# NACA

# RESEARCH MEMORANDUM

## SUMMARY OF RECENT THEORETICAL AND EXPERIMENTAL WORK ON BOX-BEAM VIBRATIONS

By John M. Hedgepeth

Langley Aeronautical Laboratory
Langley Field, Va.

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS
WASHINGTON
June 14, 1955

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

RESEARCH MEMORANDUM

# SUMMARY OF RECENT THEORETICAL AND EXPERIMENTAL WORK ON BOX-BEAM VIBRATIONS

By John M. Hedgepeth


## SUMMARY

A discussion of various secondary effects which have an important influence on the vibration characteristics of box beams is presented. Means of incorporating these effects in vibration analyses of actual built-up box beams are discussed. Comparisons with experiment are given; good agreement between theory and experiment is obtained when the secondary effects are included.


## INTRODUCTION

The need of accurate vibration modes and frequencies of aircraft is well recognized. The reliability of a flutter-speed calculation, for example, is intimately related to the accuracy of the vibration modes and frequencies - especially the frequencies - used in the analysis. Furthermore, these modes and frequencies are often useful in the analysis of other dynamic problems, such as landing impact and gust loads.

The purpose of this paper is to summarize the results of research into the vibration characteristics of one of the main types of aircraft structures - the box beam. Most of the information to be discussed is recent; some of it is old and is included in order to present a complete story.

## DISCUSSION

Shown in figure 1 is a drawing of a typical box beam which may be thought of as being the main load-carrying structure of a wing. It is composed of covers built up of sheet stiffened by stringers, and spars made up of flanges connected by webs. There are usually a number of ribs to provide cross-sectional stiffness.

The vibration analyses of beams like the one shown in figure 1 ordinarily have used elementary beam bending and torsion theory. The accuracy obtained from elementary theory was considered to be good enough for long beams. However, it is known that, in the deformation of box beams, certain secondary effects arise which are not taken into account in the elementary theory. These secondary effects can have an important influence on the vibration characteristics of box beams, particularly when the higher modes are desired.

In bending vibrations, the secondary effects that are generally recognized are transverse shear, shear lag, and longitudinal (rotary) inertia. Transverse shear arises from the fact that shear deflections of the box occur under load because of the finite shear stiffness of the webs. These shear deflections are in addition to the bending deflections considered in the elementary theory. Shear lag is caused by the finite shear stiffness of the covers which permits the direct-stress-carrying material in the middle of the cover to carry less than its full share of the load. Both of these effects tend to reduce the apparent stiffness of the box and, hence, to reduce the natural frequencies. Longitudinal inertia arises from the fact that, when the beam vibrates, inertia forces are developed because of accelerations in the longitudinal direction. This effect - which is often called rotary inertia in the bending problem - is of course not included in the elementary theory and tends to reduce the natural frequencies.

The quantitative influence of these effects on the bending frequencies of vibration of uniform, thin-walled box beams was investigated by Budiansky and Kruszewski (ref. 1). Reference 1 shows that the effects of transverse shear and shear lag are of comparable magnitude and can be quite important, particularly for short beams. On the other hand, the effect of longitudinal inertia is negligible for the shallow box beams that are typical of aircraft wing construction.

For torsional vibrations, the secondary effects that are generally recognized are restraint of warping and longitudinal inertia. When a beam is twisted, the cross sections tend to distort out of their own plane or to warp. In the elementary theory, this warping is allowed to occur without restraint. However, if the warping varies along the length, as it does in a vibration mode, direct stresses are created in the longitudinal direction which act to restrain the warping. The effect of this restraint of warping is to make the beam stiffer than it would be otherwise and, consequently, to raise the natural frequency. This warping motion also causes inertia forces in the longitudinal direction. This longitudinal inertia effect is not included in the elementary theory.

