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CALCULATION OF AIRPLANE PERFORMANCES WITHOUT THE AID OF POLAR DIAGRAMS

By Martin Schrenk

From the 1927 Yearbook of the
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CALCULATION OF AIRPLANE PERFORMANCES WITHOUT THE AID OF POLAR DIAGRAMS.*

By Martin Schrenk.

I. Introduction

In all known methods for the calculation of flight performances, use has hitherto always been made of the airplane polars from which the characteristic coefficients c_a^3/c_w^2, c_a/c_w and $c_{w\,min}$ are derived. These coefficients constitute the basis for all further calculations.

The wing drag, from which the coefficients are obtained by means of the polars, is resolved, however, into two components, which are of very different character and which are very differently affected by structural changes in the airplane. These two components are the induced drag and the profile drag. The latter is proportional to the wing area, the coefficient of the profile drag and the dynamic pressure. The former, on the contrary, is proportional to the square of the lift and inversely proportional to the dynamic pressure multiplied by the square of the span. These complex relations obscure the constructor's view of

*"Zur Berechnung der Flugleistungen ohne Zuhilfenahme der Polare," Seventy-Fourth Report of the D.V.L. ("Deutsche Versuchsanstalt für Luftfahrt," 1927 Yearbook of the D.V.L., pp. 104-112.

The Seventy-Ninth Report of the D.V.L., also by Martin Schrenk, and included in this Yearbook, pp. 145-151 (N.A.C.A. Technical Memorandum No. 457), constitutes the continuation and completion of this report.

the interdependence between the dimensions of his airplane and the effects of the air resistance.

For good profiles the profile-drag coefficient is almost constant in the whole range which comes into consideration for practical flight. This is manifest in the consideration of the Göttingen airfoil tests and is confirmed by the investigations of the writer (measurements of the profile drag during flight by the Betz method), concerning which a detailed report will soon be published. The following deductions proceed from this fact. The formulas developed on the assumption of a constant profile-drag coefficient afford an extensive insight into the influences exerted on the flight performances by the structure of the airplane.

II. Symbols.

1. Airplane without Power Plant.

G, full load, kg.

F, wing area, m².

b, span, m.

t, mean chord (F/b). On a biplane this is the sum of the mean upper and lower chords referred to the span of the longer wing $\frac{F_o + F_u}{b}$.

f_w, equivalent flat-plate areas (W/q), m²:
- f_{wp}, equivalent flat-plate area of wings, m²;
- f_{wr}, equivalent flat-plate area of non-lifting parts, m²;
- f_{ws}, total equivalent flat-plate area $(f_{wp} + f_{wr})$, m².

κ, reduction coefficient (W_D/W_E) for the induced drag of a biplane, as compared with a monoplane of like span (according to Prandtl*).

*"Ergebnisse der Aerodynamischen Versuchsanstalt zu Göttingen," Report II, p.9 ff.

c_a, lift coefficient

c_w, drag coefficient (c_{wp}, c_{wr}, c_{ws}, c_{wi}).

2. Power Plant.

N, engine power, HP.

η, propeller efficiency (referred to gliding flight polar with like c_a).

p, static pressure of atmosphere, kg/m².

T, absolute temperature in degrees C.

3. Performances

v, horizontal speed (also maximum horizontal speed) m/s.

w, climbing speed (also maximum climbing speed) m/s.

w_s, vertical speed of descent (also minimum vertical speed of descent) m/s.

w_h, vertical speed of ascent (also maximum vertical speed of ascent) m/s.

H, flight altitude, km.

ρ, air density (γ/g), kg s²/m⁴.

q, dynamic pressure ($\rho\ v^2/2$), kg/m².

The subscripts denote:

σ, sea level;

a, critical altitude (at which the engine power begins to fall off);

g, ceiling;

i, quantities associated with the induced drag (e.g., c_{wi}, b_i).

III. Derivation of the Formulas

1. Rate of Climb

The basis is the well-known expression: climbing speed = vertical speed of ascent - vertical speed of descent.

$$w = \frac{75\ N\ \eta}{G} - \frac{c_w}{c_a^{3/2}} \sqrt{\frac{G}{F}\ \frac{2}{\rho}} \qquad (1)$$

in which

$$c_w = c_{wr} + c_{wp} + c_{wi} = c_{ws} + c_{wi} = \frac{f_{ws}}{F} + \kappa\ \frac{c_a^2}{\pi}\ \frac{F}{b^2} \qquad (2)$$

c_{ws} is the coefficient of the frontal drag, i.e., of the drag or resistance of all the non-lifting parts combined with the profile drag, both assumed to be constant in the flight range chiefly under consideration. c_{wi} is the coefficient of the induced drag and depends on the square of the coefficient of lift.

From the conditions of vertical equilibrium there is further introduced

$$c_a = \frac{G}{F}\ \frac{2}{\rho v^2} \qquad (3)$$

and by introducing equations (2) and (3) into equation (1), we obtain

$$w = \frac{75\ N\eta}{G} - \frac{\rho\ v^3\ f_{ws}}{2\ G} - \frac{2\ \kappa\ G}{\pi\ \rho\ v\ b^2} \qquad (4)$$

This is the general equation for climbing speed, on which all three components of the speed along the flight path v depend (the first component on account of the interchangeableness of $N\eta$ with v).

In order to obtain the maximum value for w, the first component, the vertical speed of ascent must be made independent of the horizontal speed, since this dependence is only empirically known.

If the vertical speed of ascent now remains constant, the climbing speed will be the greatest for the minimum value of the vertical speed of descent (sum of components 2 and 3). This follows from a simple minimal calculation

$$\frac{d_{ws}}{d\,v} = \frac{3\,\rho\,v^2\,f_{ws}}{2\,G} - \frac{2\,\kappa\,G}{\pi\,\rho\,v^2\,b^2} = 0$$

$$v = \frac{(G/b)^{1/2}\,\kappa^{1/4}}{(\rho/2)^{1/2}\,(3\,\pi\,f_{ws})^{1/4}} \qquad (5)$$

This is the speed at which the minimum vertical speed of descent is attained.* If introduced into equation (4), it yields

$$w_{max} = \frac{75\,N\,\eta}{G} - 0.75\left(\frac{\rho}{2}\right)^{-1/2}\left(\frac{G}{b_i}\right)^{1/2}\frac{f_{ws}^{\,1/4}}{b_i} \qquad (6)$$

in which $b_i = b/\sqrt{\kappa}$, the induced span of the biplane, i.e., the span of a monoplane having the same induced drag.

