NATIONAL ADVISORY COMMITTEE
FOR AERONAUTICS

REPORT No. 477

EFFECT OF VISCOSITY ON FUEL LEAKAGE BETWEEN LAPPED PLUNGERS AND SLEEVES AND ON THE DISCHARGE FROM A PUMP-INJECTION SYSTEM

By A. M. ROTHROCK and E. T. MARSH

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AERONAUTIC SYMBOLS

1. FUNDAMENTAL AND DERIVED UNITS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Metric</th>
<th>English</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>l</td>
<td>meter</td>
</tr>
<tr>
<td>Time</td>
<td>t</td>
<td>second</td>
</tr>
<tr>
<td>Force</td>
<td>F</td>
<td>weight of 1 kilogram</td>
</tr>
<tr>
<td>Power</td>
<td>P</td>
<td>horsepower (metric)</td>
</tr>
<tr>
<td>Speed</td>
<td>V</td>
<td>kilometers per hour</td>
</tr>
<tr>
<td></td>
<td></td>
<td>meters per second</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Unit</th>
<th>Abbreviation</th>
<th>Symbol</th>
<th>Unit</th>
<th>Abbreviation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>W</td>
<td>Weight</td>
<td>mg</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>g</td>
<td>Standard acceleration of gravity</td>
<td>9.80665 m/s² or 32.1740 ft./sec.²</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>m</td>
<td>Mass</td>
<td>W/g</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I</td>
<td>Moment of inertia</td>
<td>mk²</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. GENERAL SYMBOLS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>v</td>
<td>Kinematic viscosity</td>
</tr>
<tr>
<td>ρ</td>
<td>Density (mass per unit volume)</td>
</tr>
<tr>
<td>g</td>
<td>Standard density of dry air, 0.12497 kg/m³-s² at 15°C and 760 mm; or 0.002378 lb.-ft.-⁴ sec.²</td>
</tr>
<tr>
<td>Specific weight of &quot;standard&quot; air, 1.2255 kg/m³ or 0.07651 lb./cu.ft.</td>
<td></td>
</tr>
</tbody>
</table>

3. AERODYNAMIC SYMBOLS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>Lift, absolute coefficient</td>
</tr>
<tr>
<td>D</td>
<td>Drag, absolute coefficient</td>
</tr>
<tr>
<td>Dₚ</td>
<td>Profile drag, absolute coefficient</td>
</tr>
<tr>
<td>Dᵢ</td>
<td>Induced drag, absolute coefficient</td>
</tr>
<tr>
<td>Dₛ</td>
<td>Parasite drag, absolute coefficient</td>
</tr>
<tr>
<td>C</td>
<td>Cross-wind force, absolute coefficient</td>
</tr>
<tr>
<td>R</td>
<td>Resultant force</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>iₛ</td>
<td>Angle of setting of wings (relative to thrust line)</td>
</tr>
<tr>
<td>iₛₜ</td>
<td>Angle of stabilizer setting (relative to thrust line)</td>
</tr>
<tr>
<td>Q</td>
<td>Resultant moment</td>
</tr>
<tr>
<td>Ω</td>
<td>Resultant angular velocity</td>
</tr>
<tr>
<td>Ven</td>
<td>Reynolds Number, where l is a linear dimension (e.g., for a model airfoil 3 in. chord, 100 m.p.h. normal pressure at 15°C, the corresponding number is 234,000; or for a model of 10 cm chord, 40 m.p.s. the corresponding number is 274,000)</td>
</tr>
<tr>
<td>Cᵥ</td>
<td>Center-of-pressure coefficient (ratio of distance of c.p. from leading edge to chord length)</td>
</tr>
<tr>
<td>α</td>
<td>Angle of attack</td>
</tr>
<tr>
<td>ε</td>
<td>Angle of downwash</td>
</tr>
<tr>
<td>αₚ</td>
<td>Angle of attack, infinite aspect ratio</td>
</tr>
<tr>
<td>αᵢ</td>
<td>Angle of attack, induced</td>
</tr>
<tr>
<td>αₛ</td>
<td>Angle of attack, absolute (measured from zero-lift position)</td>
</tr>
<tr>
<td>γ</td>
<td>Flight-path angle</td>
</tr>
</tbody>
</table>
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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

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SUMMARY

Test data and analysis show that the rate of fuel leakage between a lapped plunger and sleeve varies directly with the density of the fuel, the diameter of the plunger, the pressure producing the leakage, and the cube of the mean clearance between the plunger and sleeve. The rate varies inversely as the length of the lapped fit and the viscosity of the fuel. With a mean clearance between the plunger and sleeve of 0.0001 inch the leakage amounts to approximately 0.2 percent of the fuel injected with gasoline and as low as 0.01 percent with diesel fuel oils. With this mean clearance an effective seal is obtained when the length of the lap is three times the diameter of the lap. The deformation of the sleeve and plunger under pressure is sufficient to change the rate of leakage appreciably from that which would be obtained if the clearance was constant under pressure. The rates of fuel injection with a commercial fuel-injection pump showed little variation for a range of fuel viscosities from that of gasoline to that of S.A.E. 80 lubricating oil.

