ANALYSIS OF EXPERIMENTAL SEA-LEVEL TRANSIENT DATA AND ANALOG METHOD OF OBTAINING ALTITUDE RESPONSE FOR TURBINE-PROPELLER ENGINE WITH RELAY-TYPE SPEED CONTROL

By George Vasu and George J. Pack

Lewis Flight Propulsion Laboratory
Cleveland, Ohio

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SUMMARY

Correlation has been established between transient engine and control data obtained experimentally and data obtained by simulating the engine and the control with an analog computer. This correlation was established at sea-level conditions for a turbine-propeller engine with a relay-type speed control.

The behavior of the controlled engine at altitudes of 20,000 and 35,000 feet was determined with an analog computer using the altitude pressure and temperature generalization factors to calculate the new engine constants for these altitudes. Because the engine response varies considerably at altitude some type of compensation appears desirable and four methods of compensation are discussed.

INTRODUCTION

The problem of controlling a turbine-propeller engine during transient conditions of operation is difficult because of two characteristics inherent in this type of power plant. First, the engine must operate near limiting values of both rotative speed and turbine-inlet temperature for optimum performance. Second, the moment of inertia of the rotating parts is large compared with that of a reciprocating engine of similar power output. The first condition imposes upon a control the requirement to maintain set values of controlled parameters within small ranges. The second characteristic increases the difficulty of changing engine speed. It is therefore necessary to know the exact nature of the engine response characteristics in order to design a control that will permit satisfactory operation of the engine over broad ranges of flight conditions.
The NACA Lewis laboratory has undertaken an investigation of the transient behavior of one type of turbine-propeller engine and several experimental control systems. The objectives of this program are: (1) to determine useful mathematical forms for describing the dynamic characteristics of typical engines and controls; (2) to evaluate control parameters; and (3) to determine the altitude compensation required by control systems.

An engine and its control can, in general, be characterized by equations that, when solved, describe the system response. In these equations are constants that apply to the engine and fix its behavior for a particular operating point. The constants change with power setting, altitude, and flight speed. As a result, many solutions may be required to investigate several conditions of operation. Nevertheless, if the equations are linear and of low order, classical methods can be used to obtain solutions.

Another method exists that gives the approximate response of linear systems and is usually employed when the equations are of high order. This method consists in determining the frequency-response characteristic of each of the components of a control system and then combining the characteristics of the components to form an over-all characteristic from which the approximate transient response can be determined. The dynamic characteristics of the engine have been shown in references 1 and 2 to be described by such a frequency-response representation of a linear system. Unfortunately, the dynamic characteristics of the control investigated herein cannot be so simply represented because it employs a relay whose action is nonlinear. Furthermore, ordinary methods of solving nonlinear equations entail very tedious calculations. It has therefore been deemed advisable to use an analog computer to simulate this system and thereby obtain transient solutions that predict the action of the prototype under various conditions.

This report shows correlation between actual transient data obtained from the engine and solutions to a simulated system set up on an analog computer. Included in the scope of this study is a description of the simulator and block diagrams essential to an understanding of the system. In order to achieve the required correlation, it is necessary to include transient data of engine and control operation. Data are presented for response of engine speed to changes in set speed when the power plant was being controlled by a relay-type electronic governor that acted to maintain set speed by varying propeller pitch. Responses to changes in fuel flow are not included. The data, which were obtained under sea-level conditions, were analyzed to determine system constants. These constants were then used in the analog phase of the investigation. Correlation obtained for sea-level conditions between actual time data and analog solutions is discussed. The engine time constant can be
generalized for altitude by applying the temperature and pressure generalization factors. Based on these variations of specific engine parameters with changes in altitude, additional analog solutions are presented that show altitude effects.

Several methods of adjusting the control constants to compensate for engine variations are also discussed.

