NATIONAL ADVISORY COMMITTEE
FOR AERONAUTICS

REPORT No. 224

AN INVESTIGATION OF THE COEFFICIENT OF
DISCHARGE OF LIQUIDS THROUGH SMALL
ROUND ORIFICES

By W. F. JOACHIM

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AERONAUTICAL SYMBOLS

1. FUNDAMENTAL AND DERIVED UNITS

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2. GENERAL SYMBOLS, ETC.

$W$, Weight, $= mg$

$g$, Standard acceleration of gravity = 9.80665 m/sec.$^2$ = 32.1740 ft./sec.$^2$

$m$, Mass, $= \frac{W}{g}$

$\rho$, Density (mass per unit volume).

Standard density of dry air, 0.12497 (kg·m$^{-4}$·sec.$^2$) at 15° C and 760 mm = 0.002378 (lb·ft.$^{-4}$·sec.$^2$).

Specific weight of “standard” air, 1.2255 kg/m$^3$ = 0.07651 lb./ft.$^3$

$m k^2$, Moment of inertia (indicate axis of the radius of gyration, $k$, by proper subscript).

$S$, Area.

$S_w$, Wing area, etc.

$G$, Gap.

$b$, Span.

$c$, Chord length.

$b/c$, Aspect ratio.

$f$, Distance from $c, g.$ to elevator hinge.

$\mu$, Coefficient of viscosity.

3. AERODYNAMICAL SYMBOLS

$V$, True air speed.

$q$, Dynamic (or impact) pressure $= \frac{1}{2} \rho V^2$

$L$, Lift, absolute coefficient $C_L = \frac{L}{\rho S}$

$D$, Drag, absolute coefficient $C_D = \frac{D}{\rho S}$

$C$, Cross-wind force, absolute coefficient $C_c = \frac{C}{\rho S}$

$R$, Resultant force. (Note that these coefficients are twice as large as the old coefficients $L_c, D_c$.)

$i_w$, Angle of setting of wings (relative to thrust line).

$i_t$, Angle of stabilizer setting with reference to thrust line.

$\gamma$, Dihedral angle.

$\frac{Vl}{\mu}$, Reynolds Number, where $l$ is a linear dimension.

$e.g.,$ for a model airfoil 3 in. chord, 100 mi./hr. normal pressure, 0° C: 255,000 and at 15° C, 230,000; or for a model of 10 cm chord 40 m/sec, corresponding numbers are 299,000 and 270,000.

$C_p$, Center of pressure coefficient (ratio of distance of $C, P$, from leading edge to chord length).

$\beta$, Angle of stabilizer setting with reference to lower wing, $= (i_t - i_w)$.

$\alpha$, Angle of attack.

$\epsilon$, Angle of downwash.
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By W. F. JOACHIM

Langley Memorial Aeronautical Laboratory

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SUMMARY

The work covered by this report was undertaken in connection with a general investigation of fuel injection engine principles as applied to engines for aircraft propulsion, the specific purpose being to obtain information on the coefficient of discharge of small round orifices suitable for use as fuel injection nozzles.

Flow of the liquids tested under high pressure was obtained with an intensifier consisting of a 5-inch piston driving a direct-connected 3/4-inch hydraulic plunger. The large piston was operated by compressed air and the time required for the displacement of a definite volume by the hydraulic plunger was measured with an electrically operated stop watch. The coefficients were determined as the ratio of the actual to the theoretical rate of flow where the theoretical flow was obtained by the usual simple formula for the discharge of liquids through orifices.

Values for the coefficient were determined for the more important conditions of engine service such as discharge under pressures up to 8,000 pounds per square inch, at temperatures between 80° and 180° F. and into air compressed to pressures up to 1,000 pounds per square inch. The results show that the coefficient ranges between 0.62 and 0.88 for the different test conditions between 1,000 and 8,000 pounds per square inch hydraulic pressure. At lower pressures the coefficient increases materially.

It is concluded that within the range of these tests and for hydraulic pressures above 1,000 pounds per square inch the coefficient does not change materially with pressure or temperature; that it depends considerably upon the liquid, decreases with increase in orifice size, and increases in the case of discharge into compressed air until the compressed-air pressure equals approximately three-tenths of the hydraulic pressure, beyond which pressure ratio it remains practically constant.

