RESEARCH MEMORANDUM

THE INFLUENCE OF SURFACE INJECTION ON HEAT TRANSFER AND SKIN FRICTION ASSOCIATED WITH THE HIGH-SPEED TURBULENT BOUNDARY LAYER

By Morris W. Rubesin

Ames Aeronautical Laboratory
Moffett Field, Calif.

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SUMMARY

Existing analyses of the effect of distributed surface injection on the heat transfer and skin friction associated with the turbulent boundary layer at high speeds are correlated to eliminate, largely, the effects of Mach number and Reynolds number. It is shown that surface injection reduces greatly both skin friction and heat transfer. Data for heat transfer and skin friction at Mach numbers of 0, 2.0, and 2.7 are compared with the analyses and the agreement is rather good.

From an example employing evaporative cooling with water, it is concluded that at high Mach numbers transpiration cooling is much more effective than conventional convective cooling.

INTRODUCTION

One cooling system for high-speed aircraft experiencing aerodynamic heating that shows promise is a transpiration cooling system. The schematic diagram in figure 1 helps to indicate what is meant by a transpiration cooling system for an aircraft. In such a system the coolant passes from the interior of the aircraft through a porous outer skin and into the hot boundary layer. The system shows promise for two reasons. First, in passing through the skin, the coolant can reach the temperature of the skin because of the large amount of surface area for heat transfer existing within the pores. Thus, the coolant can reach the maximum temperature of the system and be used most effectively. In terms of a heat exchanger, this represents 100 percent effectiveness. The second contributing reason is that as the coolant passes into the hot boundary layer it cools the inner portion of the boundary layer and forms a buffer between the hot gases of the boundary layer and the skin that is being cooled. Thus, the amount of heat entering the surface is reduced by the injection of a coolant.
There are also disadvantages in a transpiration cooling system. The porous material is difficult to manufacture, and the inherent weakness of the material requires a more difficult and complex structural design. Also, the roughness of the porous materials and the effect of fluid injection are such that the normally laminar boundary layers may be tripped into turbulent boundary layers and in that way increase the amount of heat entering the body. A transpiration cooling system, therefore, would probably be considered only for cases where turbulent flow exists normally or where extremely favorable conditions exist so as to insure laminar flow. In view of these disadvantages, it is believed that only a complete systems analysis will show whether or not a sound engineering solution will employ transpiration cooling. In order to perform these systems analyses the designer will require knowledge of how surface injection affects the heat transfer and skin friction associated with boundary layers.

This paper presents available information on the effect of injection on the turbulent boundary layer. Theory and experiment are compared to determine whether or not the theoretical results can be used to extrapolate the limited amount of available data. After this comparison is made, an example of some advantages of transpiration cooling over conventional cooling systems is shown.

**SYMBOLS**

- \( C_f \)  local skin-friction coefficient
- \( F \) injection parameter, \( \rho_w v_w / \rho_l u_1 \)
- \( M \) Mach number
- \( Pr \) Prandtl number
- \( R_x \) Reynolds number based on length along surface
- \( St \) Stanton number
- \( t \) temperature
- \( T \) absolute temperature
- \( u \) velocity parallel to surface
- \( v \) velocity normal to surface
- \( w \) weight flow rate of coolant
x distance along surface from leading edge
ε surface emissivity
ηr temperature recovery factor
ρ density

Subscripts:
o zero surface injection
1 condition at outer edge of boundary layer
∞ free-stream condition
w surface condition

ANALYTICAL RESULTS

Two analyses exist at present which are concerned with the effect of the injection of air into air in a compressible turbulent boundary layer. Both are based on mixing-length theory and differ mainly in the manner in which arbitrary constants introduced in each analysis are handled. The analysis of Dorrance and Dore (ref. 1) considers the Prandtl number to be 1 and the turbulent boundary layer to extend down to the surface. The author's analysis (ref. 2) considers the Prandtl number to be 0.72, includes the existence of a laminar sublayer, and requires knowledge of its thickness. In both analyses plausible assumptions based on empirical knowledge are made to identify the arbitrary constants introduced.