The quantitative influence of these secondary effects on the torsional frequencies of uniform, thin-walled box beams was investigated

by Kruszewski and Kordes (ref. 2). Reference 2 shows that the effect of restraint of warping could be appreciable for short beams although the effects are not so pronounced as those incurred in the bending case. The effect of longitudinal inertia was unimportant for practical box beams.

Before considering the influence of these secondary effects on the bending and torsion frequencies of some actual test specimens, it is desirable to discuss how the effects of transverse shear, shear lag, and restraint of warping can be incorporated in vibration analyses of actual box beams. The results in references 1 and 2 were obtained for highly idealized beams for which exact solutions were possible. In an actual box beam, exact solutions are not feasible and some sort of simplified structure must be used.

The cross section of the box beam shown in figure 2 contains a large number of elements - stringers, flanges, and the sheet joining them. If all of these elements were treated separately in an analysis, the calculation job would be overwhelming. By lumping the properties of these elements, however, the effects of transverse shear and shear lag can be obtained without an inordinate amount of labor. The transverse-shear effect can be included simply by allowing the webs to have shear flexibility. The shear-lag effect can be duplicated by using the substitute-stringer idealization shown in figure 2. The direct-stress-carrying areas in the webs and flanges are concentrated at the corners; the direct-stress-carrying areas in the covers are concentrated as so-called substitute stringers. By properly locating the substitute stringers, it is possible to obtain frequencies within a couple of percent of those predicted by more refined theories. The use of the substitute-stringer idealization has been investigated in reference 3. For the torsion problem, the so-called four-flange box is useful in obtaining frequencies. In this idealization the direct-stress-carrying areas in both the covers and the webs are concentrated at the corners. The four-flange box does a good job of duplicating the effect of restrained warping on torsional vibrations. It might be mentioned that the substitute-stringer box can also be used for torsional vibrations with comparable success. This is fortunate since, in most actual box beams, bending and torsion will couple and the idealization used should be capable of taking all secondary effects into account.

Simplifying the cross section of the box beam is only part of what is necessary to achieve a practical vibration analysis. For a non-uniform box beam, further simplifications must be made as to the behavior in the spanwise direction. One approach is to break up the beam into a number of bays and to calculate influence coefficients. These influence coefficients then provide a basis for the vibration analysis. (See, for example, ref. 4.) Calculations have been made to

determine the number of bays necessary to obtain accurate frequencies. The results indicate that, in order to obtain the first three symmetrical bending and torsion frequencies of a free-free box beam, it is necessary to use about eight bays on the half span.

An experimental vibration-test investigation has been conducted in order to check the accuracy of methods of computing natural modes and frequencies. Tests have been carried out on two structures - one a large-scale built-up box beam and the other a somewhat smaller, hollow rectangular tube.

Consider first the built-up box beam. In figure 3 is shown a photograph of the vibration-test setup used to determine its natural modes and frequencies experimentally. The beam was 20 feet long, 18 inches wide, and 5 inches deep. The aspect ratio was about 13. The beam was hung from the gallows and was vibrated horizontally. In this way an essentially free-free condition was attained. The beam was vibrated by means of four electromagnetic shakers attached at the corners. Power was supplied by the M-G set and controlled at the left-hand cabinet in figure 3. Pickups were mounted on the beam and were used to sense the motion; their output was recorded at the right-hand cabinet.

The vibration frequencies of this beam are given in table I. In the table, for simplicity, only symmetrical bending frequencies and antisymmetrical torsion frequencies have been shown; the antisymmetrical bending and symmetrical torsion frequencies follow the same trends. For both the bending and torsion, the frequencies in the first column were obtained experimentally; the frequencies in the second column were calculated by elementary theory. The frequencies including the secondary effects are given in the last column. The word "exact" has been used for these frequencies for want of a better term. By comparing experiment and elementary theory, it can be seen that the elementary theory is accurate enough for the torsion modes and for the first bending mode. For the higher bending modes, however, the situation is different. Errors varying from 20 percent in the second mode to 70 percent in the fourth mode are experienced. When the secondary effects are included, however, the agreement is considerably improved. By looking at the bending modes, it can be seen that the large reductions in frequency due to transverse shear and shear lag have brought the calculations into satisfactory agreement with the experiment. It might be mentioned that transverse shear and shear lag were about equally responsible for this reduction. For the torsion modes, inclusion of the smaller effect of restraint of warping has helped for the first two modes and hurt for the third. Even for this third antisymmetrical torsion mode - which is the twelfth mode of the beam - the error is only 6 percent.

