RESEARCH MEMORANDUM

REDUCTION OF PROFILE DRAG AT SUPersonic VElocITIES

BY THE USE OF AIRFOIL SECTIONS HAVING

A BLUNT TRAILING EDGE

By Dean R. Chapman

Ames Aeronautical Laboratory
Moffett Field, Calif.

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SUMMARY

A preliminary theoretical and experimental investigation has been made of the supersonic aerodynamic characteristics of blunt-trailing-edge airfoils. Calculations of the drag of a family of airfoils with finite trailing-edge thickness are presented for various values of the base pressure. Theoretical expressions for the lift, pitching moment, and maximum lift-drag ratio are developed using the Busemann second-order theory for two-dimensional supersonic flow. In order to compare the theoretical estimates with experimental data, measurements were taken of the lift and drag on wings of various airfoil sections at Mach numbers of 1.5 and 2.0 and at Reynolds numbers varying from 0.2 to 1.2 million. Rectangular plan forms with an aspect ratio of 4 and a thickness ratio of either 10 or 9.1 percent were used throughout the experiments.

The experimental findings are in accord with the theoretical considerations in indicating a decrease in profile drag and an increase in lift-curve slope for properly designed airfoils with moderately blunt trailing edges. As compared to a 10-percent-thick double-wedge airfoil of equal section modulus, reductions in profile drag of 15 to 31 percent have been measured in the Mach number and Reynolds number range investigated. As compared to sharp-trailing-edge airfoils in general, the experimental results showed an increase in lift-curve slope of 17 percent for a 10-percent-thick airfoil with the maximum thickness located at the trailing edge.

The minimum drag of blunt-trailing-edge airfoils depends to a large extent on the profile shape near the trailing edge. As a result, the improper design of a blunt-trailing-edge airfoil may lead to an increase in minimum drag coefficient. It is shown that, in such cases, the maximum lift-drag ratio is not necessarily reduced since the lift-curve slope may be sufficiently improved to more than compensate for a small increase in minimum drag.
The trends to be followed in designing airfoils with lower drag and improved structural characteristics are briefly discussed in light of the present results and existing knowledge about base pressure in two-dimensional flow. It is concluded that in many cases the combined structural and aerodynamic advantages offered by blunt-trailing-edge airfoils are sufficient to warrant their use as a practical wing section.

**INTRODUCTION**

The first experimental measurements at supersonic velocities of the aerodynamic characteristics of a blunt-trailing-edge airfoil appear to have been made in 1933 by Busemann and Walchner (reference 1). In this supersonic wind-tunnel investigation a wedge airfoil was included among the various profiles tested. Since a symmetrical sharp-trailing-edge airfoil of comparable thickness was not included among the profiles investigated, very little information about the relative drag of sharp- and blunt-trailing-edge airfoils can be obtained from these early experiments. Both the theoretical and experimental results of this investigation showed, however, that the wedge airfoil produces a greater lift-curve slope than sharp-trailing-edge airfoils.

Subsequent to the work of Busemann and Walchner, and prior to the relatively recent investigation of Eggers (reference 2), practically no experimental data have been published on the characteristics of airfoils with blunt trailing edges. The investigation of reference 2 was concerned with the behavior of such airfoils at subsonic, rather than supersonic, free-stream velocities. The results showed that airfoils with maximum thickness located close to the trailing edge have remarkably good lift characteristics at subsonic supercritical velocities, but have undesirably high drag coefficients throughout most of the subsonic speed range. The high drag at low subsonic speeds has been known for many years and explains why very little attention has been paid in the past to the possibilities of blunt-trailing-edge airfoils.

At supersonic speeds there is no reason to presume that an airfoil with moderately blunt trailing edge will have higher drag than an airfoil with a sharp trailing edge. On the basis of an estimate made in reference 3 of the base pressure in two-dimensional flow, it has been concluded that the opposite, in fact, is probably more often closer to the truth. In this reference it was pointed out that the use of properly chosen airfoil sections having a blunt trailing edge would substantially decrease the pressure drag of the airfoil contour forward of the base, but would not necessarily introduce excessive base drag if the boundary layer near the trailing edge were relatively thick compared to the base height. The approximate numerical calculations given therein indicated that in some cases a properly designed blunt-trailing-edge airfoil could have from 20- to 30-percent lower profile drag than a corresponding airfoil with a sharp trailing edge. The present experimental investigation has been conducted in view of the possibilities suggested by these calculations.
Consequently, the primary purpose of the present investigation is to determine experimentally if a properly designed airfoil with moderately blunt trailing edge can have lower profile drag at supersonic velocities than a corresponding sharp-trailing-edge airfoil. Two additional purposes of the present report are: (1) to discuss qualitatively some of the more important parameters that are expected to affect the drag of blunt-trailing-edge airfoils, and (2) to make a cursory examination of the lift, lift-drag ratio, and pitching-moment characteristics of these airfoils in supersonic flow.

In practical applications, the many structural and aerodynamic factors which affect the selection of an airfoil are far too diverse to allow all-inclusive statements to be made about the superiority of one profile shape over another. Even if a selection is made on the basis of drag considerations alone without regard to lift, moment, or lift-drag ratio, then the optimum airfoil will vary with the particular structural criterion governing a given design. Notwithstanding these complications, certain simplified criteria for making drag comparisons can be used which closely represent a few of the numerous practical applications and approximately represent many others. The simplified criterion of equal section modulus is primarily used in this report, although in a few cases comparisons are made on the basis of equal thickness ratio. In most practical cases the actual drag reduction obtainable is believed to be greater than that indicated by the criterion of equal thickness ratio, since the structural properties of blunt-trailing-edge airfoils are generally superior to those of conventional sections.