The vertical speed of descent is expressed

$$w_{s\,min} = 1.06\,\rho^{-1/2}\left(\frac{G}{b_i}\right)^{1/2}\frac{f_{ws}^{\,1/4}}{b_i} \qquad (7)$$

The expression contains only constructive quantities, namely, as the most important, the span to the 3/2 power, also the weight to the 1/2 power and, lastly, the frontal-drag area to the 1/4

*At this speed it is found that the induced drag is three times the frontal drag, as has long been known.

power. The aspect ratio is therefore replaced by the span and the wing loading by a quantity which we can call "span loading." It is the load per meter of the induced span and is therefore a measure of the quantity of air acted on in the flight. Of course, the reciprocal square root of the air density is also included. Figure 1 shows this relation. Existing airplanes are mostly restricted to the space between the dotted lines.

If equation (7) is multiplied by G, it is changed into the expression for the power required to maintain horizontal flight, which will appear later in the ceiling formula.

It must now be established at what lift coefficient the minimum vertical speed of descent appears, since this value might lie so high that there would no longer be any possibility of the profile drag being constant. From equations (3) and (5) we obtain the minimum speed of vertical descent.

$$c_a = \frac{1}{t}\sqrt{\frac{3\,\pi\,f_{ws}}{\kappa}} \tag{8}$$

If the values for ordinary airplanes are here introduced, it is found that modern airplanes, with good aspect ratios and high wing loading, mostly have lift coefficients far above unity. In this case the values calculated with formula (6) would be a little too favorable under some circumstances. The following consideration then indicates the way for a more accurate solution.

Figure 2 shows the course of the profile drag in the vicinity of $c_a = 1$ for a good thick profile of medium camber according

to experiments by the writer. The polar curve bends shortly below $c_a = 1$. Below this point the profile drag decreases rapidly. Hence, for the actual polar, the point of most favorable speed of vertical descent, i.e., of minimum $c_w/c_a^{3/2}$, often lies considerably lower than indicated by equation (8). The most favorable $c_w/c_a^{3/2}$ then differs but little from the value when $c_a = 1$.* For this reason, flying at large angles of attack will in climbing not give a greater rate of climb. For such "normal" profiles the assumption is therefore made that the airplane, in climbing, should fly at $c_a = 1$. Hereby the profile-drag coefficient should remain constant. This means the replacement of the polar curve by the dotted line, a change which, due to the small influence of f_{ws}, does not affect the result. This assumption agrees with the actual procedure of most airplane pilots who, in order to retain a little reserve, do not willingly load the airplane, in climbing, beyond the "unit dynamic pressure" G/F.

Lastly, the assumption corresponds to the actual behavior of the airplane in so far as the propeller efficiency, at least of compromise propellers, increases with increase in speed, whereby the best value of η lies at a somewhat higher speed than that established by equation (5) for the minimum value of w_s. Hence, in this case also, the maximum value of w occurs at a somewhat greater speed, or smaller c_a, than that indicated by equations (5) and (8).

*These relations can be easily comprehended from Everling's nomogram ("Zeitschrift für Flugtechnik und Motorluftschiffahrt," 1922, No. 18, p.250.

For medium relations the line $c_w/c_a^{3/2}$ = constant, which passes through the point $c_a = 1$, just meets the actual polar curve. If f_{ws} is relatively large, then the $c_w/c_a^{3/2}$ found for $c_a = 1$ is somewhat too unfavorable, while in other cases it is a little too favorable. The changes are scarcely noticeable, however, and work moreover in a "corrective" sense in favor of the greatest possible reduction of the frontal drag. Then

$$c_w = c_{ws} + c_{wi} = \frac{f_{ws}}{F} + \frac{\kappa}{\pi}\frac{F}{b^2} \tag{2a}$$

and

$$w_s = \left(\frac{G}{F}\right)^{1/2}\left(\frac{\rho}{2}\right)^{-1/2}\left(\frac{f_{ws}}{F} + \frac{\kappa}{\pi}\frac{F}{b^2}\right) \tag{7a}$$

This expression contains the usual quantities G/F and F/b^2, wing loading and aspect ratio. It applies with absolute accuracy only for $c_a = 1$, but with fair approximation, however, for ordinary wing sections, as the best value on the whole. In the case of an unusual profile, other assumptions must naturally be made, which will not be further considered here, however, since they would only constitute modifications of the abovementioned assumptions. Any very great accuracy in this connection is of no importance anyway, since the induced drag in this region no longer accurately follows the quadratic law, while slight variations in the induced drag are important in comparison with similar variations in the profile drag.

2. Engine Power and Flight Altitude.

Before we can undertake the calculation of the ceiling with the help of the formulas obtained for the climbing speed, we must first establish a law, according to which the engine power will diminish with increase in altitude.

The usual assumption in Germany is that the indicated horsepower is proportional to the density of the air, while the friction horsepower remains independent of it. For a mechanical efficiency of 85%, this assumption leads to the expression

$$\frac{N}{N_0} = \frac{1}{0.85}\left(\frac{\rho}{\rho_0} - 0.15\right)^{*}$$

This expression is very inconvenient for the further calculation, on account of the algebraic sum contained in it. Moreover, it makes no allowance for the cold, as such, which will result in an impairment of the carburetion, as well as in an increase in the friction horsepower. Neither does it take into account the decrease in the revolution speed, which causes a further loss in power.

According to American experiments (Walter S. Diehl, "Engine Performance and the Determination of Absolute Ceiling" - N.A.C.A. Technical Report No. 171), the mean values of many measurements (up to $p/p_0 = 1/3$) can be assumed to be

*Brenner, "Die Steigleistungen der Flugzeuge," "Zeitschrift für Flugtechnik und Motorluftschiffahrt," 1924, No. 7, pp. 61-65.

$$\frac{N}{N_o} = \left(\frac{p}{p_o}\right)^{1.15} \text{ (T constant)} \tag{10}$$

$$\frac{N}{N_o} = \left(\frac{T}{T_o}\right)^{-6.5} \text{ (p constant)} \tag{11}$$

These laws will now be applied to the course of the air density and temperature in the standard atmosphere. At constant temperature we first have

$$\frac{N}{N_o} = \left(\frac{\rho}{\rho_o}\right)^{1.15} \tag{12}$$

The German standard atmosphere assumes a temperature reduction of 5°C (9°F) for every 1000 m (3281 ft.) increase in altitude. A reduction of 6°C (10.8°F) probably approximates the actual mean relations more closely,* whereby this represents a mean value up to an altitude of 10,000 m (32,808 ft.). Above this altitude the temperature behaves quite differently which, however, lies outside the scope of the present investigation. Therefore

$$T = T_o - 6\ H \tag{13}$$

or, with $T_o = 283°$ abs=10°C (50°F),

$$T/T_o = 1 - 0.0207\ H \tag{13a}$$

For the relation between ceiling and air density in 13a, the approximation formula is introduced which agrees very well with the mean experimental values for altitudes of 1,000 - 10,000 m (3281 - 32,808 ft.). It reads:

*Linke, "Pyknometrische Höhenformeln." A special publication of the "Institut für Meteorologie und Geophysik" at the University of Frankfort.