INTRODUCTION

Hydraulic injection systems for fuel-injection engines of either compression-ignition or spark-ignition type generally employ a lapped plunger moving in a sleeve to create the necessary hydraulic pressure for injecting the fuel into the combustion chamber or cylinder. With the use of fuel injection in the gasoline spark-ignition engine the range of the viscosities of the fuel used in fuel-injection systems has been materially increased. Consequently, it becomes desirable to know the effect of fuel viscosity, clearance between plunger and sleeve, and length of lapped surface on the leakage of fuel in the injection system.

The leakage may be objectionable for the following reasons: First, it represents a loss of fuel; second, leakage may result in uneven distribution of fuel to the different cylinders of the engines; third, leakage of fuel into the camshaft case of the pump may dilute the lubricating oil and prevent proper lubrication of the pump; fourth, leakage from the injection valves having spring-loaded lapped stems may necessitate the installation of a return tube from the injection valve to the fuel tank to prevent the fuel from wetting the engine. For these reasons the National Advisory Committee for Aeronautics has conducted an investigation to determine the effect of fuel viscosity and plunger and sleeve dimensions on the leakage between lapped plungers and sleeves. In addition, data have been obtained on the effect of fuel viscosity on the rates of fuel discharge from an injection system. This investigation is a part of the general investigation of fuel-injection systems being conducted by the Committee. (See references 1, 2, and 3.)

Previous tests on fuel leakage were conducted by Alden (reference 4) and reported in 1928. In this report Alden suggested the possibility of specifying the necessary clearance by an equation in the form

\[ d = CH_p^wD^p \]

in which

- \( d \), mean clearance between plunger and sleeve
- \( C \), constant
- \( H \), product of length of stroke by number of strokes per minute
- \( p \), density of the liquid
- \( D \), diameter of plunger
- \( p \), pressure differential through lapped fit

A sixth term must be included in the right-hand member of the equation—\( \mu \), the viscosity of the fuel. Experience at this laboratory has indicated that the product of the length of the stroke by the pump speed is not so important from the consideration of plunger seizure as the maximum plunger velocity. In the test results obtained and reported herein no attempt has been made to determine the clearance necessary to prevent seizure of the plunger, but the relationship between all the other factors, including the viscosity, to the clearance has been determined from the consideration of the amount of fuel that leaks between the plunger and the sleeve.
APPARATUS AND PROCEDURE

The tests were divided into three parts: (a) Plungers and sleeves from different fuel-injection pumps were mounted in a block and maintained under a constant hydraulic pressure so that there was a pressure gradient along the lapped surfaces which varied from the constant hydraulic pressure to atmospheric pressure and the rate of leakage between the plunger and sleeve was measured; (b) automatic injection valves were operated from an injection pump, and the amount of leakage between the injection-valve stem and sleeve was measured; (c) the rates of discharge were obtained from an injection system operating with liquids of different viscosities.

The apparatus used in the first group of tests is shown in figure 1. The sleeve was mounted in a special block containing a small reservoir. The plunger was inserted in the sleeve and held in place as shown, so that there was no motion of the plunger during the test. The liquid was supplied to the small reservoir from a larger reservoir in which the liquid was maintained under pressure by means of a hand pump. The large reservoir had a volume of 20 cubic inches in order to keep the hydraulic pressure constant within the precision to which the gages could be read. The quantity of liquid leaking between the plunger and sleeve was caught in a glass beaker and weighed. At the start of each test, care was taken to remove all air from the system.

The first drops of liquid that leaked past the plunger and sleeve were not caught because with some of the plungers and the less viscous liquids leakage started before the test pressure was reached. When the test pressure was obtained, zero time reading was taken as the first drop of liquid fell. Except in those tests for which the leakage was rapid, the liquid was caught for a time interval of 5 minutes. The constancy of the leakage (as shown by fig. 2) allows this short time interval to be used. The leakage used for this determination was slightly higher than that plotted in the following figures, owing to a higher oil temperature. Intervals were also measured between individual drops to check the constancy of the rate of leakage. The important physical constants of the six liquids tested are given in the following table.

