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TECHNICAL MEMORANDUMS

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

No. 409

KINETOGRAPHIC DETERMINATION OF AIRPLANE FLIGHT CHARACTERISTICS

By P. Raethjen

From "Zeitschrift für Flugtechnik und Motorluftschiffahrt"
December 14 and 28, 1926

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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

TECHNICAL MEMORANDUM NO. 409.

KINETOGRAPHIC DETERMINATION OF AIRPLANE FLIGHT CHARACTERISTICS.*

I. Kinetographic Flight Measurements.

By P. Raethjen.

In the "Zeitschrift für Flugtechnik und Motorluftschiffahrt," for June 27, 1925, pages 235-240, I described a method by which the flight characteristics (horizontal speed and attendant sinking speed) of the Darmstadt glider "Konsul" were determined from measurements of a gliding flight in still air. The evaluation of the flight characteristics of the Konsul was rendered very difficult by the fact that the measurements made with a theodolite and telemeter, according to a method introduced by H. Koschnieder, did not yield more than four points a minute; and especially by the fact that the time and distance coordinates of these points were very inaccurate. I was therefore obliged to introduce a correction factor and to alter the flight characteristics diagram until, by integration, it yielded the time-and-space flight path found by measurement. Nevertheless this first experiment demonstrated that an accurate flight-path measurement would enable the determination of the polar diagram from a gliding flight. Since then I have accordingly endeavored to obtain accurate flight measurements by means of a kinetograph

*Flugeigenschaftsbestimmung durch kinematographische Flugvermessung. From "Zeitschrift für Flugtechnik und Motorluftschiffahrt," December 14 and 28, 1926.

(motion-picture camera).

This method was first employed at the 1926 Rhön soaring-flight contest. The result of the measurement of a gliding flight by the "Roemryke Berge" will be given by Mr. Knott as a part of this article. I will only mention here that the object in performing this experiment was to obtain a polar diagram of the air forces by a method similar to the one employed in 1924 for determining the flight characteristics of the Konsul. Since the accelerations developed in this flight were only slight, I assumed that the air-force coefficients of unaccelerated flight were applicable. The evaluation by Mr. Knott, however, shows deviations which contain obvious indications of the effect of acceleration. The result is therefore not a real polar diagram, but only an estimate of the effect of acceleration. Though the result is to be regarded for the present, only as an indication, it is nevertheless very valuable, because the effect of the accelerations is the real object of the kinetographic measurements. (See Appendix for further details.) Even if this indication should prove to be an error caused by atmospheric disturbances, the accompanying report by Mr. Knott continues to be of interest, as the first step in kinetographic flight measurement. I will first describe the apparatus and the experimental method.

1. Experimental Task and General Method

The object of the experiment is the determination of the path followed by the center of gravity of the airplane with reference to time and space. The task is therefore to determine the time and distance coordinates of points as near together as possible in the path of the airplane. As already mentioned, these points had been previously determined by means of a theodolite and telemeter by a method introduced by H. Koschnieder. The principal disadvantages of this method were the inaccuracy and infrequency (only 3-4 per minute) of the measured points. It was especially unsatisfactory that the sighting and reading, the dictation and recording afforded many chances for errors. Hence the new kinetographic method was based on frequent points of measurement (as many as 20 per second) and on an entirely objective measuring method. For this purpose the airplane, the spatial coordinates and a clock pointer were simultaneously photographed.

The measurements were made from two fixed bases on the "Weltenseglerhang," 200 m (656 ft.) apart, covering flights from the "Kuppe" toward the "Zuckerfeld" or "Eube." The direction of photographing was nearly horizontal, which was of considerable advantage for determining the flight altitude. The fixed bases (Fig. 1) were fitted out as follows. At the basic point itself there is an Ertel kinetograph stand with a top

which can be rotated both horizontally and vertically by means of two cranks K. The kinetograph is mounted on this stand and is constantly directed toward the airplane by keeping the latter in the field of the telescopic finder V. If the orientation of the camera at the instant of exposure is known, the picture of the airplane can be photogrammetrically evaluated. The orientation of the camera was therefore fixed on the negative by the simultaneous photography of stationary reference tables (Fig. 2) together with the airplane. The pointer of a clock was also photographed simultaneously, by means of a device which will be described later.

Three of the reference tables were placed behind each kinetograph at a distance of about 2 meters (6.56 feet), as shown in Fig. 1. They were placed behind the camera, so as not to interfere with the field of vision in front. They were therefore photographed from the direction opposite to the airplane, the result being shown on the section of film in Fig. 3. The reference tables consisted essentially of U-iron frames about 1.5 m (4.92 ft.) square. Each table was divided into squares by taut iron wires, which were drawn through holes bored with a drilling-jig, in the frame at intervals of exactly 5 cm (1.97 in.) between centers. Behind this trellis there was placed a frame covered with white airplane fabric, on which the individual squares were distinguished by numbers. The numbers were written backward, because they appear in the evalua-

tion as mirror pictures. In order to avoid confusion between the wires and their shadows, the former were interrupted by white paint, so that they appear on the photograph as disconnected crosses. The tables were adjusted with the aid of a plumb line, so that the vertical wires were exactly plumb. The horizontal wires should then have been exactly horizontal, but slight deviations were found, which are attributable partly to distortions during transportation and partly to inaccuracies in drilling the frames.

The vertical wires are the reference lines for the horizontal oscillation of the camera and the horizontal wires are the reference lines for its vertical oscillation. The vertical wires are therefore the ground-plan coordinates for the measurements and the horizontal wires are the altitude coordinates.

2. The Kinetograph

This consists in part of an ordinary kinetograph, as used by motion-picture producers. It was also constructed in part at the Frankfort a. M. Institute for Scientific Photography and in part at the Research Institute of the Rhön-Rossitten Association on the Wasserkuppe.

The task of taking two pictures from opposite directions on the same film was accomplished by means of total-reflection prisms (Fig. 7). Illustrative diagrams are given in the appendix to my lecture "Accelerated Airplane Motions," delivered at

the 1926 Rhön soaring-flight contest and already sent to the "Z.F.M." for publication. You are here referred to this appendix.* The technical task was rendered more difficult by the fact that neither time nor means were available for building an entirely new measuring kinetograph. Hence an ordinary motion-picture camera was used and the special devices were added to it. The camera accordingly consists of two parts: 1st, the real measuring camera (Fig. 4) which carries both reflecting prisms and the revolving shutter; 2d, the film-driving mechanism (Figs. 5-6), which consists largely of the normal apparatus with the addition of the device for photographing the clock (Fig. 10). The two parts must be separated to introduce the film, but can always be fastened together again by the four wing nuts.

The principal difficulty in arranging the reflecting prisms lay in the circumstance that the individual image fields B_1 and B_2 were smaller than the lens apertures (Fig. 7). Hence the whole aperture could not be used on the boundary between the two fields. In order to eliminate this fault as much as possible, the optical axes A_1' and A_2' were so placed that they did not fall in the middle but on the edge of the image fields. Since even then the images mutually overlapped one another, long tubes were placed in front of both object lenses

*The abovementioned lecture was published in the "Z.F.M." for Dec. 29, 1926, and the translation of said appendix is included in the present Technical Memorandum.

and a partition was put between the image fields B_1 and B_2 , though this partition had to leave a free space for the revolving shutter in front of the prism. The partition S naturally covered a strip of the film. Hence the photograph of the clock face was projected from the back side on this strip B_3 (Figs. 3 and 7) which, moreover, runs around the image field B_1 (Fig. 3). The crosses F_1 and F_2 (Fig. 3) would have to be located exactly on the optical axes A_1' and A_2' or at least at the corresponding distance from one another, if the pivot of the camera (stationary top) were on the optical axis A_2 . Unfortunately, we had to dispense with such a Cardanic suspension and use a simple Ertel tripod. With this support, the vertical axis of rotation of the stationary top passes through the optical axis A_2 , but not the horizontal axis of rotation, which lies 6 cm (2.36 in.) below the optical axis A_2 . The diagram of the optical axes passing in opposite directions through the pivot can therefore, strictly speaking, be utilized only for the horizontal rotation of the camera, but not for the vertical. The lateral rotation is made about a point on the optical axis A_2 , but, in changing the vertical direction, the optical axis moves as a tangent about a circle K (Fig. 8). This motion produces an error $\Delta_1 h$ in the vertical coordinates h on the reference table, which is a function of h , of the distance a and of the radius r (6 cm = 2.36 in.) of the circle of rotation.

The following formulas (Fig. 8) apply here.

$$r + \Delta_1 h = \frac{r}{\cos \varphi} \quad (1)$$

$$\tan \varphi = \frac{h + \Delta_1 h}{a} = \frac{h}{a}, \quad \text{hence approximately}$$

$$\frac{1}{\cos \varphi} = \sqrt{1 + \frac{h^2}{a^2}} \quad (2)$$

hence

$$\Delta_1 h = r \left(\sqrt{1 + \frac{h^2}{a^2}} - 1 \right)$$

or approximately

$$\Delta_1 h = \frac{r}{2} \frac{h^2}{a^2} \quad (3)$$

This error $\Delta_1 h$ can be partially offset by locating the cross wires of the image B_2 a little to one side of the optical axis A_2' (Fig. 7), so that the two axes F_1 and F_2 of the reticles (in contradistinction to the optical axes) do not lie parallel, but make a small angle $\Delta \varphi$ with one another (Fig. 9). In this case we have

$$\Delta_2 h = -a \Delta \tan \varphi = -a (1 + \tan^2 \varphi) \Delta \varphi \quad (4)$$

$$\Delta_2 h = -a \left(1 + \frac{h^2}{a^2} \right) \Delta \varphi \quad (5)$$

The task is now to give $\Delta \varphi$ such a value that the total correction $\Delta_1 h + \Delta_2 h$ shall be as small as possible within the values $\frac{h}{a} \leq \frac{1}{4}$ for $r = 6$ cm (2.36 in.) and $a = 200$ cm (78.7 in.). Thereby the constant $a \Delta \varphi$ may be eliminated at first, because it denotes only a shifting of the zero point on the

reference table. This constant shifting of the zero point is eliminated in the adjustment of the table. The variable error

$$\Delta h = \frac{h^2}{a} \left(\frac{r}{2a} - \Delta \varphi \right) \quad (6)$$

shall also be eliminated as far as possible. Preferably,

$$\Delta \varphi = \frac{r}{2a} = 0.015 \quad \text{is chosen.}$$

Through an error $\Delta \varphi = 0.020$ was obtained for both measuring kinetographs.

This causes an error $\frac{\Delta h}{h} = -0.005 \frac{h}{a}$, which remains for inclinations $\frac{h}{a} < \frac{1}{4}$ below the accuracy of the film reading. For greater inclinations, it can be easily introduced mathematically (though under consideration of the terms of higher order).