It is emphasized that the airfoil shapes investigated in these preliminary tests aim solely at demonstrating certain principles, and do not aim at providing a near optimum airfoil section. The comparisons given herein attempt to illustrate only the approximate magnitude of drag reduction that may be possible in some cases. In considering the general possibilities of blunt-trailing-edge airfoils as practical wing sections, the structural characteristics must always be kept in mind. Moreover, in viewing the experimental results of this investigation it should be remembered that future research undoubtedly will provide means of achieving lower drag while still maintaining the structural advantages of these airfoil sections.

**SYMBOLS AND NOTATION**

- $A$: cross-sectional area of airfoil profile $\left[ \int_{0}^{c} (y_u+y_l) \, dx \right]$
- $c$: airfoil chord
- $c_d$: section drag coefficient
Symbol | Definition
--- | ---
c_{d_{\text{min}}}| minimum section drag coefficient (profile drag at zero lift)
c_{d_f}| section friction drag coefficient
c_{d_w}| pressure drag coefficient at zero lift for section forward
| of base (wave drag)
c_l| section lift coefficient
c_m| section pitching-moment coefficient taken about midchord
| position
(Ac_{d_{\text{min}}})_t| increment in profile drag for a given thickness ratio
(Ac_{d_{\text{min}}})_s| increment in profile drag for a given section modulus
C_D| wing drag coefficient
C_{D_{\text{min}}}| minimum wing drag coefficient
C_L| wing lift coefficient
C_1, C_2| constants appearing in Busemann second-order airfoil theory
h| trailing-edge thickness
(L/D)_{\text{max}}| maximum lift-drag ratio
M| Mach number
p| local static pressure
P| pressure coefficient \( \frac{P - P_{\infty}}{\frac{1}{2} \rho U^2} \)
P_{b_{\text{v}}}| base pressure coefficient for vacuum \( \frac{-2}{\gamma M_{\infty}^2} \)
R| Reynolds number
t| maximum thickness of airfoil
U| velocity
x| airfoil abscissa
y| airfoil ordinate
angle of attack

airfoil trailing-edge angle, measured between chord line and airfoil tangent line at the base

to ratio of specific heats (1.400 for air)
	n ratio of trailing-edge thickness to maximum thickness (h/t)

local angle of inclination of element on airfoil surface measured relative to the free-stream direction

mass density

Subscripts

base of airfoil

upper surface of airfoil

lower surface of airfoil

free stream

THEORETICAL ANALYSIS

General Considerations

The high drag of blunt-trailing-edge airfoils at very low Mach numbers is easily explained from existing knowledge of subsonic flows. At these low velocities the minimum drag of a well-designed airfoil consists primarily of skin friction. Any increase in trailing-edge thickness will not significantly alter the skin friction, but will increase the total drag through the addition of base drag and through the elimination of some of the pressure recovery normally obtained over the rear portion of a conventional airfoil. Thus, the usefulness of blunt-trailing-edge sections appears to be restricted to applications where a low drag at subsonic speeds is not of importance.

At supersonic speeds an increase in trailing-edge thickness will not necessarily lead to a drag increase, as some simple physical considerations will show. Before presenting these considerations, though, it will be advantageous to clarify one particular concept. Throughout this report the shape of a blunt-trailing-edge airfoil will be thought of qualitatively as being formed from a sharp-trailing-edge airfoil by increasing the trailing-edge thickness while maintaining the same chord length, rather than by simply cutting off the trailing edge. This latter viewpoint (removing part of the trailing edge) would needlessly complicate
matters because of the accompanying changes in the reference area on which force coefficients are based.

A double-wedge airfoil and a corresponding blunt-trailing-edge airfoil of equal chord are illustrated in figure 1. For simplicity, a common angle is used between all flat surfaces of the blunt-trailing-edge airfoil and the chord line. In comparison with a double-wedge airfoil of the same thickness ratio, the surfaces of the airfoil with a thick trailing edge are inclined at a smaller angle with respect to the chord line, thereby reducing the pressure drag of the profile contour forward of the base. On the other hand, it is apparent that considerable base drag may be introduced by employing a blunt trailing edge. Since the skin-friction drag is essentially the same for blunt- and sharp-trailing-edge airfoils, it follows that the profile drag will be lowered if the increase in base drag is less than the afore-mentioned reduction in pressure drag.

It will be illustrative to consider a particular example in order to demonstrate that, in certain cases at least, the profile drag can be reduced by increasing the trailing-edge thickness. A comparison of the drag of a 10-percent-thick double-wedge airfoil and a 10-percent thick wedge airfoil at a Mach number of 5 will serve to establish this point. By employing the customary shock-expansion method to calculate the wave-drag components of these two profiles, and by making the obviously conservative assumption that a vacuum exists at the base of the wedge, the following drag coefficients are obtained:

<table>
<thead>
<tr>
<th>Airfoil</th>
<th>Wave drag of profile forward of base</th>
<th>Base drag</th>
<th>Profile drag</th>
</tr>
</thead>
<tbody>
<tr>
<td>Double wedge</td>
<td>0.0091</td>
<td>0</td>
<td>0.0091 + c_d_f</td>
</tr>
<tr>
<td>Wedge (blunt trailing edge)</td>
<td>0.0024</td>
<td>0.0057</td>
<td>0.0081 + c_d_f</td>
</tr>
</tbody>
</table>

(vacuum at base)

This simple example showing lower drag for the wedge airfoil clearly illustrates two facts: First, at relatively high Mach numbers the conventional double-wedge section is not the optimum section for a given thickness ratio, and, second, the use of a blunt trailing edge can reduce profile drag by a substantial amount in this Mach number range. By using a more indirect method and by considering the characteristics at a Mach number of 8, these same two results have previously been pointed out by Ivey in reference 4.

