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FOR AERONAUTICS

REPORT No. 420

AIRCRAFT SPEED INSTRUMENTS

By K. HILDING BEIJ

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### AERONAUTIC SYMBOLS

#### 1. FUNDAMENTAL AND DERIVED UNITS

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<tr>
<td>Length</td>
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<tr>
<td>Time</td>
<td>( t )</td>
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<tr>
<td>Force</td>
<td>( F )</td>
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<tr>
<td>Power</td>
<td>( P )</td>
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<tr>
<td>Speed</td>
<td>( V )</td>
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#### 2. GENERAL SYMBOLS

- \( W \): Weight = \( mg \)
- \( g \): Standard acceleration of gravity = 9.80665 m/s\(^2\) or 32.1740 ft/sec\(^2\)
- \( m \): Mass = \( \frac{W}{g} \)
- \( I \): Moment of inertia = \( mk^2 \). (Indicate axis of radius of gyration \( k \) by proper subscript.)
- \( \mu \): Coefficient of viscosity

#### 3. AERODYNAMIC SYMBOLS

- \( s \): Area
- \( S \): Area of wing
- \( G \): Gap
- \( b \): Span
- \( c \): Chord
- \( A \): Aspect ratio, \( \frac{b^2}{S} \)
- \( V \): True air speed
- \( q \): Dynamic pressure, \( \frac{1}{2} \rho V^2 \)
- \( L \): Lift, absolute coefficient \( C_L = \frac{L}{qS} \)
- \( D \): Drag, absolute coefficient \( C_D = \frac{D}{qS} \)
- \( D_0 \): Profile drag, absolute coefficient \( C_{D_0} = \frac{D_0}{qS} \)
- \( D_i \): Induced drag, absolute coefficient \( C_{D_i} = \frac{D_i}{qS} \)
- \( D_p \): Parasite drag, absolute coefficient \( C_{D_p} = \frac{D_p}{qS} \)
- \( C \): Cross-wind force, absolute coefficient \( C = \frac{C}{qS} \)
- \( \rho \): Kinematic viscosity
- \( \rho \): Density (mass per unit volume)
- \( v \): Standard density of dry air, 0.12497 kg-m\(^{-1}\)s\(^{-2}\) at 15° C and 760 mm; or 0.002378 lb-ft\(^{-1}\)sec\(^{-2}\)
- \( I \): Specific weight of “standard” air, 1.2255 kg/m\(^3\) or 0.07651 lb/cu ft
- \( \nu \): Angle of setting of wings (relative to thrust line)
- \( \phi \): Angle of stabilizer setting (relative to thrust line)
- \( Q \): Resultant moment
- \( \Omega \): Resultant angular velocity
- \( R \): Reynolds number, \( \frac{\rho V l}{\mu} \) where \( l \) is a linear dimension (e.g., for an airfoil of 1.0 ft chord, 100 mph, standard pressure at 15° C, the corresponding Reynolds number is 935,400; or for an airfoil of 1.0 m chord, 100 mps, the corresponding Reynolds number is 6,865,000)
- \( \alpha \): Angle of attack
- \( \epsilon \): Angle of downwash
- \( \alpha_0 \): Angle of attack, infinite aspect ratio
- \( \alpha_i \): Angle of attack, induced
- \( \alpha_a \): Angle of attack, absolute (measured from zero-lift position)
- \( \gamma \): Flight-path angle
REPORT No. 420

AIR SPEED INSTRUMENTS

By K. HILDING BEIJ
Bureau of Standards
NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

HEADQUARTERS, NAVY BUILDING, WASHINGTON, D. C.

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SUMMARY

This report presents a concise survey of the measurement of air speed and ground speed on board aircraft. Special attention is paid to the Pitot-static air-speed meter which is the standard in the United States for airplanes. Air-speed meters of the rotating vane type are also discussed in considerable detail on account of their value as flight test instruments and as service instruments for airships. Methods of ground-speed measurement are treated briefly, with references to the more important instruments. A bibliography on air-speed measurement concludes the report.

INTRODUCTION

The major portion of the report is devoted to air-speed measurement, and in particular, to the Pitot-static type of instrument. Owing to the inherent difficulties involved, ground-speed measurement is still in a very incomplete stage of development. Consequently, only a brief resumé of this subject is given.

Previous publications have been freely consulted and acknowledgment is made in most instances to the source publications listed in the bibliography. Much material was made available through the courtesy of the Bureau of Aeronautics of the Navy Department and of various instrument manufacturers.

The appended bibliography does not pretend to be exhaustive. Nevertheless, it is believed that the list is complete enough to be of great service to anyone who may desire to study the subject in detail.

HISTORICAL OUTLINE

The development of air-speed indicators has closely followed progress in heavier-than-air flying. On the first flight, Orville Wright carried a Richard anemometer, primarily, it is true, to measure the air distance flown, in order to furnish data required for checking propeller computations. Speed was determined from the anemometer reading and the time of flight measured by a stop watch. Toward the end of the first decade of mechanical flight, various forms of instruments began to appear. Some of these were hardly more than stalling indicators, consisting of a pressure plate deflecting against a spring. The scale was merely a red mark indicating the proper speed of flight. In England the Pitot-static tube with a liquid manometer as indicator, in Germany the Robinson cup anemometer driving a centrifugal tachometer as an indicator, and in France and Italy the pressure-plate indicators all appeared several years before the outbreak of the World War. The British, who have consistently adhered to the Pitot-static type, soon discarded the liquid manometer indicator and introduced mechanical pressure gauges with nonmetallic diaphragms of rubber or oiled silk in order to obtain the necessary sensitivity. The French and Germans attacked the problem in another way. With single and double Venturi tubes they were able to obtain higher differential pressures and hence had no diffi-
difficulty in using metallic diaphragms. The Pitot-Venturi, of the type largely developed by Zahm, was favored in the United States.

Since 1922, when Report No. 127 was issued, there have been a number of outstanding changes in the methods of measuring air speed. The disappearance of the huge war stocks, the results of research conducted during and after the war, and commercial competition, stimulated by increasingly rigid specifications by the Army and Navy air services, have brought rapid advances.

A major change has been the virtual abandonment of the use, in the United States at least, of the Venturi head in all its forms and the acceptance in its place of the Pitot-static head. A number of factors have contributed to this change. Most important, it was shown toward the end of the World War that the Venturi was not satisfactory at low speeds. The need for close tolerances in the manufacture of Venturi heads in order to obtain tubes with the same calibration characteristics has been a great disadvantage in comparison with the Pitot-static tube. Another factor in the elimination of the Venturi has been the improvement of metallic diaphragm indicators so that the measurement of the lower differential pressures delivered by the Pitot-static head is no longer a serious problem.

The calibration and the computation of altitude corrections for differential pressure instruments have been affected by the adoption in 1925 of a new standard atmosphere. A "standard atmosphere" is an arbitrary altitude-pressure-temperature relation which is so chosen as to more or less correspond to the yearly average of actual atmospheric conditions. The constants of the standard atmosphere for zero altitude are used in the calibration of air-speed indicators (of the common types), and the relation itself in computing "true" air speeds at other altitudes. Previous to 1925 the standard atmosphere used for calibrating altimeters in the United States was based on the assumption of an isothermal atmosphere at a temperature of +10° C. This was satisfactory for use at altitudes of less than about 12,000 feet. In the new standard atmosphere adopted in 1925 the atmospheric temperature is assumed to decrease linearly with altitude up to the commencement of the isothermal layer at a temperature of −55° C. This relation has been found to represent very closely the average annual conditions for a latitude of 40° in the United States.

Another change, which affects the calibration of air-speed indicators of the commonly used types, has been necessitated by the increased speeds of airplanes. As long as the speeds did not exceed 150 miles per hour it was unnecessary to take into account the compressibility of the air. Much higher speeds are not uncommon at present and, accordingly, the adiabatic formula for pressures of air coming to rest has been adopted.

Other developments affecting indicators are of interest. The first is the vertical-scale indicator designed with a view to economizing space on the instrument board. It has not met with much favor as regards air-speed indicators. The second is the adoption by the United States Army and Navy of two new standard dial and case sizes for all aircraft instruments. This change, initiated by the Navy, has involved a reduction in the size of the case of the air-speed indicator from 3½ to 3¾ inches in diameter. The advantages of uniformity and interchangeability of mountings are obvious. Nonmetallic cases have also come into general use.

**TYPES OF AIR-SPEED METERS AND THEIR USES**

Aircraft air-speed meters are, in general, composed of two distinct parts—an operating element which is exposed to the stream of air flowing past the aircraft, and an indicating element which is mounted on the instrument board. In certain cases where distant indication is impracticable on account of the characteristics of the operating element, both element and indicator are built into one unit. Since the operating element must necessarily be mounted in an exposed position, usually at some distance from the pilot's seat, such combined instruments operate under serious disadvantages. Further the indicator must be very carefully housed and protected from rain, ice, and dust; and the scale must be large and open so as to be readily legible. Illumination for night flying is complicated. Finally, the instrument is bound to offer greater head resistance and introduce mounting difficulties. Such instruments, therefore, are not in common use at the present time although occasionally employed for special purposes.

Air-speed meters may be conveniently classified according to the type of operating element, as follows:

1. Differential pressure.
   (a) Pitot-static.
   (b) Venturi.
   (c) Pitot-Venturi.

2. Mechanical.
   (a) Rotating element.
   (b) Deflecting element.

3. Thermal (hot-wire).

For the purposes of this report, the above classification will be followed.

In the first type of instrument, which is most commonly used in aircraft, the differential pressures produced by the airstream in tubes of special forms, called pressure heads or nozzles, furnish a means for determining air speed. The pressures produced are measured by sensitive gauges calibrated in speed units. The second type comprises instruments with rotating fans or windmills of various forms, the speed of rotation giving a measure of air speed; and instruments in which the impact pressure of the moving air
causes an element to deflect against a restraining spring, the amount of deflection being a function of the air speed. In the third type, the heat loss from a hot wire placed in the air stream furnishes a measure of the air speed.

It is often convenient to classify air-speed meters according to the effect of variations of the air density on the operating element. The mechanical type of instrument with a rotating element is the only type which is unaffected by air density, and that only when the element rotates with a negligible resisting torque. The indication corrected for instrumental errors is known as the true air speed. Vane and cup anemometers with very little friction are examples. The indications of other types given above are affected by the air density, including instruments with a rotating element which rotates against considerable torque, briddled anemometers, pressure-plate instruments, and the thermal types.

The instruments unaffected by the air density are of especial value for flight testing, for navigation, and for use in airships. The instruments affected by the air density, especially the commonly used differential pressure type, are of particular value on airplanes. The indication of the instruments of the latter type in which the differential pressures are proportional to the product of the air density and the square of the air speed, at least within certain limits, is frequently called the "indicated speed." It so happens that the forces supporting an airplane in flight are also functions of the same quantity and hence the indicated speed is of unique value for airplanes. The stalling speed, for example, is always given by the same value of the indicated speed regardless of the altitude of flight.

The aircraft air-speed meter may be used as a service instrument to indicate the speed of flight, as one of the testing instruments for evaluating the performance characteristics of aircraft, or as a navigating instrument. The most suitable type depends not only on the purpose for which data on air speed are desired but also to some extent on the type of aircraft on which it is to be used.

In service use on airplanes the chief function of the air-speed meter is to assist the pilot in securing the desired performance of his airplane, whether it be maximum speed, most economical speed, or speed for greatest duration of flight. Also, in connection with other instruments, it helps the pilot to maintain level flight under unfavorable weather conditions or at night, and warns him of any dangerous loss of speed which might result in a stall or of excessive speeds in diving which might overstress the airplane. The differential pressure type is ideally suited for this function.

The importance of accurate measurement of the air speed in flight testing is obvious. Here the true air speed is desired. On account of its otherwise desirable characteristics the Pitot-static type is most frequently used, although density corrections are required for evaluating the true speed. Air logs (vane-type anemometers) giving true air distance, and with the aid of a stop watch true air speed, offer many advantages, particularly when slow speeds, as those in the neighborhood of the stalling speed, are to be measured.

For navigation true air speed is required in order to determine wind speed and hence ground speed. Air logs are a useful aid to dead reckoning in long-distance flights.

On airships a true air-speed indicator is the most suitable for all purposes. On account of interference effects in the neighborhood of the airship, a suspended operating element is very desirable.

**DIFFERENTIAL PRESSURE AIR-SPEED METERS**

**OPERATING ELEMENTS**

A. GENERAL THEORY OF PRESSURE HEADS

Certain general conclusions regarding the performance of pressure heads of any type may be conveniently derived by the method of dimensional analysis. (Reference 61.) It is assumed that the axis of the pressure head is in the line of the air flow. It can be shown that the differential pressure is given by the following equation:

\[ p - p_0 = K \rho V^2 \cdot \phi \left( \frac{DV \rho}{\mu \rho V^2} \right) \]  

(1)

where

- \( p \) = total pressure.
- \( p_0 \) = static pressure.
- \( p - p_0 \) = differential pressure developed by the head.
- \( K \) = dimensionless constant depending on the form of the pressure head.
- \( \rho \) = air density.
- \( V \) = air speed.
- \( \mu \) = air viscosity.
- \( E \) = modulus of compressibility of air.
- \( D \) = characteristic linear dimension of the pressure head, e.g., the throat diameter of a Venturi tube.
- \( \phi \) = a functional symbol.

For an ideal gas,

\[ E = \gamma p_0 \]

where \( \gamma \) is the ratio of the two specific heats of the gas.

Also,

\[ \frac{\gamma p_0}{\rho} = a^2 \]

where \( a \) = speed of sound in the undisturbed medium. Hence, equation (1) may also be written in either of the following two equivalent forms, assuming the air to have the properties of an ideal gas.

\[ p - p_0 = K \rho V^2 \cdot \phi \left( \frac{DV \rho}{\mu \rho V^2} \right) \]  

(1a)

\[ p - p_0 = K \rho V^2 \cdot \phi \left( \frac{DV}{\mu \frac{a}{V}} \right)^2 \]  

(1b)
Equation (1) states that the differential pressure is proportional to the product of the air density and the square of the speed, and that the factor of proportionality \( K \) depends on the geometrical shape of the pressure head. It is evident, also, that an instrument of this type will read true air speed only for the one particular density \( \rho \) which is adopted for calibrating the indicator. Hence, the readings will vary with air density and thus with altitude.

Further, the equation states that the differential pressure is some function \( \phi \) of the variables \( \frac{D V^2}{\rho} \) and \( \left( \frac{E}{\rho V^2} \right) \). The form of the function \( \phi \) is not given by the dimensional analysis and must be determined either from theoretical considerations, or if this is not possible, by experiment.

The quantity \( \frac{D V^2}{\rho} \) is called the Reynolds Number. Since the air viscosity \( \mu \) enters the equation only through this factor, it may also be called the viscosity factor. Also, the absolute size of the pressure head, as specified by the linear dimension \( D \), enters only through this factor so that it gives a measure of the scale effect. An examination shows that its effect on the differential pressures developed is nil if changes in either the air viscosity or the absolute size of the head for geometrically similar pressure heads do not affect the differential pressure for a given air density and air speed. Thus a viscosity effect and a scale effect are always associated and if changes in viscosity do not influence the differential pressure, then nothing which enters only in the Reynolds Number can have any effect and it may be omitted from the equation. In this case, equations (1), (1a), and (1b) become

\[
\begin{align*}
p - p_0 &= K \rho V^2 \cdot \left( \frac{E}{\rho V^2} \right) \\
p - p_0 &= K \rho V^2 \cdot \left( \frac{\gamma p_0}{\rho V^2} \right) \\
p - p_0 &= K \rho V^2 \cdot \left( \frac{a}{V} \right)^2
\end{align*}
\]

The quantity \( \frac{E}{\rho V^2} \) or its equivalents \( \gamma \frac{p_0}{\rho V^2} \) and \( \left( \frac{a}{V} \right)^2 \) may be termed the compressibility factor because the air compressibility appears here only. If the effect of the compressibility can be neglected, then the equations reduce to the familiar simple form

\[
p - p_0 = K \rho V^2
\]

In the left-hand members of equations (1) to (3), the total pressure head \( p \), represents the pressure developed in a device such as a Pitot or a Venturi tube placed in a moving stream of air. This pressure is the algebraic sum of the static or barometric pressure of the undisturbed air and a pressure which is developed only as a result of the relative motion of the air and the tube. Since it is the second of these pressures only which is a function of the air speed, it is necessary to measure the static head independently in order to evaluate the difference. This is the purpose of the static tube.

In the case of a combined Pitot and Venturi, the expression \((p - p_0)\) becomes \((P + p_0) - (S + p_0) = P - S\) where \(p_0\) is the static pressure, \(P\) the impact pressure or velocity pressure in the Pitot tube, and \(S\) the differential pressure (suction) of the Venturi. Since the Venturi tube produces a suction, \(S\) is less than the static pressure, and it is evident that the combined Pitot-Venturi head produces a greater differential pressure than can be obtained by either tube used with a static head.

**B. PITOT TUBES**

**THEORY OF THE PITOT TUBE**

(a) The pressure developed.—Neglecting the air-viscosity term and making certain assumptions as to the nature of the air flow, Buckingham (reference 32) has derived for the Pitot tube the relation

\[
V = \sqrt{\frac{2 \gamma p_0}{\gamma - 1} \rho \left( \frac{p}{p_0} \right)^{\frac{\gamma - 1}{\gamma}} - 1}
\]

which may be written in the form

\[
p - p_0 = p_0 \left[ \left( 1 + \frac{(\gamma - 1) \rho V^2}{2 \gamma p_0} \right)^{\frac{\gamma}{\gamma - 1}} - 1 \right]
\]

The symbols are defined in the preceding section. The validity of neglecting the effect of viscosity on heads intended for use on aircraft has been verified by experiments (reference 53) which show that the viscosity effect is inappreciable except for very small tubes (less than 0.01 inch in diameter) at very low speeds. The inclusion of the compressibility factor, however, has been justified experimentally. (Reference 45.) The formula is, therefore, applicable to the Pitot tubes of aircraft air-speed meters. For speeds greater than the speed of sound, the formula does not hold.

If the right-hand member of equation (4) is expanded by the binomial theorem, the equation becomes

\[
p - p_0 = \frac{1}{2} \rho V^2 \left[ 1 + \frac{\rho V^2}{4 \gamma p_0} + \cdots \right]
\]

which is of the same form as equation (2a) if we put

\[
\phi \left( \frac{\gamma p_0}{\rho V^2} \right) = \left[ 1 + \frac{\rho V^2}{4 \gamma p_0} + \cdots \right]
\]

The factor in brackets may therefore be regarded as a correction factor which takes into account the effect of air compressibility. Computation will show that the correction amounts to less than 1 per cent for speeds
less than about 150 miles per hour, so that with this restriction we may use the simple formula

\[ p - p_0 = \frac{1}{2} \rho V^2 \]  

(3)

In aeronautics the quantity \( \frac{1}{2} \rho V^2 \) is generally called the impact or velocity pressure rather than the differential pressure given by formula (4).

(b) Calibration formulas.—In 1925 the Air Services of the United States Army and Navy adopted the adiabatic formula (4) for calibration of air-speed meters. As the indicators can be calibrated to be accurate at but one air density \( \rho \) and static pressure \( p_0 \) which can be arbitrarily selected, the sea-level values fixed by the standard atmosphere are used. These and other constants of the standard atmosphere are given in Table I. (See reference 17.) The subscript \( s \) will be used to denote standard sea-level values and the subscript \( i \) to denote indicated values. Then from (4)

\[ p - p_i = p_s \left[ 1 + \frac{(\gamma - 1) \rho_i V_i^2}{2 \gamma p_s} \right]^{\gamma - 1} - 1 \]  

(5)

In this equation the quantity \( F V_i^2 \) is dimensionless so that the numerical value of the factor \( F \) depends on the units in which the speeds \( V_i \) are to be expressed. The units of the differential pressure \( p - p_i \) depend only on the units adopted for \( p_s \). Values of \( F \) and of \( p_s \) are given in Tables II and III. (See reference 57.)