According to these considerations, we can therefore operate with the two reticles F_1 and F_2 (Fig. 3) as though their respective axes were strictly parallel and as if the pivot of the stationary top were exactly on the reticle axis F_2 . A slight correction would be necessary under certain conditions, only for inclinations exceeding 1 : 4, hence for altitude coordinates h exceeding 50 cm (19.7 in.).

In order to avoid changes in the mutual adjustment of the lenses and the reflecting prisms, the base plate, which supports them, was made exceptionally strong. The lens on the side toward the table was a Zeiss-Tessar with an aperture of 1 : 3.5 and a focal length of 10 cm (3.94 in.). On the airplane side,

a Xenar lens made by Joseph Schneider, with an aperture of 1 : 3.5 and a focal length of 30 cm (11.8 in.), was used.

The great focal length was chosen in order to enable the measurement of the coordinates of the airplane image for the single-station method and for the determination of the angular position of the airplane, the angle of orientation for the airplane axes. On the table side, it was better for the focal length not to be too long, so as not to have to make the table divisions so small.

The most difficult task in making the measuring kinetograph was the invention of a device for the simultaneous photography of the clock. Since there was no room in the camera in front of the film, the clock had to be projected on the back side of the film (Fig. 7). In this connection a device, which had already been used in ordinary kinetographs, was found very convenient. It carries a tube R (Fig. 6), in which there is ordinarily a telescope for observing the object while taking the pictures. The third lens was mounted in this tube and at its upper end a second shutter was introduced which was coupled with the first and exposed the clock only at the instants when the exposures were made for the airplane and the coordinate tables. Above this shutter is the clock face which is photographed, with the pointer, on a diapositive film by transmitted daylight. In order that no disturbance can be created by the sun, an adjustable mirror S (Fig. 1) is so placed that the

sun cannot shine in during the whole operation.

The double pointer Z (Fig. 10), over the U-shaped dial, is so operated by an electric synchronous motor, that it ordinarily makes half a revolution per second and therefore makes one-second periods. The pointer shaft itself drives, by means of a gear, a seconds recording disk S (Fig. 10), which has 36 division marks on its periphery. After one pointer revolution, these marks change, in rotation, the place P at which they are photographed. Hence they automatically record the seconds of the pointer Z. The synchronous motors are driven by a three-phase alternating current supplied from a central station consisting of a constant-speed motor driving a rotary, reversing switch. This rotary switch interrupts and reverses the polarity of a 60 volt direct-current circuit and thereby furnishes a nonsinusoidal alternating current for the synchronous motors. This current-reversing motor runs synchronously with the two three-phase motors at both measuring places and is, so to speak, the central clock for both.

Fig. 11 shows this central clock with its storage batteries, and Fig. 12 shows the inside mechanism of the clock. It is so arranged that it can be enclosed in a protecting case. The current-reversing motor M drives, through a worm gear, a recording drum R. Marks were made on the soot-coated drum at uniform time intervals (every 1/5 and every 3 seconds) with a stylus S (Fig. 12) actuated by an electromagnet and controlled

by a clock U. On its front side, this clock has the plugs and switches for the different wires, a three-wire cable for each measuring place, with two direct-current voltages of 60 and 6 for field magnets, armature and recording mechanism.

The time-mark record enables the control of the revolution speed of the current-reversing motor and consequently of the revolution speed of the clock pointers at the measuring stations. The recording drum has a total running time of 7 minutes and enables the evaluation of the revolution speed of the motor every $1/5$ second. The contact clock U is an ordinary alarm clock, which carries a contact point on the balance wheel and on the balance lever and makes contacts at every oscillation period (every $1/5$ sec.) and at every period of the balance wheel (every 3 sec.). The balance wheel of an ordinary alarm clock does not oscillate in every period to within 0.01 sec. and consequently this clock is not really accurate enough for kinetographic measurements. It is to be replaced later by a good pendulum clock. This was not possible at the time, as the clock had to be set up out of doors for every experiment.

3. Evaluation of the Experiments

a) Determination of the ground plan.- For the evaluation, the ground plan of the flight path is first drawn on a scale of 1 : 500. This is generally done as follows: The sighting lines from the two base points are drawn in the ground plan and

their intersection point is the ground-plan position of the airplane (Fig. 13). The ground plan must first show accurately the two base points and the reference tables in their relative positions. For this purpose the distances between them must be accurately measured. This is done by a combined measurement with a theodolite and a kinetograph in the following manner.

First the two real base points are plotted as the oscillation centers of the cameras at their measured horizontal distance from each other. Then at each base the location of the reference tables is plotted with reference to the oscillation center by measuring the distances a from the first and last vertical line of each table to the oscillation center B (Fig. 14). Since the distance b between the two wires is 1.4 m (55 in.), the triangle including the oscillation center and the table is determined by its three sides. The distances c between the outer wires of adjacent tables are likewise measured and determine the relative position of the three triangles a b to one another.

Each base is thus plotted by itself and it only remains to locate them exactly with reference to each other. This must be done with the measuring kinetograph, in order to avoid possible constant errors of angle in the kinetograph itself. Some central landmark is then selected and photographed from each base with the corresponding kinetograph, the same as in a flight test. A definite ground-plan coordinate of the central

landmark is thus obtained at each base between the vertical wires on the reference tables. This ground-plan coordinate is plotted as point A in Fig. 14. The same central landmark is also measured with the theodolite by the well-known "two-station method" from both bases and plotted in the 1 : 500 ground plan. Then the reference tables are plotted as shown in Fig. 14, so that the point B coincides with the base point and the line AB passes through the central landmark. Thereby the base lines of the reference tables are preferably plotted on a larger scale (e.g., 1 : 2 has been found satisfactory) and then reflected into the measuring field in front, in order to save room on the drawing board.

For the determination of the flight-path points in the ground plan, it is first necessary to obtain simultaneous values of the coordinates. Since the clock pointers move synchronously, this is rendered possible by the clock photograph. The ground-plan coordinates of a series of film pictures are first taken from one measuring place, e.g., once a second, or every 16th picture, thereby correcting the deviation of the airplane from the cross wires in the photograph of the table. Thus the oblique cross F_2 of the small image field B_2 is corrected corresponding to the deviation of the center of gravity of the airplane from the cross F_1 (Fig. 3) of the larger image field. The ground-plan coordinates thus obtained are tabulated with the corresponding clock-time readings. From the other measuring

place there are generally no photographs which coincide exactly in time with the ones in the table. Hence, for every picture in the table from one measuring place, two pictures from the other measuring place must be utilized, one of which was taken shortly before and the other shortly after the one from the first measuring place. Between these two, the ground-plan coordinates will be interpolated in direct dependence on the time values. Since the successive exposures are only $1/16$ second apart, linear interpolation is possible within the accuracy of the time measurements. Through this interpolation, we therefore obtain the ground-plan coordinates of the second measuring place, which belong to the table of the first measuring place. Each of the two coordinates is plotted from the respective measuring place, as a straight line from the base B to the corresponding coordinate point of the reference table, and the intersection point of these lines is a point in the flight path.

If, for any reason, one of the two measuring places is missing, or if the clock fails, the determination of the flight path is nevertheless possible in the "single-station method." This single-station method deduces the distance between the airplane and the camera, from the magnitude coordinates of the airplane pictures, the dimensions of the airplane itself and the focal length of the lens. Under present conditions the calculation of this distance is very simple, because the geometric problem is

almost two-dimensional, since the optical axis (sighting line) lies nearly in the plane of the airplane (fuselage-wing plane).

The general three-dimensional problem reads: "The lengths c and d of two vectors (fuselage c and wing d), which are perpendicular to each other, are given. The cartesian components c_1, c_2, d_1, d_2 of these vectors, which lie in a plane perpendicular to the optical axis, are photographed." The first equations therefore express the relations between the components and the lengths of the vectors

$$\left. \begin{aligned} c_1^2 + c_2^2 + c_3^2 &= c^2 \\ d_1^2 + d_2^2 + d_3^2 &= d^2 \end{aligned} \right\} \quad (8)$$

There to is added the perpendicularity condition between the vectors c and d

$$c_1 d_1 + c_2 d_2 + c_3 d_3 = 0 \quad (9)$$

Lastly, the relation between the natural-size components and the image components, which latter will here be designated as "image angles." These are obtained by dividing the coordinate lengths of the film image by the focal length b . The axes of the cross F_1 can be used advantageously as the cartesian system of the film image (Fig. 3). These coordinates are therefore

$$b \delta_1, \quad b \delta_2 \quad \text{and} \quad b \gamma_1, \quad b \gamma_2,$$

in which b denotes the focal length and γ, δ the "image

angles." We then have for the distance a of the airplane:

$$c_1 = a \gamma_1, \quad c_2 = a \gamma_2 \quad (10)$$

$$d_1 = a \delta_1, \quad d_2 = a \delta_2 \quad (11)$$

Since the image coordinates γ and δ are known in these seven equations (8 and 11), they contain seven unknowns. The distance of the airplane can therefore be determined from these equations. The equations (8 - 11) naturally also enable the determination of the components $c_1, c_2, c_3, d_1, d_2, d_3$ and, with the aid of the orientation of the camera, the evaluation of the spatial position angles of the airplane axes.