If the above calculation were performed for an airfoil with thickness ratio much less than 10-percent or for a Mach number much less than 5, then the overly crude approximation of a vacuum at the base would indicate a higher drag for the wedge profile. In order for the use of a thick trailing edge to reduce the profile drag of thinner airfoils, or of airfoils in the lower supersonic Mach number range, the base drag must necessarily be considerably less than that represented by a vacuum.
Calculation of Profile Drag for Various Base Pressure Coefficients

Because very little is known about the base pressure in two-dimensional flow, the subsequent theoretical analysis will consist primarily of calculating the drag reduction that is possible for various base pressure coefficients. By comparing these results with the small amount of experimental data that are available, some indication can be obtained of the actual profile-drag reductions that may be expected.

In order to obtain a simple expression for the wave drag of the profile forward of the base, the linearized supersonic airfoil theory will be employed at present. If desired, a slightly more refined drag analysis could be made by using the conventional second-order theory of Busemann. Such an analysis, however, is not necessary for drag calculations since the drag coefficients of blunt- and sharp-trailing-edge airfoils differ even when only first-order terms are considered. The local pressure coefficient is then

\[ P = C_1 \theta \]  

(1)

where

\[ C_1 = \frac{2}{\sqrt{M_{\infty}^2 - 1}} \]  

(2)

and \( \theta \), the local angle of inclination, is measured positive for elements facing the oncoming wind. The particular airfoil sections that will be used in the drag calculations consist of straight-side symmetrical contours, as illustrated in figure 1. At present only conditions at zero angle of attack will be considered, so that by symmetry \( \theta_u = \frac{dy}{dx} = -\theta \) and the pressure drag of the contour forward of the base becomes

\[ c_{d_w} = 2C_1 \int_0^1 \left( \frac{dy}{dx} \right)^2 d\left( \frac{x}{c} \right) \]  

(3)

The profile drag is the sum of the base drag, skin-friction drag, and pressure drag of the profile forward of the base. For an airfoil of thickness ratio \( t/c \) and trailing-edge thickness \( h = \eta t \), the pressure drag from equation (3) becomes

\[ c_{d_w} = \frac{(t/c)^2}{\sqrt{M_{\infty}^2 - 1}} (2-\eta)^2 \]  

(4)

and the profile drag is
Here a minus sign is needed for the term involving \( P_b \) since a negative value of \( P_b \) corresponds to positive base drag. Equation (5) assumes that the flow does not separate at any point forward of the base. The subscripts 1 and 2 will be used to denote the double-wedge and the blunt-trailing-edge airfoil, respectively. It follows from equation (5) that

\[
c_d = \frac{(t/c)^2}{\sqrt{M_\infty^2 - 1}} \left( (2-\eta)^2 - P_b \frac{h}{c} + c_{d_f} \right)
\]  

(5)

The fractional difference in drag between the two airfoils is defined as

\[
\Delta c_d = \frac{c_{d_2} - c_{d_1}}{c_{d_1}}
\]  

(8)

With this definition, negative values of \( \Delta c_d \) correspond to a decrease in profile drag of the blunt-trailing-edge airfoil as compared to the double-wedge airfoil. If the two airfoils (fig. 1) are compared on the basis of equal thickness ratio \( t_2 = t_1 = t \) and, if it is assumed that \( c_{d_{f_1}} = c_{d_{f_2}} \), then substitution of equations (6) and (7) into (8) yields

\[
\frac{(\Delta c_d)_t}{c_{d_1}} = \frac{-\eta + \frac{\eta^2}{4} + \frac{(-P_b \sqrt{M_\infty^2 - 1})\eta}{4(t/c)}}{1 + \frac{c_{d_f} \sqrt{M_\infty^2 - 1}}{4(t/c)^2}}
\]  

(9)

The subscript \( t \) indicates that the thickness ratio is the same for both airfoils. If the airfoils are compared on the basis of equal section modulus \( t_2^2 = t_1^2 \frac{2-\eta}{2-\eta^3} \) and equation (8) becomes
It can be seen from equations (9) and (10) that the value of \( \eta \) which gives the greatest reduction in profile drag will depend only on the parameter \( \frac{(-P_b) \sqrt{M_\infty^2 - 1}}{t/c} \). Hence, the magnitude of the drag reduction will increase if the product \( \frac{|P_b| \sqrt{M_\infty^2 - 1}}{M_\infty^2 - 1} \) is decreased, or if the thickness ratio \( t/c \) is increased. The vacuum pressure coefficient \( P_{bv} \) and the product \( P_{bv} \sqrt{M_\infty^2 - 1} \) are shown in figure 2 as a function of the Mach number. The maximum drag reductions possible in comparison to a 10-per cent-thick double-wedge airfoil have been calculated for \( P_b/P_{bv} = 0, 1/4, 1/2, 3/4, \) and 1, and for \( c_{df} = 0.0028 \). This value of the skin-friction coefficient corresponds to laminar flow at a Reynolds number of 1 million. The results are shown in figures 3 and 4. Figure 3 represents the case of equal thickness ratio, and figure 4 the case of equal section modulus. For each curve in these figures the corresponding range of \( \eta \) is indicated. Beyond a Mach number of about 2 the value of \( \eta \) producing the greatest drag reduction increases as the Mach number is increased, and decreases as the base drag is increased. Moreover, for a fixed value of \( \eta \) the greatest drag reductions are obtained at relatively high Mach numbers. This is to be expected since the quantity \( \frac{|P_b| \sqrt{M_\infty^2 - 1}}{M_\infty^2 - 1} \) decreases with increasing Mach number, as shown in figure 2. Thus, the greater drag reduction at high Mach numbers is explained qualitatively by the fact that under the assumed conditions the base drag coefficient decreases more rapidly with increasing Mach number than does the pressure drag coefficient of the airfoil contour forward of the base. It should be remembered that in an actual case the ratio \( P_b/P_{bv} \) probably will change considerably as the Mach number is changed.

