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REPORT No. 411

THEORY OF WING SECTIONS OF ARBITRARY SHAPE

By THEODORE THEODORSEN

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SUMMARY

This paper presents a solution of the problem of the theoretical flow of a frictionless incompressible fluid past airfoils of arbitrary forms. The velocity of the 2-dimensional flow is explicitly expressed for any point at the surface, and for any orientation, by an exact expression containing a number of parameters which are functions of the form only and which may be evaluated by convenient graphical methods. The method is particularly simple and convenient for bodies of streamline forms. The results have been applied to typical airfoils and compared with experimental data.

INTRODUCTION

The theory of airfoils is of vital importance in aeronautics. It is true that the limit of perfection as regards efficiency has almost been reached. This attainment is a result of persistent and extensive testing by a large number of institutions rather than of the fact that the important design factors are known. Without the knowledge of the theory of the air flow around airfoils it is well-nigh impossible to judge or interpret the results of experimental work intelligently or to make other than random improvements at the expense of much useless testing.

A science can develop on a purely experimental basis only for a certain time. Theory is a process of systematic arrangement and simplification of known facts. As long as the facts are few and obvious no theory is necessary, but when they become many and less simple theory is needed. Although the experimenting itself may require little effort, it is, however, often exceedingly difficult to analyze the results of even simple experiments. There exists, therefore, always a tendency to produce more test results than can be digested by theory or applied by industry. A large number of investigations are carried on with little regard for the theory and much testing of airfoils is done with insufficient knowledge of the ultimate possibilities. This state of affairs is due largely to the very common belief that the theory of the actual airfoil necessarily would be approximate, clumsy, and awkward, and therefore useless for nearly all purposes.

The various types of airfoils exhibit quite different properties, and it is one of the objects of aerodynamical science to detect and define in precise manner the factors contributing to the perfection of the airfoil. Above all, we must work toward the end of obtaining a thorough understanding of the ideal case, which is the ultimate limit of performance. We may then attempt to specify and define the nature of the deviations from the ideal case.

No method has been available for the determination of the potential flow around an arbitrary thick wing section. The exclusive object of the following report is to present a method by which the flow velocity at any point along the surface of a thick airfoil may be determined with any desired accuracy. The velocity of the potential flow around the thick airfoil has been expressed by an exact formula, no approximation having been made in the analysis. The evaluation for specific cases, however, requires a graphical determination of some auxiliary parameters. Since the airfoil is perfectly arbitrary, it is, of course, obvious that graphical methods are to some extent unavoidable.

Curiously enough, the theory of actual airfoils as presented in this report has been brought into a much simpler form than has hitherto been the case with the theory of thin airfoils. In the theory of thin airfoils certain approximations have restricted its application to small cambers only. This undesirable feature has been avoided, and the results obtained in this report have a complete applicability to airfoils of any camber and thickness.

The author has pointed out in an earlier report that another difficulty exists in the theory of thin airfoils. It consists in the fact that in potential flow the velocity at the leading edge is infinite at all angles except one. This particular angle at which the theory actually applies has been defined as the ideal angle of attack. In the present work we shall not go any further into this theory, since it is included in the following theory as a special case of rather limited practical importance.

THEORY OF THICK AIRFOILS

In the theory of functions there is a theorem by Riemann [1] which shows that it is always possible to transform the potential field around any closed contour into the potential field around a circle. The direct transformation of an airfoil into a circle may,

[1] Handbuch der Physik, Band III, p. 245, Fundamentalsatz der konformen Abbildung.

for analytical purposes, conveniently be performed in two steps. The first step is to transform the airfoil into a curve which ordinarily does not differ greatly from a circle by the transformation

$$\zeta = z' + \frac{a^2}{z'} \tag{I}$$

where ζ is a complex quantity defining the points in the plane describing the flow around the airfoil and z' is another complex quantity defining the points in the plane describing the flow around the almost circular curve. The constant a is of dimension length and is merely a geometrical scale factor. In the following theory, attention is directed to the fact that the shape of the curve resulting from transformation (I) is arbitrary, since the airfoil shape is arbitrary. At a later point we shall transform this curve into a circle.

The z' and the ζ planes are shown superposed in Figure 1. It will be noticed that at great distances

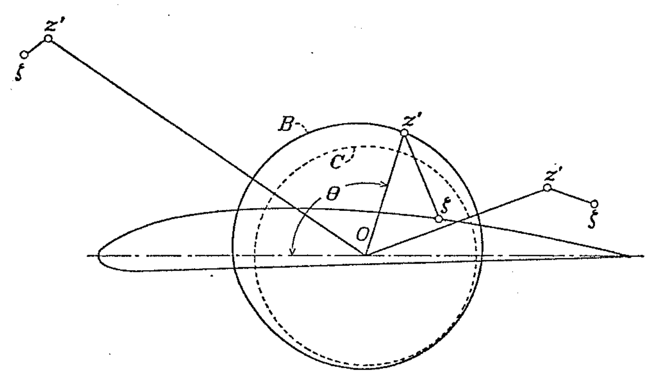


FIGURE 1.—Showing the transformation from a noncircular curve B into an airfoil.

from the origin $z' \to \zeta$; that is, both flows are similar at infinity. In particular, the "angle of attack," defined as the direction of flow at infinitity with respect to some fixed reference line in the body, is identical in both flows. Near the origin the two flows are entirely different; one value of z' is, however, uniquely associated with a given value of ζ by the relation (I).

We shall, at a later point, determine the flow in the z' plane. At present we shall determine the appearance of the airfoil when the almost circular curve B is given, or what amounts to the same thing, we shall determine the curve B when the airfoil is given. In Figure 1, C is a circle of unit radius. Since the matter of dimensions is rather important, we shall avoid confusion in the following by adhering to this length as unity. The curve B is uniquely given by the relation $z' = ae^{\psi + i\theta}$ where ψ is a known or unknown real function of the angle θ where θ varies from zero to 2π and i is the imaginary unit. Since the airfoil surface corresponds to the surface of the curve, the former is given from relation (I) as

$$\zeta = a\, e^{\psi + i\theta} + a\, e^{-\psi - i\theta}$$

or

$$\zeta = a(e^{\psi} + e^{-\psi}) \cos\theta + i\, a(e^{\psi} - e^{-\psi}) \sin\theta$$

This relation may further be conveniently expressed in hyperbolic functions

$$\zeta = 2a \cosh\psi \cos\theta + 2ia \sinh\psi \sin\theta$$

Since $\zeta = x + iy$, the coordinates of the airfoil (x, y) are given by

$$\begin{aligned} x &= 2a \cosh\psi \cos\theta \\ y &= 2a \sinh\psi \sin\theta \end{aligned} \tag{II}$$

We obtain a relation between θ and the coordinates of the airfoil as follows:

$$\cosh\psi = \frac{x}{2a \cos\theta}$$

$$\sinh\psi = \frac{y}{2a \sin\theta}$$

and since $\cosh^2\psi - \sinh^2\psi = 1$

$$\left(\frac{x}{2a\cos\theta}\right)^2 - \left(\frac{y}{2a\sin\theta}\right)^2 = 1$$

or developed

$$2\sin^2\theta = p + \sqrt{p^2 + \left(\frac{y}{a}\right)^2} \tag{III}$$

where

$$p = 1 - \left(\frac{x}{2a}\right)^2 - \left(\frac{y}{2a}\right)^2$$

Similarly we obtain a relation between ψ and the coordinates of the airfoil by using the equation

$$\left(\frac{x}{2a\cosh\psi}\right)^2 + \left(\frac{y}{2a\sinh\psi}\right)^2 = 1$$

or developed

$$2\sinh^2\psi = -p + \sqrt{p^2 + \left(\frac{y}{a}\right)^2} \tag{IV}$$

Since ψ is generally small for wing sections it may be more conveniently expressed for purposes of calculation as a series in terms of $\frac{y}{2a\sin\theta}$, as follows:

We have

$$\begin{aligned} e^{\psi} &= \sinh\psi + \cosh\psi \\ &= \sinh\psi + \sqrt{1 + \sinh^2\psi} \\ &= 1 + \sinh\psi + \frac{1}{2}\sinh^2\psi + \cdots \\ \psi &= \log_e\left(1 + \sinh\psi + \frac{1}{2}\sinh^2\psi + \cdots\right) \\ &= \sinh\psi - \frac{1}{6}\sinh^3\psi + \cdots \\ &= \frac{y}{2a\sin\theta} - \frac{1}{6}\left(\frac{y}{2a\sin\theta}\right)^3 + \cdots \end{aligned} \tag{IVa}$$

$$[\text{for } \psi < \log_e 2]$$

We are now in a position to reproduce the conformal representation of an airfoil in the z' plane, since for each point of the airfoil (x, y) both θ and ψ have been determined.

