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RESEARCH MEMORANDUM

A THEORETICAL AND EXPERIMENTAL STUDY OF

WIND-TUNNEL-WALL EFFECTS ON OSCILLATING AIR FORCES FOR

TWO-DIMENSIONAL SUBSONIC COMPRESSIBLE FLOW

By Harry L. Runyan, Donald S. Woolston,
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NATIONAL ADVISORY COMMITTEE
FOR AERONAUTICS

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SUMMARY

A recently published analytical investigation of the effects of wind-tunnel walls on air forces on an oscillating wing at subsonic speeds (NACA TN 2552) demonstrated the possibility, under certain conditions, of the existence of large tunnel-wall effects, associated with an acoustic resonance phenomenon. In the present paper the integral equation defining the problem is treated further and is presented in a form adapted to calculations. Application is made to the case of a pitching airfoil in a given wind tunnel at a Mach number of 0.7 for frequencies ranging from zero to beyond the frequency of acoustic resonance. The calculated lift and phase angle are compared with the results of experimental measurements made in the Langley 4.5-foot flutter research tunnel. Good agreement is obtained between the calculated and the experimentally determined magnitudes of the lift and phase angles.

INTRODUCTION

In the evaluation of results obtained by measurement of the air forces on an oscillating wing in a wind tunnel, the question of the effect of the tunnel walls arises. A theoretical treatment of the effect of wind-tunnel walls for the case of two-dimensional incompressible flow has been made by several investigators and reported in references 1, 2, and 3. In these papers the influence of the tunnel walls is found to be comparatively small for most cases. Extension of the treatment of the problem to include the effects of compressibility of the fluid has been reported in reference 4. It is shown in this reference that, for certain conditions of wing frequency, tunnel height, and Mach number, the oscillating wing could excite the air in the tunnel to the point of resonance and that, in the absence of damping, infinite transverse velocities could exist. In the neighborhood of such a resonant condition one would expect very large wall effects and it would be extremely difficult to approximate the free-air condition.

In reference 4 an integral equation is derived which relates the downwash distribution to the lift distribution for an airfoil of infinite aspect ratio oscillating in a wind tunnel. A main purpose of the present paper is to present a method of solution of this equation and to demonstrate some applications. The method of solution is one of collocation, similar to that used by Possio (ref. 5) and Frazer (ref. 6), in which the downwash is satisfied at selected control points along the wing chord. Accuracy of the solution is increased as the number of control points is increased, and in the present investigation successive calculations based on one, two, and three control points have been made.

Another objective of this paper is to make comparisons with experimental results and for this purpose measurements have been made of the magnitude and phase relationships of the lift on a two-dimensional wing oscillating in pitch. Measurements were made at a constant Mach number (0.7) for a range of frequencies of oscillation. These results are compared with the results of calculations made for the same conditions.

As stated in reference 4, the integral equation for the case of no tunnel walls checks the results of Possio. For the case with tunnel walls and for the limiting steady-flow case of zero frequency, it is possible to obtain a mathematical check with some existing results; this is shown in an appendix.

SYMBOLS

A_n coefficients in series expression for lift distribution (eq. 11)

a axis of rotation measured from midchord, positive rearward, based on half-chord

b wing half-chord, ft

c velocity of sound, ft/sec

h displacement of wing in vertical translation

H height of tunnel

$H_n^{(2)}$ Hankel function of the second kind

k reduced frequency, $b\omega/U$

$K(M,z) + K(M,z,H)$ kernel of integral equation

$L(x_o)$, $\overline{L}(\theta_o)$ lift distribution, lb/ft/unit span

M Mach number, U/c

$p = \dfrac{MkH}{2\pi\beta}$

$R_n = \sqrt{(x - x_o)^2 + \beta^2(nH)^2}$

U stream velocity in chordwise direction, ft/sec

$w(x)$ vertical induced velocity (perpendicular to chord), ft/sec

x, x_o, ξ chordwise coordinates

y vertical coordinate

$z = k(x - x_o)$

α angular displacement of wing in pitch

$\beta = \sqrt{1 - M^2}$

$\epsilon = \dfrac{|x - x_o|}{\beta H}$

$\mu = \dfrac{Mk}{\beta^2}$

ω circular frequency of oscillation, radians/sec

ω_{res} circular frequency at resonance, radians/sec

ρ fluid density, slugs/ft^3

Subscripts:

n 0, 1, 2, . . .

Following equation (6) variables ξ, x, x_o, H, and y are considered as nondimensional length quantities based on the half-chord b.

ANALYSIS

This section is given in two parts: First the integral equation is given in a form suitable for computation, and second the method of calculation is shown. The basic assumptions in the derivation of the integral equation of reference 4 were the usual ones of linearization, nonviscous fluid, and thin bodies. In addition, it was assumed that the tunnel walls form perfect reflecting surfaces for the disturbances.

Integral Equation

The integral equation of reference 4, which related downwash distribution to lift distribution, may be written as

$$w(x) = \frac{i}{4\rho U\beta} \lim_{y \to 0} \int_{-b}^{b} L(x_o) e^{\frac{-i\omega(x-x_o)}{U}} \left\{ \int_{-\infty}^{x-x_o} e^{\frac{i\omega\xi}{U\beta^2}} \frac{\partial^2}{\partial y^2} H_o^{(2)}\left(\frac{\omega}{c\beta^2}\sqrt{\xi^2 + \beta^2 y^2}\right) d\xi + 2\int_{-\infty}^{x-x_o} \sum_{n=1}^{\infty} (-1)^n e^{\frac{i\omega\xi}{U\beta^2}} \frac{\partial^2}{\partial y^2} H_o^{(2)}\left(\frac{\omega}{c\beta^2}\sqrt{\xi^2 + \beta^2 (y - nH)^2}\right) d\xi \right\} dx_o \qquad (1)$$

where $w(x)$ is the downwash and $L(x_o)$ is the lift distribution. The two integrals within the brackets comprise the kernel of the integral equation. The first integral within the brackets corresponds to the case of no tunnel walls. The second integral, containing the infinite summation, is the additional part of the kernel arising from the effect of the tunnel walls.