SYMBOLS

The following symbols are used in this report:

- $K_e$: engine gain, (rpm/deg)
- $K_f$: feedback or stabilizing network gain, dimensionless
- $K_p$: gain associated with pitch-change mechanism, (deg/rpm)
- $N$: speed, (rpm)
- $N_f$: feedback signal from stabilizing network, (rpm)
- $N_{f,m}$: maximum value of feedback signal, (rpm)
- $N_s$: set speed, (rpm)
- $P$: differential operator $\frac{d}{dt}$, (sec$^{-1}$)
- $t$: time, (sec)
- $X$: one-half of relay dead zone, (rpm)
- $Y$: relay output when energized, (rpm)
- $\beta$: propeller pitch, (deg)
- $T_e$: engine time constant, (sec)
- $T_f$: feedback or stabilizing network time constant, (sec)
- $T_\beta$: time constant associated with pitch-change mechanism, (sec)

Subscripts:
- $i$: initial value
- $f$: final value
DISCUSSION OF ANALOG AND BLOCK DIAGRAMS

Electronic Analog

The computer used in this investigation was of the electronic-analog type consisting of numerous components that perform basic mathematical operations. These units can be interconnected to simulate desired systems. A photograph of the computer is presented in figure 1.

All inputs and outputs are voltages that permit the functions to be displayed on oscilloscopes. The analog is so constructed that any desired type of initial disturbance can be used as a forcing function. Disturbances most commonly used, however, are step or ramp functions, which can be applied in the form of a recurrent wave so that transient solutions of the system will appear as stationary patterns by synchronizing oscilloscope sweep with the recurrent frequency of disturbance.

There are several advantages to the use of this type of computer. Time constants and gains can be varied by merely moving a potentiometer to a new setting. Because of the all-electronic nature of this analog, it is possible to operate on a short time base so that a system solution is presented in a small fraction of the time required by the physical engine and control. As a result, wide ranges of control and engine constants can be quickly investigated. A particular control system can therefore be investigated to determine optimum values of control constants quite easily with the computer, whereas in the actual system the engine may have to be stopped before adjustments can be made, or in many cases parts may have to be replaced or remachined to make a control-constant adjustment. Furthermore, the analog settings for engine constants can be varied to simulate operation at different flight conditions and the response determined. In the actual system this would necessitate either wind-tunnel or flight investigations. For preliminary studies where the response may be unsatisfactory under some conditions, flight or wind-tunnel investigation could be extremely dangerous. In these cases, analog solutions serve as a guide in determining proper control constants, thus eliminating much costly testing.

Block Diagram of Prototype System

A descriptive block diagram of the system under consideration is presented in figure 2. This block diagram serves as an aid in describing control action and also serves as a key to a realistic network of analog components. From this block diagram, it can be seen that engine speed is controlled by automatically varying propeller pitch. Power level is manually controlled by adjusting fuel flow.
Engine speed is sensed by an alternator that has an output frequency proportional to speed. The speed signal, as well as a signal from the speed-setting device, is applied to a discriminator that produces an output voltage proportional to the difference between set speed and actual speed. This signal, which is proportional to speed error, is then electronically amplified and used to operate a relay that, in turn, actuates the pitch-change motor in such a direction as to cause engine speed to approach the required set speed. If actual engine speed overshoots the set point, the discriminator output polarity is reversed and the relay operates to reverse the direction of the pitch change motor, tending to bring speed back toward the set point.

In order to permit sufficient damping in this system, a compensating or stabilizing network is used. During transients, propeller pitch changes much faster than engine speed can follow. As soon as a signal is applied to the pitch-change motor, and propeller pitch starts to change, a signal is applied to the stabilizing network. The output of this network builds up exponentially and subtracts from the speed-error signal, thus reducing the input to the relay to zero long before the speed error becomes zero. As soon as the input to the relay is zero, the relay opens and the signal to the stabilizing network is removed, allowing the output of the stabilizing network to decay exponentially. When the constants of the network are properly matched to those of the engine, propeller pitch and speed approach their proper values with a minimum amount of overshoot and oscillations.

Block Diagram of Analog System

Presented in figure 3 is another block diagram, which indicates the mathematical form of the elements shown in figure 2. The discriminator is essentially a device the output of which is proportional to the speed error, which is the difference between desired or set speed $N_s$ and actual engine speed $N$. The discriminator can therefore be simulated by a subtractor component in the analog. Another subtractor unit is used to simulate that part of the controller network that subtracts the stabilizing or feedback voltage from the discriminator output. For analog purposes, the controller amplifier gain is combined with other gains in the two loops of the system.