The curves $\psi=$ constant are ellipses in the ζ plane

$$\left(\frac{x}{2a\cosh\psi}\right)^2+\left(\frac{y}{2a\sinh\psi}\right)^2=1$$

The foci are located at $(\pm 2a, 0)$. The radius of curvature at the end of the major axis is $\rho=\frac{(2a\sinh\psi)^2}{2a\cosh\psi}$

or
$$\frac{\rho}{2a}=\frac{(\sinh\psi)^2}{\cosh\psi}\cong\psi^2$$

$$\psi\cong\sqrt{\frac{\rho}{2a}}\text{ (for small }\psi)$$

This relation is useful for the determination of ψ near the nose and the tail.

The leading edge, corresponding to $\theta=0$, is located at

$$2a\cosh\cong\psi 2a\left(1+\frac{\psi^2}{2}\right)=2a+a\psi^2=2a+\frac{1}{2}\rho$$

Thus we see that the length $4a$ corresponds to the distance between the point midway between the nose and the center of curvature of the leading edge to the point midway between the tail and the center of curvature of the trailing edge.[2]

To establish the magnitude of the velocity at any point (x, y) on the airfoil, we start in customary manner with the velocity around a circle in 2-dimensional flow. Contrary to usual practice we will, however, make the radius of the circle equal to ae^{ψ_0} where ψ_0 is a small constant quantity. This quantity is shown later in this report (equation (e)), to represent the average value of ψ taken around the circle C.

The potential function of the flow past this circle is

$$w=-V\left(z+\frac{a^2e^{2\psi_0}}{z}\right)-\frac{i\Gamma}{2\pi}\log\frac{z}{ae^{\psi_0}} \qquad \text{(V)}$$

(reference 1, p. 83) and the velocity[3]

$$\frac{dw}{dz}=-V\left(1-\frac{a^2e^{2\psi_0}}{z^2}\right)-\frac{i\Gamma}{2\pi z} \qquad \text{(VI)}$$

where Γ is the circulation. This expression must vanish at the rear stagnation point[4] (Kutta condition) whose coordinate is $z=-ae^{\psi_0+i(\alpha+\epsilon_T)}$, where α is the angle of attack and ϵ_T is shown to be the angle of zero lift.

[2] The choice of axes is entirely arbitrary. It is a matter of convenience only to choose the axes so that the airfoil appears as nearly elliptical as possible, thereby making the "almost circular" curve B as nearly circular as possible by means of the single transformation I. It will be seen that the evaluation of the important integral appearing in the appendix is then most easily accomplished. In fact, the transformation I itself is only a matter of convenience to permit the ready evaluation of this integral.

[3] $\frac{dw}{dz}$ actually equals $u-iv$, the image of the velocity vector about the x-axis.

[4] It is worthy of mention to note that the theory outlined in this report may actually be applied to smooth *bodies* of arbitrary shape if the *circulation* is specified. The term "wing sections" has been used in the title to imply bodies with sharp (or nearly sharp) trailing edges, whose circulation is or may be considered fixed by the Kutta condition or some equivalent assumption.

We obtain $\Gamma=-\frac{2\pi z}{i}V\left(1-\frac{a^2e^{2\psi_0}}{z^2}\right)$

$$=4\pi Vae^{\psi_0}\left(\frac{e^{i(\alpha+\epsilon_T)}-e^{-i(\alpha+\epsilon_T)}}{2i}\right)$$

$$=4\pi Vae^{\psi_0}\sin(\alpha+\epsilon_T) \qquad \text{(VII)}$$

This flow around the circle may now be transformed into the flow around any other body. In the particular case in which the flow at infinity is not altered the circulation will not be altered and the force experienced by a body at the origin will remain at the fixed value $L=\rho V\Gamma$.

We will now transform this circle, defined as $z=ae^{\psi_0+i\varphi}$ into our curve B defined by the relation $z'=ae^{\psi+i\theta}$. For this purpose we employ the general transformation $z'=ze^{\sum_n (A_n+iB_n)\frac{1}{z^n}}$ which leaves the flow at infinity unaltered, the constants being determined by the boundary conditions. By definition

$$z'=ze^{\psi-\psi_0+i(\theta-\varphi)}.$$

Consequently

$$\psi-\psi_o+i(\theta-\varphi)=\sum_n(A_n+iB_n)\frac{1}{z^n} \qquad \text{or}$$

$$\psi-\psi_o+i(\theta-\varphi)=\sum_n(A_n+iB_n)\frac{1}{r^n}(\cos n\varphi-i\sin n\varphi)$$

where z has been expressed in polar form

$$z=r(\cos\varphi+i\sin\varphi)$$

and by De Moivre's theorem

$$\frac{1}{z^n}=\frac{1}{r^n}(\cos n\varphi-i\sin n\varphi)$$

Equating the real and imaginary parts we obtain the two Fourier expansions:

$$\psi-\psi_o=\sum_n\left[\frac{A_n}{r^n}\cos n\varphi+\frac{B_n}{r^n}\sin n\varphi\right] \qquad \text{(a)}$$

and

$$\theta-\varphi=\sum_n\left[\frac{B_n}{r^n}\cos n\varphi-\frac{A_n}{r^n}\sin n\varphi\right] \qquad \text{(b)}$$

The values of the coefficients $\frac{A_n}{r^n}, \frac{B_n}{r^n}$, as well as the quantity ψ_0, may be determined from (a) as follows:

$$\frac{A_n}{r^n}=\frac{1}{\pi}\int_0^{2\pi}\psi\cos n\varphi d\varphi \qquad \text{(c)}$$

$$\frac{B_n}{r^n}=\frac{1}{\pi}\int_0^{2\pi}\psi\sin n\varphi d\varphi \qquad \text{(d)}$$

and

$$\psi_o=\frac{1}{2\pi}\int_0^{2\pi}\psi d\varphi \qquad \text{(e)}$$

The quantity $\theta-\varphi$ is necessary in the following analysis. Let us eliminate the coefficient $\frac{A_n}{r^n}$ and $\frac{B_n}{r^n}$ in (b) by means of (c) and (d).
We obtain

$$(\theta-\varphi)_c=\sum_n \cos n\varphi_c \frac{1}{\pi}\int_0^{2\pi}\psi \sin n\varphi d\varphi$$
$$-\sin n\varphi_c \frac{1}{\pi}\int_0^{2\pi}\psi \cos n\varphi d\varphi$$

The subscript c is added to indicate that the angles so distinguished are kept constant while the integrations are performed. The expression may be simplified

$$(\theta-\varphi)_c=\frac{1}{\pi}\sum_n\int_0^{2\pi}\psi\,(\sin n\varphi \cos n\varphi_c-\cos n\varphi \sin n\varphi_c)\,d\varphi$$
$$=\frac{1}{\pi}\sum_n\int_0^{2\pi}\psi \sin n(\varphi-\varphi_c)\,d\varphi$$

But

$$\sum_n \sin n\,(\varphi-\varphi_c)=\frac{1}{2}\cot\frac{(\varphi-\varphi_c)}{2}-\frac{\cos\,(2n+1)\,\frac{(\varphi-\varphi_c)}{2}}{2\sin\frac{\varphi-\varphi_c}{2}}$$

Therefore,

$$(\theta-\varphi)_c=\frac{1}{2\pi}\int_0^{2\pi}\psi\cot\frac{(\varphi-\varphi_c)}{2}d\varphi$$
$$-\frac{1}{2\pi}\int_0^{2\pi}\psi\frac{\cos\,(2n+1)\,\frac{(\varphi-\varphi_c)}{2}}{2\sin\frac{(\varphi-\varphi_c)}{2}}\,d\varphi$$

The latter integral is identically zero. (See Wilson, E. B. Advanced Calculus, p. 368. Follow method of exercise 10.)
Then

$$(\theta-\varphi)_c=\frac{1}{2\pi}\int_0^{2\pi}\psi\cot\frac{(\varphi-\varphi_c)}{2}\,d\varphi \qquad \text{(VIII)}$$

For purposes of calculation this integral is expressed in convenient form in the appendix.

