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RESEARCH MEMORANDUM

ON THE RANGE OF APPLICABILITY OF
THE TRANSONIC AREA RULE

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**NATIONAL ADVISORY COMMITTEE
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WASHINGTON

August 23, 1954

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RESEARCH MEMORANDUMON THE RANGE OF APPLICABILITY OF
THE TRANSONIC AREA RULE

By John R. Spreiter

SUMMARY

Some insight into the range of applicability of the transonic area rule has been gained by comparison with the appropriate similarity rule of transonic flow theory and with available experimental data for a large family of rectangular wings having NACA 63AXXX profiles. In spite of the small number of geometric variables available for such a family, the range is sufficient that cases both compatible and incompatible with the area rule are included.

INTRODUCTION

A great deal of effort is presently being expended in correlating the zero-lift drag rise of wing-body combinations on the basis of their streamwise distribution of cross-section area. This work is based on the discovery and generalization announced by Whitcomb in reference 1 that "near the speed of sound, the zero-lift drag rise of thin low-aspect-ratio wing-body combinations is primarily dependent on the axial distribution of cross-sectional area normal to the air stream." It is further conjectured in reference 1 that this concept, known as the transonic area rule, is valid for wings with moderate twist and camber. Since an accurate prediction of drag is of vital importance to the designer, and since the use of such a simple rule is appealing, it is a matter of great and immediate concern to investigate the applicability of the transonic area rule to the widest possible variety of shapes of aerodynamic interest.

The experimental data contained in reference 1 and many subsequent papers have shown that this simple rule is often remarkably successful for a wide variety of shapes ranging in complexity from simple bodies of revolution to models of complete airplanes. Furthermore, important reductions in the transonic drag of wing-body combinations have been realized by indenting the body so that the axial distribution of cross-section area corresponds to that of smooth bodies of revolution having low drag.

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supersonic speeds. On the other hand, it is important not to overlook the fact that there are a number of test results on equivalent bodies for which the correlation of drag rise by the area rule is unsatisfactory. Inasmuch as the models tested are generally of complex geometry, and only the original model and an equivalent body of revolution are tested, it is difficult to ascertain whether these discrepancies are attributable to viscous phenomena or to the fact that the drag rise may depend on other geometric parameters than the axial distribution of cross-section area.

It is the purpose of this note to examine in further detail the applicability of the area rule. Despite the fact that most of the emphasis in the tests relative to the area rule has been on wing-body combinations, there exists such a scarcity of experimental data of a sufficiently systematic type that the present discussion will be confined to rectangular wings without bodies. Transonic drag data are available from bump tests in the Ames 16-foot high-speed wind tunnel for a large family of rectangular wings having NACA 63AXXX sections, aspect ratios varying from 0.5 to 6.0, thickness ratios from 2 to 10 percent, and both symmetrical and cambered profiles. These results are reported in references 2, 3, and 4 and have been studied by McDevitt (refs. 3 and 5) who showed in a convincing manner that the experimental data can be correlated successfully by means of the transonic similarity rules. These same results will be used herein to evaluate one phase of the transonic area rule. The relationship between the two rules is naturally of interest and will also be explored.

Since the transonic area rule is considered to apply equally to all low-aspect-ratio wing-body combinations, detailed examination of such a limited class of aerodynamic shapes as a family of rectangular wings is not without value inasmuch as limitations revealed in special applications must appear as a limitation in the general case. The restricted range of the investigation is compensated somewhat by the fact that the geometric simplicity increases the chances of understanding the underlying causes. Although the results can only be said to apply with surety to the specific cases investigated, the method of approach is not restricted and may be applied similarly to other cases as more data become available and as understanding increases. In this way, the present discussion may be considered more as suggestive than definitive.

PRINCIPAL SYMBOLS

A	aspect ratio
\bar{A}	$[(\gamma + 1) M_0^2 \tau]^{1/3} A$
b	wing span

$$C_{D_{0w}} = \frac{D_{0w}}{qS_p}$$

$$\Delta C_{D_0} = \Delta \frac{D_0}{qS_p}$$

$$C_L = \frac{L}{qS_p}$$

$$\bar{C}_L = \frac{[M_0^2(\gamma + 1)]^{1/3}}{\tau^{2/3}} C_L$$

C_{l_i} ideal lift coefficient

c wing chord

D drag

D_0 drag at zero lift

D_{0w} wave drag at zero lift

$$\Delta \left(\frac{D_0}{qc^2} \right) = \left(\frac{D_0}{qc^2} \right)_{M_0} - \left(\frac{D_0}{qc^2} \right)_{M_{ref}}$$

f function of indicated variables

g_1 dimensionless function describing thickness distribution

g_2 dimensionless function describing camber distribution

$$H = \frac{h}{c}$$

h maximum camber of wing

$$\bar{h} = \frac{h}{t}$$

L lift

M_0 Mach number

M_{ref} reference Mach number less than the critical

q dynamic pressure, $\frac{\rho_0}{2} U_0^2$

R	Reynolds number
R_{ref}	reference Reynolds number corresponding to M_{ref}
S	area
S_c	cross-section area
S_m	maximum cross-section area
S_p	plan-form area
s	dimensionless area distribution function
t	maximum thickness
U_0	free-stream velocity
x,y	Cartesian coordinates in plane of wing where x extends in the direction of the free-stream velocity
Y	dimensionless function describing plan form
Z	ordinates of wing profiles
Z_u	ordinates of upper surface of wing profiles
Z_l	ordinates of lower surface of wing profiles
α	angle of attack
$\bar{\alpha}$	$\frac{\alpha}{t/c}$
γ	ratio of specific heats, for air $\gamma = 1.4$
ξ_0	$\frac{M_0^2 - 1}{[(\gamma + 1) M_0^2 \tau]^{2/3}}$
ρ_0	density
τ	$\frac{t}{c}$

