REPORT No. 17

IN FOUR PARTS

AN INVESTIGATION OF THE ELEMENTS WHICH CONTRIBUTE TO STATICAL AND DYNAMICAL STABILITY, AND OF THE EFFECTS OF VARIATION IN THOSE ELEMENTS

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17 I
17 II
17 III
17 IV
18
19 I
19 II
20
21

CONTENTS OF REPORT NO. 17.

PART I.

PART II.

PART III.

PART IV.

REPORT No. 17.

PART I.

By Alexander Klemin

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INTRODUCTION.

This report is the result of experiments conducted at the aerodynamical laboratory of the Massachusetts Institute of Technology during the summer of 1917. The work is divided into two sections, the first dealing with static, the second with dynamic, effects. The outlines of the statical experimentation were determined after consultation with Lieut. Col. V. E. Clark, to whom the authors' best thanks are due.

The work on statical conditions, in turn, falls under two heads. In the first place, working from the lift, drag, and performance curves of a standard military tractor biplane as a basis, the portion which each element of the machine contributes to the lift and drag forces was determined by testing each element separately and in all combinations of special interest. As a continuation of this work the length of body, size of tail, and angular setting of the same were varied, changing one at a time, thus determining the effect of any such changes on the lift and drag. Incidentally it has been possible to secure data on the downwash from the wings and its effect on the forces contributed by the tail.

Secondly, by computing the moments about the center of gravity of the machine due to the air pressure on each element, a vector diagram for the airplane can be built up from its component parts, and rules can be laid down for the travel of the vectors and for the initial balancing up of the machine without the necessity of a wind tunnel test in the very early stages of a design. Moments about the center of gravity were also calculated for each of the changes in size, setting, etc., of the tail surfaces, in order to secure definite data on the effect of such changes on the statical stability of the airplane.

The second main section deals with dynamical stability. The resistance derivatives and damping moments were determined for each of the cases, and the length of period and time required to damp to a certain degree were thus calculated, giving the effect of variations in the tail surface on the safety and comfort of the airplane, so far as the longitudinal motion is concerned. Some progress was also made in finding the proportion of damping contributed by the various parts of the machine.

DETAILS OF MODELS TESTED AND METHODS OF TESTING.

The standard machine selected for investigation was a Curtiss JN2 advanced training machine, this type being selected because so much similar work had already been done on it by Dr. J. C. Hunsaker. Drawings of the machine are shown in figure 1, and a table of dimensions is given herewith:

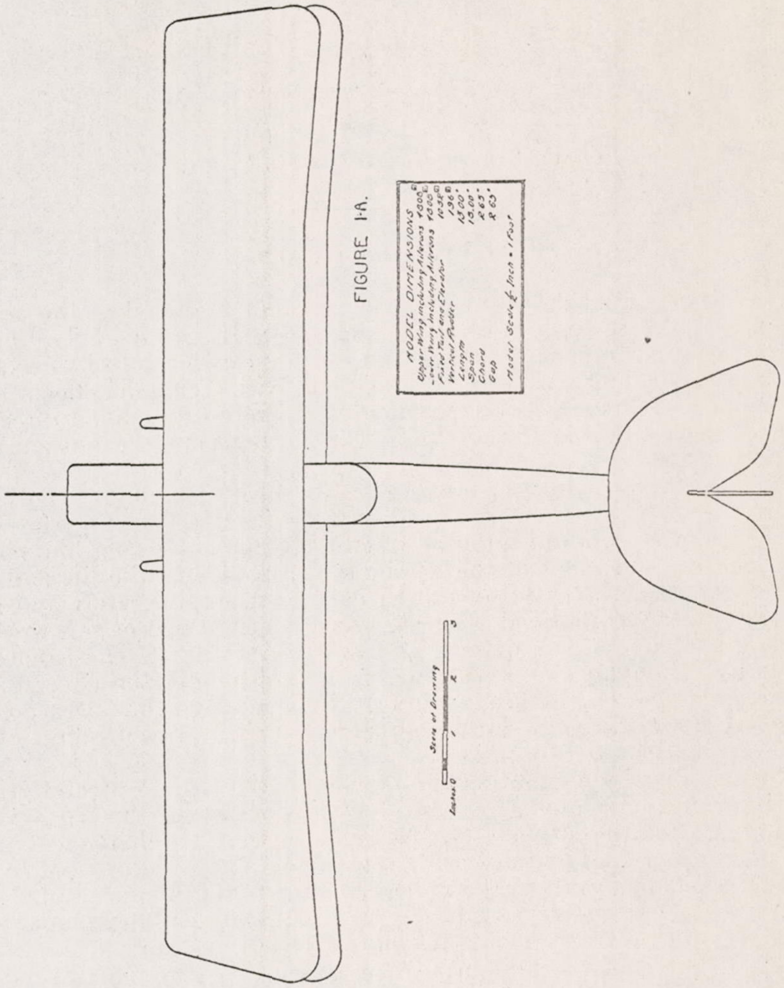

FIGURE 1-A.

Weight fully loaded	pounds..	1,800
Horsepower		110
Span, both wings	feet..	36
Chord of wings	do....	5.0
Gap between wings	do....	5.0
Stagger	do....	1.0
Length of body	do....	26.0
Area of wings	square feet..	364
Area horizontal tail surface	do....	42.0
Area vertical tail surface	do....	7.8
Wing curve	Eiffel..	36

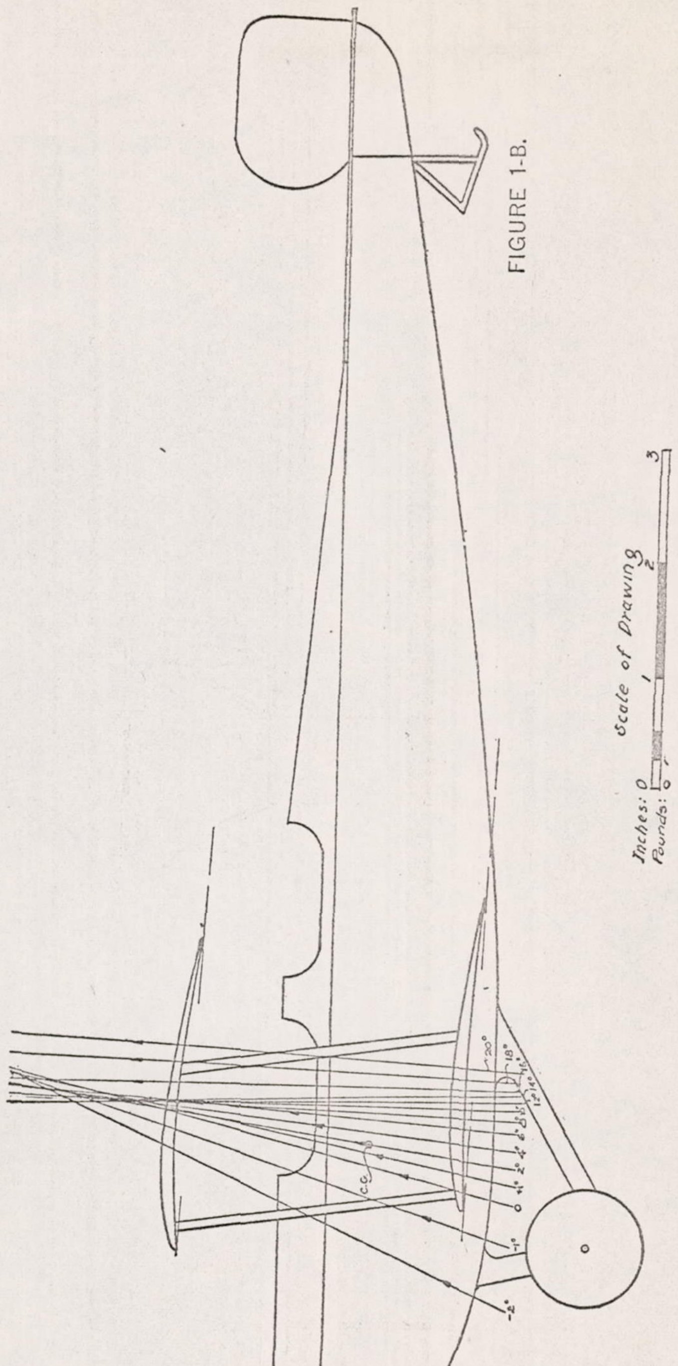

FIGURE 1-B.

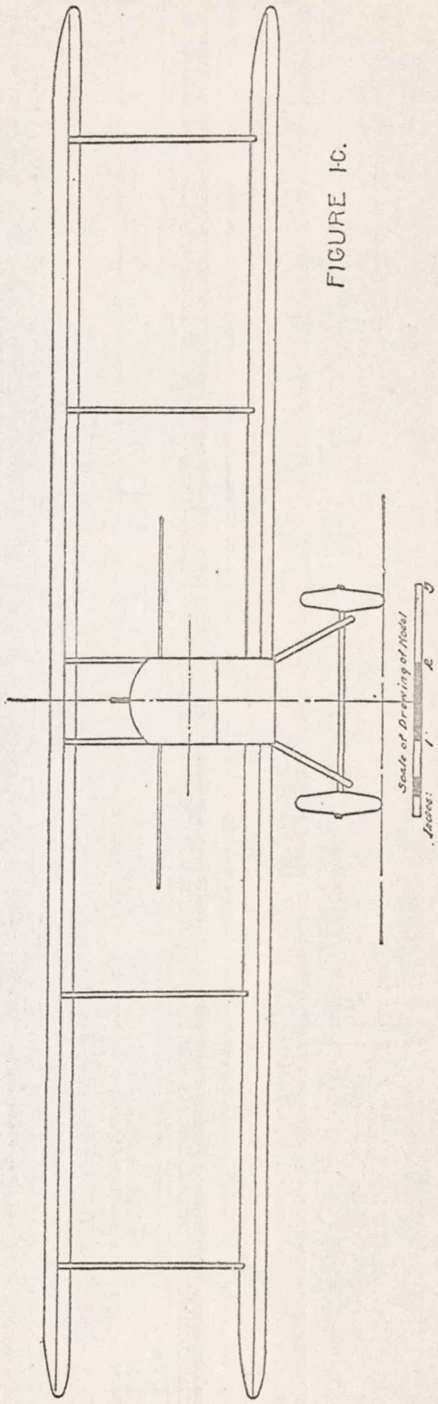

FIGURE 1-C.

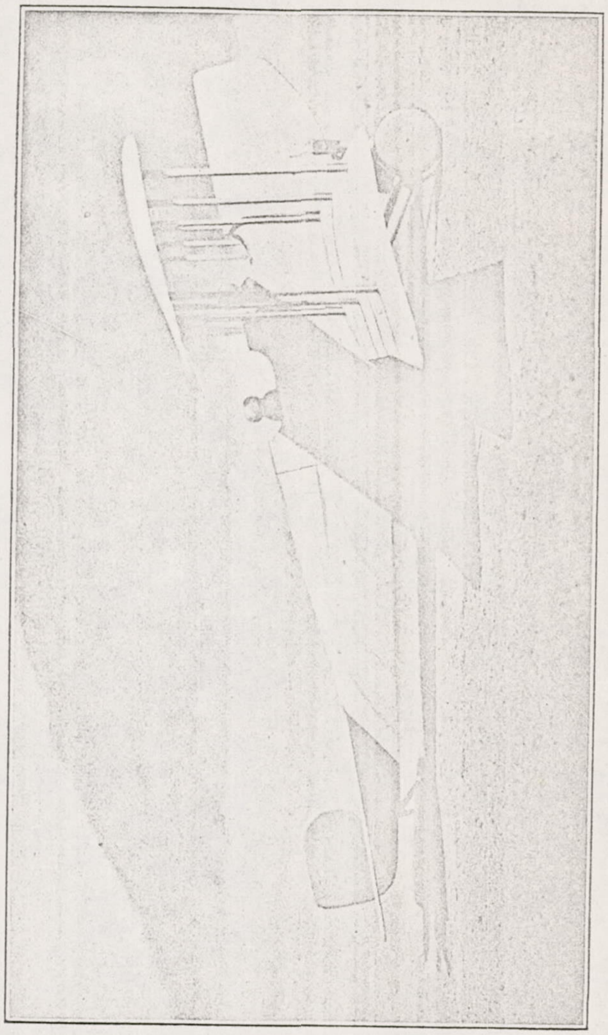

Fig. 2.

Fig. 3

280-2

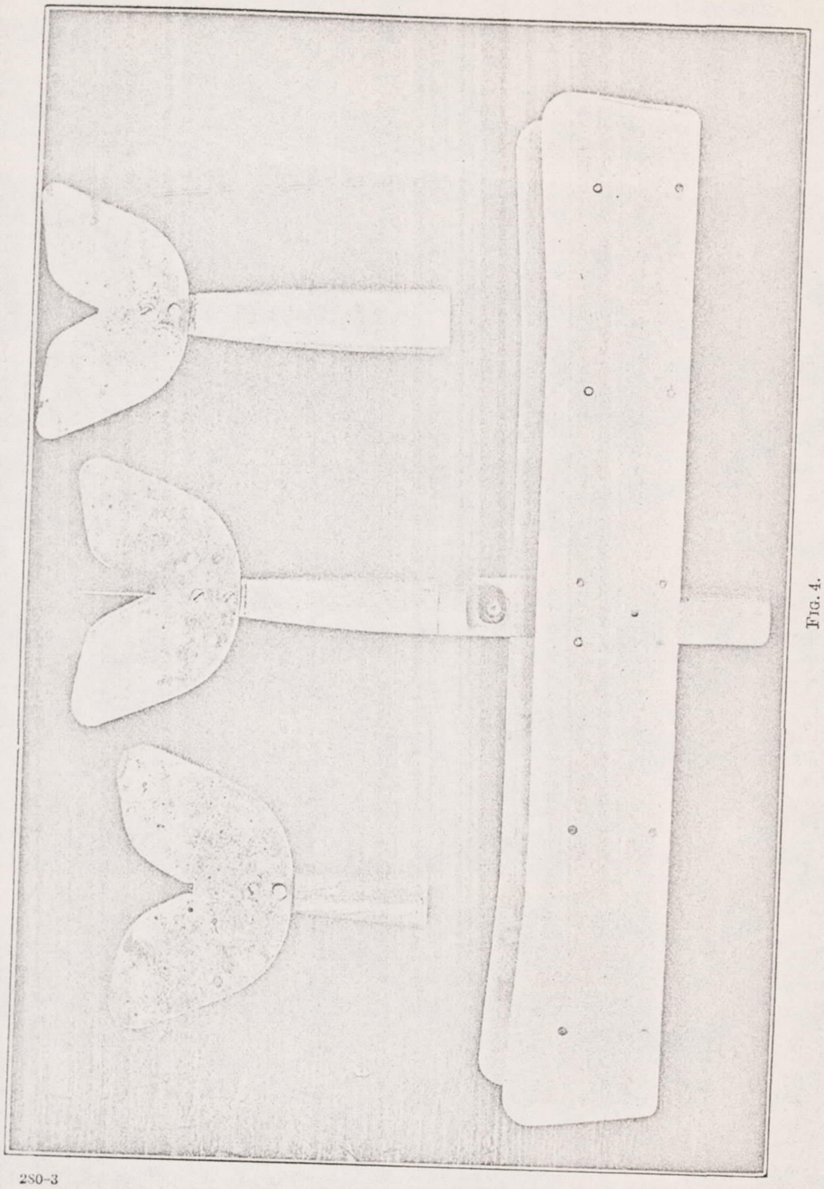

Fig. 4.

280-3

The model was made to a scale of one-half inch to a foot. The wings were made of aluminum, thus combining lightness with the greatest possible accuracy of working and freedom from warping. They were machined roughly and then scraped by hand to the desired section, the working tolerance being 0.003 inch. The tail surfaces were made of brass, and were simply cut from a sheet $\frac{1}{16}$ inch thick, no attempt being made to reproduce exactly the camber of the tail on the actual machine. The body was made of pine, and the chassis was built up from brass wire, with solid wood wheels. The wings were maintained in their proper position with respect to each other by 12 round struts 0.087 inch in diameter, and made of steel wire. In order to prevent the struts from working loose in the aluminum wings steel bushings were pressed into the wing planes, and these bushings were drilled and tapped to take the ends of the wire struts. By threading these struts oppositely on their two ends, an easy and delicate means of adjustment was provided for the elimination of any decalage or warp in the wing cellule. No bracing wires were used, and the propeller was not in place during the tests. It has been found that a model thus made gives results comparable with those for the full-sized machine, the gain due to the omission of wires and propeller being counterbalanced by the loss caused by the use of round, instead of stream-line, struts. The wings were made in the shop of Mr. George F. Day, and under his supervision. Other parts of the model were constructed, and the assembly was carried out, by Mr. W. H. Phillips, and by Messrs. Carl Selig and Edward Tighe, model and instrument makers at the Institute of Technology.

In order to make it possible to vary the length of the body, and consequently the moment arm of the tail, the body was sawn in two just behind the rear cockpit, and the two portions were dowelled together. Two additional rear halves were then made so that either could be fitted on in place of the standard one, their lengths being such as to make the distance from the center of gravity of the machine to the leading edge of the tail 10 per cent greater and 10 per cent less, respectively, than in the standard machine. Two additional tail surfaces were also made up, geometrically similar to that normally used, but one 10 per cent larger, the other 10 per cent smaller. The three bodies are hereinafter referred to as long, medium (standard JN–2), and short, and the three tails, which were tested in various combinations with them, as large, medium (standard JN–2), and small. Figures *2* and *3* show the model with medium body, and figure *4* illustrates the three bodies and tails.

The static tests were carried out in the customary fashion, the forces being measured by weighing on the aerodynamic balance, to pitching moments by the torsional strain which had to be set up in a calibrated wire in order to balance them. The apparatus, and the method of procedure, has been described in detail elsewhere.[1] All static tests were made with a wind speed of 30 miles per hour which has been found to give the best results in the Massachusetts Institute of Technology laboratory. The method of testing for damping, and calculating the dynamic stability, will be taken up in connection with the discussion of the results obtained under those heads.

[1] The New Four-Foot Wind Channel, with a Description of the Weighing Mechanism; Report of the British Advisory Committee for Aeronautics, 1912-13, pp. 59-71.

TEST OF EIFFEL 36 WING ALONE.

As a preparatory step, a test of the Eiffel 36 wing alone was made, and the resultant curves (Ky, Kx, and L/D) are plotted in figure 5. Each of the two wings was tested separately, the results checking within 2 per cent at all points, and within 1 per cent at practically all angles, indicating that the accuracy of manufacture was such that the

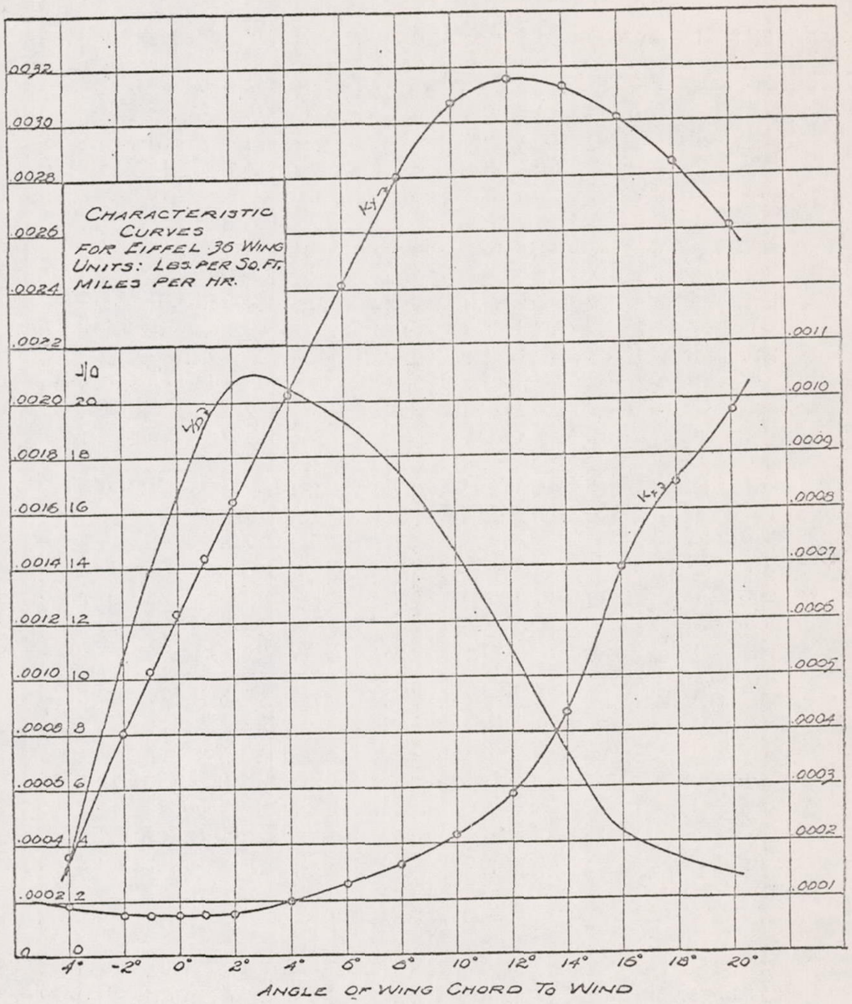

Fig. 5.

variations in profile exerted a negligible influence on the aerodynamic characteristics of the wing. The performance was exceptionally good, the maximum K_y being 0.00315 and the highest L/D 21. The good L/D is in large part chargeable to the raked wings, the high aspect ratio (7.2), and the slightly flattened tips, due to the presence of the ailerons. The corresponding values secured by Eiffel[1] for

[1] Nouvelles Recherches sur la Resistance de l'Air et l'Aviation, by G. Eiffel.

this wing were 0.00295 and 16.1. The discrepancy seems unjustifiably large, especially as the Eiffel tests were made under the better conditions as regards the speed of wind and size of model.

CHARACTERISTICS AND PERFORMANCE CURVES FOR STANDARD JN-2.