From the curves in figure 4 it is apparent that, for airfoils of approximately 10-per cent-thickness ratio, significant reductions in drag can be achieved, provided the ratio \( P_b/P_{bv} \) is less than about one-half at Mach numbers near 1.5, or less than about three-fourths at Mach numbers near 2.5. At Mach numbers near and beyond about 4, appreciable drag reductions can be achieved for these relatively thick airfoils even if a vacuum exists at the base. The experimental measurements of references 1, 5, and 6 indicate a value of approximately 0.6 for the ratio \( P_b/P_{bv} \). These tests, however, were conducted in the low supersonic Mach number range on wedge airfoils with predominately laminar boundary layers which were relatively thin compared to the thickness of the trailing edge. If the boundary layer were very thick compared to the trailing-edge thickness, then the base drag would have been virtually zero. Hence, by using a moderate amount of bluntness it is
to be expected that the ratio $\frac{P_b}{P_{b\nu}}$ can be held considerably below 0.6. It is concluded, therefore, that the proper use of a blunt trailing edge on 10-percent-thick airfoils will enable drag reductions to be achieved in the low range as well as in the high range of supersonic Mach numbers.

For thinner airfoils the preceding conclusion is still valid, although the percentage drag reductions are less for a given base pressure coefficient. Thus, for airfoils of 5-percent-thickness ratio, $\frac{P_b}{P_{b\nu}}$ would have to be a little less than one-fourth in order to achieve the same percentage drag reduction that is possible for 10-percent-thick airfoils with a $\frac{P_b}{P_{b\nu}}$ of one-half. Experiments are needed to answer the question of whether or not this ratio can be held to values of approximately one-fourth at the Reynolds numbers encountered in practical applications.

In addition to illustrating the conditions under which the use of bluntness will decrease the profile drag, equations (9) and (10) also illustrate the conditions under which the improper use of bluntness may lead to an increase in drag. For example, with $\frac{P_b}{P_{b\nu}} = \frac{1}{2}$, a wedge airfoil of 10-percent-thickness ratio at a Mach number of 1.5 will have approximately 13-percent higher drag than a double-wedge airfoil of the same thickness ratio. In general, airfoils with excessive thickness at the trailing edge will have considerably higher drag at low supersonic Mach numbers than airfoils with sharp trailing edges.

Lift and Pitching Moment

Apart from the effect of trailing-edge bluntness on profile drag, the accompanying effect on the lift characteristics at supersonic velocities can also be of practical importance. The conventional considerations of two-dimensional perturbation theory applied to sharp-trailing-edge profiles show that the lift-curve slope is independent of the airfoil shape even if second-order terms are considered. This statement, however, must be modified in order to apply to blunt-trailing-edge airfoils. Although the foregoing analysis of drag characteristics was restricted to symmetrical profiles of straight-line segments, the subsequent analysis of lift and pitching-moment characteristics is not restricted to any particular airfoil contour.

To the second order in angular deflections the pressure coefficient, according to the Busemann second-order airfoil theory, is

$$P = C_1\Theta + C_2\Theta^2$$

where $\Theta$ and $C_1$ are as previously defined and
The coordinates used in the calculations are shown in figure 5. Integrating the pressure coefficient over the airfoil contour yields the lift coefficient

\[ c_l = \int_0^1 (P_l - P_u) \, d\left(\frac{x}{c}\right) \]  

(13)

In this equation the small negative contribution of the base pressure to the lifting forces has been neglected, since it amounts to less than about 1 percent for \( t/c=0.05 \) and less than about 2 percent for \( t/c=0.10 \). The variable \( x \) is measured along the chord line which is arbitrarily defined as passing through the leading edge and bisecting the base at the trailing edge. On the upper surface the local angle of inclination is \( \theta_u = (dy/dx) u - \alpha \), and on the lower surface it is \( \theta_l = -(dy/dx) l + \alpha \). Substituting equation (11) into (13) and carrying out the detailed integration yields

\[ c_l = 2C_1\alpha + C_2 \int_0^1 \left\{ \left(\frac{dy}{dx}\right)_u^2 - \left(\frac{dy}{dx}\right)_l^2 \right\} d\left(\frac{x}{c}\right) \]  

(14)

\[ = 2C_1\alpha + C_2 \int_0^1 \left[ \left(\frac{dy}{dx}\right)_l^2 - \left(\frac{dy}{dx}\right)_u^2 \right] d\left(\frac{x}{c}\right) + 2 \frac{C_2\alpha}{c} \int_{x=0}^{x=c} (dy_u - dy_l) \]  

(15)

Since the last term in this equation is independent of angle of attack,

\[ \frac{dc_l}{d\alpha} = 2C_1 \left( 1 + \frac{C_2 h}{C_1 c} \right) \]  

(16)

The effect of trailing-edge bluntness for any airfoil contour, therefore, is simply to increase the section lift-curve slope by the factor \( 1+(C_2/C_1)(h/c) \). This expression was, in fact, given many years ago by Busemann (reference 1) for the case of a wedge airfoil. The above analysis simply brings to light a fact which is implicit in Busemann's equations, though not explicitly stated; namely, equation (16) expressing
the increase in lift due to bluntness does not depend on the airfoil shape forward of the trailing edge.

This increase in lift may be quite appreciable, particularly at relatively high Mach numbers, as is indicated by the curves in figure 6. In this figure the increment \( \frac{C_L}{C_D} \) is plotted as a function of Mach number for two cases: \( h/c = 0.05 \) and \( h/c = 0.10 \), which represent, for example, fully blunt \( h/t = 1 \) airfoils of 5- and 10-percent thickness ratios, respectively. From the curves in figure 6, it can be seen that at a Mach number of 5, for example, the theoretical increase in lift-curve slope amounts to as much as 30 percent for a fully blunt airfoil of 10-percent-thickness ratio. Hence, at these relatively high Mach numbers the effect of trailing-edge bluntness on lift-curve slope can be of considerable practical importance.