Equation (1), although identical with the integral equation of reference 4, differs slightly in appearance, in that the constant factor outside the integral is different in order to conform more conveniently to existing tables for the subsequent numerical work.

The dimensional form of the equations is retained, but later a nondimensional form is given.

It is convenient to make use of a well-known differential equation satisfied by the Hankel functions, so that one can introduce in equation (1) the identity

$$\frac{\partial^2}{\partial y^2} H_o^{(2)}\left(\frac{\omega}{c\beta^2}\sqrt{\xi^2 + \beta^2 y^2}\right) \equiv -\beta^2 \frac{\partial^2}{\partial \xi^2} H_o^{(2)}\left(\frac{\omega}{c\beta^2}\sqrt{\xi^2 + \beta^2 y^2}\right) - \frac{\omega^2}{\beta^2 c^2} H_o^{(2)}\left(\frac{\omega}{c\beta^2}\sqrt{\xi^2 + \beta^2 y^2}\right) \tag{2}$$

and then there is obtained for the downwash

$$\begin{aligned} w(x) = \frac{-i}{4\rho U\beta} \lim_{y \to 0} \int_{-b}^{b} L(x_o) e^{\frac{-i\omega(x-x_o)}{U}} \Bigg\{ & \beta^2 \int_{-\infty}^{x-x_o} e^{\frac{i\omega\xi}{U\beta^2}} \frac{\partial^2}{\partial \xi^2} H_o^{(2)}\left(\frac{\omega}{c\beta^2}\sqrt{\xi^2 + \beta^2 y^2}\right) d\xi \, + \\ & \frac{\omega^2}{\beta^2 c^2} \int_{-\infty}^{x-x_o} e^{\frac{i\omega\xi}{U\beta^2}} H_o^{(2)}\left(\frac{\omega}{c\beta^2}\sqrt{\xi^2 + \beta^2 y^2}\right) d\xi \, + \\ & 2\sum_{n=1}^{\infty} (-1)^n \Bigg[\beta^2 \int_{-\infty}^{x-x_o} e^{\frac{i\omega\xi}{U\beta^2}} \frac{\partial^2}{\partial \xi^2} H_o^{(2)}\left(\frac{\omega}{c\beta^2}\sqrt{\xi^2 + \beta^2 (y - nH)^2}\right) d\xi \, + \\ & \frac{\omega^2}{\beta^2 c^2} \int_{-\infty}^{x-x_o} e^{\frac{i\omega\xi}{U\beta^2}} H_o^{(2)}\left(\frac{\omega}{c\beta^2}\sqrt{\xi^2 + \beta^2 (y - nH)^2}\right) d\xi \Bigg] \Bigg\} dx_o \end{aligned} \tag{3}$$

The integrals of equation (3) which contain partial derivatives of Hankel functions can be integrated twice by parts to obtain

$$\int_{-\infty}^{x-x_o} e^{\frac{i\omega\xi}{U\beta^2}} \frac{\partial^2}{\partial\xi^2} H_o^{(2)}\left(\frac{\omega}{c\beta^2}\sqrt{\xi^2 + \beta^2(y - nH)^2}\right)d\xi \equiv$$

$$- \frac{\omega}{c\beta^2} e^{\frac{i\omega(x-x_o)}{U\beta^2}} \frac{x - x_o}{\sqrt{(x - x_o)^2 + \beta^2(y - nH)^2}} H_1^{(2)}\left(\frac{\omega}{c\beta^2}\sqrt{(x - x_o)^2 + \beta^2(y - nH)^2}\right) -$$

$$\frac{i\omega}{U\beta^2} e^{\frac{i\omega(x-x_o)}{U\beta^2}} H_o^{(2)}\left(\frac{\omega}{c\beta^2}\sqrt{(x - x_o)^2 + \beta^2(y - nH)^2}\right) -$$

$$\frac{\omega^2}{U^2\beta^4}\int_{-\infty}^{x-x_o} e^{\frac{i\omega\xi}{U\beta^2}} H_o^{(2)}\left(\frac{\omega}{c\beta^2}\sqrt{\xi^2 + \beta^2(y - nH)^2}\right)d\xi \tag{4}$$

The last integral of equation (4) may be written in two parts as

$$\int_{-\infty}^{x-x_o} e^{\frac{i\omega\xi}{U\beta^2}} H_o^{(2)}\left(\frac{\omega}{c\beta^2}\sqrt{\xi^2 + \beta^2(y - nH)^2}\right)d\xi = \int_0^{\infty} e^{\frac{-i\omega\xi}{U\beta^2}} H_o^{(2)}\left(\frac{\omega}{c\beta^2}\sqrt{\xi^2 + \beta^2(y - nH)^2}\right)d\xi +$$

$$\int_0^{x-x_o} e^{\frac{i\omega\xi}{U\beta^2}} H_o^{(2)}\left(\frac{\omega}{c\beta^2}\sqrt{\xi^2 + \beta^2(y - nH)^2}\right)d\xi \tag{5}$$

The first integral on the right side of equation (5) will be treated in a following section in accordance with the errata of reference 4 (see evaluation of S_3 following eq. (16)).

The second integral on the right side of equation (5) has not been integrated in closed form and must be handled by approximate methods. A practical assumption which is often made in the analysis of the effect of wind-tunnel walls is that the tunnel height is considered large compared to the wing chord. With this assumption, the argument of the Hankel function in equation (5) can be written as (in the limit as $y \longrightarrow 0$)

$$\frac{\omega}{c\beta^2}\sqrt{\xi^2 + \beta^2(nH)^2} = \frac{\omega}{c\beta^2}\beta nH\sqrt{\left(\frac{\xi}{\beta nH}\right)^2 + 1} \approx \frac{\omega}{c\beta} nH$$

provided that

$$\left(\frac{\xi}{\beta nH}\right)^2 \ll 1$$

This approximation implies that the airfoil images, and in particular the closest image $n = 1$, are a sufficient distance from the airfoil so that the actual distance $\sqrt{\xi^2 + \beta^2(nH)^2}$ may be replaced by the vertical distance βnH of the image above the airfoil. Of course, this approximation does not hold for Mach numbers close to or equal to unity.