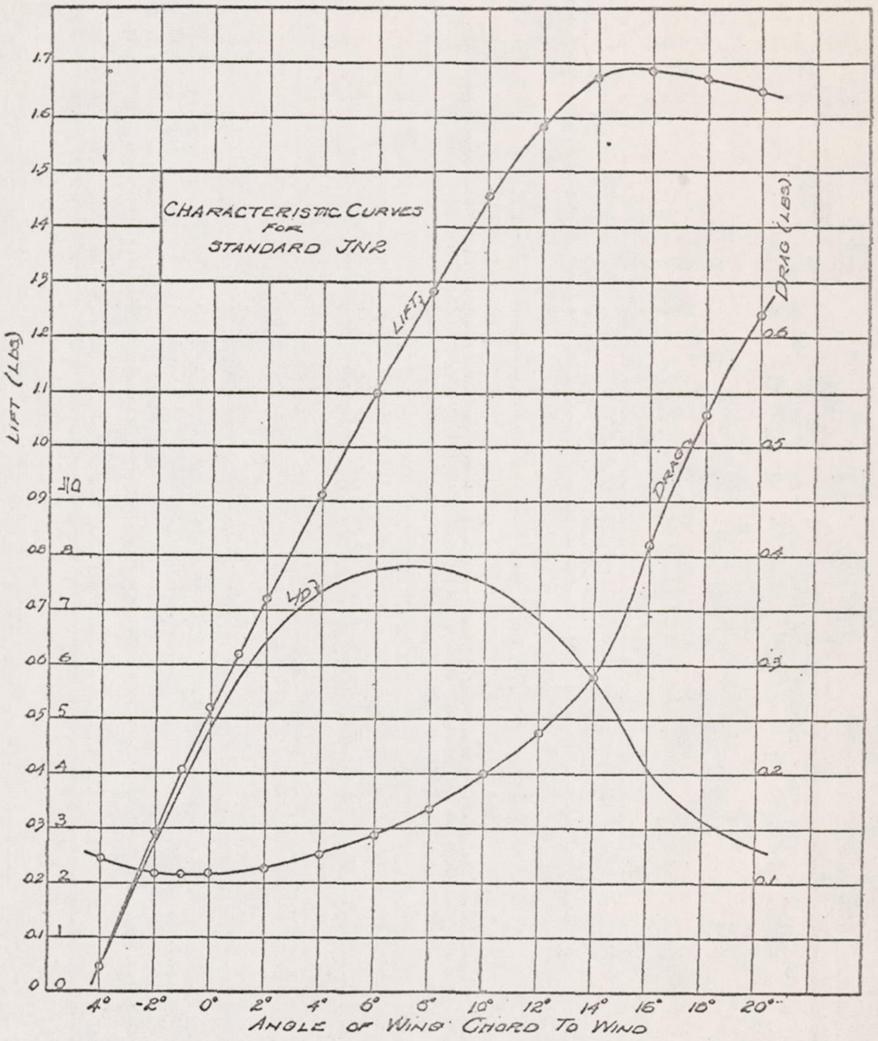

FIG. 6.

Figures 6 and 7 represent, respectively, the characteristic curves (lift, drag, and $\frac{\text{lift}}{\text{drag}}$) and the performance curves for the standard machine with the customary tail setting ($-3\frac{1}{2}°$ to the wing chord). The angle of zero lift for the complete machine is $-4\frac{1}{2}°$, whereas that for the single Eiffel 36 wing is $-6°$. The burble point for the complete machine is at 15°, the maximum lift being 1.69 pounds. The break in the curve just at the burble point is somewhat more abrupt than the corresponding bend for the wing section alone, but the falling away at higher angles is less rapid. The maximum L/D is 7.8, at $7\frac{1}{2}°$, and the minimum drag is 0.105 pound, at $-1°$.

The characteristics thus obtained furnish the basis for the computation of the performance curves. The speed required for sustentation and the lift on a model of 1/24 scale at 30 miles per hour and a like angle of incidence are connected by the formula: $V = \frac{30}{24}\sqrt{\frac{W}{L}}$, or, for $W = 1{,}800$ pounds, $V = \sqrt{\frac{55.03}{L}}$. A curve of angle of incidence against speed may be plotted from values thus obtained, and shows that the minimum speed possible is just below 41 miles per hour,

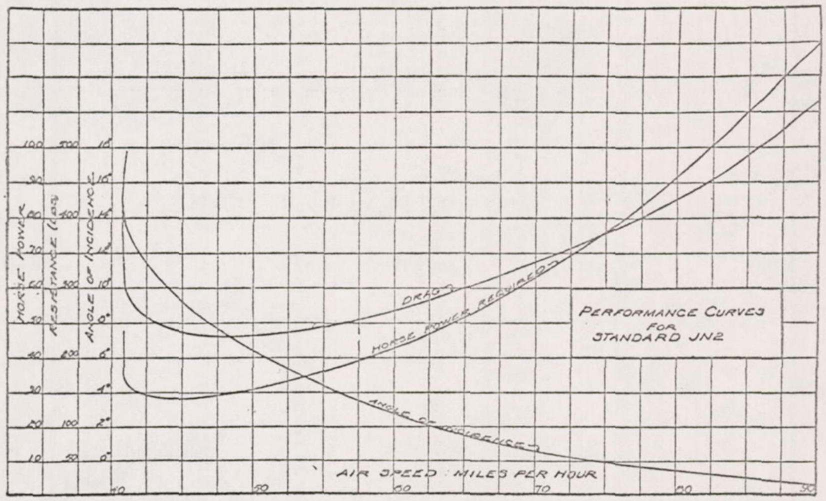

FIG. 7.

and that an angle of incidence of 0° corresponds to a speed of 74 miles per hour, which is about the usual performance of this type of machine. Points on the curve of drag against speed are secured by dividing the weight of the machine by the L/D at any given angle of incidence, and laying off the resultant at the speed appropriate to the angle of incidence in question. The minimum resistance is 230 pounds at 47 miles per hour, and indicates a best gliding angle of 1 in 7.8. The minimum horsepower required is 28, at 45 miles per hour. With an engine developing slightly over 90 horsepower, such as was used in this machine, and a propeller efficiency of 80 per cent, a speed of 74 miles per hour should be secured. The angle of incidence at the maximum speed will then be 0°. Dr. J. C. Hunsaker found[1] a maximum speed of 73 miles per hour for this machine, using a different model, with wooden wings.

[1] Experimental Analysis of Inherent Longitudinal Stability for a Typical Biplane; First Annual Report of the National Advisory Committee for Aeronautics, p. 33.

REPORT No. 17.

PART II.

STATICAL ANALYSIS.

By Alexander Klemin and Edward P. Warner and George M. Denkinger.

LIFT AND DRAG CONTRIBUTED BY BODY AND CHASSIS, TESTED WITHOUT WINGS.

This series of experiments comprised tests of each of the three bodies alone, of the medium tail alone, of the medium body with chassis attached, and of the medium body with chassis and medium tail.

A comparison of the tests of the three bodies indicates nothing except that such changes as were made in length of body affect neither lift nor drag to an extent affecting aerodynamic efficiency in design. The curves drawn for the three cross and recross in a highly irregular fashion, the difference between them always lying well within the limits of probable experimental error, which error is, of course, a relatively large percentage of the force involved when that force is very small.

In figure 8 is plotted the lift of the body and the lift due to chassis alone when in combination with body (obtained by subtracting the lift of the body alone from the lift of body and chassis together). In figure 9 are given the corresponding curves for resistance. The points marked on the body curves are those obtained for the medium body.

The lift due to the body is zero at $+\frac{1}{2}°$ (all angles referred to the line of the top longerons as datum). It is nearly directly proportional to angle at all angles from $-8°$ to $+20°$ (i. e., the lift curve is virtually a straight line). It shows a tendency, however, to increase rather more rapidly at large angles than at small. It should never be forgotten that these values for lift, as well as those for resistance due to the body, will be materially modified by the addition of the wings, the downwash from which members will decrease the lift. The quantitative values of this effect will be discussed later.

The lift due to the chassis is always positive and is virtually constant. Although no test was made on the chassis alone, the natural assumption is that the apparent chassis lift is the result of the formation of eddies and screening of the rear portion of the body, and that there is no dynamic lift on the chassis itself. This effect is hardly worth considering on the full-scale machine, the lift from this source being always less than 5 pounds.

The resistance of the body is, as would be expected, a minimum at 0° and increases rapidly and almost symmetrically with any change of angle in either direction. The resistance due to the chassis, on the other hand, is least at a large negative angle, where the chassis is screened by the forward portion of the body, and

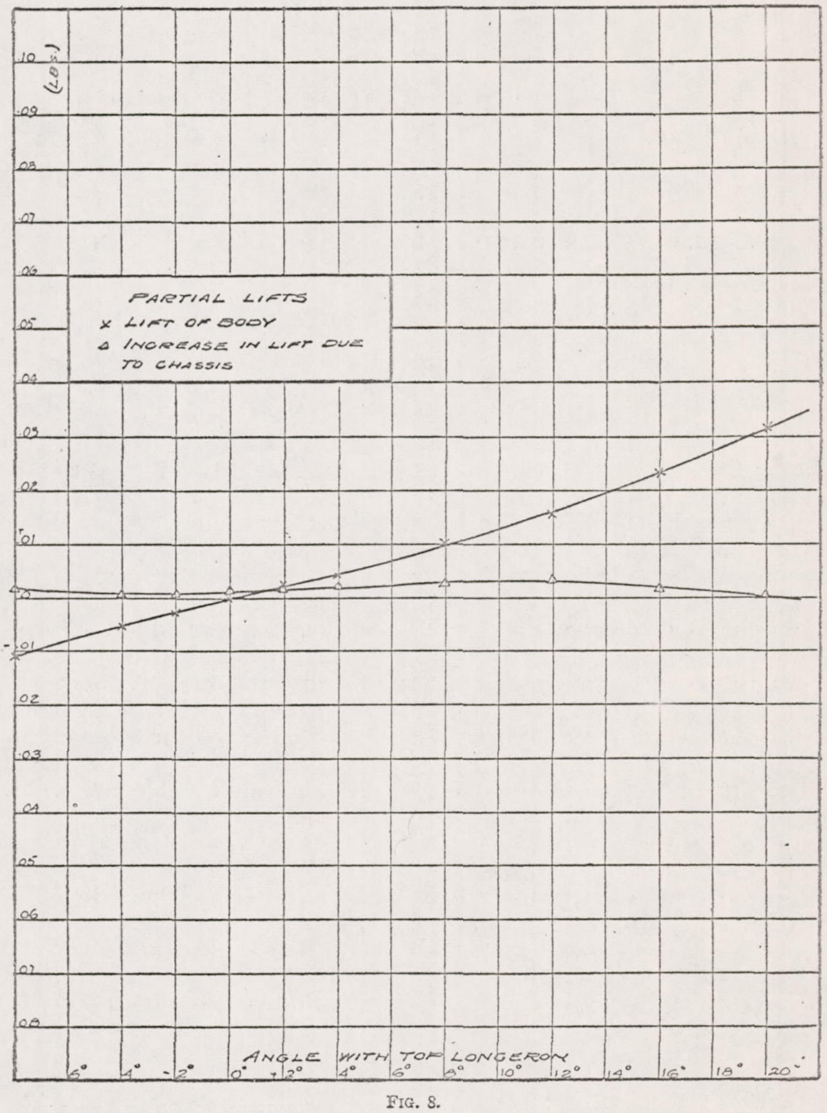

FIG. 8.

increases at a decreasing rate up to an angle (referred to the top longerons) of about 2°. After this it is virtually constant until an angle of 16° is reached, where it begins to fall off again. The maximum resistance due to the chassis is practically identical with the minimum resistance of the body.

LIFT AND DRAG CONTRIBUTED BY TAIL, TESTED WITHOUT WINGS.

In figure 10 are plotted the lift and drag of the medium tail alone and of the medium tail when in combination with the body and chassis. The latter figures were obtained by a method of differences, analogous to that used for finding the lift and drag due to the chassis.

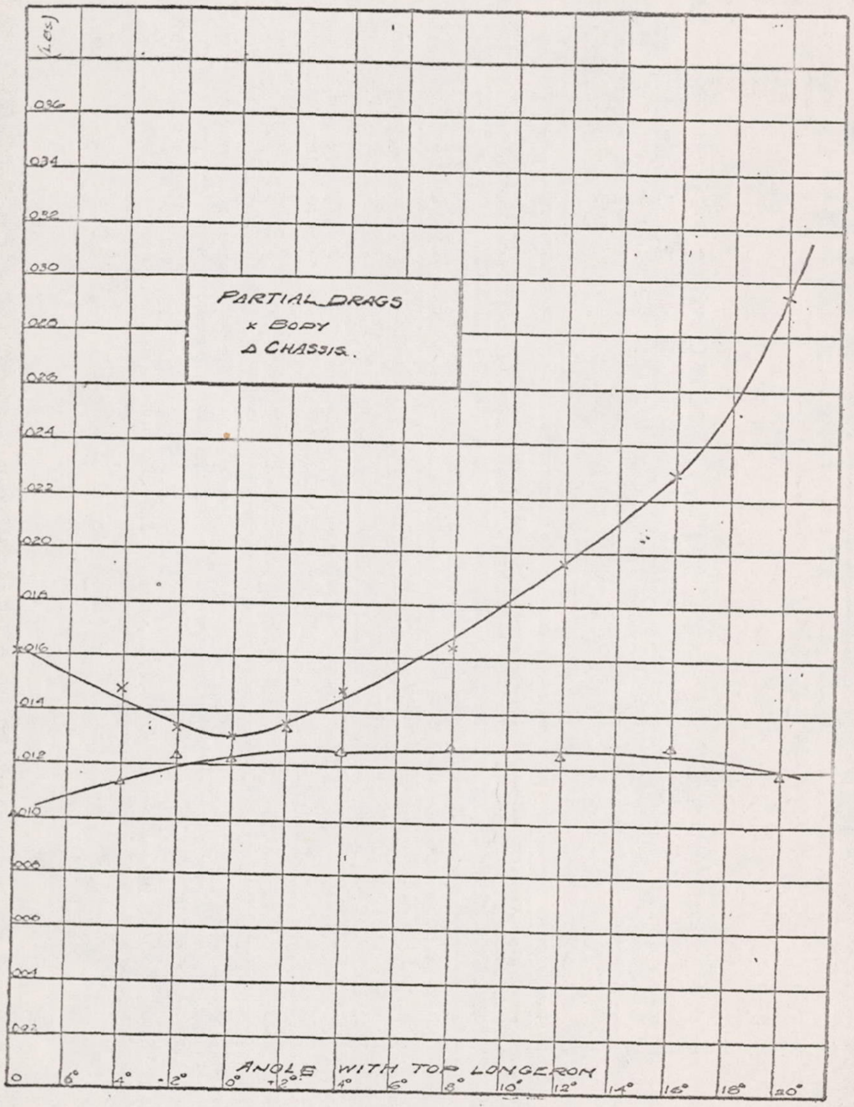

FIG. 9.

The lift of the tail alone follows a straight-line equation very closely, and is, of course, symmetrical about a zero angle of incidence, the surface itself being symmetrical in respect of the upper and lower surfaces. Dividing the lift at an angle of 6° by the area of the surface, the square of the speed, and the angle of incidence in degrees, we find that $Ky = 0.000139i$, which is the equivalent [4] of the lift coefficient on the rectangular flat plate of aspect ratio 3. The drag coefficients, however, are somewhat higher than those appropriate to this aspect ratio, with the net result that the L/D ratio is

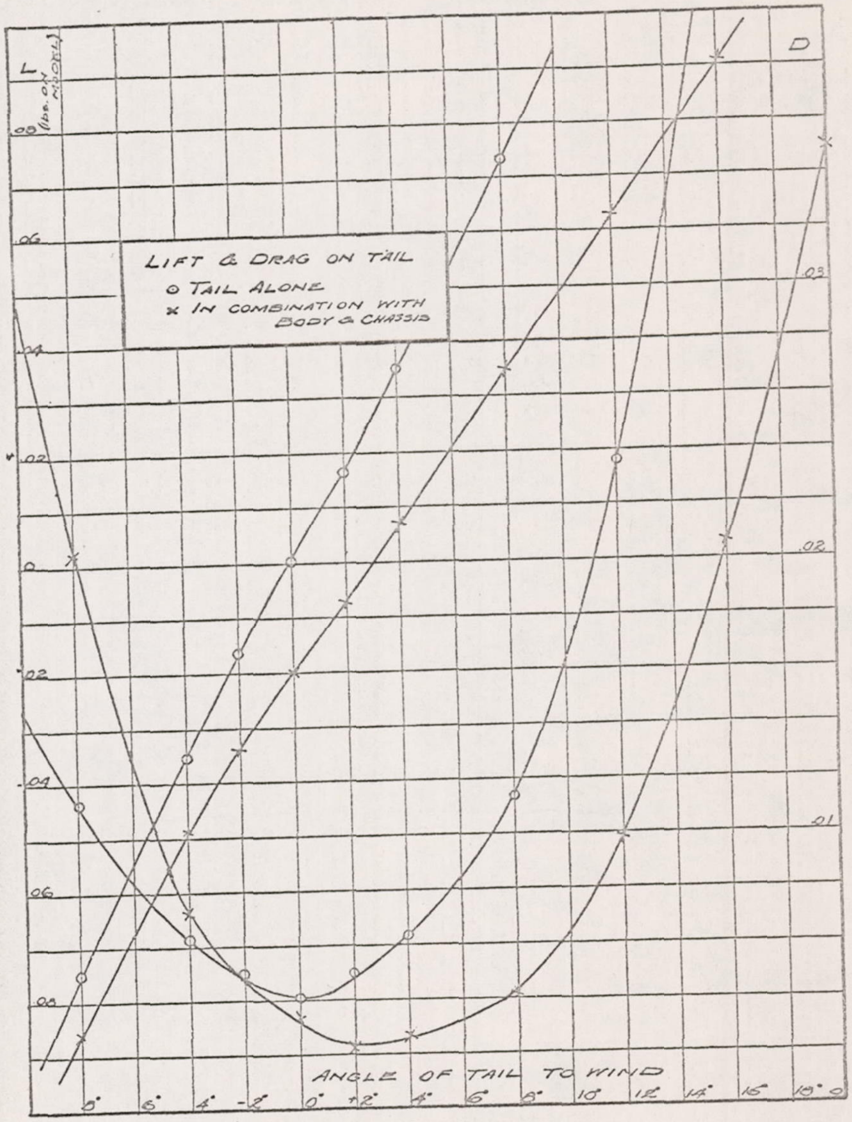

FIG. 10.

about 13 per cent lower than it should be for a rectangular plate of aspect ratio 3. The much improved lift, in view of the fact that the maximum chord of the tail is nearly as great as its maximum span, may be assigned to the raked extremities and rounded corners, as well as to the fact that the thickness was greater in proportion to the area than for the plates tested by Eiffel, and that the edges were rounded off smoothly.

The lift due to the tail in the presence of the body is also nearly proportional to the angle (measured from the angle of zero lift as a datum point), but shows a tendency to increase somewhat more rapidly at large angles than at small. The curve cuts the axis of zero lift at an angle of 3°, and the slope of the curve is about 0.75 of the slope of the lift curve for the tail alone. This change in slope may be attributed to three causes. The most obvious is that a considerable part of the tail (about 7 per cent) is actually resting on top of the body, and is virtually nonexistent, in so far as aerodynamical effects are concerned. The second is the decrease in speed of the air which has passed over the body, and the third is that, as observed by Eiffel,[1] the angle of downwash increases less rapidly than the angle of incidence. (NOTE.—This phenomenon is probably less marked than Eiffel's experiments would indicate, as he failed to take account of the second of the causes which we have mentioned). The cause for the downwash when the wings are not present is not apparent, as an upcurrent would seem more probable from the shape of the body and position of the tail.

The drag due to the tail, because of the downwash noted above, has its minimum at an angle of 2°. It is not symmetrical about this angle, increasing much more rapidly at negative than at positive angles. The minimum drag due to the tail is very small, being barely half the minimum value for the tail alone and less than 20 per cent of the minimum for the body, but it increases more rapidly than any other component, so that at 20° it is materially larger than that for either body or chassis. The drag curve for the tail in combination with body and chassis is less regular than for tail alone, the values increasing less rapidly at small, and much more rapidly at large, angles.

It should be noted that great caution must be exercised in drawing conclusions from tests of the tail, since the elevator position is neutral throughout, as is the custom in practically all wind tunnel tests, and the lift and drag are therefore considerably different from those which would arise in actual flight.

THE EFFECT ON LIFT AND DRIFT OF INTERFERENCE BETWEEN THE WINGS OF A BIPLANE COMBINATION.

In figure 11 are given the lift and drag curves for a single wing plane of the Curtiss JN-2 (with all values doubled to make them comparable with the total lift and drag for the two wings and for the complete assembly), for a biplane combination made up with the same stagger and gap as in the actual machine, for the complete machine with the tail set, as in practice, at $-3\frac{1}{2}°$ to the chord of the wings, and for the complete machine with the tail removed. To avoid confusion among so many curves the observed points have been omitted from the drawing. Every point lies within 0.005 pound of the curve to which it pertains.

The drag curves for the various arrangements do not, of course, permit of any deductions as to the biplane effect on K_x and L/D, since the effect of the struts is unknown. It may fairly be assumed,

[1] Nouvelles Recherches sur la Resistance de l'Air et l'Aviation, by G. Eiffel.

however, that these struts have no important effect on the lift, and interesting data may be obtained as to the effect which the biplane arrangement without overhang has on the lift of an actual machine. It is easily conceivable that the biplane correction for a real wing, with raked tips and with ailerons cut "on the bias" and rounded off at the corners, may be materially different from that for wings with square ends. It also permits of a comparison of the biplane lift corrections for the Eiffel 36 wing with those for other wings which have been tested as biplane combinations, and notably for the R. A. F. 6 section tested by Dr. J. C. Hunsaker.[1] [2]

The biplane lift corrections at the practical angles of flight range from 0.820 to 0.937, the large values corresponding to the large angles of incidence, and the correction ratio growing less with angle until, at about 8°, it reaches a minimum and thereafter increases in magnitude as the angle becomes smaller. This is strictly in accordance with the results of previous experiments. The maximum values of the lift coefficient, to be used in computing the landing speed, are in the ratio 0.937, as against 0.955, obtained by Dr. Hunsaker for both the R. A. F. 6 and Curtiss wings. The latter tests differed from the present ones, in addition to the points already mentioned, in that there was no stagger, the gap/chord ratio was 1.2 instead of 1, and the aspect ratio was less. Tests made by Dr. Hunsaker at the Massachusetts Institute of Technology, and by the staff of the National Physical Laboratory,[3] indicate that there is a loss of about 5 per cent consequent on the reduction of the gap/chord ratio from 1.2 to 1, and a gain of about 2 per cent from the use of a 20 per cent stagger. The exact result of changing aspect ratio in a biplane is uncertain, but it is probable that a decrease in this ratio increases the biplane lift factor slightly. Taking all these modifications into account, the lift correction obtained by us may be regarded as coinciding very closely with Dr. Hunsaker's results, that from the present experiments being a trifle the higher, and we therefore draw the conclusion that the biplane coefficients may be considered as virtually independent of the plan form of the wings. The effect of changes in section, and especially the gain from making up the two wings of different sections, remains to be further investigated. There is, of course, some loss in lift, especially at large angles, due to turbulence about the struts, although this should be slight enough not to affect the validity of the conclusions which we have based on the assumption that the strut effect was nil. Any such effect would be relatively more pronounced on the model than on the full-scale airplane with stream-line struts. Taking account of all such disturbing factors, a correction coefficient of 0.95 may be used in finding the maximum lift for a biplane combination with a gap/chord ratio of approximately 1, and a stagger of from 10 to 25 per cent. The effect of chassis, body, and tail on the landing speed will be discussed in the next section.

