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CONSIDERATIONS ON PROPELLER EFFICIENCY
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Efficiency has been adopted as a simple and practical criterion of the excellence of machines for transforming energy. It occupies a commanding position in our technical thinking on account of its simplicity and clearness. Yet some cases, which at first seem to be particularly well suited for the application of the efficiency idea, are found on closer examination to present serious obstacles to a suitable definition of efficiency. This occurs when the propeller cannot be considered alone, but when the mutual interference between propeller and airplane must also be taken into consideration. These difficulties are so great when the joint action of propeller and airplane is considered, that the aerodynamic laboratory at Göttingen originally abandoned the idea of applying the efficiency conception to the presentation of test results. These difficulties and the methods by which they can be overcome are outlined below. Moreover, this report is intended to call forth suggestions from other sources regarding a suitable definition of efficiency in these difficult cases.

The efficiency of a machine is defined as the ratio of the useful energy to the total energy put into the machine. On ac--

count of the losses of energy in the machine the useful energy is always smaller than the energy put in and accordingly the efficiency is always less than unity. In most cases as, for example, in a winch, the difference between the useful and the consumed energy is quite obvious. These considerations are also easily understood in the case of a propeller running alone. The energy transmitted from the engine through the shaft to the propeller is consumed, $M$ being the torque of the propeller shaft and $\omega$ the angular velocity of the propeller, the energy consumed per second is

$$L_1 = M \omega$$

It is the task of the propeller to move something at a speed $v$, a force $P$ being thereby exerted in the direction of motion. The energy

$$L_2 = P v$$

is required for this purpose. Hence when a force $P$ (the thrust) is exerted by the propeller moving at a speed $v$ in the direction of this force, it develops the energy $P v$ which is obviously the useful energy, since it is the energy required of the propeller. Accordingly, the efficiency is

$$\eta = \frac{P}{M} \frac{v}{\omega} = \frac{TV}{QW}$$

(1)

One could imagine an arrangement to consist of a propeller towing a vehicle at the end of a very long rope. The distance
between the propeller and the vehicle may be assumed to be so

great that the propeller is not affected by the disturbances

engendered by the vehicle and vice versa that the flow about the

vehicle is not altered by the propeller. The propeller thrust

\( P \) must be equal and opposite to the vehicle resistance or drag

\( W \). The former pertains to the propeller alone, while the latter

is a characteristic of the vehicle. However, when the propeller

is very near the vehicle, as is always the case in practice, it

is disturbed by the vehicle and the vehicle is disturbed by the

propeller. The propeller is generally situated in a zone of

reduced speed, either when occupying a position at the nose of a

fuselage, where the flow is slightly compressed, or in the wake

of a ship. Sometimes, however, it may happen to work in a zone

of increased velocity, that is, beside an airship. Since \( v \) is

the speed of the vehicle at the point where the propeller is lo-

cated and \( v' \) is the velocity of the flow relative to the vehi-

cle or to the propeller, the latter works under the same condi-

tions as if it were moving freely at the velocity \( v' \).* If,

under these conditions, \( P \) is the propeller thrust and \( \eta \) the

efficiency (determined in free motion), the propeller, according

to equation (1), absorbs the energy

\[
Mw = \frac{P v'}{\eta}
\]

*In order to simplify matters, the velocity \( v' \) is assumed to be

constant throughout the entire zone swept by the propeller. In

fact, \( v' \) differs from point to point, thus rendering the con-

ditions more confused. The above theoretical considerations, how-

ever, can be satisfactorily based on a constant average speed.
Assuming the useful energy to be the product of the vehicle drag
(= propeller thrust $P$) by the speed $v$, the resulting efficiency
is

$$\eta' = \frac{P}{M \omega} = \eta \frac{V}{V'}.$$ 

This determination of the useful energy leads to an apparent in-
crease in the propeller efficiency, when the propeller works in
a zone of reduced speed and vice versa. That there is something
wrong about this follows from the fact that the efficiency $\eta'$,
thus determined, may increase indefinitely and hence even exceed
unity, provided the arrangement is such as to maintain the veloc-
ity $v'$, in the propeller zone, at a sufficiently small value
as compared with the undisturbed velocity $v$. This contradiction
is explained by the fact that in all these cases the propeller
simultaneously affects the drag of the vehicle. The origin of
this latter influence is connected to a certain degree with the
disturbance exerted by the vehicle on the propeller. The drag
of the vehicle increases in general when the propeller works in
a zone of reduced speed and vice versa. The apparent gain re-
sulting from the location of the propeller in the zone of lower
speed on account of lower engine power required for the produc-
tion of a given thrust is partly absorbed by the increased vehi-
cle drag, which requires increased thrust.