Therefore the second integral of equation (5) can be expressed as

$$\lim_{y \to 0} \int_0^{x-x_o} e^{\frac{i\omega\xi}{U\beta^2}} H_o^{(2)}\left(\frac{\omega}{c\beta^2}\sqrt{\xi^2 + \beta^2(y - nH)^2}\right)d\xi = H_o^{(2)}\left(\frac{\omega}{U}\frac{MnH}{\beta}\right)\int_0^{x-x_o} e^{\frac{i\omega\xi}{U\beta^2}}\,d\xi$$

$$= H_o^{(2)}\left(\frac{\omega}{U}\frac{MnH}{\beta}\right)\frac{\beta^2 U}{i\omega}\left[e^{\frac{i\omega}{U\beta^2}(x-x_o)} - 1\right] \qquad (6)$$

It is convenient to introduce nondimensional length quantities, based on the half-chord b, in the subsequent work. Accordingly, ξ, x, x_o, H, and y are in the subsequent work considered as nondimensional quantities.

The right side of equation (6) can then be written in nondimensional form as follows:

$$H_o^{(2)}\left(\frac{kMnH}{\beta}\right)\frac{\beta^2}{ik}\left[e^{\frac{ik(x-x_o)}{\beta^2}} - 1\right]$$

where

$$k = \frac{b\omega}{U}$$

and these expressions may be used to express equation (3) as

$$w(x) = \frac{\omega b}{\rho U^2} \int_{-1}^{1} L(x_o) \left[K(M,z) + K(M,z,H) \right] dx_o \tag{7}$$

where

$$K(M,z) = \frac{1}{4\sqrt{1 - M^2}} e^{-iz} \left\{ e^{\frac{iz}{\beta^2}} \left[-H_o^{(2)}(\mu R_o) + iMH_1^{(2)}(\mu R_o) \frac{x - x_o}{|x - x_o|} \right] + \right.$$

$$\left. i\beta^2 \left[\frac{2}{\pi\beta} \log_e \frac{1 + \beta}{M} + \int_0^{\frac{w}{M}} e^{iu} H_o^{(2)}(M|u|) du \right] \right\} \tag{8}$$

and where

$$K(M,z,H) = \frac{1}{4\sqrt{1 - M^2}} e^{-iz} \, 2 \sum_{n=1}^{\infty} (-1)^n \left\{ \left[-H_o^{(2)}(\mu R_n) + \frac{iM(x - x_o)}{R_n} H_1^{(2)}(\mu R_n) \right] e^{\frac{iz}{\beta^2}} + \right.$$

$$\left. ik \int_0^{\infty} e^{\frac{-ik\xi}{\beta^2}} H_o^{(2)}\left(\frac{kM}{\beta^2} \sqrt{\xi^2 + \beta^2 (nH)^2} \right) d\xi + \beta^2 \left(e^{\frac{iz}{\beta^2}} - 1 \right) H_o^{(2)}\left(\frac{MknH}{\beta} \right) \right\} \tag{9}$$

in which use has been made of

$$\mu = \frac{\omega b}{c\beta^2} \qquad u = \frac{\omega \xi}{U\beta^2} \qquad w = \frac{Mz}{\beta^2}$$

$$R_o = x - x_o \qquad R_n = \sqrt{(x - x_o)^2 + \beta^2(nH)^2}$$

Although the kernels $K(M,z)$ and $K(M,z,H)$ are similar, it may be noted that $K(M,z)$ is singular since, at $x = x_o$, R_o is zero, and for small argument, $H_o^{(2)}(x)$ behaves as $\log_e x$ and $H_1^{(2)}(x)$ behaves as $1/x$. The argument R_n of the Hankel functions in $K(M,z,H)$ does not approach zero and therefore the terms in the bracket for this part of the kernel are not singular. It will be seen later, however, that the series representation of $K(M,z,H)$ becomes large as resonant frequencies are approached and is infinite at the resonant condition.

The integral equation for the case of no tunnel walls checks the results of Possio. For the case with tunnel walls and for the limiting steady-flow case of zero frequency, it is possible to obtain a mathematical check with some existing results; this is shown in an appendix.

Method of Solution

A method of using equation (7) to determine the aerodynamic forces on a wing oscillating in the presence of tunnel walls is now discussed. The method under consideration is one of collocation, similar to that used in references 5 and 6 for the case of no tunnel walls. The method involves writing the expression for the downwash at selected control points, that is, reducing equation (7) to a set of linear algebraic equations. The unknowns in the set of simultaneous equations are the various coefficients of the expression which is assumed for the loading.

This expression which is assumed for the present case is a trigonometric series expansion which satisfies the Kutta condition at the trailing edge and which has the proper type of infinity at the leading edge. This expression is

$$\frac{L(x_o)}{\rho U^2} = A_o \cot \frac{\theta_o}{2} + \sum_{n=1}^{\infty} A_n \sin n\theta_o = \overline{L}(\theta_o) \tag{10}$$

where $x_o = -\cos\theta_o$ and the A_n's are unknown coefficients to be determined in accordance with the downwash $w(x)$, which is known from the motion of the wing. It is desirable to rewrite equation (7) in terms of the new variable θ_o, and equation (7) then assumes the form

$$w(x) = Uk\left[\int_0^{\pi} \bar{L}(\theta_o)K(M,z)\sin\theta_o\, d\theta_o + \int_0^{\pi} \bar{L}(\theta_o)K(M,z,H)\sin\theta_o\, d\theta_o\right]$$