It was previously remarked that little can be deduced from a comparison of the drift curve for the biplane with that for the monoplane, since the effect of the struts can not be readily determined.

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[1] Stable Biplane Arrangements, by J. C. Hunsaker; Engineering, Jan. 7, 1916.
[2] Aerodynamic Properties of the Triplane, by J. C. Hunsaker and T. H. Huff; Engineering, July 21, 1916.
[3] Determination of the Effect on Lift and Drift of a Variation of the Spacing in a Biplane; Report of the British Advisory Committee for Aeronautics, 1911-12, p. 73.

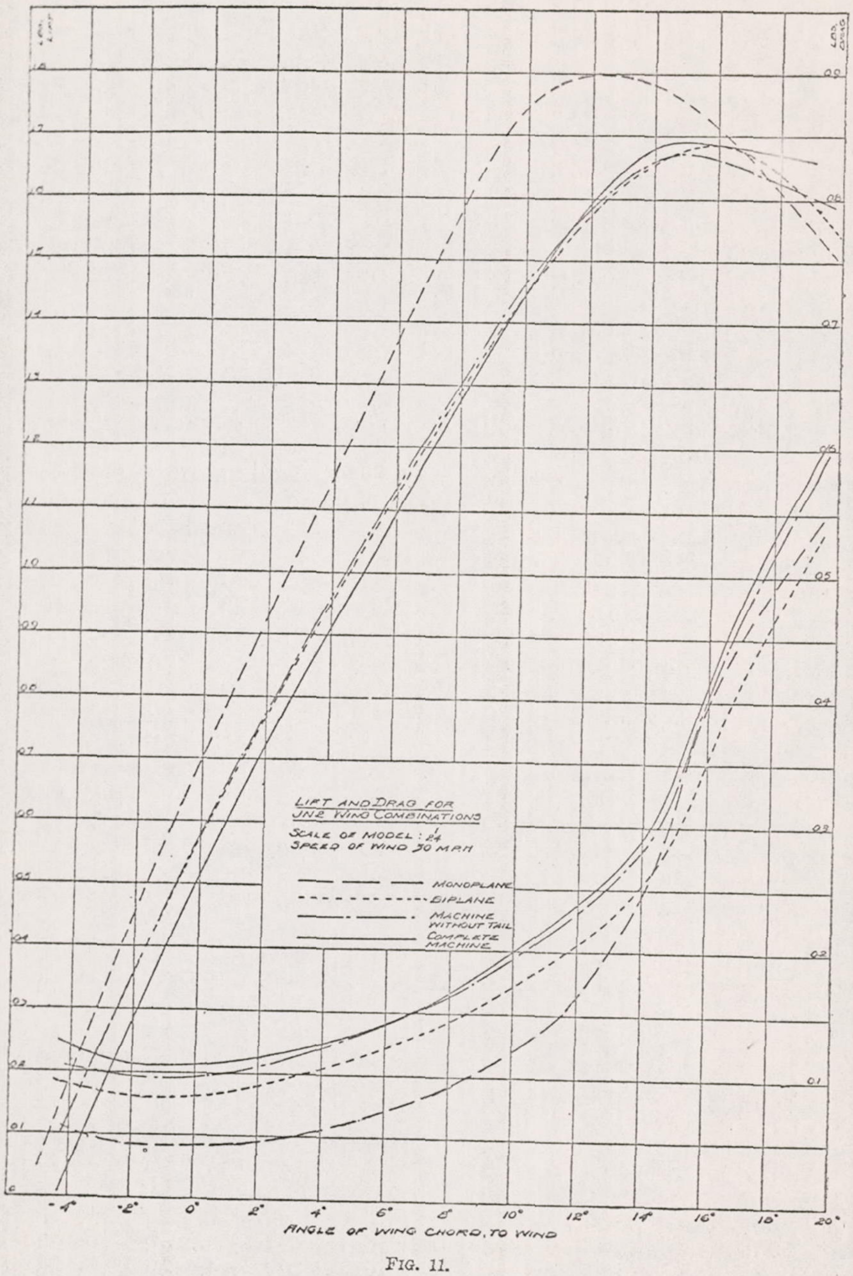
LIFT AND DRAG FOR
JN2 WING COMBINATIONS
SCALE OF MODEL: 1/24
SPEED OF WIND 30 M.P.H
MONOPLANE
BIPLANE
MACHINE WITHOUT TAIL
COMPLETE MACHINE
-4° -2° 0° 2° 4° 6° 8° 10° 12° 14° 16° 18° 20°
ANGLE OF WING CHORD, TO WIND

FIG. 11.

It will be noted, however, that the distance between the two curves constantly grows smaller as the angle increases, and that they finally actually cross each other at an angle of $14\frac{1}{2}°$. It is thus evident that the ratio of biplane to monoplane drift grows con, stantly less, and that, at angles of incidence larger than about 12°-the drag per unit area is actually less for the biplane combination than for the monoplane. This is what might be expected, in view of the screening of the upper plane by the lower, and is in accordance with the indications of other experiments of a similar nature, but it is a striking fact that the relative decrease in the drag of the biplane combination should be so marked as to give it, at large angles, an L/D superior to that for the monoplane, and the possibility of decreasing the angle at which this change occurs perhaps opens a field for future investigation.

LIFT AND DRAG CONTRIBUTED BY THE ADDITION OF BODY CHASSIS AND TAIL TO A BIPLANE COMBINATION.

At very small angles the lift curves for the biplane combination and for the machine without tail are practically coincident, showing that the lifting effect of the body and chassis is nil, or, in other words, that the downwash from the wings, acting on the rear of the body, is roughly sufficient to balance the lift arising from direct dynamic pressure on the lower surface of the body. As the angle of incidence increases, however, the two curves diverge, the separation first becoming noticeable at about −1°, and the lifting effect thus indicated increases in magnitude until, at 10°, the lift due to the body and chassis is about 0.015 pound. This figure is, of course, in excess of the lift which must be furnished by the body to replace that lost because of the containing of the part of the lower wing (about 5 per cent of its total area) within the body. The two curves cross at about 15°, indicating that the body exerts an effect opposed to the lift of the wings from there on, but the flow about the wings is so unsteady at these large angles that the measurement of the forces is comparatively inaccurate, and it would be highly unsafe to generalize on conclusions drawn from such small differences between large quantities as those with which we are dealing, and based on one or two points from a single test.

The manner in which lift is affected by the addition of a tail will be discussed more extensively at a somewhat later point, in connection with other tests under varying conditions with respect to the tail. It is sufficient to note here that the tail has a considerable effective negative lift at negative angles, that this decreases steadily until, at about 11°, the effect becomes zero, and that at larger angles it gives rise to an increasing positive lift.

The additional drag caused by the addition of body and chassis ramains almost constant, increasing very slowly, except at very large angles, where the increase becomes more rapid. It has, at 0°, a value of 0.015 pound, as against a minimum drag of 0.080 pound for the biplane combination, and 0.105 pound for the complete machine. At an angle of 12° this resistance has increased from 0.015 pound to 0.025. It will be noted that the drag caused on the complete machine by the addition of body and chassis is materially less than their parasite resistance when tested separately—about 60 per cent of that quantity under conditions of minimum resistance, to speak statistically. This reduction, which is of considerable importance in the determination of probable performance for a machine, may be attributed chiefly to decreased skin friction because of the decreased relative velocity of the turbulent air along the surface of the body. The same phenomenon will later be noted in connection with the drag of the tail.

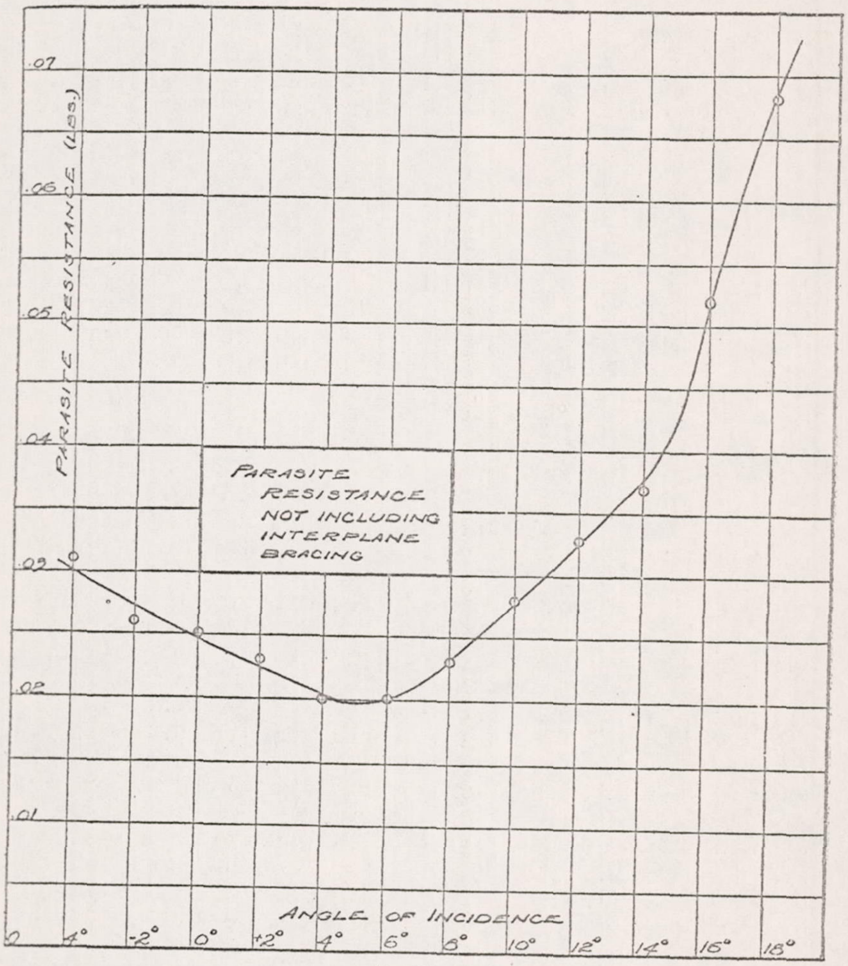

FIG. 12.

TOTAL PARASITE RESISTANCE.

In figure 12 is shown a curve of the total parasite resistance with the exception of that due to the interplane bracing. The coefficient of resistance due to body, chassis, and tail is constant within 20 per cent at all angles from 0° to 9°. Beyond the latter point the coefficient begins to increase very rapidly, but this increase would be partly counterbalanced, in an orthogonal biplane, by the decreasing resistance of the struts and wires with increasing angle. In a staggered biplane such as the JN2, this counterbalancing effect would not appear, as the struts are more nearly normal to the wind for a large angle of incidence than for a small.

The parasite resistance coefficient for the entire machine, exclusive of the interplane bracing, at 4° the units being pounds per square foot per mile per hour, is $0.020 \times \frac{(24)^2}{(30)^2} = 0.0128$. The parasite resistance coefficient for 8 struts, 5 feet long and $1\frac{1}{2}$ inches wide, having a fineness ratio 3, together with 4 similar struts $2\frac{1}{2}$ feet long, is 0.0028,[1] and the coefficient for the interplane wires roughly 0.0040,[2] making a total of about 0.02. No allowance has been made for the resistance of fittings.

EFFECT OF ANGULAR SETTING OF THE TAIL ON LIFT AND DRIFT AT VARIOUS WING INCIDENCES.

For the tests reported in this and the following section the medium body was used with the tails inclined to the wing chord successively as follows: Large tail, $-1°$, $-2°$, $-3\frac{1}{2}°$; medium tail, $-2°$, $-3\frac{1}{2}°$, $-5°$; small tail, $-3\frac{1}{2}°$, $-5°$, $-7°$. Different ranges of angles were adopted so that, as far as could be estimated in advance, the static longitudinal stability would not be excessive, nor would the instability be very great, in any test.

The results are given by four sets of curves, figures 13 to 16, inclusive Each of the first three gives the L and D curves for the three settings of some one tail. Figure 16 is a collection of the L and D curves for the three tails at $-3\frac{1}{2}°$ to the wings, and is designed particularly to show the results of varying the size of tail. An averaging of results for the three sets of graphs shows, what would be expected, that the lift increases steadily as the negative angle of the tail with respect to the wings decreases. The amount of this increase, for a given variation in tail setting, does not vary appreciably with angle of incidence, except at very large angles, and ranges from 0.010 to 0.015 pound per degree of tail angle, the larger values occurring on the small tail. The variations in effect are so small, however, that little significance should be attached to the latter fact. At angles close to and beyond the burble point, the curves spread out somewhat, the apparent effect of the change in setting becoming greater, and this has the effect of causing the burble point to occur at a larger angle of incidence as the tail and wing chords become more nearly parallel. As a concrete example, we may consider the landing speed, which was found to be 41 miles per hour for the standard machine. With the tail set at $-1°$ instead of at $-3\frac{1}{2}°$, this value would be decreased by $\frac{1}{2}$ mile an hour—a gain hardly worth taking into consideration.

At small angles the change in total drag is almost too small to determine, although a decrease in relative tail angle has a tendency to decrease the drag. At intermediate angles (the exact range differing for the three tails) the three curves merge together. At some

[1] Research on Struts of Varying Fineness Ratio; Report of the British Advisory Committee for Aeronautics, 1912-13, p. 111.

[2] Experiments on the Resistance of Wires; Report of the British Advisory Committee for Aeronautics, 1912-13, p. 126.

angle between 6° and 12° they separate, and the drag is thereafter greatest for the least angle of setting, just as is always the case with

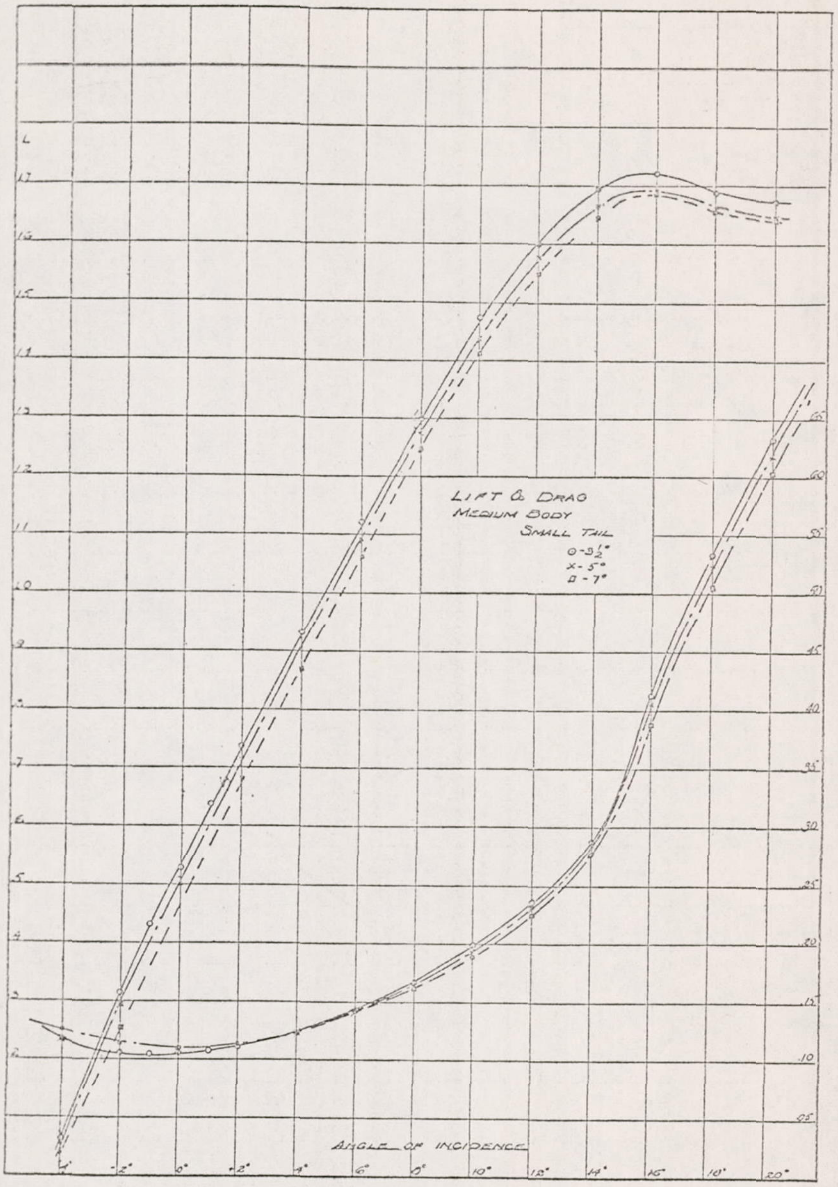

FIG. 13.

the lift—this is explainable by the fact that as the angle of setting of the tail increases its zero incidence occurs at greater angles of the wing chord.

Since the angle of maximum speed with a 90 horsepower engine for the machine under investigation corresponds very closely to the angle of minimum drag, and hence to the point where the slope of

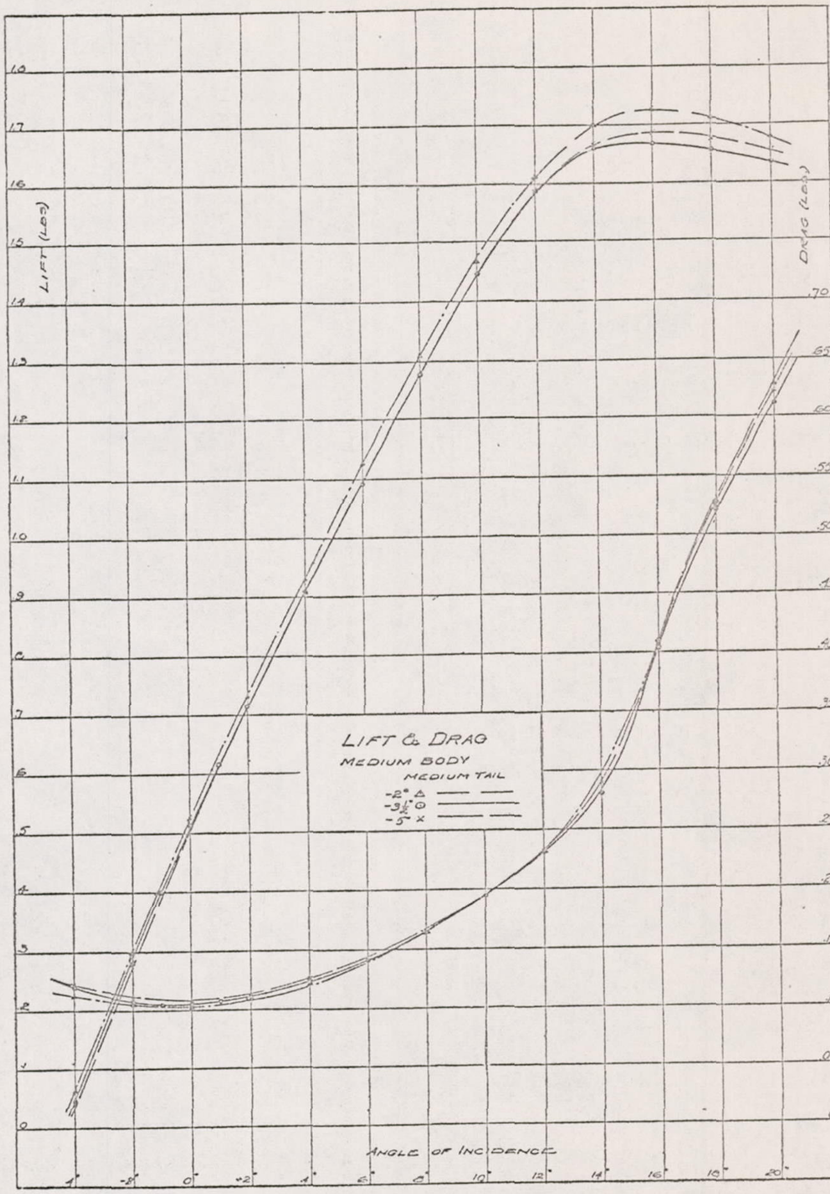

FIG. 14.

the drag curve is zero, the maximum speed is unchanged by what is in effect a shifting of the lift curve to the left. What change there is will be due to the change in drag, but this is so slight that no variation in tail setting within the bounds of reason is likely to alter the maximum speed by more than 1 mile per hour. The effect on climbing speed will be somewhat greater, as the angle of incidence for best climb corresponds to the rising portion of the drag curve, but even

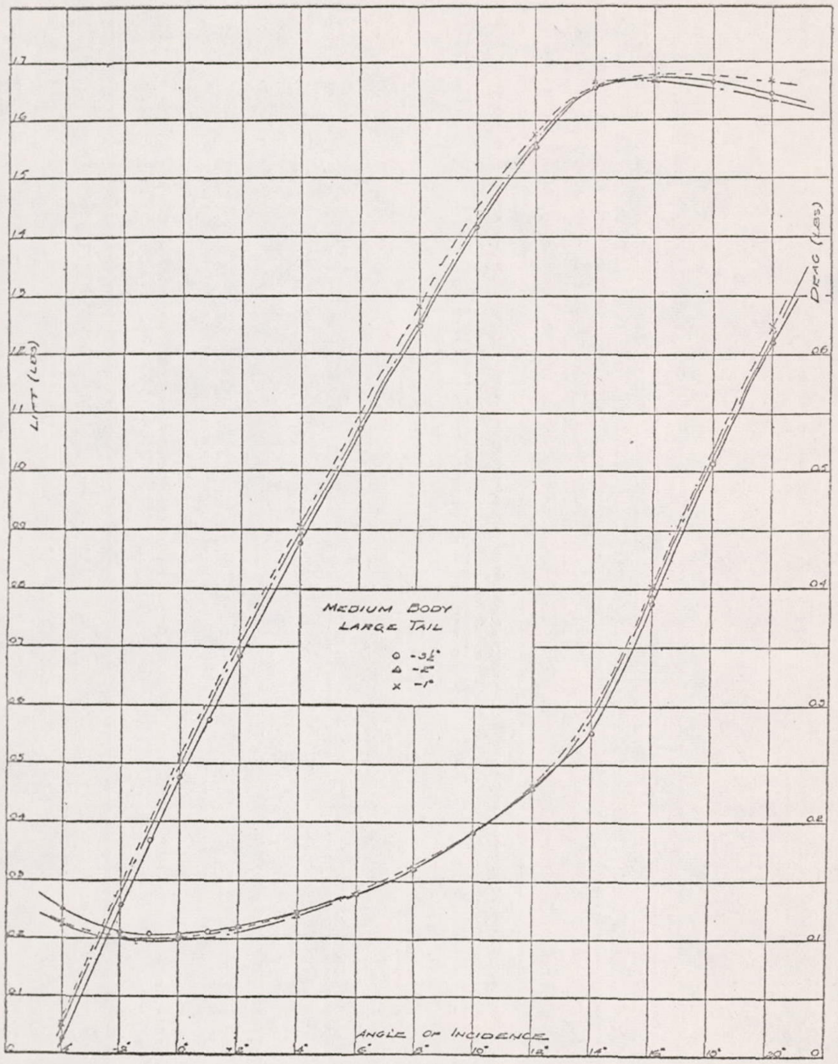

Fig. 15.

here it is not considerable (probably never enough to change the horsepower required at any point by more than 3 horsepower).