DIMENSIONAL CONSIDERATIONS

The forces on a body moving through an infinite fluid are dependent on a number of parameters equal to that necessary to describe the problem. Thus, it is well known from dimensional considerations that the drag D of a body can be expressed as the product of a pressure, say the dynamic pressure $q = \frac{\rho_0}{2} U_0^2$, a characteristic area S , and some function of Mach number M_0 , Reynolds number R , the geometry of the body, and gas properties such as the ratio of the specific heats γ , etc. This can be written symbolically as follows:

$$D = qSf(M_0, R, \text{geometry, gas properties}) \quad (1)$$

(Note that S does not necessarily refer to the plan-form area, but, rather, to any combination of geometric lengths having the dimensions of area.) The geometry of a body can be described by a number of dimensionless parameters, the number of which depends on the complexity of the shape. If these considerations are applied to the present family of rectangular wings having NACA 63AXXX profiles, the description is particularly simple since the wing plan form is determined by specifying the aspect ratio A , the profile by the thickness ratio $\tau = t/c$ and the camber ratio $H = h/c$, and the inclination of the wing by the angle of attack α . In this way, equation (1) can be rewritten in the following more explicit form:

$$\frac{D}{qS} = f(M_0, R, A, \tau, H, \alpha, \text{gas properties}) \quad (2)$$

If, as in nearly all problems of aerodynamic practice, all measurements are made in the same fluid, air, the gas properties can be represented by merely a set of constants and therefore disappear as parameters in equation (2) leaving only

$$\frac{D}{qS} = f(M_0, R, A, \tau, H, \alpha) \quad (3)$$

If attention is confined to the drag at zero lift D_0 , as is the case with the transonic area rule, further simplification results because α is then a dependent parameter and may be eliminated even though it is not a constant for variously cambered wings. Then

$$\frac{D_0}{qS} = f(M_0, R, A, \tau, H) \quad (4)$$

The proof of the foregoing step is as follows: The relationship for lift analogous to equation (3) is

$$C_L = \frac{L}{qS} = f(M_0, R, A, \tau, H, \alpha) \quad (5)$$

where f is, of course, a different function of the indicated variables. This can be solved for α

$$\alpha = f(M_0, R, A, \tau, H, C_L) \quad (6)$$

and substituted in its place in equation (3) to produce

$$\frac{D}{qS} = f(M_0, R, A, \tau, H, C_L) \quad (7)$$

In this form, it is clear that the restriction to zero lift fixes the value of the last parameter, thereby leaving D_0/qS dependent only on the remaining parameters as in equation (4).

Although dimensional considerations such as the foregoing display the parameters or factors that influence the drag, they do not provide any information on the nature of the functional relations involved. Thus, it may be that some of the parameters are more important than others, or that a parameter is of importance over a certain range of values but of negligible importance in another range, or that a parameter is important or not depending on the value of another parameter, etc. Examples are that D_0/qS depends on M_0 at transonic and supersonic speeds but not at subsonic speeds, D_0/qS depends on A for small A but not for large A (if S refers to the plan-form area), etc. Since there exists a great body of literature pertinent to the nature of these dependencies which is probably quite familiar to the readers, no further amplification appears necessary here.

TRANSONIC AREA RULE

Theoretical Considerations

The customary verbal statement of the transonic area rule is given in the INTRODUCTION. It should be noted that the term "drag rise" does not refer to the increase in drag force D_0 , but to the increase in the ratio D_0/q between a subcritical reference Mach number M_{ref} and a transonic Mach number M_0 . If $S_c(x)$ represents the streamwise or axial distribution of cross-section area, the transonic area rule states that the variations $\Delta(D_0/q)$ with Mach number M_0 are the same for all low-aspect-ratio wing-body combinations having the same $S_c(x)$. Note that the rule in this form relates a very wide class of wings, bodies, etc., but that all related configurations must have the same longitudinal length c . This restriction can be removed easily by recasting the statement in dimensionless form using c as the characteristic length. Thus, we can say that the variation of $\Delta(D/qc^2)$ with Mach number M_0 is the same for all low-aspect-ratio wing-body combinations having the same

$S_c(x/c)/c^2$. From a slightly different point of view, the area rule states that, near the speed of sound, $\Delta(D/qc^2)$ is a function of M_0 and $S_c(x/c)/c^2$, or

$$\Delta\left(\frac{D_0}{qc^2}\right) = \left(\frac{D_0}{qc^2}\right)_{M_0} - \left(\frac{D_0}{qc^2}\right)_{M_{ref}} = f\left[M_0, \frac{S_c(x/c)}{c^2}\right] \quad (8)$$

The axial distribution of cross-section area $S_c(x/c)$ can be written in the form

$$S_c(x/c) = S_m s(x/c) \quad (9)$$

where S_m is the maximum cross-section area and $s(x/c)$ is an area-distribution function. Application of the transonic area rule to the drag results of the present family of rectangular wings is facilitated particularly by the fact that $s(x/c)$ is the same for all wings. As a result, the area distribution $S_c(x/c)$ of any wing in this family is specified by stating the value of the maximum cross-section area S_m , so that

$$\Delta(D_0/qc^2) = f(M_0, S_m/c^2) \quad (10)$$

Since the chord c and thickness t of each wing are constant across the span b , the maximum cross-section area is equal to the product of the span and thickness

$$S_m = bt \quad (11)$$

and

$$\frac{S_m}{c^2} = \frac{b}{c} \frac{t}{c} = A\tau \quad (12)$$

whence equation (10) can be rewritten for this family of airfoils as

$$\Delta\left(\frac{D_0}{qc^2}\right) = f(M_0, A\tau) \quad (13)$$

The remarkable simplicity of these statements is emphasized by comparison with the functional relation revealed by dimensional consideration alone. Thus, if the unspecified area S is replaced by c^2 , equation (4) yields

$$\Delta\left(\frac{D_0}{qS}\right) = \Delta\left(\frac{D_0}{qc^2}\right) = f(M_0, R, A, \tau, H) - f(M_{ref}, R_{ref}, A, \tau, H) \quad (14)$$

where R_{ref} refers to the value of Reynolds number associated with the subcritical reference Mach number M_{ref} , and the symbol f refers in each case to the appropriate function of the indicated variables. In the customary discussion of drag-rise data, M_{ref} and R_{ref} are constants and no

longer appear as parameters in equation (14). Further simplification occurs in most cases because the wind-tunnel or flight test technique determines a specific relation between the Mach and Reynolds numbers. As a result, either M_0 or R can be removed as parameters since the value of either is determined by that of the other. Since the present problem is more closely connected with effects of compressibility than of viscosity, it is appropriate to retain M_0 as the significant parameter. In this way, equation (14) reduces to