We shall now resume the task of determining the velocity at any point of the surface of the airfoil.

The velocity at the surface of the circle is $\frac{dw}{dz}$ (see equation (VI) and footnote). For corresponding points on the curve B in the z' plane and on the airfoil in the ζ plane the velocities are respectively $\frac{dw}{dz}\cdot\frac{dz}{dz'}$ and $\frac{dw}{dz}\cdot\frac{dz}{dz'}\cdot\frac{dz'}{d\zeta}$.

The quantities ζ and z' are related by the expression

$$\zeta=z'+\frac{a^2}{z'}$$

Hence

$$\frac{d\zeta}{dz'}=1-\frac{a^2}{z'^2}=\frac{1}{z'}\left(z'-\frac{a^2}{z'}\right)=\frac{1}{z'}\,(ae^{\psi+i\theta}-ae^{-\psi-i\theta})$$
$$=\frac{1}{z'}\,[a(e^{\psi}-e^{-\psi})\cos\theta+ia\,(e^{\psi}+e^{-\psi})\sin\theta]$$
$$=\frac{1}{z'}\,[2a\sinh\psi\cos\theta+2ia\cosh\psi\sin\theta]$$

Using the relations (II),

$$2a\sinh\psi=\frac{y}{\sin\theta}\text{ and }2a\cosh\psi=\frac{x}{\cos\theta}$$

we obtain

$$\frac{d\zeta}{dz'}=\frac{1}{z'}\,(y\cot\theta+i\,x\tan\theta). \qquad \text{(IX)}$$

It now remains to find the ratio $\frac{dz}{dz'}$. From the relation

$$z'=z\,e^{\Sigma(A_n+iB_n)\frac{1}{z^n}}$$

we obtain

$$\frac{dz'}{dz}=z'\left[\frac{1}{z}+\frac{d}{dz}\sum_n(A_n+i\,B_n)\frac{1}{z^n}\right]$$

or

$$\frac{dz'}{dz}=z'\left(\frac{1}{z}+\frac{d}{dz}[(\psi-\psi_o)+i\,(\theta-\varphi)]\right)$$
$$=z'\frac{d}{dz}\,(\psi+i\,(\theta-\varphi)+\log z)$$

But

$$z=ae^{\psi_0+i\varphi}$$

from which,

$$\frac{1}{z}=\frac{d}{dz}\,(\log z)=\frac{d}{dz}\,(\log a+\psi_o+i\varphi)=\frac{d}{dz}\,(i\varphi)$$

Therefore

$$\frac{dz'}{dz}=z'\frac{d}{dz}\,(\psi+i\,(\theta-\varphi)+i\varphi)$$
$$=z'\frac{d}{dz}\,(\psi+i\theta)$$

This expression may be written

$$\frac{dz'}{dz}=z'\frac{d}{d\theta}\,(\psi+i\theta)\cdot\frac{d\theta}{dz}$$

But we have

$$\frac{1}{z}=i\frac{d\varphi}{dz}$$

or

$$\frac{dz}{z}=id\,\varphi=i\,d(\varphi-\theta)+i\,d\theta$$

and

$$\frac{dz}{d\theta}=i\,z\left(1+\frac{d(\varphi-\theta)}{d\theta}\right)$$

Hence

$$\frac{dz'}{dz}=\frac{z'}{z}\frac{d}{d\theta}(-i\psi+\theta).\frac{1}{1+\frac{d\epsilon}{d\theta}}$$

where

$$\epsilon=\varphi-\theta$$

or

$$\frac{dz'}{dz}=\frac{z'}{z}\frac{1-i\psi'}{1+\epsilon'} \tag{X}$$

where ϵ' and ψ' indicate $\frac{d\epsilon}{d\theta}$ and $\frac{d\psi}{d\theta}$, respectively.

Equations (IX) and (X) give now

$$\frac{d\zeta}{dz'}\cdot\frac{dz'}{dz}=\frac{d\zeta}{dz}=\frac{1}{z'}(y\cot\theta+ix\tan\theta)\frac{z'}{z}\frac{1-i\psi'}{1+\epsilon'}$$

$$=(y\cot\theta+ix\tan\theta)\frac{1}{z}\frac{1-i\psi'}{1+\epsilon'} \tag{XI}$$

Because we are interested more in the magnitude than in the direction of the velocity we will write for the numerical value of this expression

$$\left|\frac{d\zeta}{dz}\right|=\frac{\sqrt{(y^2\cot^2\theta+x^2\tan^2\theta)(1+\psi'^2)}}{ae^{\psi_0}(1+\epsilon')} \tag{XIa}$$

The quantity $\left(\frac{y}{2a}\right)^2\cot^2\theta+\left(\frac{x}{2a}\right)^2\tan^2\theta$ is readily seen to be equal to (by relation (II))

$$\left(\frac{y}{2a\sin\theta}\right)^2+\sin^2\theta$$

or also

$$\sinh^2\psi+\sin^2\theta$$

Hence

$$\left|\frac{d\zeta}{dz}\right|=2\frac{\sqrt{\left[\left(\frac{y}{2a\sin\theta}\right)^2+\sin^2\theta\right](1+\psi'^2)}}{e^{\psi_0}(1+\epsilon')} \tag{XIb}$$

The numerical value of the velocity at the surface of the circle is obtained by equations (VI) and (VII) as follows:

Substituting the general point $z=ae^{\psi_0+i(\alpha+\varphi)}$, where α is the angle of attack as measured from the axis of coordinates, in equation (VI)

$$\frac{dw}{dz}=-V(1-e^{-2i(\alpha+\varphi)})-2iV\sin(\alpha+\epsilon_T)e^{-i(\alpha+\varphi)}$$
$$=-V[1-\cos2(\alpha+\varphi)+2\sin(\alpha+\epsilon_T)\sin(\alpha+\varphi)$$
$$+i(\sin2(\alpha+\varphi)+2\sin(\alpha+\epsilon_T)\cos(\alpha+\varphi))]$$

$$\left|\frac{dw}{dz}\right|^2=V^2[4\sin^2(\alpha+\epsilon_T)+8\sin(\alpha+\epsilon_T)\sin(\alpha+\varphi)$$
$$+4\sin^2(\alpha+\varphi)]$$

$$\left|\frac{dw}{dz}\right|=2V[\sin(\alpha+\varphi)+\sin(\alpha+\epsilon_T)]$$

Replacing φ by $\theta+\epsilon$ (ϵ_T, the angle of zero lift, is the value of $\varphi-\theta$ at the tail[5]), we have

$$\left|\frac{dw}{dz}\right|=2V[\sin(\alpha+\theta+\epsilon)+\sin(\alpha+\epsilon_T)]$$

For a point on the airfoil we have, then,

$v=\left|\frac{dw}{dz}\right|\cdot\left|\frac{dz}{d\zeta}\right|$ and from (XI), finally

$$v=V\frac{[\sin(\alpha+\theta+\epsilon)+\sin(\alpha+\epsilon_T)](1+\epsilon')e^{\psi_0}}{\sqrt{(\sinh^2\psi+\sin^2\theta)(1+\psi'^2)}} \tag{XII}$$

where the various symbols have the following significance:

- v is the velocity at any point (x, y) of the airfoil.
- V is the uniform velocity of flow at infinity.
- y is the ordinate of the airfoil as measured from the x-axis, where to fix the system of coordinates $(2a, 0)$ is the point midway between nose and center of curvature of the nose, and $(-2a, 0)$ is the point midway between the tail and center of curvature of the tail.
- α is the angle of attack as measured from the x-axis as indicated in Figure 6.
- $y, \theta, \psi, \psi', \epsilon$, and ϵ' are all functions of x.

Equation (XII), expressing the value of the velocity at any point of an airfoil of any shape, is surprisingly simple when the complex nature of the problem is considered. It has the distinct advantage of being exact; no approximations have been made in the preceding analysis.