$$\frac{\rho}{\rho_o} = 0.896\ H \tag{14}$$

or

$$H = -\ 20.9\ \lg \frac{\rho}{\rho_o} \tag{14a}$$

From (13a) and (14a) we obtain

$$\frac{T}{T_o} = 1 + 0.433\ \lg \frac{\rho}{\rho_o} \tag{15}$$

For convenient calculation, this expression is converted into an exponential function of the form

$$\frac{T}{T_o} = \left(\frac{\rho}{\rho_o}\right)^x$$

The plotting of the function for x on logarithmic paper gives a straight line, showing the exponent to be constant. The function then reads

$$\frac{T}{T_o} = \left(\frac{\rho}{\rho_o}\right)^{0.20} \tag{16}$$

The relation of the engine power to the temperature at constant air density must now be determined, since formula (11) shows the effect of changes in density produced by temperature changes at constant pressure. If the temperature had no effect, we would then have, according to the law for gases,

$$\frac{N}{N_o} = \left(\frac{\rho}{\rho_o}\right)^{1.15} = \left(\frac{T}{T_o}\right)^{-1.15} \text{ (p constant)} \tag{11a}$$

Actually, however,

$$\frac{N}{N_o} = \left(\frac{T}{T_o}\right)^{-0.5} \tag{11}$$

The direct effect of cold on the engine power is therefore

$$\frac{N}{N_o} = \frac{\left(\frac{T}{T_o}\right)^{-0.5}}{\left(\frac{T}{T_o}\right)^{-1.15}} = \left(\frac{T}{T_o}\right)^{0.65} \tag{17}$$

This expression shows the effect of temperature on carburetion and on the friction horsepower.

Equation (16) holds good for the standard atmosphere. The effect of T in the atmosphere is therefore expressed by

$$\frac{N}{N_o} = \left(\frac{\rho}{\rho_o}\right)^{0.65 \times 0.2} = \left(\frac{\rho}{\rho_o}\right)^{0.13} \tag{18}$$

The last step is now to combine the effects of p and T in formulas (12) and (18),

$$\frac{N}{N_o} = \left(\frac{\rho}{\rho_o}\right)^{1.15+0.13} = \left(\frac{\rho}{\rho_o}\right)^{1.28} \tag{19}$$

This reduction in power occurs at constant R.P.M.

In the previously mentioned American report (N.A.C.A. Technical Report No. 171), data are also given on the observed decrease in the R.P.M. of engines with increase in altitude. According to this report, its mean value is

$$\frac{n}{n_o} = \left(\frac{p}{p_o}\right)^{0.10} \tag{20}$$

Conversion to the air density in the standard atmosphere by means of equation (16) gives

$$\frac{p}{p_0} = \frac{\rho}{\rho_0} \; \frac{T}{T_0} = \left(\frac{\rho}{\rho_0}\right)^{1.2}$$

$$\frac{n}{n_0} = \left(\frac{\rho}{\rho_0}\right)^{0.12} \tag{21}$$

This reduction in the R.P.M. results, in the most unfavorable case, in a corresponding loss in power.* With this unfavorable assumption we then have

$$\frac{N}{N_0} = \left(\frac{\rho}{\rho_0}\right)^{1.28+0.12} = \left(\frac{\rho}{\rho_0}\right)^{1.40} \tag{22}$$

This is the engine-power formula according to which the following calculations are made. In Figure 3 it is compared with the customary assumption (equation (9)). For a moderate decrease in the air density, formula (22) is therefore more unfavorable than the customary assumption, which is especially due to the introduction of the reduction in the R.P.M. The two expressions agree at $\rho/\rho_0 = 0.36$. This corresponds, however, to a flight altitude, which hardly comes in question for an engine without a supercharger. This assumption is therefore more unfavorable than

*The American report and its German translation in J.F.W., 1925/19 here draw a false conclusion, in that they confuse the total power generated by the engine with the power absorbed by the propeller and thus allow twice for the effect of the air density. According to this method our formula would read

$$\frac{N}{N_0} = \left(\frac{\rho}{\rho_0}\right)^{1.28+0.36} = \left(\frac{\rho}{\rho_0}\right)^{1.64}$$

which is much too unfavorable.

the usual one within the customary limits. At a still higher altitude the formula apparently undergoes an inadmissible extrapolation.

A certain verification of formula (22) is obtained by applying the decrease in the R.P.M., according to equation (21) to the thrust horsepower. This can be put roughly proportional to the air density and the third power of the R.P.M. Hence

$$\frac{N}{N_0} = \frac{\rho}{\rho_0}\left(\frac{n}{n_0}\right)^3 = \left(\frac{\rho}{\rho_0}\right)^{1.36}.$$

This expression does not include the reduction in the power absorption, which is effected by the increase in the degree of advance in climbing at constant dynamic pressure. This is difficult to determine numerically, but would probably not be placed too high with

$$\frac{N}{N_0} = \left(\frac{\rho}{\rho_0}\right)^{0.64}$$

We thus return to equation (22).

If the power of an engine is constant up to a certain altitude, equation (22) is subtracted from this critical altitude H_a, with the critical power N_0 on the ground = N_a at the critical altitude.

3. The Ceiling

It is now easy, with the help of equations (1), (8), and (22), to develop a simple formula for the ceiling. For normal

engines it is

$$\begin{aligned} w &= w_h - w_s \\ &= w_{ho} \frac{N}{N_o} \frac{\eta}{\eta_o} - w_{so} \left(\frac{\rho}{\rho_o}\right)^{-0.5} \\ &= w_{ho} \left(\frac{\rho}{\rho_o}\right)^{1.4} \frac{\eta}{\eta_o} - w_{so} \left(\frac{\rho}{\rho_o}\right)^{-0.5} \end{aligned}$$

Herein η denotes the propeller efficiency at the altitude corresponding to the air density ρ at the time. For the ceiling

$$w = o = w_{ho} \left(\frac{\rho_g}{\rho_o}\right)^{1.4} \frac{\eta_g}{\eta_o} - w_{so} \left(\frac{\rho_g}{\rho_o}\right)^{-0.5}$$

from which we obtain

$$\left(\frac{\rho_g}{\rho_o}\right)^{1.9} = \frac{w_{so}}{w_{ho}} \frac{\eta_o}{\eta_g}$$

$$\frac{\rho_g}{\rho_o} = \left(\frac{w_{so}}{w_{ho}} \frac{\eta_o}{\eta_g}\right)^{0.53} \qquad (23a)*$$

This formula therefore with the knowledge of the characteristic vertical speeds near the ground and of the efficiency at the ceiling, renders it possible to calculate the air density at this altitude. From this the ceiling can be determined according to any assumption for the atmosphere.