INTRODUCTION

As far as is known, no data has been published giving the coefficient of discharge for liquid fuels discharged under high pressures through small orifices. It has been necessary therefore, in the design of an injection valve, to assume a value for the coefficient in order to find approximately the orifice size capable of discharging enough fuel, under the pressure employed, to meet the engine requirements. This research was undertaken in order to determine, for various conditions similar to those met in engine service, the coefficient of discharge of orifices suitable for use in fuel injection valves.

In order to determine the effects of pressure, temperature, and back air pressure upon the coefficient of discharge the work was arranged so that each of these influences could be varied independently. The pressure tests determined the coefficient of discharge for pressures up to 8,000 pounds per square inch, the liquids being discharged at 80° F. into air at atmospheric pressure. Coefficients were obtained for Diesel engine fuel oil discharged through orifices having diameters of 0.015 inch, 0.020 inch and 0.025 inch and for gasoline and water discharged through a 0.020-inch orifice. The temperature tests determined the coefficient at 80° F., 110° F., and 180° F. for Diesel engine fuel oil discharged through a 0.020-inch orifice. Pressures up to 8,000 pounds per square inch were used, the fuel being discharged into air at atmospheric pressure. The tests on the effect of back air pressure determined the coefficient
for the discharge of Diesel engine fuel oil at 80° F. into air at pressures up to 1,000 pounds per square inch using a 0.020-inch orifice. Two air chambers were used in order to determine the effect of a change in chamber size. In the tests with the small air chamber, hydraulic pressures up to 8,000 pounds per square inch and compressed air pressures up to 750 pounds per square inch were used. For the large air chamber hydraulic pressures up to 2,045 pounds per square inch and compressed air pressures up to 1,000 pounds per square inch were used.

METHODS AND APPARATUS

The method employed in the determination of all coefficients consisted in timing the flow of a known volume of liquid and determining the coefficients as the ratio of the actual to the theoretical rate of flow. Photographs of the discharge apparatus are shown in Figures 1 and 2. In order to obtain a continuous flow of liquid under high pressure and of sufficient quantity to permit reasonably accurate timing, an intensifier operated by compressed air was used. This apparatus consisted primarily of an air cylinder 5 inches in diameter in axial alignment with a hydraulic cylinder three-fourths inch in diameter, the air cylinder piston being connected directly to the hydraulic plunger. By using air pressures up to 195 pounds per square inch in the air cylinder, working hydraulic pressures up to 8,000 pounds per square inch were obtained. The air cylinder and piston were standard Liberty engine parts, the cylinder being mounted on a casting so as to permit a working stroke of 9 3/4 inches. The hydraulic plunger and cylinder were of hardened and ground tool steel, each separately lapped to obtain highly polished working surfaces. The fit between these two parts was such that when thoroughly lubricated a force of about 5 pounds was required to maintain relative movement. The orifices through which the liquids were discharged were also of hardened and ground tool steel. The holes were lapped to a high polish and means employed to secure sharp entering and exit edges.

The timed volume of the liquid discharged was 3.485 cubic inches less the leakage past the hydraulic plunger. This leakage was determined by a separate test for all liquids, pressures, and temperatures, and though found to be practically negligible, in most cases considerably less than 1 per cent, was included in the coefficient calculations. The discharge time of this quantity of liquid was obtained with an electrically operated one-hundredth-second stop watch. Starting and stopping of the watch was controlled by an electric contactor located between the air and hydraulic cylinders. The contactor was operated, through a follower, by two shallow notches on the hydraulic plunger. Thus, as the hydraulic plunger descended and the follower moved into the first notch, the contactor closed the electric circuit and started the stop watch. As the follower moved out of the notch, the circuit was broken and the watch stem permitted to return. The stopping of the watch at the completion of the timed discharge volume was accomplished in like manner by the second notch. It may be noted that the watch was operated by two successive "makes" of the circuit, thus giving the same electrical action at the start and finish of the timed stroke. Since the liquids to be discharged were at rest at the beginning of the plunger movement and uniform flow was desired during the timed discharge, about 12 per cent of the total liquid volume was permitted to be discharged before starting the stop watch. Tests showed that this predischarge was more than ample for the apparatus used.