We shall note some of the properties of this important relation. Because y is generally small, the term $\frac{y}{2a\sin\theta}$ is of influence chiefly near the leading edge, where $\sin\theta$ is small. It is noticed, however, that if $\frac{y}{\sin\theta}=0$ for $\theta=0$, equation (XII) yields in all cases $v=\infty$. This means that the velocity at the nose becomes infinite for $\sinh\psi=0$ (thin airfoils). This fact has been pointed out in an earlier report. (Reference 2.) The quantity $\frac{y}{2a\sin\theta}$ or $\sinh\psi$ is thus of considerable significance in the theory of thick airfoils.

The velocity near the tail is obtained by putting $\theta=\pi+\Delta\theta$ and $\epsilon=\epsilon_T+\epsilon'\Delta\theta$. Where $\Delta\theta$ is a small angle, in equation (XII)

$$\left|\frac{v}{V}\right|=\frac{e^{\psi_0}(1+\epsilon')[\sin(\theta+\alpha+\epsilon)+\sin(\alpha+\epsilon_T)]}{\sqrt{(\sinh^2\psi+\sin^2\theta)(1+\psi'^2)}}$$

we get

$$\left|\frac{v}{V}\right|\cong\frac{e^{\psi_0}(1+\epsilon')[-\Delta\theta+\alpha+\epsilon_T+\epsilon'\Delta\theta+\alpha+\epsilon_T]}{\sqrt{(\psi^2+\Delta\theta^2)(1+\psi'^2)}}$$

$$=\frac{e^{\psi_0}(1+\epsilon')^2\Delta\theta}{\sqrt{(\psi^2+\Delta\theta^2)(1+\psi'^2)}}$$

$$=\frac{e^{\psi_0}(1+\epsilon')^2}{\sqrt{\left[1+\left(\frac{\psi}{\Delta\theta}\right)^2\right](1+\psi'^2)}} \tag{f}$$

[5] It should be pointed out that the rear stagnation point is *chosen* to be on the x-axis at $\theta=\pi$. The curvature at the tail is, as far as the specification of the ideal circulation is concerned, to be considered as a mechanical imperfection.

ψ near the tail may be expressed as

$$\psi_T+\psi'\,\Delta\theta+\frac{1}{2}\psi''(\Delta\theta)^2+\quad\cdots$$

or

$$\frac{\psi}{\Delta\theta}=\frac{\psi_T}{\Delta\theta}+\psi'+\frac{1}{2}\psi''\Delta\theta+\quad\cdots$$

The quantity $\frac{\psi_T}{\Delta\theta}$ is infinite if ψ_T is different from zero at $\Delta\theta=0$. The velocity is in this case zero, indicating the presence of the rear stagnation point. If, on the other hand, ψ_T is zero, that is, if the tail is perfectly sharp,

$$\frac{\psi}{\Delta\theta}=\psi' \text{ for } \Delta\theta=0$$

and the velocity at the tail is

$$v_T=V\frac{e^{\psi_0}(1+\epsilon')^2}{(1+\psi'^2)}$$

or

$$v_T^2=V^2\frac{e^{2\psi_0}(1+\epsilon')^4}{(1+\psi'^2)^2} \tag{g}$$

(For the Clark Y, v_T^2 is about 0.88 V^2 near the tail.)

We obtain the front stagnation point by letting $v=0$ in equation (XII). Hence

$$\alpha+\theta+\epsilon_N=-(\alpha+\epsilon_T)$$

$$\theta=-(2\alpha+\epsilon_N+\epsilon_T)$$

In a previous report (reference 2)

$$\alpha_I=-\frac{\epsilon_N+\epsilon_T}{2}$$

has been defined as the ideal angle of attack. It is seen that, for this angle of attack, θ is zero or the stagnation point occurs directly at the nose.

Equation (XII) may also be applied to strut forms, and for such symmetrical shapes takes even a simpler form.

PRACTICAL APPLICATION OF RESULTS

We will now apply Formula (XII) to the typical case of the Clark Y airfoil and calculate the velocities at points of the airfoil surface. The detailed method of procedure is as follows.

1. The axis of coordinates is drawn through the points $(2a, 0)$ and $(-2a, 0)$ located respectively at the point midway between the nose and the center of curvature of the nose and the point midway between the tail and the center of curvature of the tail. (See fig. 6.) The radius of curvature at the leading edge is 1.75 per cent chord.

2. The points (x, y) of the upper and lower surfaces of the airfoil are determined with respect to this axis.

3. $\sin^2\theta$, $\sin\theta$, and θ are determined by the relation

$$2\sin^2\theta=p+\sqrt{p^2+\left(\frac{y}{a}\right)^2}\qquad\text{where } p=1-\left(\frac{x}{2a}\right)^2-\left(\frac{y}{2a}\right)^2$$

4. ψ is given by the relation

$$\psi=\left(\frac{y}{2a\sin\theta}\right)-\frac{1}{6}\left(\frac{y}{2a\sin\theta}\right)^3+\quad\cdots$$

5. ψ is plotted as a function of θ

$$\psi_0=\frac{1}{2\pi}\int_0^{2\pi}\psi d\varphi\;\cong\;\frac{1}{2\pi}\int_0^{2\pi}\psi d\theta$$

6. Determine $\epsilon_c=-\frac{1}{2\pi}\int_0^{2\pi}\psi\cot\frac{(\varphi-\varphi_c)}{2}d\varphi$ by formula shown in the appendix:

$$\epsilon_c=-\frac{1}{\pi}[0.628\psi'_c+1.065(\psi_1-\psi_{-1})+0.445(\psi_2-\psi_{-2})$$
$$+0.231(\psi_3-\psi_{-3})+0.104(\psi_4-\psi_{-4})]$$

where ψ'_c is the slope of the ψ curve at $\varphi=\varphi_c$ ψ_1 the value of ψ at $\varphi=\varphi_c+\frac{\pi}{5}$, ψ_2 at $\varphi=\varphi_c+\frac{2\pi}{5}$, etc.

ψ_{-1} the value of ψ at $\varphi=\varphi_c-\frac{\pi}{5}$, etc.

7. From the ϵ versus θ curve and from the ψ versus θ curves ϵ' and ψ' are determined.

8. Determine F by the relation

$$F=\frac{(1+\epsilon')e^{\psi_0}}{\sqrt{\left[\left(\frac{y}{2a\sin\theta}\right)^2+\sin^2\theta\right](1+\psi'^2)}}$$

9. $(\theta+\epsilon)$ is determined in radians and degrees.

10. $\sin(\theta+\alpha+\epsilon)+\sin(\alpha+\epsilon_T)$ is now calculated where α is the angle of attack as measured from the axis of coordinates.

11. $\frac{v}{V}=F\cdot[\sin(\theta+\alpha+\epsilon)+\sin(\alpha+\epsilon_T)]$

12. $\frac{P}{q}=1-\left(\frac{v}{V}\right)^2$ (pressure)

The entire calculation, properly arranged, can be quite accurately obtained in a very short time.

COMPARISON WITH EXPERIMENTAL RESULTS

In order to compare the theory with experimental results, the geometric angle of attack α_G as measured in the wind tunnel must be corrected for a number of items, such as finite span and effect of wall interference. We may, however, obtain approximately the apparent or effective angle of attack α_A (in radians as measured from the angle of zero lift) by taking the quotient of the area of the pressure-distribution curve and 5.5, since it is known that this value of the lift coeffi-