The most important part of equation (23) is the expression w_h/w_s. It can be designated as the "power ratio" and stands in

*If N_o were constant up to the ceiling the expression would read

$$\frac{\rho_g}{\rho_o} = \left(\frac{w_{so}}{w_{ho}} \frac{\eta_o}{\eta_g}\right)^2.$$

the following relation to the excess power w/w_s.

$$\text{Excess power} \quad \frac{w}{w_s} = \frac{w_h}{w_s} - 1.$$

The air density at the ceiling is therefore a function of the excess power and depends further only on the efficiency near the ground and at the ceiling.

In order to obtain immediately a comprehensive view of the altitude relations, the atmosphere formula (14) is also introduced. We then have

$$H_g = 10.9 \lg\left(\frac{w_{ho}}{w_{so}} \frac{\eta_g}{\eta_o}\right) \tag{24}$$

This formula is represented in Figure 4, where the curves for the critical altitudes $H_a = 0$, 2, 4, and 6 km (0, 6562, 13123, and 19685 ft.) are plotted. For this case the equation reads

$$H_g = H_a + 10.9 \lg\left[\frac{w_{ho}}{w_{so}} \frac{\eta_g}{\eta_o} \left(\frac{\rho_a}{\rho_o}\right)^{1/2}\right] \tag{24a}$$

in which ρ_a denotes the "critical air density" corresponding to the critical altitude H_a.

Still one more step remains to be taken, in order to complete the picture, namely, the introduction of the values for w_h and w_s from Section II, 1. According to equations (1) and (8),

$$w_{ho} = \frac{75\ N_o\ \eta_o}{G} \quad w_{so} = 0.75 \left(\frac{\rho_o}{2}\right)^{-1/2} G^{1/2}\ b^{-3/2}\ \kappa^{3/4}\ f_{ws}{}^{1/4}.$$

Herewith

$$\frac{w_{ho}}{w_{so}} \frac{\eta_g}{\eta_o} = 70.8 \frac{N_o \eta_g \rho_o^{1/2}}{\left(\frac{G}{b}\right)^{3/2} \kappa^{3/4} f_{ws}^{1/4}}$$

and hence

$$\frac{\rho_o}{\rho_g} = 9.55 \left(\frac{N_o \eta_g \rho_o^{1/2}}{\left(\frac{G}{b}\right)^{3/2} \kappa^{3/4} f_{ws}^{1/4}} \right)^{0.53} \tag{25}$$

Further, with the atmosphere according to equation (14),

$$H_g = H_a + 20.4 + \lg \left(\frac{N_o \eta_g \rho_a^{1/2}}{\left(\frac{G}{b}\right)^{3/2} \kappa^{3/4} f_{ws}^{1/4}} \right) \tag{26}$$

or

$$H_g = H_a + 20.4 + \lg \left(\frac{N_o \eta_g \rho_a^{1/2}}{\left(\frac{G}{b_i}\right)^{3/2} f_{ws}^{1/4}} \right) \tag{26a}$$

If we take into account the limitation of the applicability of these formulas, which is given by equation (8) and the considerations following it, they enable an especially deep insight into the mechanics of high-altitude flight. The most important place is occupied by the critical altitude, above which the engine power can be no longer kept constant.

This coefficient stands before the logarithmic term. The logarithmic term itself contains, in the numerator, the characteristic quantities for the power plant, namely, the "critical power" of the engine, the "critical air density" and the efficiency at the ceiling. The denominator contains the corresponding quantities for the airplane in gliding flight, namely, the "span load-

ing," the "reduction coefficient" of the biplane and the "total equivalent flat-plate area" with different exponents corresponding to their effect on the ceiling. From a different viewpoint, equations (25) and (26) contain the quotient of the thrust horsepower divided by the power required to maintain horizontal flight, or a proportionate coefficient thereof.

At constant engine power, on the assumption of constant propeller efficiency and perfect applicability of the altitude formula (14), the ceiling would increase to $2/0.53 = 3.8$ times the value corresponding to an engine power, which decreases from sea level according to formula (22). As a matter of fact, these assumptions are far from applicable to the flight altitudes coming under consideration. Nevertheless, we can get an idea of what ceiling can be attained by keeping the engine power constant.

4. The Maximum Speed

The expression for the maximum horizontal speed follows from the one for the climbing speed (equation (4)) with $w = 0$:

$$75\ N\eta = \frac{\rho}{2} f_{ws}\ v^3 + \frac{2}{\pi\rho}\left(\frac{G}{b_i}\right)^2 \frac{1}{v} \tag{27}$$

or, the available power on the propeller = the power required to overcome the frontal drag + the power required to overcome the induced drag. This equation of the fourth degree in v is very inconvenient to use. We can obtain an approximation by considering that the share of the induced power, in comparison with the

frontal-drag power (at least near the ground), is generally negligible. On eliminating this term, we obtain

$$v = \sqrt[3]{\frac{75\ N\eta}{\frac{\rho}{2}\ f_{ws}}}\ , \tag{28}$$

the well-known formula for the maximum speed, although without taking the induced drag into consideration. In this expression there occurs a special quantity, the "frontal-area power" $\frac{N}{f_{ws}}$, which, along with the propeller efficiency, determines the attainable maximum speed.

If the effect of the induced drag is not to be disregarded, it must be determined approximately with the speed obtained from equation (28) and be subtracted from the available power. A new calculation of v with this smaller power according to equation (28) gives a very accurate value.

IV. Nomograms

Nomographic curves have two objects. On the one hand, they often save the work of repeated numerical calculations and, on the other hand, they furnish a comprehensive survey of all the relations and numerical effects of the critical quantities. The latter object is of decisive importance for designing, because numerical calculations alone seldom furnish a quick survey.

Nomograms for flight performances have often been published. They are all derived more or less directly from the polars of the

airplane. This means that the polars are first calculated from the assumed dimensions of the airplane and this work must be repeated for every change in the aspect ratio. This method results in an overvaluation of the polars and especially of the aspect ratio comprised therein. The true relations between the constructive quantities are obscured.

The formulas in the present report avoid this roundabout way. Hence, they enable the production of convenient and comprehensive nomograms, a few examples of which follow.

1. Nomograms for Climbing Performances

This is based on formulas (6), (7), (8), and (23). It is constructed on the principle of linear addition and proportional tables.

The skeleton of the nomograms forms a reclining "Z" with two further parallel scales below at the left. This part solves the equation

$$\frac{w_s}{\left(G\,\frac{\rho_o}{\rho}\right)^{1/2}} = 3\,\frac{f_{ws}^{\,1/4}}{b_i^{\,3/2}}$$

(in which $\rho_o = 0.125$) by drawing two parallels with the corresponding values for b_i, f_{ws}, and $G\,\frac{\rho_o}{\rho}$. If $c_a > 1$ (for ordinary profiles), which can be read above in the middle, we must resort to equation (7a), which contains the following expression

$$w_s = 4\left(\frac{G}{F}\right)^{1/2}\left(c_{ws} + \frac{1}{\pi}\,\frac{F}{b_i^{\,2}}\right).$$

The values of this equation are plotted on the three arms of the "Z." The solution is effected with the aid of two parallel lines through the corresponding values.