PROPERTIES OF LIQUIDS USED

<table>
<thead>
<tr>
<th>Liquid</th>
<th>Viscosity, poises</th>
<th>Specific gravity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aviation gasoline, standard grade</td>
<td>0.0042</td>
<td>0.74</td>
</tr>
<tr>
<td>Hydrogenated safety fuel</td>
<td>0.0045</td>
<td>0.87</td>
</tr>
<tr>
<td>Water</td>
<td>0.0065</td>
<td>1.00</td>
</tr>
<tr>
<td>Laboratory Diesel fuel</td>
<td>0.022</td>
<td>0.85</td>
</tr>
<tr>
<td>Auto Diesel fuel</td>
<td>0.10</td>
<td>0.80</td>
</tr>
<tr>
<td>S.A.E. 30 lubricating oil</td>
<td>3.07</td>
<td>0.90</td>
</tr>
</tbody>
</table>

The effect of pressure on the viscosity of three of the liquids is shown in figure 3. A rolling-ball viscosimeter was used to obtain the data for the two fuels. The viscosimeter was not calibrated over the range necessary to obtain viscosity curves of the gasoline and S.A.E. 30 lubricating oil. Figure 4 shows the esti-
EFFECT OF VISCOSITY ON FUEL LEAKAGE BETWEEN LAPPED PLUNGERS AND SLEEVES

mated diameters of the injection tubes necessary to insure laminar flow for a constant volume rate of flow with the different viscosities used in the tests. The values are for a 0.020-inch discharge orifice and a pump speed of 750 r.p.m., and are based on the dimensions of the injection pump used in the rate-of-discharge tests. This pump and the method by which the computations were made are described in reference 1. The curve shows that to insure laminar flow in the injection tube with the lighter liquids, tube diameters too large to be practicable would be necessary. With the lighter liquids the friction losses even with turbulent flow are sufficiently small, however, that they need not be considered.

All liquids except safety fuel were tested in six plungers and sleeves to determine the effects of pressure, viscosity, plunger diameter, length of lap, and clearance of lap on the leakage. The plungers are listed below in the order of size and manufacturer's number.

<table>
<thead>
<tr>
<th>Manufacturer's no.</th>
<th>Plunger and sleeve no.</th>
<th>Outside diameter of plunger</th>
<th>Mean diameter of plunger and sleeve</th>
<th>Length of lap</th>
<th>Material, (a) plunger (b) sleeve</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>Inch 0.725</td>
<td>Inch 0.297</td>
<td>25/32s</td>
<td>(a) Steel, (b) steel.</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>0.625</td>
<td>0.297</td>
<td>25/32s</td>
<td>(a) Steel, (b) phosphor bronze.</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>0.625</td>
<td>0.441</td>
<td>25/32s</td>
<td>Do.</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>0.784</td>
<td>0.354</td>
<td>25/32s</td>
<td>(a) Steel, (b) steel.</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>0.784</td>
<td>0.472</td>
<td>25/32s</td>
<td>Do.</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 5 shows the form of the plungers and sleeves and the method by which they were mounted. (See also fig. 1.) Plunger 3 was in the same position relative to its sleeve as plunger 2, and plunger 5 was in the same position as plunger 4 when the leakage tests were made. With the method used in clamping sleeves 2 and 3, the final lapping of the plunger and sleeve was completed after the sleeve was assembled in the block.

Plunger and sleeve 1 had been used in a fuel-injection pump for a few hours with Laboratory Diesel fuel before the leakage tests were made. Plunger and sleeve 2 had been used for about 750 hours with the diesel fuels, and plunger and sleeve 3 for about 75 hours with the same fuels. Plunger and sleeve 4 had been used for approximately 30 hours with gasoline and with hydrogenated safety fuel; plunger and sleeve 5 about 5 hours; and plunger and sleeve 6 approximately 200 hours, both with the same fuels as 4. No difficulties had been experienced with any of the assemblies. In most of these previous tests the fuel pumps were run at 750 r. p. m.
As seen from the tabulated values, plunger 4 is elliptical in cross-section. This lack of circularity is shown more definitely in the second set of measurements than in the first. This difference is not caused by discrepancy in the measurements but by the fact that although in each set of measurements the diameters were taken at right angles to each other, those in the two sets were not necessarily taken in the same position. The dimensions of the second set were apparently taken close to the major and minor axes of the ellipse and those of the first at about 45° to these axes. The sleeve is circular within 0.000005 inch, that is, within the precision of the measurements. Plunger 5 shows a very high degree of circularity, the diameters taken at right angles to each other never varying by more than 0.000003 inch. Sleeve 5, however, shows a maximum difference in diameters in the same plane of 0.000025 inch. With the exception of this one position, and another in which the difference reaches 0.000010 inch, the recorded difference does not exceed 0.000004 inch. Plunger 3 shows a maximum difference in diameters of 0.000010 inch and sleeve 3, 0.0000012 inch.