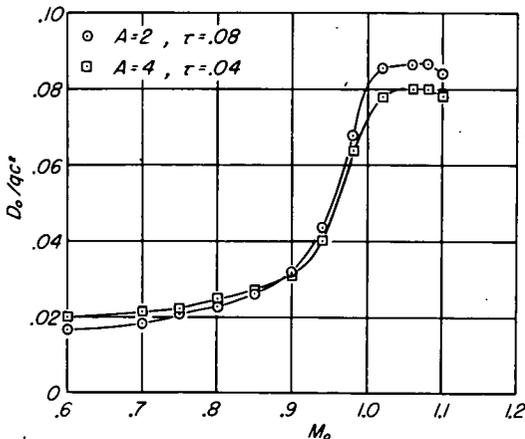
$$\Delta \left(\frac{D_0}{qc^2} \right) = f(M_0, A, \tau, H) \quad (15)$$

Comparison of equations (13) and (15) highlights the fact that the area rule affirms, in dimensionless terms, that the drag-rise parameter $\Delta(D_0/qc^2)$ for the present family of wings depends on Mach number and the product of aspect ratio and thickness ratio $A\tau$ (or the maximum cross-section area parameter S_m/c^2) but is independent of camber ratio H , and aspect ratio A or thickness ratio τ taken separately.

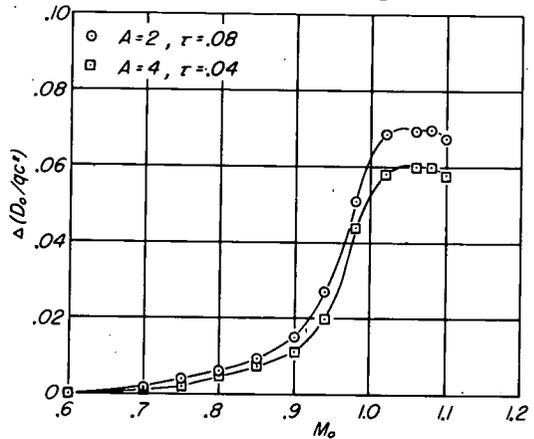
Comparison with Experiment

Application to wings having identical area distributions.- The applicability of the transonic area rule to the present family of rectangular wings can be examined in several ways. Perhaps the most obvious way is to actually compare the variation of $\Delta(D_0/qc^2)$ with M_0 for two or more wings having identical area distributions. For the present family of wings, this means comparing wings having the same S_m/c^2 or $A\tau$. The transonic area rule predicts that the variation of $\Delta(D_0/qc^2)$ with M_0 should be the same for all such wings.

An example of such a comparison is shown in sketches (a) and (b). The experimental data are from references 2 and 3. Both wings have



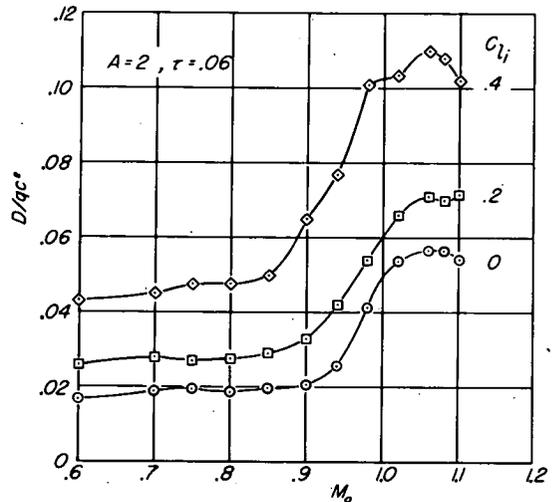
Sketch (a)



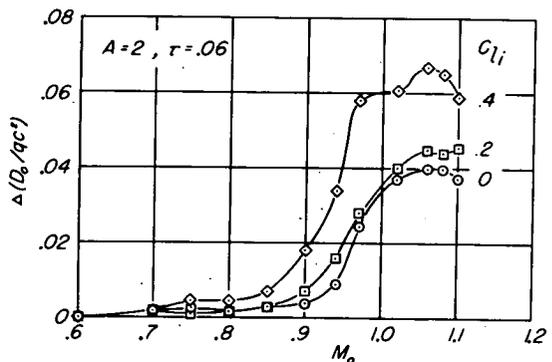
Sketch (b)

symmetrical sections ($H = 0$) and $A\tau = 0.16$ but one has an aspect ratio of 2 and thickness ratio of 0.08; whereas, the other has an aspect ratio of 4 and thickness ratio of 0.04. The first sketch shows the total drag and the second the drag rise determined by subtracting the value of D_0/qc^2 at $M_0 = 0.6$. Although the two curves in sketch (b) are not identical as predicted by the area rule, they are closely related. Innumerable reasons could be advanced to explain the differences between the two curves; perhaps there are viscous effects which may significantly affect the drag rise, perhaps the measurements are not sufficiently accurate or the flow field sufficiently uniform, or perhaps the aspect ratio or thickness ratio is too large, etc. In any case, this particular comparison would probably be scored in favor of the transonic area rule.

Another comparison, this time among three wings of aspect ratio 2, thickness ratio 0.06, but different amounts of camber is shown in sketches (c) and (d). The amount of camber is specified by the ideal lift coefficient C_{li} in accordance with the NACA scheme for airfoil section designation. Again, the basic data are presented in the first sketch and the drag rise in the second. In this case, however, the variation of $\Delta(D_0/qc^2)$ with M_0 is definitely not the same for the three wings and the use of the area rule could lead to serious error. At a Mach number of unity, where the area rule is supposed to be most accurate, the drag rise of the wing with greatest camber is nearly twice that of the uncambered wing.