From the data given above the differential pressures expressed in terms of inches and centimeters of water corresponding to air speeds expressed in miles per hour, knots, and kilometers per hour have been computed. These are given respectively in Tables IV, V, and VI in the columns headed “Adiabatic.” These tables, presented at the end of this report, give pressure values for small intervals of speed for a considerably higher range than any previously published.

**TABLE I**

<table>
<thead>
<tr>
<th>Constants of the Standard Atmosphere</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard pressure</td>
</tr>
<tr>
<td>Standard temperature</td>
</tr>
<tr>
<td>Standard absolute temperature</td>
</tr>
<tr>
<td>Standard gravity</td>
</tr>
<tr>
<td>Standard specific weight</td>
</tr>
<tr>
<td>Standard density</td>
</tr>
<tr>
<td>Standard temperature gradation</td>
</tr>
</tbody>
</table>

**CONVERSION FACTORS**

1 meter = 39.3700 in. = 3.280833 ft.
1 kilogram = 2.204622 lb.

**TABLE II**

<table>
<thead>
<tr>
<th>Air Speed ( V ) expressed in—</th>
<th>Differential Pressure ( p - p_i ) expressed in—</th>
<th>( \frac{1}{2} \rho_i V^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Miles per hour</td>
<td>Inches of water</td>
<td>1.3510 X 10⁻⁴</td>
</tr>
<tr>
<td>Do</td>
<td>Centimeters of water</td>
<td>4.0307 X 10⁻⁴</td>
</tr>
<tr>
<td>Feet per second</td>
<td>Inches of water</td>
<td>1.2499 X 10⁻⁴</td>
</tr>
<tr>
<td>Do</td>
<td>Centimeters of water</td>
<td>3.8273 X 10⁻⁴</td>
</tr>
<tr>
<td>Knots</td>
<td>Inches of water</td>
<td>1.6514 X 10⁻⁴</td>
</tr>
<tr>
<td>Do</td>
<td>Centimeters of water</td>
<td>4.9184 X 10⁻⁴</td>
</tr>
<tr>
<td>Kilometers per hour</td>
<td>Inches of water</td>
<td>5.8104 X 10⁻⁴</td>
</tr>
<tr>
<td>Do</td>
<td>Centimeters of water</td>
<td>16.576 X 10⁻⁴</td>
</tr>
<tr>
<td>Meters per second</td>
<td>Inches of water</td>
<td>6.5334 X 10⁻⁴</td>
</tr>
<tr>
<td>Do</td>
<td>Centimeters of water</td>
<td>19.060 X 10⁻⁴</td>
</tr>
</tbody>
</table>

**TABLE III**

<table>
<thead>
<tr>
<th>Values of Standard Static Pressure ( p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inches of water at 15° C</td>
</tr>
<tr>
<td>Cm of water at 15° C</td>
</tr>
<tr>
<td>Inches of mercury at 0° C</td>
</tr>
<tr>
<td>Mm of mercury at 0° C</td>
</tr>
<tr>
<td>Pounds per sq. in.</td>
</tr>
<tr>
<td>Grams per sq. cm</td>
</tr>
</tbody>
</table>

The values have been checked against several other tables, particularly those of Zahm (reference 56), and Zahm and Lowden (reference 57), as far as these extended, and by the method of second-order differences.

For reference, the values of the differential pressure have also been computed from the relation

\[ p - p_i = \frac{K}{2 \rho_i V^2} \]  

(3a)

in which \( p - p_i \) is the differential pressure, \( \rho_i \) the standard density (given in Table I), \( V \) the speed and \( K \) a constant depending only on the units chosen for \( \rho_i \), \( V \), and \( p - p_i \). The values are given at the end of this report in Tables IV, V, and VI in the columns headed “Incompressible.” The values of \( \frac{1}{2} \rho_i V^2 \) are given in Table VII for \( p - p_i \) in various units.

(c) Density effect.—For densities and pressures differing from the standard sea-level values, the indicated air speed will obviously not be the true air speed. If we substitute the actual values of the density, pressure, and the differential pressure in equation (4) the true air speed may be computed. To determine the differential pressure from the indicated air speed which is observed, formula (5a) may be used or convenient tables such as Tables IV to VI. However, for practical purposes a much simpler method may be adopted.

**TABLE VII**

<table>
<thead>
<tr>
<th>Air Speed ( V ) expressed in—</th>
<th>Differential Pressure ( p - p_i ) expressed in—</th>
<th>( \frac{1}{2} \rho_i V^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Miles per hour</td>
<td>Inches of water</td>
<td>4.9390 X 10⁻⁴</td>
</tr>
<tr>
<td>Do</td>
<td>Centimeters of water</td>
<td>12.499 X 10⁻⁴</td>
</tr>
<tr>
<td>Feet per second</td>
<td>Inches of water</td>
<td>2.8273 X 10⁻⁴</td>
</tr>
<tr>
<td>Do</td>
<td>Centimeters of water</td>
<td>8.2733 X 10⁻⁴</td>
</tr>
<tr>
<td>Knots</td>
<td>Inches of water</td>
<td>2.8104 X 10⁻⁴</td>
</tr>
<tr>
<td>Do</td>
<td>Centimeters of water</td>
<td>6.5334 X 10⁻⁴</td>
</tr>
<tr>
<td>Kilometers per hour</td>
<td>Inches of water</td>
<td>10.576 X 10⁻⁴</td>
</tr>
<tr>
<td>Do</td>
<td>Centimeters of water</td>
<td>19.060 X 10⁻⁴</td>
</tr>
<tr>
<td>Meters per second</td>
<td>Inches of water</td>
<td>6.5334 X 10⁻⁴</td>
</tr>
<tr>
<td>Do</td>
<td>Centimeters of water</td>
<td>19.060 X 10⁻⁴</td>
</tr>
</tbody>
</table>
In equation (4a) the terms of the series after the first are quite small within the present range of aircraft speeds. Consequently, the part of the density correction derived from these terms may be regarded as a small quantity of the second order which can be neglected. The correction, therefore, may be computed for the first term only; that is, we may use equation (3).

For standard conditions, equation (3) becomes

$$p - p_i = \frac{1}{2} \rho_i V_i^2$$

(6)

Under conditions other than standard, say at an altitude $H$, we have

$$p - p_H = \frac{1}{2} \rho_H V_H^2$$

(6a)

where $V_H$ is the true air speed. If the differential pressures of (6) and (6a) are equal,

$$\frac{1}{2} \rho_H V_H^2 = \frac{1}{2} \rho_i V_i^2$$

or

$$V_H = \sqrt{\frac{\rho_i}{\rho_H}} V_i$$

(7)

The factor $\sqrt{\frac{\rho_i}{\rho_H}}$ is then the correction factor which reduces indicated to true air speed. Assuming the air density to be proportional to the static (barometric) pressure and inversely proportional to the absolute temperature, then

$$\frac{\rho_i}{\rho_H} = \frac{T_H}{T_i}$$

where $T$ = absolute temperature. Equation (7) becomes

$$V_H = \sqrt{\frac{\rho_i T_H}{\rho_H T_i}} V_i = CV_i$$

(7a)

The errors resulting from the use of the correction factor $C$ in equation (7a) in place of equation (4) are very small, except for high speeds and for high altitudes (low densities). At 250 miles per hour with an altitude of 30,000 feet the error is approximately 1 per cent. At 300 miles per hour the error is about 1 per cent at 17,000 feet, and about 1.6 per cent at 30,000 feet. These altitudes are given in terms of the standard atmosphere.

A chart giving the correction factor $C$ for various values of the air pressure $p_H$ and air temperature $T_H$ is given in Figure 1. The diagonal lines give the value of the correction factor. A somewhat similar chart is given as Figure 19 in reference 62 in which the correction factors differ slightly from those in Figure 1, owing to the adoption of a new standard atmosphere since its publication. To use the chart, enter on the left with the air pressure and from the bottom with the air temperature. At the intersection of these ordinates follow the diagonal thus determined upward and to the edge of the chart where the correction factor can be read. The indicated air speed (corrected for scale errors if necessary) multiplied by this factor gives the true air speed. For example, given an air pressure of 560 mm of mercury, an air temperature of $-12^\circ$ C., and an indicated air speed of 180 knots, to find the true air speed. The given air pressure and air temperature are found to determine an intersection on the chart very close to a diagonal which is read on the right edge as correction factor 1.11. The true air speed is $1.11 \times 180$ or 200 knots.

The chart given in Figure 2 differs only in that the ordinate is in terms of the standard altitude instead of air pressure. It is convenient to use when an altimeter is used to determine the air pressure, avoiding the necessity of converting from standard altitude to air pressure. In addition to correcting the altimeter for scale error, it is necessary that the dial of the instrument be adjusted at the time of take-off to read the altitude corresponding to the atmospheric pressure. For many instruments now available this adjustment is made by setting the dial so that its zero corresponds to the fixed reference mark provided for the purpose. This adjustment, or an equivalent but less conveniently made correction, is essential in order to have the indication correspond to the pressure in the standard atmosphere. The procedure of using this chart is similar to that given for Figure 1. Thus, assume as before, a standard altitude of 8,200 feet (corresponding to 560 mm of mercury), an air temperature of $-12^\circ$ C., and an indicated air speed of 180 knots, to find the true air speed. Entering the chart on the left with the altitude and from the bottom with the temperature gives an intersection about midway between two diagonals marked 1.10 and 1.12 on the right. As before, the true air speed is $1.11 \times 180$ or 200 knots.

Another correction chart which is given in Figure 3 is a graphical representation of equation (7). The relation between indicated air speed and air density is given in the chart for true air speeds in the range from 40 to 300 miles per hour in steps of 10 miles. The abscissas are also given in terms of the altitude corresponding to the air density in the standard atmosphere and for convenience the abscissa scale is evenly divided in altitude units. The standard values of the differential pressure corresponding to the indicated air speed are presented in the chart as an auxiliary ordinate. The chart can be used to determine true air speed (a) if the air density and indicated air speed are known and (b) if the altitude in the standard atmosphere and indicated air speed are known. In the latter case the chart gives correct results only if the temperature of the air at the standard altitude does not differ from the standard temperature. The true air speed
Figure 1.—Chart for obtaining true air speed from the indicated air speed at a given air pressure and temperature. The true air speed equals the indicated air speed times the correction factor obtained from the chart.
Figure 2.—Chart for obtaining true air speed from the indicated air speed at a given standard altitude and air temperature. The true air speed equals the indicated air speed times the correction factor obtained from the chart.
is in error about one-sixth of 1 per cent for each degree centigrade deviation from the standard temperature. If an altimeter is used to determine the altitude it must be set in the manner discussed in the preceding paragraph. If so set and if the altimeter has reasonably low scale errors (under 2 per cent), the error in the indicated air speed as 160 miles per hour. This altitude and air speed intersect in the chart at a point between two diagonals reading 180 and 190 at the right, giving 189 miles per hour as the true air speed. It is seen also that the density corresponding to this altitude is 0.0548 pounds per cubic foot.

**Figure 3.** Chart for obtaining the true air speed for a given indicated air speed and standard altitude.

The "true" air speed will remain of the same order of magnitude as that due to air temperature.

To use the chart, enter at the left with the indicated air speed, pass horizontally to the vertical representing the altitude or density, and then follow the diagonals down to the right-hand side of the chart where the true air speed may be read off. As an example, take the standard altitude as 11,000 feet and the indicated air speed as 160 miles per hour. This altitude and air speed intersect in the chart at a point between two diagonals reading 180 and 190 at the right, giving 189 miles per hour as the true air speed. It is seen also that the density corresponding to this altitude is 0.0548 pounds per cubic foot.

**Figure 3.** Chart for obtaining the true air speed for a given indicated air speed and standard altitude.

**Design of Pitot-Static Pressure Heads for Aircraft**

Many factors must be considered in the design of Pitot-static tubes for aircraft in order to obtain a pressure head which will give satisfactory service. The relative importance of the factors will depend in great measure on the purpose for which air speed is
to be measured and whether it is to be measured on airplanes or airships.

The ideal pressure head should—

(a) Be of a design which produces the theoretical difference in pressure.
(b) Produce a pressure difference large enough to be readily and accurately measurable.
(c) Be easily reproducible without the necessity for individual calibrations.
(d) Be insensitive to yaw or pitch.
(e) Be insensitive to gusts; that is, indicate the true average air speed.
(f) Have low head resistance.
(g) Be easily mounted without interfering with other equipment.
(h) Be unaffected by atmospheric conditions, dust, rain, etc.
(i) Be sufficiently rugged to withstand the wear and tear incident to service.

As previously stated, viscosity and scale effects have been shown to be negligible for Pitot tubes of practicable dimensions within the range of aircraft speeds. Within wide limits the Pitot tube will accurately develop the total head as given by theory regardless of form or dimensions. That tubes of the sizes commonly used develop the theoretical pressure in turbulent flow is a fact based on the results of experience.

The situation with regard to the static tube is quite different. In order to accurately measure the static pressure, an opening in the tube is required at a point where the air flow is undisturbed, and the opening must be set so there is no component of air motion perpendicular to its plane. If a tube is set in an air stream, parallel to the direction of flow, the distribution along the length of the tube of the pressure normal to its surface is of the nature shown in Figure 4.

With obvious modifications in some of these requirements, the list applies to all types of operating elements for air-speed meters and furnishes a basis for comparison in the selection of a type for any desired conditions of service.

(a) Design to produce theoretical pressure.—The ratio between the differential pressure actually produced by the tube, when mounted to align properly with the air stream, and the theoretical differential pressure is called the constant of the Pitot-static pressure head. It is desirable to have this constant equal to unity, which is practically equivalent to stating that the static tube measures the true static pressure as will appear later.

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now commercially available with indications down to 20 miles per hour.

(c) **Reproducibility.**—One of the chief advantages of the Pitot-static pressure head is that it can be readily duplicated so that individual calibrations are not required. It is, therefore, suitable for use not only as a primary standard but also as a general-service instrument.

![Graph](image)

**Figure 5.**—Effect of yaw on performance of Pitot-static tube. The pressure developed is \( p_i \), the static pressure \( p_s \), and the impact pressure \( \frac{1}{2} \rho V^2 \). The curve \( S \) is for the static tube, the curve \( P \) for the Pitot tube, and the curve \( PS \) is for a combination of both.

(d) **Effect of inclination.**—Inclination of the pressure element to the direction of the wind stream affects the performance of both the Pitot tube and the static tube. The effects are shown qualitatively by the graphs in Figure 5. (See references 39, 47, and 50.) The curve marked \( P \) represents the ratio of the pressure in excess of static pressure \( (p_i) \) developed by a typical Pitot tube at various angles of inclination or yaw to the impact pressure at zero inclination, and the curves marked \( S \) and \( PS \) pertain in a similar manner to a static tube and a Pitot-static tube.

It will be noted that the change in the Pitot pressure due to yaw is very small for angles less than about 15 degrees, and then as the angle of yaw increases the Pitot pressure decreases at an accelerated rate. The effect depends to some extent on the form of the tube. In Prandtl's work (reference 50) on tubes with hemispherical ends, a minimum yaw effect was observed when the diameter of the opening was about three tenths that of the tube itself.

![Graph](image)

**Figure 6.**—Performance of a Pitot-static tube suspended from an airplane.

As might be expected, the static tube is considerably more sensitive to inclination than the Pitot. The curve \( S \) shows, qualitatively, the decrease in the pressure as the angle of inclination increases. As this decrease is at first more rapid than for the Pitot tube, the result is that the differential pressure increases for small inclinations as shown by curve \( PS \). At a certain angle depending on the form of the tubes, the differential pressure reaches a maximum and then decreases at a gradually increasing rate as the inclination becomes greater.

Slotted openings have a number of advantages. Those made circumferentially on the tube are not so much affected by inclination (yaw) as slots cut parallel to the axis of the tube.

Since the static holes are disposed symmetrically about the tube, it follows that when inclined to the direction of the wind stream, there must be flow into some and out of other openings. Anything which disturbs this flow such as a bend in the tube or the attachment of the supporting tube in close proximity to the static openings will in general increase the error due to inclination.
Inclination to the wind stream of a pressure element mounted on an airplane may be caused by yaw or pitch of the airplane. Any irregular small changes in yaw or pitch will be of minor moment. A steady angle of yaw will only be experienced when sid-slip and turning, but ordinarily the air speed need not be determined at such times. The angle of pitch, however, is a function of the speed and of the load. The pressure element is, or should be, installed so that its axis is parallel to the fore-and-aft axis of the airplane when it is flying level at normal speed under normal loading. At low speeds, and with heavy loading, the increased angle of incidence may result in a perceptible inclination error amounting to as much as 5 per cent.

For flight testing it may be preferable to use either a swiveling pressure element or a suspended head. The swiveling element is rotatable about a transverse horizontal axis and is furnished with directive vanes. It corrects for changes in angle of incidence only. The suspended head automatically faces into the wind, as will appear from the descriptions given later.

(e) Gusts.—Under service conditions gusts may cause some unsteadiness in the indications of air speed, but the errors in the mean speed shown are so small that they need not be considered. In flight testing the question should not arise, since the work will be done only under very favorable weather conditions.

(f) Head resistance.—The added head resistance due to a Pitot-static tube rigidly mounted to an aircraft is less than that of any other differential pressure tube and also small compared with other types of operating elements.

(g) Installation.—The mounting of a Pitot-static head presents no difficulties from the mechanical standpoint. Care must be taken, while the airplane is on the ground, not to strike and damage the tube.

(h) Blocking of tubes.—A commercial air line must maintain a regular schedule of flights independent of any save the most extreme weather conditions. This is particularly true as regards the air mail services. Military flying must also be accomplished under very adverse conditions. Hence it is of the greatest importance that the performance of the air-speed pressure head, which of necessity must be mounted in an exposed position, shall be influenced by the weather to the smallest possible degree. The chief weather factors which may affect the pressure head are rain, freezing rain or ice accumulations, snow, and spray. These may on occasion block the tubes and render the air-speed meter inoperative. A further accumulation of data from actual experience is desirable, particularly data concerning ice formation.

Some data of considerable value have been obtained from flight tests made by the National Advisory Committee for Aeronautics and from laboratory tests made by the Engineering Division of the United States Army Air Service. (References 24 and 38.)

The Air Service tests were made on Pitot-static and Venturi pressure heads at an air speed of about 60 miles per hour. A special 6-inch wind tunnel mounted in a temperature chamber was used. A fine spray of water was introduced to simulate rain, and at a temperature of $-15^\circ$ C. this spray formed ice coatings on the pressure heads under tests.

As regards Pitot tubes, it was found that the larger the diameter, the longer the time required to freeze over and block the opening. Beveling the edge of the tube on the outside seemed to have no effect, but beveling inside ($15^\circ$ to $20^\circ$ bevel) delayed the freezing over. As long as even a fine hole existed in the ice cap on the nose of the tube, the Pitot pressure appeared to be unaffected. An electrically heated tube did not freeze over. The tubes froze over at the tip before any ice accumulation at a bend inside was sufficient to block the tube. On Venturi tubes, even a slight amount of ice seriously affected the pressure head. Rain had practically no effect on either Pitot or Venturi tubes, though Pitot tubes required a bend of at least $30^\circ$ upward not more than five diameters from the tip in order not to become clogged. (See fig. 10 B.) Drain holes with total area not exceeding one-seventeenth of the Pitot tube cross-sectional area did not appreciably affect the differential pressure.