At the upper left, ρ_0/ρ is plotted against H, according to data by Süring, as the mean value for Europe at 50° north latitude. The reduction coefficient κ for a biplane with equal upper and lower wings ("symmetrical biplane") is also plotted against h/b according to Prandtl (Göttingen "Ergebnisse" Report II, p. 9 ff.), and the value of c_a, according to equation (8) for constant c_{wp} in the vicinity of this "best lift coefficient," is plotted against $f_{ws}/\kappa t^2$.

The vertical speed of ascent $w_h = 75\ N\eta/G$ is plotted on the middle line and near it the corresponding values of the power loading for an efficiency of 0.6 (with reference to the gliding-flight polar at the same c_a), corresponding to the actual mean relations. The connecting line for w_s and w_h intersects the prolonged right arm of the "Z" at the point which gives the climbing speed w.

On this right upper scale, there are still to be found the values of ρ/ρ_g belonging, according to formula (23), to the corresponding ratios of the power required for vertical flight to the power required to maintain horizontal flight, from which values the ceiling can be found for any air-density course.

The nomogram thus renders it possible to include all the climbing relations with three lines and a few simple slide-rule

auxiliary calculations. The somewhat inconvenient drawing of parallels is avoided by the use of a celluloid sheet with engraved parallel lines or of a small auxiliary device with parallel slides.

2. Nomogram for Ceilings*

Here the form of the logarithmic rectangular tables was chosen. Its basis is formula (25)

$$\frac{\rho_a}{\rho_g} = 9.55 \frac{N_o \eta_g \rho_a^{1/2}}{\left(\frac{G}{b}\right)^{3/2} \kappa^{3/4} f_{ws}^{1/4}},$$

which gives perfectly accurate values, however, only when c_a does not greatly exceed, according to equation (8), the value 1 with ordinary profiles. In return, this expression gives a general view of the effect of the individual quantities. The arrangement of the nomogram is perfectly symmetrical, corresponding to the structure of the formula. The fixed values of the airplane are at the top, the fixed engine values at the bottom, and between these a scale for the air-density relations, supplemented by the relation between the air density and flight altitude already employed in the preceding nomogram.

The simplest way to use the nomogram is with the aid of a celluloid sheet with a rectangular system of lines. Three arms of each cross must pass through the corresponding fixed values. The fourth arm then determines the corresponding point on the reference line, which can be noted with the aid of the millimeter

*This nomogram was constructed by Erik Thomas.

spacing, since the celluloid sheet does not allow direct marking. The connecting line to this point on the reference line cuts the middle line at the desired point.

3. Nomogram for Maximum Speeds

The construction of a nomogram for the equation of the fourth degree with v^3 and v^{-1} (equation (27)) seems difficult at first. A surprisingly simple solution is obtained, however, if the two performance components are so plotted against v, that their sum can be read directly.

For this purpose, equation (28) (with $\rho_o = 0.125$) is conveniently written as follows:

$$\frac{N\eta}{f_{ws}} = \frac{1}{1200}\ \frac{\rho}{\rho_o}\ v^3 + 0.068\ \frac{\rho_o}{\rho}\left(\frac{G}{b_i}\right)^2 f_{ws}^{-1} v^{-1}$$

Now the power for each square meter of the total equivalent flat-plate area stands on the left, while the correspondingly assumed power components, with reference to the total equivalent flat-plate area, stand on the right. Hereby the most important component, the power per unit of equivalent flat-plate area, independently of the magnitude of the equivalent flat-plate area, and the corresponding curves in the nomogram bear, as designation, only the air-density ratio for the temporary flight altitude.

The values corresponding to the frontal-area performance are plotted on the left margin. It is therefore necessary to calcu-

late, on the slide rule, only the two fixed values $\frac{N}{f_{ws}}\eta$ and $\frac{\rho_o}{\rho}\left(\frac{G}{b_i}\right)^2 f_{ws}^{-1}$ and we can then find directly, by trial with the compasses or with a scale drawn on a celluloid sheet, the speed at which the delivered power equals the absorbed power. Simultaneously this nomogram gives a very good survey of the speed relations at high altitudes.

V. Examples

We will now apply this method to a few airplanes which participated in the 1925 Lilienthal contest, in order to demonstrate, on the one hand, the applicability of the method and, on the other hand, to make a definite presentation of the absolute values of the constants involved.

In order to avoid any obscuring of the results through biplane effects, we have chosen the three monoplanes B II ("Sausewind"), U 10 (low-wing) and U 8 (high-wing).* First the air densities at the ceiling were determined on the basis of an estimated equivalent flat-plate area of non-lifting parts and of a profile with the coefficient $c_{wp} = 0.01$. Comparison with the measured air density shows the good agreement of the calculation. Next the accurate values of these equivalent flat-plate areas were determined from the maximum speeds flown, with the aid of the estimated equivalent flat-plate areas of non-lifting parts, whereby it appears that the estimated equivalent flat-plate areas were

*Madelung, "Der Wettbewerb um den Otto-Lilienthal-Preis," 1925 Yearbook of the W.G.L.

mostly too small.* Table I contains the results of the calculations carried out with the aid of the nomograms in Figs. 5 and 7.

TABLE I.

	1	2	3	4	5	6	7	8	9
Airplane type	G kg	b m	$\frac{G}{b}$	F m^2	f_{wp} (c_{wp}=0.01)	f_{wr} Estimated	f_{ws} m^2	$t=\frac{F}{b}$	c_a best
B 11	570	9.4	61	12.4	0.125	0.225	0.35	1.32	1.37
U 10	595	10.7	56	15.2	0.15	0.45	0.55	1.42	1.5
U 8	1040	14.3	73	23.0	0.23	0.70	0.93	1.63	1.5

	10	11	12	13	14	15	16	17	18	19
Airplane type	c_{ws}	$\frac{F}{b^2}$	$\frac{G}{F}$	w_s m/s	N HP.	$\frac{G}{N}$	η_{st} Estimated	w_h m/s	γ_g Calculated	γ_g Measured
B 11	0.028	0.14	46	2.0	65	8.8	0.65	5.5	0.73	0.75
U 10	0.036	0.133	39	2.0	63	9.4	0.60	4.8	0.79	0.80
U 8	0.040	0.112	45	2.1	91	11.4	0.60	4.0	0.89	0.89

	20	21	22	23	24
Airplane type	$10^{-3}\left(\frac{G}{b}\right)^2 f_{ws}^{-1}$	v km/h Measured	$\frac{N\eta}{f_{ws}}$	η_{hor} Estimated	f_{ws} Calculated
B 11	11.8	183	112	0.65	0.38
U 10	5.7	146	61	0.60	0.62
U 8	5.7	146	61	0.60	0.93

*See footnote, page 26.