Tests were also made to determine the rate of leakage past the plungers and sleeves of the port-controlled injection pump (fig. 6) used for the tests reported in references 1, 2, and 3. In the second series of tests (in which the leakage from an automatic injection valve was measured) two different injection valves were connected to the same fuel-injection pump. The dimensions of the valve sleeves were as follows:

<table>
<thead>
<tr>
<th>Injection valve no.</th>
<th>Diameter of sleeve</th>
<th>Length of lap</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td>0.132</td>
<td>.180</td>
</tr>
<tr>
<td>15-C</td>
<td>0.142</td>
<td>.189</td>
</tr>
</tbody>
</table>

Both injection valves were spring loaded and hydraulically operated by means of the fuel pressure (fig. 7). The fuels used were hydrogenated safety fuel and AUTO Diesel fuel. The maximum injection pressure was measured with a check valve mounted in the injection tube close to the injection valve.

The rate-of-discharge data were obtained with the apparatus described in reference 2. This apparatus
consists of a slotted disk that intercepts the fuel discharged from the injection valve during 1° of pump rotation. Means are provided for changing the phase relation of the disk with respect to the pump camshaft. Gasoline was omitted in these tests because of the fire hazard presented in conjunction with the electrically operated timing apparatus.

The values of the leakage as obtained were subject to evaporation errors which were not constant because of the variations in time, amount of liquid leaked, amount of surface exposed to the air, and air temperature. The following table shows the values for evaporation from the beaker with different liquids. It was impossible to estimate the quantity of liquid evaporating from the plunger at the low-pressure end or from the liquid as it fell from the plunger to the beaker. As the data show, the error caused by evaporation affects only the curves for gasoline. In the examination of data presented later, it will be noticed that the leakage of the gasoline was both greater and less than that of water. For those plungers and sleeves in which the leakage rate was greatest the gasoline showed leakage values less than those for water, whereas with those cases with the slowest rate of leakage the rate with gasoline was in general greater than that for water. The indication is that if a correction for evaporation could be made, the gasoline in each case would have shown more leakage than the water.

<table>
<thead>
<tr>
<th>Fluid</th>
<th>Weight of fluid in beaker</th>
<th>Quantity evaporated</th>
<th>Beaker remaining stationary for 5 minutes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Grams</td>
<td>Grams</td>
<td>Grams</td>
</tr>
<tr>
<td>Gasoline</td>
<td>30.26</td>
<td>30.27</td>
<td>0.13</td>
</tr>
<tr>
<td>Water</td>
<td>20.04</td>
<td>19.27</td>
<td>0.77</td>
</tr>
<tr>
<td>Auto Diesel</td>
<td>50.04</td>
<td>49.99</td>
<td>0.00</td>
</tr>
<tr>
<td>S.A.E. 30 lubricating oil</td>
<td>47.37</td>
<td>47.37</td>
<td>0.00</td>
</tr>
</tbody>
</table>

**RESULTS AND DISCUSSION**

Figure 8 shows the variation of leakage with pressure for the six plungers and sleeves for all the liquids except the hydrogenated safety fuel. In all cases it was necessary to express the leakage of the S.A.E. 30 lubricating oil by the enlarged ordinates. With plunger and sleeve 1, the relation between the rate of leakage and the pressure is linear except for the S.A.E. 30 lubricating oil. The same is, in general, true for plunger and sleeve 2, with the exception of an irregularity shown for water. The reason for this irregularity cannot be explained. Water was the only liquid for which the data could not be reproduced within the accuracy of the curves. The two curves for water in figures 8 (b) and 8 (c) show the data for separate tests. For plunger and sleeve 3, with all liquids the rate first increased and then decreased, reaching a maximum between pressures of 1,500 and 3,000 pounds per square inch. For plungers 4, 5, and 6, the percentage increase in the leakage was in every case greater than the percentage increase in pressure. Plunger and sleeve 1 showed the greatest rate of leakage, 6 was next and 4 third. The other three showed leakages of the same order of magnitude. The high rate of leakage with plunger and sleeve 1 can only be explained by a poor lapped fit, since they had been used for only a short time. With plungers and sleeves 4, 5, and 6, the order of leakage varied directly with the time of operation. It is probable that the change in the leakage rates was caused by operating with the hydrogenated safety fuel and the gasoline which have lubricating qualities inferior to diesel fuel oil. From the analysis presented later, it can be shown that the mean clearance with plunger and sleeve 6 was approximately twice that of 5. Plunger and sleeve 2, which had the greatest amount of running time, but always with diesel fuel, showed one of the smallest rates of leakage.