No freezing over was found at the static holes if these were at least five diameters from the tip of the tube. Where the static openings were circular holes, there was a tendency for water films to form which were sufficient to affect the readings appreciably. Slotted openings were not so affected. Very little water or ice accumulated inside of the static tube.
It was recommended that for protection against rain Pitot-static tubes should have an upward bend of 30° or more not over five diameters from the tip, drain holes in the Pitot tube, and slotted static openings (circumferentially placed) not less than one-sixteenth inch by three-sixteenths inch in size. For protection, it was recommended that the Pitot tube should be heated or made so large that it does not freeze over under the conditions of use, and the static openings should be located at least five diameters from the tip. It is obvious that a compromise is necessary, and that the upward bend recommended will have to be placed more than five diameters from the tip on pressure heads where the tubes are concentric. Heated tubes are expensive and require additional equipment. Probably a minimum diameter can be adopted on the basis of service data such that the tube will not freeze over before the airplane is forced down.

The data obtained from the flight tests by the National Advisory Committee for Aeronautics confirm the laboratory results. On Pitot-static heads the ice accumulated in practically the same way as in the wind-tunnel tests. Trouble has been experienced occasionally on account of clogging of the pressure element by sand or dust as the airplane takes off. Coutinho reports such an incident at the start of his transatlantic flight. The Badin Venturi was choked with sand so that the indicator was useless during the flight across the ocean. If much flying is to be done under these conditions, a long trapped Pitot tube may be required.

The choking of Pitot tubes by insects has been reported from tropical countries. A modified design and regular inspection should prevent any difficulties.

(i) Ruggedness. Pressure heads, since they have no moving parts, are not subject to wear. However, damage may result from corrosion or striking the tube while the airplane is on the ground. The possibility of serious damage is slight in either case. Corrosion, if very severe, may affect the static holes, but with ordinary commercial tubes under ordinary conditions of service the corrosion will be negligible. If the edge of the Pitot tube is bent over, or the tube is dented or bent in any way affecting the static holes, the pressures will be in error. Reasonable care will eliminate the possibility of any such accident.

(j) Interference. As an aircraft moves through the air, air currents are set up near the aircraft. If a Pitot-static head or any type of air-speed meter is to indicate accurately the speed relative to the undisturbed air, it must be located in undisturbed air. No position can be found on an airplane where the pressure element will not be affected by disturbances due to the wings, struts, or other parts of the airplane. The errors caused by these disturbances depend not only on the location of the element but also on the design of the airplane and must be determined by experiment for each particular case. In general it has been found that the errors are likely to be greater on monoplanes of the internally-braced-wing type where the most convenient and practicable location for the pressure head is immediately in front of the leading edge of the airplane.

On biplanes the most satisfactory position for mounting the pressure head (reference 14) is about one-third to two-thirds of the gap above the lower wing, forward and slightly to the side of a strut. The interference is less in this position than in any other which is convenient and practicable.

On internally braced monoplanes the pressure element has in the past been usually mounted 2 to 4 feet forward and above the leading edge of the airplane. The interference is considerable in this position. The velocity field in the neighborhood of a wing has recently been investigated by Lapresle. (Reference 26.) His data indicate that the theoretically correct speed is obtained at all angles of attack up to the stalling angle in a position from 0.2 to 0.3 chord length back of and about 0.4 chord length above the trailing edge of the wing. The distance back is measured along the chord extended, and the distance up, at right angles to the chord. Flight tests were conducted by the Pioneer Instrument Co. on an airplane with a Göttingen No. 387 wing in which a Pitot-static tube was mounted 0.2 chord back and 0.4 chord above the trailing edge of the wing. The indications of the air-speed meter were compared with a calibrated trailing Pitot-static tube and were found to have no error in excess of 1.5 miles per hour. This may point to the solution of the problem of where to mount the tube on a monoplane.

For special purposes on airplanes and for service use on airships, the operating element is often suspended below the aircraft at such a distance that the interference (or position) effect is negligible.

The magnitude of the interference effect may be determined by the following procedure. The over-all errors of an installed instrument may be determined by calibrating it on a speed course. The over-all error will include errors (a) due to the fact that the tube constant may not be unity, (b) due to varying angles of incidence of the tube, (c) due to scale errors of the indicator, and (d) those caused by the interference of the aircraft. The tube constant and indicator errors can be determined in the laboratory. It is difficult to separate the other two factors since the inclination of the air stream to the axis of the commonly used tube is affected by the aircraft structure. Due to this fact the interference error, or perhaps more exactly, the error due to both factors, ordinarily varies with the speed of flight of a given airplane.

The mounting position of the tube for minimum position error at selected speeds for each design of airplane is determined by flight tests usually made over a speed course. The interference error can not ordinarily be reduced below from 1 to 5 per cent.
DESCRIPTION OF PITOT-STATIC PRESSURE HEADS

The forms of "standard" Pitot-static tubes adopted by the National Advisory Committee for Aeronautics, the Bureau of Standards, and the National Physical Laboratory (England) for use in wind-tunnel investigations are illustrated in Figure 8. The originals have been calibrated with all possible accuracy, and duplicates are made to the exact dimensions of the originals in order to avoid any possibility of scale effect. The Bureau of Standards type follows quite closely the dimensions of the National Physical Laboratory tube. It has been found that this form of tube is quite sensitive to variations in the shape of the nose and to slight variations in the position of the static holes. (Reference 55.) The third tube is very similar to that developed by Prandtl except that static holes are used instead of a slot. These tubes are not suitable for general service on aircraft.

An early form of Pitot-static head is that shown in Figure 9C. This was developed in England during the World War and is still in extensive use. It is known as the Mark IV A pattern. This head has been shown to develop accurately the theoretical pressure difference and it is not greatly affected by inclination. Possible disadvantages are the method of mounting immediately in front of a strut (forward and to one side is better), the small size of the static holes, and the insufficient provision for excluding water from the Pitot tube. A modification of this pressure head known as the Mark V A is available for use in localities in which insects get into the pressure tube. The Pitot tube connects to the pressure tubing through a number of small holes and is removable for cleaning.

A pressure head which has been and still is widely used in the United States is the Pioneer head shown in Figure 9A and 10C. It differs from the Mark IV A head in an important respect, in that there is an upward bend of a considerable portion of the head which effectively prevents any water from entering the pressure lines. Also the static holes are somewhat larger and therefore not so likely to be clogged by water films.

The pressure head now specified by the Bureau of Aeronautics of the Navy Department is of particular interest since it has been designed with due regard to all the factors which might influence its performance under the severe service conditions encountered in naval aviation. Dimension drawings of the head are given in Figure 9B and a photograph in Figure 10A. The concentric tube form has been adopted and the dimensions reduced to the minimum consistent with other considerations in order to keep down the head resistance. The upward bend is to prevent the passage of water. The horizontal offset permits mounting against the side of a strut and at the same time brings the dynamic and static openings forward and to the side of the strut at a point where interference is a minimum. The Pitot tube is of the trapped type. The main tube is one-half inch outside diameter with a slight inside bevel at the opening. This form and size were chosen to delay the effects of ice formation. The main tube is stopped by a rigid diaphragm 2 inches back of the opening and a closed inner tube projects forward through this diaphragm. A small slot is cut in the side of the inner tube to transmit the pressure. Three small weep holes are provided in the outer tube to permit the escape of the trapped moisture, which is facilitated by the excess pressure of the air in the inside of the tube over that at the outside at this point of the tube. The static openings comprise two rows of three slots each, slots being chosen to prevent stoppage or interference by films of water during a rain. The curvature of the head starts less than two diameters back of the static holes, which would lead one to expect that the static pressure would indicate a little too high, or in other words the differential pressure would be slightly low. A concomitant scale effect is to be expected so that the exact placing of the static slots with respect to the bend in the tube must be considered.

The results of wind-tunnel calibrations of two pressure elements of this type are plotted in Figures 11 and 12. The differential pressures delivered by the heads are expressed as a proportion of the true differential pressure. It should be noted that here, as elsewhere, the proportional errors in pressure are twice those in air speed. The difference in performance in the two heads is mainly due to differences in the static holes, those of the laboratory model being the more carefully made.

Two Pitot-static tubes now being used to some extent are the Kollsman tube shown in Figure 10B and the Pioneer tube, Figure 10D. Both designs show
especial attention to the prevention of stoppage by rain and freezing rain. Note that no static hole is placed directly in the rear of the connecting tubing of the head shown in Figure 10D.

The Badin Pitot-static tube shown in Figure 10F is part of the Badin air-speed recorder to be described later. The static pressure is secured from the hole in the center of the disk which is kept parallel with the air stream by the vane.

The pressure heads so far mentioned are designed for biplanes, although they are used on monoplanes also whenever it is convenient or possible to install them in positions where satisfactory performance is obtained. The Aircraft Control pressure head, Figure 9D and the Pioneer head, Figure 10E are in common use on many monoplanes. These are intended for mounting on the leading edge of the wing, ordinarily about 2 feet forward.

The extended tube reduces to some extent the great interference due to the wing. The Pitot static head is set 3 or 4 inches below the supporting tube so that water can not enter the pressure lines. One theoretical disadvantage of the form illustrated is that the vertical tubes cause interference at the static openings. This interference has been reduced somewhat by eliminating the openings at the top of the head. However, a static tube asymmetrical about the horizontal plane would be expected to have an unusually large inclination error in pitch. Consequently, the speed factor would vary considerably with angle of incidence, that is with speed and loading.

It is believed that considerable improvement both in performance and construction would be secured in tubes adapted for mounting on the leading edge of a monoplane if a tube such as shown in Figure 9A or 9B, but of proper length, were bent so as to project forward and downward.

Electrically heated tubes (fig. 13) have been introduced in order to prevent formation of ice. Undoubtedly these tubes will not be clogged but sufficient data from experience are not available to determine whether the increased complexity due to wiring and electrical connections are warranted by the results obtained. It must be remembered in this connection that when ice of the dangerous form has accumulated, the aerodynamic characteristics of the airplane are changed considerably so that the air-speed indicator is no longer to be relied upon to indicate stalling speed. The value of the new stalling speed may be very different from the normal value when the airplane is ice-free.

![Figure 9](image)

The suspended head developed by the National Advisory Committee for Aeronautics is shown in Figure 14A. The over-all length is about 28 inches. The head is connected to the recording instrument by rubber tubing of one-eighth inch inside diameter. The rubber tubes are inclosed in a flexible metal conduit of circular cross section. The conduit supports the weight of the head. Suspension lengths varying from about 30 to 70 feet have been used.
A number of commonly used Pitot-static tubes. A. Navy head; B. Kollsman head; C and D. Pioneer heads; E. Pioneer head for monoplanes; and F. Badin head with swivelling static head (obtained from hole in center of disk)
The committee has also developed a combined flight-path-angle and air-speed recorder, all self-contained within a suspended head. (Reference 19.) The over-all length is 41.8 inches and weight about 18 pounds. A round-nosed Pitot-static tube with an annular slot for the static opening is used.

The new head recently designed at the Bureau of Standards for the flight test section of the Bureau of Aeronautics at Anacostia is shown in Figure 14B. This head was made as light as possible and designed for use with a special suspension cable in an effort to minimize the extra drag. Its over-all length is 18 inches. The cable is made of rubber, has a streamlined section, and contains a stranded steel wire and two air tubes. Flight tests have shown that the assembly is satisfactory up to speeds of about 80 miles per hour. Lag, due to the small bore of the tubing, is rather large but this fact is considered of minor importance for the particular uses for which this head is designed. For general use a somewhat larger head of similar design with a modified suspension cable is under consideration.

The Pioneer Instrument Co. has constructed a similar cable for use with its suspended head except that it is of circular cross section. This is shown in Figure 14C. The theoretically greater head resistance of the circular cable is offset by the greater care needed in lowering the streamlined cable.

An early English suspended head is a Pitot-static tube suspended in a frame so as to be free to rotate about a horizontal axis. The frame is prolonged downwards to support a 10-pound weight. This head
has been used in airship tests. A later form, shown in Figure 15, is considerably simplified and consists of a static tube only. (Reference 52.) It is to be used in conjunction with a swiveling Pitot head mounted in the conventional position.

**Theory of the Venturi**

A satisfactory derivation on theoretical grounds of the differential pressure-speed relation for Venturi air-speed pressure heads has not been achieved beyond that presented in equation (1). The simplifying assumptions usually made in the mathematical treatment of the problem are such that the final results represent actual conditions only to a limited extent.

Hence, it is necessary to resort to experiment to determine the form of the function \( \phi \left( \frac{DV}{\mu}, \frac{E}{\rho V^2} \right) \) of equation (1) and the value of the constant \( K \).

For a certain range of moderate speeds, which varies according to the particular tube under consideration, the differential pressure may be expressed in the form of equation (3):

\[
\Delta p = K\rho \frac{V^2}{2}
\]

where \( K \) is a constant depending only on the form of the pressure head, \( p_s \) is the pressure at the throat of the Venturi; and \( p_0 \) is the static pressure. This relation may also be applied to double Venturi tubes and to the Pitot-Venturi except that in the latter case \( p_0 \) must be replaced by the Pitot pressure. The approximate speed range within which the equation holds is given in Table VIII for a number of types which have been used to a considerable extent in the United States. The lower limit of the speed range is determined by the viscosity effect. At speeds below this minimum the differential pressure falls off sharply. The upper limit of the speed range depends on a number of factors, including the magnitude of the compressibility effect. Available data (references 62 and 63) indicate that at least within the speed range given in Table VIII, equation (3) applies with approximately the same accuracy as for Pitot-static tubes. Deviation from this relation may be expected at greater speeds as well as at lower speeds.

Since the differential pressure developed by a Pitot static head in the same speed range is given by

\[
\Delta p = \frac{1}{2} \rho \frac{V^2}{2}
\]

the ratio of the differential pressures of a Venturi head to a Pitot static head is given by

\[
\frac{p_s - p_0}{p - p_0} = 2K = C
\]

where \( C \) is called the "efficiency" of the Venturi or Pitot-Venturi tube. Values of \( C \) are given in Table VIII. The approximate differential pressure delivered by any of the tubes listed for any given speed (within the proper limits for the tube) may be found by multiplying the differential pressures for Pitot-static tubes in Table VIII by the constant \( C \). For accurate work with the Zahm types, reference should be made to the original calibration tables. (Reference 33.) For high accuracy with other types individual calibrations are necessary.
Density Effect

Within the speed ranges given in Table VIII density corrections may be applied to the indicated speeds of Venturi and Pitot-Venturi instruments by using the same correction factor as for Pitot-static indicators.

TABLE VIII

[Speed range in which Venturi and Pitot-Venturi pressure heads follow the \( \rho^{1/2} \) law. (Efficiency \( C \) is constant)]

<table>
<thead>
<tr>
<th>Pressure head</th>
<th>Type</th>
<th>For details see</th>
<th>Minimum speed m.p.h.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Navy Zahm</td>
<td>Pitot-Venturi</td>
<td>17(a)</td>
<td>Above 150 6.35</td>
</tr>
<tr>
<td>Army Zahm</td>
<td>Pitot-Venturi</td>
<td>17(b)</td>
<td>Above 200 6.35</td>
</tr>
<tr>
<td>Toussaint-Lepere</td>
<td>Pitot-Venturi</td>
<td>17(c)</td>
<td>Above 240 7.52</td>
</tr>
<tr>
<td>Badin</td>
<td>Single Venturi</td>
<td>17(d)</td>
<td>Above 300 7.38</td>
</tr>
<tr>
<td>Badin</td>
<td>Double Venturi</td>
<td>17(e)</td>
<td>Over 110 11.6</td>
</tr>
<tr>
<td>Bruhn</td>
<td>Double Venturi</td>
<td>17(f)</td>
<td>80-150 11.6</td>
</tr>
</tbody>
</table>

1 The values of speed are given to the nearest 5 miles and for zero altitude in the standard atmosphere. (Reference 62 and 63.)
2 Based on experimental results for very few heads.

The curves in Figure 16 show the minimum air speed at various altitudes for which the simple density correction factor may be used at various indicated air speeds without appreciable error. If the point corresponding to the air speed and altitude for which a correction is desired falls above or to the left of the curve for the tube in question, then a calibration is required to determine the correction. Curves are given by Eaton and MacNair for the Zahm Pitot-Venturi tubes. (Reference 62.)

Descriptions of Venturi Pressure Heads

Venturi heads are still in common use in Germany and France. The Bruhn double Venturi, Figure 17D, is the usual form adopted for all ranges of speed in Germany although a single Venturi tube appears to be coming into favor. In France the Badin head, Figure 17C, is employed for speeds up to 200 kilometers per hour and Pitot-static heads for higher speeds. Typical calibration curves for these heads are given in reference 62. For accurate work it is necessary to calibrate each head individually since slight variations in dimensions may have appreciable effects on the calibration characteristics, especially when low speeds are to be measured.
The American Zahm Pitot-Venturi was made in two forms differing in size and materials. The Navy type, which was the larger, is shown in Figure 17A, and the smaller Army type in Figure 17B. For both of these tubes the calibration is closely represented by the relation

\[ V = 17.89 \sqrt{2} \frac{h_i}{h} \]

when the air pressure is 760 mm of mercury and the temperature is 16° Centigrade. \( V \) is the velocity in miles per hour and \( h_i \) is the differential pressure in inches of water. (See reference 62.)

The characteristics of the Zahm pressure heads at low speeds have been investigated in detail by Eaton and MacNair, who give curves for the density effect. (Reference 62.)

**Advantages and Disadvantages of Venturi and Pitot-Venturi Heads**

The single Venturi was first introduced for the reason that it gives considerably higher differential pressures than the Pitot-static and, consequently, a satisfactory indicator with metal diaphragm was more easily constructed. The Pitot-Venturi and the double Venturi were natural developments of this same idea. The double Venturi was also adopted for the purpose of measuring low speeds on airships, and for airplanes because the very high differential pressures produced made it possible to neglect the small static pressure variations in the cockpit and so dispense with a static tube.

However, these advantages are no longer of any significance and in the United States pressure heads with Venturi tubes are practically obsolete. Satisfactory indicators for use with Pitot-static tubes are available for the whole range of airplane speeds. The supposed advantages of the Venturi, and in particular the double Venturi, for measurement of low speeds have been shown to be largely illusory. The viscosity effect at low speeds is large and varies greatly with the design of the tube so that individual calibrations of the tubes are required. The magnitude of the effect is shown by the tube efficiency (C in equation (9)) which decreases very rapidly with decreasing speed. The Bardin double Venturi tube, for example, has an efficiency of nearly 13 at 65 miles per hour which drops to 2 at 5 miles per hour. The decrease in efficiency is considerably less for Pitot-Venturi heads but is still quite large.

In the range of speeds for which the differential pressure follows the law expressed by equation (8), the advantage of higher pressures is outweighed by the sensitiveness of Venturi tubes to relatively small changes in dimensions. This necessitates either very careful manufacturing methods to produce tubes of exactly the same size and shape, or else individual calibrations. In either case it is obvious that the cost will be increased. This same sensitivity is also apparent under conditions where ice may form on the tube. Even a small quantity of ice, which would have no effect whatever on a Pitot tube, will cause such great errors in the Venturi pressure that the air-speed meter will be useless.