First equation (9) is squared and equation (8) inserted in it:

$$(c_1 d_1 + c_2 d_2)^2 = (c^2 - c_1^2 - c_2^2) (d^2 - d_1^2 - d_2^2)$$

Then equations (10) and (11) are introduced:

$$\begin{aligned} a^4 (\gamma_1 \delta_1 + \gamma_2 \delta_2)^2 &= \{c^2 - a^2 (\gamma_1^2 + \gamma_2^2)\} \{d^2 - a^2 (\delta_1^2 + \delta_2^2)\} \\ a^4 \{(\gamma_1 \delta_1 + \gamma_2 \delta_2)^2 - (\gamma_1^2 + \gamma_2^2) (\delta_1^2 + \delta_2^2)\} &+ \\ &+ a^2 \{c^2 (\delta_1^2 + \delta_2^2) + d^2 (\gamma_1^2 + \gamma_2^2)\} = c^2 d^2 \end{aligned}$$

or, after removing the coefficient brackets from a^4 :

$$-a^2 (\gamma_1 \delta_2 - \gamma_2 \delta_1)^2 + a^2 \{c^2 (\delta_1^2 + \delta_2^2) + d^2 (\gamma_1^2 + \gamma_2^2)\} = c^2 d^2 \quad (12)$$

The coefficient of a^4 apparently disappears,* when the optical axis lies in the plane of the vectors c and d . Since this is generally almost the case and since the coefficient of a^4 is consequently very small, equation (12) can be solved only as a linear equation of a^2 . A division by the very small quantity $(\gamma_1 \delta_2 - \gamma_2 \delta_1)^2$ would burden the solution of the quadratic equation of a^2 with great errors. It is therefore,

$$a^2 = \frac{c^2 d^2}{c^2(\delta_1^2 + \delta_2^2) + d^2(\gamma_1^2 + \gamma_2^2) - a^2(\gamma_1 \delta_2 - \gamma_2 \delta_1)^2} \quad (13)$$

It must be borne in mind that the right-hand term in the denominator, which contains a^2 , plays the role of a correction term, on account of its small coefficient. Therefore, a first approximation \bar{a} for the distance a can be determined as:

$$\bar{a}^2 = \frac{c^2 d^2}{c^2(\delta_1^2 + \delta_2^2) + d^2(\gamma_1^2 + \gamma_2^2)} \quad (14)$$

This first approximation is sufficient when the optical axis falls in the plane of the vectors c and d . If this condition is only approximately fulfilled, the second approximation $\bar{\bar{a}}$ suffices, which is obtained by substituting in equation (13), at the right in the denominator, \bar{a} for a and by developing the geometrical series as far as the first term:

*The equation $\frac{\gamma_1}{\gamma_2} = \frac{\delta_1}{\delta_2}$ shows that the fuselage axis and the wing spar lie in the same straight line on the picture.

$$\bar{a}^2 = \bar{a}^2 + \bar{a}^4 \frac{(\gamma_1 \delta_2 - \gamma_2 \delta_1)^2}{c^2(\delta_1^2 + \delta_2^2) + d^2(\gamma_1^2 + \gamma_2^2)} \quad (15)$$

The distance a is converted into a horizontal distance by means of the vertical coordinates on the reference table. This horizontal distance is then plotted in the ground plan on the corresponding sighting line, which passes through the base point B and the corresponding coordinate point on the ground plan of the reference table. (See H. Knott's ground plan of the "Roemryke Berge" flight.)

b) Determination of the altitude.— First the zero line must be located on every table, i.e., the number which, as a coordinate, belongs to an object on a level with the measuring place. This zero line was likewise determined by photographing the central landmark. As explained in the description of the camera (Fig. 9), it does not lie at the same height as the lens, but at an angle of $\Delta\varphi$ below it. As demonstrated above, we can proceed, notwithstanding the altitude evaluation, as though the angle $\Delta\varphi$ were 0. Therefore, the zero line on the tables is also determined on this plan.

From the photograph of the central landmark H , we obtain an altitude coordinate H' on one of the tables. From the measurement of the vertical angle φ toward the central landmark, we obtain a corresponding height h' up to the zero line on the table (Fig. 15) with the aid of the horizontal dis-

tance a' taken from the ground plan. If we subtract the height h' from the altitude coordinate H' of the central landmark H , we obtain the zero coordinate N' . This differs a little under different lateral coordinates, since the horizontal wires on the tables are not strictly horizontal. Still the inclination of the zero line to the wires is determined by a round measurement with the theodolite from the camera stand. In this manner the zero coordinates on the three tables are determined and plotted for all the lateral coordinates. The determination of the altitude h of a point in the flight path above the measuring place is now a simple calculation in proportion. It bears the same ratio to the coordinate height h' as the ground-plan distance a of the flight-path point to the ground-plan distance a' of the lateral coordinate.

4. Focal Lengths and Accuracy of Measurement

The accuracy of measurement is chiefly determined by the focal lengths. Knowledge of the focal lengths is especially necessary in the transfer of the coordinates measured in the image field of the airplane (deviations of the center of gravity of the airplane from the cross wires) to the image field of the reference table. The transfer is effected by shifting the oblique cross F_2 (Fig. 3). The magnitude of this displacement bears the same ratio to the amount of the deviation in the image field of the airplane, as the corresponding dis-

tances of the image plane from the main plane of the lens. This distance is the focal length itself for photographs taken at an infinite distance. This is 292 mm (11.5 in.) for the airplane photographs in both cameras. On the other hand, the image distances for the table photographs differ for the two cameras at the 2-meter position of the lens. For every camera, therefore, corrections were made for the corresponding displacement of the oblique cross F_2 in the table image field from the coordinates in the airplane image field (Fig. 3). This evaluation is made on a projected image magnified about fifteen-fold. It was found that an evaluation of the film to 0.02 mm (0.0008 inch) was the utmost possible. This accuracy cannot be attained, however, in the present arrangement of camera and reference table. Hence an evaluation to 0.05 mm (0.002 in.) was considered satisfactory, this value just corresponding to the thickness of the photographed wires and to the thickness of the cross wires. The accuracy of the evaluation can be considerably increased by evaluating all the film images and thus determining 16-20 flight-path points per second. The accuracy of the result is considerably increased by averaging so many measurements.

In the ground plan the flight-path points can be determined by the two-station and also by the single-station method as applied at each camera, hence as intersection points of two circles and two straight lines in the same point. The two-station

method will generally give the most accurate result, but a systematic deviation in the application of the single-station method would at least show any disturbance in the experiment, e.g., any error in the synchronism of the clock pointer.

The altitude of the flight-path point is also determined from both measuring places and systematic discrepancies in these two determinations would likewise show errors in measuring. Such errors were manifested in the first measurements with the "Roemryke Berge" and "Westpreussen" by a displacement of the zero coordinate on the reference tables. It was, however, possible to determine the zero coordinate from the measurement itself, as will be explained by Mr. Knott.

In concluding, I wish to disclaim any idea that the kinetograph described in Section 2 is a technically perfect instrument. As already mentioned, the described form was necessitated by circumstances. A satisfactory kinetographic instrument had to be produced in a short time with very few technical resources.

The equipment of the measuring station with a wooden stand and makeshift reference tables was due to lack of funds. In the evaluation of the first flight measurements it was possible, only with the greatest pains, to eliminate errors due to changes in the position of the stand and tables. It is therefore essential, for very accurate measurements, to create perfectly stable measuring stations. Moreover, the evaluation of the "Roemryke

Berge" flight shows that the determination of the altitude and time are not yet sufficiently accurate for accelerated flights. For these measurements we should have an angular accuracy of 0.0002° to 0.0001° and a time accuracy of $0.005''$ to $0.002''$. Concrete measuring bases and great focal lengths on the table side, an accurate contact clock and a constant motor with considerable reserve power are the resources which render these goals attainable.

This improvement in the kinetographic records must be accompanied by accurate records made on the airplane, of the longitudinal inclinations, dynamic pressure and angle of attack. The first experiments with "Askania" dynamic-pressure recorders were unsatisfactory. Accurate records can probably be best obtained by photographing the instruments. A simultaneous series of photographs taken on the airplane, of the horizon, clock, dynamic-pressure indicator, angle-of-attack vane, and possibly of the control stick, should supplement the kinetographic records made from the ground stations. Only when this is accomplished, can free-flight experiments equal wind-tunnel experiments in accuracy.

II. "Roemryke Berge"

By H. Knott

S y m b o l s

- c_w drag coefficient of airplane,
 c_{w_1} drag coefficient of wing section or profile,
 c_a lift coefficient of airplane,
 γ density of air (kg/m^3),
 g acceleration due to gravity (m/s^2),
 F wing area (m^2)
 G weight of airplane (kg),
 v measured tangential speed (m/s),
 v_h measured horizontal speed (m/s),
 \bar{v} speed in steady flight (m/s),
 \bar{v}_h horizontal speed in steady flight (m/s),
 $\bar{\epsilon}$ angle of glide in steady flight,
 $\epsilon = -\tan\phi$ spatially measured angle of flight,
 t time (seconds),
 λ correction factor ($\%$),
 a constant (s^2/m^2),
 b constant (m^2/s^2).

The remarkable flights of the "Roemryke Berge," with Nehring as pilot in the 1926 Rhön soaring-flight contest, created a desire to test the flight characteristics of this glider.

This was the first opportunity to try the above-described method of "kinetographic flight measurements" developed by Mr. P. Raethjen. The intention was to determine the polar curve of the glider by an experiment in which the glider was to pass through all the speeds at a low acceleration. In the evaluation, however, it was found that the flight was affected by influences which made a direct determination of the polar impossible, so that the work shows only an estimate of these influences and the calculation of an \bar{E} -curve, which is to be regarded as an energy balance of the tested flight.

The Experiment

On the morning of August 8, 1926, the velocity of the wind over the "Wasserkuppe" was almost zero (less than 1 m/s). For days there had been high-pressure weather. The day seemed suitable for the test flight and toward 9 o'clock a.m., the Roem-ryke Berge started. Nehring, as pilot, was instructed to push the glider slowly from the minimum to the maximum speed and then gradually slow down again, while maintaining as straight a course as possible. The measurements were made from the ground with the aid of the measuring kinetograph. The dynamic-pressure recorder used on the glider was a special instrument made by the Askania Works. Unfortunately, the clockwork for rapid recording proved very unreliable, rendering it impossible to identify the time. Nevertheless, the diagram furnishes a good record of

the speeds in the range of the most favorable angle of glide, so that it seemed appropriate to publish it (Fig. 19). The speed range went only to 20 m/s (65.6 ft./sec.), so that the maximum speed could not be checked with the dynamic-pressure recorder.

Evaluation

The evaluation was made in the manner already described by Mr. P. Raethjen. Fig. 16 shows the ground plan of the flight path. The reference tables were drawn half their natural size, as reflected in front of the measuring places. The points, which were obtained by the two-station method, are represented as full circles. They were determined by the intersection of two steel wires, which are to be regarded as measuring wires M_1 and M_2 . The steel wires can be wound around pins which fix the position of the vertical axes of the kinetographs.

On account of the unfavorable location of the first measuring stand and the lack of experience in following an airplane in the finder, the first 19 seconds could not be photographed from the measuring stand I, so that they had to be calculated by the single-station method from measuring stand II, in the manner described above by Mr. P. Raethjen. The calculated points are represented by crosses. The two-station method is naturally more accurate than the single-station method. The scattering of the single-station points is due to the impossibility of accurately measuring the airplane coordinates, since

one wing tip is hidden by the fuselage. At long distances the single-station method gives only inaccurate results, since the determination of the magnitudes is difficult, due to the smallness of the pictures. We therefore stopped calculating by the single-station method the distances of points on the flight path which had already been definitely located by the two-station method. When measuring points were lacking on one film, the measuring rays of the other film were made to intersect with the ground plan (represented by the cross lines in the flight path), in order to be able to determine the flight distance and the altitude for these intersection points.