The relay that operates the pitch-change motor is the basic non-linearity of this system. The relay will assume one of three positions, depending on the strength and polarity of the amplifier output signal. When the amplitude of the incoming signal is small, the relay remains in an opened position, thus introducing a dead zone into the system. For signals outside the dead zone in a positive direction (underspeed), the relay closes in a corresponding direction and applies a constant voltage
to the pitch-change motor, which acts to decrease blade angle. For negative signals outside the dead zone (overspeed), relay action is such as to increase blade angle. This action is simulated on the analog by a number of components so interconnected as to give an output-against-input plot, as shown in figure 4. Relay dead zone is plus or minus X and when the "relay" is closed, the output is plus or minus Y, depending on the input signal polarity.

It has been shown that because of relay action, the pitch-change mechanism has either zero signal or a constant plus or minus signal applied to the unit. Therefore, because it is either stopped, or once started is running at constant speed, the pitch-change mechanism can be simulated by an integrator, as shown in figure 3.

The stabilizing network of the controller is a simple lag circuit and is simulated on the computer by a corresponding component.

Analysis of engine data indicates that the engine is essentially a first-order lag system. It follows that the engine proper can be electrically simulated by another lag component of the computer.

Electrical interconnections for the analog setup correspond to the interconnecting lines shown in the analog block diagram. The constants needed for settings on the analog are those gains and time constants shown in figure 3. The manner in which the values of these constants are obtained will be discussed later.

EQUIPMENT AND PROCEDURE

Apparatus

Experimental data were obtained from operation of a turbine-propeller engine mounted on a conventional sea-level static test stand. The engine used in this investigation has an axial-flow compressor with pressure ratio (maximum) of 6. Normal engine speed during flight is between 11,000 and 13,100 rpm. Two planetary-reduction-gear systems in series with an over-all ratio of 11.351 to 1 are used to reduce engine speed to a satisfactory propeller shaft speed. This engine (less propeller) has a moment of inertia of 4 (slug)(sq ft).

A 13-foot 6-inch, four-bladed propeller was used, which has a 24-volt direct-current motor in the assembly to provide means for changing blade angle. The pitch-change mechanism operated at 40° per second. Pitch changes were stopped by a solenoid-operated brake on the motor.
The control system was previously described herein.

Instrumentation

Transient data were recorded by use of photopanel techniques. Variables measured were: engine speed, propeller pitch, fuel flow, torque, and speed setting. Instruments indicating values of these variables were photographed with a 35-millimeter motion-picture camera, operating at 20 frames per second. Inasmuch as transient data were of primary importance, consideration was given not only to steady-state accuracy of sensing units but also to their ability to respond rapidly during transients. Standard-type instruments were selected with special attention paid to the manner of installation in order to obtain fast response.

Speed was measured by two independent methods. The output of a direct-current generator driven by the engine was measured with a small voltmeter. A circuit was used that allowed full-scale deflection of the meter for small changes in revolution per minute at any speed operating point. In steady state, the accuracy is 2 percent of the range used. The transient response of the system is primarily determined by the meter, which has a natural frequency of 2 cycles per second, and was slightly underdamped.

This system was then cross-checked with another system consisting of a standard aircraft tachometer indicator driven by a three-phase generator mounted on the engine. This type of indicator has a steady-state accuracy of approximately 1/3 percent of full scale in the normal speed operating range. Its transient response was slightly better than that of the direct-current system.

Blade angle was recorded by application of a self-balancing bridge. A potentiometer mounted on the propeller hub was so connected that its moving arm assumed a position proportional to blade angle. The potentiometer comprised two arms of a bridge. The self-balancing unit located on the photopanel board had a steady-state accuracy of 1/2 percent of full scale and could follow blade-angle changes up to 44° per second as a limit.