A very extreme case may explain these conditions. Figure 1
represents a vehicle with a propeller arrangement in which the
space behind the propeller is completely enclosed. Of course
this arrangement has no practical value. It was adopted only to afford the clearest possible illustration of mutual interference. Under these conditions the propeller acts like a compression pump. It produces in the enclosed space a positive pressure \( p \). If \( F = \frac{D^2 \pi}{4} \) is the propeller disk area, then the force acting on the propeller in the axial direction, that is, the thrust, is

\[ P = p F \]

according to the previous definition. However, the same force is exerted on the vehicle in the opposite direction by the pressure \( p \), so that the propeller thrust is exactly counterbalanced. The propeller acts like a piston which compresses the enclosed fluid. The forces thereby engendered are entirely reciprocal, like the internal stresses in a body, and have no external effect. On the other hand, no power is theoretically required in this case for driving the propeller, disregarding losses, since no fluid flows through it. If the force \( P \), acting between the propeller and the vehicle, is multiplied by the speed \( v \), and if this product is regarded as the useful energy, the magnitude of this apparent useful energy is found to exceed materially the energy actually developed.

Hence the force acting between the propeller and the vehicle is not a suitable criterion for the rational evaluation of a propeller arrangement. We shall therefore look for a better way to express the useful energy. In order to distinguish the force
acting between the propeller and the vehicle from propeller thrusts otherwise defined, we shall call it the "thrust-bearing thrust" or the "dynamometer-hub thrust," since it is transmitted to the vehicle by the thrust bearing and may be measured by a dynamometer hub (on an airplane). In general, when a vehicle moves at a speed $v$, and thereby overcomes a resistance $W$, the work done (or the energy consumed) per second is $Wv$. Under these conditions the mutual interference between vehicle and propeller becomes negligible. The energy required to move the vehicle forward without propeller, e.g., to tow it at the end of a long rope, may be considered the useful energy. Hence the resistance or drag of the vehicle, when not affected by the propeller, is taken as the propeller thrust. The propeller is then credited or debited with the drag variations which it actually engenders. The efficiency thus defined

$$\eta = \frac{Wv}{M\omega} = \frac{D\omega}{Q\omega}$$

is called "propulsive efficiency" in shipbuilding parlance. It affords in general quite an accurate idea of the economical value of any given propeller arrangement. Thus, for example, the fact that a propeller arrangement is unsatisfactory is expressed by a proportionally reduced propulsive efficiency. However, one can also imagine cases in which the propeller arrangement reduces the vehicle drag without simultaneously exerting an unfavorable influence on the propeller, or cases in which the working condi-
tions of the propeller are improved by the presence of the vehicle, without a corresponding increase in the vehicle drag. In such cases even this definition of efficiency does not afford a comprehensive view of the conditions, and the efficiency may exceed unity in some cases. Such a case is illustrated by the following example.

In order to obtain a better quantitative view of the conditions, we shall replace the vehicle by a screen which absorbs part of the energy of the fluid flowing through it and consequently reduces the velocity of the fluid. The screen is first assumed to be at rest with the fluid flowing against it at a velocity $v_1$. At a certain distance behind the screen, where the pressures are again equalized, the velocity of the fluid is assumed to be $v_2$ (Fig. 2). If $M$ is the mass of the fluid passing through the screen per second, the resistance of the screen, according to the law of momentum, is

$$W = M (v_1 - v_2).$$

If the screen is assumed to be moving at a speed $v_1$ (contrary to the direction of flow hitherto considered) in the fluid at rest, a useful energy $Wv_1$ must be expended according to our last definition. We now arrange the propeller so as to include the whole fluid retarded by the screen. In order to produce a thrust equal to the screen resistance $W$, the propeller must, according to the law of momentum, accelerate the fluid again
from the velocity \( v_2 \) to the velocity \( v_1 \). Since, owing to a well-known consideration* of the propeller theory, the velocity at which the fluid passes through the propeller is