At high speeds the compressibility and other factors affect the calibration in a practically unknown manner so that pressure heads with Venturi tubes should not be used for the upper ranges of airplane speeds. It is clear from the above discussion that at the present time there is no warrant for using pressure heads with Venturi tubes on aircraft. There does not seem to be any advantage in their use except, possibly, for experimental purposes under unusual conditions and only then if a complete calibration of the pressure head can be made over the speed and density range for which it is to be used.

**D. Calibration of Pressure Heads**

**Laboratory Calibration**

When the question of calibration is considered, the great superiority of the Pitot-static tube over all other types of operating elements for general service is strikingly evident. Only in special cases do other types offer compensating advantages of sufficient value to warrant their use. In the first place, for a given design of Pitot-static tube one calibration suffices, since it has been demonstrated that duplicates can be made with ordinary machine-shop methods which will have the same calibration characteristics as the original to a high order of accuracy. But even a single calibration is not required unless the pressure head is to be used for test work, or the yaw errors are required. For by following well-established principles, a Pitot-static head can be designed which will produce the theoretical differential pressure with errors considerably less than those due to interference. In practice, therefore, the calibration errors of the pressure head are included in the speed factor which must necessarily be determined for each type of air plane and mounting if air speed is to be measured with any pretension to accuracy.

Calibrations may be made in the wind tunnel or on a whirling-arm apparatus, the latter probably being the more accurate at low speeds. Wind tunnel calibrations may be extended to cover a greater speed range by towing the tube in a water channel. The results are reduced to equivalent values for air by the principle of dynamical similarity.

A high order of accuracy may be obtained by the whirling-arm apparatus but only at the expense of careful and painstaking experimental work. The chief source of error is due to the swirl of air set up by the movement of the arm. For a detailed description of the method of testing, the reader is referred to an early paper by Bramwell, Relf, and Fage describing the calibration of the N.P.L. standard head. (Reference 43.)
Flight Calibration

The methods which have been used to determine the errors of an air-speed meter unit as installed in an airplane are—

(a) Comparing the indicated air speed with the speed of the airplane as determined by timing over a speed course.

(b) Comparing the indicated air speed with that given by an instrument suspended from the airplane at a sufficient distance to make the interference errors negligible.

(c) Flying in formation with an airplane whose speed factor has been previously determined.

(d) By comparison with an element mounted on a rod projecting far forward from the wing or strut.

(a) Calibration over speed course. (See reference 29).—A speed course consists merely of two ground stations, easily identified from the air, at an accurately known distance apart. The course should be marked by some prominent topographical feature such as a straight highway or railway. The time of flight may be measured either from the ground stations or on board the airplane. Measurement of time of flight from the ground stations requires an observer at each station, electrical or visual intercommunication, and appropriate timing apparatus. If wind corrections are also to be determined from the ground, additional equipment is necessary.

When all measurements are made on the airplane itself, no additional personnel and no equipment other than markers are required. Therefore, this procedure is generally adopted.

The length of the speed course should be sufficient so that the unavoidable slight errors in fixing the time of transit will be negligible in comparison with the total time of flight. For average use a length of about 2 miles is satisfactory. The course should be level and chosen to have a direction such that the probability of cross winds is a minimum.

The weather is usually the controlling factor as regards accuracy. The effect of a wind, steady in direction as well as intensity, can be readily determined, but such a wind is rare in nature. Generally it is necessary to wait for a favorable opportunity.

In making a run at one indicated air speed, the time of traverse is measured in both directions. When obtained by the pilot or an observer in the airplane a stop watch is used, which to insure accuracy should be rated. The traversing flight is made at low altitude, so that the air density, determined by the air pressure and air temperature, is practically that at the ground level. Sighting is more accurately done at low altitudes both by the pilot and by ground observers. The ground speed is obtained by dividing the length of the course by the time of traverse. The average ground speed in the two directions eliminates the effect of a head wind provided the wind speed remains constant. If there is a component of the wind perpendicular to the course, the airplane should be flown so that its longitudinal axis is always parallel to the course. The cross wind will then drift the airplane off the course but correct results will be obtained if the time interval of the traverse is measured in the usual way.

The average ground speed thus determined is the true air speed $V_H$ of the airplane for zero wind. The true indicated air speed $V_i$ is given by equation (7a),

$$V_i = V_H \sqrt{\frac{p_s T_H}{p_H T_s} \frac{V_H}{C}}$$  \(7a\)

in which, as before stated, $p_s$ and $T_s$ are the standard values of the air pressure and air temperature and $p_H$ and $T_H$ the values at the altitude of flight. The two latter quantities may be observed on instruments in the airplane or at a ground station if its elevation and the altitude of flight do not differ more than 100 feet. The factor $C$ may be computed or more conveniently obtained from a chart such as Figure 1.

The correction to be applied to the reading $V_i$ of the air-speed meter is given by the difference $V_i - V_T$. This correction includes the effect of all errors of the air-speed meter.

Air-speed meters are not usually calibrated on a speed course at speeds near the stalling speed of the airplane owing to the danger involved in flying at the required low altitude. Usually method (b) or (c) is used.

The French have developed methods and equipment for measuring the air speed from ground stations. (Reference 22.) Somewhat similar arrangements were used on a speed course for high-speed trials laid out on Kent Island during the summer of 1929 by the Bureau of Aeronautics with the cooperation of the Bureau of Standards. The method consists essentially of photographing the airplane at each end of the course, including markers to indicate the position of the airplane with reference to the start and finish of the course. The moment of taking the photographs is recorded upon a chronograph by means of contacts in the shutter of the cameras. The speed is determined from the time interval thus measured and the length of the course, corrected for the deviation of the airplane from the start and finish line as shown by the photographs.

(b) Suspended head method.—In this method of calibrating an air-speed meter an instrument is used as standard in which the operating element is suspended below the airplane such a distance that interference is nil or at least negligible. This method compared to the speed course method is less dependent on weather conditions, requires no particular terrain, and, most important of all, it can be used even at low speeds.

Several types of suspended heads have been used. In England the suspended static head (reference 52) has been developed for this work. (See fig. 15.) As long

References:

22. Similar arrangements were used on a speed course for high-speed trials laid out on Kent Island during the summer of 1929 by the Bureau of Aeronautics with the cooperation of the Bureau of Standards.
as a Pitot tube faces accurately into the wind stream and there are no disturbing eddies ahead of it, it measures the true total head. Hence, it is necessary only to suspend the static tube, thus eliminating the necessity of an additional suspended tube. To avoid inclination errors, a swiveling Pitot tube is required. In the United States, a combined Pitot-static suspended head is favored. The N. A. C. A. head, shown in Figure 14 A, and the Pioneer head, Figure 14 C, are examples. Recently, the Bureau of Standards has constructed a suspended head (fig. 14 B) for the Flight Test Branch of the Bureau of Aeronautics.

Other instruments which have been used for the same purpose are the Barr and Stroud air log (fig. 40) and the Bureau of Standards electric air-speed meter (fig. 42).

The length of the suspending cable is important. The relation for one airplane (reference 52) between the percentage increase in indication and the length of the suspended cable is shown in Figure 6. The indication becomes greater as the length of suspended cable increases, a result that may be considered as typical for airplanes. Note that the error becomes practically zero for a length of about 30 feet; this is usually considered the minimum for accurate work.

On airships the position error varies considerably according to the distance of the air-speed element from the nose of the airship. Experiments on the R33 (reference 4) gave the results shown in Figure 7 for a Pitot-static head suspended from the control car. Roughly, all disturbances are avoided by using a suspension about 40 feet long.

(c) Flying in formation.—This method is practically independent of weather conditions and easily carried out. The airplane is flown in close formation for several minutes with one containing a calibrated air-speed meter. Simultaneous readings of the speed on each airplane are obtained. The accuracy of the result is, of course, limited by the original calibration of the reference instrument. Errors from other sources can be kept quite small.

(d) Instrument mounted on forward projecting rod.—Mounting a Pitot-static tube or the element of other types of air-speed meters forward of the airplane is similar in principle to the use of a suspended head. The use of a suspended head is preferable for mechanical reasons, since in the former method the element must be mounted from one and one-half to two wing chord lengths in front of the leading edge of the wing. It is used whenever the suspended head type is not practicable, as in obtaining readings when very close to the ground.

PRESSURE TUBING

In the United States copper tubing of one-fourth inch, or more commonly, three-sixteenths inch, outside diameter is used for connecting the pressure head to the indicator. Brass fittings of the type illustrated in Figure 18 are soldered to the tubes wherever connections are required. Aluminum tubing is favored in England and is sometimes used in the United States. Its only disadvantage is that the type of fittings now standard can not be used on account of the fact that soldering is not possible. Rubber tubing should never be used except for temporary installations as in flight testing.

Connecting tubing may get out of order on account of leaks or stoppage. With carefully made connections and tubing well supported so that it can not vibrate or chafe, no trouble should be encountered with leakage. Stoppage usually results from water in the lines, either blown in through the pressure head or formed by condensation. To avoid trouble from this cause, the tubing should lead upward from the pressure head and then steadily downward to a low point at the fuselage under the instrument board. A tee fitting at the low point fitted with a removable cap or a valve serves as a drain.

Inspection of the tubing is a relatively simple matter and should be done periodically. Provided the indicator case is air-tight, leaks in the lines may readily be detected. The tubes are disconnected at the fittings adjacent to the pressure head and a slight pressure is maintained on the Pitot line sufficient to make the indicator read about half of its maximum range. If the reading drops off, there is a leak in the tubing. Similarly, by applying a slight amount of suction, the static line may be tested. Pressure or suction must be applied with extreme care, as it is very easy to ruin an indicator merely by blowing too hard in the tubes.

By disconnecting the tubing near the indicator and blowing through from this point any stoppage may be detected. It is well to try the drain first, so that if any water has accumulated it will not be spread through the lines.

The time lag in indication with tubing of the diameter now standard and of the lengths commonly used is negligible under the usual conditions of use. (Reference 20.) The use of tubing of a diameter less than one-eighth inch should be avoided.
INDICATORS FOR DIFFERENTIAL PRESSURE INSTRUMENTS

A. DESCRIPTION OF TYPES OF INDICATORS

The indicating element of the differential pressure-type air-speed meter is essentially a sensitive pressure gauge. Indicators may be either liquid-column manometers or mechanical pressure gauges. In the mechanical gauges, pressure is measured by the deflection of a diaphragm or combination of diaphragms which may be metallic or nonmetallic.

LIQUID-MANOMETER TYPE

Before and during the early part of the World War the liquid manometer was extensively used in Great Britain. A few years after the war it was revived for a brief time in the United States. In its simplest forms it consists of a U-tube with the columns connected to opposite sides of the pressure nozzle. A scale graduated in speed units is mounted adjacent to one column and the height of the liquid column is an indication of the air speed.

These indicators are now obsolete except for occasional special test work. Obvious disadvantages are their fragility, large size, and susceptibility to acceleration errors.

NONMETALLIC DIAPHRAGM TYPE

In an effort to secure sufficient sensitivity when using a Pitot-static tube, the nonmetallic diaphragm gauge was introduced by the British. Favorite materials were rubber and varnished or doped silk. In instruments with rubber diaphragms, the deflection of the diaphragm was utilized to provide a measure of the air speed exactly as with metallic diaphragms (Ogilvie). The oiled-silk diaphragm on the other hand was used merely as a sort of scale pan to transfer the load to a flat spring which formed the measuring element. (Clift.) Nonmetallic diaphragms are no longer used, in the United States at least, as metallic diaphragms of ample sensitivity are available. Their chief disadvantages were the rapid deterioration of rubber with age, and the generally very large temperature errors. (Reference 12.)

METALLIC DIAPHRAGM TYPE

(a) Diaphragms.—The metallic pressure element may be in the form of a bellows in a single piece such as the sylphon or hydron type or made up of single corrugated diaphragms fastened together alternately at the center and at the rim. The simplified form consisting of two corrugated diaphragms joined at the rim is most commonly used in present-day air-speed meters except in more or less special instruments of low range.

The form of the corrugated diaphragm is based on experience rather than theory since the problem of its elastic action has not been solved. One theory indicates that the depth of the corrugations should vary, the flatter or shallower ones being located at the center and the edge of the diaphragm. So far as known, however, there is no instrument in production with a diaphragm of this form. The shape of corrugation varies considerably, each manufacturer apparently having his own ideas as to what the proper shape should be.

Nickel silver and phosphor bronze have been found by experience to be the most suitable material for diaphragms. Other materials (steel, brass, silver, etc.) have been used to a very limited extent, chiefly in experimental instruments.

Diaphragms may be made by spinning or by stamping (pressing). The latter method is preferred when the number required warrants the expense of a stamping machine, not only because it results in a more uniform product but also because of its relative cheapness. In the nature of the process it is obvious that diaphragms made from the same sheet material will vary greatly in elastic characteristics. Some method of selection is necessary. One method in use by manufacturers is to make up a large stock of diaphragms of form and material shown by experience to be satisfactory, and then to sort these into groups, depending on their deflection for a given load. When diaphragms are desired for a specific purpose, selection is made from the group most nearly approximating the desired characteristics and the mechanism is adjusted to compensate for any deviations from the desired performance. Although this method may seem crude it works very well in practice. No matter what operations may be used to form a diaphragm, internal stresses are necessarily introduced and the diaphragm is not usable without further treatment. Essentially this treatment consists of low-temperature annealing for a fairly long time, or mechanical seasoning in combination with it. Manufacturers have developed these methods empirically in the effort to meet increasingly stringent specifications and the processes used are regarded more or less as trade secrets. The results are “seasoned” diaphragms whose characteristics are not subject to changes with time, to any appreciable extent.

The present method of joining and soldering the diaphragms to form a capsule is a decided improvement. The two rims are held in contact at a point near the edge and solder applied to the rim between this point of contact and the outer edge. (See fig. 20.) This reduces the amount of solder required and eliminates stresses normal to the rim, due to the differential pressure.

The suitability of a diaphragm depends on its elastic properties, as evidenced by the pressure-deflection relation, and the effect of temperature, hysteresis, drift, etc. These will be discussed later in connection.
with the testing of indicators. The elastic properties depend on the material used, the treatment of the material in manufacture, and the form of the diaphragm.

(b) Mechanism.—The mechanism serves a threefold purpose; it converts the linear motion of the center of the diaphragm capsule to circular motion of a pointer, it provides magnification, and it secures an evenly or nearly evenly divided scale. Also there may be added arrangements for adjustment and temperature compensation. With the introduction of the present-day improved diaphragms, it has been possible to avoid complications in the mechanism, and consequently, this part of the instrument is standardized, at least in its general aspects, as far as the circular-dial type of indicator is concerned. A special form of mechanism is required for vertical-dial instruments.

Figure 19 shows the parts of an indicator mechanism of the circular dial type in diagrammatic form and Figure 20 a photographic view. Photographs of three typical instruments are shown in Figure 21. A wire bridge or metal lug B, attached to the center of the diaphragm presses against the short arm SA of a bell crank. The long arm LA of the crank works against what is essentially a cam surface on the sector S. This surface may be the curved arm of the sector, or a slot of suitable form. The sector engages a pinion on the pointer shaft on which is also fixed a restraining hairspring. Balance is secured by proper proportioning of the parts with added weights (W, for example) when required. Sometimes one of the bell-crank arms is made of a bimetallic strip to secure temperature compensation. On indicators of high range a restraining spring may be added, operating directly on the diaphragm through the lug B.

This simple mechanism has proved itself satisfactory both from the manufacturer's and the user's standpoint. It provides many possibilities for initial adjustment (by bending the bridge or any one of the lever arms) and does not get out of order easily.

In the vertical dial type of instrument (fig. 22) the mechanism is the same as far as the bell crank. This, however, instead of driving a sector drives a second bell crank to which is attached the link arrangement for changing rotational motion into the straight line motion of the tip of the pointer. The hairspring is inserted on one of the staffs of the link motion. The link arrangement is a potential source of weakness since it must be comparatively large and light and therefore is apt to be flimsy.

The development of instruments having a uniform dial size of 2¾ inches in diameter (fig. 21) was initiated...
by the United States Navy and adopted as standard by the Army and Navy air services. These instruments have met with great favor and are extensively used. The use of dials 3/4 inches in diameter is also common. Instruments with larger dials are available for passenger cabins of transport airplanes. The vertical scale instrument (fig. 22) has not definitely established itself and is not in general use. The scale is approximately 5 inches long, the face of the dial being 6 by 1 1/2 inches in size.

Common ranges are 120, 140, and 160 miles per hour with lowest speed of 40 miles per hour. Higher (250 miles per hour) and lower ranges (80 to 100 miles per hour) are available but are usually considered special. The Navy has standardized on ranges of 30 to 160 and 40 to 260 knots and the Army on ranges of 30 to 180 and 40 to 300 miles per hour. American practice tends toward scales of a single revolution or slightly less, divided in 5-mile or 5-knot intervals, whereas the English tendency is toward scales as long and open as possible even though more than one complete revolution of the pointer may be required. Night illumination is provided by radium paint on the major scale divisions, on the pointers, and on the figures, or by indirect instrument panel lighting.

One idea resulting from the more general attention paid to instrument-board arrangement is of considerable merit if the air-speed meter is being used to indicate level flight. That is to graduate the dial so that at normal flying speed the pointer will be level and pointing toward the right. Then the instrument will indicate more conveniently as a flight-attitude indicator, since if the airplane is diving the pointer moves down, and if nosing up the pointer also will move up.

The composition case, of bakelite or similar material, has almost entirely displaced the metal case with the resulting advantages of cheapness, light weight and noncorrosiveness.

**Air-Speed Recorders**

At present air-speed recorders are used in flight testing to a limited extent, and in flight research. All of those described below are designated for use with a Pitot-static tube.

The air-speed recorder developed by the National Advisory Committee for Aeronautics for use in flight investigations is shown in Figure 23. The diagram shows the details of the pressure element and of the method of recording, which is optical. The drum
containing the recording film is rotated by an electric motor, the speed of which is kept sufficiently constant by means of a centrifugal governor.

**Figure 24.** — Badin air-speed recorder

In the Badin air-speed recorder (fig. 24), the metal diaphragm capsules are inclosed in an air-tight case so as to subject them to the differential Pitot-static pressure. The stuffing box through which the motion of the capsules is transmitted is a shaft in a long bearing. The record is made on a clock-driven drum.

**Figure 25.** — Toussaint-LePère air-speed recorder modified by substituting metallic for the nonmetallic diaphragms

A Toussaint-LePère type recorder modified for the Bureau of Aeronautics by the Bureau of Standards is shown in Figure 25. The Pitot tube is connected to the lower set of capsules and the static tube to a like set above. A rigid rod connects the two sets. Relative motion of the capsules is transmitted to the recording arm through a lever pinned to the stiff rod. The record is made on a smoked chart which is attached to a clock-driven drum.

An air-speed recorder constructed at the Bureau of Standards is shown in Figure 26. The instrument is similar in principle to the Badin recorder just described. The stuffing box consists of a shaft in a long bearing but with the shaft bearing only at three narrow surfaces. It is leak-tight for differential pressures up to one inch of water. The sensitivity varies; the motion of stylus is 0.06 inch for the speed interval 0 to 20 miles per hour and 0.35 inch for 150 to 160 miles per hour. The height of the recording drum is 3½ inches and the weight of the instrument, 4½ pounds.

**Indicators of True Air Speed**

As has been explained, in order to obtain the true air speed, the indication of a Pitot-static meter must be corrected for deviation of the density of the air from the standard density. This in general involves measurement of the air pressure and temperature and the computation of the density and the correction, or more simply the use of a chart such as shown in Figure 1. Two types of differential pressure instru-
by a Venturi tube and the one marked B by an aneroid barometer mechanism.