$$= Uk\left(I^{I} + I^{II}\right) \tag{11}$$

The integral I^{I} of equation (11) to be discussed first is the integral expression as first derived by Possio for the condition of no tunnel walls and has been treated by several investigators. As pointed out previously, the kernel $K(M,z)$ of I^{I} is singular at $x = x_o$. Since, in handling the complete integral I^{I}, numerical or graphical means are necessary, it is convenient to isolate the singularity and express it in an integrable form. This has been done by Schwarz (ref. 7) who expanded $K(M,z)$ in the vicinity of $z = 0$ in the form

$$K(M,z) = \frac{F(M)}{z} + iG(M)\log_e|z| + K_1(M,z)$$

where $F(M) = \frac{-\beta}{2\pi}$, $G(M) = \frac{1}{2\pi\beta}$, and $K_1(M,z)$ is a tabulated function which is no longer singular. Substitution of this expression in the integral I^{I} of equation (11) gives

$$I^{I} = \int_0^{\pi} \bar{L}(\theta_o)\sin\theta_o K_1(M,z)\,d\theta_o +$$

$$\int_0^{\pi} \bar{L}(\theta_o)\sin\theta_o\left[\frac{F(M)}{z} + iG(M)\log_e|z|\right]d\theta_o \tag{12}$$

With the use of equation (10), the integral of equation (12) containing $K_1(M,z)$ becomes

$$\int_0^\pi \bar{L}(\theta_o)\sin\theta_o K_1(M,z)d\theta_o = A_o\int_0^\pi (1+\cos\theta_o)K_1(M,z)d\theta_o +$$

$$\sum_{n=1}^{\infty} A_n\int_0^\pi \sin\theta_o \sin n\theta_o K_1(M,z)d\theta_o \tag{13}$$

By utilizing the values of $K_1(M,z)$ tabulated by Schwarz, the indicated integrations of equation (13) may be readily carried out by numerical or graphical means.

Putting $z = -k(\cos\theta - \cos\theta_o)$ gives for the second integral of I^I in equation (12)

$$\int_0^\pi \bar{L}(\theta_o)\sin\theta_o\left[\frac{F(M)}{z} + iG(M)\log_e|z|\right]d\theta_o$$

$$= -\frac{1}{k}F(M)\int_0^\pi \frac{\bar{L}(\theta_o)\sin\theta_o}{\cos\theta - \cos\theta_o}d\theta_o +$$

$$iG(M)\int_0^\pi \bar{L}(\theta_o)\sin\theta_o \log_e\left|\cos\theta - \cos\theta_o\right|d\theta_o +$$

$$iG(M)\log_e k\int_0^\pi \bar{L}(\theta_o)\sin\theta_o\, d\theta_o$$

$$= -\frac{1}{k}F(M)I_o + iG(M)I_1 + iG(M)I_2\log_e k \tag{14}$$

The integrals I_o, I_1, and I_2 have been evaluated by Frazer (ref. 6) and have the following values:

$$I_o = -\pi A_o + \pi\sum_{n=1}^{\infty} A_n\cos n\theta \tag{15a}$$

$$I_1 = -\pi\left(A_o + \frac{1}{2} A_1\right)\log_e 2 - \pi A_o \cos\theta + \frac{\pi}{4} A_1 \cos 2\theta +$$

$$\frac{\pi}{2}\sum_{n=2}^{\infty} A_n\left[\frac{\cos(n + 1)\theta}{n + 1} - \frac{\cos(n - 1)\theta}{n - 1}\right] \tag{15b}$$

$$I_2 = \pi\left(A_o + \frac{1}{2} A_1\right) \tag{15c}$$

With the use of these expressions in equation (14) and with the results of the integration of equation (13), the integral I^I of equation (11) is now expressed solely in terms of the unknown coefficients A_n. It remains now to obtain I^{II} of equation (11) in a similar form.

The kernel $K(M,z,H)$ of I^{II}, given in equation (9), is the sum of four infinite series; namely

$$K(M,z,H) = \frac{e^{-iz'}}{2\beta}\left[-e^{\frac{iz}{\beta^2}}\sum_{n=1}^{\infty}(-1)^n H_o^{(2)}(\mu R_n) + \right.$$

$$\beta^2\left(e^{\frac{iz}{\beta^2}} - 1\right)\sum_{n=1}^{\infty}(-1)^n H_o^{(2)}\left(\frac{MknH}{\beta}\right) +$$

$$ik\sum_{n=1}^{\infty}(-1)^n\int_0^{\infty} e^{\frac{-ik\xi}{\beta^2}} H_o^{(2)}\left(\frac{kM}{\beta^2}\sqrt{\xi^2 + \beta^2(nH)^2}\right)d\xi +$$

$$\left. e^{\frac{iz}{\beta^2}}\sum_{n=1}^{\infty}(-1)^n \frac{iM(x - x_o)}{R_n} H_1^{(2)}(\mu R_n)\right]$$

$$= \frac{e^{-iz}}{2\beta}\left(C_1S_1 + C_2S_2 + C_3S_3 + C_4S_4\right) \tag{16}$$

where the S_n's denote the indicated infinite summations and the C_n's the respective multipliers.

The series S_1 and S_2 of equation (16), which involve the Hankel function $H_o^{(2)}$, may be put in a more rapidly convergent form according to reference 8 by introducing the variables p and ϵ, where

$$p = \frac{MkH}{2\pi\beta}$$

and

$$\epsilon = \frac{|x - x_o|}{\beta H}$$

Accordingly, the series S_1 and S_2 can be written as

$$S_1 = \sum_{n=1}^{\infty} (-1)^n H_o^{(2)}(\mu R_n)$$

$$= \sum_{n=1}^{\infty} (-1)^n H_o^{(2)}\left(2\pi p\sqrt{\epsilon^2 + n^2}\right)$$

$$= \frac{1}{2\pi}\left(\frac{2i \exp\left(-\pi\epsilon\sqrt{1 - 4p^2}\right)}{\sqrt{1 - 4p^2}} + 2i\sum_{n=1}^{\infty}\left\{\frac{\exp\left[-\pi\epsilon\sqrt{(2n + 1)^2 - 4p^2}\right]}{\sqrt{(2n + 1)^2 - 4p^2}} + \frac{\exp\left[-\pi\epsilon\sqrt{(2n - 1)^2 - 4p^2}\right]}{\sqrt{(2n - 1)^2 - 4p^2}}\right\} - \pi H_o^{(2)}(2\pi\epsilon p)\right) \tag{17}$$

and

$$S_2 = \sum_{n=1}^{\infty} (-1)^n H_o^{(2)}\left(\frac{MknH}{\beta}\right)$$

$$= \sum_{n=1}^{\infty} H_o^{(2)}(2\pi np)$$

$$= \frac{1}{2\pi}\left[-\pi + 2i\left(\gamma + \log_e 2p\right) + 4i\sum_{n=1}^{\infty}\left(\frac{1}{\sqrt{(2n - 1)^2 - 4p^2}} - \frac{1}{2n - 1}\right)\right] \tag{18}$$

where Eulers' constant $\gamma = 0.577215$.