Sketch (c)



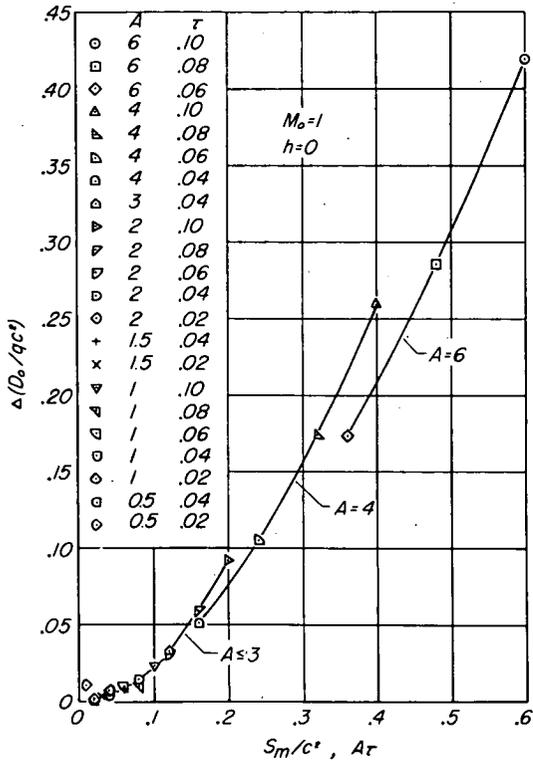
Sketch (d)

Wings having similar area distribution. - Although a certain number of additional comparisons of the type described in the preceding section can be made using the data of references 2 through 5, the number is definitely limited because the test program was not designed to preserve a single value for $A\tau$ for all wings. It is furthermore not practical to carry out extensive programs of such a type because it necessitates the testing of very thin wings of high aspect ratio and thick wings of low aspect ratio. As mentioned in the derivation of equation (13), however, all members of the present family of wings have similar distributions of

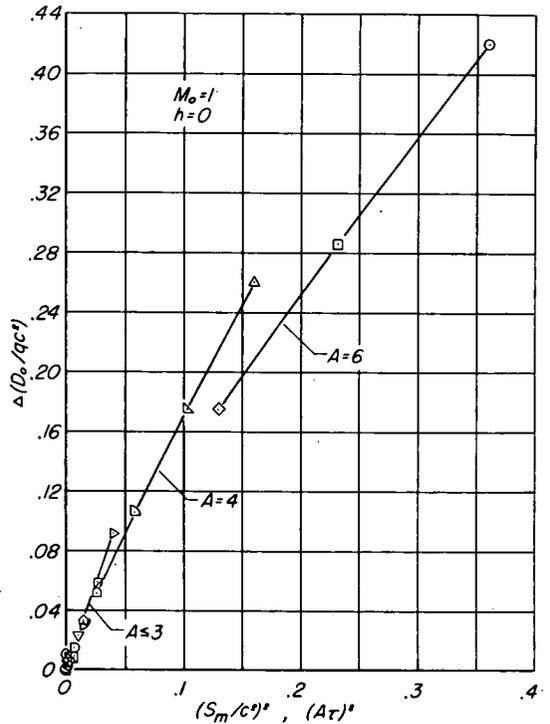
cross-section area (a single area-distribution function $s(x/c)$), and the transonic area rule can be extended to include such cases by introducing S_m/c^2 or $A\tau$ as a second parameter. In exchange for being able to correlate the drag rise of bodies having not only identical, but also similar area distributions, we incur the complications of a dependence on two parameters rather than only one. Thus, whereas the curves representing the variation of $\Delta(D_0/qc^2)$ with M_0 for all wings having identical S_m/c^2 or $A\tau$ should coincide to form a single line, those for a family of wings having similar area distributions should form a family of lines. Simplicity can be regained, however, by restricting attention to a single Mach number and ascertaining the variation of $\Delta(D_0/qc^2)$ with S_m/c^2 or $A\tau$. Since the customary statement of the area rule restricts attention to near-sonic speeds, the most appropriate single Mach number to select for such a comparison is unity.

Sketch (e) shows the variation of $\Delta(D_0/qc^2)$ with S_m/c^2 for all the uncambered wings of references 2 and 3. It can be seen that all these results fall near to a single curved line for wings of all aspect ratios up to 3 but that those for wings of aspect ratios 4 and 6 depart from this line in a systematic manner.

The same results are replotted in sketch (f) versus the square of S_m/c^2 rather than the first power. It can be seen that the curved line



Sketch (e)



Sketch (f)

of sketch (e) for wings having aspect ratios less than 3 is now a straight line, indicating that the sonic drag rise for these wings is proportional to the square of the maximum cross-section area. Such a dependence is consistent with the formulas of linearized compressible flow theory for the wave drag of nonlifting slender wings and bodies at supersonic speeds.

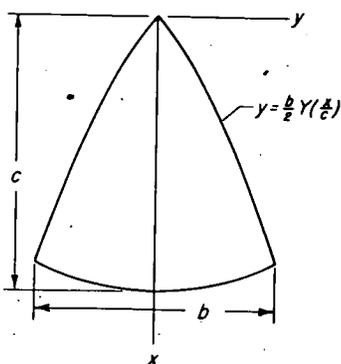
These plots can be interpreted as showing that the sonic drag rise of the uncambered members of the present family of wings depends on the cross-section area in accordance with the area rule, provided the aspect ratio is about 3 or less. On the other hand, the results for wings of higher aspect ratio can only be interpreted as indicating that some parameter other than the cross-section area must be involved. Inasmuch as the geometry of wings of the present family is described completely by the two parameters, aspect ratio and thickness ratio, it is clear that these parameters must assume importance in some form other than their simple product Ar for wings of larger aspect ratio. One could seek the new relation empirically, but the transonic similarity rules provide a theoretical basis for proceeding. Some properties of transonic similarity rules are reviewed in the following section, although the reader is referred to the original references for further details.