Fuel flow was measured by means of a 0 to 200-inches-of-water bellows-type differential pressure gage installed to read pressure drop across a sharp-edged orifice positioned in the main fuel line. The indicator, as well as pressure transmission lines, were liquid filled to eliminate effects of compressing air in the system and thereby improved transient-response characteristics. Steady-state accuracy of
this gage is ±2 percent of full scale. Its natural frequency is 10 cycles per second and the system appeared to be slightly underdamped.

Torque was measured with a Bourdon tube type gage also having liquid-filled lines. Steady-state accuracy of the gage was ±1 percent of full scale and it had a natural frequency of 20 cycles per second.

The speed-setting device was part of the control unit and consisted of a readily variable circuit element the position of which was calibrated in terms of percentage of maximum speed.

**Procedure**

Data presented herein show responses of the controlled engine to approximate step changes in set speed with fuel flow held constant. These transients were obtained by rapidly moving the speed-setting control to a new position. Because a finite time was involved in this move, the input disturbance was in reality a ramp function having a rise time of from 0.1 to 0.5 second. Magnitude of input disturbances varied from 200 to 1500 rpm. The investigation was performed at several fuel settings within the normal speed range of 70 to 100 percent of maximum. The control constants \( K_f \) and \( T_f \) were varied through a broad range to obtain different amounts of damping.

**CORRELATION OF ENGINE AND ANALOG DATA**

Several experimental runs were simulated on the analog. Of these runs, two typical ones are presented, one to show a case where speed is approximately critically damped and another to represent the underdamped case. The response of propeller pitch and engine speed to changes in set speed is shown in figures 5 and 6 for the two cases. From the transient data, the dynamic characteristics of the engine needed for an analysis of a control system were determined. The engine behavior is described by plots of amplitude ratio against frequency and phase angle against frequency for speed to propeller pitch, which are shown in figures 7 to 10. These figures were obtained by analyzing the time-response curves according to the methods of reference 1, with a rolling-sphere type of harmonic analyzer, giving the complex ratio of speed to blade angle at particular frequencies.

The frequency plots (figs. 7 to 10) for the engine indicated that its dynamic response is that of a lag and that any second-order effects are small. At some of the higher frequencies, a scattering of points is attributed to the type of instrumentation used and to inaccuracies in performing the frequency analysis of the transient data. Data
obtained on this engine by sinusoidal methods presented in reference 2 and subsequent transient data obtained with better instrumentation, some of which was analyzed by new techniques, also indicate that the engine is primarily first order.

Two constants are necessary to define a first-order response, the gain and time constant. The time constant is determined as the reciprocal of the break frequency in radians per second. The engine gain is the ratio of the change in steady-state speed to the change in steady-state blade angle around the particular operating point. For the engine operating point chosen, the engine time constant is approximately 5 seconds and the engine gain as determined from steady-state data is 429 rpm per degree.

To define a relay control of this type requires knowledge of the relay dead zone as well as the gain of the stabilizing network and its time constant. These characteristics were determined from bench tests on the control. The dead zone was determined to be approximately 30 engine rpm. The maximum value of the feedback signal is variable from approximately 40 to 80 volts, which corresponds to 480 to 960 engine rpm. The feedback time constant was adjustable from 0 to approximately 10 seconds. The time constant of the integrator representing the pitch-change motor was determined from slopes of curves of blade angle against time.

It is necessary to determine certain relations that exist in the prototype system and to maintain these relations in the analog in order to obtain simulated responses that will hold for the original system. A simulated response can be obtained for linear systems by doing two things. First, maintain the loop gains for the analog equal to the loop gains for the prototype system. Second, maintain the same ratios between time constants in the prototype and analog systems. In nonlinear systems, the loop gains may change continuously because of the nonlinearities. Analog constants have therefore been obtained using an instantaneous gain obtained for the instant when the input to the relay is at the edge of the dead zone. At this moment the relay gain is \( Y/X \).