It should be noted that the series becomes infinite when

$$4p^2 = (2n - 1)^2$$

or where

$$\frac{\omega H}{c} = \pi\beta(2n - 1)$$

This equation gives the conditions for resonance in a tunnel in which the air is oscillating in a transverse mode, and under which conditions the forces on the wing are zero.

Series S_3 was evaluated by utilizing the alternate expression for S_1 (eq. (17)) for the series of Hankel functions and integrating the resulting expression as follows:

$$S_3 = \int_0^\infty e^{\frac{-ik\xi}{\beta^2}} \sum_{n=1}^{\infty} (-1)^n H_o^{(2)}\left(\frac{kM}{\beta^2}\sqrt{\xi^2 + \beta^2(nH)^2}\right)d\xi$$

$$= -\frac{1}{2}\int_0^\infty e^{\frac{-ik\xi}{\beta^2}} H_o^{(2)}\left(\frac{kM\xi}{\beta^2}\right)d\xi +$$

$$\frac{2i\beta}{M}\sum_{n=0}^{\infty}\frac{1}{\sqrt{(2n + 1)^2\left(\frac{\pi\beta}{M}\right)^2 - (kH)^2}}\int_0^\infty e^{-\xi\left(\frac{M}{\beta^2 H}\sqrt{(2n+1)^2\left(\frac{\pi\beta}{M}\right)^2 - (kH)^2} + \frac{ik}{\beta^2}\right)}d\xi$$

$$= \frac{-\beta}{\pi k}\log_e\frac{1 + \beta}{M} + \frac{2i\beta}{M}\sum_{n=0}^{\infty}\frac{1}{\left(\frac{\pi}{\beta H}\right)^2\left[(2n + 1)^2 - 4p^2\right] + \left(\frac{k}{\beta^2}\right)^2}\left[\frac{M}{\beta^2 H} - i\frac{k/\beta^2}{\frac{\pi\beta}{M}\sqrt{(2n + 1)^2 - 4p^2}}\right] \qquad (19)$$

Series S_4 was evaluated in a direct manner by employing tables of the Hankel function and by using for large values of the argument the approximation

$$H_1^{(2)}(\mu R_n) \approx \sqrt{\frac{2}{\pi\mu R_n}}\, e^{-i\left(\mu R_n - \frac{3}{4}\pi\right)} \tag{20}$$

With the aid of these series S_1, S_2, S_3, and S_4, the kernel $K(M,z,H)$ may be evaluated. The integral I^{II} of equation (11) may be expressed in terms of the unknown coefficients A_n upon substitution of the assumed expression for $\bar{L}(\theta_o)$ and by graphical integration.

The integrals of equation (11) are determined in this manner for a selected number of control points and equated to the expression for the downwash.

The expression relating the downwash to the motion of a plunging and pitching wing is

$$w(x) = \dot{h} + U\alpha + b(x - a)\dot{\alpha} \tag{21}$$

With the assumption of harmonic motion, equation (21) is used to calculate $w(x)$ for values of x appropriate to each of the control points. A set of simultaneous equations can then be written, the number of which corresponds to the number of control points assumed and to the number of terms retained in the series for $\bar{L}(\theta_o)$. The unknown coefficients may now be determined by solving these simultaneous equations. It may then be shown (see, for instance, ref. 9) that the total lift and moment are given by

$$\left.\begin{aligned} \frac{-L}{\pi\rho bU^2} &= \frac{1}{2}\left(A_o + \frac{1}{2}A_1\right) \\ \frac{M_\alpha}{\pi\rho b^2U^2} &= \frac{1}{8}\left(A_o + \frac{1}{2}A_1\right) \end{aligned}\right\} \tag{22}$$

where M_α is the moment about the midchord.

Summary of procedure for calculating tunnel-wall effects.- As an aid in calculating the effects of wind-tunnel walls on an oscillating wing the following summary of the procedure is given. It is assumed that the Mach number, speed of sound, wing chord, frequency of oscillation, and tunnel height are known.

(1) Select the position x of each of the control points for which equation (11) will be written. First, treat I^{I} of equation (11) as given by equation (12).

(2) For each value of x, evaluate the first integral on the right side of equation (12), in the form given by equation (13), by numerical or graphical means.

(3) Evaluate the second integral of equation (12) in the form given by equation (14) by use of equation (15). Add the results to the results of step (2) to obtain I^{I} in terms of the unknowns A_n.

(4) For I^{II} of equation (11), first evaluate $K(M,z,H)$ given by equation (16) by use of the series given in equations (17) to (20).

(5) For each value of x substitute the values of $K(M,z,H)$ from step (4) in I^{II} of equation (11) together with $\overline{L}(\theta_o)$ from equation (10) and perform graphical or numerical integration to give I^{II} in terms of the A_n. Combine these results with the results of step (3) for the corresponding values of x.

(6) From equation (21), with the assumption of harmonic motion, calculate $w(x)$ for values of x appropriate to each of the control points.

(7) Equate results of steps (5) and (6) for corresponding values of x to obtain a set of simultaneous equations in the unknowns A_n.

(8) Solve the equations from step (7) for the A_n's.