In résumé, it is apparent that the effect of tail setting on the efficiency of such a machine as the Curtiss JN2 is quite negligible, and that the tail angle should be chosen purely from considerations of stability.

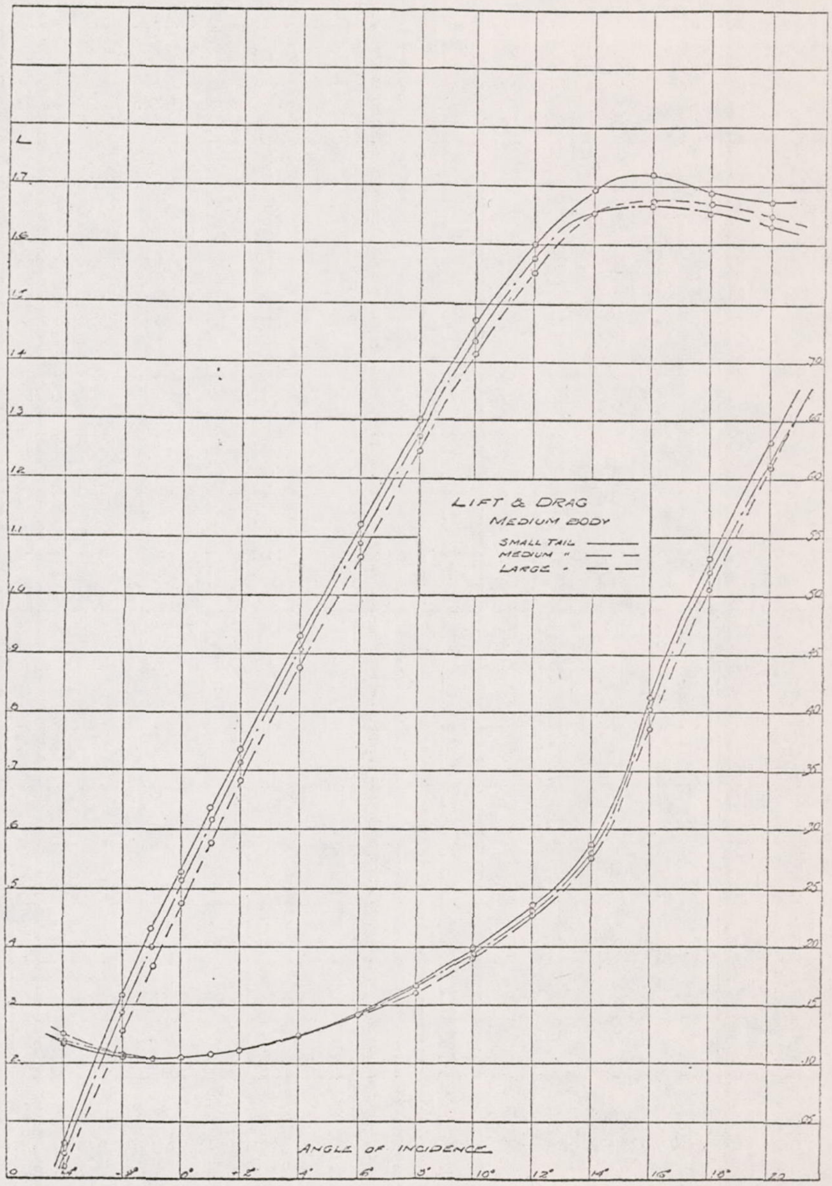
L
LIFT & DRAG
MEDIUM BODY
SMALL TAIL
MEDIUM "
LARGE "
ANGLE OF INCIDENCE

FIG. 16.

EFFECT OF VARYING SIZE OF TAIL, KEEPING ANGLE OF SETTING CONSTANT.

The curves for the machine with the three tails already described, the tail being set at $-3\frac{1}{2}°$ to the wing chord in every case, are plotted in figure 16. These curves show that the lift for the whole machine throughout ranks inversely as the sizes of the tails—that is, it is greatest for the small tail and least for the large tail. The spacing between the three curves is nearly constant. The arrangement of the curves in this order is what would be expected at small angles, where the total force due to the tail is downward and the negative effect is naturally least for a tail of small area, but the reason for such behavior at large angles is less obvious. While it would be impossible to draw definite conclusions without making an exhaustive investigation of the pressure distribution over the surface of the tail, the most probable hypothesis to account for the phenomenon is that the downwash from the wings is less felt near the body than out in the open and that the farther away from the body one gets the greater the downwash angle becomes. The mean downwash angle will then be larger for the large tail than for the small, and the lift (taking account of sign) will always be less for a large tail than for a small one.

The drag, too, is largest for the small tail at angles equal to and greater than 2°. From $-2°$ to $+2°$ the curves merge together, and at negative angles greater than $-2°$ the drag for the small tail is least. This, too, may be accounted for by the hypothesis stated above in conjunction with the fact that at the points of maximum downwash (i. e., the parts farthest away from the body) there is probably an actual negative drag on the tail, due to eddying and the existence of pressure on the top of the tail. This is analogous to the force which when a pair of plates are exposed in tandem tends to draw the rearmost forward into the wind.

EFFECT OF VARYING LENGTH OF BODY AND SIZE OF TAIL AT THE SAME TIME, KEEPING CONSTANT MOMENT OF TAIL SURFACE ABOUT THE CENTER OF GRAVITY.

The reason for adopting this method of testing relates especially to the pitching moments, but the results can be used to show the way in which lift and drag are affected by the variation of the distance between tail and wings.

Figure 17 represents the lift and drag for the machine with the medium and short bodies, each carrying the large tail at an angle of $-3\frac{1}{2}°$ to the wing chord, while figure 18 gives similar data for the medium and long bodies in conjunction with the small tail. In the case of the first, the lift for the two bodies is virtually identical at angles less than 5°. At this point the two lift curves diverge, the lift for the short body being the greater, and the divergence becomes steadily greater as the angle of incidence increases until at 16° there is a difference in lift of over 0.03 of a pound, so that the landing speed would be somewhat reduced by shortening the body, quite aside from the fact that the weight of the machine would be markedly decreased by a reduction of 10 per cent in the length of the body.

The drag for the two is identical within the experimental error up to an angle of 10°, beyond which angle the curves separate in the same way as for the lift, the drag being greater for the small body.

In the case of the small tail, the lift is about 0.01 pound more with the medium body than with the long one at all angles from −4° to

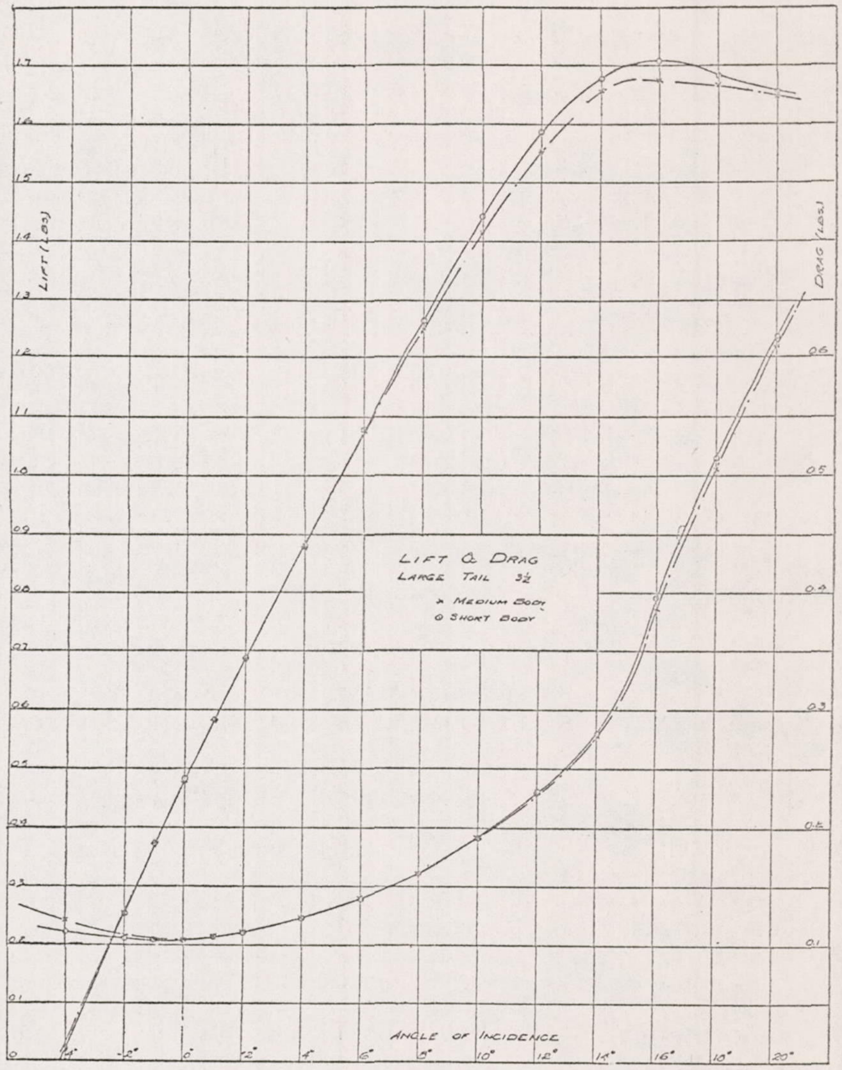

Fig. 17.

12°. The two curves then come together, being virtually coincident at angles beyond 14°. The drag for the medium body is greater than for the long at all angles, the difference being very small at small angles, and increasing steadily to over 0.01 pound at 18°.

These results, like those of the last section, at first sight seem quite unreasonable, and their fair interpretation requires an examination into the actual conditions of flow about, and in the rear of, a wing.

Photographic investigations of the flow about a wing section in a water channel, carried out at the National Physical Laboratory [1] show that the fluid behind the wing, especially at large angles of

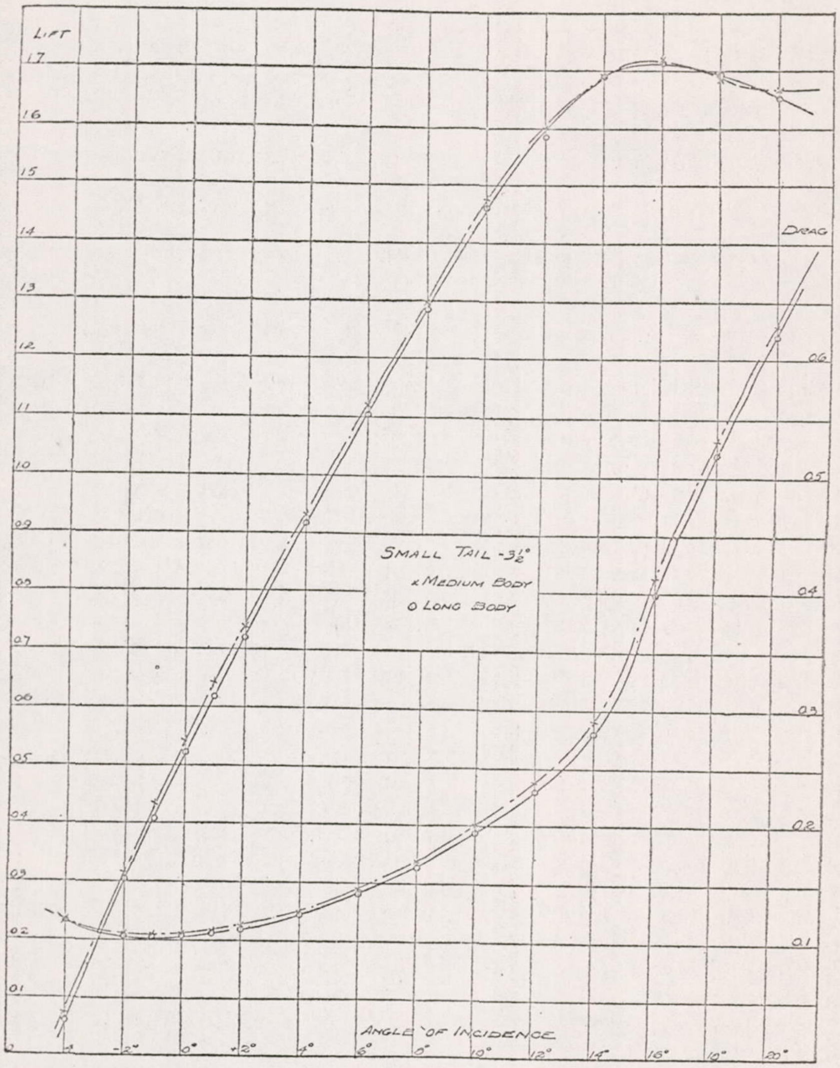

FIG. 18.

incidence, forms marked eddies, and, on the dissipation of these, takes up a wave motion extending backward for a considerable distance. It is, therefore, probable that there is some point or points where the downwash angle is a maximum, and a motion in

[1] Photographic Investigation of the Flow Round a Model Aerofoil, by E. Relf; Report of the British Advisory Committee for Aeronautics, 1912–13, p. 133.

either direction from these points will result in a decreased downwash and increased lift and drag. It appears that this is what has happened in the present case, and it would undoubtedly be found that, if the body should be shortened up still farther than was done in the tests with the large tail, the lift would be a maximum for some length, beyond which any further shortening would diminish it.

The gains in efficiency consequent on shortening the body depend chiefly on the reduction in weight, permitting also a further reduction in area. The direct gain in lift is small, as it was for changes in the angle of tail setting, although it is by no means negligible. The application to other airplanes of the results obtained from this particular set of tests is not to be recommended, however, since the effect of changing the body length might be quite different when a different machine, using a different wing section, was affected.

A QUANTITATIVE DISCUSSION OF THE FORCES ON THE TAIL AND THE EFFECTS OF DOWNWASH.

Although we have now examined the characteristic curves for the complete machine in a considerable number of cases (11 in all), as well as for the machine without the tail, we have not yet made any attempt to correlate the figures for tail effect, or to secure any measure of the downwash angle, and this subject will be treated next.

Enough has been done to make it evident that no single figure or formula can express the degree of downwash, which varies with distance from wings, angle of incidence, type of body, and is not even the same on all parts of the tail at a given time. Any formulæ that are given, therefore, must be accepted with due reservation, as representing a "mean effective" downwash which, if it actually corresponded to the conditions of flow, would give rise to the same tail effects as those observed. It is further obvious that the figures thus secured will not apply to the effects of the tail on the drag curve, as the eddying flow above and behind the tail actually results in its having a negative drag at times.

In figure 19 are plotted the lifts due to each of the three tails when attached to the medium body at an angle at $-3\frac{1}{2}°$ to the wing chord. The wavy curves, drawn in full lines, pass through all the points with the exception of one or two which were obviously very far off. Although the peculiar shape of these curves may be due in some part to observational errors, which would show forth very much exaggerated on such a plot as this, it will be noted that the curves roughly parallel each other, and it is probable that the irregularities represent approximately a condition actually present. Such irregularities may be accounted for on the hypothesis stated in connection with the tests of different body lengths, the lift due to the tail varying in an irregular manner with the angle of incidence, since the length and amplitude of the fluid waves back of the wing change with the angle, and the position of the tail with respect to the wave form is consequently altered. As a measure of simplification, however, and for possible use in the framing of empirical rules, ideal curves have been drawn with all irregularities removed, and these lie within 0.005 pound of the more exact curves at all points. These faired plots curve slightly upward, the curvature being greatest near the middle of the curve.

The curves for the other six cases in which the medium body was employed were plotted in a similar manner, and led to the same conclusions, but lack of space has prevented their inclusion herewith.

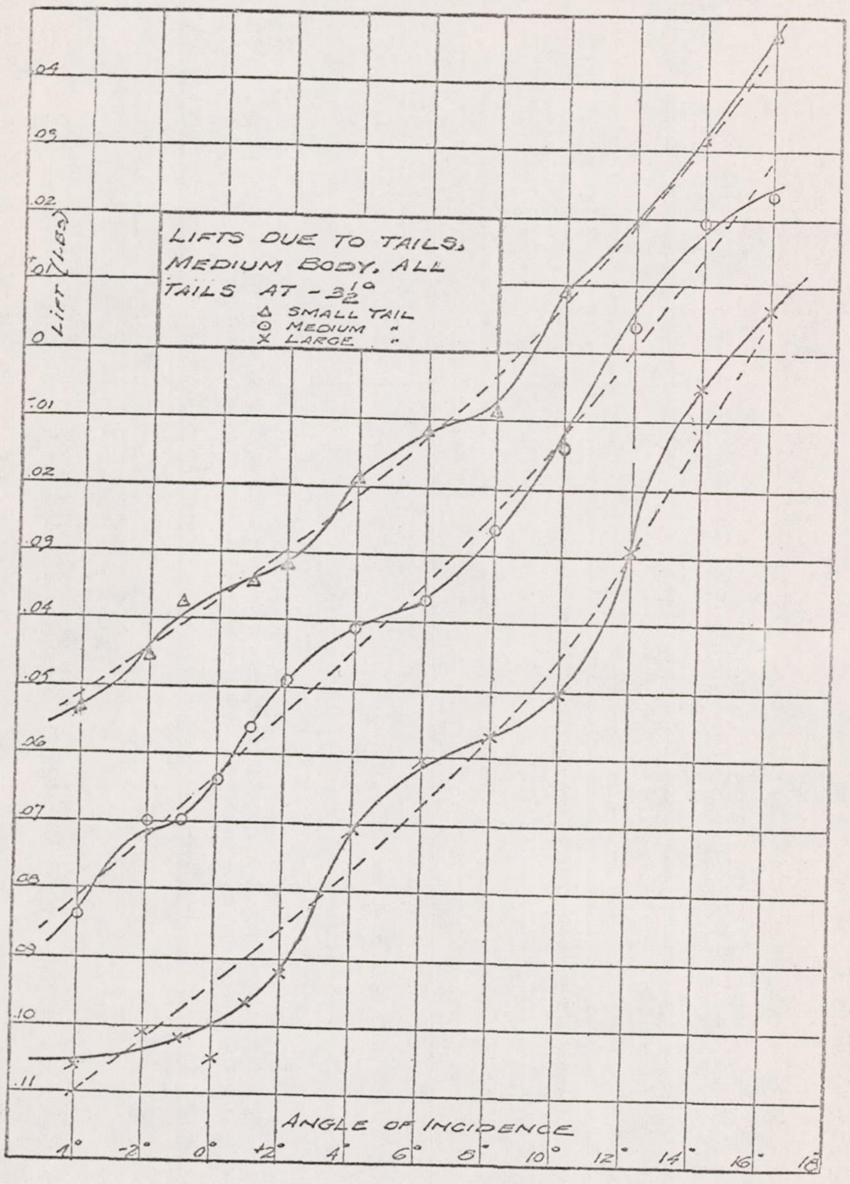

FIG. 19.

An incomplete investigation of the effective downwash angle (i. e., the difference between the angle of the tail to the wind and the angle of incidence at which the tail, tested alone, would give the same lift as that which it actually contributes to the machine) indicates that, as was also shown by Eiffel,[1] the graph of downwash angle against angle of incidence, can be at least approximately represented by a straight line. (In the present experiments, a broken line, its two portions meeting at an angle of incidence somewhere between 6° and 10°, gave greater accuracy, though at the sacrifice of simplicity.) Eiffel's formula $\propto = 1 + \frac{1}{2}i$, does not, however, suit

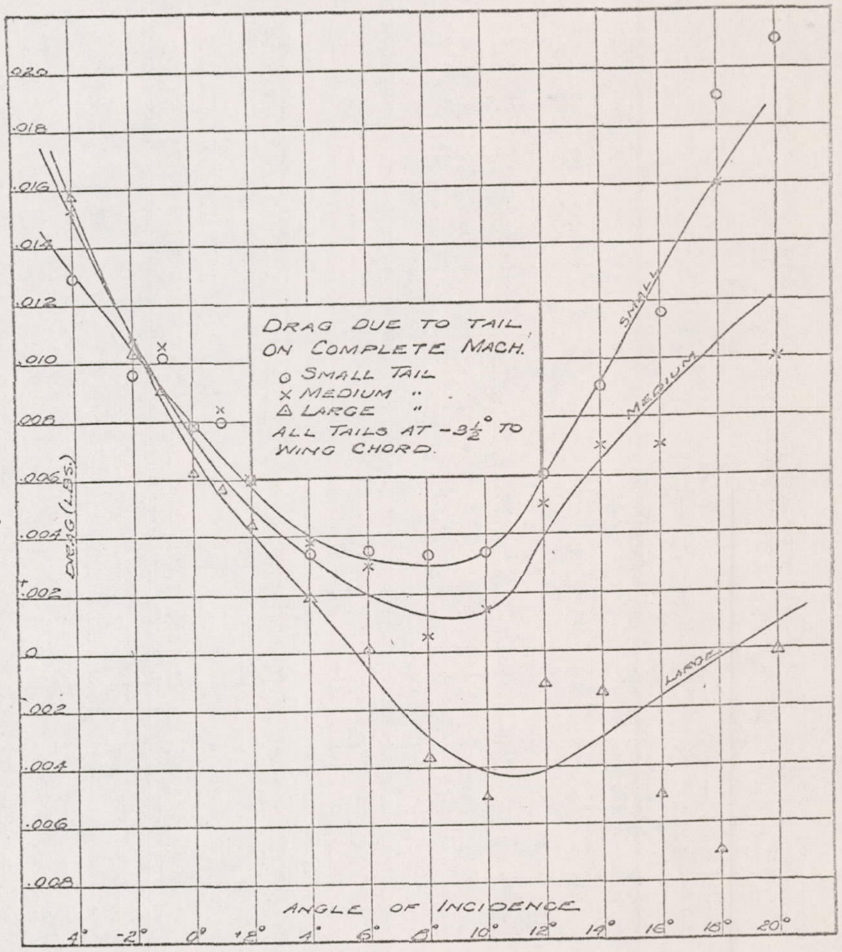

FIG. 20.

our results so well as one with a larger constant term, the discrepancy doubtless being due to the presence of the body and to the use of the flat tail, in place of two wings in tandem. The equation of the straight line plot for the mdium tail set at $-3\frac{1}{2}°$, for example, is: $\propto = 3\frac{1}{2} + \frac{3}{8}i$, and this is a fair average of the results obtained. They are not given in extenso, as they were not sufficiently consistent for comparison to be useful.