The built-up box beam which has been discussed had a large amount of internal stiffening in the form of stringers and bulkheads. The other

specimen - a hollow rectangular tube - was considerably simpler and was tested for the purpose of checking theories before the vibration equipment necessary for testing large-scale built-up structures was obtained. The results for this hollow rectangular tube which is shown in figure 4 serve to illustrate an effect which may be important for box beams with little internal stiffening. This tube was approximately 8 feet long, 7 inches wide, and 2 inches deep. The aspect ratio was about 14. It was made by welding 1/4-inch plates together along the corners.

The results of vibration tests are given in table II. Again in this table only the symmetrical bending frequencies and antisymmetrical torsion frequencies are presented. Along with the experimental frequencies are given the "exact" calculated frequencies for which the secondary effects of transverse shear, shear lag, and restraint of warping have been included. For the first two symmetrical bending modes, the "exact" frequencies are fairly good; for the rest of the modes shown here, however, the errors are large - especially for the torsion modes. The reason for these large discrepancies is tied up with the occurrence of large cross-sectional distortions which were not taken into account in the calculations. The nature of these distortions is illustrated in figure 5.

When a box is undergoing a bending vibration, the inertia forces tend to bend the covers and webs in a manner like that shown at the top of figure 5. This distortion, in turn, causes additional inertia forces which tend to raise the effective mass of the beam and lower the frequency. For torsional vibrations there occurs a similar bending of the webs and covers. More importantly, however, the cross section undergoes an overall shear. Both of these effects tend to raise the effective mass moment of inertia and, hence, to lower the torsional frequencies. In an ordinary box, the presence of stiffening members, such as stringers and ribs or bulkheads, tends to prevent these cross-sectional distortions. In the tube, however, there were no such stiffeners, and the frequencies were therefore greatly reduced.

It should be mentioned that one of the present trends in structural design is in the direction of eliminating stringers and bulkheads and letting the skin carry most of the load. Therefore, this effect of cross-sectional flexibility may indeed become important for actual box beams, just as it is for the hollow tube.

Studies of the effects of cross-sectional distortions on the bending and torsional frequencies of box beams have been carried out. The effect of the local bending of the covers and webs has been presented in reference 5; an analysis of the effect of the overall shear of the cross-section has also been performed. Results of applying the corrections indicated by these studies to the frequencies of the hollow tube are shown by the frequencies in the last column of table II for both

symmetrical bending and antisymmetrical torsion. The agreement with experiment is now satisfactory for all the bending modes. The agreement for the torsional modes is surprisingly good considering the large effects taken into account by the approximate correction.

## CONCLUDING REMARKS

From the comparisons between theory and experiment that have been discussed for the box beams, it can be concluded that the secondary effects of transverse shear and shear lag can have an important influence on the bending vibrations of box beams. This is true even for long beams if the higher modes are desired. The effect of restraint of warping on the torsional modes is not nearly so important. The effects of cross-sectional distortion can be quite large unless stiffening members are used to minimize the distortion. In any event, the methods developed for taking all these effects into account are very successful.

Langley Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., April 22, 1955.

## REFERENCES

1. Budiansky, Bernard, and Kruszewski, Edwin T.: Transverse Vibration of Hollow Thin-Walled Cylindrical Beams. NACA Rep. 1129, 1953. (Supersedes NACA TN 2682.)

2. Kruszewski, Edwin T., and Kordes, Eldon E.: Torsional Vibrations of Hollow Thin-Walled Cylindrical Beams. NACA TN 3206, 1954.

