REPORT No. 73

THE DESIGN OF WIND TUNNELS AND WIND TUNNEL PROPPELLERS

INCLUDING

SOME EXPERIMENTS ON MODEL WIND TUNNELS

NATIONAL ADVISORY COMMITTEE
FOR AERONAUTICS

PREPRINTS FROM FIFTH ANNUAL REPORT

WASHINGTON
GOVERNMENT PRINTING OFFICE
1919
REPORT No. 73

THE DESIGN OF WIND TUNNELS AND WIND TUNNEL PROPELLERS
INCLUDING
SOME EXPERIMENTS ON MODEL WIND TUNNELS

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

PREPRINTS FROM FIFTH ANNUAL REPORT

WASHINGTON
GOVERNMENT PRINTING OFFICE
1919
REPORT No. 73

THE DESIGN OF WIND TUNNELS AND WIND TUNNEL PROPELLERS

BY

EDWARD P. WARNER, F. H. NORTON,
and C. M. HEBBERT

Aerodynamical Laboratory, National Advisory Committee for Aeronautics, Langley Field, Va.
THE ELEMENTARY THEORY OF THE FLOW OF AIR THROUGH WIND TUNNELS.

If the air flowing through a wind tunnel and back through the room from the exit to the entrance of the tunnel followed Bernouilli's theorem with exactness, there would be no change in the energy possessed by a given particle of air, except for the loss due to friction, as the kinetic energy lost on issuing from the tunnel would be restored in the form of pressure energy. The power required to maintain the flow would then be

\[ P = m \times h_f \]

where \( h_f \) is the head (in feet of air) lost by friction and \( m \) is the mass of air flowing per second. As the same mass of air must pass every point in the tunnel, the product of mean air speed by cross-section area must be a constant for its whole length, neglecting compressibility and changes in temperature during the passage. Since the major part of the frictional losses occur in the reduced section of the tunnel (provided that it is not very short and that the diffuser is not so constructed as excessively to hamper the travel of the air from the tunnel back into the room) \( h_f \) would be practically independent of the size and angle of the exit cone, and the power consumed would also be independent of these factors.

As a matter of fact the conditions of flow are not simple enough to permit the direct application of Bernouilli's theorem. Borda has shown that the loss of energy when fluid moving at high velocity in a pipe is discharged abruptly into a large room or reservoir is equal to the kinetic energy initially possessed. The kinetic energy is not converted into pressure energy as the theory indicates that it should be, and it is therefore profitable to use an exit cone of considerable length, in order that part of the kinetic energy may be saved by conversion into the potential form before the sudden discharge into the room. The length to which it is desirable to prolong the cone is limited by the growing loss by friction within the cone itself. A more rapid conversion of the kinetic energy by increasing the vertex angle of the exit cone is forbidden by the unwillingness of the air to change its course suddenly and follow the walls of the exit cone. If the vertex angle be made too large the effect is almost the same as that of an abrupt increase in cross section. Eiffel, as the result of an elaborate theoretical and experimental research on tunnels having exit cones generated by straight lines, has come to the conclusion that the vertex angle of the exit cone should be not more than 7°, and that the diameter at the large end of the exit cone should be three times that at the small end. It is necessary to base the dimensions of a tunnel on a compromise, as the arrangement which would give the absolute maximum of efficiency would have to be housed in a building of prohibitive size. The overall length can be materially reduced at the cost of a slight increase in power, and the first cost of the building, depending on its dimensions, must be balanced against the cost of operation, which varies with the power of the motor and so with the efficiency of the tunnel. The relations to be observed among the various dimensions of the tunnel and the angles of the cones will be discussed more fully elsewhere. Knowing the power consumed by a tunnel, its diameter, and the speed of the air, the total losses can easily be computed for that particular speed,
and the magnitude of the figure thus obtained will serve as a measure of the efficiency of operation of the tunnel. Since, however, the losses vary with the speed, they can not be compared directly for two tunnels unless they are run at the same speed. The factor most commonly used for comparisons between tunnels is the ratio of the kinetic energy possessed by the air passing through the tunnel in unit time to the work done by the motor in unit time. This is sometimes called the "over-all efficiency," but it is herein alluded to as the "energy ratio." The term efficiency in this connection is misleading, as the two quantities introduced into the ratio are not directly connected, but merely happen to have the same dimensions and so to be convenient for the purpose. Furthermore, the value of the ratio is very commonly more than 1, and is sometimes very much more.

To determine the manner in which the power consumed varies with speed, and so determine the validity or otherwise of the above relation, as well as to find the relation which must be preserved among the various factors in order that geometrically similar tunnels may be strictly comparable, the Theory of Dimensions may be used. The method pursued need not be gone into in detail, as it has been described many times before, and it will suffice to summarize the results. It appears that, if the compressibility of the air and the action of gravity on it be assumed to be of negligible importance at the speeds employed, the power consumed is proportional, for geometrically similar tunnels, to the cross-section area and to the cube of the speed, provided that $\frac{VD}{\nu}$ where $V$ is the air speed, $D$ the tunnel diameter, and $\nu$ the coefficient of kinematic viscosity, is maintained constant. Experiments conducted with a model tunnel at Langley Field and fully described elsewhere in this report, as well as those carried on by Durand, Castellazzi, and others, show that the "energy ratio" varies but little with changes of $\frac{VD}{\nu}$ and it is therefore safe to apply the results of model experiments to full-sized tunnels, even though the speeds may not be strictly in inverse ratio to the diameters. In general, the "energy ratio" increases as $\frac{VD}{\nu}$ increases, and it therefore requires less power to drive a tunnel than would be predicted from a direct application of the results of tests on a model of the tunnel and propeller.

The useful work done by a propeller is equal to the product of the thrust by the speed of flow of the fluid through the propeller disk. The thrust of a wind tunnel propeller is then

$$m \times h_f \times V',$$

where $V'$ is the speed of the air past the propeller, and this equation holds good whether Bernouilli's theorem is followed or not, so long as $h_f$ is the total loss of head from all causes.

$$m = \frac{\rho}{g} \times A' \times V'$$

$A'$ being the cross-section area at the propeller, and the propeller thrust is therefore equal to the weight of a column of air having a height equal to the total loss of head and a cross-sectional area equal to the disc area of the propeller. Since the power is proportional to the cube of the speed, the thrust varies as its square.