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[1] Nouvelles Recherces sur la Resistance de l'Air et l'Aviation, by G. Eiffel.

In Figures 20 and 21 have been plotted the effect which the presence of the tail has on the total drag of the airplane. The first shows this effect for the three tails in connection with the medium body, each of the tails being set at 3½°, and the second relates to the medium-sized tail set at its three different angles.

The first set of curves manifests more clearly a point to which we have already called attention, that the drag is least for the largest tail, clearly indicating a region of negative drag. The drag due to the medium tail falls very nearly to zero, and that for the large tail actually becomes negative over a considerable range of angles. The drag increases rapidly as the angle of minimum resistance is departed from in either direction.

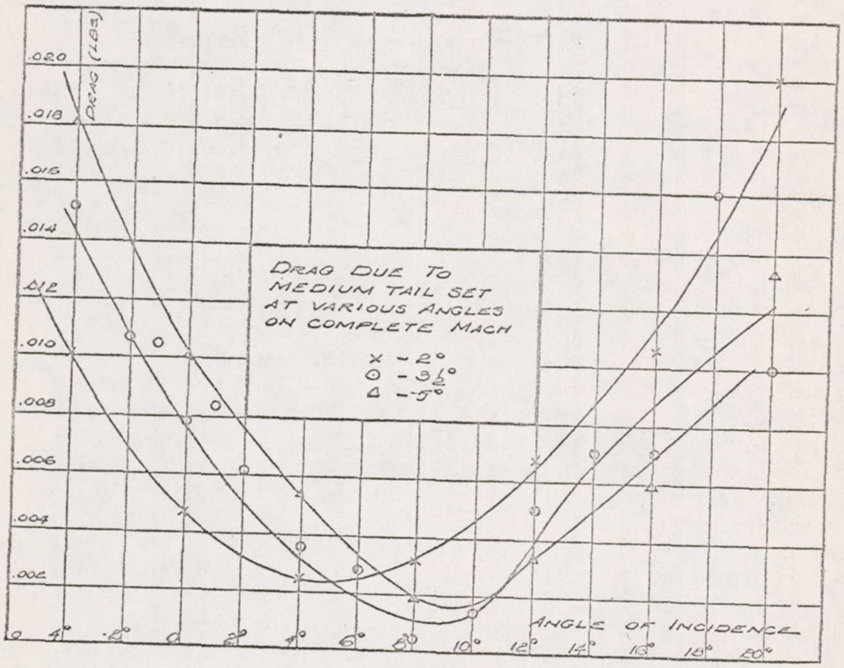

FIG. 21.

In the case of the curves showing the effect of varied angle, it is evident that, as might be foreseen, the three curves are very nearly parallel, simply being displaced horizontally with respect to each other by an amount roughly corresponding to the change in angle of setting. The minimum values all lie between 0.001 and 0.0024 pounds, the difference being well within the probable experimental error in view of the indirect method by which the figures were obtained. Taking an average value, we find that the drag due to the tail is a minimum at or near that angle of incidence at which the angle of the tail to the path of the flight is +4°. In other words, the angle of minimum drag and the angle of zero lift due to a symmetrical tail tested in the presence of the wings are very far from coinciding, the latter being the greater by several degrees.

EFFECT OF SIZE AND SETTING OF TAIL ON STATICAL LONGITUDINAL STABILITY.

We are now able to consider the stability of the airplane and the manner in which it is affected by variations in the design. This stability is best investigated with reference to the moments about the center of gravity of the machine, as the popular vector diagram, while it possesses the merit of simplicity, does not give a true criterion of stability except at the angle where the machine is in equilibrium with the elevator neutral. In order to insure statical stability at all angles the pitching moment must always decrease (the positive sign being given to stalling moments) as the angle of incidence increases, or, in other words, the slope of the moment curve must be negative throughout the range of normal flight angles. On the other hand, it is obvious from a moment's consideration, as well as deducible from Bairstow's solution of the general stability equations, that the slope of the curve should not be excessive, as too much statical stability results in a very short pitching period, which is uncomfortable for the pilot. Ultra-stable machines are also subject to the disadvantage that they require large elevators, moved through a considerable range of angle, to balance them at angles of incidence far removed from the normal.

The complete machine was tested under 11 different conditions, as already described in detail in connection with lift and drag. The moments about the spindle were measured with a calibrated torsion wire, according to the usual procedure. Since so much depended on the flow of air from the wings to the tail, and since it was feared that the straight spindle generally employed might unduly interfere with this flow, it was discarded and an offset spindle, bent through right angles at three points and passing into the bottom instead of the side of the body, was substituted. The position chosen for the spindle gave a center of rotation, about which the moments were measured, just above the trailing edge of the lower wing. The moments about the center of gravity were computed by a process explained in detail by Dr. Hunsaker's paper,[1] and which need not be gone into here. The center of gravity has been chosen, in every case, in such a position that the machine was in equilibrium at an angle of incidence of 2°. This necessitated using a different position for the center of gravity in each case, the extreme movement being about one-fourth inch on the model, corresponding to 6 inches on the machine.

The resulting curves are plotted in figures 22 to 25, the moments being reduced to foot-pounds per unit mass (slug). The mass of the Curtiss JN2, ready for flight, is 55.9 slugs. It will be seen that they are very similar in general shape, and that there are no abrupt decreases in slope except in the case of the medium tail at −2°. In this case the discrepancy with the other curves is very probably due to an error. The stability, represented by the slope of the moment curve, is always least at or near an angle of incidence of 3°.

The curves speak for themselves, and it is difficult to draw any specific criteria for stability, especially since the degree of statical

[1] Experimental Analysis of Inherent Longitudinal Stability for a Typical Biplane, by J. C. Hunsaker: First Annual Report of the National Advisory Committee for Aeronautics, p. 36.

stability to be wished for is not definitely known. All of the cases tested give satisfactory stability. It is apparent that a decrease of 10 per cent in the size of the tail has an effect equal to that of a decrease of 2° in the angle of setting, and a consideration of all factors of stability, control, etc., would seem to point to the use of a tail of

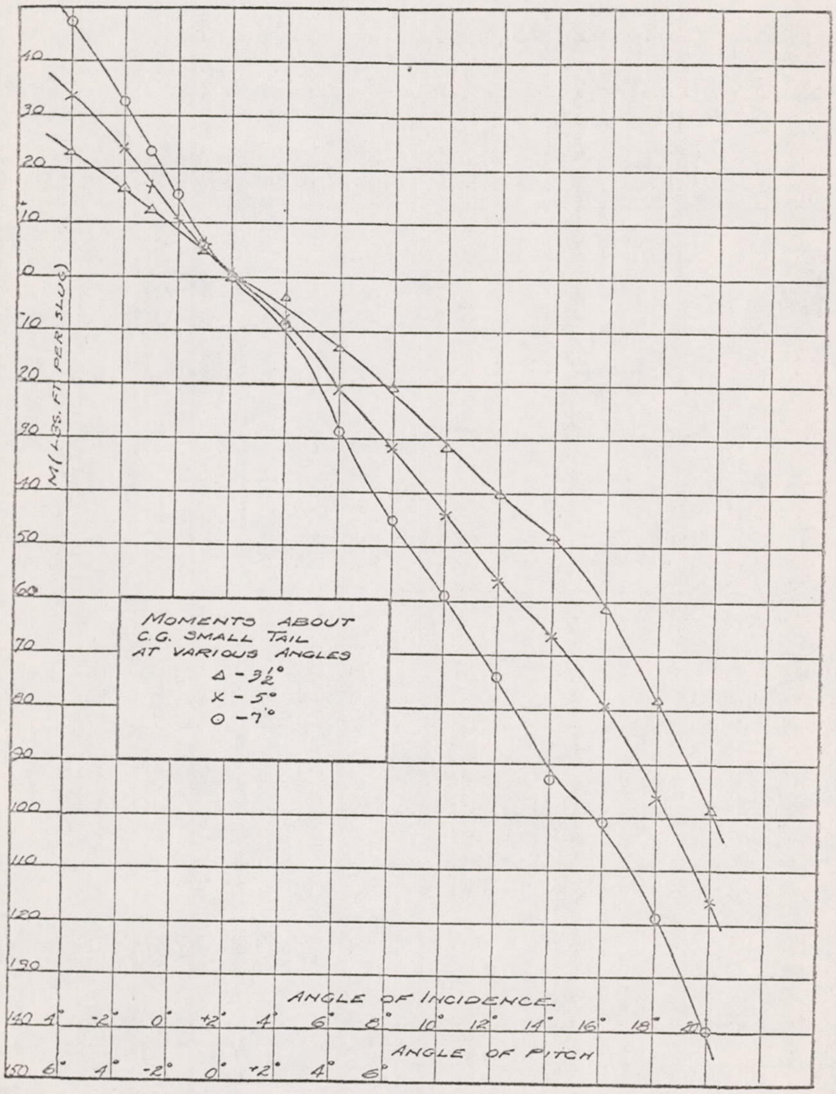

FIG. 22.

large size set at a small angle relative to the wings. This recommendation is fortified by the decreased drag from such an arrangement, this factor alone being enough to balance the slight increase in weight. Even with a tail of the present size the angle might be decreased to −2° without prejudicial results, and the ease of motion would probamoved back nearer the 14° vector. A backward movement of the center of gravity, too, has the effect of decreasing the stability, since the change in moment arm is the same for every vector, and the moments are consequently most increased where the force is greatest;

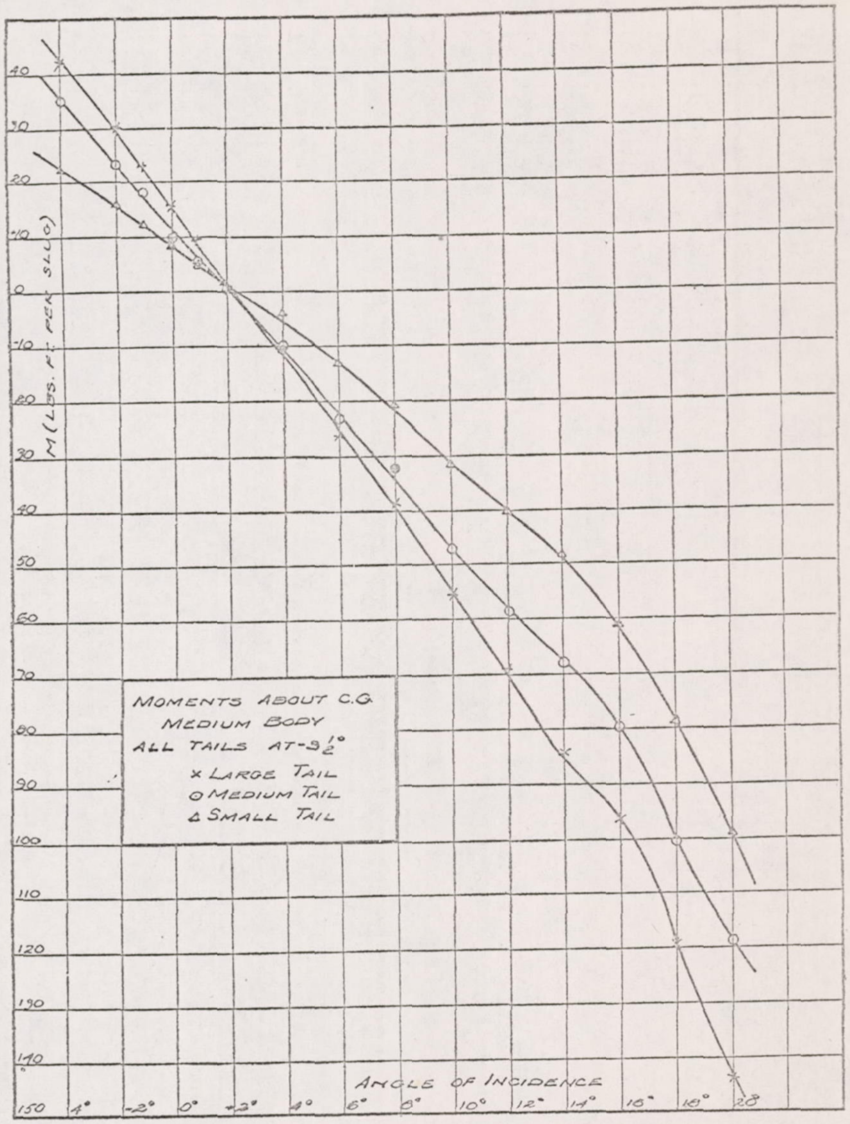

FIG. 25.

i. e., at large angles of incidence. The result is to flatten the moment curve.

For the sake of completeness, and to facilitate comparison with other machines, the vectors for the JN-2 have been superposed on the side view in figure 1, and a vector diagram for the biplane combination has been drawn in figure 26.

EFFECTS OF LENGTH OF BODY ON STABILITY.

Figure 27 shows the moments for the JN2 equipped with the standard body and with the long and short bodies, the tails used being such that the product of their area by their distance from the center of gravity of the machine was the same in the three cases. It would then appear that, if the angle of downwash were the same in the three cases, the moment curves should be sensibly identical, and this is actually the case. The short and medium bodies gave moments so

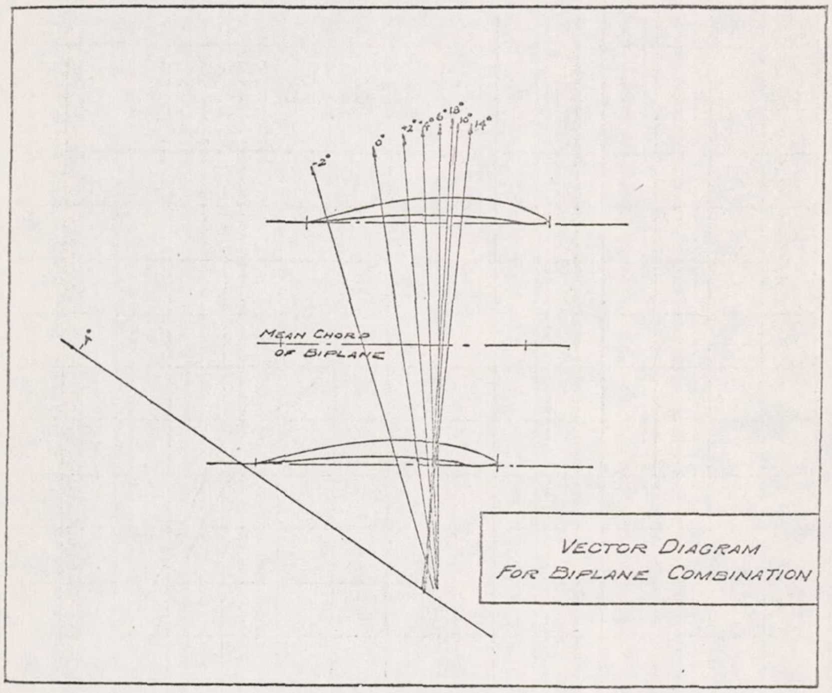

FIG. 26.

nearly the same at all angles that one curve represented both sets of points, while the stability of the long body combination with the small tail was slightly less. In a previous section we have discussed the angle of downwash, and deduced that it varies somewhat with the length of body, and that the effectiveness of the tail surface also varies with its distance away from the body. These and other similar effects are all small, however, and it appears that they virtually balance each other in respect of moments.

In figure 28 are plotted the moment curves for the long and medium bodies in combination with the small tail at an angle of $-3\frac{1}{2}°$ to the wing chord. The stability is greatest for the long body, as would obviously be the case, but the effect of changing length is less than might be expected, and the inevitable conclusion is that, so far as statical longitudinal stability is concerned, a considerable decrease in the length of the body over present practice is permissible, and may

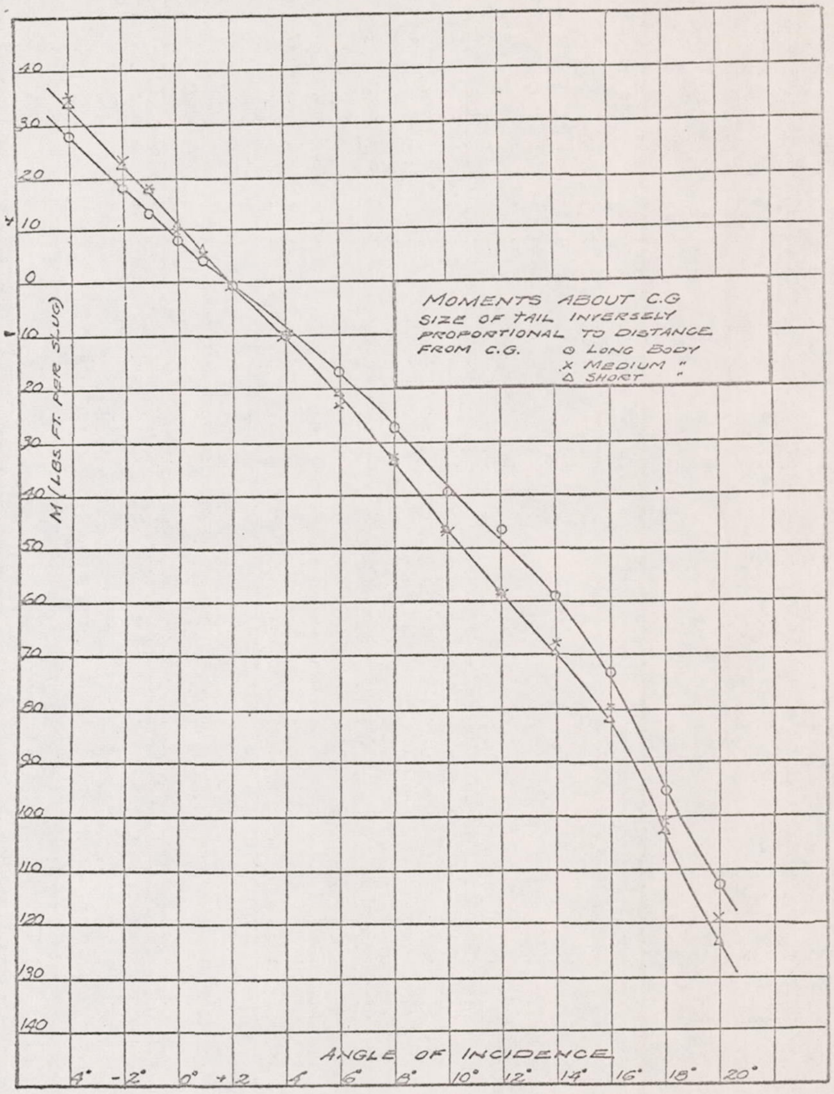

FIG. 27.

be strongly desirable. Of this, too, we can speak with more certainty in connection with the determination of damping coefficients and the study of the periodicity and damping of the general longitudinal motion.

THE EFFECTS OF THE VARIOUS ELEMENTS OF AN AIRPLANE ON LONGITUDINAL STABILITY AND THE PLACING OF THE FORCE VECTORS.

Although the above subject was not extensively investigated, tests were made for the single wing, for the biplane combination, and for

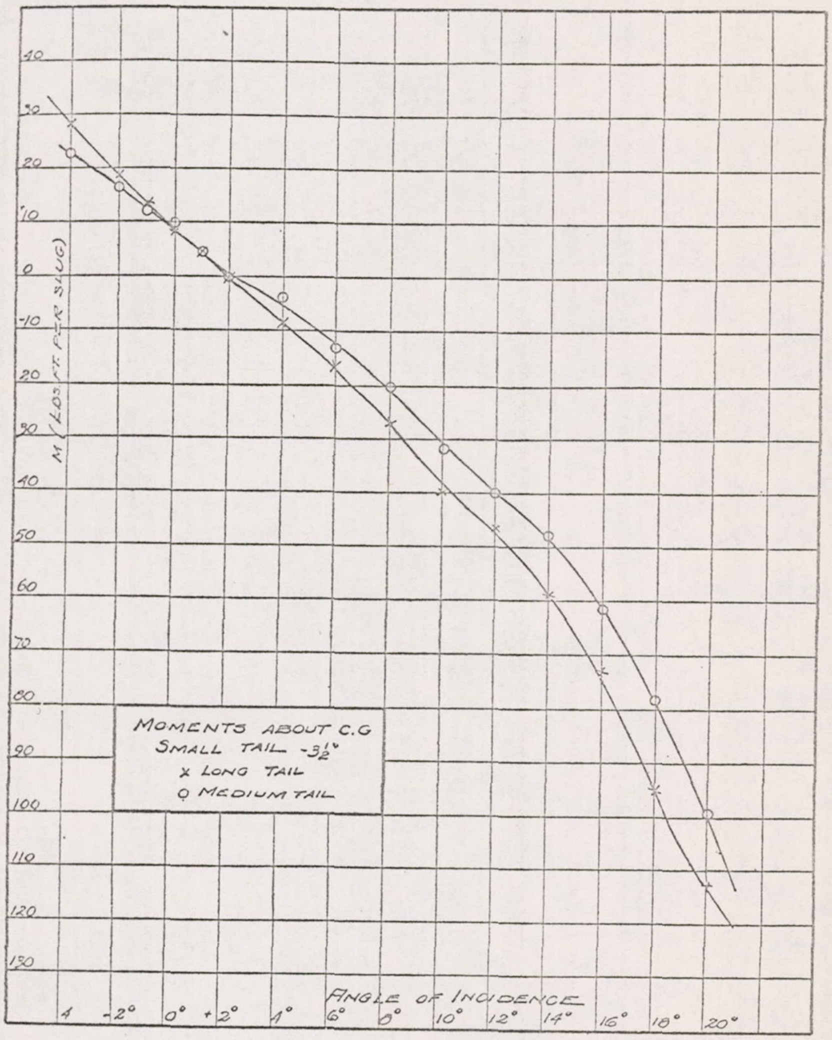

Fig. 28.

the machine complete except that the tail was lacking. The results of these experiments have been plotted in two different ways. In figure 29 we have plotted the travel of the center of pressure of the single wing and of the biplane combination, the latter being defined as the point of intersection of the force vector and a line parallel to the chord of the wings and midway between them. The chord of the biplane combination is limited by the lines connecting the leading and trailing edges, respectively, of the upper and lower wings. Secondly, figure 30 shows the moments of the biplane combination and of the machine with tail removed, the moments being referred to the point located as the center of gravity of the standard machine, with medium tail set at $-3\frac{1}{2}°$. The difference between the above two quantities is also plotted, this representing the effect of body and chassis.