On the basis of maintaining instantaneous loop gains, the gain of the stabilizing network can be determined by the following equation:

\[
K_p = \left( \frac{\text{Maximum feedback, (rpm)}}{\text{Dead zone, (rpm)}} \right)^X_Y
\]

where \( X/Y \) is the ratio of relay dead zone to relay output.
The engine and feedback time constants determined were set into the analog directly without any further calculations. The other constants to be set into the analog were calculated from the preceding information by means of the following equation:

\[
\frac{K_p K_e}{\tau_\beta} = \left(\text{Pitch change rate, (deg/sec)}\right) \left[\text{Engine gain, (rpm/deg)}\right] \frac{X}{Y} \text{ Dead zone, (rpm)}
\]

The term \(\frac{K_p K_e}{\tau_\beta}\) represents the gain around the outer loop from relay output to relay input.

A comparison between results obtained with the analog and data experimentally obtained is shown in figures 11 to 14 for sea-level conditions. These figures indicate that the experimental speed response is slightly slower than the analog response. This discrepancy can be partly attributed to the pitch-change motor starting and stopping characteristics, which have been omitted in the analog representation. Some of the discrepancy may also be due to uncertainties in experimental data and the harmonic analysis of the time data. Gains and time constants that were set into the analog were calibrated and were accurate to 2 percent. In view of the correlation obtained and the action of the system under special conditions where the type of response is known, it is believed that the over-all simulation of the control system with the analog is satisfactory. The method can therefore be extended to investigate action of the control under conditions other than those obtained in the sea-level test stand.

**ALTITUDE RESPONSE FROM ANALOG**

As previously mentioned, the constants of the engine vary with flight speed, power setting, and altitude. As the engine constants change, the response of the complete system will also change and may become undesirable or possibly dangerous. In order to avoid unwanted types of response, it is desirable and may become necessary to add some type of compensation that will automatically change the control constants to give a reasonable response under all flight conditions. This report considers the altitude phase of the control problem.

The operation of the system at several altitudes was determined with the analog. Control constants were chosen that gave a "critically" damped speed response at sea level for a 300-rpm step change in set
speed. This response was obtained by setting the feedback time constant equal to the engine time constant and then varying the feedback gain until the desired speed response was obtained. The critically damped case was chosen not because it is necessarily optimum but because it is sufficiently well defined so that it can be used in studying how control constants must be varied with altitude. It should be kept in mind that the response will become less damped for larger step changes in set speed and more damped for smaller step changes because the system is nonlinear. For comparison purposes, a 300-rpm step change was used to obtain the responses shown in figures 15 to 18.

New engine constants for each altitude were determined by applying the altitude pressure and temperature generalization factors. The temperature and pressure generalization factors can be applied to the engine time constant with reasonable accuracy up to an altitude of 35,000 feet. The generalized engine constants were used with the control constants fixed at the sea-level settings to determine how the engine response would change with a variation in altitude. The results obtained for operation at sea level, 20,000, and 35,000 feet, as simulated on the analog, are shown in figures 15 and 16 for a 300-rpm step change in set speed. These figures are reproductions of actual photographs as taken from the analog oscilloscopes. The curves shown on these figures indicate that when the control constants are set to obtain a critically damped response at sea level, the system response at altitude will have a considerable amount of overshoot and some oscillation. The change in response is the result of the engine's responding much more slowly at altitude than at sea level.

Because the engine response at an altitude of 35,000 feet is quite different from its response at sea level, several methods of compensating the control to more nearly maintain the sea-level response were investigated. At each altitude, the control constants were changed from the sea-level settings according to certain methods to try to bring the speed response back to a critically damped condition. The responses obtained at an altitude of 35,000 feet using various methods of compensation are presented in figures 17 and 18. The sea-level response is also shown for comparison purposes.

The first method used was to keep the ratios of \( \frac{\tau_f}{\tau_e} \) and \( \frac{\tau_p}{\tau_e} \) constant with altitude. This method gives a type of response that is very similar to the sea-level response, but, because all parts of the system are slowed down in proportion to the generalization factor applied to \( \tau_e \), the response will also be that many times slower. This method neglects the change in engine gain with altitude, which is small.
A second method for adjusting the control constants is to set the feedback time constant equal to that of the engine for a given altitude and then to vary the feedback gain until the critically damped response is obtained. From these settings the amount of feedback signal needed can be calculated. This type of compensation appears to be satisfactory in that the speed of response is almost as fast as at sea level. Fast response is obtained by a large overshoot in propeller pitch. The amount of overshoot in propeller pitch can be greater at altitude than at sea level without causing excessive torque; however, overshoot should not be so great that propeller blades will stall.