Skin Friction

A comparison of the effects of distributed injection on skin friction, as determined by the two analyses, is made in figure 2. The ordinate is the local skin-friction coefficient divided by the local skin-friction coefficient for zero injection and the abscissa is the dimensionless injection parameter $F$ divided by half the local skin-friction coefficient for zero injection. The injection parameter $F$ is the coolant mass-flow rate per unit area normal to the surface divided by the mass flow per unit area of the main airstream. The shaded areas on this figure represent the numerical results obtained over a large range of the parameters: Mach number, Reynolds number, and the ratio of wall to free-stream temperature. For instance, for the analysis of the Dorrance
and Dore calculations were made in which the Mach number ranged from 0
to 20, the ratio of wall temperature to free-stream temperature ranged
from 1 to 3, the Reynolds number ranged from $10^7$ to $10^9$, although higher
Reynolds numbers also fall within this shaded region. In the author's
analysis, the Mach number ranged from 0 to 8, the ratio of wall to free-
stream temperature ranged from 1 to 3, and the Reynolds number ranged
from $10^6$ to $10^8$. The effect of these parameters is largely eliminated
by this type of coordinate system. Note that calculations with both
$F$ constant along the body and $F$ proportional to the local skin-friction
coefficient have been plotted on this figure. Both analyses, although
yielding different results, show that the effect of injection on skin
friction can be very large; reductions down to 1/5 of the zero-injection
skin-friction coefficient are shown.

Heat Transfer

The calculated effect of injection on heat transfer is shown in
figure 3. In this figure the ordinate is the ratio of the local Stanton
number to the local Stanton number for zero injection and the abscissa
is the blowing parameter $F$ divided by the local Stanton number for
zero injection. The results of the analysis of Dorrance and Dore are
not plotted here as they would result in a curve identical to that shown
in figure 2. The reason for this is that the Prandtl number of 1 used
in their analysis results in an exact Reynolds analogy between skin
friction and heat transfer.

The region shown, representing the author’s analysis, is quite
similar to the region in figure 2 for the skin-friction relationship,
even though the Prandtl number is 0.72 and no exact Reynolds analogy
exists. Apparently the effect of Prandtl number is largely absorbed
in the choice of coordinates for figure 3. In effect, the results of
heat transfer can be considered essentially identical to those of skin
friction for both analyses when plotted as in this figure. Thus, the
analyses predict that heat transfer is also reduced considerably by
surface injection.

The relatively small difference between the two analytical results
should not be considered as an indication of the certainty of these
results. Other analyses, based on equally plausible flow models, could
yield results that differ greatly from these results. Ultimately, the
worth of these analyses can be assessed only through a comparison with
experimental data. Agreement between analysis and data, however, should
not imply a verification of the physical assumptions of the theory, but
should be considered simply as providing a systematic means of extending
the range of applicability of the limited amount of data now available.
Mickley, Ross, Squyers, and Stewart (ref. 3) obtained skin-friction and heat-transfer data while injecting air into the boundary layer on a flat plate. The free-stream air flows were at speeds below 60 feet per second. Data were obtained for constant values of the blowing parameter $F$ along the plate and for values of $F$ varied proportionately to the skin-friction coefficient.

Skin friction. - The skin friction was measured by surveying the boundary layer with impact-pressure probes and then calculating the momentum thickness of the boundary layer at several stations along the plate. The local skin-friction coefficient was determined from the difference between the local momentum-thickness gradient and the local injection parameter. Because this difference was often small compared with the individual terms, errors in the momentum thickness or local injection rate produced larger errors in the skin-friction coefficient. The data, therefore, scatter considerably. Another factor requiring mention is that the plate under zero injection was not aerodynamically smooth, the skin-friction coefficient being in general about 15 percent higher than on a smooth plate.

