 1.); $\frac{1}{2} + a + x_\alpha = 2d$

r_α $\dfrac{\text{Mass radius of gyration referred to shear center}}{\text{Half-chord}}$

κ ratio of mass of cylinder of air of diameter equal to chord of wing to mass of wing, both taken for equal length along span

ω_α angular frequency of uncoupled torsional vibration about shear center

ω_h angular frequency of uncoupled bending vibration

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TABLE I

DATA PLOTTED IN FIGURE 2

Missile	Λ (deg)	L (in.)	c (in.)	d	GJ (kip-in.^{2})	Flight result (a)
1	0	10.37	9.63	0.18	67.3	N
2	0	12.92	8.12	.18	52.2	N
3	0	14.63	6.88	.18	21.9	F
4	0	14.63	6.88	.18	145.0	N
5	0	10.37	9.63	.25	71.6	N
6	34	8.63	10.67	.18	106.0	N
7	34	12.51	7.90	.18	31.8	N
8	34	15.25	6.56	.18	24.6	N
9	34	15.25	6.56	.18	109.7	N
10	34	17.63	5.69	.18	11.4	F
11	34	17.63	5.69	.18	95.2	N
12	45	10.11	9.10	.18	55.8	N
13	45	14.75	6.88	.18	18.3	N
14	45	18.28	5.74	.18	13.0	F
15	45	20.63	4.88	.18	6.8	F
16	45	14.75	6.88	.25	20.4	F
17	45	14.75	6.88	.25	147.6	N
18	52	11.61	7.94	.18	32.5	N
19	52	16.88	5.88	.18	9.7	F
20	52	16.88	5.88	.18	91.6	N
21	52	21.00	5.00	.18	7.6	F

[a]N - No failure
F - Failure

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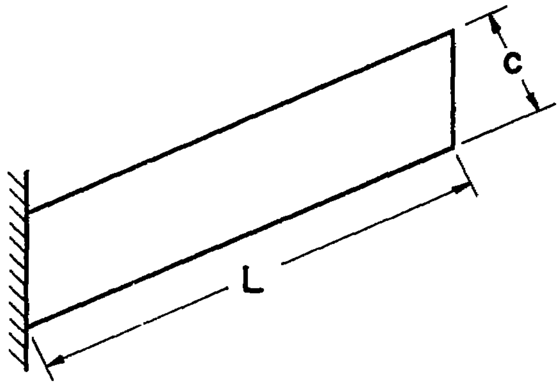
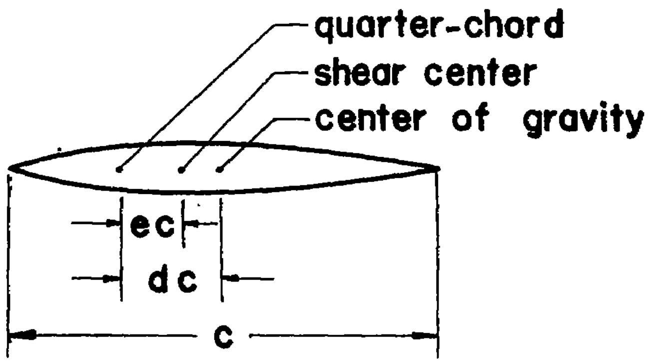
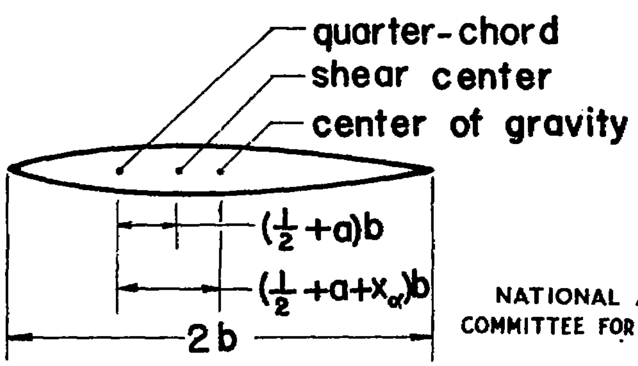


(a) Uniform sweptback wing.