TRANSONIC SIMILARITY RULES

Statement of Rule

Transonic similarity rules are derived from the nonlinear equation of inviscid flow theory and are known for thin wings (e.g., ref. 6) and slender bodies of revolution (ref. 7), but not for wing-body combinations with pointed noses. In contrast to the transonic area rule which relates the zero-lift drag rise of families of bodies having identical or similar axial distributions of cross-section area, the transonic similarity rules relate the aerodynamic properties of much more highly restricted families of bodies. Even for wings alone, the restrictions imposed on the members of a single family are much more severe than for the area rule, since all members of a single family must have affinely related plan forms, affinely related thickness distributions, and affinely related camber distributions. To be more explicit: if the plan form is given by

$$\frac{y}{b/2} = Y(x/c) \quad (16)$$

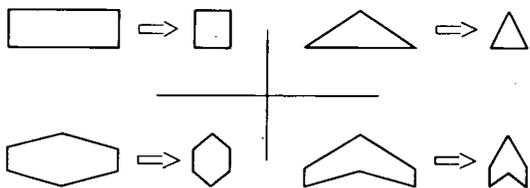


Sketch (g)

as indicated in sketch (g), it is required that $Y(x/c)$ be a single function for all wings of a given family. Furthermore, if the ordinates of the wing surface are given by

$$\frac{Z}{c} = \frac{t}{c} \left[\pm \frac{1}{2} g_1 \left(\frac{x}{c}, \frac{y}{b} \right) + \frac{h}{t} g_2 \left(\frac{x}{c}, \frac{y}{b} \right) - \frac{\alpha}{t/c} \frac{x}{c} \right] \quad (17)$$

where the plus sign is associated with the ordinates of the upper surface Z_u and the minus sign with those of the lower surface Z_l , it is required that $g_1(x/c, y/b)$ and $g_2(x/c, y/b)$ be single functions for all wings of a given family. The first of these restrictions requires that related plan forms be obtainable one from another by a differential lengthening of lateral and longitudinal dimensions. Thus rectangular plan forms constitute one family, triangular plan forms with straight trailing edges another, etc. Examples of related plan forms are illustrated in sketch (h).



Sketch (h)

The relationship expressed in equation (17) requires that the thickness distribution and camber variations must be the same for all wings of a particular family. The magnitudes of the maximum thickness ratio t/c and camber thickness ratio h/c , as well as the angle of attack α , may be different for various members of a single family.

From the present point of view, one of the most significant properties of the family of wings described above is that all members have the same dimensionless area-distribution function $s(x/c)$. This can be shown as follows:

$$\begin{aligned} S(x/c) &= \int_{-Y(x/c)}^{+Y(x/c)} (Z_u - Z_l) dy = \int_{-Y}^{+Y} t g_1 \left(\frac{x}{c}, \frac{y}{b} \right) dy \\ &= bt \int_{-Y/b}^{+Y/b} g_1 \left(\frac{x}{c}, \frac{y}{b} \right) d \left(\frac{y}{b} \right) = bt \bar{g}(x/c) = S_m s(x/c) \quad (18) \end{aligned}$$

where \bar{g} represents the integral of the preceding expression and is proportional to $s(x/c)$, since S_m is proportional to bt for a family of wings. Because of this fact, a family of wings suitable for correlation by the transonic similarity rules is also part of a family of bodies suitable for correlation by the area rule.

For wings of such a family, the similarity rules of inviscid, small-disturbance, transonic flow theory provide that the wave drag D_w be given by

$$D_w = qS_p \frac{\tau^{5/3}}{[M_o^2(\gamma+1)]^{1/3}} f \left\{ \frac{M_o^2 - 1}{[(\gamma+1)M_o^2\tau]^{2/3}}, [(\gamma+1)M_o^2\tau]^{1/3} A; \frac{h}{t}, \frac{\alpha}{t/c} \right\}$$

$$= qS_p \frac{\tau^{5/3}}{[M_o^2(\gamma+1)]^{1/3}} f(\xi_o, \bar{A}, \bar{h}, \bar{\alpha}) \quad (19)$$

where

$$\tau = t/c$$

$$\xi_o = \frac{M_o^2 - 1}{[(\gamma+1)M_o^2\tau]^{2/3}}$$

$$\bar{A} = [(\gamma+1)M_o^2\tau]^{1/3} A$$

$$\bar{h} = h/t$$

$$\bar{\alpha} = \alpha/\tau$$

and S_p is the plan-form area of the wing. Equation (19) is not in the proper form for comparison with the area rule since the latter is concerned with conditions at zero lift and, hence, imposes an indirect requirement on $\bar{\alpha}$. We can proceed toward the desired form, however, by introducing the transonic similarity rule for the lift coefficient

$$C_L = L/qS_p$$

$$C_L = \frac{\tau^{2/3}}{[M_o^2(\gamma+1)]^{1/3}} f(\xi_o, \bar{A}, \bar{h}, \bar{\alpha}) \quad (20)$$

and defining the reduced lift coefficient

$$\bar{C}_L = \frac{[M_o^2(\gamma+1)]^{1/3}}{\tau^{2/3}} C_L = f(\xi_o, \bar{A}, \bar{h}, \bar{\alpha}) \quad (21)$$

Now since \bar{C}_L is a function of the same four variables that appear in equation (19), $\bar{\alpha}$ can be replaced with \bar{C}_L in the latter equation, whence

$$D_w = qS_p \frac{\tau^{5/3}}{[M_o^2(\gamma+1)]^{1/3}} f(\xi_o, \bar{A}, \bar{h}, \bar{C}_L) \quad (22)$$

The condition of zero lift eliminates the last parameter, leaving only

$$\begin{aligned}
 D_{Ow} &= q S_p \frac{\tau^{5/3}}{[M_o^2(\gamma+1)]^{1/3}} f(\xi_o, \bar{A}, \bar{h}) \\
 &= q S_p \frac{\tau^{5/3}}{[M_o^2(\gamma+1)]^{1/3}} f \left\{ \frac{M_o^2 - 1}{[(\gamma+1)M_o^2\tau]^{2/3}}, \right. \\
 &\quad \left. [(\gamma+1)M_o^2\tau]^{1/3} A, h/t \right\} \quad (23)
 \end{aligned}$$

Application to Rectangular Wings and Comparison with Transonic Area Rule

The restrictions introduced in the derivation of the transonic similarity rule for zero-lift wave drag are such that they permit the direct application of equation (23) to the present family of rectangular wings having NACA 63AXXX profiles. It is natural to compare this functional relation with the corresponding relations of equations (10) and (13) given by the transonic area rule.

$$\Delta \left(\frac{D}{qc^2} \right) = f(M_o, S_m/c^2) = f(M_o, A\tau) \quad (24)$$

At first glance, the two sets of relationships appear to bear only slight resemblance. It can be seen upon closer examination, however, that some of the apparent differences are superficial and of little or no significance. For instance, equation (23) is concerned with wave drag D_{Ow} whereas equation (24) is concerned with drag rise. It is evident, however, that the two rules are actually concerned with the same quantity, since the drag rise can be considered to be an approximation for the wave drag under the assumption that the friction drag coefficient is independent of Mach number.