The ground plan (Fig. 16) shows slight oscillations which might indicate measuring errors. The slight curves are, however, real because, according to Nehring, the glider was longitudinally and laterally unstable and very difficult to hold to a rectilinear flight path. The deviations of the measured points from a smooth ground-plan curve do not exceed 2 m (6.56 ft.) at the point of intersection by the two-station method. Nevertheless, the accuracy of the flight-path determination must be considerably greater, because the ground-plan path is smoothed out at numerous points and especially, because the intersections of the individual measuring lines with the ground-plan path are considerably more blunt than the intersections of the measuring lines with one another. The ground plan now forms the basis for the altitude determination. The altitudes

were calculated by Mr. Raethjen's method and plotted in Fig. 2 against the corrected flight path, as was likewise the flight time, which was determined from the soot record of the operating motor. There were some difficulties at first, in that the evaluations from the measuring base II did not agree with those from base I. The error was due to the fact that the reference points had changed position in the interval (about 14 days) between the calibration and the experiment. The fact that many reference points of the landscape could be found on the film itself, led to a satisfactory result after recalibrating the reference points. The following table contains the results of the calculation and evaluation for the "Roemryke Berge."

Symbols

- s flight distance on ground plan (m),
- h_I altitude above measuring base I (m),
- h_{II} altitude above measuring base II (m),
- Δh difference in altitude between the measuring bases = 6.7 m (about 22 ft.),
- t flight duration (sec.).

Table

No.	s m	h _I +Δh m	h _{II} m	t s	No.	s m	h _I +Δh m	h _{II} m	t s
1	34.0	-	31.4	0	31	391.5	11.9	12.2	37.00
2	57.5	-	29.3	0.98	32	396.5	-	11.6	37.24
3	79.5	-	29.3	2.00	33	410.0	9.9	10.0	28.02
4	96.5	-	30.9	3.00	34	427.3	7.6	7.8	29.09
5	111.0	-	32.1	4.06	35	430.8	-	6.5	29.52
6	124.5	-	33.5	5.13	36	451.0	4.2	-	30.19
7	136.5	-	34.5	6.13	37	468.0	1.0	1.0	31.22
8	149.0	-	35.4	7.22	38	486.8	- 2.8	- 2.5	32.32
9	158.5	-	35.2	8.24	39	492.5	-	- 3.7	32.58
10	169.3	-	34.7	9.28	40	508.5	- 6.1	- 5.9	33.38
11	181.0	-	34.2	10.27	41	511.3	-	- 6.2	33.46
12	193.5	-	33.7	11.42	42	534.3	- 9.8	- 9.1	34.40
13	204.5	-	33.4	12.51	43	537.0	-	-10.2	34.67
14	216.5	-	32.7	13.56	44	554.0	-	-13.2	35.46
15	228.0	-	31.1	14.59	45	559.0	-	-13.0	35.72
16	240.3	-	30.2	15.66	46	580.5	-16.9	-	36.53
17	252.3	-	28.5	16.78	47	535.3	-	-17.8	36.78
18	264.5	-	27.2	17.78	48	608.0	-20.8	-20.2	37.62
19	276.5	-	26.1	18.85	49	612.3	-	-21.0	37.87
20	286.5	25.1	25.0	19.63	50	631.8	-23.6	-22.9	38.62
21	289.3	-	24.5	19.90	51	662.3	-24.8	-25.3	39.68
22	300.0	22.9	23.0	20.70	52	686.0	-25.5	-25.2	40.76
23	302.4	-	22.2	21.00	53	710.5	-25.3	-26.3	41.78
24	313.0	20.6	20.7	21.76	54	736.3	-25.9	-23.8	42.89
25	315.8	-	20.4	22.06	55	760.8	-24.6	-24.9	44.02
26	326.5	18.7	19.0	22.82	56	780.3	-21.4	-21.5	45.08
27	342.0	16.7	17.1	23.85	57	803.3	-18.2	-18.7	46.09
28	358.8	15.5	15.4	24.88	58	827.8	-16.5	-16.1	47.21
29	375.8	13.7	14.2	25.95	59	846.3	-	-16.1	48.34
30	381.0	-	13.6	26.02					

Hopf's equations for accelerated flight were used for the further evaluation (Fuchs and Hopf, "Aerodynamik," 1932, p.346).

$$c_w \frac{\gamma}{2g} F v^2 = - G \sin \varphi - \frac{G}{g} \frac{dv}{dt} \tag{1}$$

$$c_a \frac{\gamma}{2g} F v^2 = G \cos \varphi + \frac{G}{g} v \frac{d\varphi}{dt} \tag{2}$$

On dividing equation (2) by equation (1), we obtain

$$\frac{c_w}{c_a} = \bar{\epsilon} = \frac{-\sin \varphi - \frac{1}{g} \frac{dv}{dt}}{\cos \varphi + \frac{1}{g} v \frac{d\varphi}{dt}}$$

As the first approximation

$$\bar{\epsilon} = \left[-\tan \varphi - \frac{1}{g \cos \varphi} \frac{dv}{dt} \right] \left[1 - \frac{1}{g \cos \varphi} v \frac{d\varphi}{dt} \right] \quad (3)$$

In our computation we put

$$-\tan \varphi = \epsilon$$

$$\frac{d\varphi}{dt} = - (1 - \epsilon^2) \frac{d\epsilon}{dt} \quad (4)$$

$$\frac{1}{\cos \varphi} = 1 + \frac{\epsilon^2}{2} \quad (5)$$

We then obtain

$$\bar{\epsilon} = \left[\epsilon - \frac{1}{g} \left(1 + \frac{\epsilon^2}{2} \right) \frac{dv}{dt} \right] \left[1 + \frac{1}{g} \left(1 + \frac{\epsilon^2}{2} \right) v (1 - \epsilon^2) \frac{d\epsilon}{dt} \right] \quad (3a)$$

Equation (3a) is approximately written

$$\bar{\epsilon} (1 - \lambda) = \epsilon - \frac{1}{g} \frac{dv}{dt} \quad (3b)$$

whereby the correction factor

$$\frac{v_h}{g} \frac{d\epsilon}{dt} = \lambda \quad (6)$$

is introduced. Then

$$v = \frac{v_h}{\cos \varphi}$$

in which v_h is the measured horizontal speed.

$$\frac{dv}{dt} = \frac{1}{\cos \varphi} \frac{d v_h}{dt} + v_h \frac{\tan \varphi}{\cos \varphi} \frac{d\varphi}{dt}.$$

If we introduce the values from equations (4)-(6), we have approximately

$$\frac{1}{g} \frac{dv}{dt} = \frac{1}{g} \frac{d v_h}{dt} - \epsilon \lambda \quad (7)$$

λ being a small correction factor to be added in per cent in the vertical flight-path curves. In Fig. 18, λ was plotted against the time and was taken into account when it was equal to or greater than 3%.

Equation (3b) presents the task of plotting ϵ and $\frac{1}{g} \frac{dv}{dt}$ against the time, in order to be able to deduce $\bar{\epsilon} (1 - \lambda)$ from the difference between the two curves. $\bar{\epsilon}$ belongs to a \bar{v}_h which can be calculated from equations (2a) and (8).

$$c_a \frac{\gamma}{2g} F \frac{v_h}{\cos \varphi} = G \cos \varphi + \frac{G}{g} \frac{v_h}{\cos \varphi} \frac{d\varphi}{dt} \quad (2a)$$

For unaccelerated flight, we have

$$c_a \frac{\gamma}{2g} F \frac{\bar{v}_h^2}{\cos \bar{\varphi}} = G \cos \bar{\varphi} \quad (8)$$

If we introduce the values from equations (4), (5), and (7), we obtain approximately

$$\frac{v_h^2}{\bar{v}_h^2} = 1 - \lambda$$

hence,

$$\bar{v}_h = v_h \left(1 + \frac{\lambda}{2} \right) \quad (9)$$

The evaluation was made by drawing polygon outlines through the altitude and time points, whose sections, according to the conditions of the altitude and time curves, extended over about 1 second each. The polygons yield, between the corner points, constant gliding-angle values ϵ and speed values v_H . These values were plotted as step curves against the time in seconds (Fig. 18). In the determination of these step curves, care was taken that, for longer time intervals, they actually corresponded to the measured loss in altitude, i.e., that the integrals for longer intervals were relatively more accurate than the separate values for the separate polygon sides. This was done so that the relatively large errors in the speed and angle of glide during 1 second should not enter into the result for longer flight periods and distances. The polygons and the altitude diagram of the flight path show that Nehring flew part of the time (contrary to his instructions) with a very rapidly changing angle of attack, which he explained as due to the longitudinal instability of the glider. These fluctuations therefore require a smoothing out of the step curves, since it is obvious that the step curves themselves cannot be integrated.

Moreover, the altitude determination which, especially on the latter part of the flight-path curve, can be made exactly only to within 30 cm (11.8 in.), requires the mean value for about 5 seconds, since for shorter periods the errors in measuring the altitude are no longer small in comparison with the loss

in altitude of the glider. These mean values were obtained from second to second over each 5-second period. The smoothed-out curves of Fig. 18 are therefore the curves for these mean values. The mean speed values were obtained for the quantity v_h itself, so that the flight distance integral

$$\int v_h \, d t$$

is maintained. The mean values of the spatial gliding angle were obtained over the quantity $v_h \epsilon$, so that the sinking-altitude integral

$$\int v_h \epsilon \, d t$$

is maintained.

The dynamic-pressure diagram was then plotted for the determination of the v_h curve (Fig. 19). As already mentioned, no time identification was possible. The fluctuations in the basis are not ascribable to gusts, but to the fact that the pressure recorder continued to run during transportation and therefore shows the effect of shaking. Nevertheless, the base line, which was recorded before the start, can be easily recognized. In the pressure diagram, the bend, which lies between the 20th and the 30th second in the speed curve in Fig. 3, is very manifest. Also the minimum pressure agrees, within the accuracy of the pressure recorder, with the minimum measured flight speed, according to the calibration by the Askania Works. The high speeds obtained by the flight-path measurements in the first 3 seconds after the start, contradict the pressure diagram. Apparently

they are not real since, in this field, the single-station method is subject to large errors. A shifting of the ground-plan path, within the accuracy of the single-station method (dotted curve, Fig. 16) would eliminate these differences between the measured speeds and the dynamic pressure. The mean values of $\frac{d v_h}{dt}$ for 5 seconds were again determined mathematically from the v_h curve. From these values a curve was drawn according to equation (7) for the value $\frac{1}{g} \frac{dv}{dt}$.