(b) Notation of this report.



(c) Notation of Theodorsen and Garrick.

Figure 1.- Symbols for wing dimensions.

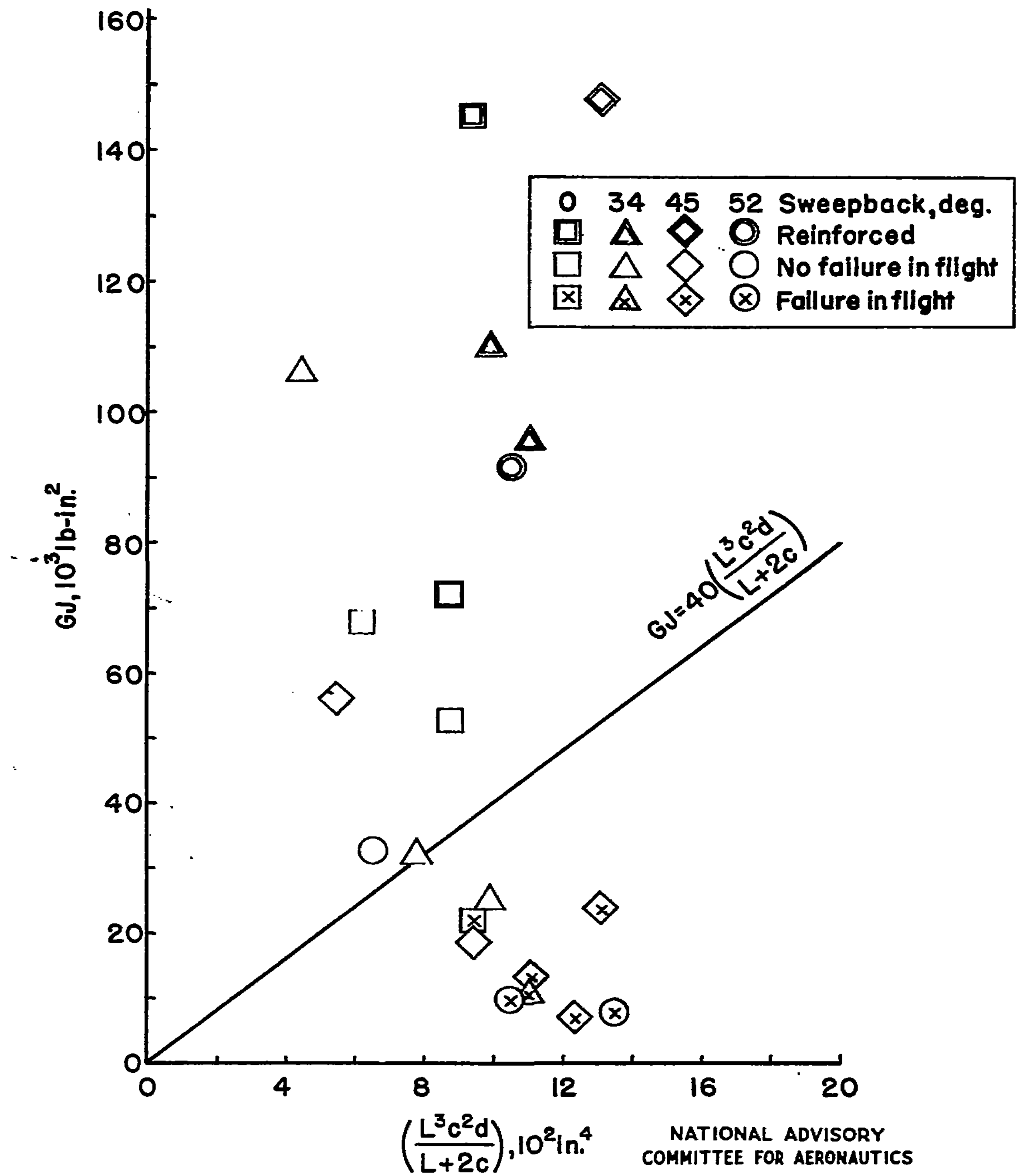


Figure 2.- Comparison of torsional stiffness criterion with flight test experience.

