RESEARCH MEMORANDUM

EXPERIMENTAL INVESTIGATION OF A PRELOADED
SPRING-TAB FLUTTER MODEL

By

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An experimental investigation was made of a preloaded spring-tab flutter model to determine the effects on flutter speed of aspect ratio, tab frequency, and preloaded spring constant. The rudder was mass-balanced, and the flutter mode studied was essentially one of three degrees of freedom (fin bending coupled with rudder and tab oscillations). Inasmuch as the spring was preloaded, the tab-spring system was a nonlinear one. Frequency of the tab was the most significant parameter in this study, and an increase in flutter speed with increasing frequency is indicated. At a given frequency, the tab of high aspect ratio is shown to have a slightly lower flutter speed than the one of low aspect ratio. Because the frequency of the preloaded spring tab was found to vary radically with amplitude, the flutter speed decreased with increase in initial displacement of the tab.

INTRODUCTION

Fin-rudder-tab flutter has been found to be a significant problem in airplane design. An investigation of tab flutter has consequently been made at the Langley Memorial Aeronautical Laboratory of the National Advisory Committee for Aeronautics. The results of flutter tests of a vertical tail assembly for a medium bomber are reported in reference 1. Interest in the effects to be obtained with a preloaded spring in the rudder-tab-control circuit led to the present investigation, in which tests were made of a fabricated tab-flutter model of rectangular plan form.

The idea of the preloaded spring tab is many years old. A description of this mechanism is found in reference 2. Before an attempt is made to analyze the preloaded spring tab, a brief statement concerning the functions of a nonpreloaded spring-tab type of control is considered desirable. One of the features of the nonpreloaded spring-tab control, which is equipped with a weak spring,
is that it enables a pilot to attain lightness of control at high speeds without close aerodynamic balance and the attendant risk of overbalance. This type of control is similar to a free servotab control. However, at low speeds, a stiff spring (irreversible control) is desirable to retain positive action with appropriate pilot "feel" as obtained in the geared-tab control. The use of a stiff spring is also advantageous because it reduces the danger of flutter to which a weak spring-tab control is subject. The preloaded spring-tab control (very stiff at small amplitudes) has been suggested as a means of combining the desirable features of the non-preloaded spring tab at both low and high speeds. The increase in stiffness can make the system sufficiently irreversible at small amplitudes to eliminate the necessity of mass balance. The resultant saving in weight may be appreciable. Such a preloaded mechanism presents a nonlinear problem dependent on the amplitude of the tab.

Except for this nonlinearity of the spring constant, the flutter parameters for a preloaded spring-tab system are the same as those for a nonpreloaded spring-tab system. Some of these parameters are fin, rudder, and tab natural frequencies and stiffnesses, moments of inertia, mass and aerodynamic balancing, tab product of inertia, gear ratio of tab movement compared to rudder movement, rudder and tab aspect ratios, and damping. Only a few of these parameters were investigated for the present study with the main emphasis on the effect of the preloaded spring type of control. A theoretical tab-flutter analysis was not undertaken at this time.

SYMBOLS

d_{cg} distance from center of gravity of tab to tab hinge line, inches

f tab experimental frequency with rudder stationary for 3.25° tab deflection, cycles per second

f_1 approximate experimental tab frequency for 8.25° tab deflection, cycles per second

f_2 tab calculated natural frequency for 8.25° tab deflection, cycles per second

I_r rudder moment of inertia about hinge line, inch-pound-second^2

I_t tab moment of inertia about hinge line, inch-pound-second^2
K  product of inertia, inch-pound-second\(^2\) \(\int xy \, dm\), where \(x\) is distance from tab-hinge line to element mass \(dm\) in tab, and \(y\) is distance from rudder hinge line to same element of mass

\(k\)  spring constant of spring in preloaded spring mechanism, pounds per inch

\(k_1\)  piano-wire spring constant, pounds per inch

\(k_2\)  approximate combined spring constant for 8.25° tab deflection, pounds per inch

\(n\)  gear ratio, tab deflection divided by rudder deflection

\(v_f\)  flutter speed at sea level, miles per hour

\(Z\)  static unbalance of tab, inch-pounds \(\int S \, dx\), where \(S\) is unbalance of tab per unit of tab spanwise distance and \(x\) is spanwise distance