As far as the section pitching-moment curve is concerned, an equally simple result is obtained using the second-order theory. The pitching moment about midchord is

\[
C_M = \int_0^1 (P_m - P_l) \left( \frac{x}{c} - \frac{1}{2} \right) \, d\left( \frac{x}{c} \right) \quad (17)
\]

The algebraic details of substituting equation (11) into (17) and integrating will be omitted, as they are the same as encountered in calculating the lift coefficient. The resulting expression, which applies to an arbitrary airfoil shape, is

\[
\frac{dC_m}{d\alpha} = \frac{2C_L}{c^2} \left( A - \frac{hc}{2} \right) \quad (18)
\]

where \( A \) is the cross-sectional area of the blunt-trailing-edge airfoil. Thus, the derivative \( \frac{dC_m}{d\alpha} \) is simply proportional to the difference between the cross-sectional area of the blunt-trailing-edge airfoil and the area of a simple wedge having the same trailing-edge thickness. Since most airfoils have a greater area than a simple wedge of the same base height, it follows from equation (18) that the effect of bluntness is to move the center of pressure closer to the midchord position. In the special case of zero thickness at the trailing edge the moment-curve slope is proportional to the airfoil cross-section area, as was pointed out in reference 7.

**Maximum Lift-Drag Ratio**

The preceding analysis has shown that the minimum drag coefficient can be reduced by properly using bluntness at the trailing edge, and that by so doing the lift-curve slope always is slightly increased. Consequently, the accompanying change in maximum lift-drag ratio would
also be expected to be of importance. Since the lift-curve slope of sharp- and blunt-trailing-edge airfoils differs only when second-order terms are considered, the analysis which follows must consider terms of equal order throughout. Only profiles symmetric about the chord line will be considered here, as the algebra would otherwise become unduly involved without significantly affecting the final result.

The section drag coefficient to second order in angular deflection terms is

\[ c_d = \int_0^1 (P_l \theta_l + Pu \theta_u) d \left( \frac{x}{c} \right) + c_{db} + c_{df} \]  

(19)

Substituting equation (11) and noting that for symmetrical airfoils

\[ \left( \frac{dy}{dx} \right)_u = - \left( \frac{dy}{dx} \right)_l = \frac{dy}{dx} \]

\[ c_d = 2C_1 \int_0^1 \left[ \left( \frac{dy}{dx} \right)^2 + a^2 \right] d \left( \frac{x}{c} \right) + 6C_2 \int_0^1 \left[ a^2 \frac{dy}{dx} + \frac{1}{3} \left( \frac{dy}{dx} \right)^3 \right] d \left( \frac{x}{c} \right) + c_{db} + c_{df} \]

\[ = 2C_1 a^2 + 6C_2 a^2 \frac{h}{2c} + 2C_1 \int_0^1 \left[ \left( \frac{dy}{dx} \right)^2 + \frac{C_2}{C_1} \left( \frac{dy}{dx} \right)^3 \right] d \left( \frac{x}{c} \right) + c_{db} + c_{df} \]

For simplicity, the base drag coefficient will be taken as being approximately independent of \( \alpha \), then

\[ c_d = 2C_1 a^2 \left( 1 + \frac{3C_2 h}{2C_1 c} \right) + c_{dmin} \]  

(20)

Since

\[ c_l = 2C_1 \alpha \left( 1 + \frac{C_2 h}{C_1 c} \right) \]

the drag-lift ratio is approximately

\[ \frac{C_d}{c_l} = \frac{1}{1 + \frac{C_2 h}{C_1 c}} \left[ \alpha \left( 1 + \frac{3C_2 h}{2C_1 c} \right) + \frac{c_{dmin}}{2C_1 \alpha} \right] \]  

(21)

The minimum of this function occurs when
and the maximum lift–drag ratio for small values of $h/c$ is, accordingly,

$$\alpha^2 \approx \frac{c_d_{min}}{2C_1} \left(1 + \frac{3C_2h}{2C_1c}\right)$$

(22)

If the changes in minimum drag are small, this leads to the following approximate result: The percentage improvement in maximum lift–drag ratio is equal to one-half the percentage improvement in minimum drag plus one-fourth the percentage improvement in lift-curve slope. Consequently, it is possible for a blunt-trailing-edge airfoil to have a higher minimum drag coefficient, yet still have a higher maximum lift–drag ratio than a corresponding sharp-trailing-edge airfoil. For such a case to occur, it is necessary that the percentage increase in lift-curve slope exceed twice the percentage increase in minimum drag.

Parameters Affecting the Theoretical Characteristics of Blunt–Trailing–Edge Airfoils

The preceding theoretical calculations apply strictly only for two-dimensional flow. On the basis of existing knowledge it would be expected that the calculations of lift–curve slope would represent actual conditions reasonably well as long as three-dimensional effects, such as tip effects, are not large. In general, variations in airfoil–thickness ratio, type of boundary-layer flow, or shape of the airfoil contour forward of the base should not have an appreciable effect on the lift characteristics in two-dimensional flow. Such variations, however, may have a pronounced effect on the drag. The calculations made earlier, which illustrated lower drag for blunt-trailing-edge sections, were concerned only with specific flow conditions; namely, airfoil contours of straight sides, thickness ratio of 10 percent, and laminar flow in the boundary layer. Since the analysis has shown that sizable drag reductions may result under these specific conditions, the question immediately arises as to what may be expected when other conditions exist.

One parameter that is expected to have a significant effect on the drag of blunt-trailing-edge airfoils is the condition of the boundary layer just forward of the base. A change from laminar to turbulent boundary-layer flow is known to have a large effect on the base drag of bodies of revolution. In fact, negative base drag coefficients have actually been measured (reference 8) on certain highly boattailed bodies having a turbulent boundary layer approaching the base. This phenomenon
is accompanied by marked changes in the schlieren photographs of the axially symmetric flow. (See reference 8.) The trailing shock wave, which normally stands downstream of the boat-tailed base for laminar flow, moves upstream as transition is effected and attaches to the rim of the base, thereby reducing the base drag. The condition of the boundary-layer flow, therefore, should definitely be viewed as an important variable.