If the factors causing departures from Bernouilli's theorem are neglected, the useful work done in moving the air against friction will be, as already mentioned, independent of the degree of expansion of area in the exit cone, and so of the diameter of the propeller. Under these conditions, in fact, the advantage in respect of power consumed would rest with the short exit cone and small propeller, as the propeller efficiency is highest for a large value of the 'slip function' and this is obtained by making the speed of the air through the propeller high and keeping down the diameter of the propeller. Assuming that the output of work is the same in all cases, the thrust will be inversely proportional to the speed of air through the propeller, or directly proportional to the disk area.
DESIGN OF WIND TUNNELS AND WIND TUNNEL PROPELLERS.

LAWS OF SIMILITUDE FOR WIND TUNNEL PROPELLERS.

It is obvious from a study of the Drzewiecki theory of propeller action that a series of propellers of similar blade form and width-diameter ratio, all working at the same true angle of attack, will give thrusts approximately proportional to $N^2D^4$, where $N$ is the engine speed in revolutions per unit time and $D$ the propeller diameter. This proportion can be demonstrated by the Theory of Dimensions to hold exactly true for geometrically similar propellers of perfect rigidity, but it is very nearly correct even where propellers of different pitches are concerned. It has been shown that the thrusts of a series of propellers designed to drive the same wind tunnel or geometrically similar tunnels, is proportional to $N^2D^4$, and also to the cross-section area, which, in turn, varies as $D^2$. It follows from these two relations that $N^2D^4$ must be a constant, and the peripheral speed of the propeller required to draw air through a wind tunnel at any particular speed will therefore be quite independent of the diameter of the propeller if the power required is independent of that diameter. It follows as an obvious corollary that, if the power required is not independent of the degree of expansion in the exit cone, the peripheral speed of the propeller will be least under the same conditions as those for which the power required has its minimum value.

It is easily demonstrable that the stresses, both those due to centrifugal force and those due to bending by the air pressure, in a series of geometrically similar propellers depend only on the peripheral speed, and that they vary as the square of that quantity. There is therefore a limiting peripheral speed which can not be exceeded with safety. For wooden propellers, it is unsafe to run the peripheral speed much beyond 60,000 feet per minute, or 305 meters per second, and it is better to stay well inside this figure. In the case of an airplane or airship where large power must be taken on a single propeller the peripheral speed can be reduced by gearing down, as the engine speed decreases more rapidly than the propeller diameter increases. In the wind tunnel, it has just been shown that this is not the case, and that the peripheral speed, and so the stress, actually increases if the propeller diameter is enlarged beyond a certain point. There is then a clearly defined upper limit to the power which it is safe to apply to driving the propeller in any given wind tunnel, and therefore a limit to the maximum speed attainable. This maximum can only be raised by reducing the losses and so improving the over-all efficiency of the plant.

Since the power required to secure a given speed with a given "energy ratio" is proportional to the cross-sectional area of the tunnel, and is also proportional to $VN^2D^4$, the propellers in a series of tunnels of different diameters operating at the same speed and having the same "energy ratio," all work at the same value of $N^2D^4$, and so of the peripheral speed. This leads to the rather astonishing conclusion that the peripheral speed necessary to produce a given air speed depends only on that air speed and on the energy ratio, and is not at all affected by the size of the tunnel or of the propeller (except indirectly, in so far as these factors have an effect on the energy ratio). For any value of the energy ratio, then, there is a limiting air speed which can not be exceeded without running the peripheral speed up beyond the limits of safety, and this speed is the same for large tunnels as for small, although the actual power consumed of course varies with the tunnel diameter. In order to realize the highest possible wind speed the power coefficient of the propeller must be made as large as possible. This can be done by using many blades and by making them of high-lift sections set at relatively large angles of attack. If the velocities desired are too high to be obtained in this way, it will be necessary to use two or more propellers arranged in tandem, acting like a multi-stage compressor.

It has been shown that

$$ P = K_1D_1^2V_1^3 $$

and also that

$$ P = K_2V_2N^2D_2^4 $$

where the subscripts 1 and 2 denote, respectively, the conditions existing in the experimental chamber and at the propeller, and $K_1$ and $K_2$ are experimental constants depending on the
type of tunnel and propeller. Since \( D_2^2 V_1 = D_2^2 V_2 \), if the velocity across the exit cone at the propeller is uniform, the first of these relations may be written

\[ P = K_1 D_2^2 V_1^2. \]

Dividing this by the second of the relations above,

\[ K_2 N^2 D_2^2 = K_1 V_1^2 \]

and

\[ \frac{V_1}{N D_2} = \sqrt{\frac{K_2}{K_1}} \]

The ratio of the air speed to the peripheral speed is thus a constant for a given tunnel, and its value for any particular tunnel depends only on the type of installation—not at all on its size.

Values of \( \frac{V_1}{N D_2} \) for a few tunnels are tabulated herewith:

<table>
<thead>
<tr>
<th>Name</th>
<th>( V_1 \text{ (m. sec.)} )</th>
<th>( N \text{ (r. p. s.)} )</th>
<th>( D_2 \text{ (m.)} )</th>
<th>( \frac{V_1}{N D_2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eiffel, Autueil</td>
<td>31.8</td>
<td>3.83</td>
<td>3.80</td>
<td>2.18</td>
</tr>
<tr>
<td>Leland Stanford, Jr.</td>
<td>24.0</td>
<td>6.77</td>
<td>3.35</td>
<td>1.06</td>
</tr>
<tr>
<td>Langley Field, model</td>
<td>41.5</td>
<td>68.3</td>
<td>0.010</td>
<td>1.00</td>
</tr>
<tr>
<td>N. P. L., 4-foot</td>
<td>15.24</td>
<td>22.5</td>
<td>1.676</td>
<td>0.40</td>
</tr>
<tr>
<td>Curtis, 4-foot</td>
<td>34.5</td>
<td>22.92</td>
<td>2.44</td>
<td>0.62</td>
</tr>
<tr>
<td>Curtis, 7-foot</td>
<td>42.8</td>
<td>20.00</td>
<td>3.66</td>
<td>0.58</td>
</tr>
<tr>
<td>McCook Field</td>
<td>221.0</td>
<td>29.50</td>
<td>1.52</td>
<td>4.92</td>
</tr>
</tbody>
</table>

1 This tunnel was square and the ratio of \( V_1 \) to \( V_2 \) is therefore equal to the ratio of the cross-section areas and not to that of the square of diameters at the minimum section and at the propeller.