A comparison of these data with the analyses is shown in figure 4. The ordinate is again the ratio of the local skin-friction coefficient to its value for zero injection, and the abscissa is the injection parameter divided by half the local skin-friction coefficient for zero injection. The skin-friction data decrease considerably with increased injection, the reduction being as high as 90 percent of its initial value at the highest injection rate. The roughness of the plate is not expected to alter these results significantly. It can be concluded, therefore, that within the scatter of the data, there is general agreement between the analyses and the data for skin friction.

Heat transfer. - Heat-transfer measurements were made in the investigation of reference 3 by employing heaters placed locally within the porous plate. The local heat-transfer coefficients were calculated from a heat balance on the individual elements of the plate containing heaters. Thus, these data were obtained in a somewhat more direct fashion than the skin friction. Because heat-transfer data is not affected greatly by surface roughness (ref. 4), these heat-transfer data are considered to be reliable. These data are compared with the analyses in figure 5. The ordinate is the ratio of local Stanton number to its value for zero injection and the abscissa is the injection parameter divided by the Stanton number for zero injection. Data are shown for both constant and varying injection parameter along the plate.
Although the data show considerable scatter, a marked decrease in Stanton number with increased injection rate can be discerned. The agreement between data and analyses is again good, the data in general lying between the analytical results.

High-Speed Data

Several tests have been performed recently to determine the effect of surface injection on the turbulent boundary layer at supersonic speeds. (See, for example, refs. 5 and 6.) All these tests are of a preliminary nature, where thoroughness has been sacrificed to expedite obtaining the results. Tests of limited accuracy, however, still supply significant results when there are expectations of large changes in the quantity measured, and when no data exist on the subject.

Skin friction.- Two sets of skin-friction data were obtained at the Ames Aeronautical Laboratory at a free-stream Mach number of 2.7, with air injection. One set was obtained on a porous frustum of a cone (made of sintered woven stainless steel) preceded by a solid ogive. The other set was obtained on a porous flat plate made of sintered powdered stainless steel.

The cone frustum data were obtained by direct force measurements. The average skin-friction drag over the cone frustum was determined from the measurements of total drag, base drag, and fore pressure drag, with estimations made for the influence of the skin friction on the solid nosepiece and of the boundary-layer trip ahead of the porous portion. The injection rate along the cone was nearly uniform. The skin-friction coefficient for zero injection was about 25 percent higher and showed less Reynolds number dependence than is expected on a smooth body. These results were not surprising, since the cone appeared to be aerodynamically rough. This measured skin-friction coefficient, nevertheless, is used as the reference value in the correlations that follow.

The flat-plate data were obtained by boundary-layer surveys with an impact pressure probe. The local skin-friction coefficient was determined from the derivative of the momentum thickness with respect to distance along the plate minus the local injection parameter F. At the higher injection rates this difference becomes small compared with the magnitude of the individual quantities, and errors in the momentum thickness or the local injection rate produce larger errors in the skin-friction coefficient. For the zero injection case, however, it was found that the data agreed with data obtained on a solid smooth surface.

The data from the two tests are plotted in figure 6. The ordinate is the ratio of the skin-friction coefficient to its value for zero injection and the abscissa is the injection parameter F divided by
half the skin-friction coefficient for zero injection. Although average skin-friction coefficients are used for the cone data and local skin-friction coefficients are used for the flat-plate data, it can be shown analytically that essentially the same curves should result when the data are plotted on this coordinate system. On examining the data, it is found that the two sets of data obtained in different ways agree very well with each other, even though the cone was initially rough. Both sets of data show a considerable reduction in skin-friction coefficient with increasing injection rate and are in good agreement with the analytical results, especially the analysis of reference 2. The reduction in skin friction shown here at $M = 2.7$ is quite similar to that determined at $M = 0$.

Heat transfer.- Two sets of data are available showing the effect of surface injection on heat transfer in the turbulent boundary layer at supersonic speeds. One set (ref. 5) was obtained on a porous frustum of a cone made of sintered powdered stainless steel preceded by a solid steel nosepiece. These data were obtained at $M = 2.02$ with nitrogen, helium, and water as coolants. The water data will not be reported here because the amount of evaporation taking place during the tests was not known; thus correlation of these data with the gas data is impossible. The other set of data (ref. 6) was obtained on a porous flat plate at $M = 2.7$. Air was used as a coolant in these tests.