\(x_0\)  half maximum amplitude of preloaded spring system, inches

\(F_0\)  preload force in preloaded spring system, pounds

\(m\)  mass of preloaded spring system, pounds-second\(^2\) per inch

\(M\)  Mach number

APPARATUS AND TESTS

Description of Model

A test model with a rectangular plan form was constructed for this experimental investigation. The model consisted of a horizontal and vertical tail assembly with the horizontal surface serving primarily as a support for the vertical fin-rudder-tab combination (fig. 1). The total weight was 325 pounds with the rudder and tab weighing approximately 30 pounds when mass-balanced. The moment of inertia of the rudder was 20.35 inch-pound-second\(^2\). A diagram showing the tab linkage is given in figure 2. Cross-sectional views of the test configurations used in the present investigation are shown in figure 3.
In one group of tests, a tab with aspect ratio of 2.78 was used with the model arranged as follows (fig. 3):

Configuration 1 - Geared tab, \( n = 0.5 \), where \( n \) is the ratio of tab deflection (relative to the rudder) to rudder deflection

Configuration 2 - Preloaded spring tab, control link in place, varying tab moment of inertia

Configuration 3 - Preloaded spring tab, no rudder control (control link removed), varying preloaded spring

Configuration 4 - Preloaded spring tab, piano-wire spring in rudder control (piano-wire spring system replacing control link), varying piano-wire spring constant

In a second group of tests a tab with aspect ratio of 6.25 and with the same area as the other tab was used with the model arranged as follows (fig. 3):

Configuration 5 - Preloaded spring tab, no rudder control, varying preloaded spring constants and tab moments of inertia.

Configuration 6 - Preloaded spring tab, piano-wire spring in rudder control, varying both piano-wire and preloaded spring constants.

In addition, the model was provided with a cable and spring weight system which could simulate the restraint in the control system of a full-scale airplane.

Instrumentation

Six midget accelerometer pickups and one rudder and one tab inductance position indicator were installed in the flutter model (fig. 4). A pickup was in each corner of the rudder, one in the front upper fin, and one in the middle of the right side of the horizontal stabilizer. One position indicator was at the lower front corner of the rudder, and the other at the middle of the tab. These position indicators and pickups were used in conjunction with bridges, amplifiers, and a recording oscillograph. The recording equipment, which includes that used in shaking the model, is shown in figure 5.
Installation

The model was mounted in the rear part of the test section of the Langley high-speed 7″ by 10-foot tunnel. Two mounts constructed of steel streamline tubing were securely bolted to the floor of the tunnel, one on either side of the model. The model was supported on four cantilever steel leaf springs rigidly bolted to the mounts and hinged on ball-bearings to the model at the ends of the horizontal stabilizer (fig. 6). Cables connected to the top rear of the rudder provided a means to start and stop flutter. These cables were brought together by a pulley system over the tunnel section. Another cable connected to a spring-trip mechanism was used to give the tab an initial displacement and sudden release.

Leads to the electronic equipment were brought out through the tunnel-floor turntable. A steel safety net was installed approximately three feet behind the model to minimize damage of the tunnel in case the model failed.

Preliminary Vibration Tests

Natural frequencies and modes of the tab-flutter model were first determined in preliminary vibration tests with the model on rigid mounts on a bedplate in the laboratory. For this study, a moving coil shaker was used. The results are given in table I.

Prior to the first wind-tunnel tests the frequencies were checked and confirmed with the model mounted in the tunnel. (See fig. 6.) The frequency of the model mounted on vertical leaf springs, which simulated fuselage side bending, was determined to be 600 cycles per minute. The rudder was mass-balanced for all tests. After a relatively weak piano-wire spring system was substituted for the control link, the rudder frequency was low compared with the tab frequency.

Curves showing decrements in amplitude for the various spring combinations were obtained by deflecting and releasing the tab. A typical record is shown in figure 7 giving the shape of the decrement curve and the change in frequency with amplitude for a preloaded spring tab.

Because the tab oscillated at different amplitudes in the various configurations, it was convenient for purposes of correlation to reduce all the tab natural-frequency data to frequencies at the tunnel tripping displacement of 8.25°. This procedure led to the frequency analysis of the preloaded spring system given
in the appendix in which three approximations to the experimental conditions were considered. These approximations were obtained by three methods: Method A is based on the maximum displacement intercept of the spring system; method B, on the differential equation for one-quarter cycle of the motion; and method C, on equal strain energy. Results of these analyses are given in figure 8, in which frequency ratio \( \frac{1}{2} \frac{\sqrt{k}}{\sqrt{m}} \) is plotted as a function of preloaded-spring-force ratio \( \frac{kx_0}{F_0} \).