3. Davenport, William W., and Kruszewski, Edwin T.: A Substitute-Stringer Approach for Including Shear-Lag Effects in Box-Beam Vibrations. NACA TN 3158, 1954.

4. Levy, Samuel: Computation of Influence Coefficients for Aircraft Structures With Discontinuities and Sweepback. Jour. Aero. Sci., vol. 14, no. 10, Oct. 1947, pp. 547-560.

5. Budiansky, Bernard, and Fralich, Robert W.: Effects of Panel Flexibility on Natural Vibration Frequencies of Box Beams. NACA TN 3070, 1954.

TABLE I

FREQUENCIES OF BUILT-UP BOX

| SYMMETRICAL BENDING | | | | ANTISYMMETRICAL TORSION | | | |
|---|---|---|---|---|---|---|---|
| MODE | EXP., CPS | ELEM., CPS | "EXACT", CPS | MODE | EXP., CPS | ELEM., CPS | "EXACT", CPS |
| 1 | 18.1 | 18.4 | 18.0 | 1 | 64.7 | 63.0 | 63.3 |
| 2 | 84.7 | 101.0 | 86.4 | 2 | 194.0 | 189.0 | 194.0 |
| 3 | 176.0 | 247.0 | 181.0 | 3 | 313.0 | 315.0 | 332.0 |
| 4 | 271.0 | 458.0 | 285.0 | | | | |

TABLE II

FREQUENCIES OF HOLLOW TUBE

| SYMMETRICAL BENDING | | | | ANTISYMMETRICAL TORSION | | | |
|---|---|---|---|---|---|---|---|
| MODE | EXP., CPS | "EXACT", CPS | CORR., CPS | MODE | EXP., CPS | "EXACT", CPS | CORR., CPS |
| 1 | 68.7 | 70.2 | 70.2 | 1 | 301.0 | 377.0 | 316.0 |
| 2 | 342.0 | 348.0 | 328.0 | 2 | 455.0 | 1133.0 | 485.0 |
| 3 | 572.0 | 761.0 | 586.0 | 3 | 648.0 | 1911.0 | 705.0 |