According to equation (3b), the difference between ϵ and $\frac{1}{g} \frac{dv}{dt}$, when increased by $\lambda\%$, gives the aerodynamic angle of glide, which corresponds, in unaccelerated flight, approximately to the same speed according to equation (9). Equations (3a) and (9) therefore enable the determination of an angle-of-glide diagram for unaccelerated flight from the curves v_h , ϵ and $\frac{1}{g} \frac{dv}{dt}$. The result of this determination is introduced with the plain curve into the $\bar{\epsilon}/\bar{v}_h$ diagram in Fig. 20. We may conclude from the great fluctuations of this curve that the measurement admits of no possibility of the direct determination of $\bar{\epsilon}$. Our present task is to inquire into the causes of these fluctuations in the $\bar{\epsilon}$ curve. For comparison I have introduced into the $\bar{\epsilon}$ diagram (Fig. 20) the long-dash curve calculated according to the wind-tunnel measurements (Göttingen profile 426) for 1000 m (3280 ft.) altitude and placed at my disposal by Mr. Koch of Darmstadt. With the aid of this $\bar{\epsilon}$ curve, a curve $\frac{1}{g} \frac{dv}{dt}$ was determined according to

equation (3b) and likewise long-dashed in Fig. 18, which satisfies the measured values ϵ and the theoretical $\bar{\epsilon}$ diagram (Fig. 20). Furthermore, the values v_h were integrated from the 15th to the 38th second over this curve dv/dt and a corresponding long-dash curve v_h was likewise plotted in Fig. 18. The deviations of this curve v_h from the actually measured speeds (step curve) are obviously greater than the errors in measuring. The deviations of the measured $\frac{1}{g} \frac{dv}{dt}$ values (plain curve) from the theoretical values are so great up to the 10th second that one is inclined to suspect very great disturbances in the experiment. It has not been possible, however, to discover any such sources of error, except from the start up to the 5th second, a region in which, as already remarked, errors may occur by the single-station method. We are somewhat inclined, however, to ascribe even these deviations to errors in the single-station method.

The experimental results can, in general, be vitiated by various causes in the following orders of magnitude. The time may be regarded as accurate to within 0.02 second; the speed, therefore, by determining the mean value for 5 seconds, up to 4% time error.

The determination of the flight-path point on the ground plan may be regarded as accurate to within 1 m (3.28 ft.), making, on an average, 75 m (246 ft.) in 5 seconds, an error of 1 to 1.5%. The altitude determination will have an accuracy of

20 cm (7.87 in.) at short distances and of 30 cm (11.8 in.) at long distances. This might cause errors of about 4% in the ϵ values for aerodynamic sinking speeds of 5 m (16.4 ft.) in 5 seconds at short distances and for 8 m (26.25 ft.) in 5 seconds at long distances. These errors must be regarded, however, as maxima. They do not suffice, even when all added together, to explain the deviations.

The atmospheric disturbances are more difficult to estimate. At low speeds, even slight horizontal gusts can greatly affect the ϵ values. This is impossible, however, at high speeds. The surmise would rather be justified that the flight had been affected by ascending and descending air currents. These currents must have changed rapidly, however. Their order of magnitude would be about as follows:

At the 10th second,	60 cm (23.6 in.)/sec. up;
" " 20th "	30 cm (11.8 in.)/sec. down;
" " 25th "	0 cm/sec;
" " 30th "	70 cm (27.6 in.)/sec. down;
" " 36th "	45 cm (17.7 in.)/sec. up.

An up-wind might be caused by thermal currents, although at 9 o'clock in the morning they would hardly be expected to be so strong. On the other hand, down-winds of 30 and 70 cm/sec. have never been observed in the lee of the Wasserkuppe with a 5 m (16.4 ft.)-per-second wind. The flight might well be subjected to an atmospheric flow up to the 15th second, but a down-

wind of 70 cm/sec. at the 30th second seems impossible.

The conclusion cannot be escaped that these deviations are due at least in part to the fact that the air-force coefficients are dependent not only on the angle of attack but also on the state of acceleration. This conclusion is supported by the circumstance that the theoretical and measured \bar{c} values coincide in the field of the most favorable gliding angle, because it is in this field that they are least affected by the speed. The second acceleration d^2v/dt^2 seems to be especially effective. Hence this is also plotted in Fig. 18, though of course there may be considerable errors in this third differentiation.

The deviations before the 15th second are hard to explain. In each case I have tried to determine a mean curve \bar{c} in the diagram \bar{c}/v_h (short-dash curve in Fig. 20). This curve has at least the value of an energy balance for the accelerated flight under consideration, in so far as the corresponding values $\bar{v}_h \bar{c}$ furnish a mean horizontal force. It is also of interest to see how far an \bar{c} diagram can be evaluated from the experiment under consideration.

The following evaluation can be made for the determination of the \bar{c} values.

$$c_w = c_{w_1} + \text{constant } c_a^2$$

$$c_a = \frac{g}{F} \frac{\gamma}{2g} \frac{1}{v^2}$$

$$\frac{c_w}{c_a} = \bar{c} = a \frac{1}{v^2} + \frac{b}{v^2} \quad (11)$$

in which a is a constant of the dimension s^2/m^2 and b is a constant of the dimension m^2/s^2 .

According to the method of the least squares, the values of a and b can be calculated, which determine, for the measured $\bar{\epsilon}$ values, the least error-squares of $\bar{\epsilon} \bar{v}_h$ (equation 11). The "error equations," according to which the constants a and b were determined, therefore read

$$\hat{Q}_h = a \bar{v}_h^3 + \frac{b}{\bar{v}_h} - \bar{\epsilon}_h \bar{v}_h \quad (12)$$

The method of the least squares therefore furnishes, as "normal equations" for the determination of the constants a and b , the two equations.

$$[\bar{v}^3 \bar{v}^3] a + \left[\bar{v}^3 \frac{1}{\bar{v}} \right] b - [\bar{v}^3 \bar{\epsilon} \bar{v}] = 0 \quad (13a)$$

$$\left[\bar{v}^3 \frac{1}{\bar{v}} \right] a + \left[\frac{1}{\bar{v}} \frac{1}{\bar{v}} \right] b - \left[\frac{1}{\bar{v}} \bar{\epsilon} \bar{v} \right] = 0 \quad (13b)$$

In the field between the 10th and 40th seconds the constants were

$$a = 0.000104 \text{ s}^2/\text{m}^2,$$

$$b = 8.26 \text{ m}^2/\text{s}^2.$$

The curve according to equation (11) is plotted as a short-dash-line in Fig. 20. The result is not at all satisfactory as a flight-characteristic determination for unaccelerated flight conditions, but the investigation nevertheless shows what possi-

bilities kinetographic flight measurements offer and at the same time indicates the field for which this method of measuring was developed, namely, the investigation of accelerated flight conditions. In the evaluation it was found that records can be made on the airplane itself, along with an exact ground measurement, which is only possible when the measuring bases, by being concreted, render impossible any change in the reference stations. Along with an exact measurement of the dynamic pressure an angle-of-attack recorder is very important, as likewise the recording of the rudder and elevator deflections. In this way it will probably be possible to learn more about the problem of accelerated flight.

A P P E N D I X

Kinetographic Flight Measurements*

By P. Raethjen

In the Research Institute of the Rhön-Rossitten Association on the Wasserkuppe, the investigation of accelerated airplane motions is receiving special attention.** These investigations must consist principally of the mechanical analysis of airplane motions in free flight, because the unsteady air flow conditions are very difficult to obtain in a wind tunnel.

For the mechanical analysis of an airplane motion, the motion of the airplane in space and the motion of the air about the airplane must both be known. This constitutes a double task. The flow of the air against the airplane must be recorded by instruments carried on the airplane itself. In general, a dynamic-pressure recorder and an angle-of-attack recorder suffice for the longitudinal motions; and two each of these instruments in curving flight. Moreover, the motion of the airplane must be measured as the "motion of a rigid body" with respect to both space and time. This measurement can even be made photogrammetrically on the airplane itself from the airplane itself. In this case the continuous photography of the landscape, characterized by measured points, would suffice.

*"Kinematographische Flugvermessung." From "Zeitschrift für Flugtechnik und Motorluftschiffahrt," Dec. 28, 1926, pp. 547-549.
**P. Raethjen, "Beschleunigte Flugzeugbewegungen," from "Z.F.M." Dec. 28, 1926, pp. 537-547.

This is a very elegant method, but necessitates the carrying of heavy apparatus and a complicated evaluation.

Another possibility is the two-station method from the ground. In order to obtain accurate results, this method must be carried out photogrammetrically. The two-station method is a simple trigonometrical method. By measuring the spatial angle (azimuth and altitude angle) from two base points M and M' , the position of the airplane in space is found as the intersection point of two lines (Fig. 13). For the measurement of accelerated airplane motions, I have developed the two-station method as a kinetographic-photogrammetric method. This method consists essentially in taking continuous motion pictures, at both base points, of the airplane and of a reference table R divided into squares and located behind the camera (Fig. 13). Thereby the camera is continuously directed (like a theodolite) toward the airplane. The camera is mounted so it can be rotated horizontally and vertically by means of cranks. Thus, with the aid of a telescopic finder it can be kept continuously directed toward the airplane. With a sufficient focal length of the object lens, large enough images of the airplane can be obtained, even at distances of 1000-2000 m (3280-6560 ft.), to determine the position of the three airplane axes. Behind the camera, there is a vertical reference table divided into squares, which is photographed simultaneously with the airplane. This is made possible by the apparatus

shown diagrammatically in Figs. 7 and 21, in which there are two object lenses O_R and O_A , with their optical axes parallel, which throw images on a system of mirrors $H J$ and the same film F . The two optical axes are indicated on the film F by cross wires. If the camera is rotated, the image of the reference table R continuously shows the direction in space assumed by the optical axes. Any deviation of the position of the airplane from this optical axis can be determined from the picture of the airplane. The accuracy of this method might theoretically be still further increased, since it increases with increasing focal length of the lens and with increasing distance of the reference table R . With this apparatus, angles can be measured to within 0.0005 of a degree, i.e., at a distance of 1000 m (3280 ft.), the position of the airplane can be determined to within 0.5 m (1.64 ft.).

The chief difficulty lies in the time identification of the photographs for the two measuring stations. I have adopted the method of photographing at both stations a synchronously running clock pointer (Fig. 21, clock pointer Z). The image of this pointer, which makes one revolution per second, is projected on the back side of the film. Fig. 3 is a section of a film showing the three image fields, one above the other: the square field of the airplane, the field of the reference table with the numbers, and the U-shaped field of the clock scale with the seconds pointer in the corner. It is a double pointer, which

completes a period every half-revolution. Hence the scale is an open U, instead of a closed circle like ordinary clock dials. On the film is seen only one or both tips of the pointer, which appear as shadows on the U-shaped scale. The central portion of the pointer is concealed under the dial. This arrangement was adopted to save space.

The pointer is actuated at both measuring stations by electrically synchronized motors, which are in turn operated from a central station (Fig. 13). The central motor M, therefore runs synchronously with the motors M_1 and M_2 , with which it is connected by three wires. In order to record the revolution speed of these motors, a recording drum at the central station is driven by the motor M. On this drum, time marks are registered by clockwork at regular intervals of 0.2 and 3.0 seconds. A telephone line between the two stations enables their harmonious cooperation.