The travel of the center of pressure is closely similar for the single wing and for the biplane combination (with struts, of course, included) is very similar, but the biplane center of pressure is slightly farther back through the greater part of the range, the maximum separation in this portion being about $1\frac{1}{2}$ per cent of the chord.

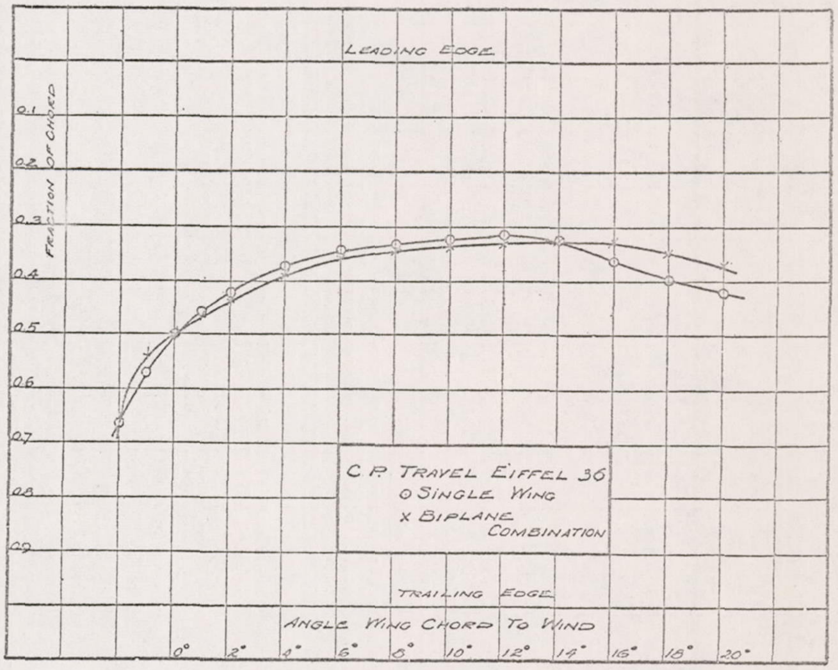

FIG. 29.

The biplane curve turns less abruptly as the angle increases, so that at large angles the center of pressure is farther back for the single wing. The dotted line in this figure represented the position of the center of gravity of the machine under standard conditions.

From figure 30 we see, as is equally obvious from a cursory inspection of the vector diagram for the complete machine, that the biplane combination exerts a diving moment about the center of gravity at all angles of incidence. The machine without the tail exerts an even greater diving moment at all points, indicating that the sign of the moments due to the body and chassis is always negative. This is due chiefly to the resistance of the chassis, centered far below the center of gravity. The moment due to the addition of the tail is zero at between 10° and 11°. This angle of zero pitching moment checks with a fair degree of accuracy with the angle of zero tail lift, already determined. Although the slope of the pitching moment curves for the biplane combination and for the machine without the tail is everywhere negative, it must not be inferred that this indicates stability. The slope of the curve is a satisfactory criterion only when the moments are related to a point at which the system is in equilibrium at some normal angle of incidence, and this is not the case here, as the moments are everywhere negative. If a moment axis be chosen such that the moment about it is zero anywhere between $-2°$ and $+20°$, it will be found that the curve has a positive slope through at least a part of its range.

In order to define the position of the center of gravity of the machine, and to furnish a guide to designers in choosing a position for that point which will give equilibrium at the desired angle of

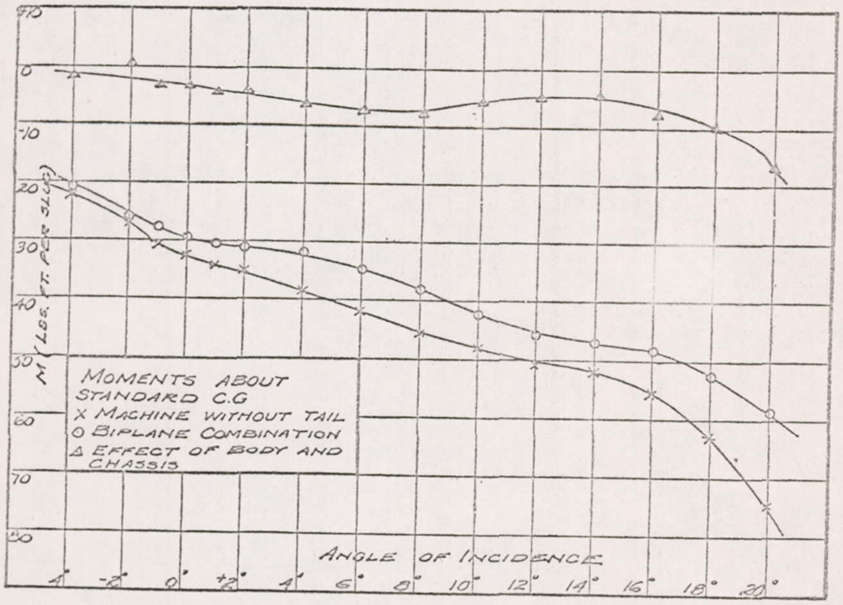

Fig. 30.

incidence, a line has been drawn connecting the center of pressure of the two wings at 2°. The horizontal distance between the middle of this line and the 2° force vector was then measured and multiplied by a proper scale ratio to convert it to full size, thus giving the distance, in a horizontal line, from the mean center of pressure of the wings to the center of gravity, assuming that the airplane flies at 2° incidence with the elevator neutral. The same process was carried through for each of the cases, both for 2° and for 4°, and the distances are tabulated herewith. The center of gravity was assumed to lie on the line of thrust, but the vectors for the angles in question are so nearly vertical that any reasonable raising or lowering of the center of gravity relative to the wings will affect its fore-and-aft location only a very slight degree. The center of the line connecting the individual centers of pressure of the wings was used to locate the datum plane in preference to the center of pressure of the biplane combination, as it is much easier to secure information on the center of pressure travel for a single wing of a given section than to secure similar information for a combination of two wings.

Body.	Tail.	Angle of tail to wing chord.	Distance from center of pressure to center of gravity.	
			2°.	4°.
		°	*Inches.*	*Inches.*
Medium	Small	3½	13	8
Do	do	5	14	9
Do	do	7	18	12
Do	Medium	2	10	5
Do	do	3½	13	8
Do	do	5	15	9
Do	Large	1	8	4
Do	do	2	10	5
Do	do	3½	14	8
Short	do	3½	13	8
Long	Small	3½	12	6

This table shows that the position of the center of gravity to give equilibrium at any given point small angle is nearly independent, within reasonable limits, of the length of the body and the size of the tail. It is, however, materially affected by the angle at which the tail is set. As the angle of equilibrium increases, the required position of the center of gravity approaches the center of pressure of the wings alone with great rapidity.

REPORT No. 17.

PART III.

DYNAMICAL ANALYSIS.

By Alexander Klemin and Edward P. Warner and George M. Denkinger.

FUNDAMENTAL PRINCIPLES OF DYNAMICAL STABILITY.

Before taking up in detail the dynamical stability of the Curtiss JN2, we shall briefly tabulate, for purposes of reference, the well-known principles on which the treatment of dynamical stability depends, and shall discuss the methods of applying those principles.

It has been found by Bryan [1] and other investigators that the general equations of motion of an airplane, with symmetry taken into account, reduce to two sets of equations of the fourth degree in λ, λ being the logarithmic increment or decrement of the oscillation, one of which equations corresponds to symmetric, or longitudinal, the other to asymmetric, or lateral, oscillations. The second of these equations does not enter into the present investigation in any form, and we shall not discuss it. Before proceeding to an examination of the first, it is necessary to describe the notation adopted.

The origin is located at the center of gravity of the airplane. The three mutually perpendicular axes of reference are fixed in the machine in such a position that they are parallel and perpendicular to the relative wind when the machine is in steady horizontal flight. They therefore change their position with respect to the earth as the airplane oscillates. When there is a change in speed of flight, however, and consequently in angle of incidence, the axes change their position in the machine. These axes are denominated the x, y, and z axes, and the forces parallel to them, respectively, are called X, Y, and Z. The x axis is parallel to the relative wind, the y axis is parallel to a line connecting the wing tips, and the z axis is vertical. The moments about these axes are denominated, respectively, L, M, and N. The components of velocity parallel to the x, y, and z axes are called u, v, and w, and p, q, and r, similarly, are the components of angular velocity about these axes, and corresponding to the moments L, M, and N.

It has been shown that the longitudinal motion may be considered as entirely independent of the side slipping velocity v and of the angular velocities of roll and yaw—p and r. This is not strictly correct in every case, a side slip having a distinct influence on the drag, and a roll affecting both lift and drag, for example, but it is necessary to make the approximation in order that the equations of motion may simplify as described above. Each of the five coefficients in the biquadratic may then be written as a function of one

[1] Stability in Aviation, by G. H. Bryan.

or more of 11 quantities—the speed u (negative when the machine is moving forward in the normal manner), the radius of gyration about the u axis K_B^2, and the nine resistance derivatives X_u, Z_u, M_u, X_w, Z_w, M_w, X_q, Z_q, and M_q, X_u representing $\frac{\delta X}{\delta u}$. It is found that X_q and Z_q are so small as to be negligible, and M_u is zero, since the moment about the center of gravity of the airplane due to air forces is zero in horizontal flight and therefore will not be affected by variations in speed.

Our eleven original quantities are thus reduced to eight, and the coefficients in the equation $A\lambda^4+B\lambda^3+C\lambda^2+D\lambda+E=O$ may then be written in the following form:

$$A=K_B^2$$

$$B=-(M_q+X_uK_B^2+Z_wK_B^2)$$

$$C=\begin{matrix} Z_w & U \\ M_w & M_q \end{matrix}+X_uM_q+K_B^2\begin{matrix} X_u & X_w \\ Z_u & Z_w \end{matrix}=M_q(Z_w+X_u)-UM_w+K_B^2(X_uZ_w-Z_uX_w)$$

$$D=-\begin{matrix} X_u & X_w & O \\ Z_u & Z_w & U \\ O & M_w & M_q \end{matrix}=-M_q(X_uZ_w-Z_uX_w)+UX_uM_w$$

$$E=-g\ M_wZ_u$$

COMPUTATIONS OF RESISTANCE DERIVATIVES.

The first step in computing the resistance derivatives for a specific airplane is to determine X, Z, and M for each angle of incidence at which the model was tested, and to plot these against the angle of pitch away from the position of equilibrium. To avoid the appearance of mass in the stability equations, all forces are reduced to pounds per unit mass. The transformation is made for X and Z by the application of the equations:

$$X=D\cos\theta-L\sin\theta$$
$$Z=L\cos\theta+D\sin\theta$$

θ being the angle of pitch. The method of obtaining M has already been described in the first part of this report, and has been carried through for all the cases under examination for an angle of incidence of 2° and angles of pitch extending from $-6°$ to $+18°$.

X_u and Z_u may be readily calculated from the fact that all the air forces on a machine vary as u^2. X therefore equals Cu^2 and Z equals Cu^2. Differentiating the first of these, we have

$\frac{\delta X}{\delta u}=2C_1u=\frac{2X_o}{U_o}$, and Z_u, similarly, equals $\frac{2Z_o}{U_o}$.

To determine X_w, Z_w, and M_w, it is necessary to consider their physical meaning. A vertical velocity w, compounded with a horizontal flight-velocity U, results in a flight path inclined to the horizontal at the angle: $-\tan^{-1}\frac{\tau w}{U}$. If the angle of the airplane with respect to the earth remains unchanged, in accordance with our assumptions, the angle of incidence will be increased by $\tan^{-1}\frac{\tau w}{U}$. Since

$$\frac{\delta X}{\delta w}=\frac{\delta X}{\delta\theta}\cdot\frac{\delta\theta}{\delta w},\text{ and }\theta=C+\tan^{-1}\frac{\tau w}{U},\ \frac{\delta X}{\delta w}=\frac{\delta X}{\delta\theta}\cdot\frac{\frac{1}{U}}{1+\frac{w^2}{U^2}}=\frac{\delta X}{\delta\theta}\cdot\frac{U}{U^2+w^2}.$$

w is always small in comparison with U, and we may, therefore, write, without serious error, $\frac{\delta\theta}{\delta w}=\frac{1}{U_o}\cdot\frac{\delta X}{\delta\theta}$ may be obtained graphically, being equal to 57.3 times the slope of the X-curve in units of force per degree. Z_w and M_w, similarly, are given by the expressions $\frac{57.3}{U_o}\cdot\frac{\Delta Z}{\Delta\theta}$ and $\frac{57.3}{U_o}\cdot\frac{\Delta M}{\Delta\theta}$.

The only remaining resitance derivative is M_q, the damping coefficient of pitching. This is secured by oscillating the model in a current of air, measuring the time required for the amplitude of the oscillations to be damped to a certain predetermined degree. The method has been fully described elsewhere,[1] and will not be gone into here.

The solution of the equation of motion for the oscillator reduces to: $\log_e\frac{\theta_o}{\theta}=\frac{\mu t}{2I}$, where t is the time required to damp the angle of swing from θ_o to θ and I is the moment of inertia of the entire oscillating mass, calculated by timing the periods of oscillation with the oscillator counterweights in two different positions, and eliminating the effect of the springs between the two equations thus secured. The maximum amplitude of oscillation is about 3° each side of the equilibrium position.

Bairstow has shown [2] that μ, the damping coefficient, is a function of $\rho l^4 v$ where l is any linear dimension of the machine, this deduction being based on a strict proportionality between the air forces and the square of the speed. The above relation, in so far as it states that the damping coefficient varies as the first power of the speed, is in close accord with the results obtained by oscillating experiments at different speeds for the complete model of an airplane; but the damping coefficient for the apparatus alone varies with the speed in a highly irregular manner, being nearly constant at speeds of from 20 to 35 miles per hour, beyond which points it changes rapidly. This behavior is in accordance with that indicated by previous tests with the same apparatus, as is shown by the positions of the observed points with respect to their curves, although it has always been assumed that the discrepancies from a straight line were caused by experimental errors, and such a line was drawn through an average of the points. In the present experiments, since μ for the complete model is considered to be directly proportional to the speed, a mechanical method of fairing the curve and obtaining M_q has been substituted for the device of plotting all the points and drawing the line by eye, as has formerly been the custom. μ was found for each case at seven different speeds, ranging from 12 to 39 miles an hour, and was divided by the speed of test, thus giving the damping coefficient at 1 mile an hour. An average of the seven values obtained for the seven different speeds was then taken to be the true value

[1] Experimental Analysis of Inherent Longitudinal Stability for a Typical Biplane, by J. C. Hunsaker: First Annual Report of the National Advisory Committee for Aeronautics, pp. 41–45.

[2] The Experimental Determination of Rotary Coefficients, by Leonard Bairstow: Report of the British Advisory Committee for Aeronautics, 1912-13, p. 176.

for 1 mile an hour, except where one or two of the values were so far from the rest as to be obviously wrong, in which case these aberrant values were omitted from consideration in making up the average. This method also has the advantage that the mean deviation of the individual values from the average gives an excellent quantitative measure of the accuracy of the run. This deviation was almost always found to be less than 4 per cent. Having found this unit damping coefficient, M_q is found by multiplying by the fourth power of the scale of the model and by the speed, dividing by the mass of the airplane and changing the sign, since M_q acts so as to resist pitching.

SOLUTION OF THE STABILITY EQUATION.

Since the motion is oscillatory, the roots of the biquadratic stability equation will be complex, and will occur in pairs. The substitution of any root in the expression $y=e^{\lambda t}$ gives the product of an exponential, the exponent corresponding to the real part of the root, and a trigonometric expression involving both sine and cosine, and therefore having the period 2π, the magnitude of the angles (in radians) corresponding to the imaginary part of the root. In order that the motion may be a damped one, the real parts of all the roots must be negative, and the condition for this is, as demonstrated by Routh [1], that all the coefficients in the equation: $A\lambda^4+B\lambda^3+C\lambda^2+D\lambda+E=O$, must be positive, and that the expression $BCD-AD^2-B^2E$, known as Routh's discriminant, must also be positive. The magnitude of Routh's discriminant is frequently taken as a criterion of the degree of stability, but it is not entirely satisfactory for this purpose, as will be shown later.

Bairstow has shown [2] that this equation can be so factored as to give approximately correct roots, since the values of the coefficients do not vary widely in modern airplanes of standard type. The solution is as follows:

$$\left(\lambda^2+\frac{B}{A}\lambda+\frac{C}{A}\right)\left(\lambda^2+\left[\frac{D}{C}-\frac{BE}{C^2}\right]\lambda+\frac{E}{C}\right)=O$$

The first factor corresponds to a short and heavily damped oscillation, the second to one of much longer period. If there is any instability, it appears in the latter motion. It is evident that, if the second motion is to be stable, $\frac{D}{C}-\frac{BE}{C^2}$ must be positive, and CD must therefore be greater than BE. This is a somewhat simpler, although less absolutely correct, criterion than is the use of Routh's discriminant.

The above product multiplied by A is: $A\lambda^4+\left(B+\frac{AD}{C}-\frac{ABE}{C^2}\right)\lambda^3+\left(C+\frac{AE}{C}+\frac{BD}{C}-\frac{B^2E}{C^2}\right)\lambda^2+D\lambda+E=O$. In order that this may be identical with the original equation, the conditions: $CD=BE$ and $AE+BD=\frac{B^2E}{C}$ must be satisfied. These conditions are incompatible unless $AE=O$, which is manifestly impossible in a statically stable

[1] Advanced Rigid Dynamics, by E. J. Routh.

[2] Invsetigation into the Stability of an Aeroplane; Report of the British Advisory Committee for Aeronautics, p. 160.

machine, as neither M_w nor Z_u can be zero in such an airplane. Bairstow's solution is therefore never perfectly correct, but it is close enough at all times to be of great value, which is all that has ever been claimed for it.

It has already been stated that Routh's discriminant is not a satisfactory measure of stability. A better quantity for this purpose can be obtained in the following manner.

The most satisfactory basis for a single expression defining the degree of stability, is the percentage of damping during one complete oscillation. A large value for this expression is to be desired, as it involves heavy damping in combination with a long period, both of which make for comfort and safety. The damping in one oscillation depends on the ratio of the period to the time required to damp the amplitude 50 per cent, both of which quantities have been determined for every case investigated. If we write our quadratic in the form $\lambda^2+a\lambda+b=O$, the period equals $\frac{4\pi}{\sqrt{4b-a^2}}$ and the time to damp one-half equals $\frac{2\log_e 2}{a}$. Then $\frac{p}{t}=\frac{Fa}{\sqrt{4b-a^2}}$, where F is a constant, and, substituting their true values for b and a, $\frac{p}{t}=\frac{F\left(\frac{D}{C}-\frac{BE}{C^2}\right)}{\sqrt{\frac{4E}{C}-\left(\frac{D}{C}-\frac{BE}{C^2}\right)^2}}$, and this expression is a maximum when $\frac{(CD-BE)^2}{C^3E}$ is a maximum. $\frac{(CD-BE)^2}{C^3E}$ will therefore serve as the desired measure of stability. A word of caution is necessary to the effect that this does not furnish a means of distinguishing degrees of instability, and that, of two machines giving negative values, the one for which the value is algebraically largest (nearest to zero), may be the more unstable of the two. Only positive values, therefore, should be taken into account. To minimize the effect of instability it is desirable that the product of the period and the time to double in amplitude, not their ratio, be a maximum.

DYNAMICAL STABILITY OF THE CURTISS JN2.

The resistance derivatives were computed and the stability discussed for each of the 11 different combinations of body and tail which were made up. The machine was also placed on the oscillator without a tail, in order to determine the amount of damping, or the proportion of M_q, due to the wings, body, and chassis. It would, however, have been useless to make complete stability calculations in this condition, as it is obvious that a machine which is unstable in a statical sense can not possess dynamical stability. The calculations have all been made, as in the case of the statical work and the reduction of the center of gravity, for an angle of incidence of 2°, corresponding to a speed of slightly over 60 miles per hour. Investigations by Dr. J. C. Hunsaker [1] have shown that the degree of stability of any given machine falls off rapidly as the speed de-

[1] Experimental Analysis of Inherent Longitudinal Stability for a Typical Biplane, by J. C. Hunsaker: First Annual Report of the National Advisory Committee for Aeronautics, p. 50.

creases, and that all typical machines investigated became unstable in respect of the long oscillation at some speed greater than the mini-

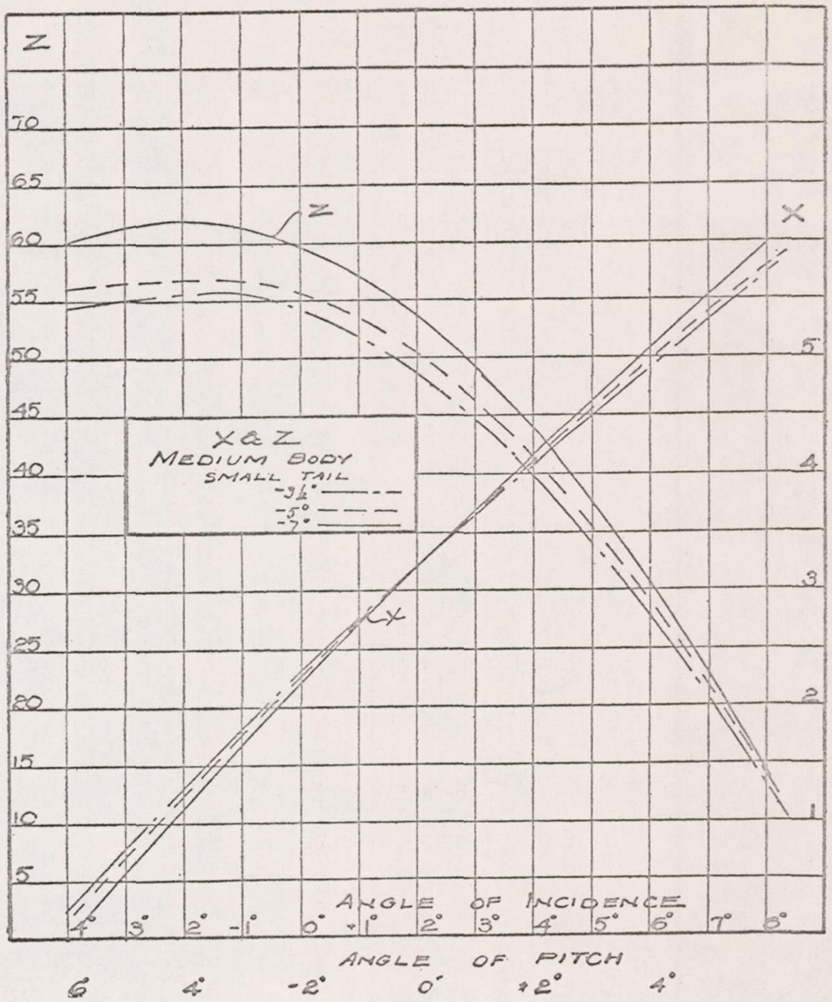

FIG. 31.

mum attainable in flight. An intermediate angle, corresponding to a good climbing speed, was therefore chosen for the present experiments.