The third method of compensation is to change only the feedback time constant. For altitude conditions, if \( T_f \) is increased approximately one-half as much as \( T_e \) increases, a response will be obtained that is similar to that at sea level. A further increase in \( T_f \) will cause an oscillating condition again. The responses obtained using this method of compensation are similar to those obtained for the second method.

Another method is to adjust only the feedback gain. For the conditions obtained at altitude, varying only \( K_f \) changed the frequency of oscillations and reduced their amplitude but did not remove them. Because a critically damped response could not be obtained, no figures are presented for this last method.

**CONCLUDING REMARKS**

Correlation has been established for sea-level conditions between actual transient engine and control data obtained experimentally and data obtained by simulating the complete system on an analog computer.

By the use of the altitude pressure and temperature generalization factors for the engine, constants were calculated that were then used in the analog to predict the behavior of the system at altitudes of 20,000 and 35,000 feet. Because the engine response varies considerably at altitude, some type of control compensation appears desirable. Several methods were tried and the following results were obtained:

1. Keeping the ratios of feedback time constant to engine time constant \( T_f/T_e \) and pitch-change mechanism time constant to engine time constant \( T_\beta/T_e \) constant with altitude resulted in similar responses at widely different altitudes but the speed of response was slower by the correction factor involved for the engine time constant \( T_e \).
2. Keeping the ratio of feedback time constant to engine time constant $\tau_f/\tau_e$ equal to unity and increasing the feedback gain $K_f$ with altitude appeared to be a satisfactory type of compensation.

3. The third method consisted in adjusting the feedback time constant $\tau_f$ to approximately one-half the value of the engine time constant $\tau_e$ at altitude. A fast response similar to that with the second method was obtained. This method appeared to be promising because it involved changing only one parameter.

4. The fourth method of varying only the feedback gain $K_f$ to adjust the control changed and reduced the frequency of oscillations but did not remove them.

Lewis Flight Propulsion Laboratory,
National Advisory Committee for Aeronautics,
Cleveland, Ohio.

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Figure 1. - Electronic analog computer used to simulate engines and controls.
Figure 2. - Block-diagram representation of relay-type control system for turbine-propeller engines.
Figure 3. - Mathematical and analog block diagram of relay-type control system for turbine-propeller engines.
Figure 4. - Relay characteristics.
Figure 5. - Experimental sea-level response of engine speed and propeller pitch to change in set speed (critically damped). \( N_1 \), 11,380 rpm; \( N_2 \), 11,560 rpm; \( \beta_1 \), 12.8°; \( \beta_2 \), 12.18°; \( N_{f,m} \), 780 rpm; \( \tau_f \), 3.2 seconds.

Figure 6. - Experimental sea-level response of engine speed and propeller pitch to change in set speed (underdamped). \( N_1 \), 11,280 rpm; \( N_2 \), 11,640 rpm; \( \beta_1 \), 12.6°; \( \beta_2 \), 11.76°; \( N_{f,m} \), 780 rpm; \( \tau_f \), 1.1 seconds.
Figure 7. - Amplitude ratio against frequency for speed to propeller pitch for engine, obtained by harmonic analysis of figure 5.

Figure 8. - Phase angle against frequency for speed to propeller pitch for engine, obtained by harmonic analysis of figure 5.
Figure 9. - Amplitude ratio against frequency for speed to propeller pitch for engine, obtained by harmonic analysis of figure 6.

Figure 10. - Phase angle against frequency for speed to propeller pitch for engine, obtained by harmonic analysis of figure 6.
Figure 11. - Comparison of speed response obtained experimentally and on analog. \( N_1 \), 11,380 rpm; \( N_4 \), 11,560 rpm; \( N_{f,m} \), 780 rpm; and \( \tau_f \), 3.2 seconds.