It will be seen that the pointer B has a spiral-shaped wire fastened to it. The indicated air speed is indicated as usual by pointer A on the graduations at the outer edge of the dial. The true air speed is indicated on the dial at the intersection of pointer A and the spiral part of pointer B.

The indication of true air speed of both instruments is that based on the standard atmosphere for which values are given in Figure 3. Correct results, neglecting instrumental errors, are obtained if the temperature of the air at the standard altitude as measured by an altimeter does not differ from the standard temperature.

**B. LABORATORY TESTS OF INDICATORS OF THE DIFFERENTIAL PRESSURE TYPE**

The purpose of testing air-speed meters is twofold. In the first place the accuracy of the instrument is to...
be evaluated and a table of corrections determined. In the second place, tests furnish a measure of the mechanical fitness of the instrument as regards design, construction, and ability to withstand the conditions of service.

![Diagram of Bureau of Standards indicator of true-relative air speed](image)

The three general categories of tests which may be applied to air-speed meters are (1) flight tests, (2) wind-tunnel or whirling-arm tests, and (3) static or laboratory tests.

(1) The chief purpose of flight tests is to determine the characteristics of the pressure head as mounted on the airplane. The methods of conducting the tests have been previously described. The effects of interference by structural parts of the airplane can be determined in this way only. The tests, however, give the total errors including those due to angle of incidence (pitch), the calibration errors of the tube, and errors in the indicator in addition to the interference errors. In order to separate the errors the other types of tests are required.

(2) Wind-tunnel or whirling-arm tests, as previously discussed, are made to determine the calibration errors of the pressure head and the errors due to yaw or pitch.

(3) Laboratory tests are designed to evaluate the performance of the indicator as a pressure gauge. The following pages, in which the discussion is limited to metallic-diaphragm indicators for use with Pitot-static pressure heads, will be concerned exclusively with this type of test. With minor changes, which are obvious from the context the same testing methods may also be used for Venturi and Pitot-Venturi instruments, as the indicators are fundamentally the same.

![Testing Equipment](image)

**TESTING EQUIPMENT**

(a) Manometers.—The errors of the air-speed indicator are determined by comparing its readings at various pressures with those of a suitable water manometer. A typical arrangement for the purpose is shown diagrammatically in Figure 29. The Pitot opening of the instrument is connected to a common header which is connected to the manometer. The pressures corresponding to air speed may be obtained by a hand-operated pump or by reducing the volume of the closed system by means of a metal bellows arrangement, all as indicated in the figure or by the rubber tubing and roller shown in Figure 30.

The simplest and most convenient manometer is a single glass tube open at the top and connected at the
AIRCRAFT SPEED INSTRUMENTS

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bottom to a reservoir as shown in Figure 29. The cross section of the reservoir should be large compared to the bore of the tube. Water is commonly used as the filling liquid, but as explained below benzol is more satisfactory if the necessary additional precautions are taken. The manometer is equipped with a scale which for convenience should be graduated both in inches and in speed units.

The indicators can be read to about 0.5 mile per hour. If an accuracy of this amount is to be obtained, the manometer must be read to 0.015 inch of water at 30 miles per hour, 0.02 inch at 40 miles, and 0.10 inch at 200 miles. This sensitivity equals 3.3, 2.6, and 0.5 per cent, respectively, of the differential pressures at these speeds.

The effect of changes in temperature of the manometer and of variations in the acceleration of gravity are ordinarily neglected. The error due to variation from the standard value of gravity will usually not exceed 0.2 per cent. The standard temperature for the manometer is +15° C. For a temperature of +30° C. the error is 0.3 per cent of the differential pressure and for +40° C., 0.7 per cent. Thus these errors are usually within the over-all error of calibration of ordinary indicators.

It is desirable to increase the sensitivity of the manometer for calibrating low-range indicators. In most cases it is sufficient to use a manometer with the tube tilted 60° from the vertical. The sensitivity is thus doubled. Using benzol instead of water increases it somewhat more and at the same time gives a better defined meniscus upon which to sight. The variations in the density of benzol make it necessary to determine the density of each lot.

The Pioneer water manometer for testing air-speed indicators is shown in Figure 30. The air pressure is controlled in this apparatus by varying the volume of the closed system by means of the rubber tubing and roller shown at the side of the cistern.

(b) Temperature control apparatus.—The current Army-Navy specifications require that the indicators be given scale error tests when at temperatures of −35° and +45° C. The higher temperature is easily secured in a temperature chamber by means of an electrical heater. The lower temperature may be obtained by using (a) an ammonia compressor, (b) a carbon dioxide compressor, (c) solid carbon dioxide ("dry ice") or (d) liquid carbon dioxide. For occasional tests the use of solid carbon dioxide is very convenient provided a source of supply is available. This refrigerant is put into the temperature chamber in the same manner as ice in a refrigerator. The temperature is controlled by a variable-speed fan blowing directly on the solid carbon dioxide. A temperature chamber of this type constructed and in use at the Bureau of Standards is shown in Figure 31.

In using an ammonia compressor it is found that the rate of cooling at temperatures below about −25° C. can be substantially increased if an auxiliary blower is used in the low-pressure line just ahead of the main compressor. This arrangement as used at the Bureau of Standards practically doubles the amount of ammonia flowing through the coils.

Equipment for using liquid carbon dioxide economically was developed at the Langley Memorial Aeronautical Laboratory and is described in reference 15.

(c) Vibration board.—The instrument board of an airplane is under continuous vibration while the engine is running. The mode of vibration varies widely on airplanes of different types. Even on one and the same instrument board there may be great variations as an instrument is added or removed, or if the engine of the airplane is changed. Accordingly, the best procedure for a laboratory test is to specify a certain mode of vibration and require that all instruments be able to withstand this vibration for a specified time. This may be called the "standard vibration" and is a translational motion in a circular path in a plane 45° from
the horizontal with a diameter (or double amplitude) of \( \frac{3}{8} \) inch and frequencies varying from 1,000 to 2,000 per minute.

In cooperation with the Bureau of Aeronautics of the Navy Department, the Bureau of Standards has designed a vibration board for general use in testing aircraft instruments including the indicators of air-speed meters. A photograph of the apparatus is shown in Figure 32. It is mounted on a heavy concrete pedestal with nonferrous reinforcement to eliminate magnetic disturbances. The motor is mounted in the base of the pedestal, as shown, and drives the board by means of a long leather belt. Nonmagnetic materials are used throughout except for the ball bearings and races.

The motor drives an eccentric which is adjustable so as to secure double amplitudes up to one-eighth inch. The eccentric rotates in a ball bearing which is attached to an aluminum plate. The motion of this plate is restrained at two parallel edges by a similar plate directly beneath through ball bearings in linear races so as to obtain a linear motion in the 45° plane with respect to the lower plate. The motion of the lower plate is restrained with respect to the base of the apparatus by similar ball bearings at the two remaining edges, thus obtaining a linear motion in the 45° plane, at right angles to that of the upper plate. The top plate has a motion with respect to the base which is the resultant of the linear motion of the bottom plate.
and its own motion with respect to the bottom plate, which is the desired vibration. Two stiff brass brackets are shown in Figure 32 mounted on the top plate to which panels containing instruments can be attached. Amplitudes are measured by an Ames dial (shown in fig. 32) and frequency by a tachometer.

**Test Methods**

(a) **Order of tests on indicators.**—In making complete tests on an indicator it is essential that a prescribed order or sequence be followed so that the effects of one test on the mechanical behavior of the instrument will not influence the following test. Also when possible, the first tests should be those concerning mechanical features which are adjustable so that the instrument can be readjusted if required, without the necessity of following through the entire testing procedure. For example, it would not be advisable to run temperature tests until after scale errors had been determined at room temperature. The various tests will be described in the sequence in which it is usually preferable that they be made.

(b) **Scale error.**—This test is made to determine whether the instrument is properly adjusted and calibrated so that it indicates according to the standard pressure—speed relation, data on which are given in Table IV. Errors may be due to incorrect zero setting, poor adjustment of the multiplying mechanism, incorrect graduation, or an unsuitable diaphragm. Errors due to the first two sources are comparatively easily eliminated by simple adjustments. Those from the last sources can not be removed by adjustment.

The test is made by comparing the indication of the instruments at various pressures with the corresponding reading of a standard manometer at a normal or room temperature, usually about 20° C. (See fig. 29.) A common procedure is to obtain readings of the instrument at the pressures corresponding to even 10-mile intervals for increasing and if desired, also decreasing pressures. With pressures increasing or decreasing the the pressure should be brought up to, or down to, the desired value without passing it. The instrument should be tapped before each reading.

A satisfactory instrument should have no errors greater than 1 1/2 per cent of the maximum scale reading and over the central part of the scale the errors should not exceed 1 per cent. Figure 33 shows typical scale errors for a satisfactory instrument and also for one with excessive errors. Average scale errors (signs disregarded) for 52 instruments tested during 1929-30 at the Bureau of Standards are shown by the lowest curve.

(c) **Friction test.**—A slight amount of friction is of little importance because air-speed indicators are always subject to vibration in service. An excessive friction error, however, would indicate poor adjustment of the mechanism, which might result in sticking or jamming or, more likely, in excessive wear. On decreasing pressures an excessive frictional error might indicate a weak hairspring.

Friction errors may be determined while the scale error test is being carried out by reading the instrument before and after tapping. The difference in readings at any point of the scale should not be over 1 per cent of the maximum reading of the instrument. A second indication of friction is furnished by the motion of the pointer. The pointer should move smoothly while the pressure is being changed at a uniform rate.

(d) **Position error.**—In order to avoid errors due to acceleration effects, the mechanism of the indicator must be statically balanced for all positions. The position error test furnishes all needed data. First, the instrument is read while in the normal position. Then without changing the pressure the indicator is tipped 90 degrees to the right or left and a second reading made. It should be tapped before each reading. The maximum difference in reading at any point of the scale due to change in position should be less than 1 1/2 per cent of the maximum reading.

This test may also be run with the scale error test if only one or two instruments are being tested at the same time. Readings need be taken only at 30-mile intervals on the scale.

(e) **Leak test.**—The purpose of this test is to check the sealing of the instrument case and fittings. The case is tested by applying suction to the static connection sufficient to produce full-scale deflection and then sealing off the instrument. Any noticeable decrease in reading indicates leakage, usually at the rim of the glass dial cover. Navy specifications permit a maximum change of 2 per cent of the reading in one minute. The connections to the diaphragm capsule are tested in the same manner by applying pressure to the Pitot connection. There should be no appreciable change in reading during a period of one minute.
It is advisable to apply these tests to the indicator periodically while it is in service. The connecting tubes are disconnected at the indicator and a short length of rubber tube connected to the instrument. By sucking or blowing, according as the tube is connected to static or Pitot side, a full-scale deflection of the instrument is produced and the rubber tube is sealed as closely as possible to the instrument by means of a pinchock. It is essential that the rubber tubing be connected leak-tight, and caution must be taken not to damage the instrument by applying excessive pressure or suction.

After testing the indicator the connecting tubes should be blown out and drained if necessary. They are then reconnected to the indicator and the leak test is again applied, this time from the point where the pressure head is connected to the tubing. In this manner leaks or breaks in the tubing may readily be detected.

(f) Vibration test.—Air-speed indicators are tested for vibration on the vibration board previously described. They are mounted with their dials in the vertical plane and oriented so as to be in the normal operating position. Thus, in the plane of the instrument dial, the instrument is subjected to the full amplitude of the board horizontally and to sevenths of the amplitude vertically. In the horizontal direction, perpendicular to the dial, it receives sevenths of the full-board amplitude.

The vibration tests of air-speed indicators at the Bureau of Standards conform to those specified in type tests by the Bureau of Aeronautics. The scale errors are determined before and after the vibration test. Each instrument is subjected to three hours of vibration at a total amplitude of one-thirty-second inch and one frequency, usually 1,500 vibrations per minute. During the first hour no differential pressure is applied. During the next two hours a pressure corresponding to approximately one-half to two-thirds full-scale reading (100 and 150 knots for the Navy standard 160 and 260 knot instruments, respectively), is applied. During the test the pointer of the instrument should not oscillate more than about 1 per cent of the maximum scale reading (2 and 3 knots, respectively, for the Navy instruments mentioned above). Also the average change in scale error due to vibration should not be more than one-half of 1 per cent of the maximum-scale reading. Lastly, no screws or other parts of the mechanism should be loosened by the vibration.

(g) Seasoning test.—This test and the drift test described below furnish measures of the elastic qualities of the diaphragms or pressure element. The importance of properly seasoning diaphragms—that is, reducing the internal stresses so that their characteristics will not change with use—has been previously discussed in the paragraph on the Metallic Diaphragm.

The test consists simply of 100 successive applications of differential pressures sufficient to cause full-scale deflection. A scale-error test is made previous to and not less than an hour after the pressure applications. The average change in scale error should not exceed one-half of 1 per cent of the maximum scale reading for instruments with properly seasoned diaphragms.

(h) Drift test.—Drift is a phenomenon associated with elastic after working and manifests itself as an increase in deflection of an elastic body under constant load with time. In an air-speed indicator, drift would be evidenced by an increase in reading with time while the indicator was subjected to a constant differential pressure. An excessive drift indicates unsuitable diaphragm material or poor design or workmanship of the diaphragm capsule.

The test is made by subjecting the instrument to a pressure causing nearly full-scale deflection (usually to an indication 10 miles less than the maximum) for one hour. The reading during this time should not increase by more than two-thirds of 1 per cent. An interval of at least one hour should follow before any other tests are made.

(i) Temperature tests.—The change in reading of an air-speed indicator due to a change in temperature is partly caused by changes in the elastic moduli of the diaphragm material and partly by changes in the dimensions of the mechanism resulting from the linear expansion of its parts. By proper design the effect of linear expansion on the mechanism can be reduced to a negligible amount. In an uncompensated instrument, therefore, the change in calibration due to temperature variations is almost wholly the result of the change of the elastic moduli of the diaphragm material, and the effect of this can be predicted with a fair degree of approximation. Several methods of compensation have been used, none of which are entirely satisfactory, as evidenced by changes in design from time to time. One method consists of mounting the diaphragms on a boss of a metal with dissimilar temperature characteristics. The differential expansion of diaphragm and boss changes the form of the diaphragm and therefore its load-deflection curve. This method has not proved satisfactory, up to the present time at least, since the form of the calibration curve is changed and also an excessive zero shift may result. A more successful method is to introduce a bimetallic element into the mechanism. For example, the longer lever arm of the bell crank may be made of a bimetallic strip. The curvature of the bimetallic arm with temperature variations is utilized to change the multiplying ratio of the mechanism in the amount required.

The tests required are scale-error tests at minus 35° C. and plus 45° C., in order. The average change in scale reading should not exceed 1 per cent of the maximum scale reading.
The error in the reading of an indicator caused by the change with temperature of the elastic moduli of the diaphragm material can be calculated approximately. For this purpose it is sufficiently accurate to assume that the relation between the differential pressure, \( P \), and the reading of the indicator, \( V \), is given by

\[
P = \frac{1}{2} \rho V^2 = KV^2
\]

(10)

Also, the deflection, \( D \), of the diaphragm element may be taken as proportional to the differential pressure and inversely proportional to Young's modulus, \( E \), of the diaphragm material; that is,

\[
D = aP = \frac{b}{E}
\]

(11)

Combining these equations, the relation between the deflection and the indication is found to be

\[
V^2 = \frac{c}{E}
\]

(12)

where \( c \) is a constant.

Hence,

\[
2V \Delta V = -\frac{c}{E} \Delta E
\]

(13)

where \( \Delta V \) represents the change in \( V \) due to a small change \( \Delta E \) in \( E \). Dividing equation (13) by equation (12), we find

\[
\frac{\Delta V}{V} = -\frac{\Delta E}{2E}
\]

(14)

The temperature coefficient \( n \) of Young's modulus is defined as

\[
n = \frac{1}{E} \frac{\Delta E}{\Delta T}
\]

(15)

in which \( \Delta T \) is the change in temperature. By substitution equation (14) becomes

\[
\frac{\Delta V}{V} = -\frac{n \Delta T}{2}
\]

(16)

This shows that the proportional change in indication due to change in the elastic modulus is proportional to one-half of the temperature coefficient.

For diaphragms made of the usual materials, the temperature coefficient of Young's modulus is about \(-0.0004\) per degree centigrade. (Reference 31.) Equation (16) then reduces to

\[
\frac{\Delta V}{V} = +0.0002 \Delta T
\]

where \( \Delta T \) is in degrees centigrade.

It will be of interest to compare this result with data obtained from instrument tests. Figure 34 shows the scale errors at two temperatures of a typical air-speed indicator which passed the specifications of the Bureau of Aeronautics of the Navy Department. As the slopes of the curves are small, the ratio \( A/B \) (indicated on the graph) will be very nearly equal to \( \Delta V/V \). From the graph,

\[
\frac{A}{B} \frac{\Delta V}{V} = 0.017
\]

which should be equal to

\[
0.0002T = (0.0002) (76) = 0.015
\]

It may be concluded, therefore, that the temperature errors of this instrument are due solely to the effect of temperature on the elastic properties of the diaphragm capsule.

In Figure 35 are given the scale-error curves for an instrument which did not meet specifications. The value of \( \frac{\Delta V}{V} \) in this case is 6.5 \( \times \) 0.060 which is to be compared with the computed value of 0.015. It is evident that the design of the instrument is faulty. The most probable source of an excessive temperature error of this type is a differential expansion of the parts of the mechanism such that the shape and thereby the pressure-deflection relation of the diaphragm is changed or that the position of the diaphragm at zero pressure is changed so as to affect the action of the multiplying mechanism.

The second class of air-speed meters includes those instruments in which the motion of the air stream is utilized directly in operating some mechanical device. Both on the basis of design principles and of practical value, this class must be subdivided into two groups of quite dissimilar characteristics.
The first and most important group includes all instruments having an element which is set into rotation by the air stream. The name anemometer is frequently used in the restricted sense for these instruments. The axis of rotation of the element may be perpendicular to the direction of the air stream, as in the Robinson cup anemometer, or it may be parallel to the direction of the air stream, as in the vane or windmill form of anemometer. The unique value of these anemometers lies in the fact that, under certain conditions easily determinable, their indications are practically independent of air density and hence are directly proportional to the true air speed.

In the second group are instruments with a member or element which is deflected against a restraining spring (or gravity) by the direct impact of the moving air. Many of the very earliest forms of aircraft air-speed meters were of this type. They are usually known as pressure-plate instruments. The Robinson cup or the vane anemometer may be converted into a deflection instrument by restraining the motion of the element by a spring. Such instruments are known as bridled anemometers.

The vane or propeller type anemometer is used on airplanes only to measure the air speeds in the lower speed range (below 40 miles per hour) in which it is difficult to measure the pressure developed by a Pitot-static tube, and then only in flight testing or for some special purpose. This type of instrument is especially useful on powered lighter-than-air craft both because of the lower speeds at which they can be operated and the fact that the indicated speed is practically independent of air density.

The cup anemometer and the pressure-plate instrument are at present but little used on aircraft. The design of the latter type of instrument is usually such as to give only a qualitative indication.

B. THE VANE ANEMOMETER

GENERAL REMARKS

In any discussion of the vane anemometer two cases must be sharply distinguished. In the first case there are no forces opposing the rotation of the vanes. Practically this condition can not be completely realized, but it can be very closely approximated if the bearing friction is reduced to a very low value and if the anemometer is not required to do any work such as driving a mechanism. In the second case there exists a resisting torque. The greater part of this torque is due to some mechanism driven by the anemometer. The bearings under these conditions must necessarily be more rugged and consequently their frictional resistance will be greater.