\[
v' = \frac{v_1 + v_2}{2}, \quad v_1 > v_2
\]

the energy \( L = Wv' \) \( < Wv_1 \) is required, provided no losses are incurred. As a matter of fact, the energy \( Mw \) actually expended by the engine must always be slightly greater on account of unavoidable losses. Moreover, the propeller can seldom include all the retarded fluid, as here assumed. It is therefore conceivable that the engine output \( Mw \) may be smaller than the useful energy defined above, an efficiency exceeding unity being thus obtained. In general, the influence of such favorable arrangements is not decisive. It merely lessens the injurious effects of the interference between the propeller and the vehicle. Hence, for most practical purposes, the propulsive efficiency is a useful criterion.**

Further difficulties are encountered on attempting to apply this efficiency idea to airplanes, the useful energy being defined as the product of the drag and speed of the undisturbed vehicle.


**The two extreme conditions represented theoretically in Figs. 1 and 2, are known in shipbuilding as the "displacement wake" (Fig. 1) and the "friction wake" (Fig. 2). An excellent description of these phenomena and of the resulting conclusions is contained in the article by Fresenius on "Das grundsätzliche Wesen der Wechselwirkung zwischen Schiffskörper und Propeller," in Schiffbau, 1921-22, p. 257.
We have thus far confined ourselves to considerations regarding the vehicle drag on the one hand, the propeller thrust on the other, and their mutual interference. However, as regards airplanes, an additional important force, the lift, must be taken into account. The lift is perpendicular to the direction of motion and does not directly affect the energy comparisons. In general, the lift is also affected by the propeller, and therefore the question arises as to whether the propeller efficiency is affected by the lift variations. One may feel inclined to neglect the effect of the propeller on the lift entirely, since, as mentioned above, the lift does not enter into direct consideration in energy calculations, the airplane resistance being the only factor which affects the requisite energy. However, it is quite obvious that this point of view does not afford means of fairly estimating the economic aspects of propeller arrangements. In fact, of two arrangements identical as to thrust, drag and requisite power, the one increasing and the other decreasing the airplane lift, the former is certainly to be preferred. Although the lift does not directly affect/calculation, it can be produced only by the expenditure of energy. Any increase in lift normally increases the drag (e.g., the induced drag). Hence, energy calculations are still indirectly affected by the lift.

Nevertheless, conditions would always be comparatively simple, if there were a definite general relation between lift and drag. Each increase in lift produced by the propeller could then
be converted into a corresponding decrease of drag or increase in thrust, and these converted values could be introduced into the energy calculations. Unfortunately, there exists no such general well-defined relation between lift and drag. However, such relations can always be established more or less arbitrarily and applied to approximately correct estimations of lift variations caused by the propeller. In this connection a few suggestions are made below. In order to facilitate their comprehension, a few test results, which have been recently published by Mr. Seiferth, are given below.*

These tests refer to the interrelation of a propeller and a wing without fuselage and tail surfaces. The test results are shown in Figures 3 and 4, where they are arranged in a form suitable for our consideration. The diagrams represent the relation between the following quantities: angle of attack $\alpha$, lift $A$, drag (minus thrust) $W$, and engine power $L$ and the corresponding nondimensional coefficients

$$c_a = \frac{A}{qF}, \quad c_w = \frac{W}{qF}, \quad \text{and} \quad c_1 = \frac{L}{qFv} = \frac{P}{qFv}$$

where $F$ is the wing area, $q = \frac{\rho v^2}{2}$ the dynamic or impact pressure and $v$ the flying speed. $c_a$ and $c_w$ are currently used symbols, whereas the power coefficient $c_1$ introduced above is not generally used. The propeller forces were usually divided by the

propeller disk area. In the present case, however, the wing area was resorted to for the purpose of establishing a simple relation between the power and the drag coefficients. As a matter of fact, the power coefficient used in this case is just equal to the reduction of the drag coefficient which would be obtained if the power were used exclusively for towing the wing, no losses and accessory phenomena resulting from mutual interference being taken into consideration. The representation in the diagrams is arranged in such a manner that the polar of the wing without propeller (always the farthest one to the right)* is plotted first, and then other polars for predetermined $c_1$ values (engine power). Lines of constant angle of attack $\alpha$ are also plotted.