Figure 12. - Comparison of propeller-pitch response obtained experimentally and on analog. \( \beta_i \), 12.6\(^\circ\); \( \beta_i \), 12.18\(^\circ\); \( N_{f,m} \), 780 rpm; and \( \tau_f \), 3.2 seconds.
Figure 13. - Comparison of speed response obtained experimentally and on analog. \( N_1 \), 11,280 rpm; \( N_2 \), 11,640 rpm; \( N_{f, m} \), 780 rpm; and \( \tau_f \), 1.1 seconds.

Figure 14. - Comparison of propeller-pitch response obtained experimentally and on analog. \( \beta_1 \), 12.6°; \( \beta_2 \), 11.78°; \( N_{f, m} \), 780 rpm; and \( \tau_f \), 1.1 seconds.
### Figure 15.

- Speed response to changes in set speed for various altitudes and without altitude compensation obtained on analog.

<table>
<thead>
<tr>
<th>Altitude (ft)</th>
<th>$K_e$ (rpm)</th>
<th>$\tau_e$ (sec)</th>
<th>$N_{f,m}$ (rpm)</th>
<th>$\tau_f$ (sec)</th>
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</thead>
<tbody>
<tr>
<td>Sea level</td>
<td>429</td>
<td>4.5</td>
<td>880</td>
<td>4.5</td>
</tr>
<tr>
<td>20,000</td>
<td>398</td>
<td>9.1</td>
<td>880</td>
<td>4.5</td>
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<tr>
<td>35,000</td>
<td>374</td>
<td>16.7</td>
<td>880</td>
<td>4.5</td>
</tr>
</tbody>
</table>

### Figure 16.

- Propeller-pitch response to changes in set speed for various altitudes and without altitude compensation obtained on analog.
Table:

<table>
<thead>
<tr>
<th>Method of compensation</th>
<th>$K_e$</th>
<th>$\tau_e$</th>
<th>$N_f,m$</th>
<th>$\tau_f$</th>
<th>Blade rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sea level</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1: Altitude, 35,000 ft; $\tau_f/\tau_e$ and $\tau_B/\tau_e$</td>
<td>374</td>
<td>16.7</td>
<td>880</td>
<td>4.5</td>
<td>1.08</td>
</tr>
<tr>
<td>2: Altitude, 35,000 ft; $\tau_f = \tau_e$ and $K_f$ scheduled for critically damped response</td>
<td>374</td>
<td>16.7</td>
<td>880</td>
<td>16.7</td>
<td>4</td>
</tr>
<tr>
<td>3: Altitude, 35,000 ft; $\tau_f$ scheduled for critically damped speed response</td>
<td>374</td>
<td>16.7</td>
<td>880</td>
<td>10</td>
<td>4</td>
</tr>
</tbody>
</table>

Figure 17. - Comparison of speed response at sea level and altitude of 35,000 feet to changes in set speed obtained on analog for three methods of compensation.
<table>
<thead>
<tr>
<th>Method of compensation</th>
<th>$K_e$ (rpm)</th>
<th>$\tau_e$ (sec)</th>
<th>$N_{f,m}$ (rpm)</th>
<th>$\tau_f$ (sec)</th>
<th>Blade rate (deg/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sea level</td>
<td>429</td>
<td>4.5</td>
<td>880</td>
<td>4.5</td>
<td>4</td>
</tr>
<tr>
<td>1: Altitude, 35,000 ft; $T_f/T_e$ and $T_\beta/T_e$ constant with altitude</td>
<td>374</td>
<td>16.7</td>
<td>880</td>
<td>16.7</td>
<td>1.08</td>
</tr>
<tr>
<td>2: Altitude, 35,000 ft; $T_f = T_e$ and $K_f$ scheduled for critically damped response</td>
<td>374</td>
<td>16.7</td>
<td>1610</td>
<td>16.7</td>
<td>4</td>
</tr>
<tr>
<td>3: Altitude, 35,000 ft; $T_f$ scheduled for critically damped speed response</td>
<td>374</td>
<td>16.7</td>
<td>880</td>
<td>10</td>
<td>4</td>
</tr>
</tbody>
</table>

Figure 18. - Comparison of propeller-pitch response at sea level and altitude of 35,000 feet to changes in set speed obtained on analog for three methods of compensation.