Another point of apparent lack of resemblance is that equation (23) does not show an explicit relationship between wave drag and maximum cross-section area, since the latter is not a function of $S_p\tau^{5/3}$ nor of ξ_o , \bar{A} , or \bar{h} taken separately. This does not mean that the transonic similarity rule is incompatible with the area rule because there are several possibilities for making such a dependence visible. Two permissible procedures are to multiply the right side of equation (23) by either ξ_o or \bar{A} . The first procedure is of no help for the discussion of conditions at $M_o = 1$, since the quantity $M_o^2 - 1$ appears in two places and an indeterminate form ensues. The second is perfectly acceptable, however, and produces the following relationships:

$$\begin{aligned}
 D_{Ow} &= q S_p A \tau^2 f(\xi_0, \bar{A}, \bar{h}) \\
 &= qc^2 \left(\frac{bt}{c^2} \right)^2 f(\xi_0, \bar{A}, \bar{h}) \quad (25)
 \end{aligned}$$

Since the maximum cross-section area S_m of the present family of rectangular wings is equal to the product bt , equation (25) can be rewritten as follows:

$$\begin{aligned}
 D_{Ow} &= qc^2 \left(\frac{S_m}{c^2} \right)^2 f(\xi_0, \bar{A}, \bar{h}) \\
 &= qc^2 \left(\frac{S_m}{c^2} \right)^2 f \left\{ \frac{M_0^2 - 1}{[(\gamma+1)M_0^2 \tau]^2/3}, \right. \\
 &\quad \left. [(\gamma+1)M_0^2 \tau]^{1/3} A, h/t \right\} \quad (26)
 \end{aligned}$$

Equation (24) can likewise be rewritten by multiplying f by the square of S_m/c^2 ,

$$\Delta \left(\frac{D_o}{qc^2} \right) = \left(\frac{S_m}{c^2} \right)^2 f(M_0, S_m/c^2) = \left(\frac{S_m}{c^2} \right)^2 f(M_0, A\tau) \quad (27)$$

This appears to be the closest that the two rules can be brought together without introducing additional simplifications. Both rules are now concerned with essentially the same quantity, D_{Ow}/qc^2 and $\Delta(D_o/qc^2)$. Each rule states that this quantity is proportional to the square of S_m/c^2 times some unknown function of certain specified parameters. The two rules disagree completely, however, as to the nature of the parameters. The transonic similarity rule specifies three parameters ξ_0 , \bar{A} , and \bar{h} , whereas the area rule specifies only two, M_0 and $A\tau$. Since neither set can be transformed into the other, it is apparent that the only way in which both rules can be universally correct is for the function f to be actually a constant and not to depend on the value of any of the five parameters. It is obvious that such is not the case, however, since it requires, for instance, that the wave drag be independent of Mach number.

There is a way out of this apparent impasse, however, if the range of validity of one or both of the rules is restricted sufficiently that f is independent of the remaining variables. Comparison of the two rules shows that the drag depends in both cases on the Mach number of the stream and the geometry of the wing and, in the case of the transonic similarity rule, on the ratio of the specific heats γ . Since nearly all problems of aerodynamic interest are concerned solely with air, however, γ is a constant and need not be retained as a parameter. The resulting simplified equation can be written in full in either of the following forms:

$$\begin{aligned} \frac{D_{Ow}}{qc^2} &= \left(\frac{S_m}{c^2}\right)^2 f \left[\frac{M_o^2 - 1}{(M_o^2 \tau)^{2/3}}, (M_o^2 \tau)^{1/3} A, h/t \right] \\ &= (A\tau)^2 f \left[\frac{M_o^2 - 1}{(M_o^2 \tau)^{2/3}}, (M_o^2 \tau)^{1/3} A, h/t \right] \end{aligned} \quad (28)$$

With regard to further simplifications, it should be noted that the derivation of the transonic similarity rule requires that the thickness and camber be small with respect to the chord, but does not restrict the Mach number or aspect ratio. The transonic area rule, on the other hand, restricts itself to Mach numbers near the speed of sound and to wings having low aspect ratios.

The functional relation given in equation (27) representing the area rule simplifies if attention is fixed on any given Mach number, since M_o appears as an isolated parameter, thus

$$\Delta \left(\frac{D_o}{qc^2} \right) = \left(\frac{S_m}{c^2} \right)^2 f(A\tau) \quad (29)$$

On the other hand, the only Mach number at which the corresponding relation given by the transonic similarity rule simplifies, irrespective of the thickness ratio, is $M_o = 1$. The resulting expression is

$$\frac{D_{Ow}}{qc^2} = \left(\frac{S_m}{c^2} \right)^2 f(A\tau^{1/3}, h/t) \quad (30)$$

This expression simplifies further for wings having symmetrical sections, since h/t is zero. For such cases, the transonic similarity rule for wave drag at $M_o = 1$ reduces to

$$\frac{D_{Ow}}{qc^2} = \left(\frac{S_m}{c^2} \right)^2 f(A\tau^{1/3}) \quad (31)$$

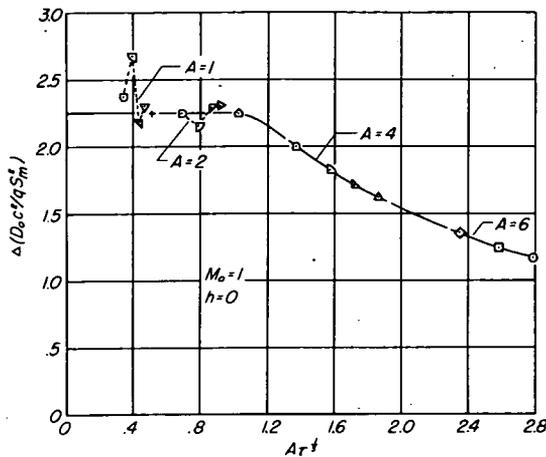
The foregoing analysis has developed certain equations relating to the drag rise or wave drag of a family of wings having relatively simple geometry. Despite the great restrictions imposed by the selection of such a family of bodies, the analysis has disclosed a number of significant points which complement those discussed in connection with the experimental data presented in sketches (a) through (f). First of all, it has been shown that the only way in which the wave drag at $M_o = 1$ can depend on the axial distribution of cross-section area (defined in the present family of wings by the value of the maximum cross-section area S_m) and still

be compatible with the transonic similarity rule for drag is for D_{Ow}/qc^2 to be proportional to the square of S_m/c^2 . This is precisely the dependence disclosed by the experimental data for low-aspect-ratio wings having symmetrical sections and shown in sketch (f). The transonic similarity rule states that the wave drag may depend on the camber, whereas the area rule states that it does not. The experimental data in sketch (d) show that camber has a significant effect on drag.