(9) Determine lift and moment from equations (22).

Application to a specific problem.- The foregoing theory has been applied to the particular case of a wing oscillating in pitch about the midchord ($a = 0$, $h = 0$ in eq. (21)). Calculations have been performed for $M = 0.7$, $H = 3.802$ feet, $b = \frac{1}{2}$ foot and $c = 531$ feet per second for various frequencies of oscillation from 0 to 60 cycles per second.

In performing the calculations, the number of terms of the series for lift distribution (eq. (10)) and thus the number of control points required to obtain satisfactory accuracy was not known. Accordingly, the calculations were performed by increasing the number of terms of the series until the solutions were in reasonable agreement. First, the initial term of the series for lift distribution $A_o \cot \frac{\theta_o}{2}$ was used together with a control point at $\frac{3}{4}$-chord position. Then two terms $A_o \cot \frac{\theta_o}{2} + A_1 \sin \theta_o$ and two control points located at $\frac{3}{4}$- and $\frac{1}{2}$-chord positions were used. Finally, the first three terms of the series and control points at $\frac{1}{4}$-, $\frac{1}{2}$-, and $\frac{3}{4}$-chord positions were utilized. In addition, calculations were made for the case of one doublet placed at the $\frac{1}{4}$-chord position and for the downwash satisfied at the $\frac{3}{4}$-chord position. The results of these calculations are shown in figure 1.

In the calculations of the series for the determination of $K(M,z,H)$ it was found that as the resonant condition was approached, more terms were required for convergence. In all cases, however, a sufficient number of terms of the series was taken to assure convergence in the third decimal place. Series S_4 was the most slowly convergent and in one case required 240 terms in order to assure convergence.

EXPERIMENT

The experimental part of the investigation of the effect of wind-tunnel walls was conducted in the Langley 4.5-foot flutter research tunnel in which a 2-foot by 4-foot rectangular section has been temporarily installed for testing two-dimensional models. This tunnel is of the closed-throat, single-return type employing either air or Freon-12 as a testing medium at pressures from 1 atmosphere down to about 1/8 atmosphere. Since Freon-12 has a speed of sound equal to about one-half that of air and since the critical tunnel frequency varies directly as the speed of sound, the experiments to be discussed were conducted in Freon-12 so that the resonant frequency could be surveyed. The Mach number of the tests was $M = 0.7$ and the Reynolds number was 4.01×10^6. The frequency range was from 0 to 57.54 cycles per second, and the amplitude was about 2.4°.

The model was of 1-foot chord having NACA 65-010 airfoil sections and completely spanned the 2-foot dimension of the test section. The gaps between the wing and the tunnel wall were sealed by end plates which rotated with the model. The wing was oscillated in pitch about the midchord by a direct-drive eccentric-cam system powered by an induction motor with variable frequency supply.

The lift on the wing was obtained by electrical integration (ref. 10) of the outputs of 12 model 49-TP NACA miniature electrical pressure gages (ref. 11) arranged to indicate the differential pressures between orifices on the upper and lower surfaces at the midspan position. The angular motion of the midspan position was indicated by resistance wire strain gages attached to a torque rod running through the center of the hollow wing. One end of the torque rod was fixed to the center of the wing and the other end was fixed to the wind-tunnel wall.

The amplitude of the fundamental frequency of both the lift and position were indicated on an alternating-current vacuum-tube voltmeter attached to the output of a variable-frequency, narrow-band-pass filter. The phase angle between lift and position was determined by use of a special electronic timing system.

RESULTS AND DISCUSSION

A comparison of analytical and experimental results obtained for the lift on a wing pitching about its midchord is shown in figures 1 and 2. Variations in the magnitude of the lift and in phase angle are given as functions of the frequency of the pitching oscillation for a constant value of Mach number.

The ordinate in figure 1, the magnitude of the lift, is presented as a ratio of the value obtained either experimentally or theoretically, with consideration of the tunnel walls, to the theoretical value for the condition of no tunnel walls. As previously mentioned, in investigating the number of control points required to give sufficient accuracy, a series of calculations was made at each pitching frequency with each succeeding calculation including an added control point. The result of each of these calculations is shown in figure 1.

The abscissa in figure 1 is the ratio of the frequency of the pitching oscillation to a frequency calculated for the resonant condition. Since in the experiment the velocity of sound of the mediums in which the tests were made differed slightly from that considered in the theoretical work, the calculated resonant frequencies were not quite the same. The resonant frequency for conditions of the calculations was 49.5 cycles per second; whereas for conditions of the experiment, the resonant frequency was 52.6 cycles per second. The pitching frequencies for the calculated and the experimental results were divided by the appropriate value of the resonant frequency.

The experimental results show the existence of large tunnel-wall effects and demonstrate the existence of a resonant phenomenon as predicted in reference 4. The theoretical results are in very good agreement with experiment with regard to predicting the value of the resonant frequency. At low values of the frequency ratio the theoretical results for the lift ratio given in figure 1 indicate good agreement between the calculations for 1, 2, and 3 control points and are in good agreement with the experiment. At values of the frequency ratio nearer unity there is a wider divergence between the various solutions and between theory and experiment. This indicates that near resonance additional control points or possibly different control-point locations would be required to give a more accurate approach to the resonant condition.

As a matter of interest, and as a partial check on the mechanics of the procedure, calculations of the aerodynamic lift and moment coefficients for the condition of no tunnel walls were performed with three control points, and excellent agreement was obtained with the tabulated results of Dietze.

In figure 2 the phase angle between lift force and position of the wing is plotted against the same frequency ratio ω/ω_{res} used in figure 1. Although the variation of phase angle with frequency in free air is almost linear, the phase angles found both experimentally and by the theory including wall effects show a very large drop as the resonant frequency is approached and change abruptly. The calculated values of the phase angle are in good quantitative agreement with those measured experimentally.