Another parameter that is expected to be important is the airfoil thickness ratio. If the thickness ratio is decreased, equations (9) and (10) indicate that the percentage drag reduction will also decrease substantially. This is easily explained on physical grounds since the drag reduction ultimately is obtained by a decrease in wave drag. The pressure drag, of course, progressively becomes a smaller fraction of the profile drag as the thickness ratio approaches zero. For very thin profiles, however, the boundary layer becomes thick compared to the trailing-edge height, and this should tend to reduce the base drag. The extent to which the profile drag can be reduced for airfoil ratios of, say, 5-percent-thickness ratio will have to be determined by future experiments.

Since the ambient air can flow laterally around the wing tip and into the dead-air region behind the base, there probably is a tip-relieving effect of a finite span. This inflow would be expected to reduce the base drag, particularly at high supersonic Mach numbers, and hence it would appear that a finite aspect ratio would be more favorable for blunt-trailing-edge wings than an infinite aspect ratio. Again, experiments are needed to establish the importance of this variable.

Some of the foregoing is, of course, conjectural in nature. The discussion of the various parameters that may affect the drag of blunt-trailing-edge wings has been given in order to emphasize the fact that there is as yet no simple answer to the question of whether blunt-trailing-edge airfoils can always be designed to have significantly lower drag than corresponding sharp-trailing-edge airfoils.

TEST METHODS

A description of the apparatus and the general procedure for testing wing models in the Ames 1-by 3-foot supersonic wind tunnel No. 1 may be found in reference 9. In order to simplify model construction as well as test methods, constant-chord wings of finite span were employed throughout the experimental phase of the present investigation. Each wing had an aspect ratio of 4 and was sting supported from the rear in the manner shown by the photograph in figure 7. The profile shape was the sole variable for the different wings tested. The dimensions of the various airfoil contours are given in figure 8. Wings 1, 2, 3, and 4, which have essentially the same section modulus, were tested only at zero angle of attack; whereas wings 5, 6, and 7, which have the same thickness ratio, were tested through the available angle-of-attack range.
Drag and lift forces were measured by means of a strain-gage balance which was shrouded from the external flow. Since the pressure in the balance chamber was greater than the free-stream static pressure, an appropriate correction for the "piston effect" has been applied to each drag measurement. This correction is based on the measured value of the pressure in the balance chamber and normally amounted to about 10 percent of the uncorrected force data. In reducing the profile drag data to coefficient form, an estimated correction of 0.0025 has been applied in each case to approximately account for the tare drag of the sting support. Because of imperfect alignment of the wings with the oncoming flow, a small lift force was measured on the symmetrical profiles with the wings nominally at zero angle of attack. Consequently, a correction based on the measured lift and linearized wing theory has been applied to the drag measurements in order to account for the small amount of drag due to lift. This latter correction usually amounted to 1 or 2 percent of the profile drag.

Since the Reynolds number of each wing is about 1 million at the highest tunnel pressure, laminar flow would be expected over the entire wing surface. This expectation was verified by the liquid-film technique, the details of which have been described in reference 10. Hence, in order to simulate the case of a turbulent boundary layer approaching the base, it was necessary to add artificial roughness to the wing surfaces. This was done by applying a narrow band of salt crystals on both sides of the wing at approximately the 25-percent-chord position. It is known that the addition of artificial roughness at supersonic speeds invariably produces a certain increment of wave drag which must be accounted for if the measured drag is to correspond approximately to conditions of natural transition. This incremental wave drag was estimated from the measured increase in profile drag caused by the addition of roughness to the double-wedge profile (wing 1). The accompanying change in friction drag was approximately accounted for by assuming low-speed skin-friction coefficients and the existence of turbulent flow over the rear half of the chord. The wave drag due to roughness, as estimated in this manner, has been subtracted from all data representing cases where artificial roughness was used.

The data presented have not been corrected for nonuniformities in the free stream. The small inaccuracies in the experimental technique, together with the fact that in the present tests no corrections have been applied for the stream nonuniformities, may introduce errors of the order of ±5 percent in the absolute value of the lift-curve slopes and drag coefficients. Such uncertainties, however, will not introduce any significant error in the difference between the force coefficients of two wings of identical plan form that have the same sting support, the same artificial roughness, and are tested in same position along the nozzle axis. In view of these common test conditions the measured increments in lift and in minimum profile drag of the various wings are believed to be practically unaffected by the possible experimental errors discussed above.
EXPERIMENTAL RESULTS AND DISCUSSION

Drag Measurements at Zero Lift

The results of drag measurements at zero lift for wings 1 and 2 at a Mach number of 1.5 are shown in figure 9. These data were taken with the wing surfaces smooth and represent the case of laminar flow in the boundary layer. In accordance with the theoretical expectations, the measurements at this Mach number show that the blunt–trailing-edge airfoil has a significantly lower drag than the double-wedge airfoil of the same section modulus. The drag reduction varies from 15 to 23 percent over the Reynolds number range encountered in the tests. The results of measurements on wings 1 and 2 at a Mach number of 2.0 are shown in figure 10. Also shown in this figure are the results for wings 3 and 4, which were obtained from wing 2 by modifying the base contour. At this Mach number, wing 2 has from 17– to 25–percent lower drag than wing 1. Wing 4 has from 25– to 31–percent lower drag than wing 1. The measured reductions in minimum drag with laminar boundary-layer flow approaching the base, therefore, are in satisfactory agreement with the theoretical considerations both at a Mach number of 1.5 and 2.0.

Some indication of the effect of finite span is given by the data for wing 1. The sum of the theoretical wave drag of this wing as calculated by the shock–expansion method, and the laminar skin-friction drag as calculated from low-speed values, is shown by the dotted lines in figures 9 and 10. These lines representing the theoretical values for two–dimensional flow are several percent higher than the corresponding measured values for the double–wedge profile. The direction of this discrepancy is the same as would occur if the flow separated from the surface downstream of the maximum thickness location. Such separation, which would tend to reduce the profile drag, was clearly shown to exist near the wing tips by the liquid–film technique.

