REPORT No. 247

PRESSURE OF AIR ON COMING TO REST FROM VARIOUS SPEEDS

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SUMMARY

The text gives theoretical formulas from which is computed a table for the pressure of air on coming to rest from various speeds, such as those of aircraft and propeller blades. Pressure graphs are given for speeds from 1 cm. sec. up to those of swift projectiles.

The present treatment, slightly modified, was prepared for the Bureau of Aeronautics, Navy Department, February 17, 1926, and by it was submitted for publication to the National Advisory Committee for Aeronautics.

PRESSURE-SPEED FORMULAS FOR MODERATE SPEEDS

A solid surface in uniform translation through a frictionless incompressible fluid, otherwise quiescent, can thereby receive at one point or more a maximum pressure increase \( \rho_0 V_0^2/2 \), where \( \rho_0 \) is the fluid density, \( V_0 \) the body's speed. One calls \( \rho_0 V_0^2/2 \) the "full impact" or "stop" pressure; and any point where it occurs a "stagnation point" or "stop point." Likewise if the body is fixed in a uniform stream, of speed \( V_0 \), the incompressible fluid comes to rest at a stop point with the pressure increase \( \rho_0 V_0^2/2 \). Here the whole stop pressure above vacuo is

\[
p_1 = p_0 + \rho_0 V_0^2/2
\]

if \( p_0 \) is the pressure in the unchecked part of the stream. In every case here treated \( p_0 \) is assumed void of gravity effect.

When a gas for which \( p/p_0 = (\rho/\rho_0)^\gamma \) comes to rest adiabatically the stop pressure is

\[
p_2 = p_0 \left[ 1 + \frac{(\gamma - 1) \rho_0 V_0^2}{2 \gamma p_0} \right] \]

as shown in hydrodynamics, \( \gamma = C_p/C_v \) being the ratio of the specific heat at constant pressure to that at constant volume. This formula is valid for engineering speeds below that of sound in the fluid; for higher and for extremely low speeds other formulas will be given presently.

Expanding (2) gives

\[
p_2 = p_0 + \rho_0 V_0^2/2 + p_0 \left( \frac{\rho_0 V_0^2}{8 \gamma p_0} + \ldots \right)
\]

which exceeds (1) by the parenthetical factor. This excess is negligible at sufficiently low speeds; but not at the speed of a fast airplane or propeller blade, as presently will be shown by some examples.

To furnish the aeronautical engineer with ready numerical values of (1), (2) for air on coming to rest, Table I has been computed for the standard values specified below it.\(^1\) Taking \( \gamma = 1.40 \), one first writes (1), (2) in the convenient working forms

\[
p_1/p_0 = 1 + 0.60471 V_0^2 \times 10^{-3}, \quad p_2/p_0 = (1 + 1.7277 V_0^2 \times 10^{-16})^{3.00}
\]

For speeds below that of sound no material error ensues from taking \( \gamma = 1.40 \) instead of the slightly different values given in physics.

\(^1\) A like table was computed for C. & R. Report No. 129, dated May 13, 1919, and one giving five speeds was published by Finzi and Soldati in 1863, in their pamphlet "Esperimenti Sulla Dinamica dei Fluidi."
The computations were made by various members of the aerodynamics staff in the Construction and Repair Aerodynamical Laboratory of the United States Navy, and checked by the aeronautics staff at the Bureau of Standards. The diagrams were made by Mr. F. A. Louden.

The importance of the pressure excess due to compression may be judged from the tenth column of the table. For speeds under 70 miles an hour the excess is less than \( \frac{3}{4} \) per cent of the impact pressure computed for air without compression. At 100 miles an hour, it is 0.41 per cent; at 150 miles, 0.86 per cent; at 300 miles, 4 per cent; at 800 miles, 31 per cent. The last is about the speed of sound and of some propeller tips, while 300 miles is attained by fast airplanes in diving.