Heating of the liquids was controlled in all tests by inclosing the liquid reservoir and discharge apparatus in a cabinet and circulating hot air around all parts until the test temperature was reached. This temperature was then maintained by control of the air temperature, which was read from thermometers through glass windows in the cabinet door. The temperature of the liquids was obtained by thermocouples and a potentiometer. Figures 1 and 2 show the arrangement of the apparatus except for the electric heating elements and circulating fan in the back of the cabinet.

The pressure in the compressed air supply tank was raised at the beginning of a test to the maximum value required and the hydraulic pressure controlled by successively lowering the air pressure. The pressure in the air chamber into which the liquids were discharged was maintained at atmospheric pressure in all tests except those discharging into compressed air.
The test procedure, after the desired working conditions had been obtained, was as follows: A valve control, which operated the hydraulic needle valves, was first raised to its limit of travel. This simultaneously opened the inlet valve and closed the discharge valve. The liquid under test was then pumped by hand from the reservoir into the hydraulic cylinder, thus raising the plunger and piston against the air pressure. When the plunger and piston reached the top of their stroke the stop-watch switch was closed and the valve control brought rapidly to its lower limit of travel. This simultaneously closed the inlet valve and opened the discharge valve, thus discharging the liquid through the orifice and into the air chamber. The hydraulic pressure obtained during the timed portion of the test was read from the hydraulic gage and the time of discharge obtained automatically as previously described.

RESULTS

The greater part of the data obtained in this investigation is given in Figures 3 to 8, inclusive. The data obtained in part-range check tests covering certain pressure and temperature effects are not given because these are practically the same as those presented. Considerable data covering high-pressure discharge into compressed air are also not given because they duplicate, within the limit of experimental error, that for high-pressure discharge into air at atmospheric pressure.

In order to present the data in a form that could be used to calculate discharge quantities and velocities without unnecessary complication, the effects of the compressibility of the liquid and its change of specific gravity with pressure are included in the coefficients. When these variables are taken into account in the calculations, the coefficient of discharge is found to be the same at the lower discharge pressures and to depart uniformly as the pressures increase until at 8,000 pounds per square inch it is about 4 per cent higher than the coefficient based on simple computation.

Pressure tests.—The results of the pressure tests are given in Figures 3, 4, and 5. The data presented in Figure 3 are the experimental and computed results of a test of Diesel engine fuel oil at 80° F. discharged into air at atmospheric pressure. The observed hydraulic pressures and recorded times are plotted as obtained during the test. The times required for the discharge volume to leak past the hydraulic plunger at various pressures are also plotted as obtained. These two curves give the experimental data from which the per cent of leak taking place during the test and the coefficient of discharge are calculated. The per cent of leak at any hydraulic pressure is calculated as the ratio of the discharge time to the
leak time, multiplied by 100. The coefficient of discharge is calculated as the ratio of the actual to the theoretical rate of flow. The actual rate of flow is calculated from the experimental data and the theoretical rate by means of the usual hydraulic formula,

\[ v = \frac{2gh}{\sqrt{1 - \frac{v^2}{g}}}. \]

The effects of orifice size and of different liquids are given in Figures 4 and 5, the data for these tests being obtained and computed in the same way as that for Figure 3. Three orifice sizes were tested, the diameters and lengths being 0.015 by 0.050 inch, 0.020 by 0.060 inch, and 0.025 by 0.060 inch. The data presented in Figure 4 is for Diesel engine fuel oil at 80°F.

![Figure 4](image)

**FIG. 4.**—Effect of orifice size. Diesel engine fuel oil at 80°F discharged into air at atmospheric pressure. Orifice size: Curve 1, 0.015 inch by 0.050 inch; curve 2, 0.020 inch by 0.060 inch; curve 3, 0.025 inch by 0.060 inch.

The parallelism between the curve and the practical constancy of the coefficients for discharge pressures above 1,000 pounds per square inch are worthy of note.

The data on the effect of different liquids, Figure 5, are for Diesel engine fuel oil, gasoline and water discharged through a 0.020-inch orifice, the test temperature and pressures being the same as for Figure 4. The parallelism and proximity of the curves for gasoline and Diesel engine fuel oil are noteworthy and probably indicate that the effect of the lesser density of the gasoline is nearly counteracted by its lesser viscosity.