Figure 3 shows the results with a propeller located behind the wing above the trailing edge, and Figure 4, the result for the corresponding arrangement of the propeller below the trailing edge (See the silhouettes in the diagrams). The lines of constant angle of attack indicate that, in the first case, the thrust of the running propeller not only reduces the drag, but that, for a constant angle of attack, it also considerably increases the lift, whereas in the second case the lift is only

*During the tests with the propeller running, disturbances are also caused by the engine. Consequently, in order to show only the propeller action in the diagrams, the adopted initial curve is in each case the polar of the wing with engine installed. The two polars without propeller are slightly different, since the upper surface is not affected by the engine in quite the same way as the lower surface. Instead, considerations might have been based on the wing alone, the joint effect of engine and propeller being considered as the propeller effect.
slightly increased and sometimes even reduced.

We shall now consider the problem of moving a wing of given weight \( G \) forward at a speed which will develop just enough lift to support it. If the drag of the fuselage and other airplane components is added to the wing or if the airplane is intended to rise or to fall, the corresponding allowances can be easily made in the usual manner. We shall disregard such minor details in our theoretical considerations, since they do not affect the substance of these considerations. We shall first imagine the wing alone pulled through air unaffected by the disturbing influence of the propeller. For a fixed wing area \( F \), and air density \( \rho \), the requisite \( c_a \), namely,

\[
c_a = \frac{G}{\frac{\rho}{2} F v^2}, \quad C_l = \frac{c_a v_0}{\rho F}
\]

is deduced from the weight \( G \), to be carried and the requisite speed \( v \), whence the requisite angle of attack \( \alpha \), for example, point \( P \) of the polars for \( c_a = 0.87 \) and \( \alpha = 6^\circ \) (Figs. 5 and 6), is derived from the polars (Figs. 3 and 4). Motion is imparted by the force

\[
W = c_w \frac{\rho}{2} F v^2, \quad \beta = c_w \frac{\rho}{2} v^2
\]

c\( w \) being represented by the distance \( P_3 P_1 \) in Figures 5 and 6. In the case of Figure 3, \( c_w = 0.085 \), and in that of Figure 4, \( c_w = 0.077 \).
We shall now impart forward motion by means of a propeller, located in one case above the wing, according to Figures 3 and 5 (arrangement I); and in the other case below the wing, according to Figures 4 and 6 (arrangement II). If the angle of attack \( \alpha \), remains constant, an unexpected increase in lift is obtained, at least for the first arrangement (Figs. 3 and 5), in addition to the thrust required for overcoming the drag. Thus point \( P_2 \) is reached. The increased lift coefficient (\( c_a = 0.95 \) in Fig. 5) produces a lower flying speed. One might simply neglect this undesired change, but the estimate of such a lift-increasing propeller would then be unwarrantably poor. Thus the requisite engine power for arrangement I would be

\[
L_1 = c_1 \frac{D}{2} F v^3 = 0.145 \frac{D}{2} F v^3.
\]

For arrangement II, it would be

\[
L_2 = c_1 \frac{D}{2} F v^3 = 0.111 \frac{D}{2} F v^3.
\]

Consequently the efficiency,

\[
\eta = \frac{C_W}{C_1},
\]

in one case would be

\[
\eta_1 = \frac{0.083}{0.145} = 0.57,
\]

and in the other case it would be

\[
\eta_2 = \frac{0.077}{0.111} = 0.69.
\]

As a matter of fact the requisite motion can also be imparted in
case I with much less power by reducing the angle of attack so as to eliminate the useless surplus lift and thus work at the point $P_2$. In other words, we do not maintain the angle of attack $\alpha$, but the lift coefficient $c_a$. The power coefficients are then

$$c_{l1} = 0.128$$

for the first arrangement and

$$c_{l2} = 0.115$$

for the second arrangement. The resulting efficiencies are accordingly

$$\eta_1 = \frac{0.083}{0.128} = 0.65$$

and

$$\eta_2 = \frac{0.077}{0.115} = 0.66.$$"
connected with a disproportionate decrease in drag and a correspondingly lower propeller efficiency is required. Hence, the introduction of the distance $P_4 P_e$, as the useful thrust for the calculation of the useful work, is obviously excessive, since the true drag is considerably reduced by the decrease in the angle of attack. The drag coefficient and the engine-power coefficient actually obtained in our example are $c_w = 0.24$ and $c_l = 0.29$, respectively, the apparent efficiency being consequently