In both sets of tests, the amount of heat transferred to the model was determined by measuring the temperature rise of the coolant as it passed from the inside of the model to the outer surface of the porous skin. Average heat transfer was determined on the cone, whereas local values were determined on the flat plate. In the flat-plate tests pains were taken to separate the individual effects of injection on the Stanton number and on the temperature-recovery factor. Because of this the flat-plate data will be discussed first in terms of Stanton number and of recovery factor, and then comparison will be made of the overall cooling effects of both sets of tests.

A comparison of the flat-plate data and the analyses is made in figure 7. The ratio of Stanton number to its value for zero injection is plotted against the injection parameter divided by the Stanton number for zero injection. The data points represent the reduction experienced by the local Stanton numbers, averaged over all the tests. The data show a marked decrease in local Stanton number with increased injection. The reduction in Stanton number, however, is not as large as the analyses indicate or as was shown by the $M = 0$ data. This point should not be emphasized because a saving feature appears. This is shown in the next figure.

The effect of surface injection on the temperature-recovery factor is shown in figure 8. Here the ratio of recovery factor to its value
for zero injection is plotted against the injection rate divided by the local Stanton number for zero injection. The data correlate quite well on this type of plot and show a reduction with increased injection, the reduction being as much as 20 percent. Neither analysis, even the one for Fr = 0.72, predicts this reduction.

The heat transfer to a surface, now, depends on both the reduction in Stanton number and the reduction in recovery factor. This combined effect is shown in figures 9(a) and (b). In these figures the ordinate is a dimensionless grouping composed of the wall temperature, the coolant's initial temperature, and the recovery temperature under zero injection conditions. This is a parameter known to the designer and one which he must design for. The abscissa is the injection parameter divided by the Stanton number for zero injection, an average value for the cone data and local value for the flat-plate data. The data for the cone and the flat plate with nitrogen or air as the coolant (fig. 9(a)) agree well with each other and with the analytical results. The data, however, are a little lower in general at the lower values of injection parameter. At higher values of injection there is excellent agreement. It is noted that the surface temperatures in this case are much lower than would be produced by a conventional heat exchanger of 100 percent effectiveness.

The data shown in figure 9(b) give some indication of how analyses based on air-to-air injection predict the behavior transpiration cooling systems employing helium. The data were obtained on the cone. It is seen that the data, like the data for air at these injection rates, lie a little below the analytical values. The analytical values were determined by using the assumption that helium injection affects the boundary layer in the same manner as air injection, but that helium acts as a more effective coolant because of its high specific heat. Although it appears that the analyses for air agree with data for helium as the coolant almost as well as they do for air, it must be cautioned that the data shown in figure 9(b) were not obtained at sufficiently high rates of coolant flow. This is seen from the curve representing the conventional heat exchanger of 100 percent effectiveness, which does not differ greatly from the curves predicted by the analyses.

CONCLUDING REMARKS AND PRACTICAL APPLICATION

From the figures shown we can conclude that there is a general agreement between existing experiment and analysis for both skin friction and heat transfer under conditions of surface injection with air. The effect of other gases as coolants is at present somewhat inconclusive. All the experimental data are too limited in their range of variables and accuracy to allow formulation of empirical laws at this stage. At
present, therefore, it is necessary to rely on the analyses in extrapolating the available experimental data to conditions which the designer must face. From what has been shown, the analysis of either reference 1 or 2 can be used with some degree of confidence.