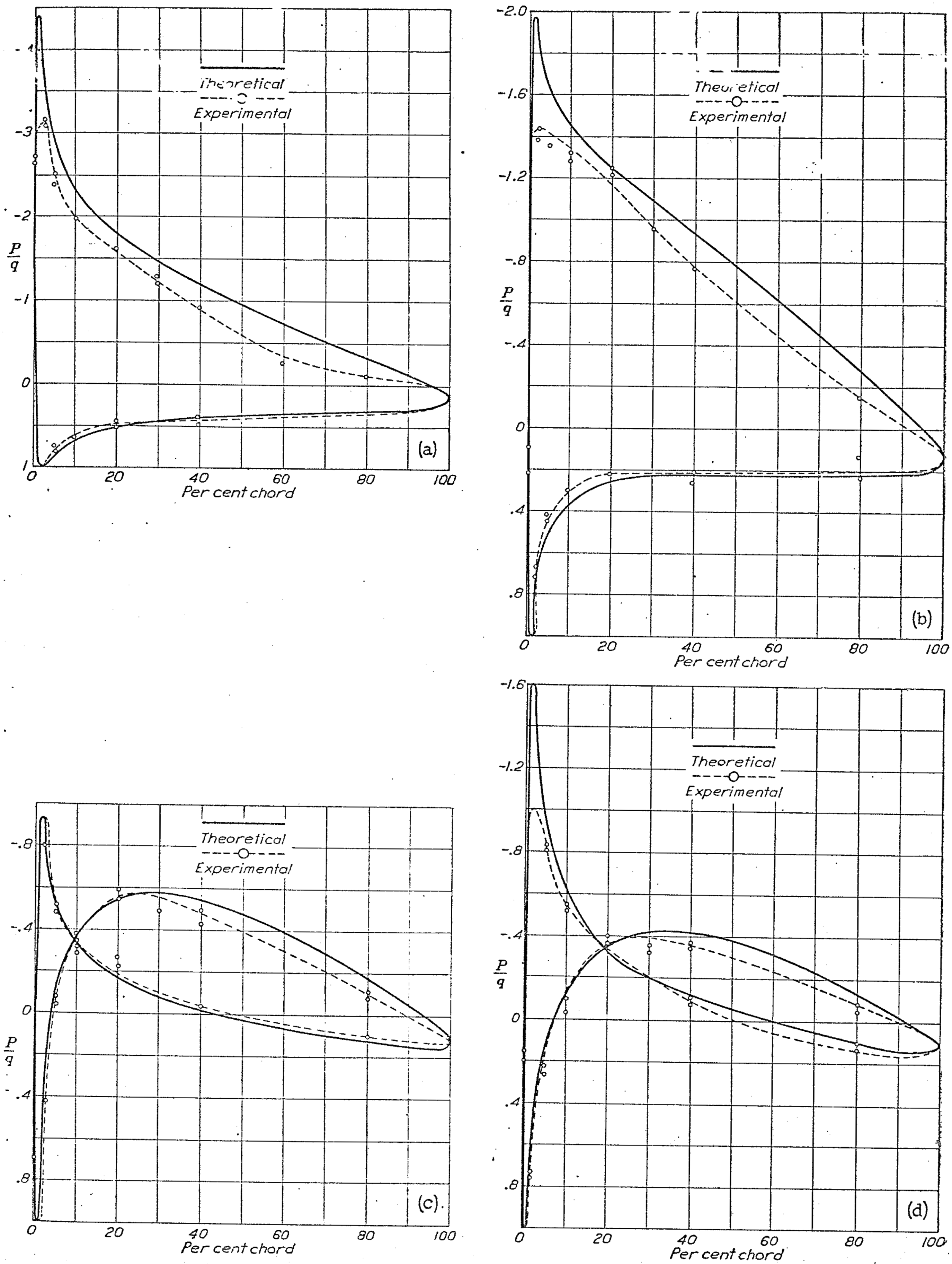


FIGURE 2.—Pressure-distribution curves along x-axis of Clark Y; $\frac{P}{q}$ against per cent chord

(a) $\alpha=9°33'$. (b) $\alpha=5°\ 19'$. (c) $\alpha=-1°\ 16'$. (d) $\alpha=-3°\ 15'$

cient is very nearly realized in most cases. This has been done in Table III, and the angle of attack α, which should be substituted in the Equation (XII), is given in the last column. The pressure distribution curves, Figures 2a, b, c, d, and 3a, b, c, d, were obtained by application of Equation (XII) to the Clark Y airfoil. Numerical results are shown in Tables I, II, and III. The experimental values are from original data sheets for N. A. C. A. Technical Report No. 353, and are not entirely consistent due to difficulties experienced in these experiments. After the theoretical pressure distribution curves have been obtained, the moments about any required axis may be found. Table IV gives some of these results and Figure 4 shows the comparison with experimental data taken from N. A. C. A. Technical Report No. 312.

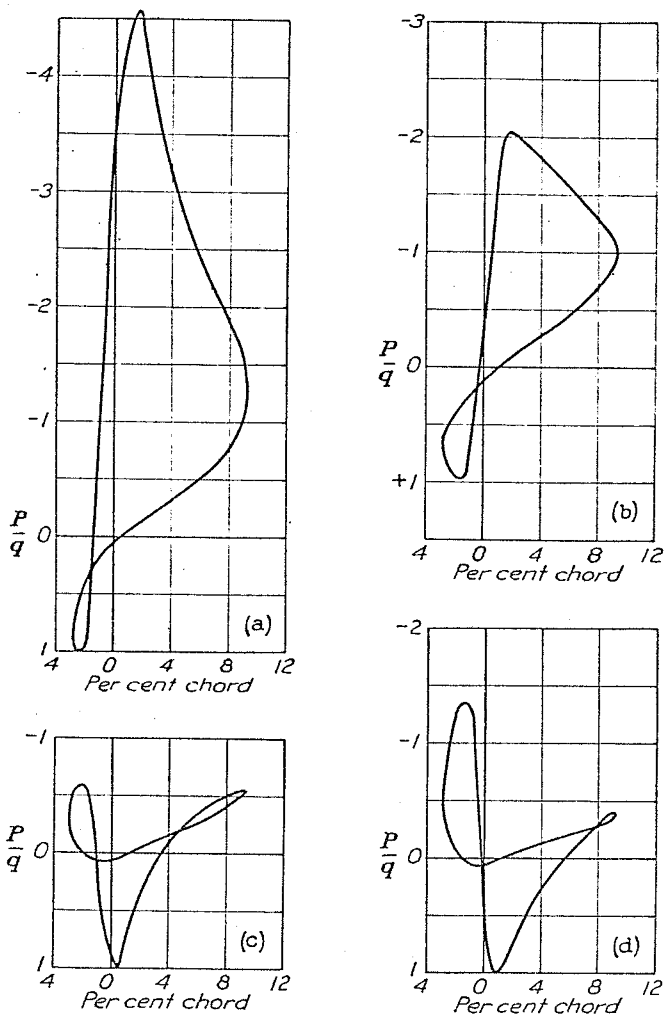


FIGURE 3.—Theoretical pressure distribution along y-axis of Clark Y
(a) $\alpha=9° 33'$. (b) $\alpha=5° 19'$. (c) $\alpha=-1° 16'$. (d) $\alpha=-3° 15'$

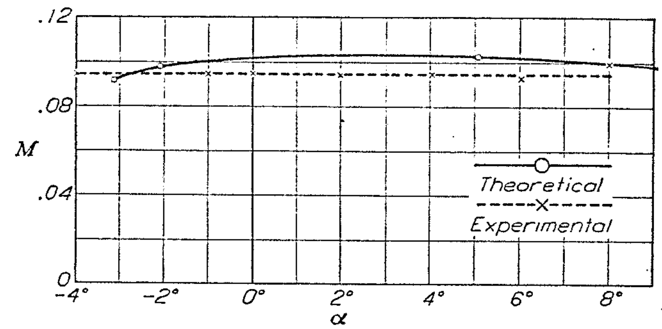


FIGURE 4.—Moment against angle of attack

LANGLEY MEMORIAL AERONAUTICAL LABORATORY,
NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS,
LANGLEY FIELD, VA., *October 15, 1931.*

APPENDIX

EVALUATION OF THE FORMULA

$$\epsilon_c=(\varphi-\theta)_c=-\frac{1}{2\pi}\int_0^{2\pi}\psi\cot\frac{(\varphi-\varphi_c)}{2}\,d\varphi$$

Although the above integrand becomes positively and negatively infinite around $\varphi=\varphi_c$, it is readily verified that for ψ finite, throughout the range $0-2\pi$, the integral remains finite, the positive and negative infinite strips exactly canceling each other.