It will be noted that the highest value of \( \frac{V_1}{N D_2} \) in this table, with one exception, is 2.50, and this value was obtained in a tunnel of very efficient type in combination with a propeller having a total blade width equal to one-third of its circumference. Analysis by the Drzewiecki method leads to the belief that it will be possible to raise \( \frac{V_1}{N D_2} \) to 3, but that this figure can hardly be exceeded with propellers resembling those now in use. The exception mentioned above, the small tunnel at McCook Field, has a fan of special type and will be discussed later.

If the allowable peripheral speed be taken as 285 meters per second, \( N D_2 \) is 90.6 meters per second. If \( \frac{V_1}{N D_2} \) be assumed to be 3 the limiting value for \( V \) is 271.8 meters per second, or 607 miles an hour. This is a considerably higher speed than has yet been attained, or than is ever likely to be desired in connection with the study of aircraft. If higher speeds should be needed they can be secured either by the use of a multiplicity of propellers in series or, up to a certain point, by the use of a fan with an abnormally large hub and short blades entirely filling the periphery of the hub, as in the McCook Field tunnel,\(^1\) where the hub diameter is

\(^1\) Studies in high speed aerodynamic phenomena, by F. W. Cattell and E. N. Fales; Automotive Industries, Aug. 28, 1916, p. 422.
two-thirds of the total diameter. If \( \frac{V}{N D} \) is raised to 5, a value only a little higher than that in the McCook Field tunnel, the limiting air speed for the peripheral speed given above is increased to 453 meters per second, or 1,012 miles an hour.\(^1\)

The assumption has so far been made that the air has a free passage across the whole area swept by the propeller. Of course the hub always blocks off a part of this area, but it has usually been an insignificant fraction. If the propeller diameter is \( n \) times the hub diameter, the proportion of the area blocked off is \( \frac{1}{n^3} \), and the speed of the air across the propeller blades, assuming a uniform distribution everywhere outside the hub, is increased in the ratio \( \frac{1}{1 - \frac{1}{n^3}} \).

If the propeller be made, as is the common practice, with a constant blade width, and if the lift coefficient be assumed constant all along the blade, the portion of the total thrust given by the part of the blade inside of any given point is very nearly proportional to the cube of the radius at that point. For example, one-eighth of the thrust would be given by the inner half of the blades if they extended clear to the center, with no hub at all. The use of a hub, or the covering up of part of the blades with a "spinner" therefore decreases the thrust in the ratio \( 1 - \frac{1}{n^3} \). Since useful power is equal to the product of the thrust by the speed across the propeller disk, the net change in power, due to hub or spinner, is

\[
\frac{1 - \frac{1}{n^3}}{1 - \frac{1}{n^2}} = \frac{n^3 - 1}{n^3 - n} = 1 + \frac{1}{n^2 + n/3}
\]

The increase in power coefficient by the use of a spinner, the propeller pitch being adjusted to give the same angle of attack of the blades with as without the spinner, is 5 per cent for a spinner or hub one-quarter the diameter of the propeller, 17 per cent when the ratio is one-half, and 27 per cent when, as in the McCook Field tunnel, it is two-thirds. Furthermore, the use of a very large hub makes it possible to use more blades and make their total width a larger fraction of the circumference of the circle swept by the blades. In the McCook Field fan there are 24 blades, and their total width is approximately equal to the circumference of the hub.

Where very high speeds are desired, as in the calibration of air-speed meters, a throttling insert has sometimes been used to reduce the section of a large tunnel. The effect is to increase the speed, but usually much less than is expected. If the “energy ratio” remained constant, halving the diameter of the tunnel would increase the speed available with a given expenditure of power by 59 per cent. A change of this sort usually, however, diminishes the energy ratio unless the tunnel is of the type combining a long straight portion with conical ends, and permitting of the extension of the cones back into the straight cylindrical part. The use of a throttling insert in a tunnel with a short experimental chamber, like those used by Eiffel and Crocco, is almost certain to lead to a large drop in energy ratio, and the increase of speed by halving the diameter in such a laboratory would probably be less that 50 per cent. Furthermore, it is necessary for best results that the propeller ordinarily used be replaced by one especially designed for use in conjunction with the throttling insert. If the diameter of the tunnel be halved the area at the smallest section is divided by four, and, even with an increase of 59 per cent in speed at the throat or in the experimental chamber, the speed of the air past the propeller is reduced by 60 per cent. Since the propeller diameter and its normal rotational speed to develop the rated power are unchanged, the propeller for use with the throttling insert must have a much smaller effective pitch than that employed with the full section, if the maximum of efficiency is to be obtained.

\(^1\) In this analysis the change of density of the air, due to decrease of static pressure with increasing speed, is neglected. This does not lead to a very large error, as both the propeller thrust and the frictional resistance to the passage of the air increase with the air density, the former varying more rapidly than the latter.
It has just been shown that the gain in speed by reducing the diameter by the use of a throttling insert is disappointingly small. This leads naturally to a study of the best size of wind tunnel to be employed, and of the relation between speed and size which should be sought.

In the construction of aerodynamical laboratories, as the attempt has been made to approach ever more nearly to full-flight conditions, two divergent schools of practice have grown up. The first, best represented by the National Physical Laboratory in England, has constantly increased the diameter of the wind stream, and so increased the size of model which may be tested, but has remained content with relatively moderate wind speeds. The second, on the other hand, has concentrated its efforts on the pumping of the air across a small section at enormous velocity.

In comparing the merits of the high speed and the large diameter tunnels, there are three points which must be borne in mind. In the first place, the highest possible value of $LV$ ($LV$ being the criterion of dynamic similarity) is to be obtained with a minimum expenditure of power. Secondly, the interference between the model and its support is to be reduced to a minimum, and, finally, that disposition should be favored which enables us to secure the greatest accuracy in the construction of the models.

It has been shown that

$$P = KDV^3 = K_1D^3V^3$$

where $D$ is the diameter of the tunnel and $K_1$ is a constant.

In order to avoid interference between the model and the walls of the tunnel, the ratio of maximum span to tunnel diameter must not exceed a certain value (usually about 0.4). Setting $L$, the span of the model, proportional to $D$, we can then modify the above equation:

$$P = K_2D^3V^3 = K_2\left(\frac{LV}{D}\right)^3.$$  

The power required to drive the fan will therefore be least, for any given value of $LV$, in that tunnel where the diameter is largest and the speed is smallest.