THEORY OF THE VANE ANEMOMETER

The following theoretical treatment of the vane anemometer is due to Ower and Duncan (references 21, 79, and 81), whose papers should be consulted for a more detailed presentation. The ideal anemometer to which the discussion applies is made up of two or more thin vanes set symmetrically around the axis of rotation. At any section of a vane taken parallel to the axis of rotation, the tangent of the angle between the vane element and the axis is proportional to the distance of the section from the axis. It is assumed that the direction of the air stream is parallel to the axis of rotation.

(a) Case I. Zero resisting torque.—It is evident that the anemometer will rotate without slip and therefore the relative wind will be directed along the vane. Referring to Figure 36, $RR$ is the axis of rotation and $AA$ is a section of one vane by a plane parallel to the axis at a distance $r$.

Let $\theta$ = angle between vane element and a line parallel to the axis.

$n$ = number of vanes.

$N$ = number of revolutions in unit time.

$\omega$ = angular velocity at section $AA$.

$V$ = wind speed.

$v$ = peripheral speed at section $AA$.

$V_R$ = relative wind speed at section $AA$.

Then from the figure it is evident that

\[ v = \omega r = 2\pi r N \]

and

\[ v = V \tan \theta \]

so that

\[ V = 2\pi r N \cot \theta \]

But

\[ 2\pi \cot \theta = p, \]

where $p$ is the pitch of the vane.

Hence

\[ V = pN \] (17)
Accordingly, for this case, the number of revolutions in unit time is directly proportional to the air speed, or in other words, the total number of revolutions in any given time is directly proportional to the air distance traveled. This result is independent of the air density. The graph for equation (17), a straight line passing through the origin, is represented by curve I in Figure 37.

On the basis of such experimental results, the resultant force $F$ on the vane in a steady air stream may be written

$$F = k\phi A \rho V_R^2$$

where $k$ is a coefficient, known when $\phi$ is known. $A$ is the area of the vane. $\rho$ is the air density.

It is assumed that there is no mutual interference between the blades, that the slowing up of the air entering the vane circle may be disregarded, and that $\phi$ is sufficiently small so that it may be taken equal to $\tan \phi$.

The component of $F$ in the direction of rotation of the vane ($v$ in Fig. 38) times $r$, the radius of the center of pressure is the torque $T$. Hence, since there are $m$ vanes,

$$T = mk\phi A \rho r V_R^2 \cos(\theta - \phi + \gamma)$$

and substituting $(V^2 + v^2)$ for $V_R^2$

$$T = mk\phi A \rho (V^2 + v^2) \cos(\theta - \phi + \gamma)$$

Also, from the velocity triangle, it is found that

$$v = V \tan (\theta - \phi)$$

(b) Case II. Resisting torque not zero.---In this case the section $AA$ of a vane, Figure 38, is taken through the center of pressure of the vane, which with sufficient precision may be assumed to coincide with the geometrical center. Since the anemometer rotates with slip, the resultant wind $V_R$ will make an angle $\phi$ with the vane element. The resulting wind force $F$ will be at an angle $\gamma$ with the normal $N$ to the resultant wind. Assuming the equivalence of the vane to a flat plate, the value of $\gamma$ as a function of $\phi$ may be determined from the results of wind-tunnel tests on flat plates.

The relation between vane speed $v$ and air speed $V$ at one air density for a typical anemometer is given as curve II in Figure 37. The upper portion of the curve is practically a straight line which very nearly passes through the origin if extended. At the lower portion of the curve, the frictional torque is no longer negligible and the curve bends downward and intersects the horizontal axis. The point of intersection corresponds to the lowest air speed at which the anemometer will rotate.

Since the angle $\theta$ is a constant of the instrument which can be measured, the angle $\phi$ can be determined from equation (20) and the curve of $v$ against $V$. It is seen that $\phi = \theta$ at the air speed $V_o$ when the vane speed $v$ is just zero, that $\phi$ is large compared to $\theta$ at air speeds just greater than $V_o$, and that, for the straight line portion of the curve, $\phi$ is small compared with $\theta$.

To compute the frictional resisting torque $T$ from the above equations it is further necessary to know the values of $k$ and $\gamma$ as functions of $\phi$. These are obtained from the results of wind-tunnel tests on flat plates. (See reference 21 or 79 for details.)

### Density Effect

When instruments of this type are to be used on aircraft it is necessary to consider the effect of change in air density. Ower and Duncan (reference 81) have shown that if air speeds at two densities are chosen for a given anemometer so that the torques $T$ are equal, it follows that the angle $\phi$ is the same in the two cases and that the values $k$ and $\gamma$ are also equal.
since they are functions of $\phi$ only. At air densities $\rho_1$ and $\rho_2$ and air speeds $V_1$ and $V_2$ the torques equal

$$
T_1 = mk_1\phi_1\kappa \rho_1 (V_1^2 + \rho_1^2) \cos (\theta - \phi_1 + \gamma_1)
$$

$$
T_2 = mk_2\phi_2\kappa \rho_2 (V_2^2 + \rho_2^2) \cos (\theta - \phi_2 + \gamma_2)
$$

When $T_1 = T_2$, then $\phi_1 = \phi_2$, $k_1 = k_2$, and

$$
\rho_1 (V_1^2 + \rho_1^2) = \rho_2 (V_2^2 + \rho_2^2) \quad (21)
$$

From equations (20) and (21) it follows that

$$
\frac{V_1^2}{\rho_1} = \frac{V_2^2}{\rho_2} \quad (22)
$$

These equations can be used to obtain the air speed-vane speed relation at any density $\rho_1$ if the relation is known at density $\rho_2$. The general procedure will be described after considering the case where a straight line relation exists between the two speeds.

The straight line portion of a curve of the vane speed $v$ against the air speed $V$ (curve II Figure 37) can be represented by the equation

$$
v = p + qV
$$

in which $p$ and $q$ are constants. For a calibration at density $\rho_1$ the straight line portion of the curve is

$$
v_1 = p_1 + q_1V_1 \quad (23)
$$

and for density $\rho_2$

$$
v_2 = p_2 + q_2V_2 \quad (24)
$$

Substituting for $v_1$ and $V_1$ in equation (23) the values from equation (22) there is obtained

$$
v_2 = p_1 \sqrt{\frac{\rho_1}{\rho_2}} + q_1V_2 \quad (25)
$$

from which it follows that

$$
p_2 = \sqrt{\frac{\rho_1}{\rho_2}} p_1
$$

$$
q_2 = q_1.
$$

Thus if the relation between $v$ and $V$ is known at one air density, the straight line portion of the curve can be easily obtained for other values of the density by means of equations (23) and (25). To carry this out graphically, extend the straight line of the known $v$-$V$ relation at density $\rho_1$ so as to obtain the intercept $p_1$ on the $V$-axis. For density $\rho_2$ the intercept is $p_2 = \sqrt{\frac{\rho_1}{\rho_2}} p_1$ and the straight line part of the curve is a line through this intercept, parallel to the one for density $\rho_1$.

For the nonlinear portion of the curve at low speeds the effect of change in air density must be determined by the general relations given in equation (22). The values at low speeds so determined are of somewhat uncertain accuracy since it has not been definitely established that all of the assumptions underlying equation (22) are valid at such speeds. However, for practical purposes, in aeronautics at least, the curve so determined is useful in order to show the lower limit of the speed to which the linear relation between air speed and vane speed extends. An inspection of equation (22) shows that

$$
v_2 = \sqrt{\frac{\rho_1}{\rho_2}} v_1
$$

$$
V_2 = \sqrt{\frac{\rho_1}{\rho_2}} V_1
$$

From equation (26) it follows that a straight line through the origin of coordinates and a particular value of $v_1$ also passes through $v_2$. Further, since the slopes of the lines determined by equations (26) and (27) are the same for corresponding values of $v_1$ and $V_1$, this line also passes through $V_2$. Computing $v_2$ or $V_2$, using either equations (26) or (27), then fixes their position on the line. In a similar manner, corresponding values of $v_2$ and $V_2$ are determined for other corresponding values of $\rho_1$ and $V_1$.

Curve II, Figure 37 for density $\rho_2$, ($=0.5 \rho_1$) was constructed in the manner just outlined.

Since the calibration curve depends on the friction in the instrument, each instrument must be individually calibrated. As the friction may change with time, due to aging of the lubrication or wear in the bearings, repeated calibrations may be required at intervals. The vane anemometer is ordinarily used as an aircraft instrument only within the range for which the calibration curve is linear.

### C. Robinson Cup Anemometer

Since 1846 when the cup anemometer was developed by Robinson as an instrument for measuring wind speed, it has been the subject of continued and extensive research. This interest has been due to the fact that the cup anemometer, since its inception, has been the standard instrument for meteorological observation. Despite all efforts, only a limited success has been attained in developing the theory of the anemometer, for the complexity of the problem has prevented a complete solution. The torque required to set the anemometer in rotation is furnished by the greater resistance of the cups with concave faces toward the wind than those with convex faces toward the wind. As the anemometer rotates the cups are continually changing their orientation with respect to the wind so that the torque on any given cup, and on the instrument as a whole, is continually varying. On a single cup the torque varies irregularly between a maximum positive value and a minimum negative value in each revolution. The relative velocity varies from the inner to the outer edge of the cup, disturbances are set up by each cup and its arm which change the forces on the other cups, frictional resistances affect the motion, and at high rotational speeds the...
AIRCRAFT SPEED INSTRUMENTS

supporting system may be deformed by the stresses set up so that the shape of the anemometer is changed. Accordingly, it has been found necessary to determine by calibration the characteristics of each form and size of cup anemometer. Even so, appreciable differences in performance may be found for instruments of the same design and size if there is any considerable difference in friction.

References to some of the more important papers on the Robinson cup anemometer are given in the bibliography. Attention is directed particularly to the recent work of Patterson (reference 78) in Canada, and Ferguson and Covert (reference 76) in the United States on the three-cup anemometer.

The ratio of the air speed \( V \) to the linear speed \( v \) of the cup centers of a Robinson cup anemometer is known as the anemometer factor \( f \). Obviously it is desirable to make the factor a constant, if possible, so that

\[
V = f v
\]

The calibration curve would then be a curve passing through the origin, as curve I in Figure 38, although not necessarily a straight line as is the case for a frictionless vane anemometer.

Conceivably an anemometer with constant factor might be constructed if the proper geometrical form and dimensions could be ascertained and if the friction could be made negligible. Actually, it has been found that neither condition can be satisfied exactly. For example, for four-cup instruments of certain proportions, a relation of the form

\[
V = a + bv
\]

or

\[
f = \frac{V}{v} = b + \frac{a}{v}
\]

is found by experiment. That is, the factor varies, with the speed of the cups approaching the value \( b \) asymptotically as the speed attains large values. This relation does not hold for low air speeds. The calibration curve is similar to curve II in Figure 37, for the vane anemometer with friction, but usually with a curvature upward at low speed instead of downward.

At the present time (1931) the three-cup form of anemometer, whose design is based on the work of Patterson and Ferguson, is used by the Weather Services of the United States and Canada, while in Europe the four-cup form is used. In aircraft only the four-cup form has been used. The relative advantage of the two forms as to degree of constancy of factor \( f \) is debatable, but it appears that a constant factor \( f \) is not obtained with either.

As an aircraft air-speed meter the cup anemometer is less desirable than the vane type chiefly because it is more difficult to construct and its axis of rotation is perpendicular to the air stream which leads to difficulties in mounting on the aircraft.

Variation in air density causes a shift in the relation between cup speed and air speed which may be represented by that shown in Figure 37, curve II for \( P_1 \) and \( P_2 \). Wilke (reference 73) has derived a relation for the effect of density based on the assumption that the friction in the mechanism and the coefficients of wind resistance of the cups are independent of speed. This gives a difference in the slope of the straight line portion of the relation at two densities between cup speed and air speed. The data obtained by Pinkerton (reference 82a) on the effect of density on an anemometer driving a magneto indicates a reduction of about 5 per cent in indication for a reduction in air density to two-thirds of the normal value. The data are insufficient to establish the validity of any theoretical relation such as Wilke's but they sustain the opinion, however, that the effect for a cup anemometer driving a revolution counter is of the same order of magnitude as for the vane anemometer.

D. PRESSURE-PLATE INSTRUMENTS

In the early days of aviation the air speed indicator commonly used consisted of a hinged, flat plate exposed perpendicularly to the air stream. The force acting on the plate was balanced by means of a spring and the deflection of the plate used to indicate the air speed. The difficulty of obtaining distant indication is the chief reason for its disappearance from airplanes. At the present time, however, instruments of this type are being used to some extent on gliders, since they are reasonable in cost, sufficiently accurate for glider operation and are more suitable than the Pitot-static type instrument since indications can be obtained at much lower air speeds.

TRANSMITTING AND INDICATING ELEMENTS

A. GENERAL REMARKS

Cup and vane anemometers are essentially air logs: that is, one revolution of such an anemometer corresponds to a definite travel of the anemometer in the air stream. For an ideal anemometer this travel or distance per revolution is a constant, while for a vane anemometer with friction or a cup anemometer not having a constant factor the travel is a function of the air speed and also of the air density. If an air log is desired, a revolution counter will serve as the indicator, which preferably should be connected to the cup or vane shaft through gearing so that the indication is directly in miles (or other units of distance) rather than number of revolutions. Air logs for aircraft must be distant-indicating since the cups or vanes must be mounted in a position where they will not be affected by the propeller slip stream, or aircraft structure, which ordinarily means some distance away from the cockpit or cabin. Further, it is desirable that the wind-driven element do as little work as possible in order to reduce the effect of changes in air density.
This requires the use of an outside source of power for operating the revolution counter or indicator.

In order to indicate air speed, an anemometer must be connected with some form of tachometer or its equivalent. The necessity for distant indication and for drawing no power from the cups or vanes is the same as for the air logs. Although a variety of methods have been used, mostly electrical in nature, distant-indicating tachometers of a suitable type are difficult to design.

A number of air logs and air-speed indicators will be described which differ almost entirely in the method of obtaining distant indication. The choice of the type of wind-driven element is largely based on the relative ease in installation and simplicity in mechanical design.

Additional mile. A cam on the pointer shaft operates the counter once every revolution; that is, every 100 miles. The weight of a typical installation is approximately 2.5 lbs.

The indication is given in true air miles; i.e., practically independent of air density since the amount of work done by the vane is very small. It is, of course, obvious that in order to obtain distance flown relative to the ground the indication must be corrected for the effect of the wind velocity. In addition to possible scale errors, the indicated air miles may be in error due to the fact that it is difficult or impractical to install the transmitter in a position where it is subjected to an air stream undisturbed by the proximity of the aircraft structure. This difficulty is common to all air logs and air-speed instruments and has been discussed in the section on the "Design of Pitot-static pressure heads for aircraft." The best solution is in general to calibrate the instrument in flight for each particular type of installation.

At low air speeds such as are obtained in airships the Venturi does not develop sufficient suction. The air log has been modified for this case so that electrical instead of pneumatic transmission is used. In this form of the instrument the vane makes an electrical contact every air mile by means of suitable reduction gearing. When the contact is made, a solenoid in the indicator is energized which forces a gear forward one tooth by means of a pawl. The pointer or revolution counter giving the indication is operated by this gear. This type of instrument is not self-contained since it requires the use of a battery.

**Barr & Stroud Air Log**

This instrument, although strictly an air log (since time must also be measured in order to obtain the air speed) is designed for use in measuring the air speed.
of aircraft during tests at speeds below that at which the use of the Pitot-static tube is possible. The instrument is shown in Figure 40 and is of substantially the same design as that developed by H. L. Stevens and D. A. Jones of the Royal Aircraft Establishment. (Reference 25.) The instrument is of the suspended head type. Distant indication is secured electrically by means of a revolution counter operated by a solenoid. By means of a 20 to 1 reduction gear an electrical contact is made at the head at intervals of 20 revolutions of the windmill or vanes. The head is kept into the wind by a 6-inch ring and cross vanes.

Tests made on this instrument illustrate a point which is of importance on all vane-type instruments when used in aircraft. The instrument was tested in a wind tunnel at two temperatures differing by 26° C. and was found to have practically no change in the number of contacts per minute at an air speed of 40 miles per hour but had about 3 per cent less at 30 and 7 per cent less at 20 miles per hour at the lower temperature. This difference is most probably due to congealing of the lubricating oil, and not to the difference in the air density as in this case the number of contacts per minute would have been increased at the lower temperature.

C. CENTRIFUGAL TYPE

Examples of this type in which the wind-driven element operates a centrifugal mechanism are more or less obsolescent. In the Morell instrument (references 6 and 8) the centrifugal element is directly driven by a four-cup anemometer, and in the Horn instrument, by a propeller. The complete instrument forms an integral unit. The Morell air-speed meter was used prior to about 1922 to a very considerable extent in Germany as a service instrument on air-

planes. The disadvantages of this type of instrument in comparison with the Pitot-static or Venturi instrument are obvious. Ordinarily the "indicated" and not true air speed is desired in the operation of airplanes and further the dial is of necessity integral with the instrument and hence is out on a strut at some distance away from the pilot, out of his direct line of vision, and difficult to read under many circumstances.

For use in measuring true air speeds at low speeds this design is not desirable since true air speed is not indicated under all conditions on account of the variation in the power required to operate the tachometer.

D. COMMUTATOR TYPES

COMMUTATOR-CONDENSER TYPE

In this instrument the indication depends essentially upon the rate of charge and discharge of an electrical condenser through a milliammeter. The rate is controlled by means of a commutator rotated by a propeller in the air stream. The instrument is of the suspended head type, that is, the propeller, commutator, and condenser are installed in, or attached to, a streamlined body arranged to be lowered into a region of air which is undisturbed by the aircraft.

It is perhaps interesting to note that Robinson cups were used as the windmill in the first model of this instrument constructed at the Bureau of Standards. (Reference 80.)

The wiring diagram of the latest model of this type of air-speed meter constructed by the Bureau of Standards for the Bureau of Aeronautics of the Navy Department for use on the airship Los Angeles is shown in Figure 41. The instrument consists essentially of a battery, an indicator, and an electrical condenser E, all electrically connected in series. The commutator reverses the polarity of the mica condenser eight
times per revolution of the vane, the entire resulting charge and discharge passing through the milliammeter which serves as the indicator. The mazda lamps $M$ and resistances $B$ and $C$ form an automatic voltage regulator. Resistance $A$ serves to adjust for the proper voltage, the indication of which is obtained on the indicator when the switch is in the "check" position. Resistance $D$ is added to bring the voltage indication within the range of the indicator.

The automatic voltage regulator is of some interest. It is seen in Figure 41 that the regulator is a network composed of two tungsten lamps and two resistances. The resistance characteristic of the tungsten filament lamps is taken advantage of in a unique way. In one particular arrangement, the output voltage was 5 volts plus or minus 1.2 per cent for a variation in input voltage from 9 to 15 volts. The output voltage is low at 9 volts input, goes to a maximum at 12 volts input, and is low again at 15 volts input. The variation in output voltage is proportional to the square of the range in input voltage, thus for a range of 10.5 to 13.5 volts the output would have a variation of plus or minus 0.3 per cent. The power efficiency, defined as the power output divided by power input, is very low in the circuits thus far devised, not exceeding 2 per cent. It has been possible to raise the ratio of output to input voltage by the use of nickel filament ballast resistances, but unfortunately the change caused a sharper top on the curve of output against input voltage.