VALIDITY OF FORMULA

The validity of \( 2 \) is here assumed without proof; viz, the compression is assumed to occur without sensible heat transfer. At speeds above 150 miles an hour, for which the density increment is no longer negligible, the compression of the air filament from \( p_0 \) to \( p_2 \) may occur in very brief time. To illustrate, suppose air streaming at 200 feet a second across a rod 1 inch in diameter. From both theory and experiment one knows that the speed is sensibly unchecked at points 1 foot before the rod and 1 foot behind it. Hence a particle traversing this range must receive its maximum compression in about \( \frac{1}{10} \) second. The dissipation of compression heat in this case may be assumed negligible, both because of suddenness and because the heating or cooling of any filament is paralleled by that of its immediate neighbors, thus lessening the temperature gradient.

It is commonly assumed also that for usual wind tunnel speeds the stop pressure on a large body equals that on a like small one in like conditions. In 1902 the writer found the impact pressure in the nozzle of a \( \frac{3}{8} \)-inch pitot the same as in a pipe 5 inches in diameter when both were pointed upstream in a 6-foot wind tunnel maintaining a steady 40-mile wind. (Reference 1.) In December, 1925, he found the impact pressure in a square-ended glass tube of \( \frac{3}{8} \)-inch bore, pointed into a 40-mile wind, equal to that in a like tested neatly pointed hypodermic tube 0.01 inch in diameter, \( ^{2} \) truly to \( \frac{1}{3} \) inch of water.

The arrangement for this latter test is shown in Figure 1. A glass U-tube of \( \frac{3}{8} \)-inch bore with arms in a horizontal plane and pointing upstream, is held by a sheet metal clamp mounted on the spindle of the aerodynamic balance in the 4-foot wind tunnel. With both ends wide open in a 40-mile wind the U-tube was adjusted, by canting the balance, first till the small piston of alcohol there shown just moved forward, then till it just moved backward. The amount of cant was indicated by an Ames dial gauge at the tip of the balance beam. Now, with the glass arms in their neutral position, and one plugged with a hypodermic needle of \( \frac{1}{16} \)-inch bore, as shown, the piston rested in the same equilibrium position as before in the same wind. Since the dial gauge showed that a cant of 1 in 10,000 is sufficient to move a piston 3 inches long, of alcohol of specific gravity 0.81, the differential pressure between the fine and coarse nozzle can not exceed about \( \frac{1}{3} \) inch of water.

For the medium speeds listed in Table I many experiments have shown that the pitot impact pressure equals the reservoir pressure; that is, the pressure of stagnant air from which the stream would issue with the speed \( V_0 \) through a perfect nozzle. For such speeds therefore no corroborative data need be presented. For swifter flows Dr. Briggs furnishes some unpublished measurements made by himself and Dr. Buckingham showing that the static air pressure in a reservoir equals the pitot pressure in its fair discharge nozzle at exit speeds of 400 to 1,000 feet a second, but progressively exceeds it for higher speeds up to that of sound, though the excess is but a few per cent. Check measurements with improved apparatus will be made by them before publication.

1 Inside diameter 0.0397; outside, 0.0411 inch.
PRESSURE OF AIR ON COMING TO REST FROM VARIOUS SPEEDS

LOW-SPEED FORMULA

Table I therefore is valid for bodies of all but microscopic size. For small bodies at low speeds a viscous pressure may have to be added to the inertia pressure. As shown in hydrodynamics, the nose impact pressure on a sphere of radius \( a \), fixed in a boundless uniform stream of liquid, of viscosity \( \mu \), is

\[
P_i = \frac{1}{2} \rho_s V_s^2 + \frac{3}{2} \frac{\mu V_s}{a}
\]

The ratio of this to \( \frac{p_s}{\rho_s V_s^2/2} \), found for an inviscid liquid is

\[
y = \frac{p_i}{p_s} = 1 + \frac{3}{R}
\]

where \( R = a V_s/\nu \) is Reynolds Number. On plain section paper \( y \) plots against \( R \) as an hyperbola asymptotic to the lines \( y = 1, R = 0 \).