The best c_a (at which the minimum vertical speed of descent would be obtained on the assumption of a constant profile drag) lies far above unity. Hence the calculation is made with c_{ws} and F/b^2. In the other case, the results would be as given in Table II.

TABLE II.

Airplane type	w_s	γ_g	γ_g Measured
B 11	1.95 (instead of 2.0)	0.72 (instead of 0.73)	0.75
U 10	1.9 (instead of 2.0)	0.77 (instead of 0.79)	0.80
U 8	1.9 (instead of 2.1)	0.85 (instead of 0.89)	0.89

The results of this calculation are more favorable on the whole, but the numerical differences are so small, that they can be often disregarded in consideration of the inaccuracy of all the assumptions in the constructional computations. Columns 10, 11, and 12 in Table I are thus eliminated and the calculation becomes shorter and clearer.

*To simplify the use of the nomogram in the speed calculation, the sea-level speed was put equal to the measured speed, which was almost exact. The brake horsepower on the ground can be thereby introduced.

VI. Summary

The following formulas for the flight performances were obtained by the resolution of the polar curves into their components, the profile drag and induced drag, and by taking as a basis the American data regarding the dependence of the engine power on the altitude of flight.

Climbing speed (or rate of climb):

$$w = \frac{75\ N\eta}{G} - 1.06\ \rho^{-1/3} \left(\frac{G}{b_i}\right)^{1/2} \frac{f_{ws}^{1/4}}{b_i}$$

Ceiling:

$$H_g = H_a + 20.4 + \lg \frac{N_o\ \eta_g\ \rho_a}{\left(\frac{G}{b_i}\right)^{3/2} f_{ws}^{1/4}}$$

Maximum horizontal speed:

$$\frac{75\ N\eta}{f_{ws}} = \frac{\rho}{2} v^3 + \frac{2}{\pi\rho}\left(\frac{G}{b_i}\right)^2 f_{ws}^{-1} v^{-1}$$

Herein occur the hitherto unused quantities: $b_i = b/\sqrt{\kappa}$, the induced drag (on multiplanes); G/b_i, the span loading; f_{ws}, total equivalent flat-plate area (including profile drag); H_a, the critical altitude (up to which the engine power can be kept constant); ρ_a, the corresponding critical air density; N/f_{ws}, power per unit of equivalent flat-plate area. The hitherto customary fixed values "aspect ratio" and "wing loading" are eliminated in these performance calculations, as also the values c_a/c_w and $c_a^{3/2}/c_w$ taken from the polar curves. The wing loading still serves only for calculating the landing speed.

Since the calculation is based completely on the constructive quantities G, b_i, and f_{ws}, it shows the constructor directly the effect of these quantities. No further simplification appears possible.

The formulas enable a series of conclusions, of which the following are the most important.

1. For the climbing speed, the power loading is the most important, when there is not too little excess power. Airplanes, designed to climb swiftly at a low altitude, must therefore have a low power loading.

2. For the ceiling, the critical altitude of the engine and the performance ratio of the airplane are the most important. A high ceiling requires high-altitude engines and a good performance ratio on the ground.

3. The performance ratio is the ratio of the power required for vertical flight to the power required to maintain horizontal flight. A reduction of the latter by half has the same effect on the ceiling as doubling the former. The climbing speed near the ground is therefore generally but little affected by it.

4. The most important quantity for holding down the power required for horizontal flight is the span loading, in comparison with which the total equivalent flat-plate area plays only a very subordinate role, for which reason the old Wright airplanes and

others flew, in spite of this small engine power and great drag.

5. For maximum horizontal speed, the power per unit of flat-plate area is most important. Swift airplanes therefore require minimum frontal drags. The wing dimensions are important only in so far as they affect the profile-drag component. Keeping down the profile-drag coefficient is just as effective as reducing the wing area.

6. For equal spans, the biplane seems to be superior, especially with regard to the ceiling, though not so much so with respect to maximum speed near the ground (with reservation as to the solution of the profile-drag problem by further flight tests).

7. With respect to the speed range, most wings have much too good aspect ratios. Increasing the chord brings the "best c_a" more into the realm of actual flight and only slightly impairs the flight performances. The landing speed is, however, simultaneously much reduced.

8. The designing of a practical airplane is accomplished on the basis of the maximum span, while taking into account its contemplated use, wing loading and propeller efficiency. The chord is determined, on the one hand, by the "best c_a" and, on the other hand, by the landing speed. Thus the best dimensions of the wings are determined with reference to the flight performances.

Translation by Dwight M. Miner,
National Advisory Committee for Aeronautics.

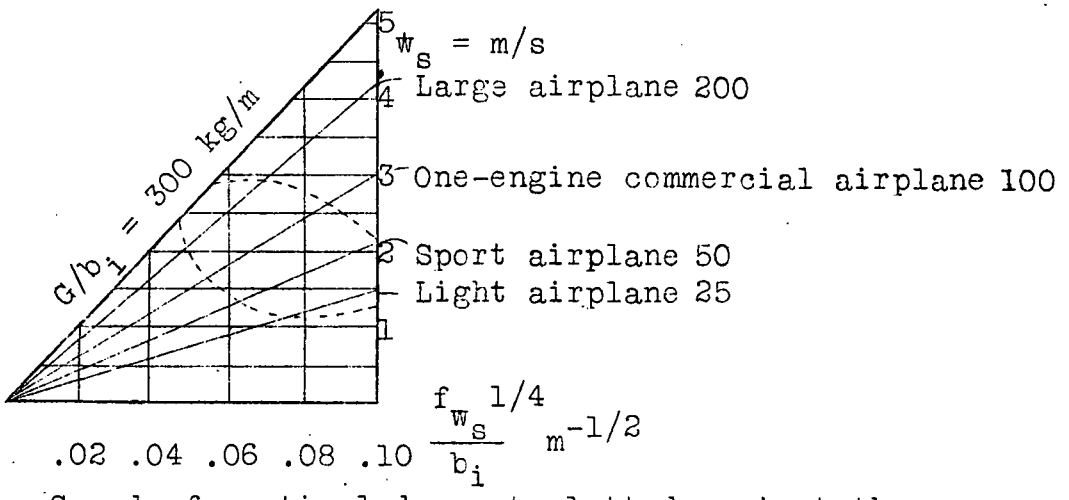


Fig.1 Speed of vertical descent plotted against the span loading.

$a = \frac{c_w}{c_a}3/2$ = constant for mean ratio.

b = replacement by straight line up to c_a=1

c = profile drag polar.