Since the contact resistance at the brushes may vary, the effect on the indication of changes in resistance in the circuit is of importance. Neglecting the inductance in the circuit, which is largely due to that of the moving coil in the indicator, the charge passing through the indicator as the commutator revolves from segment to segment is given by the well-known expression for the flow of electric charge in an electrical circuit containing a capacity and a resistance. This is

$$q = CE(1 - e^{-\frac{t}{CR}})$$

in which $q$ is the charge transferred in the circuit in the time $t$, $C$ is the capacity of the condenser, $r$ is the resistance in the circuit, and $E$ is the voltage applied at the instant from which time $t$ starts. If $t$ is the time interval of contact on the condenser, $q$ is the charge passing through the indicator per contact. It should be noted that the effective voltage, $E$, applied to the condenser is twice the voltage output of the regulator, as the polarity of the condenser is changed from complete charge in one direction to complete charge in the opposite direction. Equation (31) shows that if the quantity of charge transferred through the indicator per contact is to be practically the same for variations in the time of contact $t$ with the speed of the commutator, $e^{-\frac{t}{CR}}$ must be small. If this condition obtains the calibration curve plotted between revolutions per minute and milliamperes will be practically a straight line. It is important that even a small deviation from the straight line should not be caused by variable contact resistance, as this would cause a variable calibration. It has been found possible to construct the commutator so that its resistance does not exceed 50 ohms as determined by actual measurements during a 250-hour duration test. In addition to this resistance there is also in the circuit, 33 ohms in the moving coil of the instrument and 100 ohms added resistance to prevent sparking at the commutator when the contact is made. The total circuit resistance is thus less than 200 ohms. Assuming $r = 200$, $t = 2 \times 10^{-3}$, and $C = 0.5 \times 10^{-6}$,

$$e^{-\frac{t}{CR}} = 2 \times 10^{-9}.$$  

If the resistance should increase to 750 ohms,
$e^{-\frac{t}{\tau}} = 0.005$ and the indication would be one-half of 1 percent low. It is easily possible for carbon brushes with low-contact pressure on a silver commutator to develop a resistance as high as 4,000 ohms. Silver brushes on a silver commutator have a tendency to streak a silver deposit across the insulation short-circuiting the commutator. A combination of one silver brush and one carbon brush running in the same track on the commutator has eliminated both of these difficulties.

**Other Commutator Types**

Two types of air-speed meters, other than the commutator-condenser type, have been constructed in phase. This current drives a synchronous motor in the indicator unit, which in turn operates a centrifugal mechanism similar to that in the Morell indicator. As the windmill has to overcome the resistance of at least four brushes bearing on the commutator and a slip ring, two sets of Robinson cups are used to furnish the power so as to minimize the effect of changes in air density. The indication is sensibly...

*Figure 42.—Commutator-condenser type air-speed meter. The reel is shown in the upper left, the indicator at the upper right, and the suspended head in the lower part of the figure.*
independent of changes in electrical resistance at the brushes, provided that there is sufficient power available to drive the motor and centrifugal mechanism.

In the Stover-Lang instrument the commutator driven by the windmill closes an electric circuit once in every revolution. The current actuates a solenoid in the indicator, which by means of a pawl operates a chronometric tachometer mechanism. This adds up the number of contacts for uniform short-time intervals, usually one second. The number of the contacts, which is proportional to the speed, is indicated on the dial of the instrument. This mechanism has been extensively used for measuring the speed of aircraft engines in which, of course, the commutator is directly connected to the engine.

Neither the Favre-Bulle nor the Stover-Lang type of instrument has advanced much beyond the experimental stage. This is partly due to the small demand for a true air-speed indicator and partly due to the relative complication of the design as compared to other types.

E. MAGNETO TYPE

There is the possibility of obtaining a distant indicating air-speed indicator by having the wind-driven element operate a direct-current generator or magneto. The voltage developed is proportional to the air speed and is indicated on a suitable voltmeter. In addition to the necessity of keeping the brush friction low, there is the added disadvantage of the effect of change in the contact resistance between the brushes and commutator. A circuit with a large amount of electrical resistance must be used in order to minimize this effect, and as a result a comparatively sensitive voltmeter is required. This type of instrument has the advantage over the electrical instruments previously described in that the use of a battery is not required. The magneto instrument is being used at present to measure the wind velocity at a number of meteorological stations. The standard-type Robinson cups are used to turn the generator and a rather large-size voltmeter is used as the indicator. At the present time generators of comparatively small size and weight and voltmeters with an extreme pointer motion of 270° are available, which may make this type of instrument a practical possibility for use on aircraft.

F. FREQUENCY TYPE

An electrical frequency type air-speed meter was constructed by the Bureau of Standards in 1926 for the Air Corps of the United States Army. The instrument is shown in Figure 43. The transmitter consists of a permanent magnet type 3-phase alternating-current generator. The indicator is designed so that the alternating current produces a rotating magnetic field which exerts a torque on an aluminum rotor. This torque is balanced by a hairspring. The design has the advantage of eliminating the use of the troublesome brush and commutator arrangement, but it appears that an indicator of this type has the inherent disadvantage of requiring the use of a rotor which has an excessive inertia compared to the operating torque obtained at all but the very highest air speeds.

G. MISCELLANEOUS TYPES

One of the disadvantages of moving surface instruments is the difficulty in measuring gusts owing to their relatively large inertia. A reduced time lag in the indication can be obtained by using a pressure plate and spring combination having as high a natural frequency as possible and at the same time having the proper amount of damping. An instrument of this type has been constructed by Sherlock and Stout (reference 83) for studying wind gusts of comparatively high velocity. In order to obtain the indication, the motion of the pressure plate varies the air gap in a magnetic circuit in which the field is produced by an alternating current. The resulting changes in current must be measured in an oscillograph in order to obtain the minimum time lag, but as finally arranged a direct-current indicating instrument was used, the alternating current being first rectified by means of copper-oxide rectifiers.

An interesting method of obtaining distant indication of wind velocity was developed by Friez. A multiple blade rotor consisting of 32 aluminum blades is exposed to the wind and is restrained from rotating more than one revolution for the desired range of wind velocity by means of a suitable helical spring. The transmitter consists of a Selsyn motor arrangement. In order to obtain an indication the well-known property of motors of this type is utilized; i.e., that electrically connected rotors move in synchronism. The Selsyn motor used as the indicator is equipped with a dial graduated in wind velocity and its rotor with a pointer. It is difficult to reduce friction in the instrument so that its
effect is not excessive, and further, the indication for a
given wind speed varies periodically with the direction
of the wind in angular steps equal to the angle between
two blades. In order to operate the instrument, alter­
ning current must be available, the motors ordinarily
available being designed for 110-volt, 60-cycle, single­
phase current.

THE HOT-WIRE ANEMOMETER

GENERAL REMARKS

Since the hot-wire anemometer was first proposed
about 25 years ago by a number of investigators work­
ing independently, the instrument has been success­
fully applied to the measurement of slowly moving
air currents and fluctuations in the velocity which are
too rapid to measure by other devices owing to their
greater inertia. The hot-wire anemometer is essen­
tially a research instrument. Its lack of ruggedness
and other characteristics bar it from consideration as
a service instrument on aircraft and even as a research
instrument if the differential pressure or moving sur­
face types of instruments are suited for the purpose.

THEORY

The rate of cooling of a heated wire in an air stream
depends, among other things, upon the relative speed
of the air stream and the wire, and hence the heated
wire may be used to measure this speed. The con­
vective cooling of wires has been studied extensively,
in particular by L. V. King. (References 85, 87, and
88.) As a result of his work, King derived two ap­
proximate formulas for the heat loss, the formulas
being valid for different speed ranges. The following
notation will be used:

\[ H = \text{heat lost per unit length of wire} \]
\[ K = \text{thermal conductivity of air} \]
\[ s = \text{specific heat of air} \]
\[ \rho = \text{density of air} \]
\[ V = \text{speed of air stream} \]
\[ T = \text{temperature difference between the wire and the unheated wire} \]
\[ r = \text{radius of wire} \]
\[ \gamma = \text{Euler's constant} = 0.5771 \]

Then King's formulas are (a) for low speeds:

\[ H = 2\pi K \frac{T}{\log b/a} \]

(32)

where

\[ b = Ke^{-\gamma} (S\rho V) \]

and (b) for high speeds:

\[ H = [K + 2\sqrt{\pi K r s (\rho V)^{1/2}}] T \]

(33)

For values of \( VR \) greater than 0.00935 cm\(^2\)/sec., accord­
ing to King, the high-speed formula is to be used, and
for smaller values the low-speed formula. If we as­
sume that the smallest wire which can be used on
aircraft has a radius of 0.0005 inch, then the critical
value of the velocity \( V \) is about one-sixth of a mile
per hour. Therefore the high-speed formula is the
only one which need be considered as far as aircraft
speeds are concerned. The speed \( V \) represents the
total speed of the air stream past the wire, neglecting
the effects of convection currents set up by the heated
wire itself. These convection currents need only be
considered when very low speeds are to be measured.

If we assume that the specific heat \( s \) and the thermal
conductivity \( K \) of the air are independent of air tem­
perature and density, then equation (33) may be
written:

\[ H = A + B (\rho V)^{1/2} \]

(34)

where \( A \) and \( B \) are constants depending upon the tempera­
ture difference \( T \), the characteristics of the wire,
and physical properties of the air. This equation
brings out clearly one of the characteristics of the hot­
wire anemometer which is that the heat loss is a func­
tion of the product of the air density by the air speed.

METHODS OF MEASUREMENT

There are a number of methods of measurement
which may be used with the hot-wire anemometer.
In one of the simplest methods, the electrical resis­
tance, and therefore the temperature of the wire, is
maintained constant by means of a Wheatstone bridge,
and the consequent variations in the current furnish
the measure of air speed. The essentials of the elec­
trical circuit are shown in Figure 44 A. The hot wire,
\( H \), is connected as one arm of a Wheatstone bridge,
the other three arms of which are adjusted so that the
bridge is balanced when the resistance (temperature)
of the wire is at the desired value at zero air speed.
Any change in the air speed will change the tempera­
ture and hence the resistance of the hot wire and the
change in current required to again balance the bridge
is the measure of the air speed. The change in current
may be measured by an ammeter in series or a high­
resistance voltmeter in parallel with the wire, or by
the measurement of the current in the external circuit
as shown in Figure 44 A. The accuracy can be greatly
increased by using a potentiometer in the manner des­
cribed by Simmons and Bailey. (Reference 97
and 98.) If \( \mathcal{I} \) is the current necessary to maintain a
constant wire temperature, then as the resistance is
constant in this method, equation (34) reduces to

\[ \mathcal{I}^2 = K_1 + K_2 (\rho V)^{1/2} \]

(35)

since the heat loss is proportional to the square of the current.
Putting \( \mathcal{I}_0 \) equal to the current necessary to
maintain the temperature when \( V=0 \), then

\[ \mathcal{I}^2 = \mathcal{I}_0^2 + K_2 (\rho V)^{1/2} \]

(36)

In this formula \( \mathcal{I}_0 \) is readily measured, and a second
measurement at a known air speed \( V \) will furnish all
the data needed to determine the value of the constant $K_2$. Note that the current $i$, and therefore the deflection of the indicator is proportional to the fourth root of the air speed. The result is that the scale is very much compressed at the upper end.

In the second, or constant-voltage method (often erroneously called constant-current method) the hot wire also forms one arm of a Wheatstone bridge with a constant voltage impressed upon the bridge. The circuit is essentially the same as that shown in Figure 44 A except that the indicator $I$ is omitted and an indicator for checking the applied voltage must be provided. At zero air speed the bridge is balanced. Any change in the forced convection changes the temperature of the wire and throws the bridge out of balance. The "out of balance" current is a measure of the air speed.

Huguenard, Magnan, and Planiol (references 94 and 96) have developed a hot-wire anemometer for measuring wind velocities based on the electrical circuit shown in Figure 44 B. The hot wire $H$ is exposed to the wind stream while the galvanometer $G$ is shunted by a shielded fine platinum wire $S$. When $F$ is exposed to air of zero velocity, the voltages $B_1$ and $B_2$ are adjusted so that no current flows through $S$. The resistance $R$ is large compared with the resistance of $S$ so that the current in $S$ due to voltage $B_2$ is substantially constant. When $H$ is exposed in an air stream its temperature falls, and therefore its resistance decreases and the electric current increases. At the same time the additional current heats the wire $S$, increasing its resistance and thus shunts proportionally more of the current through the galvanometer $G$ than that due to the change in air speed. The advantage of this method of measurement is that if $H$, $S$, $B_1$, and $B_2$ are properly chosen, the current through $G$ is approximately proportional to the air speed instead of the 4th root of the air speed. An evenly divided air-speed scale is thus obtained.

Dryden and Kuethe (references 99 and 100) used the circuit shown in Figure 44 C in order to measure the integrated fluctuations in air speed in a wind tunnel in the frequency range 1 to 100 cycles per second. The hot wire is at $H$ in the figure and has a resistance which is small compared to that of the rest of the circuit so that the current through it is very nearly constant at all speeds. The potential drop across the fixed resistance $R$ is measured by a potentiometer and serves to control the current in the circuit. The alternating change in potential across the hot wire due to variations from the mean wind speed is amplified by a resistance-coupled amplifier and finally measured by an alternating current milliammeter. This current is proportional to the square root of the mean square deviation of the resistance from its mean value. The amplitude of the sine curve of resistance giving the same square root of the mean square value was computed and converted to amplitude of fluctuation in air speed by means of a calibration curve of electrical resistance against air speed. The parts of the circuit not shown are described in reference 99.

**SOURCES OF ERROR**

One inherent defect of hot-wire anemometers is the progressive change or aging of the wire which necessitates frequent recalibration. This defect is very troublesome in many cases. The aging depends upon the material of the wire, its dimensions, and the temperature at which it is used. Satisfactory materials are platinum, nickel, and gilded iron. Platinum has been used more extensively than other materials since it is practically unaffected by atmospheric conditions even at high temperatures. Nickel in a pure form is suitable for lower temperatures. The purity appears to be an important consideration. Thorough annealing is necessary.

The effect of aging is greater for fine wires. However, the fundamental advantage of the hot wire in measuring air speed is its comparatively low lag which has led to its use in measuring wind speed and to fluctuations in air speed in wind tunnels as described above. The lag decreases with decrease in the diameter of the wire, which has required in the latter case
at least that the wire diameter be as small as practicable (0.0007 inch). The necessity of low lag means a maximum aging effect. It is of interest to note that welding the wire to the support, instead of soldering, greatly reduces the variation in the calibration with time. (Reference 100.)

Since the heat loss of the hot wire depends upon the difference between the temperature of the air and the hot wire, equation (33), it is obvious that the effect of variation in the air temperature is minimized by using high-wire temperatures. This method of correction is objectionable since sufficiently high-wire temperatures greatly increase the aging effect and also increase the free-convection currents which is undesirable. It is feasible and preferable to provide compensation in the electrical circuit. (References 21 and 98.) Wire temperatures in the range 150° to 500° C. appear to be the most satisfactory.

It is seen by referring to equation (33) that the indication of the hot-wire anemometer is also a function of the air density. Changes in air density from that at the calibration require the application of a correction. No means of compensating for changes in air density have been developed as yet.

GROUND-SPEED METERS

INTRODUCTION

A. APPLICATIONS OF GROUND-SPEED MEASUREMENT

The speed of an aircraft measured relative to the earth's surface is called its ground speed. The ground speed may be measured from fixed stations on the ground or by means of instruments carried on the aircraft itself. The determination of ground speed from ground stations finds limited application in connection with flight testing, air races, and military purposes such as range finding, etc. The methods and instruments which can be used on board aircraft are of practical importance in flight testing and in navigation. The methods of measuring ground speed in flight testing have been described previously with reference to the calibration of air-speed meters. The following discussion will be limited to the measurement of ground speed on board aircraft for navigation purposes.

B. PRESENT STATUS

From the standpoint of navigation, the subject of ground-speed measurement is in a very unsatisfactory state. This condition of affairs is due to the fact that a general method, applicable under all or nearly all circumstances, has not yet been devised, and there are no indications that the problem will be solved in the near future. The methods in use are not only limited in application but are cumbersome and inaccurate. Even in the case of the most commonly adopted methods, there is no general agreement either as to the procedure to be followed or the instrumental equipment to be used. Although the ideal ground-speed meter appears to be unattainable for the present, it is greatly to be desired that by common agreement some procedure and instrumental equipment be accepted as standard to serve until something markedly better can be produced.

C. CLASSIFICATION OF INSTRUMENTS AND METHODS

It is convenient to group the methods of ground-speed measurement in two classes according as the methods are dependent on ground visibility (optical types of instruments) or independent of ground visibility (absolute types). Each class is further divisible into indirect methods which require the computation of ground speed from the results of observations on other quantities, and methods permitting the use of indicating instruments which show the ground speed on a dial or scale as soon as the observation is completed in the case of optical types, or continuously in the case of absolute meters.

DESCRIPTION OF INSTRUMENTS

A. OPTICAL TYPES

General Remarks

Instruments falling under this classification are based on two general methods. The first consists essentially of means for determining the time of flight over a known distance. The angle of drift is usually measured simultaneously. The second method, known as the wind-star method, is based on the solution of the velocity triangle from the angles of drift on two courses and the air speed.

Timing Method, Indirect

(a) Theory.—The obvious procedure, which is very useful on many occasions, is to measure the time of flight between two points on the ground whose distance can be scaled from a map, and hence derive the value of the average ground speed. It is evident that a fairly long distance must be chosen in order to reduce errors in scaling from the map. Therefore the time of flight is appreciable so that both air speed and wind speed may vary during the measurement and considerable errors may result. In order to reduce the time required to make an observation it is necessary to project a base line of known length from the aircraft to the ground and measure the time of flight over this base line. In the case of an airship such a base line is projected whenever the sun is bright enough to cast a sharp shadow of the airship on the ground. Since the sun for all practical purposes is at an infinite distance, the shadow will be the same size as the airship. Thus from the known length of the shadow and the time required for it to
pass a given point, the ground speed may be derived as follows:

Ground speed (m. p. h.) = \frac{\text{length of shadow (feet)}}{\text{time in seconds}} \times 5280

or

Ground speed (knots) = \frac{\text{length of shadow (feet)}}{\text{time in seconds}} \times 6080

The method which is generally followed in projecting a base line may be illustrated as follows: Suppose that \( A \) in Figure 45 represents a peep sight and \( B \) and \( C \) two sighting beads mounted on a frame in an aircraft. The points \( A, B, C \) lie in a vertical plane, and for convenience it is assumed that the angle \( ABC \) is a right angle, although this is not essential. It is essential, however, that the points \( B \) and \( C \) lie in the same horizontal plane. Let \( E \) and \( F \) be the points on the ground determined by the projections of the lines \( AB \) and \( AC \), respectively. Then from the figure, by similar triangles,

\[
\frac{D}{h} = \frac{H}{h}
\]

and, hence,

\[
D = \frac{d}{h} H
\]  

(37)

The ratio \( \frac{d}{h} \) is fixed by the dimensions of the sighting device and the distance \( H \) is the altitude of the airplane, which we may suppose can be determined by an altimeter, so that the timing distance \( D \) is completely determined.

Obviously, the ratio \( \frac{d}{h} \) must vary for each altitude \( H \) in order to provide a fixed timing distance. The ratio may be changed by varying either \( h \) or \( d \). The usual method in instruments is to make \( A \) (or \( C \)) movable and to provide a scale on \( AB \) (or \( BC \)) which is graduated directly in altitude.

In order to obtain ground speed, the time \( t \) is observed for the line of sight on an object on the ground to change from \( AF \) to \( AE \). If \( G \) is the ground speed,

\[
G = \frac{D}{t} = \frac{d}{h} \frac{H}{t}
\]  

(37a)

It is essential to maintain a constant air speed while measuring the time \( t \).