For \( R \) quite large, as assumed for Table I, the viscous term is inappreciable; for \( R \) small, as when a mist particle falls in air, that term is predominant.

Assuming \( p_i \) to express the impact pressure of a fine pitot at small speeds, Muriel Barker (Reference 2), of Cambridge University, plotted it against the impact pressure \( p_s \) determined by her for the point of a pitot 1 millimeter in diameter held at the center of a long brass pipe 11 millimeters in diameter conducting water in steady stream-line flow at approximately 16°C. Fixed speeds of unchecked flow from 0.82 to 11.76 centimeter-seconds at the pipe's center were used. For \( V_s < 6 \) centimeter-seconds, or for \( R < 30 \), fairied values of \( p_s \) plot as a straight line

\[
p_s = p_i/1.1;
\]

for \( R > 30 \), \( p_s = 0.5 \rho_s V_s^2 \).

No correction of \( p_s \) was made for the ratio of the diameters of the pipe and pitot. Until this has been done (5) may be regarded as but an approximate expression for the differential pressure of said pitot in a boundless stream.

HIGH-SPEED FORMULA

For speeds well above that of sound the value of (2) is doubtful; first because \( \gamma \) is then quite variable, secondly, because the pressures given by (2) are much higher than the nose pressures found in high-speed projectiles. For such speeds Rayleigh (Reference 3) derives a special formula which, with \( \gamma = 1.40 \) reduces to

\[
\frac{p_s}{p_i} = \frac{166.7 V_s^2}{C^2(7V_s^2 - C^2)^{3/4}}
\]

where \( C \) is the speed of sound in the unchecked stream. For \( V_s = C \), (6) and (2) give the same value of \( p_i/p_s \). Rayleigh's formula is indorsed in Reference 4.

PRESSURE-SPEED GRAPHS

Fig. 2 shows graphically the absolute pressures given in Columns 4, 5 of Table I; also the impact pressures got from them by subtracting unity.
With \( p = \) impact pressure plus viscous pressure, the upper curve of Fig. 3 delineates \( p/\rho_0V_0^2 \) versus \( V_0 \) for air on coming to rest against the nose of a sphere 1 millimeter in diameter. It shows the effect of viscosity at low speed and adiabatic compression at high speed. The lower curve gives \( p/\rho_0V_0^2 \) for water, in comparison with Miss Barker's readings with the one millimeter pitot. Semilog paper is used to lengthen the low-speed scale.

For ultrasonic speeds two graphs are given in Figure 3; the higher derived from (2), the lower from (6). The true impact pressure at these speeds is found by Stanton to be given by Rayleigh's formula within \( \frac{1}{2} \% \) at 2.3 times the speed of sound (Reference 4).

REFERENCES

### TABLE I

**PRESSURE OF AIR ON COMING TO REST FROM VARIOUS SPEEDS**

(Symbols defined below)

<table>
<thead>
<tr>
<th>Air speed</th>
<th>Barometric plus impact pressure in standard atmospheres</th>
<th>Impact pressure in pounds per square foot</th>
<th>Impact pressure in inches of water</th>
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<tbody>
<tr>
<td>Miles per hour</td>
<td>Knots per hour</td>
<td>Kilometers per hour</td>
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</table>

Symbols defined below:

- \( p \): Air speed (in miles per hour)
- \( V_s \): Impact pressure (in pounds per square foot)
- \( V_a \): Impact pressure (in inches of water)
- \( V_f \): Impact pressure (in pounds per square foot)
- \( V_{f0} \): Impact pressure (in inches of water)

Using \( \gamma = 1.40 \) would lower the values in column 7 and 9 less than 0.02% for speeds less than 300 miles per hour.

\[ \text{Impact pressure} = \frac{p}{\gamma - 1} \]

\[ \text{Incompressible} = \frac{p}{\gamma - 1} \]