The relative magnitude of the interference between the model and its support, the so-called "spindle effect" depends on the ratio of the spindle diameter to the linear dimensions of the model. Its reduction is a matter of very vital importance, the spindle correction undoubtedly being the largest single source of error in most wind-tunnel tests.

The bending moment in the spindle at any point (say one chord length from the wing tip) is proportional to the product of the span by the force acting on the model.

$$M = C_sLF = C_sL^2V^2 = C_sL^2V^2.$$  

If $d$ is the diameter of the spindle, the relations between bending moment, fiber stress, and deflection may be written:

$$f = \frac{Me}{I} = \frac{C_sL^2V^2}{d^3}$$

$$y = \frac{ML^2}{C_sEI} = \frac{C_sL^2V^2}{d^4},$$

if the material of the spindle be the same in all cases.

If the maximum fiber stress be limited to a definite value,

$$\left(\frac{d}{L}\right)^3 = \frac{C_sV^2}{f}$$

$$\frac{d}{L} = \left(\frac{C_s}{f}\right)^{\frac{1}{3}} \times V^2.$$  

The ratio of spindle diameter to model size, and consequently the spindle interference, will therefore be greatest in the high-speed, small, diameter tunnel.
DESIGN OF WIND TUNNELS AND WIND TUNNEL PROPELLERS.

If, as is usually the case, it is stiffness and not strength which prescribes the diameter of the spindle, and if the deflection be limited to a determined value, the required spindle size is given by the equation:

\[ \frac{d^4}{L^4} = \frac{C_d LV^2}{y} \]

\[ \frac{L}{d} = C_e \times (LV)^{\frac{1}{3}} \times V^3. \]

For a given value of \( LV \), then \( \frac{d}{L} \) will be least when the speed is low and the tunnel diameter large. The advantage of the large tunnel on this score is even greater than appears at first, as a larger spindle deflection is permissible with a large tunnel than with a small one. In fact, the permissible deflection increases nearly as rapidly as does the tunnel diameter.

In respect of the third consideration, accuracy of construction of the model, the superiority of the large tunnel, permitting the use of a large model, is so manifest as hardly to call for discussion. A model of 3-foot span can include many parts, such as fittings and wires, which it is quite hopeless to put on one of half that size.

So far, the advantage has rested with the large diameter in every particular. It has one disadvantage in that the size and weight of the balance are much increased, longer weighing arms, heavier counterweights, and a general strengthening up of the apparatus are necessitated. Furthermore, the initial cost of the building to house a large tunnel is very high. In the writer's opinion, however, the advantages far outweigh the drawbacks, and any future development of wind tunnels for model testing should proceed along the lines of increasing the diameter rather than the speed.

All that has been said against high speeds applies, of course, only to tunnels for the testing of models. Speeds equal to the speeds of flight of airplanes are essential for the calibration of instruments.

DESIGN OF WIND TUNNEL PROPELLERS BY THE DRZEWIECKI THEORY.

It is possible, if the rate of flow of the air through a wind-tunnel propeller be known, to predict the performance of the propeller by the Drzewiecki theory. Indeed, the application of that theory to wind-tunnel propellers is rather simpler than its application to the airplane, as there is no in-draught correction to contend with. If the velocity at the minimum section of the tunnel is given, the velocity through the propeller can be computed with absolute accuracy on the assumption that the distribution across the exit cone is uniform. This assumption can only justify itself in the results of the analysis derived from it as a basis.

The best way of checking the accuracy of the analytical method of design is to apply it to a propeller already working satisfactorily. This has been done with the propeller used in the model wind-tunnel experiments described in a later section of the report. The angle of the relative wind to the plane of the propeller can be computed from the wind speed, and it is then possible, knowing the angles of blade setting, to work back and find the angle of attack of each blade element. Having this, the power consumed by the propeller and its efficiency can be found in the usual way. This was done for two cases. In the first case the tunnel was of the Eiffel type, with an enlarged experimental chamber, and the calculated power checked the actual consumption within the experimental error (about 2 per cent, owing to uncertainty as to motor losses). In the second case the air stream was inclosed throughout, a cylindrical tube being carried across the experimental chamber, and the power consumed was about 15 per cent more than that calculated. It is considered that both of these tests showed a very fair check and that the use of the Drzewiecki theory for design is amply justified. The average error, both in these and in other cases which have been tried, is in the direction of underestimation of the power consumption.

---

1 The effect of spindle deflection on the accuracy of measurements is discussed in Report No. 72, on Wind Tunnel Balances.
In designing a propeller for a new tunnel it is necessary to make an estimate of the energy ratio, and so of the speed for a given power. If the estimate is too low, the propeller pitch will be made too low, and the propeller will work at an inefficiently small angle of attack. If the energy ratio is too high, the propeller pitch will be made too high, and the propeller will work at an inefficiently large angle of attack. If the propeller blades are made too narrow, or if too few blades are used, the full power of the motor will not be absorbed at the rated revolutions per minute. The speed will then fail to reach the value expected for the rotational speed realized, the angle of the relative wind to the plane of the propeller will fall below the estimated value, and the angle of attack of the blade elements will become inefficiently large. Any change of this sort from the designed conditions of operation tends to correct itself, as the larger angle of attack increases the power consumed and the thrust given by the propeller. This in turn speeds up the air and brings the angle of attack to a lower value.

It is for this reason that fairly satisfactory results have so frequently been secured with propellers chosen almost at random, but the best efficiencies can only be obtained with a propeller designed especially for the conditions under which it is to operate. The commonest faults in the design of wind-tunnel propellers have been either to overestimate the energy ratio for a projected tunnel or to underestimate the total blade width required for the absorption of the given power at the most efficient angle of attack. The result in both cases is to cause the blades to work at too large an angle of attack.

There is some doubt as to the manner in which the angle of attack should vary along the blades. Most wind-tunnel propellers in which the Drzewiecki system was used at all have been designed for a constant angle of attack, but since, as was just noted, the propellers have usually been made too small to absorb the full power of the motor, they actually work at an angle of attack larger than that desired and increasing from the tip to the root of the blade. In the design of a propeller for the Langley Field wind tunnel the opposite disposition has been deliberately chosen, the angle of attack being made largest near the tips and decreased toward the hub in order that the air may be drawn out along the sides of the exit cone and in order that the larger part of the thrust may come on the most efficient portion of the blades. No experimental data on the effect of this arrangement of the blade sections are available as yet.