In this way, the airplane is continuously photographed from both stations. The individual exposures are not simultaneous, however, but the evaluation is enabled by the time pointer. On one film the spatial angles are taken directly and on the other the angles are interpolated to correspond. This is entirely possible, due to the frequency of the exposures (16-20 per second). The time can be determined to within 0.02 second.

As already mentioned, one can determine from the photographs, aside from the flight path of the airplane (the path of

the center of gravity of the solid body), the rotation of the airplane axes about the center of gravity, especially the longitudinal inclination. The latter can be determined best by photographing the airplane from one side. The base points on the Wasserkuppe were therefore selected to one side of the starting place. At long distances the determination of the position of the airplane axes is naturally inaccurate. If, at a distance of 500 m (1640 ft.) the measurement is accurate only to within 0.25 m (0.82 ft.) (which is accurate enough, however, for the determination of the path of the center of gravity), this enables, for a fuselage 5 m (16.4 ft.) long, at best (with lateral photographs), a determination of the longitudinal altitude with an accuracy of about 3 degrees. Errors of 3 degrees are disagreeable, although the longitudinal inclination must be measured only for the determination of the up-wind. Nevertheless, even this error can be considerably reduced by averaging for the numerous individual pictures.

In any event, it is desirable to determine the altitude of the airplane by a third kinetographic photograph from the airplane, even if only with the American "Kymograph" by photographing the sun; better still, by photographing the horizon by means of a rigidly mounted kinetograph.

Briefly stated, the problem is to obtain trustworthy data which will enable a complete and reliable analysis of the combined effect of the forces of inertia and of the air. These

data can be obtained as follows:

1. Through the determination, with reference to both space and time, of the path followed by the glider (or airplane), by means of the kinetographic two-station method.

2. By the space and time measurement of the motion of the three airplane axes. This can be made from the kinetographic two-station method, but can be obtained more accurately by kinetographic photography of the horizon from the airplane.

3. By recording the dynamic pressure and angle of attack at one point (fuselage) or at two points (wing tips). This record indicates the air movements (gusts or up-wind) which might affect the picture of the airplane.

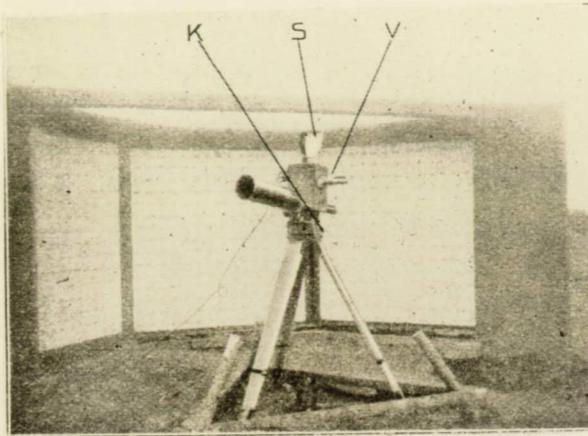
4. By recording the rudder and elevator deflections. This is necessary only on piloted airplanes. On models these deflections would be made in the desired direction by means of clockwork.

An essential condition is the time identification of all the measurements on the airplane and on the ground. This can be accomplished by simultaneous photography and registration of time marks. The take-off and landing supply two time marks which assist in the time identification.

I will add just a few remarks regarding the kinetographic photography. It can be exceedingly complicated and difficult

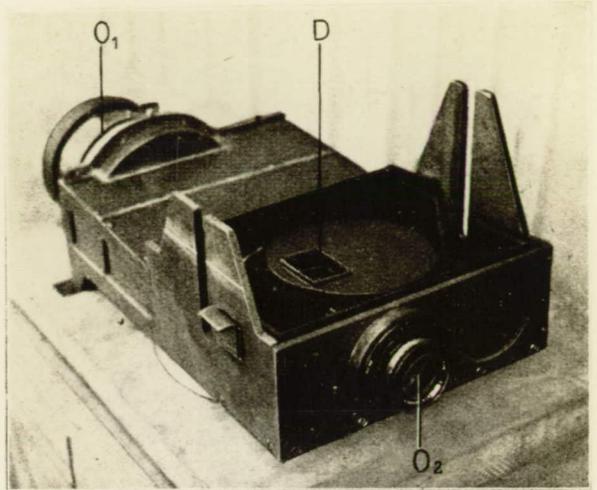
if undertaken in an unpractical manner. It is important to employ graphical methods and above all to adapt the evaluation to the theoretical problems under investigation. It will seldom be necessary to evaluate every individual picture, as a rough evaluation of the whole flight will suffice. It is necessary to evaluate exactly only the portions of the flight which are essential for the actual problems of accelerated flight conditions. On these portions, however, neither care, time nor labor must be spared, in order to obtain accurate results. Here applies the fundamental principle of all experimental sciences, that one accurate experimenter and one carefully evaluated experiment is worth more than a hundred inaccurate ones.

Translation by Dwight M. Miner,
National Advisory Committee
for Aeronautics.



K, Directing crank; S, Mirror; V, Telescopic finder.

Fig.1 Kinetograph and reference tables.



O₁ O₂, Lenses; D, Shutter.

Fig.4 Camera.

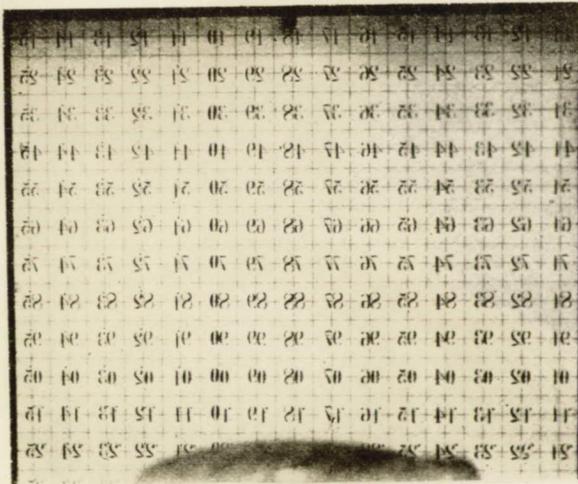
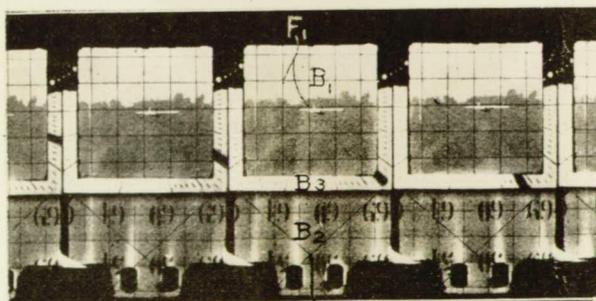
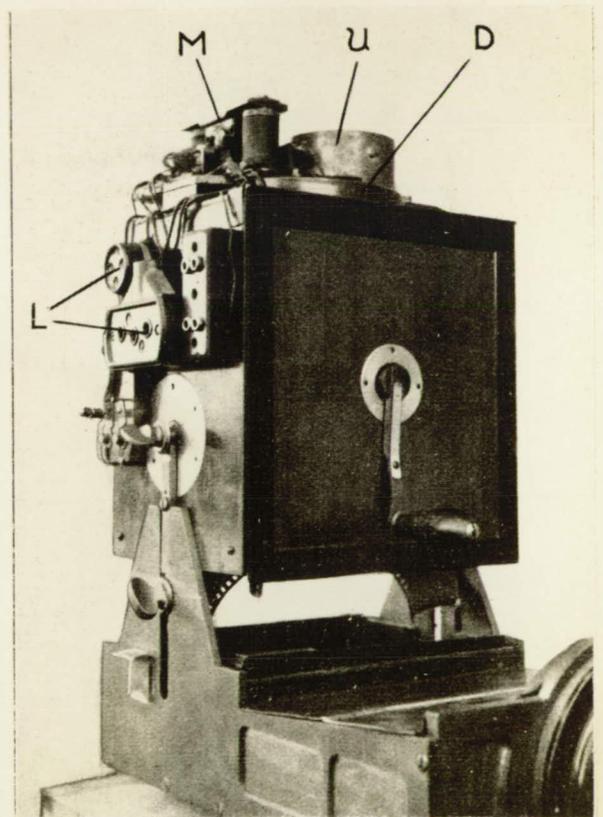


Fig.2 Reference table.



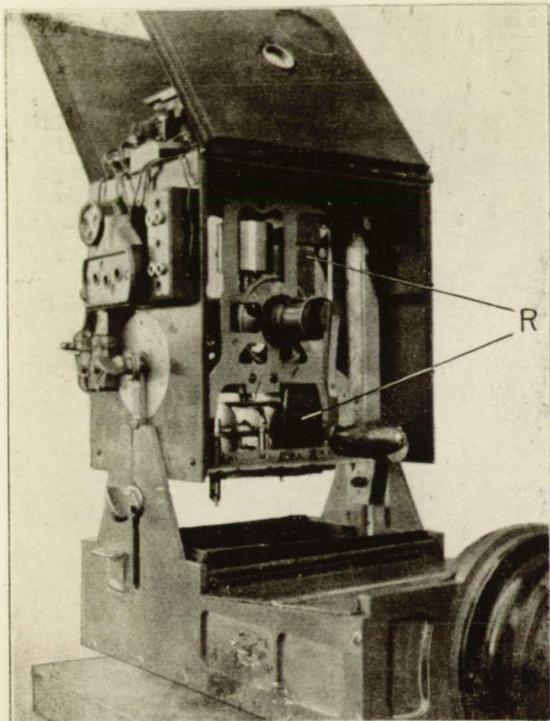
B₁ Airplane field; B₂ Reference-table field; B₃ Clock field. F₁ F₂ Reticles.

Fig.3 Section of film.



L, Electric connections; M, Synchronizing motor; U, Synchronizing clock; D, Shutter for photographing clock.

Fig.5 Film-driving mechanism.



R, Tube holding lens O₃.
Fig.6 Film-driving mechanism, open.