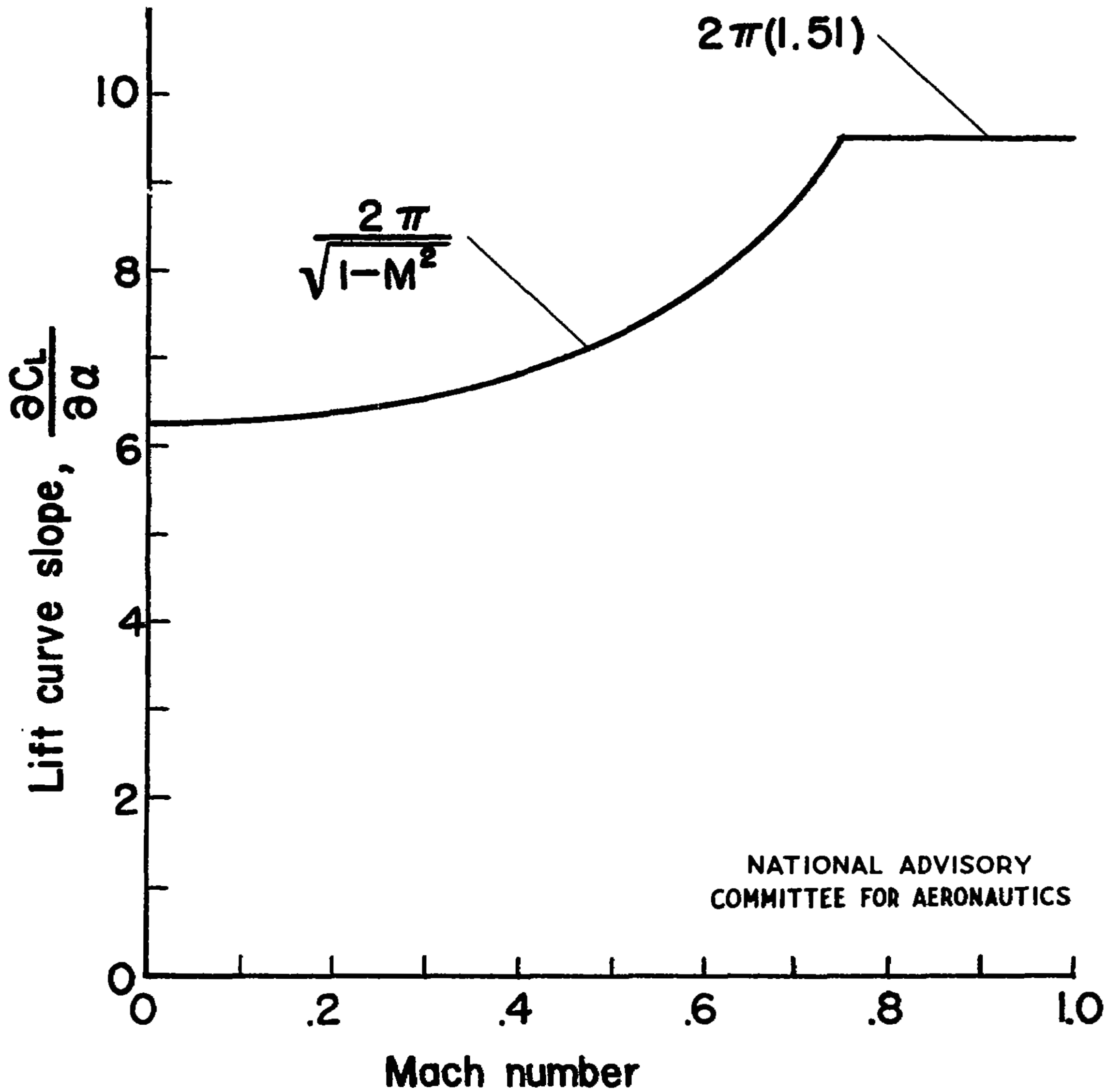


Figure 3.- Assumed variation of lift curve slope with Mach number for purposes of flutter analysis.