In order to make it easy to estimate the number of blades and the blade width required in a propeller for a tunnel, assuming that the wind speed, power consumption, and revolutions per minute are known, a number of propellers have been computed for a variety of conditions and the results expressed by a formula and a curve. The power is given by the formula

\[ P = \frac{V \times b \times D^2 \times N^2 \times n}{C \times 10^8} \]

where \( P \) is the horsepower input of the motor, \( V \) the air-speed through the propeller in meters per second, \( b \) the blade width in centimeters, \( N \) the revolutions per minute, \( n \) the number of blades, \( D \) the propeller diameter in meters, and \( C \) a constant, the magnitude of which depends on the pitch of the propeller. \( C \) is plotted against \( \frac{V}{ND^2} \) in figure 1. If English units be used,
$V$ being given in miles an hour, $D$ in feet, and $b$ in inches, a factor $10^6$ replaces $10^4$ in the denominator of the power formula given above, and $C$ is given by the dotted curve in figure 1. The theoretical basis for the derivation of this formula is the same as that for a formula derived by the writer, and previously published,¹ for the power consumption of airplane and airship propellers.

The efficiency of wind-tunnel propellers is usually very low, and the maximum attainable depends largely on the magnitude of the pitch ratio. In the propeller designed for the Langley Field tunnel the calculated efficiency is 58 per cent. In figure 2, probable propeller efficiencies have been plotted against $\frac{V^2}{ND^2}$. The efficiencies there predicted may be exceeded when the peripheral speed is low, so that thin sections can be used over the whole length of the blade, or when a very large hub or spinner is used to cover up the less efficient parts. In order to give an idea of the range of values of $\frac{V^2}{ND^2}$ employed in successful tunnels, a few are tabulated below, the data being taken from the table under "Laws of Similitude for Wind Tunnel Propellers".

<table>
<thead>
<tr>
<th>Propeller Type</th>
<th>$\frac{V^2}{ND^2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leland Stanford, Jr.</td>
<td>0.265</td>
</tr>
<tr>
<td>N. P. L., 4-foot</td>
<td>0.27</td>
</tr>
<tr>
<td>Curtiss, 4-foot</td>
<td>0.20</td>
</tr>
<tr>
<td>Curtiss, 7-foot</td>
<td>0.20</td>
</tr>
<tr>
<td>Langley Field model</td>
<td>0.25</td>
</tr>
<tr>
<td>McCook Field</td>
<td>0.48</td>
</tr>
</tbody>
</table>

THE FORMS OF ENTRANCE AND EXIT CONES.

There has been a great deal of discussion and dispute as to the best form for the cones in which the air acquires and loses its speed, and further experiment is desirable. The effect which changes in the form of these cones have on the efficiency is, however, much less than has commonly been supposed, judging from experiments recently performed at Langley Field and reported in another part of this paper.

In the absence of data to indicate the best form, most of the wind tunnels which have been constructed have used, at least on the exit side of the experimental chamber, the frustum of a right cone generated by a straight line. This was true of the N. P. L. and all their imitators, and it has been true also of most of the tunnels designed with an eye to the results of the experiments of Crocco and Castellazzi, and using long exit cones of very gradual slope. A surface of this type has at least the advantage of being easy to generate and to fabricate from wood or sheet metal. There is, however, no particular reason to believe that it is the most efficient that can be constructed from an aerodynamical point of view. Eiffel and his followers, on the other hand, have always used cones of curving form. It seems fair to assume that the loss in diverging nozzle is partially dependent on the deceleration of the fluid, and that the loss will usually be least where the deceleration is least. It is obvious, furthermore, that the flow through the exit cone will be smoothest and least turbulent when the form of the cone is smooth, and that any abrupt change of slope of the walls, such as that at the juncture of the parallel portion of the tunnel with an exit cone generated by a straight line, is liable to cause the lines of flow to break away from the contour of the tunnel wall, and to establish a region of "dead-water".

¹ Aviation and Aeronautical Engineering, Feb. 15, 1919, p. 84
and turbulence around the periphery of the exit cone. The smoothness of a curve can best be judged by taking differences or, if the equation of the curve is known, by plotting the derivatives. This was done in designing the cones for the Leland Stanford, Jr., tunnel. The plotting of the curve of acceleration for a tunnel will then serve the double purpose of indicating the smoothness of the curve and of giving the maximum rate at which the velocity of the air is changing, and so the maximum force necessary for accelerating the moving stream.

A curve of velocity against distance along the axis of the tunnel can be drawn on the assumption that velocity is inversely proportional to the square of the diameter of the tunnel. This, of course, is true only for velocity parallel to the axis, and entirely neglects the radial component. In order to obtain the acceleration from this curve, the derivative giving acceleration is written

\[ \frac{dv}{dt} = \frac{dv}{dx} \times \frac{dx}{dt} = v \times \frac{dv}{dx} \]

The acceleration at any point along the tunnel is therefore equal to the product of the ordinate of the curve just described by its slope at that point. These factors can be found graphically or, in the case of a curve for which the equation is known, analytically.

In the case of a straight cone, for example, the formula for diameter at any point is

\[ D = D_1 + (D_2 - D_1) \times \frac{x}{l} \]

where \( D_1 \) and \( D_2 \) are the diameters at the small and large ends of the cone, respectively, \( l \) the length of the cone, and \( x \) the distance from the small end. Then

\[ v = v_1 \times \frac{D_2^2}{D_1^2} = v_1 \times \left[ D_1 + \frac{x}{l} (D_2 - D_1) \right]^2 \]

and

\[ \frac{dv}{dx} = -2v_1 \times \frac{D_1^2 \times D_2 - D_1}{D_1 + \frac{x}{l} (D_2 - D_1)} \]

The acceleration is equal to the product of these expressions, or

\[ \frac{dv}{dt} = -2v_1^2 \times \frac{D_1^4 \times (D_2 - D_1)}{l \times \left[ D_1 + \frac{x}{l} (D_2 - D_1) \right]^3} \]

If the exit cone is generated by rotating about the axis of the tunnel a parabola having its vertex at the junction of the exit cone with the straight portion the formula for diameter of the cone at any point becomes

\[ D = D_1 + \left( \frac{x}{l} \right)^2 \times (D_2 - D_1) \]