Method C, based on equal strain energy, gave frequencies too high. Method B was the most rigorous mathematically, but even this method neglects the friction which is present in the actual model. The approximation based on the maximum displacement intercept (method A) gave tab natural frequencies closest to the experimental values and this method was therefore used in all calculations of the tab natural frequencies. Figure 9 shows the comparison of the results obtained by method A with experimental frequency data.

In some tests the control link was replaced with a piano-wire spring system (figs. 2 and 3). By use of a force-displacement analysis of the complete tab-rudder linkage system, the effective constant of the preloaded spring system \( k_2 \) was combined with the piano-wire spring constant \( k_1 \) to give the torsional stiffness designated as the combined spring constant \( k_2 \). With the knowledge of \( k_2 \) and the tab moment of inertia \( I_t \), the tab natural frequency \( f_2 \) of one configuration for an amplitude of 3.25° was calculated. These calculated values of the tab natural frequency agree reasonably well with the experimentally determined frequencies, \( f_2 \), as given in table II. The few cases of large deviation are probably caused by variations in damping.

Flutter Tests

Oscillograph records were made to record the natural frequencies of the model at zero airspeed for every run in the tunnel. Other oscillograph records were taken at various intervals of speed up to and beyond the initial flutter speed. For each record the tab was tripped to develop possible flutter at low speeds. In the neighborhood of flutter the tab was tripped at airspeed increments of 10 miles per hour. The average tripping displacement was 3.25°.

Although an attempt was made to extrapolate to the flutter-speed point by plotting aerodynamic damping against speed for various
speeds of one configuration, no particular success was attained because of scatter in the data. The flutter point was determined by visible means at the initial tripping displacement and is believed to be accurate within 15 miles per hour.

RESULTS AND DISCUSSION

A summary of the results is found in table II.

In the tests of configuration 1 (geared-tab case) flutter did not occur, nor was flutter expected since the spring system was comparatively stiff. Tunnel speeds up to 300 miles per hour were attained with no tendency for the model to flutter. Tunnel speeds for subsequent tests were limited to less than 200 miles per hour in order to save the model.

In the tests of configuration 2, the moments of inertia of the tab were changed successively from 0.0613 inch-pound-second$^2$ to 0.0776, 0.1370, and 0.2851 inch-pound-second$^2$ by adding distributed weight along the trailing edge of the tab. (For comparison, the moment of inertia of the tab studied in reference 1 was 0.1020 in.-lb-sec$^2$.) In this set of tests there was no flutter. The probable reason for the lack of flutter was again the relatively high spring stiffness of the system.

In order to obtain flutter in the desired speed range, the control link was removed, and the spring stiffness of the tab system was then materially reduced. With the removal of this link (configuration 3), flutter occurred during the tests. This flutter was essentially a three-degree-of-freedom flutter involving fin bending coupled with rudder and tab oscillations about their respective hinge lines (tab lagging by 30°). A typical oscillograph flutter record is shown in figure 10. This record shows the relative positions of the various components of the model and the acceleration (g units) of each of these components during the first 0.45 second of flutter. During the flutter, the tab amplitude was sufficiently large to bend the tab connecting link. This link, a duralumin tube, was replaced with one of steel.

The effect of changing the preloaded spring was investigated. Springs of 8.3 pounds per inch with 2.25 pounds preload, of 33.3 pounds per inch with 11.3 pounds preload, and of 83 pounds per inch with 21.6 pounds preload were used. Flutter speed $v_f$ of the model increased with an increase in preloaded spring constant (fig. 11). The significance of this figure is qualitative.
It was also observed that increasing the initial tab displacement caused the model to flutter at a lower speed. With an 8.3-pound-per-inch spring, flutter was encountered at 102 miles per hour for a large tab deflection; whereas at 150 miles per hour the slightest disturbance initiated flutter with rapidly increasing amplitude. This dependence of initial flutter speed on amplitude has many practical implications because the roughness of the air, the presence of gusts, and the type of maneuver have a definite effect on the initial deflections of the tab. Table II shows the change in flutter speed $v_f$ with the variation of the parameters and indicates particularly the importance of tab natural frequency.

In the tests of configuration 4 a piano-wire spring was installed in the control system (fig. 2) and thereby increased the effective combined spring constant of the tab-rudder combination as compared with the spring constant for the configuration with no rudder control. Flutter similar to that obtained in the tests of configuration 3 occurred at higher speeds. At these higher speeds this flutter was more difficult to control and emergency shutdowns of the tunnel were necessary in order to save the model.