The value of the integral for any point φ_c may be accurately obtained by the following device. We know that if ψ is a continuous function and the range φ_2 to φ_2 not too large ·

$$\frac{1}{2}\int_{\varphi_1}^{\varphi_2}\psi\cot\frac{(\varphi-\varphi_c)}{2}\,d\varphi \text{ is very nearly } \psi_A\log\frac{\sin\frac{\varphi_2-\varphi_c}{2}}{\sin\frac{\varphi_1-\varphi_c}{2}}$$

where ψ_A is the average value of ψ in the range φ_1 to φ_2. Also near $\varphi=\varphi_c$ we may write

$$\psi=\psi_c+(\varphi-\varphi_c)\,\psi'_c+(\varphi-\varphi_c)^2\frac{\psi''_c}{2}+\quad.\quad.\quad.\quad.$$

Then for s a small quantity

$$\int_{\varphi_c-s}^{\varphi_c+s}\psi\cot\frac{(\varphi-\varphi_c)}{2}\,d\varphi=2\int_{\varphi_c-s}^{\varphi_c+s}\psi'_c\,\frac{(\varphi-\varphi_c)}{2}\cdot\cot\frac{(\varphi-\varphi_c)}{2}\,d\varphi$$
$$=4\,s\psi'_c$$

(Since the even powers drop out and the $\lim\limits_{\varphi\to 0}\varphi\cot\varphi=1$).

Let us now divide the interval $0-2\pi$ into 10 parts, starting with φ_c as a reference point. (See fig. 5.)

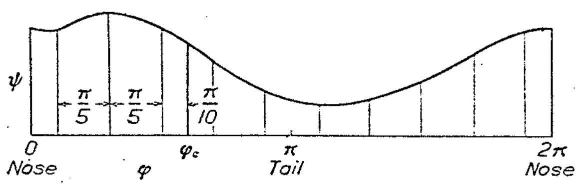


FIGURE 5.—The ψ against φ curve, illustrating method of evaluation of ϵ_c

$\varphi_c-\frac{\pi}{10}$ to $\varphi_c+\frac{\pi}{10}$, $\varphi_c+\frac{\pi}{10}$ to $\varphi_c+\frac{3\pi}{10}$, $\varphi_c+\frac{3\pi}{10}$ to $\varphi_c+\frac{5\pi}{10}$, $\varphi_c+\frac{5\pi}{10}$ to $\varphi_c+\frac{7\pi}{10}$, $\varphi_c+\frac{7\pi}{10}$ to $\varphi_c+\frac{9\pi}{10}$, $\varphi_c+\frac{9\pi}{10}$ to $\varphi_c-\frac{9\pi}{10}$, $\varphi_c-\frac{9\pi}{10}$ to $\varphi_c-\frac{7\pi}{10}$, $\varphi_c-\frac{7\pi}{10}$ to $\varphi_c-\frac{5\pi}{10}$, $\varphi_c-\frac{5\pi}{10}$ to $\varphi_c-\frac{3\pi}{10}$ and $\varphi_c-\frac{3\pi}{10}$ to $\varphi_c-\frac{\pi}{10}$.

Then,

$$\epsilon_c=-\frac{1}{2\pi}\int_0^{2\pi}\psi\cot\frac{(\varphi-\varphi_c)}{2}\,d\varphi$$

$$\cong-\frac{1}{\pi}\Bigg[\frac{\pi}{5}\psi'_c+(\psi_1-\psi_{-1})\log\frac{\sin\frac{3\pi}{20}}{\sin\frac{\pi}{20}}$$

$$+(\psi_2-\psi_{-2})\log\frac{\sin\frac{5\pi}{20}}{\sin\frac{3\pi}{20}}+(\psi_3-\psi_{-3})\log\frac{\sin\frac{7\pi}{20}}{\sin\frac{5\pi}{20}}$$

$$+(\psi_4-\psi_{-4})\log\frac{\sin\frac{9\pi}{20}}{\sin\frac{7\pi}{20}}\Bigg]$$

$$=-\frac{1}{\pi}[0.628\,\psi'_c+1.065\,(\psi_1-\psi_{-1})+0.445\,(\psi_2-\psi_{-2})$$
$$+0.231\,(\psi_3-\psi_{-3})+0.104\,(\psi_4-\psi_{-4})]$$

where ψ'_c is the slope of the ψ curve at $\varphi=\varphi_c$

ψ_1 value of ψ at $\varphi=\varphi_c+\frac{\pi}{5}$, ψ_{-1} at $\varphi=\varphi_c-\frac{\pi}{5}$,

ψ_2 at $\varphi=\varphi_c+\frac{2\pi}{5}$, ψ_3 at $\varphi=\varphi_c+\frac{3\pi}{5}$, etc.

To evaluate the above integral it is, strictly speaking, necessary to know ψ as a function of φ rather than of θ.[1] We have $\varphi=\theta+\epsilon$. For all flattened or streamline bodies, however, ϵ is small; for ordinary airfoils it is, in fact, so small that $\psi(\theta)$ may unconditionally be considered equal to $\psi(\phi)$. For the sake of mathematical accuracy we will, however, indicate how the problem may be solved also for bodies of more irregular contour by successive approximations. We have

$$\psi(\varphi)=\psi(\theta)+\epsilon\,\psi'(\theta)+\quad.\quad.\quad.$$

As a first approximation we neglect the second and all following terms of this expression. The value of ϵ thus obtained by graphical integration or otherwise is then used in the expression for $\psi(\varphi)$ and a second integration is performed, etc.

[1] The equation for ϵ is a nonlinear integral equation and to obtain its exact solution is a difficult matter; fortunately because of the small magnitude of ϵ the solution is obtainable to any desired accuracy by ordinary definite integrals.

APPLICATION OF FLOW FORMULA TO THE SPECIAL CASE OF AN ELLIPTIC CYLINDER

As a matter of interest we will assume the form of the body to be the ellipse $\left(\frac{x}{2a\cosh\psi}\right)^2+\left(\frac{y}{2a\sinh\psi}\right)^2=1$ and find $\frac{v}{V}$ for zero angle of attack, i. e., we have $\psi=\psi_o=\text{constant}$, $\psi'=0$, $\epsilon=0$, $\epsilon'=0$, $\alpha=0$.

Equation (XII) becomes

$$\left(\frac{v}{V}\right)=\frac{\sin\theta\cdot e^{\psi}}{\sqrt{\sinh^2\psi+\sin^2\theta}}=\frac{\frac{y\cdot e^{\psi}}{2a\sinh\psi}}{\sqrt{\sinh^2\psi+\left(\frac{y}{2a\sinh\psi}\right)^2}}$$

and

$$\frac{p}{q}=1-\left(\frac{v}{V}\right)^2=1-\frac{e^{2\psi}\,(2ay)^2}{(2a\sinh\psi)^4+(2ay)^2}$$

This result checks exactly with the form given by Dr. A. F. Zahm in N. A. C. A. Technical Report No. 253, Flow and Drag Formulas for Simple Quadrics, equation 14.

REFERENCES

1. Glauert, H.: Elements of Airfoil and Air-Screw Theory. Cambridge University Press, 1926.
2. Theodorsen, Theodore: On the Theory of Wing Sections with Particular Reference to the Lift Distribution. T. R. No. 383, N. A. C. A., 1931.

EXPLANATION OF THE TABLES

The first part of Table I refers to the upper surface or to positive ordinates of the Clark Y, the second part to the lower surface or to negative ordinates. Column 1 gives the location in per cent of the chord; 2 gives the ordinates with respect to the x-axis in this same unit; 3 and 4 give x and y in the present system of coordinates; 5, 6, and 7 give $\sin^2\theta$, $\sin\theta$, and θ, respectively (Equation (III)); 8 gives ψ (by equation (IVa)); 9 gives ϵ (appendix); 10 and 11 give $\frac{d\psi}{d\theta}$ and $\frac{d\epsilon}{d\theta}$ as obtained from ψ against θ and ϵ against θ curves; (See figs. 7 and 8). Column 12 gives the quantity

$$F=\frac{(1+\epsilon')\,e^{\psi_0}}{\sqrt{\left[\left(\frac{y}{2\sin\theta}\right)^2+\sin^2\theta\right](1+\psi'^2)}}\quad(e^{\psi_0}=1.11);$$

Column 14 gives $\theta+\epsilon$ in degrees. The velocity at any point x and angle of attack α is given by $v=V\,[\sin\,(\alpha+\theta+\epsilon)+\sin\,(\alpha+\epsilon_T)]\cdot F$ and the pressure, by $\frac{P}{q}=1-\left(\frac{v}{V}\right)^2$

It must be noted that α is measured from the line of flow to the x-axis as shown in Figure 6, and if otherwise measured, must be reduced to this basis.