The acceleration may then be obtained by the same steps just employed for the straight cone.

\[ v = v_1 \times \left[ D_1^2 + \left( \frac{x}{l} \right)^2 \times (D_2 - D_1) \right]^2 \]

\[ \frac{dv}{dx} = -2v_1 \times D_1^2 \times \frac{2x \times (D_2 - D_1)}{B \times \left[ D_1 + \left( \frac{x}{l} \right)^2 \times (D_2 - D_1) \right]^3} \]

\[ \frac{dv}{dt} = -\frac{4v_1^2 \times B^2 \times x \times (D_2 - D_1)}{B \times \left[ D_1 + \left( \frac{x}{l} \right)^2 \times (D_2 - D_1) \right]^4} \]

---

In figure 3, the velocity and acceleration, as well as the cone diameter, are plotted against \( x \) for cones of these two forms. The units are meters and seconds, and the curves relate to a tunnel having an exit cone tapering in diameter from 1.5 meters to 3 meters in a length of 6 meters, and a wind speed of 50 meters per second. It appears that the straight cone is far inferior, judged by the criteria laid down above, to that of parabolic form. The maximum acceleration for the first is more than two and a half times that for the second, and there is a large discontinuity in the acceleration curve for the straight cone, as might be anticipated from the discontinuity in the slope of the sides of the tunnel.

The parabolic form gives zero acceleration at the juncture of the exit cone with the experimental chamber, and this is very desirable, but it does not give a zero acceleration at the point where the air emerges from the exit cone. There is some question as to the desirability of using a reverse curve which will have tangents parallel to the axis of the tunnel at both its ends, and so securing zero acceleration at both ends of the exit cone. The air has to be slowed down some time, and there would seem to be little advantage in bringing it to a constant velocity as it leaves the retaining walls of the exit cone if it is to be decelerated again the instant that it is free from those walls. Also, the current of air, since it is to be turned through an angle of 180° and travel back through the room to the entrance of the tunnel, must acquire a radial velocity either inside the exit cone or immediately after it has left it. No gain is apparent from a construction which permits the air to acquire a certain amount of radial velocity and then straightens it out again, only to force it to turn outwards once more a few feet farther along its path. The effect of a reversal in the curve of the walls near the large end of the exit cone is certainly slight, as very good results have been obtained both with and without such a reversal.

The form of the entrance cone appears to have but little effect on the "energy ratio," and this is in accord with the results of hydraulic experiments, where it is always found that the loss in a converging nozzle is much less than that in a diverging one, and that the nozzle can converge very abruptly without seriously increasing the loss. Most of the European experiments on model tunnels have been made with straight entrance cones. While these are probably as efficient as any other type, they must have a vena contracta near the large end, causing turbulence which persists into the experimental chamber, and there is further eddying and disturbance due to the turning of the air around a sharp corner at the small end of the cone. To avoid
these difficulties and to secure as steady a flow as possible in the experimental chamber it is
the almost universal practice, in actual tunnels, to make the entrance cone of curving form. It
has been found at the National Physical Laboratory that, even if the entrance cone, or bell-
mouth, as it is called there, is curved around until a tangent to the wall at the large end is per-
pendicular to the axis of the tunnel, there still are marked and persistent eddies in the neighbor-
hood of the sharp edge. To entirely eliminate this edge it is now the practice at the N. P. L.
to carry the bell-mouth around, as shown in figure 4, until it meets the straight portion of the
tunnel. This method has not been adopted at Langley Field, as it is desired to make some
experiments on the full-sized tunnel with the normal entrance cone, but provision has been
made for building a fairing to extend clear around to the experimental chamber, as shown by
the dotted lines in figure 5, so giving the air a perfectly smooth passage.
DESIGN OF WIND TUNNELS AND WIND TUNNEL PROPELLERS.

THE THEORY OF LOSSES IN THE EXIT CONE.

The losses in the exit cone of a wind tunnel arise from three sources. The first is the friction against the walls, and is best determined by Fritzsche's formula for fluid friction. The second is the diverging angle of the cone, which, as already noted, always leads to a loss of energy as compared with the ideal conditions expressed by Bernoulli's theorem. The magnitude of this loss is determined with satisfactory accuracy by a formula devised by Fliegner. Finally, there is a loss due to the sudden release of the air from the exit cone and its passage into the room, where its velocity drops almost to zero. This loss was shown by Borda to be equal to the kinetic energy possessed by the air at the large end of the cone. These losses, and their relation to the factors entering into wind tunnel design, together with all the losses in other parts of the tunnel, have been fully discussed by Eiffel, and it is not necessary to repeat his work here. For the benefit of those designing tunnels, however, a set of curves has been plotted which make it possible to read off at once the loss in a straight conical exit cone of any type and to determine, given the limiting conditions, such as size of building to house the tunnel, the characteristics of the best exit cone for that particular case. Since from 80 per cent to 90 per cent of the total losses in a tunnel (not including those in the propeller) occur in the exit cone the problem of designing a tunnel with a high energy ratio is essentially a problem of reducing the losses in the exit cone.

In figure 6 the ordinates are the vertex angle of the exit cone, the abscissae the ratio of the cross-section area at the large end of the cone to the cross-section area where models are tested, at the throat or in the experimental chamber. The family of curves drawn in full lines are curves of equal loss, and the number which each one bears expresses the loss in the exit cone as a percentage of the kinetic energy possessed by the air at the smallest section of the tunnel. For example, if there were no losses except those in the exit cone, a tunnel having an exit cone of form corresponding to any point on the curve marked 20 would have an energy ratio of 5.

The nearly straight dotted lines running across the sheet diagonally correspond to various constant lengths of exit cone, and they are marked with the ratio of length to diameter at the small end.

To illustrate the use of this chart in choosing an exit cone a few illustrative examples will be given.

1. A tunnel is to be 2 meters in diameter. In order to keep the size and cost of the building within reasonable limits, it is desired that the length of the exit cone shall not exceed 20 meters.

Subject to this limitation, the cone is to be chosen for maximum efficiency.

The ratio of length to diameter here is 10. Passing along the dotted line bearing that number, it is seen that it cuts the curve of 16 per cent loss at two points and that it does not cut the 14 per cent curve at all, but that it approaches nearest to the latter at the point (α = 6.8°, n = 4.8). It is usually best to make n a little smaller than the value for minimum loss in the exit cone, as a reduction in n is a reduction in the diameter at the large end of the cone and so in the propeller diameter, and it has already been shown that this is favorable to propeller efficiency. It would probably be best, in this case, to take n = 4.3, α = 6.1°, or some other combination in that immediate neighborhood.