Case I. Medium body, small tail at $-3\frac{1}{2}$.
Case II. Medium body, small tail at $-5°$.
Case III. Medium body, small tail at $-7°$.
Case IV. Medium body, medium tail at $-2°$.
Case V. Medium body, medium tail at $-3\frac{1}{2}°$.
Case VI. Medium body, medium tail at $-5°$.
Case VII. Medium body, large tail at $-1°$.
Case VIII. Medium body, large tail at $-2°$.
Case IX. Medium body, large tail at $-3\frac{1}{2}°$.
Case X. Short body, large tail at $-3\frac{1}{2}°$.
Case XI. Long body, small tail at $-3\frac{1}{2}°$.

The curves of X and Z are plotted in figures 31 to 34.

TABLE I.—*Tabulation of resistance derivatives.*

Case.	U.	X_u.	Z_u.	X_w.	Z_w.	M_w.	M_q.
I	−90.8	−0.108	−0.709	+0.218	−2.76	+2.31	−130
II	−92.3	− .109	− .698	+ .229	−2.88	+2.92	−133
III	−94.5	− .114	− .681	+ .239	−2.94	+3.85	−138
IV	−90.9	− .105	− .708	+ .242	−2.83	+1.97	−135
V	−91.7	− .109	− .702	+ .234	−2.80	+3.30	−143
VI	−92.3	− .111	− .698	+ .227	−2.79	+3.96	−151
VII	−91.5	− .106	− .703	+ .216	−2.82	+2.63	−160
VIII	−93.1	− .105	− .692	+ .212	−2.83	+2.72	−167
IX	−94.0	− .110	− .685	+ .230	−2.93	+4.22	−175
X	−93.9	− .110	− .686	+ .226	−2.99	+3.36	−136
XI	−92.0	− .107	− .700	+ .226	−2.86	+2.78	−156

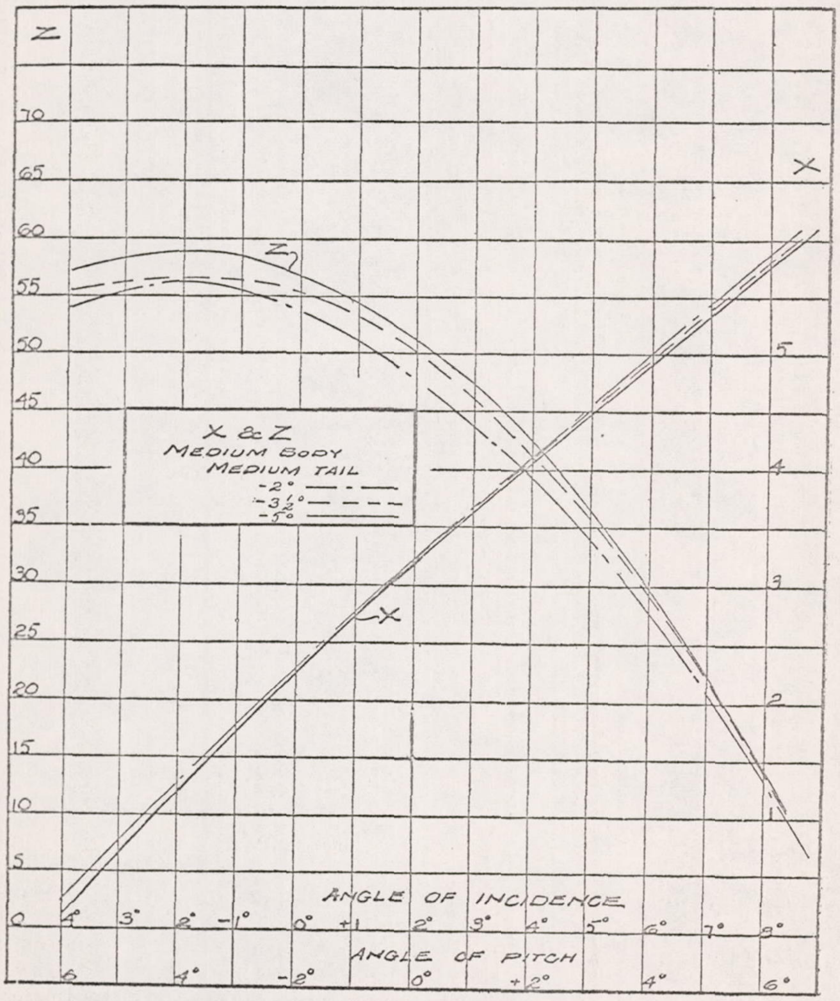

FIG. 32.

All the values above were obtained by direct calculation from the observed forces, moments, and damping times, with the exception of those for M_q. These are faired values, a few of those originally secured being slightly inconsistent with the rest. In no case did this fairing alter the value by more than 3½ per cent.

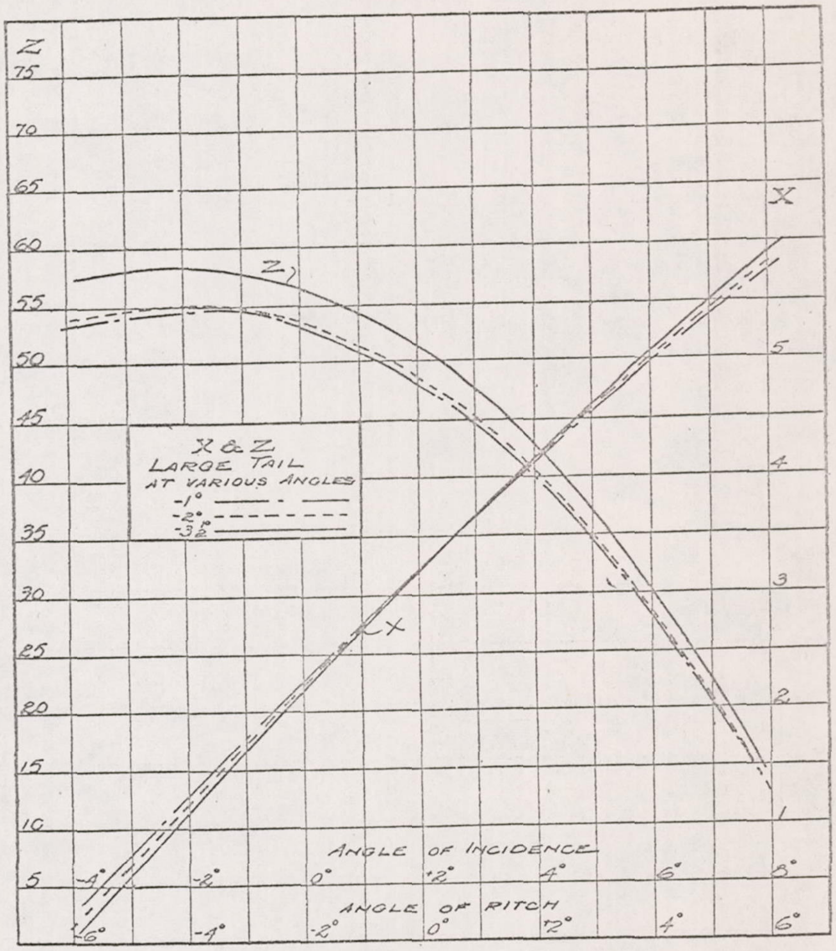

FIG. 33.

DISCUSSION OF THE RESISTANCE DERIVATIVES.

X_u varies only slightly. It increases with the angle between the wings and the tail, and increases very slightly with size of tail. It is greater for the short body than for the long. The largest and smallest values among the 11 are separated by less than 5 per cent.

Z_u is inversely proportional to U, and calls for no special comment. The maximum variation here is less than 4 per cent.

X_w, Z_w, and M_w are determined much less accurately than X_u and Z_u, as they depend on the slope of the curve, not on the value of its ordinate. X_w varies in a highly irregular manner through a range of about 12 per cent. Errors in the determination of this quantity may account for a large part of the variation.

Z_w also varies irregularly, but not so badly, showing a general tendency to increase with the absolute value of the tail angle.

The behavior of the moment curves and the variation of their slopes have already been discussed. M_w increases rapidly with increasing tail angle and with increasing size of tail. The only serious discrep-

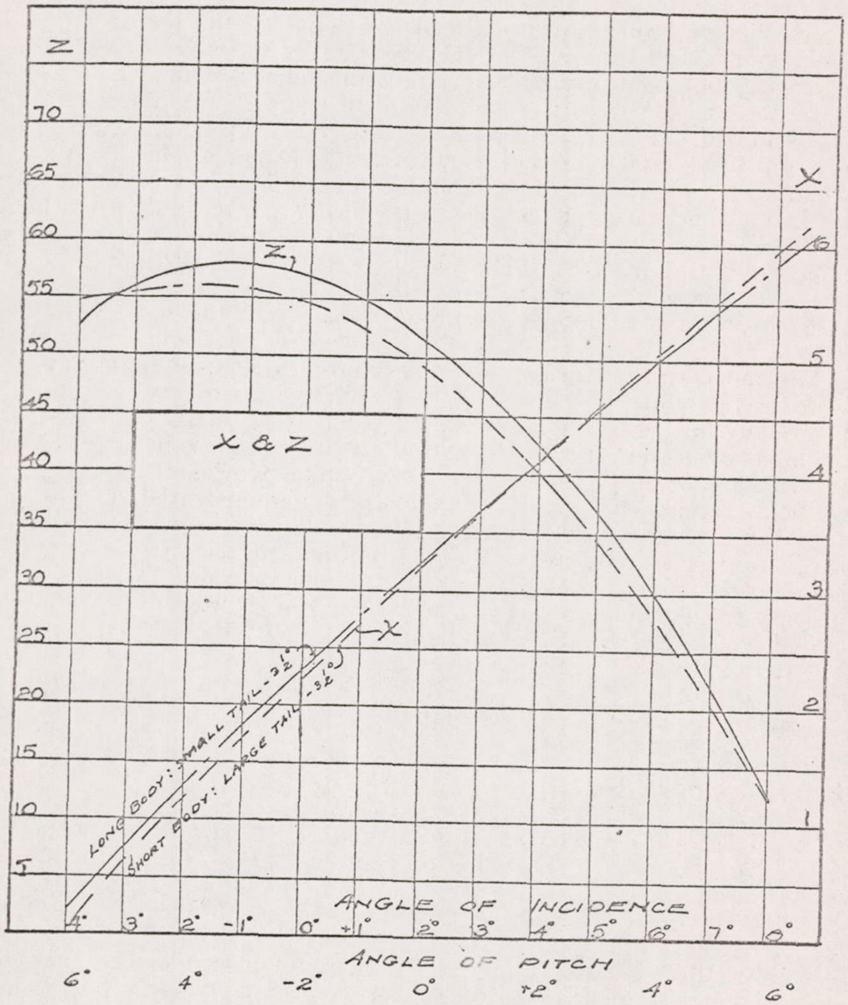

FIG. 34.

ancy here is in the values for the large tail, where the change in the slope of the moment curve due to a change in tail angle from $-1°$ to $-2°$ is almost negligible, compared with that arising from a change from $-2°$ to $-3\frac{1}{2}°$.

The most interesting of the derivatives, however, is the damping coefficient, M_q. The damping action on the tail is generally assumed to arise from the fact that when the airplane is in pitch the tail has angle of incidence due to the pitch. If the truth of this hypothesis be admitted, the damping moments should vary as the square of the distance from the tail to the center of gravity, and should also be proportional to the area of the tail. In geometrically similar machines, therefore, the damping coefficient should vary as the fourth power of a linear dimension, a fact which has already been remarked. In varying the length of the body and the size of the tail simultaneously in accordance with the convention which we adopted, the damping coefficient should be directly proportional to the length of the body, and it will be seen that this represents the actual condition within the limits of experimental error if allowance is made for the fact that the damping does not all arise from the tail. A quantitative discussion of this point (the distribution of damping between the elements of the machine) will be entered on at another place. When the tail area alone is changed the damping increases with area, indeed, but the increase in damping, especially when the large tail is substituted for the medium one, appears to be considerably more rapid than that in area. The changing of body length alone also causes a variation in damping moment more rapid than would be indicated by a strict adherence to proportionality to the square of the length.

The most striking feature of the damping coefficients, however, is their variation with angle of tail setting. In every case, even before any fairing was attempted, the value of M_q increased with the angle between the tail and the wings, a result which is in direct contravention of the damping hypothesis which we have already described. It has never been conclusively demonstrated, however, that the force on a plate at a fixed angle of incidence is the same as the instantaneous force when the angle of incidence is constantly varying,[1] and it may be that there is an inherent damping force arising directly from a change in the type of field of flow. Such a force would undoubtedly vary with the magnitude of the direct force on the tail, and would therefore give the observed result.

COMPUTATION OF K_B^2.

The radius of gyration under each case was computed on the following assumptions:

(1) Changes in tail angle have no effect.

(2) The weight of the tail is proportional to the area.

(3) The weight of the body is proportional to its length (for small variations).

(4) The radius of gyration of the part of the body behind the center of gravity is proportional to its length.

(5) The difference between the moments of inertia of the various tails about their own respective centers of gravity is negligible.

The radius of gyration for the JN2 in its standard arrangement has been very carefully calculated, and the computation has been checked by swinging the complete machine,[3] the result being very nearly 5.8 feet.

A tabulation of the other cases, as calculated from the standard radius of gyration and the assumptions above, follows:

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[1] Dynamical Stability of Aeroplanes, by J. C. Hunsaker.
[2] Effet exercé sur un aile par un vent rapidement, by Com. Lapay: La Technique Moderne, May 1, 1914.
[3] Experimental Analysis of Inherent Longitudinal Stability for a Typical Biplane, by J. C. Hunsaker; First Annual Report of the National Advisory Committee for Aeronautics, p. 46.

	K_B^2	K_B
Medium body, small tail	33.3	5.77
Medium body, medium tail	34.0	5.83
Medium body, large tail	34.8	5.90
Short body, large tail	32.3	5.69
Long body, small tail	36.3	6.02

FORMATION AND SOLUTION OF STABILITY EQUATIONS.

Each case has been treated separately, the coefficients of the biquadratic being computed from the resistance derivatives and other quantities previously given, and the period (p), time required to damp to 50 per cent of the original amplitude (t), and percentage of damping in one complete oscillation (d), being computed for both the long and the short oscillations in accordance with Bairstow's approximate solution, already described.

Case I. Medium body, small tail at $-3\frac{1}{2}°$:

$$A=33$$
$$B=226$$
$$C=598$$
$$D=82$$
$$E=53$$

$$BCD-AD^2-B^2E=82\times10^5$$

Short oscillation: $\lambda^2+6.78\lambda+17.96=O$

$$=-3.39\pm\frac{\sqrt{45.9-71.8}}{2}=-3.39\pm2.55i$$

$p=2.46$ secs. $t=0.205$ sec. $d=99.98$ per cent.

It is evident that the period of this oscillation is so short and the damping is so heavy and so complete that its existence would be hardly perceptible to the aviator.

Long oscillation: $\lambda^2+0.103\lambda+0.0885=O$

Whence

$$\lambda=-0.0515\pm0.293i$$

$p=21.4$ secs. $t=13.5$ secs. $d=66.6$ per cent.

The stability here, while much less than for the short oscillation, is still amply sufficient for safety and comfort.

Case II. Medium body, small tail at $-5°$:

$$A=33$$
$$B=233$$
$$C=684$$
$$D=92$$
$$E=66$$

$$BCD-AD^2-B^2E=107\times10^5$$

Short oscillation: $p=2.18$ secs. $t=0.199$ secs. $d=99.75$ per cent.

Long oscillation: $p=20.4$ secs. $t=13.6$ secs. $d=64.7$ per cent.

Case III. Medium body, small tail at $-7°$:

$A = 33$
$B = 240$
$C = 802$
$D = 110$
$E = 84$

$BCD - AD^2 - B^2E = 160 \times 10^5$.

Short oscillation: $p = 1.89$ secs. $t = 0.193$ secs. $d = 99.89$ per cent.

Long oscillation: $p = 19.65$ secs. $t = 13.1$ secs. $d = 64.7$ per cent.

Case IV. Medium body, medium tail at $-2°$:

$A = 34$
$B = 235$
$C = 591$
$D = 72$
$E = 45$

$BCD - AD^2 - B^2E = 70 \times 10^5$.

Short oscillation: $p = 2.7$ secs. $t = 0.201$ secs. $d = 99.9$ per cent.

Long oscillation: $p = 23.1$ secs. $t = 15.1$ secs. $d = 65.3$ per cent.

Case V. Medium body, medium tail at $-3\frac{1}{2}°$:

$A = 34$
$B = 242$
$C = 735$
$D = 100$
$E = 74$

$BCD - AD^2 - B^2E = 131 \times 10^5$.

Short oscillation: $p = 2.10$ secs. $t = 0.195$ secs. $d = 99.9$ per cent.

Long oscillation: $p = 20.1$ secs. $t = 13.5$ secs. $d = 64.4$ per cent.

Case VI: Medium body, medium tail at $-5°$:

$A = 34$
$B = 250$
$C = 819$
$D = 112$
$E = 89$

$BCD - AD^2 - B^2E = 169 \times 10^5$.

Short oscillation: $p = 1.93$ secs. $t = 0.189$ secs. $d = 99.92$ per cent.

Long oscillation: $p = 19.3$ secs. $t = 13.4$ secs. $d = 63.1$ per cent.

Case VII. Medium body, large tail at $-1°$:

$A = 35$
$B = 262$
$C = 725$
$D = 97$
$E = 59$

$BCD - AD^2 - B^2E = 141 \times 10^5$.

Short oscillation: $p = 2.43$ secs. $t = 0.185$ secs. $d = 99.99$ per cent

Long oscillation: $p = 22.1$ secs. $t = 13.25$ secs. $d = 68.5$ per cent

Case VIII: Medium body, large tail at $-2°$

$A = 35$
$B = 270$
$C = 758$
$D = 101$
$E = 60.5$

$BCD - AD^2 - B^2E = 159 \times 10^5$.

Short oscillation: $p = 2.42$ secs. $t = 0.179$ secs. $d = 99.99$ per cent.

Long oscillation: $p = 22.7$ secs. $t = 13.25$ secs. $d = 69.5$ per cent.

Case IX. Medium body, large tail at $-3\frac{1}{2}°$:

$A = 35$
$B = 281$
$C = 946$
$D = 128$
$E = 93$

$BCD - AD^2 - B^2E = 261 \times 10^5$

Short oscillation: $p = 1.90$ secs. $t = 0.173$ secs. $d = 99.95$ per cent.

Long oscillation: $p = 20.3$ secs. $t = 13.1$ secs. $d = 65.8$ per cent.

Case X. Short body, large tail at $-3\frac{1}{2}°$:

$A = 32$
$B = 236$
$C = 752$
$D = 101$
$E = 74$

$BCD - AD^2 - B^2E = 135 \times 10^5$

Short oscillation: $p = 1.99$ secs. $t = 0.191$ secs. $d = 99.93$ per cent.

Long oscillation: $p = 20.3$ secs. $t = 13.5$ secs. $d = 64.8$ per cent.

Case XI. Long body, small tail at $-3\frac{1}{2}°$:

$A = 36$
$B = 264$
$C = 737$
$D = 99$
$E = 63$

$BCD - AD^2 - B^2E = 144 \times 10^5$

Short oscillation: $p = 2.36$ secs. $t = 0.191$ secs. $d = 99.98$ per cent.

Long oscillation: $p = 21.8$ secs. $t = 13.5$ secs. $d = 67.3$ per cent.

On reviewing the above cases it is seen that from the point of view of dynamical longitudinal stability, it is evident that all these slight variations from the normal give entirely satisfactory results at the medium speed for which analyses were made. The short oscillation never gives any trouble, and, indeed, the pilot would hardly be able to perceive its existence as an oscillation. Although there are distinct variations in the period and strength of damping for the long oscillation, these variations are small in magnitude.

We shall, somewhat later, treat the effects of variations of certain derivatives on dynamical longitudinal stability, but in reviewing these 11 cases, where everything changes at once, it is only possible to draw the most general conclusions. In increasing the angle of tail setting, passing from Case I to III, IV to VI, and VII to IX, there is manifested a general tendency to shorten the period of the long oscillation and decrease the time required to damp one-half. The percentage of damping in one oscillation decreases somewhat, so that the net effect of such changes may be considered to be unfavorable. An increase in tail angle brings about a considerable increase in M_w, the static righting moment, and a slight increase in M_q, the damping moment. It is evident at once that the first of these changes will decrease the period and that the second will decrease the time required for damping.

The effect of tail area was less than might have been anticipated. There was a general tendency to decrease both period and damping time with a larger tail, the angle being kept constant.

A comparison of Cases V, X, and XI shows that the period increases as the length of the body is increased, the tail area being correspondingly decreased. The damping time, on the other hand, is absolutely identical for all three cases. When the body length in increased without changing the tail area there is, again, surprisingly little change. Such as there is is a general improvement, through a lengthening of period, a decrease of damping time, or both.

In short, it appears that considerable modifications can be made in the size, placing, and arrangement of the tail surfaces without serious adverse effect on dynamical longitudinal stability at moderate and high speeds, and that these details may be chosen primarily from the standpoints of weight, aerodynamic efficiency, maneuverability, and the possession of a sufficient degree of static stability to insure a moderately rapid recovery from a nose-dive or other abnormal attitude. The needs of lateral stability, too, must be kept in mind when changing the length of body.