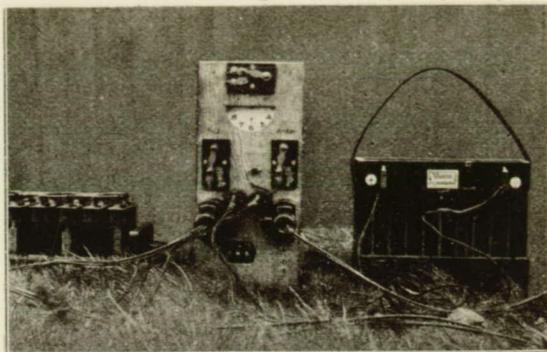
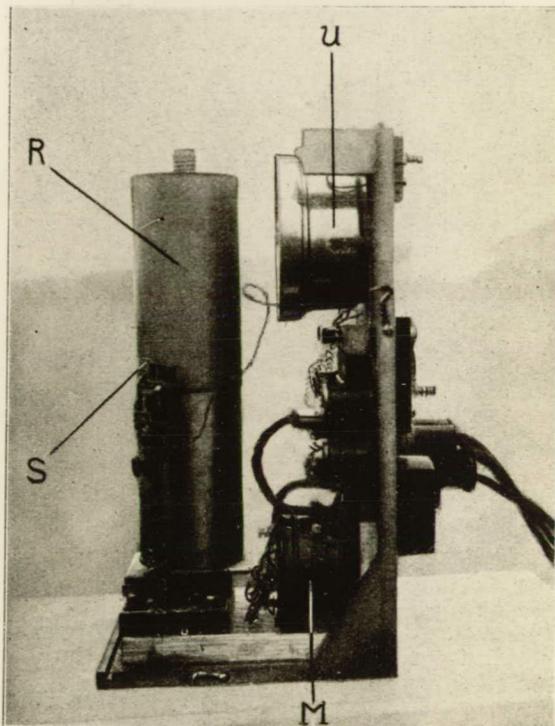
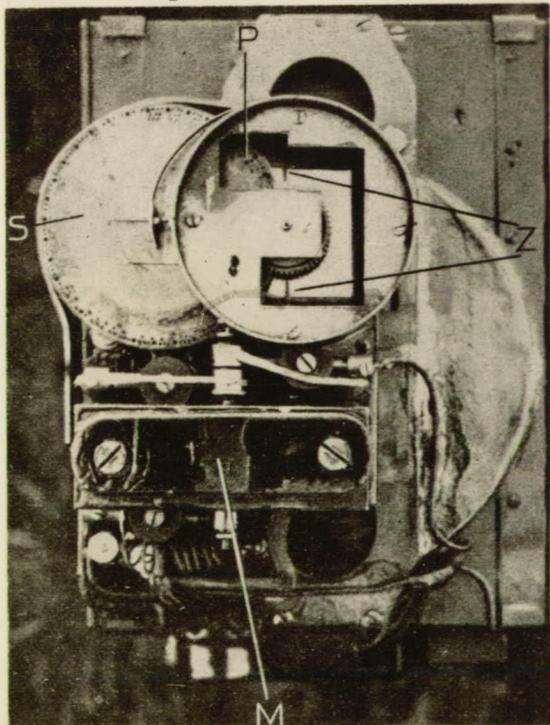


Fig.11 Central clock.



M, Motor; S, Stylus; R, Drum; U, Clock.
Fig.12 Inside of central clock.



M, Synchronizing motor; Z, double pointer; S, Seconds disk; P, Point where seconds disk is photographed.
Fig.10 Synchronizing clock.

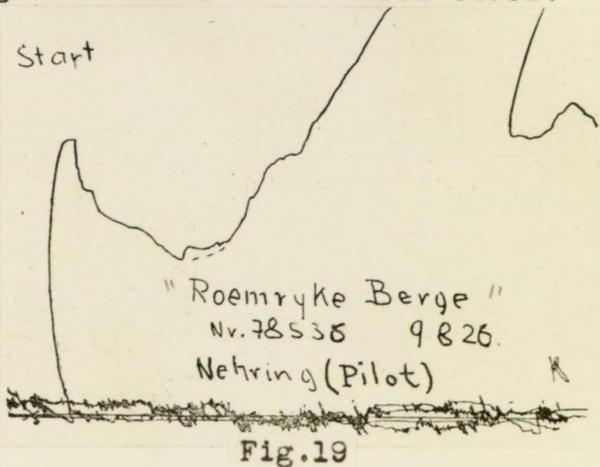
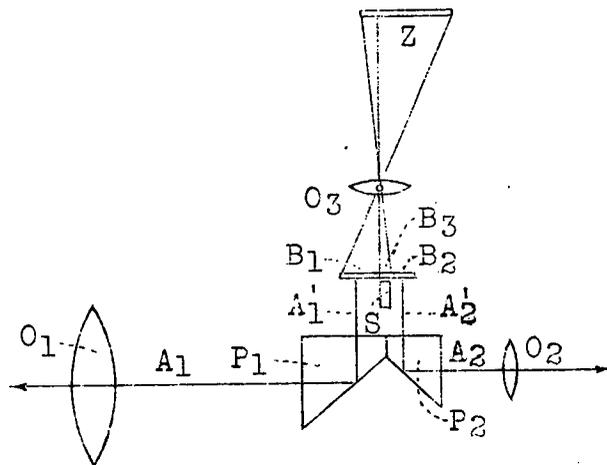


Fig.19



O_1, O_2, O_3 , Lenses

A_1, A_2 , Optical axes

P_1, P_2 , Reflecting prisms

A_1', A_2' , Reflected optical axes

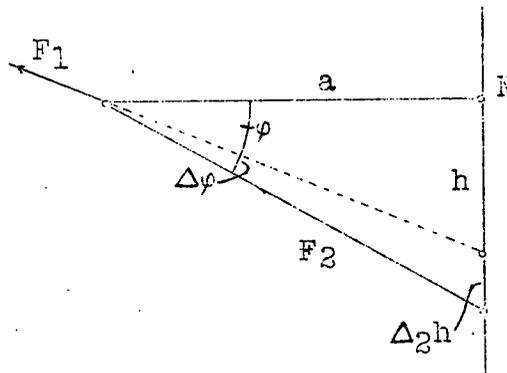
B_1 , Airplane field

S , Partition

B_2 , Table field

Z , Clock dial

Fig.7 Courses of rays in kinetograph.



F_1 , Axis of reticle in air plane field

F_2 , Axis of reticle in reference table

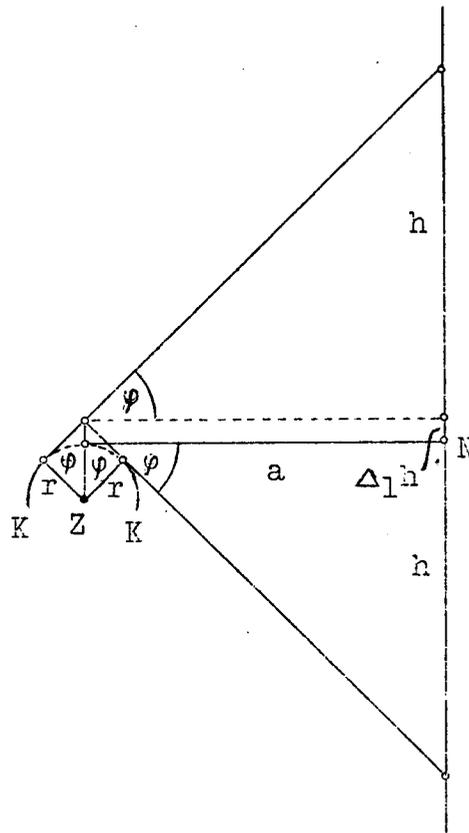
φ , Angle of reticle axis to horizon

$\Delta\varphi$, Angle between reticle axes F_1 , and F_2

a , Distance of kinetograph pivot from table

h , Coordinate altitude on table, calculated from zero point

Fig.9 Diagram of reticle axes.



K, Oscillation circle

r, Radius of oscillation circle

Z, Center of oscillation

ϕ , Oscillation angle of optical axis with reference to horizon

h, Coordinate altitude on reference table, calculated from zero point N

a, Distance between oscillation center Z and reference table

Fig.8 Vertical-turning diagram of kinetograph.

M, Central motor
 M₁, M₂, Motors
 R, Reference table

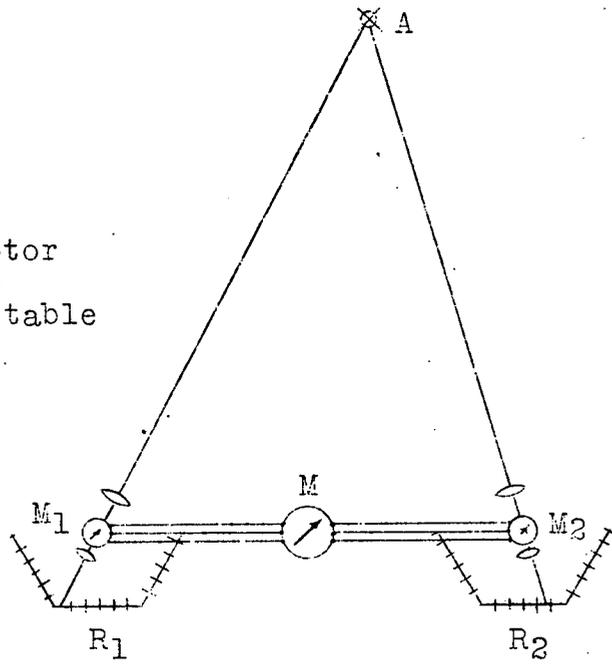
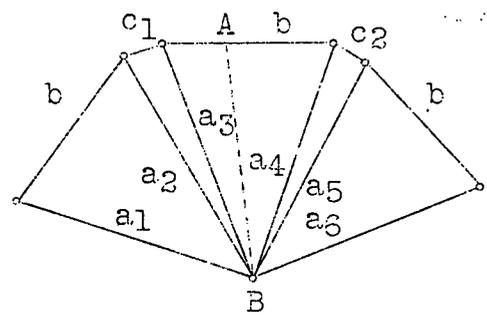
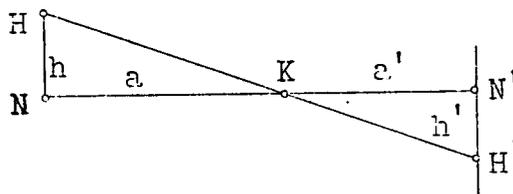


Fig.13



a, Distances of first and last wires of each table from kinetograph
 b, Distance between each pair of wires
 c, Distance between outer wires of adjacent tables
 A, Ground-plan coordinates of central landmark
 B, Pivot of kinetograph

Fig.14 Plan of a measuring base.



- H, Direction of central landmark or flight-path point
- H', Corresponding altitude coordinates
- h, Altitude of point H above measuring base
- h', Coordinate altitude above zero line N'
- N', Coordinate altitudes of all the points N in the altitude of the measuring base
- a, Distance of point H from kinetograph K
- a', Distance of point H' from pivot of kinetograph

Fig. 15 Diagram of altitude determination.