**Temperature tests.**—The effect of temperature on the coefficient of discharge was determined for Diesel engine fuel oil discharged through a 0.020-inch orifice into air at atmospheric pressure.
Tests were made at 80°, 110° and 180° F. with hydraulic pressures up to 8,000 pounds per square inch. The coefficient was uniformly lower with the higher temperatures but over the range of these tests the difference was less than 2 per cent.

 Tests on discharge into compressed air.—The results of the work on low-pressure discharge into compressed air are given in Figures 6, 7, and 8, the results for high pressure discharge being omitted because they duplicated the results for high pressure discharge into air at atmospheric pressure as previously mentioned. It was also found that the results obtained for discharge into the two sizes of air chambers tested were the same. It was noted during the preliminary tests on discharge into compressed air that the coefficient of the orifice used had increased 0.06 for the case of discharge into air at atmospheric pressure. Microscopic examination of the orifice showed that its entering edge had been rounded. This was probably caused by the flow of unstrained liquid through the orifice, the high pressure strainers having failed during extraneous work. Since all conditions in these tests were the same as those for the pressure tests it is concluded that the increase in the coefficient was due to the change in the orifice edge.

![Figure 6: Coefficient of discharge versus back-air pressure. Diesel engine fuel oil at 80° F. discharged through a 0.020-inch orifice into air at various pressures. Hydraulic pressures: Curve 1, 2,045 pounds per square inch; curve 2, 1,645 pounds per square inch; curve 3, 1,255 pounds per square inch; curve 4, 1,005 pounds per square inch.](image)

In the work with low hydraulic pressures data was obtained for discharge into compressed air ranging in pressure up to 1,000 pounds per square inch, the pressure on the liquid during a test being maintained constant. Four hydraulic pressures, namely, 1,005, 1,255, 1,645, and 2,045 pounds per square inch were used. The data are for Diesel engine fuel oil at 80° F. discharged through a 0.020-inch orifice and are presented in three ways. In Figure 6 the coefficients are calculated in the same way as those for discharge into air at atmospheric pressure and do not take into account in the calculations any decrease in the effective hydraulic pressure due to the increased back air pressure. The coefficients are plotted against the back-air pressures. The data plotted from simple computation as above are convenient to use in practice and show clearly the effect of increased back-air pressure. It may be noted that the coefficient does not change materially below compressed air pressures approximately three-tenths of the hydraulic pressure and that the deviation within this range decreases with increase in hydraulic pressure. This deviation entirely disappears with higher hydraulic pressures, since in the work on high pressure discharge into compressed air no deviation of the coefficient was noted below the critical pressure ratio of three-tenths. Above this point the coefficient and rate of discharge decrease uniformly with increase in compressed air pressure.
In order to determine the coefficient at the critical point as accurately as possible several check tests were made for each hydraulic pressure. The experimental data giving the back-air pressures and corresponding discharge times for the 1,005 pounds per square inch hydraulic pressure, curve 4 of Figure 6, are plotted over a range covering the critical point in Figure 7. These data are plotted to enlarged scales and show the degree of check obtained for several runs.

The data for discharge into compressed air have also been computed on the basis that the compressed-air pressure reduces the effective hydraulic pressure on the liquid. The theoretical rate of flow was calculated in this case by means of the hydraulic formula

\[ V = \sqrt{2gh} \]

in which \( h \) is the hydraulic-pressure head and \( h_a \) the back-air pressure head. The coefficients as thus calculated are plotted against the ratio of the back-air pressure to the hydraulic pressure in Figure 8. It may be noted that the coefficients for all four curves, calculated on the above basis, increase from about 0.74 at a pressure ratio of zero up to about 0.88 at a pressure ratio of approximately three-tenths. Beyond this ratio or the critical point, the coefficients are practically constant. A decrease in the coefficient at the higher ratios of back-air pressure to hydraulic pressure may also be noted, this decrease being considerably smaller in degree for the higher discharge pressures.