Since the underlying theory of an oscillating wing in a compressible fluid is one of acoustics, it seems reasonable to conjecture that some application of the classic principles of sound absorption to the test section of a tunnel might be a method of eliminating or greatly reducing this new type of tunnel-wall interference. Any method of sound absorption employed, of course, should not interfere with the basic flow in the tunnel. One tunnel configuration, which has been suggested for the steady-flow case and which may be desirable for the present case, is the ventilated tunnel.

CONCLUSIONS

This paper presents the results of an application of the theory as developed in NACA TN 2552 (which treats the effect of the presence of tunnel walls for oscillating two-dimensional compressible flow) and compares the results with some experiments at a Mach number of 0.7. The following conclusions may be enumerated:

1. Some experimental results of lift and phase angles made at a constant Mach number and for various oscillating frequencies show the existence of large tunnel-wall effects and demonstrate the existence of a resonant phenomenon as predicted in NACA TN 2552.

2. A method of calculating the effect of tunnel walls on the oscillating two-dimensional air forces is presented.

3. Comparison of the experimental lift with the theoretical lift which includes the effect of tunnel walls indicates good agreement throughout the frequency range.

4. A comparison of the phase angles as determined by experiment and as calculated by the theory which includes the effect of tunnel walls shows good agreement throughout most of the frequency range and shows a very large phase-angle change as the resonant frequency is approached. It is observed, however, that the theory without tunnel walls does not give either the trend or the magnitude of the experimental results.

5. At the Mach number of these tests (0.7) the effects of the tunnel walls is very great at the resonant frequency and the large effect extends over a considerable range of frequencies.

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APPENDIX

REDUCTION OF INTEGRAL EQUATION TO THE CASE OF ZERO FREQUENCY

In this appendix, the integral equation for the downwash for a wing oscillating in a compressible medium and enclosed by wind-tunnel walls is reduced to the zero-frequency condition.

If equation (7) is written as

$$w(x) = \lim_{\omega \longrightarrow 0} \frac{1}{\rho U^2} \int_{-b}^{b} L(x_o)\left[\omega K(M,z) + \omega K(M,z,H)\right] dx_o \qquad \text{(A1)}$$

and the limit is taken as $\omega \longrightarrow 0$, it will be found that all the terms of $K(M,z)$ and $K(M,z,H)$ are canceled except terms involving $H_1^{(2)}$. These terms become infinite, however, as $\omega \longrightarrow 0$ so that the asymptotic expansion for very small values of the argument may be used; therefore,

$$H_1^{(2)}(\mu R_n) \approx -\frac{2}{\pi i \mu R_n}$$

and

$$\lim_{\omega \longrightarrow 0} e^{\frac{iz}{\beta^2}} H_1^{(2)}(\mu R_n) \frac{iM(x - x_o)}{R_n} = \frac{-2Mc\beta^2(x - x_o)}{\pi\left[(x - x_o)^2 + \beta^2(nH)^2\right]}$$

The vertical induced velocity may then be written as

$$w(x) = \frac{-Mc\beta}{2\pi\rho U^2} \int_{-b}^{b} L(x_o)\left[\frac{1}{x - x_o} + 2\sum_{n=1}^{\infty} (-1)^n \frac{x - x_o}{(x - x_o)^2 + \beta^2(nH)^2}\right] dx_o \qquad \text{(A2)}$$

or

$$w(x) = \frac{-Mc}{2\rho U^2 H} \int_{-b}^{b} L(x_o)\left[\frac{1}{\frac{\pi}{\beta H}(x - x_o)} + 2\sum_{n=1}^{\infty} (-1)^n \frac{\frac{\pi(x - x_o)}{\beta H}}{\frac{\pi^2(x - x_o)^2}{\beta^2 H^2} + n^2\pi^2}\right] dx_o \qquad \text{(A3)}$$

This equation may be written as

$$w(x) = \frac{-Mc}{2\rho U^2 H} \int_{-b}^{b} L(x_o)\left[\operatorname{csch} \frac{\pi(x - x_o)}{\beta H}\right]dx_o \tag{A4}$$

The additional induced velocity due to the presence of tunnel walls for the steady case in compressible flow is given by equation (40) of reference 12. Equation (A2) can be reduced to the same form by making the approximation that the airfoil chord is small compared to the tunnel height.