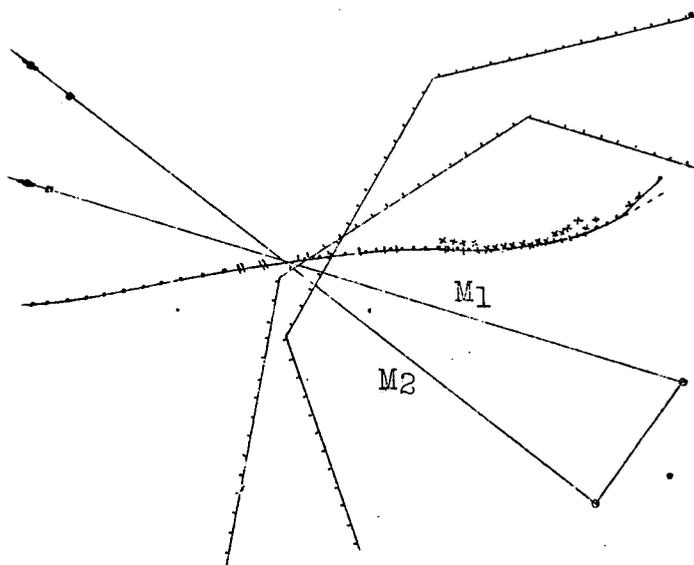


Fig. 16

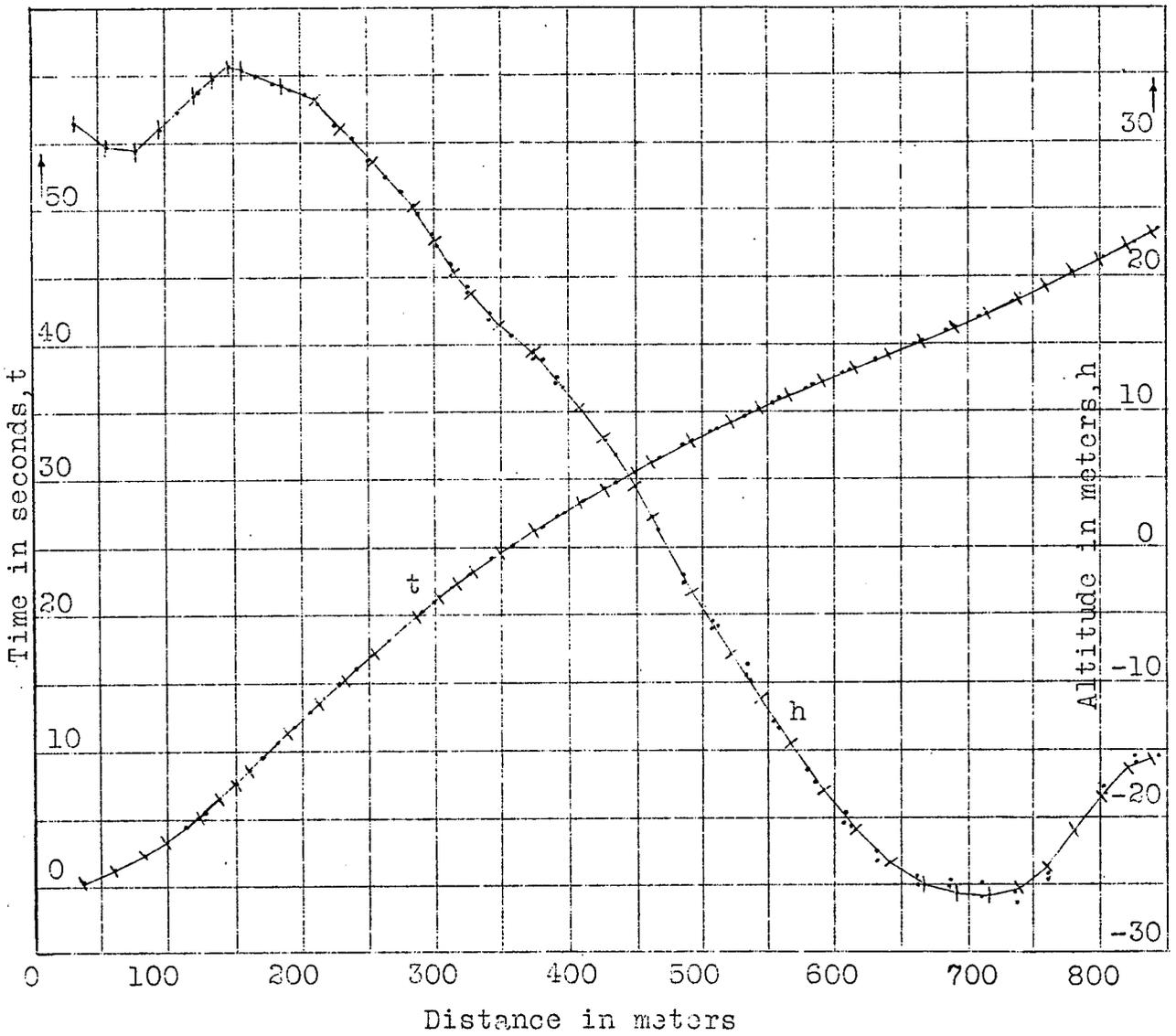


Fig.17

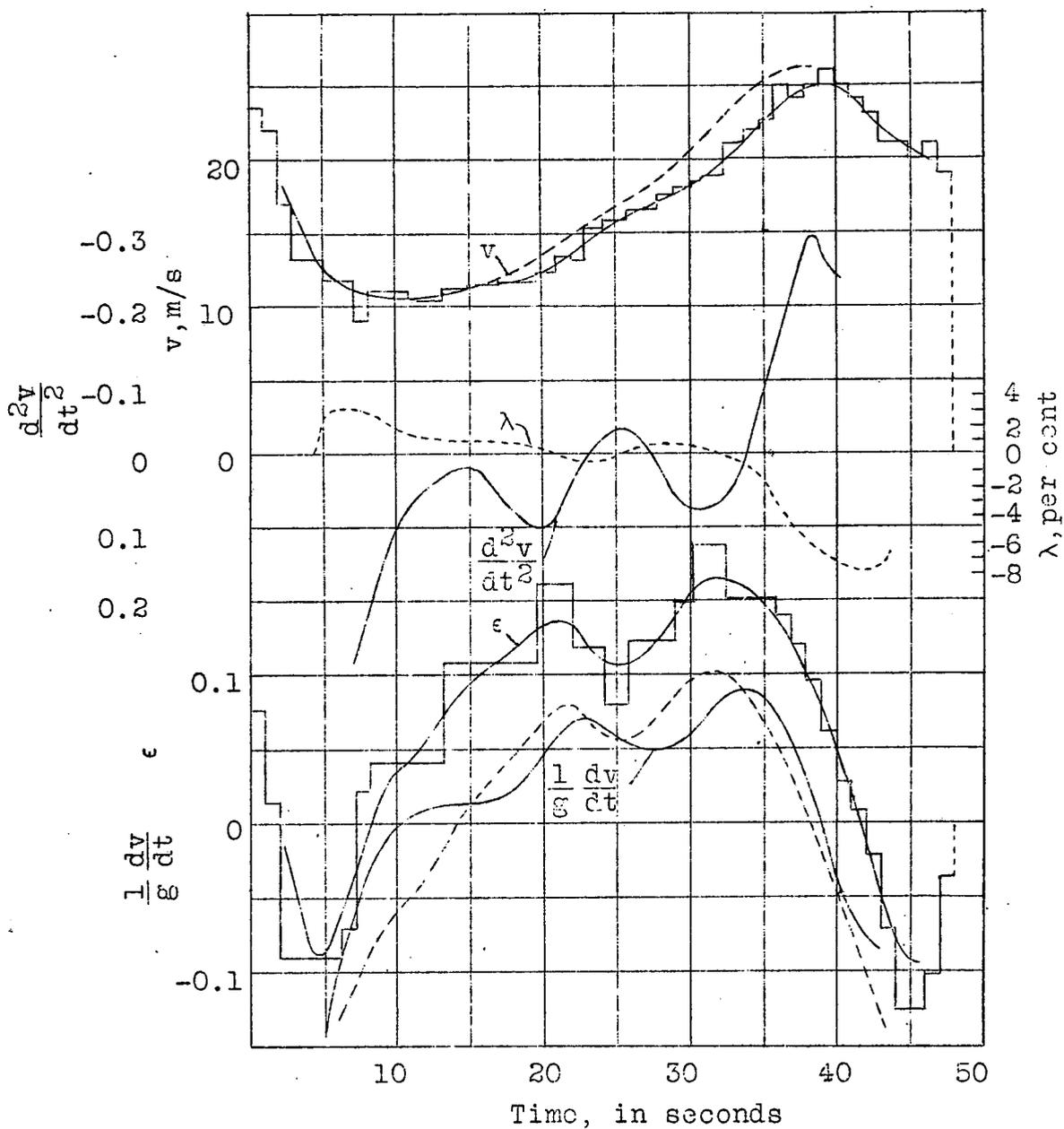


Fig. 18

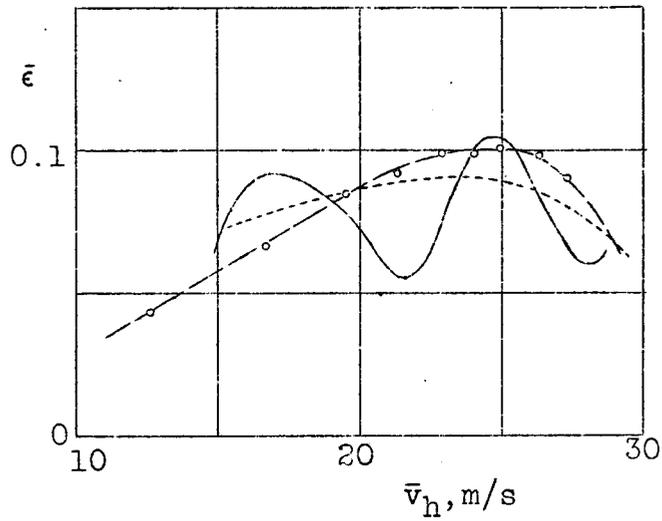
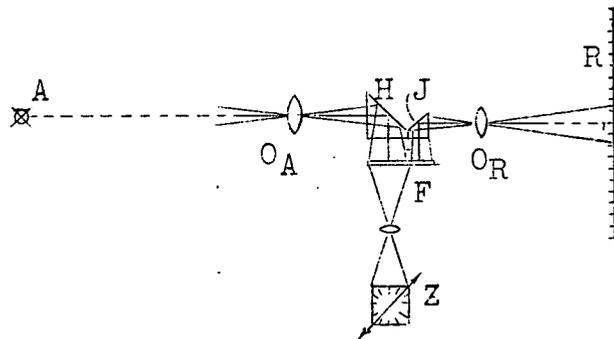


Fig.20



O_R, O_A, Object lenses
 H, J, Prisms
 F, Film
 R, Reference table
 Z, Clock pointer

Fig.21