TYPICAL BOX BEAM

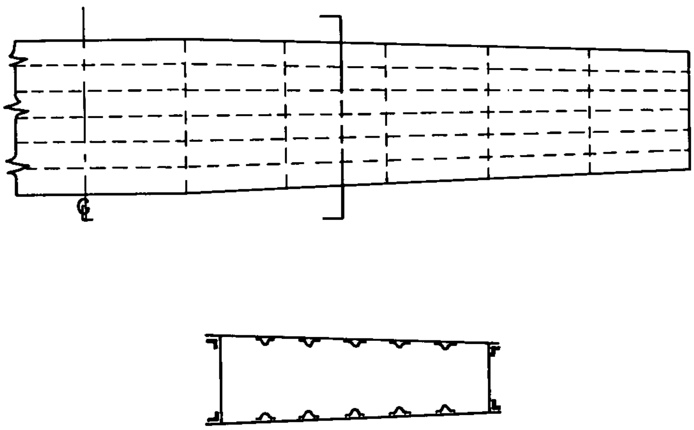


Figure 1

CROSS-SECTIONAL IDEALIZATIONS

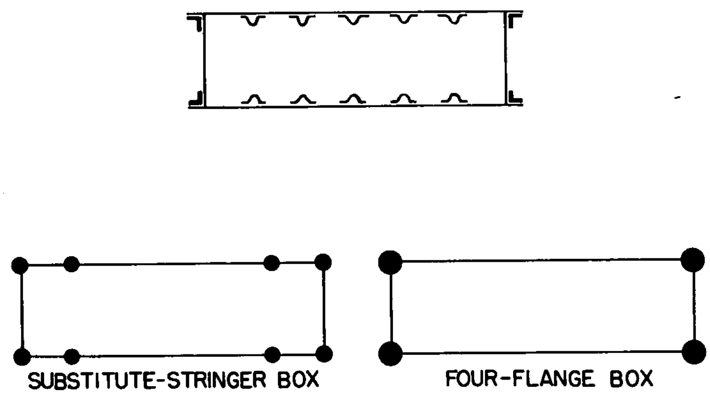


Figure 2

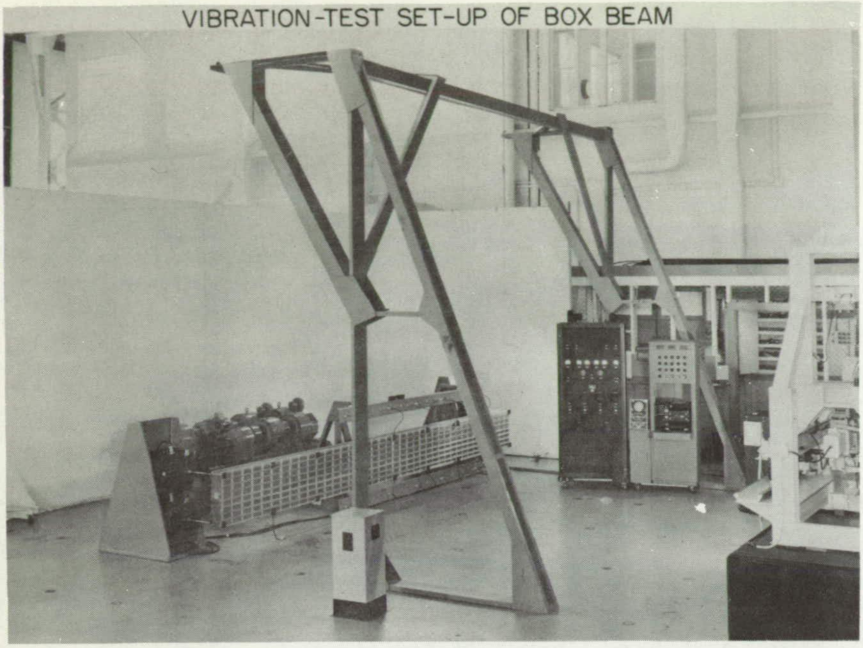


Figure 3

HOLLOW RECTANGULAR TUBE

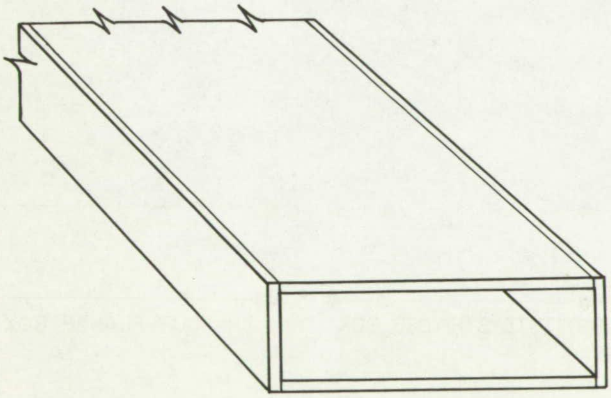

Figure 4

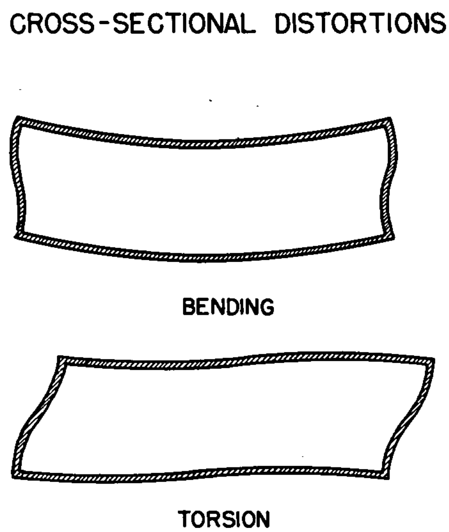


Figure 5