The experimental values of minimum profile drag for wings 1 and 2 with artificial roughness added are shown in figure 11. These data, which have been corrected for the wave drag due to roughness, are for a Mach number of 2.0 and are representative of the case of turbulent flow approaching the trailing edge. The data for M=1.5 are not presented as they show essentially the same characteristics as the curves in figure 11. It is apparent from this figure that the drag reduction of wing 2 as compared to wing 1 is not as great as for the case of laminar flow approaching the base. This result indicates that on wing 2, which does not have appreciable boattailing, the base drag for turbulent boundary-layer flow is greater than for laminar boundary-layer flow.

As was discussed earlier, the experimental results for axially symmetric supersonic flow (reference 8) have shown that, with turbulent flow approaching the base, the base drag is greatly reduced by employing a moderate amount of boattailing. In view of this known result for bodies of revolution, the angle of boattailing at the base of wing 2 was
progressively increased to form wings 3 and 4. The measured values of minimum drag for the revised base shapes at a Mach number of 2.0 are also shown in figure 11. The observed reduction in profile drag as compared to wing 2 clearly indicates the importance of properly designing the airfoil contour near the trailing edge.

Measurements at Angle of Attack

Airfoil sections composed of circular-arc segments, as illustrated in figure 8 by wings 5, 6, and 7, were used for measuring the characteristics of the blunt-trailing-edge airfoils at angle of attack. The ratio of trailing-edge thickness to maximum thickness for these three wings is 0, 0.5, and 1.0, respectively. The measured lift curves and the drag polars at a Mach number of 1.5 with smooth wing surfaces are shown in figures 12 and 13.1 The corresponding characteristics at \( M=2.0 \) are not shown as they are similar to the results for \( M=1.5 \). It may be noted from figure 12 that wing 7, with the fully blunt trailing edge, has approximately a 17-percent greater lift-curve slope than wing 5. The theoretical increase, according to figure 6, is 12 percent. The difference between the theoretical and the measured increase in lift-curve slope is attributed to the difference in viscous effects between blunt- and sharp-trailing-edge airfoils. It is known from the experimental results of Ferri (reference 11) that, even at small angles of attack, the actual lift-curve slope of a sharp-trailing-edge airfoil is less than theory indicates because of flow separation ahead of the trailing edge. At low angles of attack the flow over an airfoil with maximum thickness at the trailing edge would not separate at any point on the airfoil surface. Thus it would be expected that the lift-curve slope of blunt-trailing-edge airfoils would approach the theoretical values more closely than sharp-trailing-edge airfoils. Hence it also would be expected that the measured increase in lift-curve slope due to bluntness would be greater than the theoretical increase calculated from second-order effects in an inviscid flow.

The effect of bluntness at the trailing edge on the drag polars is illustrated by the curves in figure 13. As in figure 12, the various curves in this figure are for airfoils with a common thickness ratio of 10 percent, and for smooth wing surfaces. The principal experimental

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1The lift curves in figure 12 do not pass through the origin of coordinates because the measured data are not corrected for the small stream angle existing in the test section. Although the observed angles for zero lift of the various wings should coincide, these curves show a slight discrepancy because of small constructional differences between the wings.
results for wings 6 and 7, in comparison to wing 5, are summarized in the following table:

<table>
<thead>
<tr>
<th>Wing</th>
<th>Bluntness h/t</th>
<th>Change in minimum drag</th>
<th>Increase in lift-curve slope</th>
<th>L/D increase according to equation (23)²</th>
<th>Observed increase in (L/D)_{max}</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<td>6</td>
<td>.5</td>
<td>-8%</td>
<td>+13%</td>
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<td>+8%</td>
</tr>
<tr>
<td>7</td>
<td>1.0</td>
<td>+4%</td>
<td>+17%</td>
<td>+2%</td>
<td>+3%</td>
</tr>
</tbody>
</table>

As is evident from these data, the theoretical expectations are again substantiated by the experimental measurements. In particular, the experimental results for wing 7 prove that even in those cases where a blunt-trailing-edge wing may have higher profile drag than a conventional section, it nevertheless is possible for it also to have a higher maximum lift-drag ratio. This, of course, is attributed to the improvement in lift-curve slope, and is evident graphically in figure 13 by the intersection of the two drag polars at a lift coefficient below that which yields maximum lift-drag ratio.

General Discussion

The foregoing comparison of theory and experiment shows that the theoretical predictions are qualitatively substantiated by the wind-tunnel measurements conducted on airfoils of approximately 10-percent thickness at Mach numbers of 1.5 and 2.0. In accordance with the theoretical calculations it is expected that the improvement in lift of blunt-trailing-edge airfoils over conventional sections will progressively increase as the Mach number is increased beyond about 2.0. Unfortunately, an analogous statement about the reduction in drag cannot be made because of the present limited knowledge about base pressure in two-dimensional flow. As regards thickness-ratio effects, however, simple physical considerations make it apparent that the improvement in lift and drag must approach zero as the airfoil thickness ratio approaches zero.

The failing of theoretical calculations which indicates that the biconvex and the double-wedge profiles are optimum for specific conditions is, of course, attributed to the assumption of a sharp trailing edge which has been made in previous analyses. The inadequacy of such analyses becomes even more apparent when it is recalled that in the present experiments no attempt has been made either to develop the optimum airfoil shape forward of the base or to use the optimum amount

²The observed change in minimum drag has been used in the evaluation of the increase in L/D from equation (23).
of bluntness at the trailing edge. It is apparent from the present results that extensive experimental work is needed before optimum airfoil shapes can be specified which are satisfactory for engineering purposes. As an example of this, the results show, contrary to the calculations made in reference 12, that at moderate supersonic Mach numbers the optimum airfoil for a given section modulus is not approximately a biconvex section. This is illustrated by the blunt-trailing-edge profile of wing 6 which has about 15 percent greater section modulus than the biconvex profile of wing 5, yet has less drag. (See fig. 13.) As another example, it also may be deduced from the experimental results that the double-wedge section is not close to the optimum for a given airfoil thickness ratio even at moderate supersonic Mach numbers. In particular, a double-wedge profile of the same thickness ratio as wing 4 (9.1 percent) would have about 18 percent less drag than wing 1, since the latter wing has a double-wedge profile of 10-percent thickness ratio. At some Reynolds numbers, however, wing 4 has as much as 30 percent less drag than wing 1 (fig. 10), which would mean about 12 percent less drag than a double-wedge profile of equal thickness ratio.