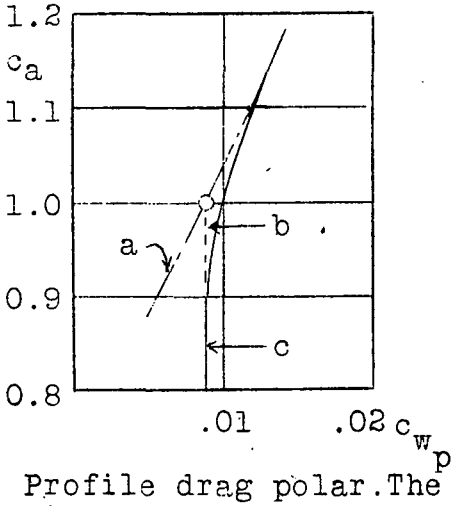


Fig.2 Profile drag polar. The line of constant flight coefficient touches the polar curve and passes through the point of intersection of the lines c_a=1 and c_{w_p} = constant.

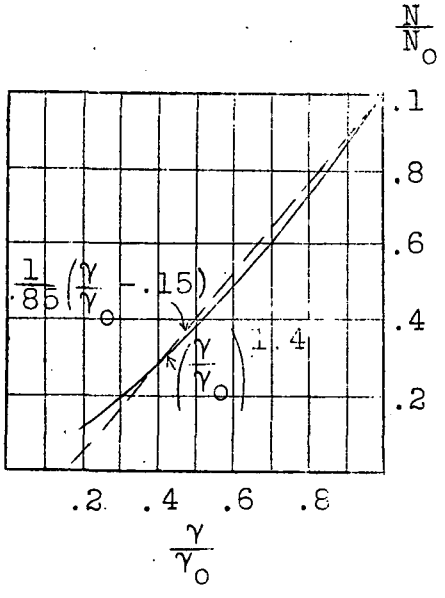


Fig.3 Engine power and flight altitude. Comparison of the customary formula with the one calculated from American experiments.

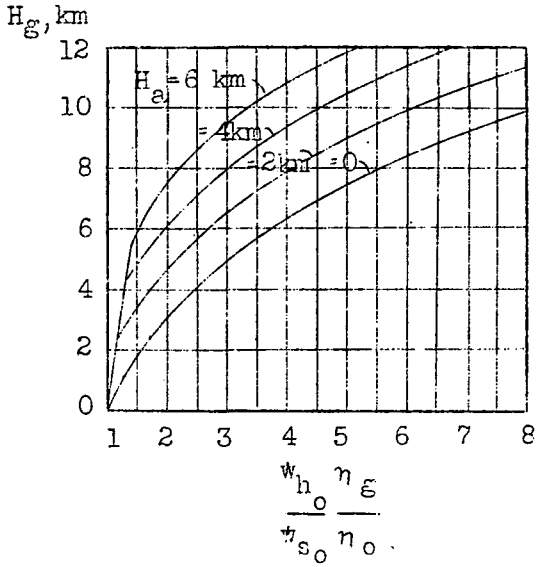
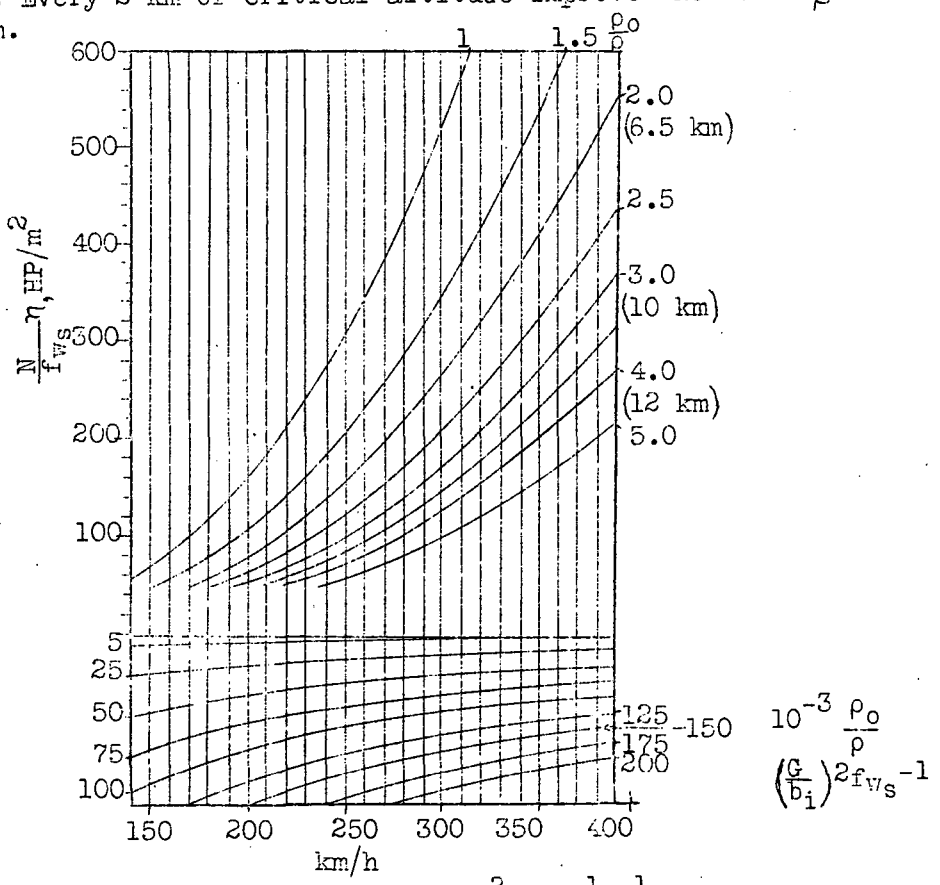


Fig.4 Ceiling plotted against the critical altitude and the performance ratio. Every 2 km of critical altitude improve the ceiling about 1.4 km.



$$75 \frac{N}{f_{w_s}} \eta = \frac{1}{16} \frac{\rho}{\rho_o} v^3 + \frac{16}{\pi} \frac{\rho_o}{\rho} \left(\frac{G}{b_i}\right)^2 f_{w_s}^{-1} v^{-1}$$

Fig.7 Nomograph for the maximum horizontal speed. The speeds are found by ascertaining the corresponding performances with the aid of compasses.