For configuration 5, the tab was replaced with one of higher aspect ratio and the moment of inertia of the tab was varied from 0.0246 inch-pound-second$^2$ to 0.1017, 0.0885, and 0.0780 inch-pound-second$^2$. The ratios of products of inertia to tab moments of inertia $K/I_t$ were determined. However, the changes in $K/I_t$ were too small to indicate the variation of $v_f$ with $K/I_t$.

In conjunction with changes in $I_t$, the effect of three different springs in the preloaded spring mechanism was investigated. For the springs with the low constant no variation of $v_f$ with $I_t$ was observed. For the spring with the higher constant, $v_f$ increased with a decrease in $I_t$ as indicated in table II. This result may be explained by the following considerations. Lowering the tab moment of inertia caused the tab frequency to increase, other things being equal; thus, $v_f$ tended to increase. However, the variation of $v_f$ with changes in $I_t$ is greater and consequently more easily observed at high frequencies than at low frequencies.

The tests of configuration 6 were made to permit a study of tab flutter with the combined spring constants varied by changing both the piano-wire and preloaded springs. This variation in turn changed the tab frequency and consequently $v_f$. Near the end of this set of tests, play in the linkage system had become appreciable and a flutter of low amplitude at 8 cycles per second was noticed at 150 miles per hour. The amplitude was approximately $10^\circ$, which was the amplitude allowed by the play in the system.
It was not possible in all cases to measure the tab natural frequency at the amplitude corresponding to the average tab tripping displacement of 8.25°. For purposes of correlating and presenting the data the natural frequencies of the tab were calculated by using a combined spring constant $k_2$. The data plotted in figure 12 show wide scatter, probably because of the variation in damping as well as the variation in the initial tripping amplitude.

The effect of aspect ratio is small. With all the flutter points on one graph, (fig. 12) there is an indication that, for the same frequency, the tab with the low aspect ratio has a higher flutter speed than the one of high aspect ratio. This effect is in agreement with the results of an analysis made by W. R. Griffin of Curtiss-Wright Corporation, in which strip theory was used. However, the possibility exists that the aspect-ratio effect is due, at least in part, to spanwise coupling.

CONCLUDING REMARKS

Results presented of tests of a preloaded spring-tab flutter model indicated that, with a rudder mass-balanced and at a low frequency compared with the tab frequency, the tab frequency appears to be the most significant parameter. Because the frequency of a preloaded spring-tab system was found to vary inversely with amplitude, the flutter speed decreased with an increase in initial displacement of the tab. Although the effect of aspect ratio was small, it was indicated that the tab with the low aspect ratio showed a tendency to flutter at a higher speed than the tab with a higher aspect ratio having the same area and frequency.

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METHODS OF CALCULATING THE NATURAL FREQUENCIES
OF A PRELOADED SYSTEM

The calculations of the natural frequencies of a preloaded spring-tab system might be approached in several ways. For example, in figure 13, the forces, displacements, and other parameters involved in such calculations are represented to illustrate three methods of approach. Method A is based on the maximum displacement intercept of the system; method B is based on the differential equation for one-quarter cycle of the motion; and method C is based on the concept of equal strain energy.

Method A.- The approximation by method A proceeds as follows: Consider first the force-displacement diagram of a preloaded spring system. (See fig. 13(a)). In place of the actual system A B C D, choose the path A O D along which the slope \( k_a \) is always finite and constant. The use of the effective spring constant \( k_a \) mathematically reduces the discontinuous preloaded spring system to a linear one in which force equals \( k_a \) times displacement. The geometry of the figure shows that

\[
k_a x_o = k x_o + F_o
\]

or

\[
k_a = k + \frac{F_o}{x_o}
\]

The frequency of the sinusoidal motion is then given by

\[
f_a = \frac{1}{2\pi} \sqrt{\frac{k_a}{m}} = \frac{1}{2\pi} \sqrt{\frac{k + \frac{F_o}{x_o}}{m}}
\]

which may be written nondimensionally as

\[
\frac{1}{2\pi f_a} \sqrt{\frac{k}{m}} = \sqrt{\frac{1}{1 + \frac{F_o}{kx_o}}}
\]
Method B.- For the analysis according to method B, the system and symbols are given in figure 13(b). The differential equation of motion may be expressed as

\[ mx + kx = -F_o \quad (x < 0) \]

\[ mx + kx = F_o \quad (x > 0) \]

The solution of the equations is

\[ x = -\frac{F_o}{k} + A \cos (\omega t + \alpha) \quad (1) \]

By proper choice of the original time, the phase angle \( \alpha \) may be eliminated.