2. A very large tunnel is to be built, and, in order that the propeller diameter may not be unreasonably large, as well as to keep down the height of the building, the propeller diameter is limited to twice the diameter of the tunnel at the minimum section.

If the ratio of diameters at the ends of the exit cone is 2, n = 4. Drawing a vertical from the scale of abscissae at this point, it is seen that it approaches nearest to the 14 per cent curve at (α = 4.5°). The length of the exit cone for this angle is 13 times the minimum diameter. It would not be advisable, under these conditions, to choose the cone for the absolute maximum of efficiency, as the length could be decreased 24 diameters at a cost of only 5 per cent increase in the total power by increasing α to 6.7°. Since the curvature of the constant

1 Note on the Calculation of the Efficiency Coefficients of Air Channels, G. Eiffel, Paris, 1918.
FIG. 7.—METHOD OF CONSTRUCTING PLASTER CONES FOR MODEL TUNNEL.

FIG. 8.
of the wall. On going still farther out the speed dropped rapidly, due to friction. The velocity in the experimental chamber near the entrance cone was constant, as nearly as could be detected, over 90 per cent of the diameter of the stream. On going farther downstream the velocity distribution became more irregular, the speed being a maximum at the center and dropping off steadily toward the edges of the stream. The ratio of the velocity 75 per cent of the way out to the edge of the stream to that at the center was 1.00 at C, 0.97 at D, and 0.96 at E. The edge of the stream was not sharply defined, even very near to the point of issuance from the
entrance cone, and at \( E \), three-quarters of the way across the chamber, the velocity dropped off in a smooth curve from very near the center of the stream out to far beyond its normal boundaries.

The velocity distribution in the exit cone was exceedingly strange. The velocity dropped off rapidly from a maximum at the center, so that the stream appears actually to contract rather than to expand in the exit cone. As in the entrance cone, there was another maximum near the wall, but it was farther from the wall than that at the entrance, and the velocity was much lower than at the center. Directly in front of the propeller the velocity at the center dropped sharply, due to the hub, and varied in an irregular manner over the rest of the section. The flow at this point was so turbulent and so varying in direction that the measurements of velocity may contain considerable errors.

In order to make direct observations on the sharpness of definition of the edge of the stream in the experimental chamber and to determine the general nature of the flow in the chamber an observer got inside and sounded the flow with a thread. It was evident that the air in the whole chamber was much stirred up, and that the flow near the nominal edge of the stream was extremely turbulent, except in the immediate neighborhood of the entrance. Even in the farthest corners of the chamber, at a distance from the center of the stream equal to more than three times its nominal diameter, there was still a distinct movement of the air. The motion everywhere was very unsteady, the direction of flow at a given point changing 60° or more almost instantaneously. The best defined part of the circulation was near the small end of the exit cone, where two strong vortices rotating in opposite direction existed in the corners of the chamber. The examination of the flow was not extended to points above and below the stream in this neighborhood, so it is not certain whether or not a complete vortex ring, surrounding the opening into the exit cone, existed. The results of this examination of the flow in the experimental chamber made it clear that the balance would have to be shielded in some way from the air currents if any accurate work was to be done. In Eiffel's tunnel partial shielding of the balance is accomplished by placing it on a platform which, however, extends across only a small proportion of the width of the room, and can hardly act as a complete protection from air-currents for the measuring instruments.

The power curve is plotted in figure 14 (curve No. 1) and the curve of speed against revolutions per minute in figure 15 (curve No. 1). The energy ratio varied too little and too irregularly to make it worth while to plot a curve. Its mean value was 0.90, making no allowance for propeller losses. If the propeller efficiency be assumed to be 57 per cent (the value calculated by the Drzewiecki method), the energy ratio for the tunnel proper becomes 1.58.

In view of the irregularities of flow found in the experimental chamber it was decided to try next the effect of inclosing the stream in a cylindrical tube during its passage across the experimental chamber. No attempt was made to make the tube air-tight, the static pressure inside the tube being equal to that in the experimental chamber, which was carefully made air-tight. Curve No. 2 in figure 14, and also in figure 15, correspond to this case, and the traverses of the stream at points corresponding with those taken for the original model are plotted in figure 16.

Comparing these traverses with those in figure 13 it is seen that the nature of the distribution in the entrance cone is practically unaffected. The velocity at point \( C \) was a little less regular than for the case of the unconstrained stream, showing an increase near the walls similar to that which characterized the entrance cone. At \( D \) and \( E \), however, the velocity was much more even with the inclosing tube than without it, being constant within 1 per cent over 75 per cent of the diameter. Evidently, from the standpoint of steadiness of flow, the inclosed type of tunnel is superior to the Eiffel type.

In the exit cone the effect of surrounding the stream with a definite boundary was still more apparent. At \( F \) the velocity three-quarters of the way from the center to the walls was 94 per cent of that at the center, as against 67 per cent in the original model. At \( G \) the corresponding figures were 82 per cent and 40 per cent. At \( H \) there was, as in the first case, a minimum at the center and two maxima, the distribution of velocity being reasonably uniform across the outer 70 per cent of the blade, which is the most effective portion.
It is reasonable to suppose, in view of the better filling of the exit cone and of the generally improved velocity distribution, that the energy ratio would be increased by inclosing the stream, and this supposition was fully justified by the power measurements. For a given rate of rotation of the propeller the wind speed was increased while the power consumption was decreased, and the power consumption for a given wind speed was decreased just about 50 per cent. The energy ratio with the inclosing tube was 1.83 for the whole installation, or, making due allowance for the propeller losses, 3.20 for the tunnel alone.
It is evident that the inclosure of the stream improves the results in every way. The results obtained in these experiments, so far as power consumption is concerned, check very well with those obtained in some similar experiments on model tunnels, carried out by Lieut. Castellazzi. Lieut. Castellazzi found that the efficiency was decreased 40 per cent by the use of an open experimental chamber. The experimental chamber used in his experiments was round in cross section and was twice as large in diameter as the entrance and exit cones where they entered the chamber, and the slightly greater loss in efficiency found in the experiments conducted at Langley Field may be accounted for by the larger size and more irregular form of the experimental chamber there employed.