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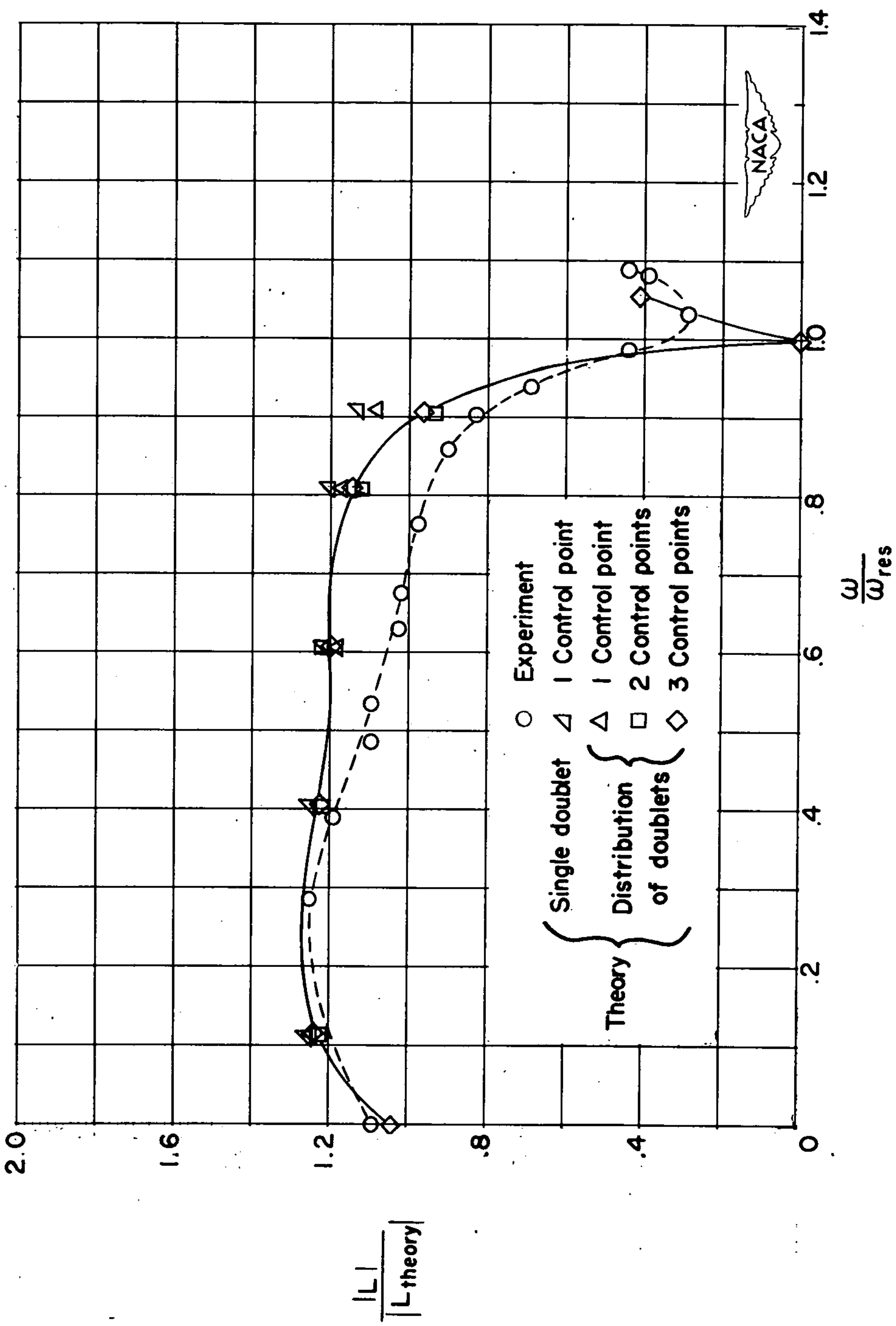

Figure 1.- Plot of ratio of magnitude of experimental and theoretical lift, including tunnel-wall effects, to the theoretical lift without tunnel-wall effects against frequency ratio of oscillation to resonant frequency. M = 0.7.

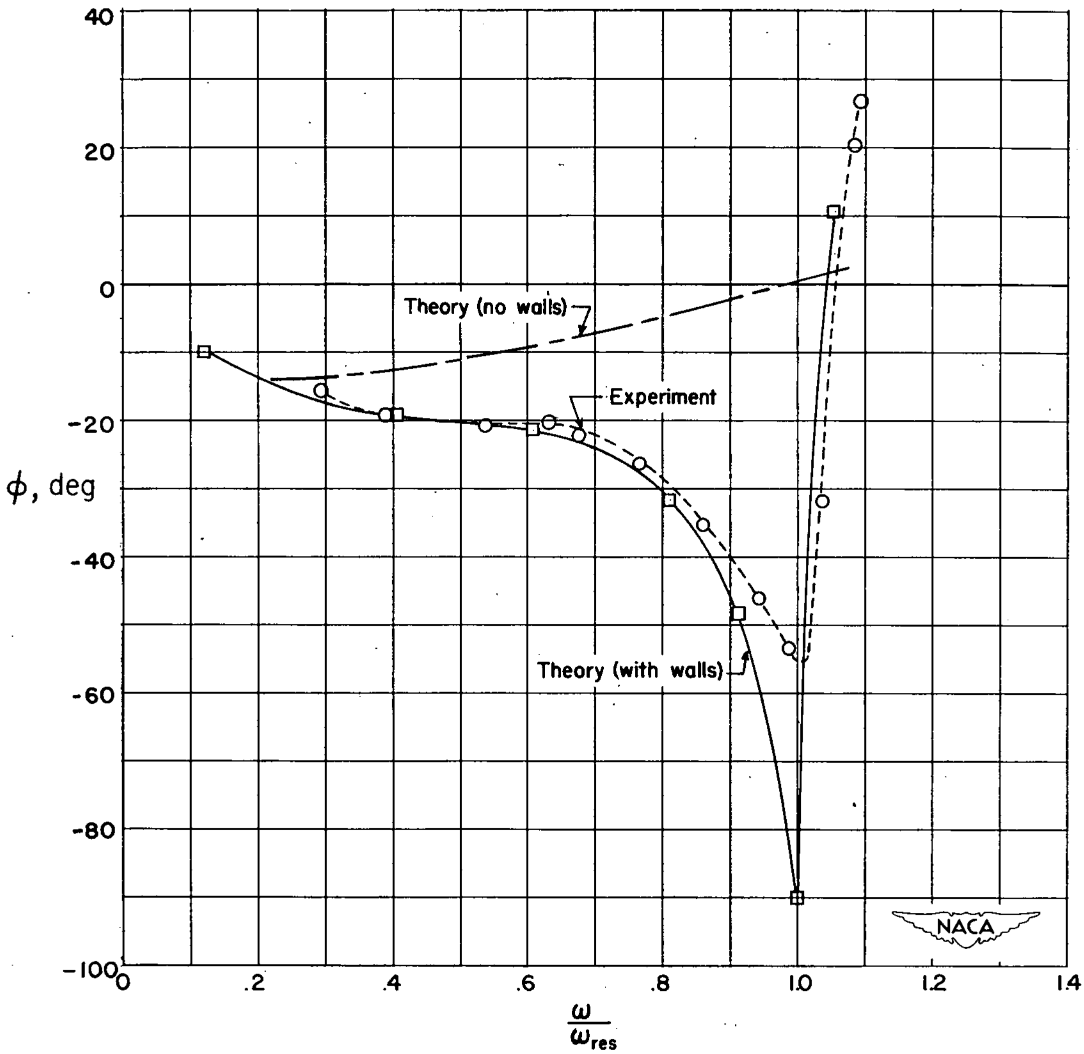

Figure 2.- Comparison of theoretical and experimental phase angle between lift force and position of wing. $M = 0.7$.

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