It is evident, both from a priori considerations and from the experimental results shown in sketch (f), that some change must occur in the relation between wave drag and maximum cross-section area as the aspect ratio becomes very large. The only possibility permitted by the transonic area rule is that $\Delta(D_{Ow}/qc^2)$ varies with S_m/c^2 in some other manner than as the square. It can be seen from sketch (e), however, that the data for wings of the present family cannot be correlated on this basis if the aspect ratio is greater than about 3. The transonic similarity rule for the zero-lift wave drag of uncambered wings at $M_0 = 1$ provides a different dependence by stating that D_{Ow}/qc^2 is equal to the square of S_m/c^2 times some function of $Ar^{1/3}$. This statement is compatible with the area rule if D_{Ow}/qc^2 is independent of $Ar^{1/3}$ for small values of the latter. If wave-drag data for wings of larger aspect ratio can be correlated successfully by considering the value of this quantity, a theoretical basis for a limit to the range of applicability of the transonic area rule has been found.

Comparison with Experiment

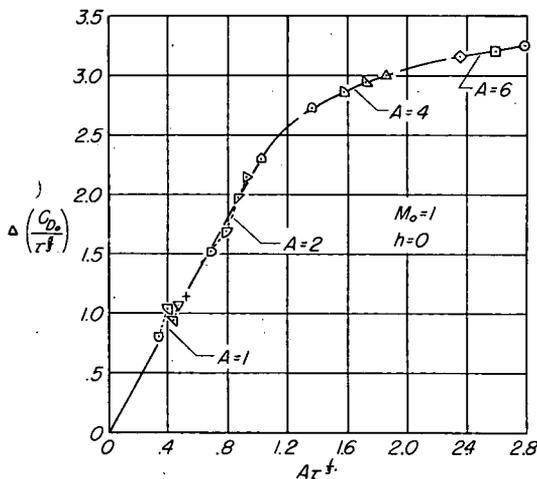
The foregoing discussion has served to focus attention on the fact that the parameter $Ar^{1/3}$, already familiar from prior papers on transonic flow (e.g., ref. 3), may be of importance in defining the limit of applicability of the transonic area rule as applied to a family of affinely related wings. The functional relation of equation (31) suggests that if $D_{Ow}c^2/qS_m^2$ is plotted as a function of $Ar^{1/3}$, all the data for the uncambered wings should fall on a single curve. Moreover, the values of $D_{Ow}c^2/qS_m^2$ should be independent of $Ar^{1/3}$ over whatever range the area rule applies. Although this method of plotting is conceptually simple, it imposes severe requirements on the accuracy of the experimental determination of wave drag because any errors are magnified as a result of dividing by the square of the maximum cross-section area. For results such as the present in which the wave drag is not actually measured, but is inferred from measurements of total drag by subtracting an estimated friction drag, greatest difficulties are experienced if the wings are thin and of low aspect ratio so that the wave drag is only a small fraction of the total drag. Consequently, data points for which the wave drag is less than half the total drag are omitted in the plot of $\Delta(D_{Ow}c^2/qS_m^2)$ versus



Sketch (i)

shown previously by McDevitt, ref. 3); second, $\Delta(D_{oc}^2/qS_m^2)$ is, at best, independent of $A\tau^{1/3}$ for values of the latter up to about unity, indicating that the drag in this range varied in accordance with the area rule as well; third, $\Delta(D_{oc}^2/qS_m^2)$ varies appreciably with $A\tau^{1/3}$ at larger values of the latter, indicating that the range of applicability of the area rule was exceeded.

Before leaving the discussion of the correlation of drag data at $M_0 = 1$ by use of the transonic similarity rule, it is worthwhile to call attention to another method of plotting which illustrates the same factors although in a slightly different manner. This procedure, which is described more completely in references 3, 5, and 6, is based on equation (23) rather than equation (26) and consists, for the zero-lift drag of a family of uncambered wings in an air stream with $M_0 = 1$, of plotting the variation of $C_{D_{ow}}/\tau^{5/3}$ with $A\tau^{1/3}$ where $C_{D_{ow}}$ represents D_{ow}/qS_p .



Sketch (j)

$A\tau^{1/3}$ shown in sketch (i). Even with this precaution, the data evidences considerable scatter for thin, low-aspect-ratio wings. (The symbols refer to the same wings as in sketch (e).)

The principal points of interest in this plot are threefold. First, except for the smallest values of $A\tau^{1/3}$ where the scatter is too large to provide any positive conclusions, the points determine essentially a single line, indicating that the sonic wave-drag characteristics of the uncambered wings of the present family can be correlated successfully by the transonic similarity rule (this has been

shown previously by McDevitt, ref. 3); second, $\Delta(D_{oc}^2/qS_m^2)$ is, at best, independent of $A\tau^{1/3}$ for values of the latter up to about unity, indicating that the drag in this range varied in accordance with the area rule as well; third, $\Delta(D_{oc}^2/qS_m^2)$ varies appreciably with $A\tau^{1/3}$ at larger values of the latter, indicating that the range of applicability of the area rule was exceeded.

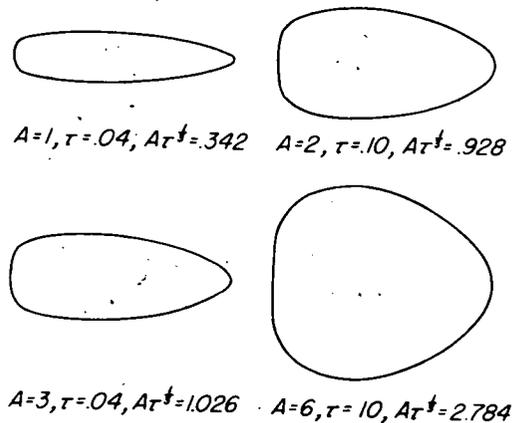
Sketch (j) shows the data of sketch (i) replotted in this manner. (Once again, the lack of wave-drag data requires the substitution of drag-rise information and the introduction of the drag-rise coefficient ΔC_{D_o} , defined as equal to $\Delta(D_o/qS_p)$.)