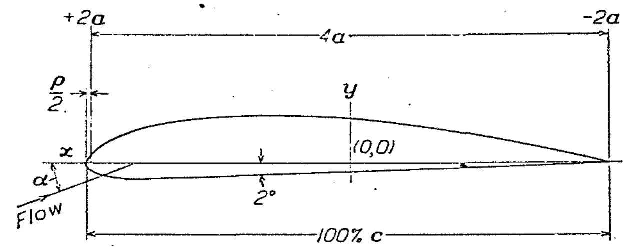


FIGURE 6.—Clark Y airfoil—showing system of coordinates

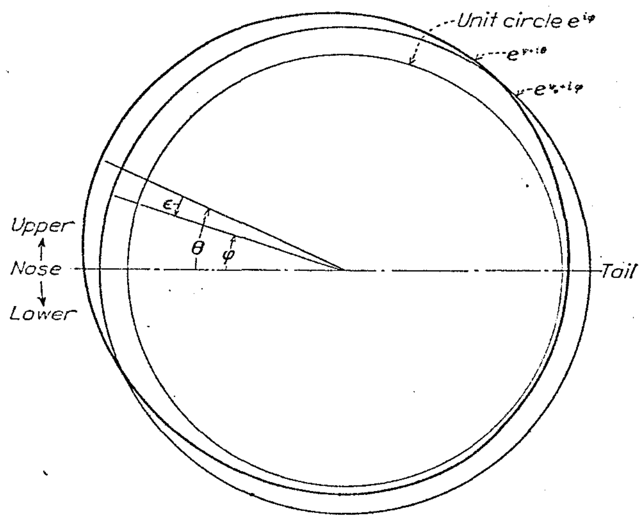


FIGURE 7.—The unit circle $z=e^{i\phi}$, the circle $z=e^{\psi_0+i\phi}$, and the corresponding curve $z'=e^{\psi+i\theta}$

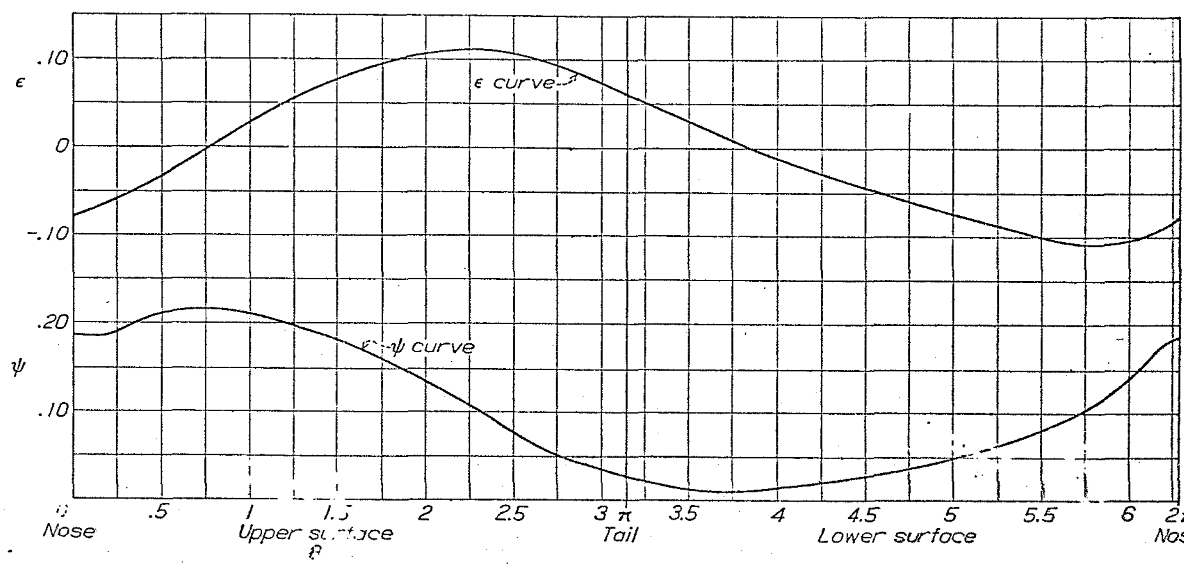


FIGURE 8.—(a) The ψ against θ curve for the Clark Y. (b) The ϵ against θ curve for the Clark Y

TABLE I
CLARK Y

UPPER SURFACE

% c	y in %	x	y	$\sin^2\theta$	$\sin\theta$	θ radians	ψ	ϵ	ψ'	ϵ'	F	$\theta+\epsilon$ radians	$\theta+\epsilon$
													° ′
0	0	2.035	0.000	0.000	0.000	0.000	0.188	−0.079(ϵ_N)	0.000	0.075	6.33	−0.079	−4 32
1.25	1.99	1.985	.0804	.0474	.218	.220	.184	−.067	.035	.080	4.20	.153	8 47
2.50	3.08	1.934	.124	.0994	.315	.321	.198	−.055	.075	.105	3.28	.266	15 17
5.0	4.59	1.833	.185	.195	.442	.458	.208	−.038	.060	.115	2.52	.420	24 4
7.5	5.61	1.732	.226	.282	.531	.559	.213	−.026	.030	.115	2.16	.533	30 33
10	6.45	1.631	.260	.365	.605	.649	.214	−.016	.020	.120	1.92	.633	36 16
15	7.70	1.430	.311	.512	.716	.798	.216	.003	.000	.120	1.66	.801	45 53
20	8.55	1.228	.345	.640	.801	.929	.214	.020	−.030	.120	1.48	.949	54 22
30	9.23	.824	.372	.837	.915	1.156	.202	.044	−.060	.105	1.30	1.200	68 45
40	9.28	.421	.374	.957	.978	1.361	.190	.063	−.065	.091	1.21	1.424	81 35
50	8.74	.0175	.353	1.000	1.000	1.571	.176	.080	−.085	.078	1.17	1.651	94 35
60	7.77	−.386	.313	.963	.982	1.761	.159	.094	−.100	.070	1.18	1.855	106 16
70	6.32	−.790	.255	.847	.920	1.972	.137	.106	−.104	.025	1.22	2.078	119 3
80	4.48	−1.193	.181	.649	.806	2.204	.112	.110	−.107	.004	1.36	2.314	132 34
90	2.40	−1.596	.097	.368	.606	2.490	.080	.106	−.110	−.015	1.78	2.596	148 43
95	1.25	−1.798	.050	.195	.443	2.683	.056	.095	−.080	−.075	2.30	2.778	159 10
100	.06	−2.001	.002	.000	.000	3.142	.030	.062(ϵ_T)	−.053	−.080	Large	3.204	183 33

LOWER SURFACE

% c	y in %	x	y	$\sin^2\theta$	$\sin\theta$	θ radians	ψ	ϵ	ψ'	ϵ'	F	$\theta+\epsilon$ radians	$\theta+\epsilon$
0	0	2.035	0.000	0.000	0.000	6.283	0.188	−0.079	0.000	0.075	6.33	6.204	−4 32
1.25	1.53	1.985	−.0617	.0387	−.197	6.085	.156	−.100	.170	.085	4.71	5.985	−17 5
2.50	1.95	1.934	−.0787	.0822	−.287	5.992	.137	−.105	.162	.055	3.64	5.887	−22 41
5.0	2.38	1.833	−.0960	.171	−.414	5.857	.116	−.110	.130	.000	2.55	5.747	−30 43
7.5	2.61	1.732	−.105	.258	−.508	5.750	.103	−.107	.110	−.015	2.09	5.643	−36 40
10	2.73	1.631	−.110	.342	−.585	5.658	.094	−.105	.093	−.032	1.81	5.553	−41 50
15	2.84	1.430	−.115	.492	−.702	5.505	.082	−.100	.080	−.046	1.49	5.399	−50 40
20	2.78	1.228	−.112	.625	−.791	5.371	.071	−.093	.075	−.048	1.33	5.278	−57 35
30	2.47	.824	−.0996	.831	−.912	5.135	.055	−.081	.058	−.050	1.15	5.054	−71 0
40	2.12	.421	−.0885	.956	−.978	4.922	.045	−.072	.043	−.050	1.075	4.850	−82 6
50	1.78	.0175	−.0720	1.000	−1.000	4.712	.036	−.058	.040	−.060	1.04	4.654	−93 21
60	1.38	−.386	−.0556	.962	−.981	4.518	.028	−.045	.030	−.060	1.06	4.473	−103 40
70	1.03	−.790	−.0416	.844	−.919	4.306	.023	−.034	.025	−.060	1.13	4.272	−115 14
80	.74	−1.193	−.0295	.645	−.803	4.075	.018	−.0[illegible]	.022	−.070	1.275	4.055	−127 39
90	.40	−1.596	−.0161	.364	−.604	3.790	.013	.0[illegible]0	.018	−.082	1.685	3.790	−142 51
95	.24	−1.798	−.0097	.191	−.438	3.595	.011	.023	−.017	−.090	2.30	3.618	−152 43
100	.06	−2.001	−.002	.000	−.000	3.142	.030	.062	−.053	−.080	Large	3.204	−176 27