$$\eta = \frac{c_w}{c_l} = \frac{0.24}{0.29} = 0.83.$$  

This value is obviously much too high and leads to a wrong estimation of the propeller. For higher values of the angle of attack, cases may even be imagined in which the efficiency, thus determined, exceeds unity. In addition to this difficulty, it should also be noted that no real efficiency can be obtained under working conditions beyond $P_e$, since these $c_a$ values are never reached by a wing without the propeller, a comparison for a constant $c_a$ thus becoming impossible. On the other hand these working conditions are of great practical importance. Thus for the take-off, an increase in the maximum $c_a$ is a very valuable contribution of the propeller and it would be highly desirable to find a reasonable definition of the propeller efficiency for these working conditions.

The whole question is to find a suitable means of determin-
ing the lift variation caused by the propeller, i.e., a method for converting it into a drag variation. In the last method described, this conversion was effected by compensating the lift variation by a change in the angle of attack, the corresponding drag variation being introduced as a compensation of the lift variation. Of course any number of more or less practical methods for the conversion of the lift variation into a drag variation may be suggested, but they are all arbitrary to a certain degree, and there is danger of using too many methods, thus making comparisons impossible. One of these methods (perhaps the most satisfactory) is given below as an example.

The difficulties encountered in the last-named method are chiefly due to the disproportionately great increase in drag in the vicinity of \( c_n \text{ max} \). These difficulties might be avoided by eliminating the profile drag in conversion calculations and using the variation in the induced drag only. It can then be claimed that an increase in lift from \( c_{a1} \) to \( c_{a2} \) is normally accompanied by an increase of

\[
\Delta c_{wi} = \frac{c_{a2}^2 - c_{a1}^2}{\pi} \frac{F}{b^2}
\]

in the induced drag, \( F \) being the wing area and \( b \) the span. Thus, when the propeller produces an increase in lift from \( c_{a1} \) to \( c_{a2} \), it can be credited with an increase in wing drag of \( \Delta c_{wi} \), and hence the propeller thrust, increased by \( \Delta c_{wi} \), can be introduced in the calculation of the useful energy. In
Figure 8 our wing diagram is again represented with the propeller influence, as in Figure 3. We have also plotted the polar of the induced drag

\[ c_{wi} = \frac{c_a^2}{\pi} \frac{F}{b^2}. \]

(During the tests the value of \( \frac{F}{b^2} \) was 1/4.5.) Let us start anew from the point \( P_1 \) and tow the wing with propeller at a constant angle of attack (point \( P_2 \)). The value adopted for the thrust would be \( c_w = P_2 P_1 \) if the lift were not estimated. However, on estimating the lift increment in the manner last described, we shall adopt the value \( c_w + \Delta c_{wi} \) for the thrust.

It follows from the present example that \( c_w = 0.083, c_{al} = 0.87 \) and \( c_{al2} = 0.95 \), whence

\[ \Delta c_{wi} = \frac{0.95^2 - 0.87^2}{4.5} = 0.010, \]

and \( c_w + \Delta c_{wi} = 0.093 \). On the other hand, since the requisite power for point \( P_2 \) is given by the power coefficient \( c_l = 0.145, \)

\[ \eta = \frac{0.093}{0.145} = 0.64, \]

or approximately the same as the result obtained by the last method. On starting from point \( P_4 \) (Fig. 9, \( \alpha = 18^\circ, c_a = 1.42 \)), where the difficulties of the last method were actually experienced, the working point \( P_5 \) with \( c_a = 1.79 \) is obtained. At the point \( P_4, c_w = 0.24, \Delta c_{wi} \) becomes 0.08, and the power coefficient at the point \( P_5 \) is \( c_l = 0.46 \). Hence the resulting
efficiency is
\[ \eta = \frac{C_w + \Delta C_{wl}}{c_l} = \frac{0.32}{0.43} = 0.70. \]

This method meets most of the practical requirements. However, there may be cases in which even this method would fail. This happens always when the airplane drag is considerably increased or especially reduced under the influence of the propeller. This quite agrees with the previously discussed cases of vehicles without lift, except that the phenomenon is more frequent and more pronounced for wings, especially near the point of maximum lift.