Although the motion is discontinuous, it is symmetrical about \( x = 0 \) so that it can be completely specified by an analysis of the motion between the point \( x = x_o \), where it starts from rest \( (t = 0) \), and the point \( x = 0 \) \( (t = \frac{1}{4f_b}) \). Substituting these limits into equation (1) gives

\[ x_o = -\frac{F_o}{k} + A \]

or

\[ A = x_o + \frac{F_o}{k} \]

and

\[ 0 = -\frac{F_o}{k} + \left(x_o + \frac{F_o}{k}\right) \cos \left[\omega \left(\frac{1}{4f_b}\right)\right] \]

or

\[ \cos \left(\frac{\omega}{4f_b}\right) = \frac{1}{1 + \frac{kx_o}{F_o}} \]
In nondimensional form this equation may be written

\[ \frac{1}{2\pi f_b} \sqrt{\frac{k}{m}} = \left( \frac{2}{\pi} \right) \arccos \left( \frac{1}{1 + \frac{kx_0}{F_0}} \right) \]

For zero preload, the nondimensional form reduces to the usual relation

\[ f_b = \frac{1}{2\pi} \sqrt{\frac{E}{m}} \]

**Method C.** In method C, the actual system ABCD (fig. 13(c)) is replaced by the straight line path HK, constructed so that the area \((N+M)\) equals area \((N' + P)\). The strain energy is equal to the area under the force-displacement curve; therefore,

\[ \text{Energy} = F_0x_0 + \frac{1}{2}kx^2 = \frac{1}{2}k_c x^2 \]

or

\[ k_c = k + \frac{F_0}{x_0} \]

The frequency may be written

\[ f_c = \frac{1}{2\pi} \sqrt{\frac{k_c}{m}} = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \sqrt{1 + \frac{2F_0}{kx_0}} \]

or, nondimensionally,

\[ \frac{1}{2\pi f_c} \sqrt{\frac{k}{m}} = \sqrt{1 + \frac{2F_0}{kx_0}} \]
REFERENCES


### TABLE I. - FREQUENCIES AND MODES
OF TAB-FLUTTER MODEL

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<td>Bottom of fin and through rudder and fin at 75 percent span measured from base</td>
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<td>Bottom of fin and diagonals from top and bottom of rudder to top and bottom rear of tab, respectively, and from bottom front to top rear of fin</td>
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<th>Configuration</th>
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**Without Rudder Control Link**

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**TABLE II.- TESTS OF A PRELOADED SPRING-TAB FLUTTER MODEL**

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Figure 1.- Preloaded spring-tab flutter model.
Figure 2. - Tab flutter model showing linkage.

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Figure 3. - Test configurations.
Figure 4. - Tab flutter model showing position of accelerometers and position indicators. (Dimensions are in inches.)
Figure 5.- Electronic instrumentation of the preloaded spring-tab flutter model.
Figure 6.- Preloaded spring-tab flutter model mounted in the Langley high-speed 7- by 10-foot tunnel.
Figure 7. Oscillograph record showing decrement curve at zero airspeed. Tab aspect ratio, $2.78$; $L_t = 0.137$; 83 pound spring with 21.6 pounds preload.
Figure 8. - Frequency ratio as a function of preloaded-spring-force ratio.
Figure 9.— Frequency as a function of tab displacement. Tab aspect ratio, 2.78; $I_t = 0.1370$; 33.3 and 83 pounds per inch spring constants.
Figure 10.- Typical flutter record. Aspect ratio, 2.78; speed, 150 miles per hour; 
$M = 0.21$; tab frequency, 6.40 cycles per second for a tab deflection of $8.25^\circ$. 
Figure 11.- Flutter speed as a function of the square root of pre-loaded spring constant. No rudder control; aspect ratio, 2.78; \( I_t = 0.137 \text{ inch-pound-seconds}^2 \).
Figure 12. - Flutter speed as a function of calculated natural frequency for a tab deflection of 8.25°. Aspect ratio, 2.78 and 6.25.
Figure 13. - Methods of calculating the approximate natural frequency of a preloaded spring system.