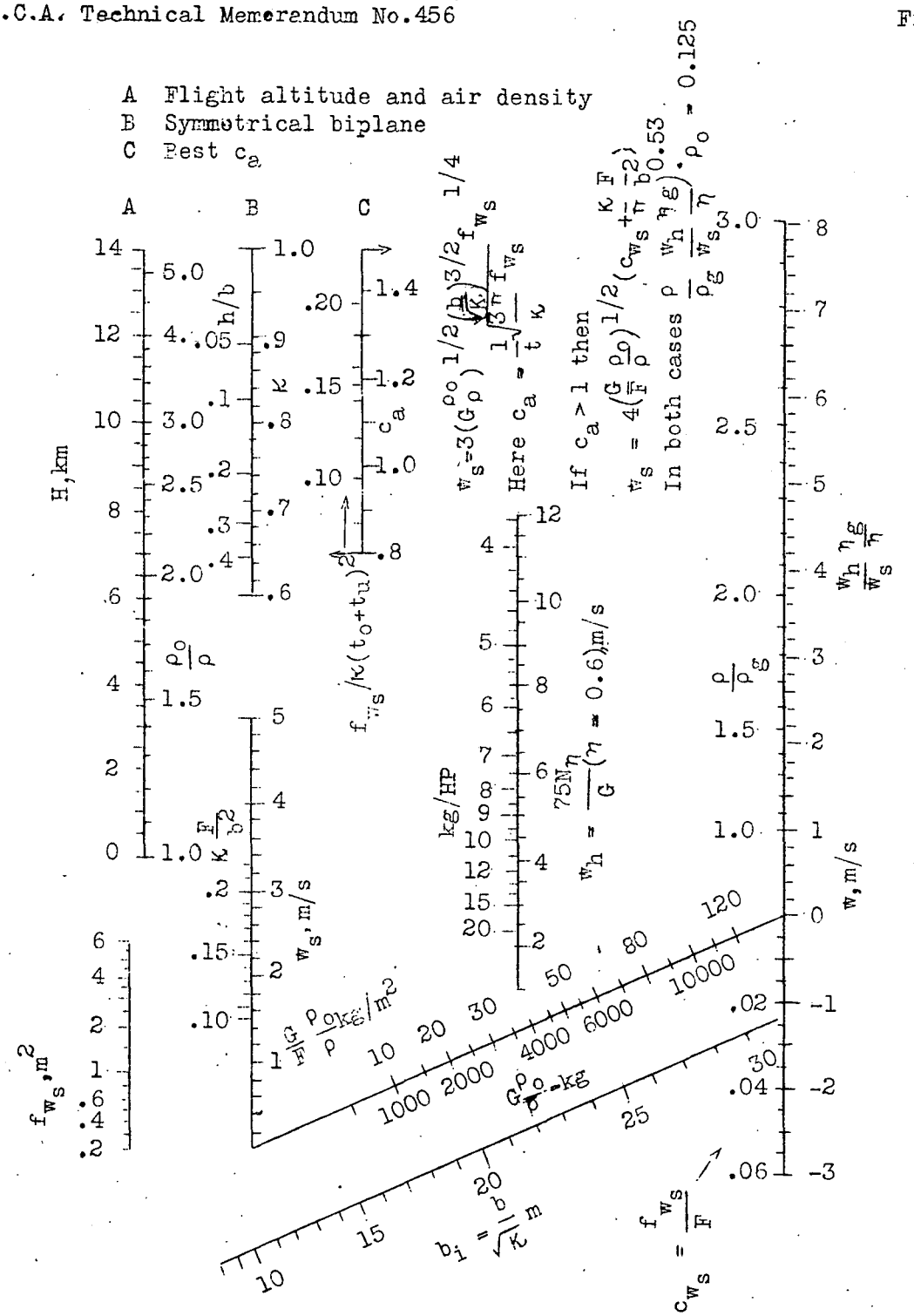


Fig.5 Nomograph for climbing performances. The performances are found by drawing two parallels.

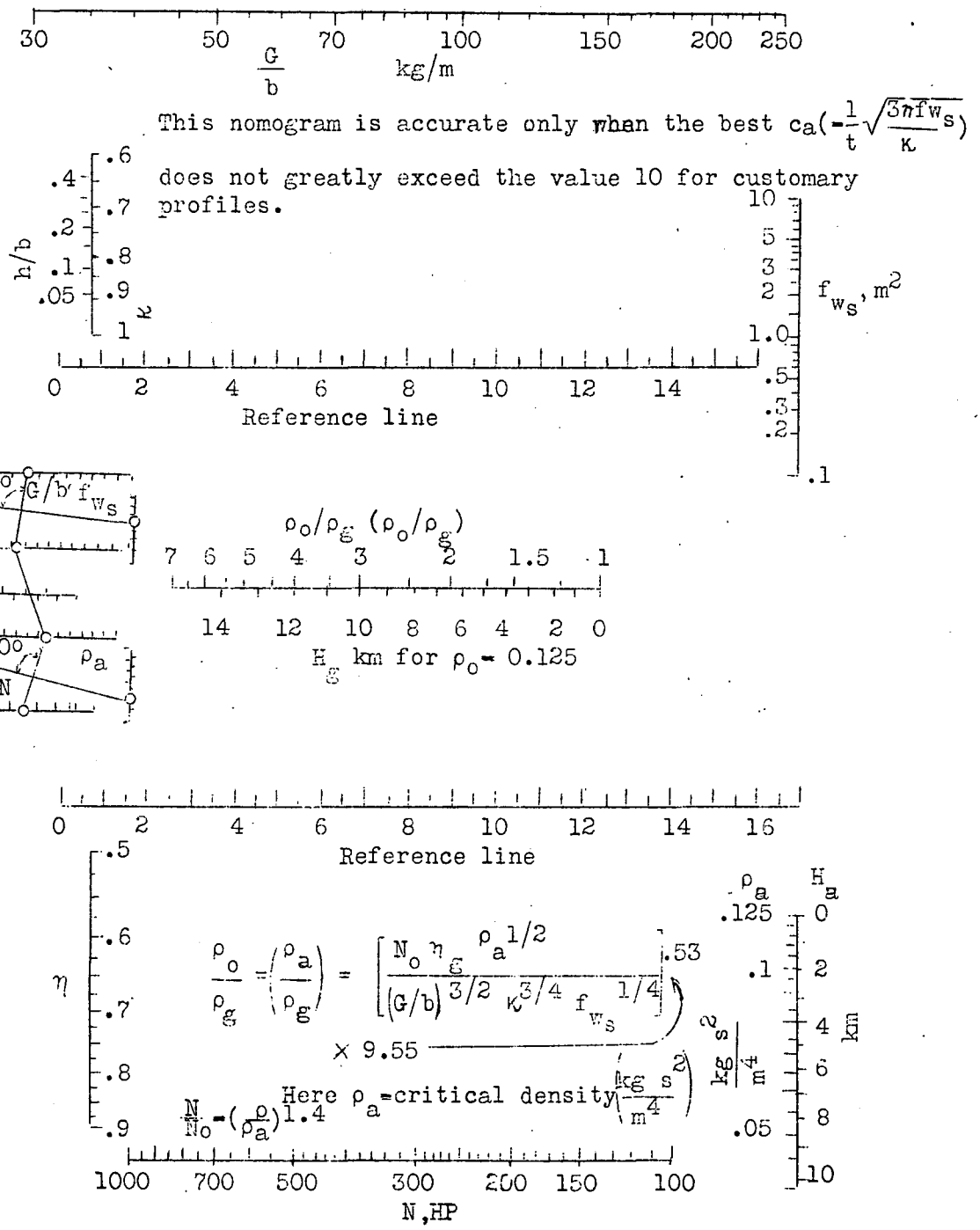


Fig.6 Nomograph for the ceiling. The ceilings are found by the construction of reference crosses.