As already mentioned, the conversion of the lift increment into a drag reduction is rather arbitrary. Under certain conditions the lift increment may be much more important than can be expressed by such a computation. The increase in the maximum lift is very important during a take-off. If such conditions are to be taken into consideration, the drag reduction and the lift increment must be determined separately. A simple means of achieving this result is shown in Figures 3 and 4. However, if it is desired to retain the efficiency idea, which offers such great advantages for judging the value of any arrangement, it is still possible to do so with a certain extension of this idea. Of course the simplicity of the ordinary efficiency idea would thus be greatly impaired, since instead of being represented by a single expression, the economical value of any arrangement would
have to be represented by two expressions, one for the drag reduction and one for the lift increment. The scalar would thus be replaced by a vector. This is illustrated by the following example.

Point $P_1$ (Fig. 5) is again chosen as the starting point, and point $P_2$ is reached by the action of the propeller. The magnitude and direction of the force exerted by the propeller is represented by the distance $P_1P_2$. This force is resolved into the drag reduction $P_1P_3 = \Delta C_W$ and the lift increment $P_3P_2 = \Delta C_a$. The power required for this purpose is given by the power coefficient $c_l = 0.145$. If the lift did not increase and if the propeller did not engender losses (with an efficiency 1) this power would result in a reduction of the drag $P_1P_1' = c_l$ (Fig. 10). By comparing the actual drag reduction and the lift increment with this theoretical drag reduction, we obtain a generalized efficiency

$$\eta = \left( \frac{P_1P_3}{c_l}, \frac{P_3P_2}{c_l} \right) = \left( \frac{\Delta C_W}{c_l}, \frac{\Delta C_a}{c_l} \right).$$

This generalized efficiency can be represented graphically by conformal increase or reduction of the figure determined by the points $P_1$, $P_2$, $P_3$, $P'$ until $P_1P_1'$ becomes equal to unity (Fig. 11). The vectorial efficiency is then represented by $P_1P_2$. If this scheme is carried through for each point of Fig. 3, and if the points pertaining to the same angles of attack on the one hand and those pertaining to equal $c_l$ values on the other hand...
are connected, Figure 12 is obtained. This diagram represents the efficiencies thus determined for all working conditions. As in the polar diagrams, in which $c_w$ is plotted at a scale five times that of the $c_a$, the scale of $\frac{\Delta c_w}{c_1}$ in this case is also five times that of $\frac{\Delta c_a}{c_1}$. Except for the largest angles of attack, these efficiencies are nearly alike. On following the curves for a constant angle of attack, the efficiency is found to reach a maximum value between $c_1 = 0.15$ (for large angles of attack) and $c_1 = 0.2$ (for small angles of attack). This is probably attributable to the propeller, since, on changing the angle of attack, the maximum value is only very slightly changed.

The adoption of such a generalized efficiency conception is not very probable, since its most valuable characteristic, that of evident simplicity, is thus somewhat impaired.

Summary

Mention is made of the difficulties encountered in defining propeller efficiency when the propeller is affected by mutual interference with a vehicle and especially with a wing. Different ways of overcoming these difficulties to some degree, at least for practical requirements, are indicated, but none of these ways is found to be entirely satisfactory.

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Fig. 1  Propeller arrangement producing maximum mutual interference between propeller and vehicle.

Fig. 2  Propeller arrangement in which the energy expended on the vehicle (screen) is partially recovered by the propeller.

Fig. 3  Test results of wing with propeller above its trailing edge.
Fig. 4  Test results of wing with propeller below its trailing edge.

Fig. 5  Example for arrangement like Fig. 3

Fig. 6  Example for arrangement like Fig. 4.
Fig. 7 Example of propeller effect near point of maximum lift.

Fig. 8 Evaluation of lift increment according to variation of induced drag $\Delta c_{wi}$ for example in Fig. 5.

Fig. 10 Example of actual propeller effect with maximum theoretically possible thrust.
Fig. 9 Evaluation of lift increment according to variation of induced drag $\Delta c_{wi}$ for example in Fig. 7.

Fig. 11 Generalized efficiency for the working point $P_2$.

Fig. 12 Diagram of the generalized efficiency for the arrangement like Fig. 3.