TABLE II
CLARK Y

$$\left|\frac{v}{V}\right| = [\sin(\theta+\alpha+\epsilon)+\sin(\alpha+\epsilon_T)]\cdot F \quad \frac{P}{q}=1-\left|\frac{v}{V}\right|^2$$

$$\alpha = 9^\circ\ 33'$$
$$\epsilon_T = 3^\circ\ 33' \quad \sin(\alpha+\epsilon_T)=\sin 13^\circ\ 6' = 0.2267$$

	Upper surface						Lower surface					
% c	$\theta+\alpha+\epsilon$	$\sin(\theta+\alpha+\epsilon)$	$\sin(\theta+\alpha+\epsilon)+\sin(\alpha+\epsilon_T)$	$\left\|\frac{v}{V}\right\|$	$\left\|\frac{v}{V}\right\|^2$	$\frac{P}{q}$	$\theta+\alpha+\epsilon$	$\sin(\theta+\alpha+\epsilon)$	$\sin(\theta+\alpha+\epsilon)+\sin(\alpha+\epsilon_T)$	$\left\|\frac{v}{V}\right\|$	$\left\|\frac{v}{V}\right\|^2$	$\frac{P}{q}$
	° ′						° ′					
0	5 1	0.0875	0.3142	1.99	3.96	−2.96	5 1	0.0875	0.3142	1.99	3.96	−2.96
1.25	18 20	.3145	.5412	2.26	5.12	−4.12	−7 32	−.1311	.0956	.450	.203	.797
2.50	24 50	.4200	.6467	2.12	4.49	−3.39	−13 8	−.2272	.0005	.002	.000	1.000
5.0	33 37	.5537	.7804	1.97	3.87	−2.87	−21 10	−.3611	−.1344	.343	.117	.883
7.5	40 6	.6441	.8708	1.88	3.55	−2.55	−27 7	−.4558	−.2291	.479	.229	.771
10	45 49	.7171	.9438	1.82	3.30	−2.30	−32 17	−.5341	−.3074	.556	.308	.692
15	55 26	.8235	1.0502	1.74	3.04	−2.04	−41 7	−.6576	−.4307	.643	.414	.586
20	63 55	.8981	1.1248	1.68	2.83	−1.83	−48 2	−.7435	−.5168	.685	.469	.531
30	78 18	.9792	1.2059	1.57	2.47	−1.47	−61 27	−.8784	−.6517	.748	.558	.442
40	91 8	.9998	1.2265	1.49	2.22	−1.22	−72 33	−.9540	−.7273	.788	.621	.379
50	104 8	.9697	1.1964	1.40	1.98	−0.98	−83 48	−.9941	−.7674	.800	.640	.360
60	115 49	.9000	1.1267	1.33	1.76	−0.76	−94 7	−.9974	−.7707	.821	.674	.326
70	128 36	.7815	1.0082	1.23	1.51	−0.51	−105 41	−.9627	−.7357	.834	.696	.304
80	142 7	.6141	.8408	1.14	1.31	−0.31	−118 6	−.8821	−.6554	.836	.698	.302
90	158 16	.3703	.5970	1.06	1.13	−0.13	−133 18	−.7278	−.5011	.844	.711	.289
95	168 43	.1957	.4224	.97	.95	0.05	−143 10	−.5995	−.3728	.857	.735	.265
100								Formula (g)		.935	.875	.125

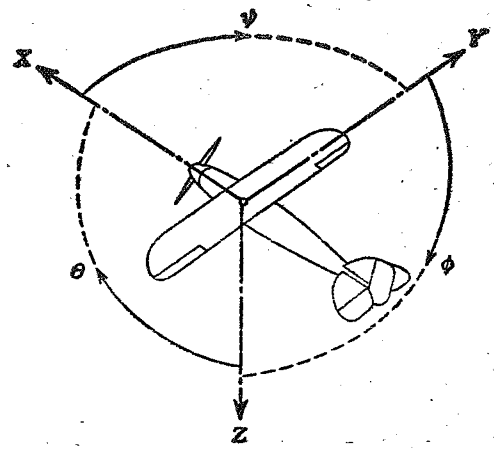
Table II gives the numerical values for Figure 2a in detail as an example. See also Table I.

TABLE III

Figure	Geometric angle as measured from chord line experimentally α_G	Area of experimental pressure distribution curve A	$\frac{A}{5.5}=\alpha_A$ radians	Apparent angle measured from zero lift $\alpha_A=\alpha+\epsilon_T$ degrees	Angle measured from chord line (to be used as a basis for comparison with α_G) $\alpha=\alpha_A-3^\circ\ 33'$
	° ′			° ′	° ′
2a	13 25	1.260	0.229	13 6	9 33
2b	7 25	.855	.155	8 52	5 19
2c	−1 40	.221	.040	2 17	−1 16
2d	−4 35	.030	.0053	0 18	−3 15

TABLE IV

Figure	M_x Moment about line $x=25$ per cent chord	M_y Moment about line $y=0$	M_T Moment about point $x=25$ per cent chord, $y=0$
2a	−0.0896	−0.0085	−0.098
2b	−.098	−.005	−.103
2c	−.098	.000	−.098
2d	−.093	.0015	−.091



Positive directions of axes and angles (forces and moments) are shown by arrows

Axis		Force (parallel to axis) symbol	Moment about axis			Angle		Velocities	
Designation	Symbol		Designation	Symbol	Positive direction	Designation	Symbol	Linear (component along axis)	Angular
Longitudinal	X	X	rolling	L	$Y \longrightarrow Z$	roll	ϕ	u	p
Lateral	Y	Y	pitching	M	$Z \longrightarrow X$	pitch	θ	v	q
Normal	Z	Z	yawing	N	$X \longrightarrow Y$	yaw	ψ	w	r

Absolute coefficients of moment

$$C_l=\frac{L}{qbS} \qquad C_m=\frac{M}{qcS} \qquad C_n=\frac{N}{qbS}$$

Angle of set of control surface (relative to neutral position), δ. (Indicate surface by proper subscript.)

4. PROPELLER SYMBOLS

D, Diameter.
p, Geometric pitch.
p/D, Pitch ratio.
V', Inflow velocity.
V_s, Slipstream velocity.
T, Thrust, absolute coefficient $C_T=\frac{T}{\rho n^2 D^4}$
Q, Torque, absolute coefficient $C_Q=\frac{Q}{\rho n^2 D^5}$

P, Power, absolute coefficient $C_P=\frac{P}{\rho n^3 D^5}$.
C_S, Speed power coefficient $=\sqrt[5]{\frac{\rho V^5}{P n^2}}$.
η, Efficiency.
n, Revolutions per second, r. p. s.
Φ, Effective helix angle $=\tan^{-1}\left(\frac{V}{2\pi r n}\right)$

5. NUMERICAL RELATIONS

1 hp = 76.04 kg/m/s = 550 lb./ft./sec.
1 kg/m/s = 0.01315 hp
1 mi./hr. = 0.44704 m/s
1 m/s = 2.23693 mi./hr.

1 lb. = 0.4535924277 kg.
1 kg = 2.2046224 lb.
1 mi. = 1609.35 m = 5280 ft.
1 m = 3.2808333 ft.