From the viewpoint of immediate practical application, an important engineering problem is that of determining how to avoid large drag increases when considerable bluntness is used on relatively thin airfoils in the low supersonic Mach number range. As was noted earlier, elementary considerations show that it will be difficult to achieve large drag reductions for very thin airfoils since the pressure drag is not a large portion of the profile drag. It is the thin sections, however, which are particularly critical as regards structural difficulties. For example, the depth of the airfoil at the hinge line of a flap is an important structural consideration. In this regard a significant improvement obviously can be obtained even with only a moderately blunt trailing edge. Hence, rather than to concentrate solely on determining optimum airfoil contours for minimum profile drag, it appears to be of equal practical importance to determine how to prevent an appreciable drag increase when employing considerable bluntness on relatively thin airfoil sections.

The fact that airfoils with blunt trailing edges have higher subsonic drag than conventional sections is a consideration that should be remembered in viewing the possibilities of applying blunt trailing edges to highly swept-back wings. The flow over the outboard regions of a highly swept wing is essentially of the subsonic type, even though the free-stream Mach number is supersonic. In such a case, a blunt-trailing-edge airfoil might increase the profile drag of the outer regions.

In discussing the possibilities of blunt-trailing-edge airfoils as a practical wing section, incidental advantages can be listed which may be of significance in some designs. For example, the improved structural characteristics near the trailing edge might allow a Fowler-type flap to be used in cases where it could not be used if a conventional airfoil section were employed. The control characteristics at high speeds can also be cited as a possible advantage. Recent experimental investigations
on a swept-back wing have indicated that the control effectiveness in the transonic range may be improved considerably by employing a blunt-trailing-edge aileron. (See, e.g., references 13 and 14.) Still another possible advantage that can be listed concerns the undesirable "flat spot" which has been observed in the hinge-moment curves of a conventional flap arrangement tested at low flap deflections and low angles of attack. (See reference 15.) This flat spot presumably would not occur for a control surface with a sufficiently blunt trailing edge, since the source of the trouble arises from separation of the flow forward of the trailing edge.

The various miscellaneous advantages such as those just mentioned, taken together with the general structural advantages and the improvement in certain aerodynamic characteristics, leave little doubt as to the practical usefulness of blunt-trailing-edge airfoils. Like many other examples of departure from conventional design, however, care must be exercised in designing airfoils with thick trailing edges. In this regard it is to be remembered that the highest Reynolds number in the present investigation is 1.2 million, and that additional experiments are needed before conclusions can be drawn about conditions at much higher Reynolds numbers.

CONCLUSIONS

The following conclusions have been obtained from a preliminary theoretical study and from an experimental investigation conducted with airfoils of approximately 10-percent-thickness ratio at Reynolds numbers between 0.2 and 1.2 million, and at Mach numbers of 1.5 and 2.0:

1. At supersonic velocities a properly designed airfoil having a blunt trailing edge produces a lower drag and a greater lift-curve slope than a conventional sharp-trailing-edge airfoil.

2. Further theoretical and experimental study of blunt-trailing-edge airfoils is needed before it is possible to specify the airfoil shape that is nearly optimum for a given structural requirement.

Ames Aeronautical Laboratory, National Advisory Committee for Aeronautics, Moffett Field, Calif.
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Airfoil 1
Symmetrical double-wedge profile

Airfoil 2
Symmetrical blunt-trailing-edge profile

Figure 1. - Airfoils considered in theoretical analysis.

Figure 2. - Limiting base pressure coefficient.
Figure 3. Estimated reductions in profile drag for equal thickness ratio; $\frac{t}{c} = 0.10$.

Figure 4. Estimated reductions in profile drag for equal section modulus; $\frac{t}{c} = 0.10$. 
\[ \theta_u = \left( \frac{dy}{dx} \right)_u - \alpha \]

Note: \( \theta \) measured positive for elements facing the oncoming wind.

**Figure 5.** Coordinates used in theoretical calculations.

**Figure 6.** Theoretical increase in lift-curve slope of blunt-trailing-edge airfoils as compared to sharp-trailing-edge airfoils.
Figure 7.—Typical model installation.

![Typical model installation diagram](image)

Double wedge

Biconvex

Circular arcs

<table>
<thead>
<tr>
<th>Wing</th>
<th>Thickness Ratio</th>
<th>h/t</th>
<th>Chord</th>
<th>Trailing-edge Half-angle, $\beta$</th>
<th>Relative Section Modulus</th>
</tr>
</thead>
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<tr>
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<td>0</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Figure 8. —Dimensions and properties of airfoils tested.
Figure 9.—Profile drag of wings 1 and 2; $M_\infty=1.5$, smooth surfaces.

Figure 10.—Profile drag of wings 1, 2, 3, and 4; $M_\infty=2.0$, smooth surfaces.
Figure 11. —Profile drag of wings 1, 2, 3, and 4; $M_\infty=2.0$, artificial roughness added.

Figure 12. —Lift curves of wings 5, 6, and 7; $M_\infty=1.5$, $Re=0.5 \times 10^6$. 

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Figure 13.—Drag polars for wings 5, 6, and 7; $M_\infty = 1.5$, $Re = 0.5 \times 10^6$. 

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