PHYSICAL CONCEPTIONS OF THE RESISTANCE DERIVATIVES.

By appropriate alterations in design, almost all the derivatives can be slightly varied one at a time and without substantial change in the others. To determine what these alterations should be, the most straightforward method is to assume variations in each of the derivatives singly, and to calculate the effects of such deviations on the long oscillation. At the same time, it is of the highest importance to have a physical conception of the nature of the derivatives, as a check on the conclusions derived from a purely mathematical treatment. The basis for such physical conceptions has been expounded with particular clearness by Dr. J. C. Hunsaker.[1]

(a) M_w, the statical moment derivative, represents the change in pitching moment with vertical velocity. If the airplane rises, the relative wind has a downward component, and the angle of incidence is diminished. If M_w is positive, it will tend to head the airplane up. Conversely, if the airplane drops the relative wind has an upward slope, the angle of incidence is decreased, and since w is now negative, M_w will tend to head the machine down. The effect of a positive M_w is therefore to maintain the airplane always at the same angle to the wind. If M_w is very large, it tends to produce violent oscillations with a short period, the condition being analogous to

[1] Dynamical Stability of Aeroplanes, by J. C. Hunsaker.

that of a ship with excessive metacentric height, or, to choose a more homely illustration, to that of a weight vibrating at the end of a very strong spring. If M_w were very small, the motion would be gentle, with a long period, but, on the other hand, the recovery after a disturbance would be insufficiently prompt.

In the calculations submitted M_w varies through a wide range, as was pointed out in connection with the statical section. Table I shows that large changes can be made in M_w by changing the angle of the tail, and that such changes have no commensurate effect on the other derivatives.

Taking the standard case (medium body, medium tail at $-3\frac{1}{2}°$) as a basis and carrying through the customary computations for various values of M_w, all the other derivatives and the speed being assumed unchanged, we have the following results:

Change in M_w.	M_w.	Period.	Time to damp one-half.	Damping in one oscillation.
Per cent.				*Per cent.*
0 (Std.)	3.30	20.1	13.5	64.4
+20	3.96	19.0	13.8	61.5
−20	2.64	21.5	13.1	67.8
+50	4.95	17.85	14.5	57.4
−50	1.65	25.6	12.0	77.2
−80	0.66	39.0	10.3	92.7

It is evident that the effect of increasing M_w is wholly unfavorable, the period being shortened and the damping decreased. The third and fifth of the above combinations appear most satisfactory, the period being long and the damping considerable, and still without sacrificing a dangerously large amount of static righting moment. If M_w be sufficiently decreased the solution of Bairstow's second factor becomes a real number, and the motion ceases to be oscillatory, becoming a dead-beat subsidence. In the case under discussion, M_w would have to be decreased to 0.11 to arrive at this condition, and so small a value would not be safe from other standpoints. A reduction of M_w to approximately 2.00 without much effect on any other derivative could be secured by the use of a tail half way between the medium and large ones in size, and set parallel to the wing chord. This is in accordance with our provisional recommendation, made in the first part of the report, that larger tails set at smaller angles should be used.

(*b*) M_q represents the rate of change of pitching moment due to angular velocity of pitch, or the coefficient of inherent damping of pitch. The effect of this quantity on longitudinal stability has apparently been very much overestimated. Varying M_q alone as we have just done for M_w, we have:

Change of M_q.	M_q.	*p*.	*t*.	*d*.
Per cent.				
0	−143	20.1	13.5	64.4
+ 10	−157	20.7	13.0	66.8
− 10	−129	19.45	14.05	61.6
+ 50	−214	22.85	12.1	73.0
− 50	− 72	16.85	16.95	49.8
−100	0	13.05	44.0	18.6

17 IV 18 19 I 19 II 20 21 22 I 22 II 23 I 23 II 23 III

Increases in the damping coefficient have exactly the opposite effect to similar changes in M_w, decreasing the damping time and lengthening the period. Considerable alterations can, however, be made without seriously altering the nature of the motion. Even when the damping coefficient is reduced to half its normal value, a change which would hardly be brought about by any modification short of the complete removal of the tail, the motion is still not uncomfortably violent, although the stability is much decreased, and the critical speed for instability would be considerably higher than that for the standard machine. When M_q is still further reduced to zero the machine is still stable, although now only slightly so. Since damping depends more on size of tail than on angle, M_q can be increased without changing M_w by increasing the size of the tail, and, what is even more important, the length of the body, while decreasing the angle of the tail to the wings. A broad, flat-bottomed body also contributes to damping.

(*c*) X_u represents the change in X with changing forward velocity. It is evident from a physical standpoint that this should be negative and as large as possible, so that any tendency to change speed will be immediately counteracted. X_u depends solely on the drag at 0° of pitch, and a high $\frac{L}{D}$ ratio is therefore unfavorable to stability. Making a quantitative study, we find that an increase of 10 per cent in X_u has no effect on the period, and decreases the damping time from 13.5 seconds to 12.4. A decrease of 50 per cent, corresponding to doubling the $\frac{L}{D}$ ratio, still leaves the period virtually unaffected, but increases the damping time to a trifle under 20 seconds, so that the damping in one oscillation is lowered from 64 per cent to barely 50. Among the five coefficients of the biquadratic, X_u enters into B, C, and D, but its effect on B is too small to be perceptible, and it influences the value of D much more than that of C.

(*d*) X_w should be large and positive for stability, as is evident from physical considerations. When the machine, in the course of its oscillation, starts to rise, it is desirable that a force be set up which will oppose the forward motion, thus decreasing the speed and checking the rise. The result of changing this derivative has been examined in the same manner as for the others already treated.

$X_w+10\%$	$X_w=+.257$	$p=20.1$ secs.	$t=13.1$ secs. $d=65.5$
$X_w-100\%$	$X_w=0$	$p=19.8$ secs.	$t=19.3$ secs $d=50.9$

Here, too, as in the case of X_u, the effect is shown mainly by a lengthening of the damping time when the derivative decreases. The change is relatively small, but may be of some importance when the degree of change is very large, as it is apt to be. When the angle of incidence decreases and the speed decreases X_w drops off with great rapidity, actually becoming negative as the critical speed is approached, and it is to the rapid change of this derivative that at least a part of the instability at large angles of incidence may be ascribed.

The means of controlling the behavior of X_w may best be shown by a brief mathematical investigation. We have already shown that

X_w is equal to the product of $\frac{\delta X}{\delta \Theta}$ by a negative constant, and that $X = D \cos \Theta - L \sin \Theta$. Differentiating, we have: $\frac{\delta X}{\delta \Theta} = \frac{\delta D}{\delta \Theta} \cos \Theta - D \sin \Theta - L \cos \Theta - \frac{\delta L}{\delta \Theta} \sin \Theta$. Since Θ is 0, $\frac{\delta X}{\delta \Theta} = \frac{\delta D}{\delta \Theta} - L$. At small angles of incidence, the drag curve is nearly horizontal, and $\frac{\delta X}{\delta \Theta}$ is consequently negative. As the angle increases, the slope of the drag curve runs up faster than the absolute value of the lift, especially as the burble point is neared, and the value of $\frac{\delta X}{\delta \Theta}$, and consequently X_w, approaches zero and finally changes sign. To minimize the decrease of X_w at low speeds, the slope of the drift curve should be small and the lift should be large in proportion. In Fig. 35 is shown a diagramatic representation of two extreme types of drag curves, of which the one marked A will obviously correspond to much the higher value of X_w at low speeds. Other features of design which are favorable to a maintenance of stability from this standpoint are: A wing section having the burble point at a small angle, the use of a variable angle of incidence, the setting of the wings at a large angle to the top longeron of the body.

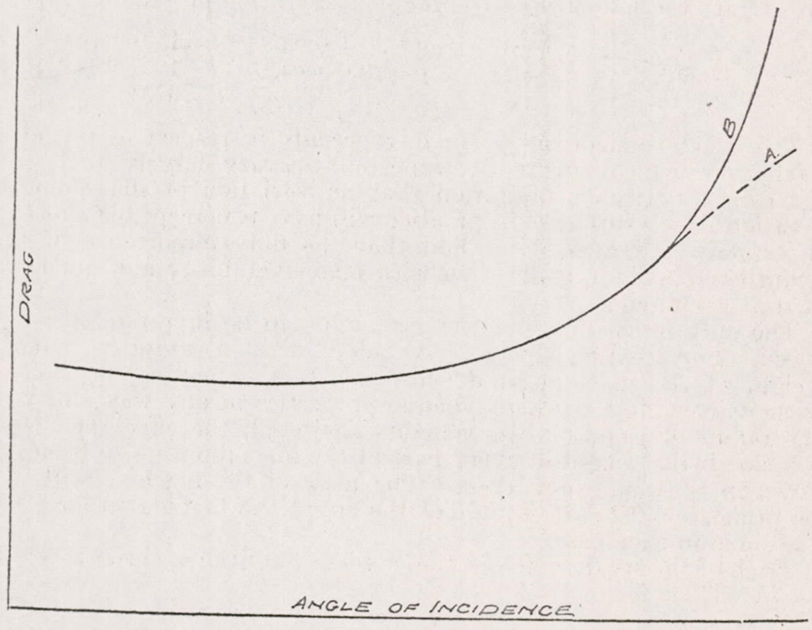

FIG. 35.

(c) In the case of Z_w, also, it is apparent that a large value is desirable, but in this case it should be negative, since the force Z acts in direct line with the velocity w, and any change in the magnitude of Z must be in the opposite direction to w (i. e., must be negative in sign) in order to damp the motion.

$Z_w+10\%$	$Z_w=-3.08$	$p=20.6$ secs.	$t=13.5$ secs. $d=65.3$
$Z_w-100\%$	$Z_w=0$	$p=13.2$ secs.	$t=19.8$ secs. $d=37.0$

It will be noticed that an increase in Z_w lengthens the period and decreases the damping time, thus markedly improving the stability. Z_w, like X_w, drops off rapidly as the angle of incidence is increased, and this is another of the elements contributing to instability at low speeds. Examining Z_w in the same manner previously employed, we see that it is proportional to $\frac{\delta L}{\delta \Theta}+D$, the first term being by far the more important. Z_w will then maintain its original high value best for machines in which the burble point is "sharp," the lift curve running up on a constant slope to within a fraction of a degree of the critical angle and then breaking suddenly. This behavior is characteristic of thick wing sections, such as are used for propeller blade elements. A sharp burble point, however, has certain disadvantages, such machines being subject to stalling and exceedingly sensitive at angles of incidence near the critical angle.

We have now examined, one by one, the effect of each of the resistance derivatives, with the exception of Z_u. It is quite useless to treat this one, as it is a function of the speed alone and nothing can be done to modify its value. The next step, therefore, is to investigate the influence of the radius of gyration on stability.

$K_B^2+10\%$	$K_B^2=37$	$p=20.1$ secs.	$t=13.6$ secs.
$K_B^2-50\%$	$K_B^2=17$	$p=20.0$ secs.	$t=12.5$ secs. $d=67.0$

The effect is surprisingly small, especially in respect of period of oscillation, which might be expected to vary largely with K_B^2. We can say without hesitation that no variation of the radius of gyration which will arise in practice will have a perceptible effect on the stability of a machine, and that the only importance of this quantity appears in connection with maneuverability and quickness of response to controls.

The only important quantity remaining to be investigated is the speed. For treating this we have adopted the assumption that the weight of the machine, and consequently the loading, is changed without changing the aerodynamic properties in any way and that the radius of gyration also remains unaltered, the effect being the same as if the weight of every part of the machine were to be scaled down in the same proportion. The mass of the machine will then be proportional to the square of the speed the flight attitude being the same in each case.

Each of the six derivatives, under these conditions, varies inversely as U. Thus, for example:

$$M_q \propto \frac{\mu l^4 U}{m} l \frac{l^4 U}{U^2} \propto \frac{1}{U}$$

The five coefficients in the stability equation then vary as follows:

A does not vary

$$B \propto \frac{1}{U}$$

$$C \propto \frac{1}{U^2}$$

D varies in an irregular manner, one term depending on $\frac{1}{U}$ and two terms on $\frac{1}{U^3}$

$$E \propto \frac{1}{U^2}$$

Proceeding to examine the effect of alterations, we have:

$U+10\%$ $U=-100.9$ $p=20.1$ secs. $t-14.5$ secs. $d=61.8$

$U+41.4\%$ (loading doubled) $U=-129.8$
$p=19.95$ secs. $t=17.15$ secs. $d=55.3$

$U-29.3\%$ (loading halved) $\mu=-64.9$
$p=20.3$ secs. $t=10.1$ secs. $d=75.2$

It is evident that, for a given flight, attitude, stability is improved by light loading and low speed,[1] and that this improvement appears mainly in the form of increased damping, the period being but little affected. This can be very simply explained on purely physical grounds. The lower the speed of the airplane the greater, relatively, is the restoring effect of any derivative dependent on w, v, or q.

The period of the long oscillation is approximately given by[2] the expression: $p=2\pi\sqrt{\frac{C}{E}}$ and since both C and E are proportional to $\frac{I}{U}$ it would not be expected that the period would change materially. We have seen that this is indeed the case. The criterion of damping, on the other hand, is: $\frac{D}{C}-\frac{BE}{C^2}$ and since $B\propto\frac{I}{U}$, $C\propto\frac{I}{U^2}$, $E\propto\frac{I}{U^2}$, and $D\propto\frac{I}{U^2}$ (approximately), this expression will decrease in value with increase in U. It is evident that pursuit machines, due to their high speed, will be peculiarly liable to instability, and special attention should be paid to their probable behavior in this respect when laying out the design of such airplanes.

ELEMENTS CONTRIBUTING TO DAMPING.

In order to make an analysis of this topic the model was tested on the oscillator with the tail removed, using both the medium and short bodies. The damping coefficient for the wings and short body was found to be 0.000067, and that for the wings and medium body 0.000070, as against a value of 0.000385 for the complete machine with medium body and medium tail at $-3\frac{1}{2}°$. The tail thus furnishes 82 per cent of the damping for the standard arrangement, and 81 per cent of that for the combination of short body and large tail. It is quite possible that the use of certain types of wings having

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[1] See also id., p. 44. [2] Id., p. 42.

a stable center of pressure motion would greatly increase the damping due to the wings.

The damping due to the tail was calculated for the airplane itself at a speed corresponding to an angle of incidence of 2°, and was found to be 117 units for the medium tail in combination with the medium body, and 111 for the large tail in combination with the short body. An independent computation of damping due to the tail, based on the customary assumption that the tail acts as a flat plate at an angle of incidence derived from its resultant path, has also been made. The effective aspect ratio of the tail was assumed to be 3, as indicated by the tests of the tail alone described in the first part of the report, and due allowance was made for the portion of the tail in contact with the body.

The values obtained by such computations were 79 and 72, respectively. These are very nearly two-thirds of the values found by experiment, and the remaining third of the damping must be derived from some other source. The discrepancy is, in fact, considerably more than a third, as we have already found that, due to decreased air-speed and the extreme complexity of flow behind the wings, the forces on a tail are much smaller than those obtained by computation from the flat-plate formula. The additional moment may well be accounted for by the hypothesis, mentioned above, of a dissipation of energy in modifying the field of flow about a plate at a constantly changing angle of incidence. The damping computed from the size and distance of the tail can be used as a basis for a stability estimate, proceeding on the assumption that the computed value forms 55 per cent of the whole M_q.

AN INVESTIGATION OF LOW-SPEED CONDITIONS.

Since, as has already been noted, typical machines become unstable at low speeds, an investigation of these conditions has been added. The angle of incidence chosen for this study was 12°, at which the investigation of Dr. J. C. Hunsaker showed the Curtiss JN–2 to be slightly unstable. As there was not sufficient time to carry out experiments on the oscillator at this angle, M_q was assumed to be directly proportional to the speed of the machine, an assumption which Dr. Hunsaker's experiments indicate to represent the facts fairly closely, but to be rather less favorable to stability than the true conditions, as M_q actually diminishes somewhat less rapidly than does the speed.

The resistance derivatives have been computed as before and are tabulated below, followed by the coefficients of the stability equation and the period and time to damp 50 per cent for the long oscillator, the motion being stable in every case.

Case.	V (M.P.H.)	U	K^2_B	X_u	Z_u	X_w	Z_w	M_w	M_q	X_uZ_w	X_wZ_u
I	42.0	−61.7	33.3	−0.154	−1.04	+0.138	−1.025	+1.94	88	0.158	−0.144
II	42.3	−62.0	33.3	− .153	−1.04	+ .112	−1.06	2.50	90	.162	.116
III	42.7	−62.6	33.3	− .150	−1.03	+ .102	−1.09	3.41	92	.163	.105
IV	41.8	−61.4	34.0	− .154	−1.05	+ .126	−1.08	1.94	91	.166	.132
V	42.2	−61.9	34.0	− .153	−1.04	+ .116	−1.07	2.44	96	.164	.121
VI	42.2	−61.9	34.0	− .153	−1.04	+ .098	−1.05	2.82	101	.161	.102
VII	42.3	−62.0	34.8	− .154	−1.04	+ .079	−1.10	2.55	108	.169	.082
VIII	42.4	−62.2	34.8	− .152	−1.03	+ .089	−1.18	2.71	111	.179	.092
IX	42.6	−62.5	34.8	− .152	−1.03	+ .069	−1.21	3.19	116	.184	.071
X	42.1	−61.8	32.3	− .153	−1.04	+ .106	−1.12	2.10	90	.171	.110
XI	42.1	−61.8	36.3	− .151	−1.04	+ .124	−1.07	2.60	105	.162	.129

Case	B	C	D	E	p	t	d
I	127	234	45	65	11.95	33.0	22.0
II	130	273	49	83	11.35	41.0	17.5
III	133	337	57	113	10.9	39.0	18.0
IV	133	241	45	65	12.05	38.0	16.0
V	138	278	50	81	11.6	40.0	18.0
VI	142	305	54	94	11.3	41.0	17.0
VII	152	302	51	85	11.85	48.0	14.5
VIII	157	316	56	90	11.8	40.0	18.5
IX	163	366	60	105	11.7	39.0	19.0
X	131	253	45	70	11.9	40.0	19.0
XI	149	300	55	87	11.7	36.0	19.0

The results at low speed are less accurate than at high, and the values of X_w, which depends on the difference of two nearly equal quantities, can not be depended on within 15 or 20 per cent.

The fact that the machine was found to be stable (although only slightly so), whereas Dr. Hunsaker found the same machine under the same conditions to be slightly unstable, is accounted for by the larger values of X_w and Z_w obtained in the present experiments, and these, in turn, were probably due to slight differences in wings of the model employed. A comparison of the characteristic curves of figure 6 with Dr. Hunsaker's curves will show that the former approximates much more nearly than the latter to the form (A), in figure 35, which was stated to be conducive to stability, and that the lift curve in the present report has the "sharper" burble point.

The difference between the cases is slight, and there is nothing to invalidate the conclusions and recommendations drawn from the high-speed and statical analyses. The stability with the large tail is somewhat poorer than with the other two, due to a lower value of X_w, which counterbalances the larger M_q. This slight disadvantage can readily be overcome, however, by a modification of the form of wings and body.

An increase in tail angle shortens the period, exactly as at high speeds. Changes in the length of body, within the limits adopted, have no harmful effects.

There seems no reason to doubt the possibility of developing, without radical changes from the present designs, an airplane which will possess a satisfactory degree of longitudinal dynamic stability at all speeds within the range of possibility, and to do this without sacrificing, to a serious extent, aerodynamic efficiency or any other desirable quality.

REPORT No. 17.

PART 4.

SUMMARIES.

By Alexander Klemin, and Edward P. Warner, and George M. Denkinger.

SUMMARY OF RESULTS OF STATICAL INVESTIGATION.

1. The Eiffel 36 wing, with raked tips and aspect ratio 7.2, gave a maximum L/D of 21 and a maximum K_y of 0.00315.

2. The aerodynamic forces on the body are not perceptibly changed by considerable changes in the length and abruptness of the run, or portion of the body in back of the largest cross section.

3. At the angle of minimum resistance, the drag of body and chassis together is slightly less than double that due to the body alone.

4. The mean biplane lift correction recommended for finding minimum flight-speed is 0.95. At small angles and high speeds correction factors of from 0.82 to 0.86 were found.

5. The lift contributed by body, chassis, and tail at large angles is negligible. At angles below 10° these elements exert a considerable negative lift.

6. The coefficient of parasite resistance varies less than 20 per cent between 0° and 9°. Its minimum value for the Curtiss JN–2, including interplane bracing, is 0.02 pounds per mile per hour.

7. The drag contributed by body and chassis, in the presence of the wings, is roughly three-fifths of that indicated by a test of these elements alone.

8. The gain in efficiency from a decrease of the angle between tail and wings is exceedingly small, a change of 2½° in the tail angle increasing the maximum speed by only 1 mile per hour, and decreasing the landing speed by one-half mile per hour.

9. Ordinary changes in tail area do not affect the landing speed perceptibly. The high speed is somewhat improved by increasing tail area.

10. Shortening the body of the JN–2 reduces the landing speed without affecting the high speed.

11. The drag due to the tail is very small except at large angles, and actually drops to a negative value under some conditions.

12. The angle of tail setting can be much decreased without serious loss of statical stability.

13. The center of gravity of the airplane is placed, for equilibrium, from 4 to 18 inches forward of the mean center of pressure of the wings. This distance is greatest when the angle of equilibrium is small and the angle of tail setting is large

SUMMARY OF RESULTS FOR DYNAMIC STABILITY.

1. M_q, the damping coefficient in pitch, increases very rapidly with the size of the tail and with the length of the body. It increases slightly with the angle of tail setting.

2. An increase in tail angle decreases the period of oscillation and the damping in one period.

3. Increased tail area shortens the period and increases the damping.

4. Increasing the length of body increases the stability slightly.

5. To secure a maximum of dynamical stability at high speed, all the resistance derivatives except M_w should be large in absolute value. M_w should be as small as is consistent with a sufficient degree of statical righting moment.

6. To secure these conditions, it is recommended that the angle between tail and wings be much decreased. A considerable shortening of the body is permissible if accompanied by an increase in the tail area which will keep the moment of area about the center of gravity of the machine constant.

7. Eighty-two per cent of the damping moment is contributed by the tail. The damping moment computed for the tail in accordance with the usual theory is about two-thirds of that found by experiment. It is recommended, for preliminary estimates, that the damping due to the tail be computed and assumed to be 55 per cent of the correct value for the whole machine.