Since the work on discharge into compressed air was limited to the use of only one size and proportion of orifice, it is thought that at present no general conclusions on the results may be drawn.
Application.—The accuracy of the results obtained in this investigation depended upon the accuracy with which the hydraulic pressures were observed, the accuracy of the stop watches, and of the measurement of a number of constants. The hydraulic pressures were read in general from a 10,000-pound hydraulic test gage which was calibrated by means of a dead-weight gage tester. It is believed that the error of the pressure readings did not exceed 5 pounds. Two one-hundredth second stop watches were used, a National Park and a Meylan. The running performance of these stop watches was determined by photographing their indicated times with those of two watches having known errors. The running error was negligible. The starting and stopping errors were not investigated, but in many hundred observations it was noted that where all test conditions were maintained constant the variation in the recorded times was generally not more than one-hundredth of a second. However, in some cases of discharge into compressed air below the critical point the variation in the recorded times was greater. Figure 7 represents the worst case in this respect. The more important constants measured were the orifice diameters and lengths, specific gravities of the liquids, and the displacement of the hydraulic plunger. At the test temperatures of 80°, 110°, and 180° F. the specific gravity of the Diesel engine fuel oil used was 0.846, 0.833, and 0.802, the corresponding Saybolt Universal viscosity being 39.0, 35.3, and 31.5 seconds. The gravitational constant, $g$, was calculated as 32.15 for the location of the laboratory of the National Advisory Committee for Aeronautics at Langley Field, Va. The displacement of the hydraulic plunger and lengths of the orifices were calculated from micrometer measurements, specific gravities were determined with a Westphal balance, and the diameters of the orifices measured on a dividing engine. Wherever practicable several check measurements and test observations were made and the limits of probable error calculated. The total limit of probable error ranged between 1 and 2½ per cent, and was greatest for the low pressures and the smallest orifice.

The results of this work are, in general, applicable to cases of continuous discharge having the same or similar conditions of operation. Intermittent discharge and the flow of other liquids through orifices of the same or different form would probably result in appreciable differences in the coefficient. For such cases the data presented herewith may only be used as an indication.

CONCLUSION

From the results of discharge into air at atmospheric pressure it is concluded that for hydraulic pressures above approximately 1,000 pounds per square inch the coefficient for a given orifice and liquid does not change materially with pressure or temperature, within the limits of these tests, but that for pressures below 1,000 pounds per square inch it trends in general toward unity. The coefficient for gasoline and Diesel engine fuel oil are practically the same, but for water it is considerably higher. An increase in orifice size within the range of this investigation consistently decreased the coefficient. A slight accidental rounding of the entering edge of an orifice in one instance caused an increase in the value of the coefficient.

From the work on discharge into compressed air it was found that the rate of discharge for Diesel engine fuel oil at 80° F. and a 0.020-inch orifice was not materially affected by discharge into dense air at pressures less than approximately three-tenths of the hydraulic pressure, but that at higher air pressures it decreased rapidly in agreement with the resulting effective hydraulic pressures.
Positive directions of axes and angles (forces and moments) are shown by arrows.

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Absolute coefficients of moment:

\[ C_L = \frac{L}{qbs}, \quad C_M = \frac{M}{qcS}, \quad C_N = \frac{N}{qfS} \]

Angle of set of control surface (relative to neutral position), \( \delta \). (Indicate surface by proper subscript.)

### 4. PROPELLER SYMBOLS

- **D**, Diameter.
- **pₑ**, Effective pitch.
- **pₚ**, Mean geometric pitch.
- **pₛ**, Standard pitch.
- **pₓ**, Zero thrust.
- **pₓ**, Zero torque.
- **p/D**, Pitch ratio.
- **V'**, Inflow velocity.
- **Vₛ**, Slip stream velocity.
- **T**, Thrust.
- **Q**, Torque.
- **P**, Power.

(If "coefficients" are introduced all units used must be consistent.)

\[ \eta = \frac{T}{V/P} \]

\[ n, \text{ Revolutions per sec., r. p. s.} \]

\[ N, \text{ Revolutions per minute., R. P. M.} \]

\[ \phi, \text{ Effective helix angle} = \tan^{-1}\left(\frac{V}{2\pi n}\right) \]

### 5. NUMERICAL RELATIONS

- 1 HP = 76.04 kg/m/sec. = 550 lb./ft./sec.
- 1 kg/m/sec. = 0.01315 HP.
- 1 mi./hr. = 0.44704 m/sec.
- 1 m/sec. = 2.23693 mi./hr.
- 1 lb. = 0.4535924277 kg.
- 1 kg = 2.2046224 lb.
- 1 mi. = 1609.35 m = 5280 ft.
- 1 m = 3.2808333 ft.