An example of the results obtained by using analytical extrapolation is shown in figure 10. Here the ordinate represents the ratio of coolant flow rate required for a transpiration cooling system to that required in a conventional system employing a heat exchanger of 100 percent effectiveness. The abscissa is the Mach number of flight. A surface temperature of $1,200^\circ$ F and $300^\circ$ F is maintained by each cooling system. Other conditions in the heat balance are that the altitude is 120,000 feet, the surface emissivity is unity, the position is 1 foot from a leading edge, the temperatures are at steady state, and the coolant is water that is evaporated. It is assumed that the effect of steam injection on the boundary layer is the same as that of air injection. No dissociation is assumed in the boundary layer. It is observed that the transpiration cooling system always requires less coolant than does the conventional system, the ordinate being always less than unity. The reduction, however, becomes significant only at the higher Mach numbers. The case with the cooler surface shows a little more advantage of a transpiration cooling system. It can be concluded, therefore, that at extremely high Mach numbers transpiration cooling may be the most effective means of attacking the aerodynamic-heating problem. In addition, the reduction in skin friction accompanying the transpiration cooling process may further increase the advantage of this type of cooling system.

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REFERENCES


SCHEMATIC DIAGRAM
TRANSPIRATION COOLING SYSTEM

FREE-STREAM FLOW

HOT BOUNDARY LAYER

POROUS SKIN

INTERIOR OF AIRCRAFT

Figure 1

EFFECT OF SURFACE INJECTION ON
SKIN FRICTION

Figure 2
EFFECT OF SURFACE INJECTION ON HEAT TRANSFER
ANALYSIS TN 3341 Pr = 0.72

\[ F = \frac{\rho_w v_w}{\rho_1 u_1} \]

\[ 0 \leq M \leq 8 \]
\[ 1 \leq \frac{T_w}{T_0} < 3 \]
\[ 10^6 \leq R_\lambda \leq 10^8 \]

Figure 3

COMPARISON OF SKIN-FRICTION THEORY AND EXPERIMENT AT M=0

\[ F = \frac{v_w}{u_1} \]

ANALYSIS TN 3341
DATA, MICKLEY ET AL
\( F = \text{UNIFORM} \)
\( \triangle \)
DORRANCE & DORE ANALYSIS

Figure 4
COMPARISON OF HEAT-TRANSFER THEORY AND EXPERIMENT AT $M=\infty$

![Figure 5](image)

COMPARISON OF SKIN-FRICTION THEORY AND EXPERIMENT AT $M=2.7$

![Figure 6](image)
COMPARISON OF HEAT-TRANSFER THEORY AND EXPERIMENT AT \( M = 2.7 \)

\[ F = \frac{\rho_w v_w}{\rho_1 u_1} \]

ANALYSIS TN 3341

DORRANCE & DORE ANALYSIS

\[ \frac{S_1}{S_{10}} \]

\[ F/S_{10} \]

Figure 7

EFFECT OF SURFACE INJECTION ON TEMPERATURE RECOVERY FACTOR AT \( M = 2.7 \)

\[ F = \frac{\rho_w v_w}{\rho_1 u_1} \]

\[ \frac{\eta_r}{(\eta_r)_0} \]

\[ F/S_{10} \]

Figure 8
EFFECT OF TRANSPIRATION COOLING ON SURFACE TEMPERATURE

COOLANT: AIR OR NITROGEN

CONVENTIONAL COOLING, 100% EFFECTIVENESS

ANALYSIS TN 3341

DORRANCE & DORE ANALYSIS

○ CONE DATA
□ FLAT-PLATE DATA

Figure 9(a)

EFFECT OF TRANSPIRATION COOLING ON SURFACE TEMPERATURE

COOLANT: HELIUM

CONVENTIONAL COOLING, 100% EFFECTIVENESS

ANALYSIS TN 3341

DORRANCE & DORE ANALYSIS

Figure 9(b)
COMPARISON OF COOLANT RATES REQUIRED FOR
TRANSPERSION & CONVENTIONAL COOLING SYSTEM

ALT = 120,000 FT X = 1 FT
ε = 1  STEADY STATE
COOLANT: EVAPORATING WATER

1.0

.8

.6

.4

.2

0

MACH NUMBER

1.0

.8

.6

.4

.2

0

Figure 10