(b) Instruments.—Many instruments varying in design details have been based on this principle. The Pioneer speed and drift meter shown in Figure 46 is a typical modern instrument. To measure ground speed, the frame is first turned so that the long wire is parallel to the track of the aircraft over the ground. The angle between this wire and the heading (fore-and-aft axis) of the aircraft is the drift angle, which is also indicated by the instrument. The movable slider carrying a cross wire is adjusted to indicate the altitude of the aircraft so that the backward drift of an object on the ground from the front wire to the rear wire always corresponds to a fixed distance. The drift of a selected object is viewed in the eyepiece and time interval is measured by a stop watch. The particular instrument here shown is provided with two altitude scales, the one in thousand feet giving a traverse of one mile, and the other in hundred feet, a traverse of four-tenths mile. Dividing this distance by the time interval gives the ground speed.

In other designs lateral compactness has been obtained by using lenses and mirrors so that observations can be made through a small hole in the bottom of the airplane and thus obviate the necessity of leaning over the side. The instruments are in general relatively bulky and heavy. See references 16 and 30 for descriptions.

A modification of the foregoing method, but identical in principle, has also been used. Referring again to Figure 45, it is evident that

\[
D = \frac{d}{h} H = \frac{H}{\tan \alpha}
\]

(38)

Therefore we may replace the frame \( ABC \) by a telescope pivoted about a horizontal axis through \( A \) and sight first in the direction \( AC \) and then in the direction \( AB \). The angle of rotation \( \alpha \) is measured on an arc graduated in altitude units. Or, more conveniently the telescope may be fixed and a rotatable reflector placed in front of its objective. An instrument of the latter type for airship use, the Goerz-Boykov drift-and-ground speed meter, is shown in Figure 47. In operation the instrument is adjusted for the drift and a reflecting prism, rotatable by knob \( A \), is set at a predetermined angle. When a suitable object appears in the field of view of the eyepiece \( E \), the stop watch \( W \)
is started and the prism reset to an angle dependent on the initial setting. When the object reappears in the field of view, the stop watch is stopped. The ground speed is obtained from the altitude of the aircraft and the time interval with the aid of the computer C. The instrument is inherently bulky and heavy. See reference 16 for further details.

**Timing Method, Indicating**

(a) **Theory.**—Any instrument of the types described in the previous section may be, and in many cases are, converted to the indicating type merely by graduating the stop-watch dial in speed units. If more than one length of base line is to be used, then the stop watch must have a scale for each length of base line.

An indicating instrument may also be obtained in still another way. Suppose an aircraft is equipped with a telescope pointing vertically downward. If, for the moment, we think of the aircraft as stationary over a point on the ground, then by a suitable device such as a rotating reflector or refractor, we can make objects in the field of view of the telescope drift across the field. Knowing the rate of this "artificial drift," we can determine by timing when a given distance on the ground has passed a crosswire in the telescope. If now the aircraft is moving, we observe the resultant of the drift due to the ground speed of the aircraft superimposed on the artificial drift. From the rate and direction of this resultant and the artificial drift, ground speed can be determined.

Four special cases will be considered.

First, let the artificial drift be at right angles to the ground-speed drift. Then if the magnitude of the artificial drift is adjusted until the direction of the observed drift is at an angle of 45° with that of the ground-speed drift, the artificial and ground-speed drifts will be equal. Hence by attaching a properly graduated tachometer to the mechanism producing the artificial drift, an indication of ground speed may be obtained.

Second, let the artificial drift be applied to one point (any suitable marker fixed relative to the aircraft) in the field of view and a means be provided for also observing the ground-speed drift. Then the apparent motion of this point may be made to coincide with the
drift of any ground object due to the motion of the aircraft and a tachometer connected to the mechanism producing the artificial drift can be used to indicate the ground speed.

As a third and perhaps more practical case, suppose that an artificial drift equal in magnitude but opposite in direction to the ground-speed drift is produced in the telescope. Then the resultant drift will be zero, that is objects seen on the ground will appear to stand still. A tachometer, as before, suitably connected will indicate the ground speed.

In the fourth case, proposed by Gatty, the artificial drift is kept constant in rate and a simple optical system provided for obtaining equality of it and the ground-speed drift. The details of the arrangement are given below.

Obviously, some method of adjusting for the altitude of the aircraft must be an integral part of any artificial-drift instrument.

(b) Instruments.—An experimental instrument of the third type, based on a suggestion by S. H. Anderson, was developed by the Bureau of Standards for the Army Air Corps in 1924. (Reference 80.) The artificial drift is obtained by rotating a hexagonal prism about a horizontal axis at right angles to the track. The manner in which the drift is produced is shown in Figure 48 in which \( AB \) represents the ground below the aircraft and \( A'B' \) the image seen in the eyepiece. The complete instrument is shown in Figure 49. The hexagonal prism is driven by a motor and the speed of rotation measured by a tachometer graduated in ground-speed units. A gear box and associated mechanism form part of the instrument for the purpose of adjusting the speed of the prism and correcting for the altitude error by independently adjusting the speed of the tachometer.

A similar instrument, except that rotating mirrors are used instead of a rotating prism has been suggested by H. M. Sylvester.

In the Gatty instrument (reference 31a) a prismatic view of the ground is obtained from the aircraft by means of two prisms arranged as shown in Figure...
50. This obviates the necessity of leaning over the side in order to make an observation. An endless celluloid or transparent strip, shown in section at A and B in Figure 50, is turned at a constant rate of speed by clockwork. The upper half of this strip is between the prism and the eye of the observer which is at E. The entire strip is marked by equally spaced crosslines. The observation is made through a peep hole C which is adjusted up or down until the artificial drift, produced by the moving strip, and the ground speed drift are equal. A knowledge of the altitude of the aircraft, the distance AC, and the horizontal speed of the strip A then serves to give the ground speed.

The principle of operation is shown in Figure 51. In order to neutralize the ground-speed drift $G$ by the constant artificial drift $S$, the angular rate of drift as viewed at C must be the same for both. If $d$ is the distance from C to the moving strip and $H$ the altitude of the aircraft, the ground speed $G$ is given by the relation,

$$G = \frac{S}{d} H$$

(39)

In the arrangements so far produced the component of the ground speed along the axis of the aircraft is measured, and not the ground speed. By means of a rotatable reticule the drift angle is also measured. Knowing the measured component of ground speed and the drift angle, the true ground speed may be obtained from a table.

Equation (39) shows that for a given ground speed the ratio $H/d$ is a constant. A minimum value of $d$ is about 8 inches. If the minimum altitude at which observations are to be made is 500 feet, $d$ will be 16 inches at 1,000 feet and 24 inches at 1,500 feet. This limitation may be met by having two or three speeds for the moving strip.

**Sources of Error in Timing Methods**

The timing methods of determining ground speed are subject to errors arising from numerous sources, some of which may be very large even under ordinary conditions of flight.

The altitude error is undoubtedly the most troublesome. Referring again to the formula for the speed

$$G = \frac{d H}{H}$$

(37 a)

and also equation (39) for the Gatty instrument, it is evident that any error in estimating the altitude $H$ of the aircraft leads to an error of equal relative magnitude in the ground speed. The altitude $H$ is the tapeline altitude or the vertical distance of the aircraft above the surface of the ground. This can be derived from the indication of the ordinary aneroid altimeter only approximately and the process requires considerable computation. To perform the computations requires a knowledge of the altimeter characteristics, the setting of the altimeter and the barometric pressure at the point of departure, and the ground elevation under the aircraft or the barometric pressure at the ground. An additional correction for the deviation of the mean air tem-
Temperature from the standard value must also be computed and applied. Necessarily the usual procedure will be to make as good an estimate as possible on the basis of the altimeter reading only. With a sonic altimeter, or its equivalent, the procedure is simplified since correction for the instrumental errors of the altimeter itself is all that is necessary.

In addition to the altitude error, large errors may arise on account of irregular movements of the aircraft during the observation, such as changes in the air speed and deviations from a straight-line course. These appear to be a maximum when observing vertically below the airplane. Errors due to the instrument itself or the stop watch are negligible in comparison to these two sources of error.

To summarize, the direct-timing methods are inherently inaccurate chiefly on account of uncertainty in the altitude of the aircraft.

**Wind-Star Method**

(a) Theory.—If the air velocity (defined by the true air speed and the aircraft heading) and the wind velocity (defined by the strength and direction of the wind) are known, the velocity triangle can be solved and the ground velocity determined. The fundamental diagram is shown in Figure 52A. Ordinarily the wind velocity is not known, so that the method primarily involves its determination and receives its name, the wind-star method, from the procedure used in its measurement. For this purpose observations of the drift of the aircraft on two or more courses are in general required.

Referring to Figure 52A, let \( a_1, g_1, \) and \( \alpha_1 \) represent the air velocity, ground velocity, and drift angle, respectively for the first course, and \( a_2, g_2, \) and \( \alpha_2 \) the same elements for the second course. If the wind velocity is the same in each case, then the third side of each velocity triangle is represented by the same wind-speed vector, \( w \).

Then in the first triangle

\[
w^2 = a_1^2 + g_1^2 - 2a_1 g_1 \cos \alpha_1 \quad (40)
\]

and in the second triangle

\[
w^2 = a_2^2 + g_2^2 - 2a_2 g_2 \cos \alpha_2 \quad (41)
\]

Since the directions of the vectors \( a_1 \) and \( a_2 \) are known, the angle \( A \) is known, and hence

\[G + \alpha_2 = A + \alpha_1 \quad \text{or} \quad G = A + \alpha_1 - \alpha_2 \]

from which we obtain

\[m^2 = a_1^2 + a_2^2 - 2a_1 a_2 \cos A \]

\[m^2 = g_1^2 + g_2^2 - 2g_1 g_2 \cos G \]

and eliminating \( m \)

\[a_1^2 + a_2^2 - 2a_1 a_2 \cos A = g_1^2 + g_2^2 - 2g_1 g_2 \cos G \quad (42)
\]

We have, therefore, three equations (40), (41), and (42) to determine the three unknowns, \( w, g_1, \) and \( g_2 \).

In order to determine the wind direction with respect to the heading of the aircraft, \( (C \text{ or } A + C) \) in Figure 52B, a further computation must be made.

\[C = B + D_1 \quad (43)
\]

in which

\[
\sin B = \frac{a_1}{w} \sin D_1
\]

The wind velocity \( w \) being thus determined, the ground velocity can be calculated for any desired air velocity.

Practically, the problem can be solved much more conveniently by a graphical construction. First, we lay off the air-velocity vector \( a_1 \), and after measuring the drift \( D_1 \), draw a line \( MN \) of indeterminate length to represent the direction of the ground-velocity vector \( g_1 \). (See fig. 52C.) Then draw the air-velocity vector...
for the second course and the line \( mn \) in a similar manner. The wind-velocity vector is then represented by the line connecting the intersection \( P \) of the lines \( MN \) and \( mn \) with the common origin of the vectors \( a_1 \) and \( a_2 \). To obtain the proper heading for any desired course, draw the line \( PQ \) in the proper direction to represent the desired course. With radius \( a \) equal to the air speed (true) to be flown, and center \( O \), swing an arc intersecting \( PQ \) at \( R \). Then the direction of \( OR \) gives the heading to be flown, and the length of \( PR \) gives the ground speed. In practice, the air speeds are maintained constant on both courses, and the courses differ by about 90 degrees.)

The above method of determining the wind velocity can be further modified in order to eliminate the necessity of flying off the course, provided the drift is not zero. Observations of the air velocity \( a_1 \) and the drift angle \( D_1 \) are obtained as just described. Then, the air speed is changed, but the same aircraft heading is obtained, and \( a_2 \) and \( D_2 \) are measured. A graph similar to that shown in Figure 52C is obtained, except that \( a_1 \) and \( a_2 \) coincide. The method is inherently less accurate owing to the smaller angle formed by the lines \( mp \) and \( MP \) and thus the greater uncertainty in the location of \( P \). The difference in the air velocities \( a_1 \) and \( a_2 \) must be as large as possible.

(b) Types of instruments.—A number of varied instruments based on the wind-star method have been devised. The simplest instrumental equipment consists of a drift sight for measuring the drift angles and a plotting board or a computer for determining the wind velocity and then the ground speed. (It is assumed that an air-speed indicator, a correction chart to obtain true air speed, and a compass are available for determining air velocity.)

Drift sights vary a great deal in details of construction, but all are designed to measure the angle between the heading of the aircraft and the direction of drift of objects or an object on the ground. The principle is illustrated in Figure 46.

Computers are available for determining graphically the wind speed by the wind-star method (fig. 52C) and the ground speed by means of the primary velocity diagram (fig. 52A). In some forms of instruments the drift sight, either recording or indicating, and the graphical computer are combined. All of these instruments are primarily navigation instruments and are therefore not described in detail here. (See references 7 and 16.)

(c) Sources of error.—The outstanding advantage of the Wind-Star method, compared to the timing method, is its independence of the altitude of flight. The troublesome and usually large errors due to incorrect altitude estimation are thus completely wiped out. Errors in measuring drift due to motion of the airplane and errors in measuring the air velocity are the same as for the other optical methods of measuring ground speed. The disadvantages, as compared with the other optical methods, are two. In the first place, the customary necessity of flying on two separate courses results in some loss of time and is an annoyance to both the pilot and navigator; secondly, a great deal more computation is required in order to obtain the ground speed. There is further the possibility of a greater error due to variation in wind strength and velocity on the two courses.
On the whole, the advantages of the method overbalance its defects. It is regarded as the most accurate available means for determining ground speed.

### B. Absolute Types

The inherent limitation on all optical methods of measuring ground speed is that ground visibility is required. A number of ways of avoiding this limitation have been proposed, but no satisfactory substitute for the optical type of instrument has appeared.

Several indirect methods are used to a slight extent. Position may be determined astronomically or by radio bearings at successive intervals and the average ground speed for each interval thus evaluated. Errors in determining position, errors due to changes in the wind, and the difficulties of observation make these methods impracticable except under unusual conditions. A more useful procedure is to obtain radio reports of wind conditions and compute the ground speed in the manner indicated in Figure 52A. This method of obtaining wind velocity also has obvious disadvantages.

As part of a system for the blind landing of aircraft short distances from a radio beacon are indicated by measuring the drop in the plate current of a radio receiving set equipped with automatic volume control when flight is toward the beacon. (Reference 28.) The rate of change in indication gives the ground speed. This method may have a limited application.

Direct methods, involving the use of instruments indicating ground speed continuously have not been successful. Methods have been proposed depending (a) upon the measurement of the time integral of the fore-and-aft horizontal accelerations, (b) upon the electromotive force apparently obtained by cutting the vertical component of the earth's magnetic field, and (c) upon the distance determined by the interference pattern of radio waves set up between two stations transmitting at two different frequencies. Many of the arrangements which have been suggested are ingenious, but thus far all have proved to be either impractical or unsound theoretically.

### Acknowledgment

—I wish to thank Dr. W. G. Brombacher for his many valuable suggestions as to the form and content of this report, for furnishing much of the data and many references, and for his careful editing of the whole report.

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TABLE IV

<table>
<thead>
<tr>
<th>Mile per hour</th>
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<th>Incompressible</th>
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<td>1.55</td>
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<td>1.59</td>
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<tr>
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<td>1.63</td>
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<tr>
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</tr>
<tr>
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<td>2.313</td>
<td>1.83</td>
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<td>1.95</td>
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<tr>
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<td>658.0</td>
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<td>1.99</td>
</tr>
<tr>
<td>450</td>
<td>673.0</td>
<td>2.604</td>
<td>2.03</td>
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</tbody>
</table>

Note: The above table provides the differential pressures for air speeds in miles per hour. The table lists the air speed in miles per hour, the differential pressure in inches of water, and the corresponding pressure in centimeters of water. The table is useful for aerodynamic calculations and wind tunnel measurements.
TABLE V
DIFFERENTIAL PRESSURES FOR AIR SPEEDS IN KNOTS

<table>
<thead>
<tr>
<th>Air speed</th>
<th>Differential pressure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Knots</td>
<td>Inches water</td>
</tr>
<tr>
<td>0</td>
<td>0.00</td>
</tr>
<tr>
<td>10</td>
<td>10.8</td>
</tr>
<tr>
<td>20</td>
<td>21.6</td>
</tr>
<tr>
<td>30</td>
<td>32.4</td>
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<tr>
<td>40</td>
<td>43.2</td>
</tr>
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<td>50</td>
<td>54.0</td>
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<td>64.8</td>
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<tr>
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<td>80</td>
<td>86.4</td>
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<tr>
<td>90</td>
<td>97.2</td>
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</tbody>
</table>

TABLE VI—Continued
DIFFERENTIAL PRESSURES FOR AIR SPEEDS IN KILOMETERS PER HOUR

<table>
<thead>
<tr>
<th>Air speed</th>
<th>Differential pressure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Knots</td>
<td>Inches water</td>
</tr>
<tr>
<td>0</td>
<td>0.00</td>
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<tr>
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<td>32.4</td>
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<td>43.2</td>
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<td>86.4</td>
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<tr>
<td>90</td>
<td>97.2</td>
</tr>
</tbody>
</table>

TABLE VI
DIFFERENTIAL PRESSURES FOR AIR SPEEDS IN KILOMETERS PER HOUR

<table>
<thead>
<tr>
<th>Air speed</th>
<th>Differential pressure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Knots</td>
<td>Inches water</td>
</tr>
<tr>
<td>0</td>
<td>0.00</td>
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<td>75.6</td>
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<td>80</td>
<td>86.4</td>
</tr>
<tr>
<td>90</td>
<td>97.2</td>
</tr>
</tbody>
</table>
Positive directions of axes and angles (forces and moments) are shown by arrows.

<table>
<thead>
<tr>
<th>Axis</th>
<th>Force (parallel to axis) symbol</th>
<th>Moment about axis</th>
<th>Angle</th>
<th>Velocities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Designation</td>
<td>Symbol</td>
<td>Designation</td>
<td>Symbol</td>
<td>Positive direction</td>
</tr>
<tr>
<td>Longitudinal</td>
<td>X</td>
<td>Rolling</td>
<td>L</td>
<td>Y → Z</td>
</tr>
<tr>
<td>Lateral</td>
<td>Y</td>
<td>Pitching</td>
<td>M</td>
<td>Z → X</td>
</tr>
<tr>
<td>Normal</td>
<td>Z</td>
<td>Yawing</td>
<td>N</td>
<td>X → Y</td>
</tr>
</tbody>
</table>

Absolute coefficients of moment

\[ C_l = \frac{L}{q b S} \quad C_m = \frac{M}{q c S} \quad C_n = \frac{N}{q b S} \]

(rolling) (pitching) (yawing)

4. PROPELLER SYMBOLS

- \( P \): Power, absolute coefficient
- \( C_p = \frac{P}{\rho n^3 D^5} \)

- \( C_s \): Speed-power coefficient
- \( \eta = \sqrt{\frac{q V^3}{P n^2}} \)

- \( n \): Revolutions per second, rps
- \( \Phi \): Effective helix angle
- \( \Phi = \tan^{-1} \left( \frac{V}{2\pi n} \right) \)

5. NUMERICAL RELATIONS

- 1 hp = 76.04 kg·m/s = 550 ft·lb/sec
- 1 metric horsepower = 0.9863 hp
- 1 mph = 0.4470 mps
- 1 mps = 2.2369 mph

- 1 lb = 0.4536 kg
- 1 kg = 2.2046 lb
- 1 mi = 1,609.35 m = 5,280 ft
- 1 m = 3.2808 ft