EFFECTS OF VARIATION IN EXIT CONE FORM.

The next series of experiments dealt with the effect of alterations in exit cone form. It was originally the intention to make a number of cones of different forms, but this plan was abandoned after two had been tried, and the experiments cover only the parabolic and straight forms of cone. These are as widely different from each other in respect of their acceleration curves as are any two forms which would be likely to be used.

The curves of power and revolutions per minute with the straight cone are plotted as curve No. 3 in figures 14 and 15. The mean energy ratio is 1.83 for the combination of tunnel and propeller, or 3.20 for the tunnel alone, values identical with those for the parabolic cone. It is evident from the curves that the effect of changing the exit cone from a parabolic to a straight form was very slight. The parabolic form seems to have a slight advantage at high values of $VD$ and to be inferior at low values, but the difference between the two curves is in no case in excess of the possible experimental error. In view of these results it appears that the efficiency of a tunnel is not affected appreciably by exit cone form or by the nature of the acceleration in the cone, but only by its length, mean angle, and total expansion ratio.

The large acceleration suddenly imposed on the air at the juncture between the parallel-sided portion of the tunnel and a straight exit cone might be expected to cause turbulence, so that the flow would be less regular than with a parabolic or other smoothly curving form. No experimental data are available on this point as yet, as the experiments were temporarily halted by an accident to the propeller before traverses and investigations of the flow had been carried out with the straight cone.

OBSERVATIONS OF THE NATURE OF THE FLOW THROUGH THE PROPELLER.

The most noticeable feature of the flow behind the propeller is the great rapidity with which the slip stream spreads. Instead of contracting, as in the case of an airplane propeller, where the direction of inflow is unrestricted, the stream expands immediately on passing clear of the cone, the air changing its direction so that there is a strong movement of the air, in a direction approximately at right angles to the axis of the tunnel, at a distance of 30 cm. back and 50 cm. out radially from the edge of the exit cone.

The flow in the throat and cones was very steady at all points except near the edges of the stream. The velocity head varied with a total amplitude of oscillation of about 2 per cent of the head and a period of from 20 to 40 seconds. On passing the propeller the pulsations of velocity became much more marked. The period of the pulsations close behind the propeller was about half a second, and the maximum velocity was estimated to be about 50 per cent greater than the minimum, although no means of measuring and making a continuous record of a rapidly varying velocity were available. On going farther away from the propeller along the lines of flow of the air the pulsations steadily increased in violence and the period lengthened until, at a distance of about 80 cm. to the rear of the propeller, the flow consisted of a violent gust about every second, the velocity in the intervals between these gusts being so low as to be hardly perceptible. These observations on the nature of the flow and its variations held in a general way for all the models tried, but the pulsations of velocity were much more marked for the case where the experimental chamber was left open than for that where it was inclosed in a tube.

1 Rendiconti dell'Instituto Centrale Aeronautico, 1917.
EXPERIMENTS ON THE EFFECT OF DISks AND SPINNERS ON THE PROPELLER.

In order to secure some idea of the effect of enlarging the hub of a propeller or of attaching a spinner, some experiments were made with disks of wall board attached in front of and behind the propeller, and also with a paper cone projecting from the propeller into the exit cone. The results of these tests do not fairly represent what might be secured with a good spinner and a propeller especially designed for it, as the propeller pitch should be increased when a spinner is incorporated or the hub is enlarged, but they will give some idea of the effect.

The effect of placing a disk in front of the central portion of the propeller, the rear not being covered and the blades not being housed in any way, was to decrease the wind speed and increase the power consumption. The inner parts of the blades acted as a centrifugal blower, taking air in from the rear and throwing it out radially. The increase in power, with a disk half the diameter of the propeller, was 9 per cent, the decrease of speed with the same disk 19 per cent. With a disk only one-fifth the diameter of the propeller the speed was decreased 5 per cent. These measurements were made at a speed of 10 meters per second and with the parabolic exit cone. The relative loss by the addition of a disk was greater with the straight cone and at high speeds, the addition of a disk four-tenths the diameter of the propeller causing an increase of 28 per cent in power and a decrease of 19 per cent in speed at a speed of 34 meters a second with the straight exit cone. The energy ratio was decreased 59 per cent. All subsequent tests were made with the straight cone, and the losses would probably be less with other forms.

The addition of another disk of equal size behind the propeller, so preventing any flow in from the rear and out toward the tips, improved the performance as compared with the single disk in front of the propeller, but remained inferior to the original case with no shielding at all. The power was increased only 6 per cent as compared with the original case without any disks, but the speed was decreased 16 per cent and the energy ratio fell off 44 per cent. When the rear disk alone was in place, so that any air thrown radially outward had to come from inside the exit cone, the power was increased 6 per cent, the velocity decreased 5 per cent, and the energy ratio decreased 19 per cent, using the model without disks as a standard in all cases. The disk behind the propeller therefore gave better results than did complete sheathing, either in the form of disks or faired by a cone in front.

The addition of a cone, having a diameter equal to two-fifths the diameter of the exit cone at its large end and an altitude of one and a quarter times its own diameter, in front of the propeller decreased the power about 2 per cent and increased the speed 7 per cent as compared with the values for the disks alone, but the energy ratio was still 30 per cent lower than for the original case. It seems strange at first that the entire blocking off of a considerable portion of the blades should increase the power consumption for a given number of revolutions per minute, but the phenomenon can be accounted for by the higher air speed past the propeller when the area of the exit cone is constricted by enlarging the hub. The theory of the effect of an enlarged hub or spinner has been discussed in another section of this report.

It appears that the addition of a spinner or the enlargement of the hub caused serious loss in every case where it was tried with the straight cone. The loss with a parabolic cone is much less, and it is likely that, with a propeller properly designed to allow for the increased velocity due to the blocking off of part of the area of the exit cone by the spinner, results as good as those in the original case could be obtained. It may even be that they could be materially improved on, but this does not seem very probable in view of the uniformly poor results shown in these experiments, where the presence of the spinner can hardly have decreased the propeller efficiency more than 10 per cent (a loss which, as already noted, could be prevented by the adoption of a propeller designed especially for the new conditions). The loss in propeller efficiency, therefore, would not be sufficient entirely to account for the decrease of energy ratio. The principal value of a very large hub is to increase the power coefficient of the propeller and make possible the reduction of the peripheral speed for a given wind speed.