The curve formed by the data points on this type of plot is perhaps somewhat simpler than that of sketch (i) since it is asymptotic to straight lines at both large and small $A\tau^{1/3}$. At small $A\tau^{1/3}$, the points define a straight line passing through the origin. Such a line is in accordance

with the area rule. At values of $A\tau^{1/3}$ larger than about unity, however, the line determined by the data points departs from this initial trend and turns toward the horizontal. This trend is contrary to the area rule but consistent with the fact that the results for wings of high aspect ratio must tend toward those for wings of infinite aspect ratio. It cannot be emphasized too much that the critical value of unity for $A\tau^{1/3}$ is determined solely on the basis of data for a very special family of rectangular wings having symmetrical profiles. Other families of wings would be represented by different curves on such a plot.

It is interesting to consider for a moment the nature of this limit and to compare it with the verbal restriction of the area rule to low-aspect-ratio wings. Although the one requires that $A\tau^{1/3}$ be small, and the other that A be small, these two statements are in better agreement, insofar as engineering applications are concerned, than might appear at first glance. This agreement results from the fact that other considerations, such as designing for structural strength or for the avoidance of excessive drag, tend to preserve a rather narrow range for the values of τ likely to be met in practice. The effectiveness of τ as a parameter is further diminished by the fact that only its cube root is involved. Consequently, the restriction to small $A\tau^{1/3}$ represents a limitation primarily on the aspect ratio and only secondarily on the thickness ratio.

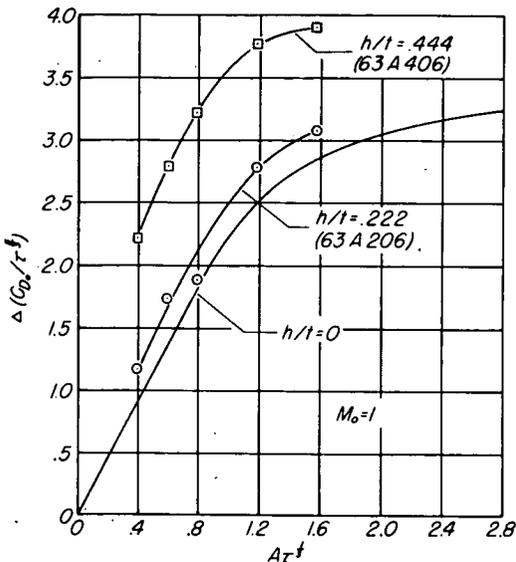
It is also interesting to examine the shapes of equivalent bodies of revolution having the same axial distribution of cross-section area as wings lying on either side of the limit. Accordingly, sketch (k) has been prepared showing the shapes of bodies of revolution equivalent to one of the wings having $A\tau^{1/3}$ much less than unity, to the two wings having $A\tau^{1/3}$ nearest unity, and to the wing having the largest $A\tau^{1/3}$ of any tested. It can be seen that the equivalent bodies are blunt and stubby rather than pointed and slender. Thus, although it has been shown that the drag-rise characteristics of the various members of the present family of uncambered rectangular wings are related to one another in the manner predicted by the transonic area rule, provided $A\tau^{1/3}$ is less than about unity, it would appear that the drag-rise characteristics of the equivalent bodies of revolution might be considerably different.



Sketch (k)

It is shown in sketch (d) that camber has an effect on the zero-lift drag rise. The transonic similarity rule suggests that the ratio of maximum camber to maximum thickness h/t might be an appropriate parameter to

use in addition to $At^{1/3}$ to correlate the sonic zero-lift wave-drag characteristics of a family of cambered wings. Accordingly, sketch (1)



Sketch (1)

has been prepared showing the influence of h/t for wings of various $At^{1/3}$. For reasons of simplicity, only three values for h/t , namely, 0, 0.222, and 0.444, are included on this plot. The important effects of camber are readily evident from this graph. It is apparent that the area rule is not applicable to wings having different camber-thickness ratios, even if the values of $At^{1/3}$ are sufficiently small to permit successful correlation of the wave drag of uncambered wings. On the other hand, it is permissible in the theory and seems to be indicated by the experimental data that the area rule is applicable to families of cambered wings providing h/t is maintained constant. This result is recognized in sketch (1) by the fact that $\Delta(CD_0/\tau^5)$ for the wings of constant h/t is approximately proportional to $At^{1/3}$.

SUMMARY OF RESULTS

The range of applicability of the transonic area rule has been investigated by comparison with the appropriate similarity rule of transonic flow theory and with available experimental data for a large family of rectangular wings having NACA 63AXXX profiles. These wings are of affinely related geometry and are hence immediately amenable to analysis by the transonic similarity rules. On the other hand, the axial distributions of cross-section area are not identical, in most cases, but merely similar. (The ratio of the local cross section to the maximum cross section is a given function.) It is shown, however, how the transonic area rule can also be used to correlate the sonic drag-rise data for such a family of wings.

It is found that the sonic zero-lift drag-rise data for the present family of wings can be successfully correlated on the basis of the area rule, provided the wing profiles are symmetrical and the product of the aspect ratio and the cube root of the thickness ratio is less than about unity. Within this range, the sonic drag rise varied as the square of the maximum cross-section area, all wings having equal chords. It is demonstrated that this is the only dependence of drag on maximum cross-section area for a family of wings like the present that is compatible with both the area rule and the transonic similarity rule.

It was found that the addition of camber greatly increased the sonic drag rise and that the application of the transonic area rule to a family of wings, some of which are cambered and others not, could lead to serious error. On the other hand, it is indicated by the transonic similarity rules and the experimental data that the area rule is applicable to families of cambered wings, provided the camber distribution, as well as the area distribution, are similar and that the ratio of the maximum ordinates of the camber and thickness distribution is maintained constant.

Ames Aeronautical Laboratory
National Advisory Committee for Aeronautics
Moffett Field, Calif., June 28, 1954

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