The rate of flow of a liquid between a lapped plunger and sleeve depends on the clearance, the viscosity and density of the liquid, the length of lapped surfaces, and on the hydraulic pressures causing the flow. Assuming that the mean clearance between the plunger and sleeve is sufficiently small so that the surfaces of the plungers and sleeves can be considered as two stationary parallel planes (this assumption is justified because the ratio of the sleeve diameter to the mean clearance is of the order of 10^3) the mean velocity \( v_a \) of the flow between the two surfaces can be expressed by

\[
v_a = \frac{p D^3 g}{12 \mu L} \tag{1}
\]

in which

- \( p \) is pressure differential through lapped fit
- \( d \), mean clearance (one half of difference between mean diameters)
- \( \mu \), coefficient of viscosity of the liquid
- \( L \), length of the lapped fit

Since

\[
v_a = \frac{m}{\pi \rho D d}
\]

in which \( \pi D d \) represents the area of flow and \( m \) the mass rate of flow. The mean clearance can be determined from

\[
d = \frac{s}{12 \frac{\mu}{\pi \rho D p}} L m \tag{2}
\]

or

\[
m = \frac{\mu}{12 \frac{D p d^3 g}{L \mu}} \tag{2a}
\]

in which

- \( D \) is the plunger diameter
- \( \rho \), density of liquid
Figure 8.—Effect of pressure on the leakage of lapped plungers and sleeves.
The analysis shows that leakage varies inversely with the viscosity but directly with the pressure and with the cube of the clearance. As the viscosity of liquid fuels increases with pressure, the percentage increase in actual leakage should increase at a slower rate than the percentage increase in pressure except for the expansion of the sleeve and contraction of the plunger. If the values of equation (2) are in the following units:

\[ \mu, \text{ lb in.}^{-1} \text{sec.}^{-1} \]
\[ L, \text{ in.} \]
\[ m, \text{ lb sec.}^{-1} \]
\[ \rho, \text{ lb in.}^{-2} \]
\[ D, \text{ in.} \]
\[ p, \text{ lb in.}^{-2} \]
\[ d, \text{ in.} \]

the equation becomes

\[ d = 0.215 \left( \frac{\sqrt[3]{\mu L m}}{\rho D p} \right) \]  

(3)

The numerical value of \( \mu \) in units of lb in.\(^{-1}\) sec.\(^{-1}\) is equal to \( 5.6 \times 10^{-5} \) \( \times \) the numerical value of the viscosity in poises. If the liquid used is water at a temperature of 60° F., equation (3) becomes

\[ d = 0.025 \left( \frac{\sqrt[3]{L m}}{D p} \right) \]  

(3a)

The general shape of the curves in figure 8 can be explained from equation (2) and from an analysis of the conditions under which the plungers were mounted. With plunger and sleeve 1 the hydraulic seal was made at the outer shoulder of the sleeve. The port in the sleeve was plugged so that the lapped length consisted of the distance between the last port and the end of the sleeve. When the hydraulic pressure was applied a pressure gradient was produced between the plunger and sleeve in the direction of the plunger axis so that, although the hydraulic pressure at the start of the lapped fit was the recorded pressure, the pressure at the outer end of the lapped fit was atmospheric. Consequently, the force varied in intensity from a maximum at the inner end of the lap to a minimum of zero at the outer end of the lap. This force tended to expand the sleeve and contract the plunger. In the hollow portion of the plunger, however, the full hydraulic pressure was maintained, tending to expand the plunger.

With plungers and sleeves 2 and 3 there was a pressure between the plunger and sleeve tending to contract the plunger and expand the sleeve, but there was also a pressure on the outside of the inner section of the sleeve tending to contract the sleeve because the hydraulic seal was maintained, not at the inner end of the sleeve, but at the position between the threaded portion and the collar. With plungers and sleeves 4, 5, and 6 there was only the hydraulic force tending to contract the plungers and expand the sleeves. With all the plungers the hydraulic force on the top of the plunger tended to expand the plunger opposing the contraction caused by the radial force.

Using the equations given by Timoshenko (reference 6, p. 533), the deformation of the sleeve or plunger is given by

\[ \delta = \frac{D p}{2E} \left[ \frac{1}{E_1} \left( \frac{n^2 + 1}{n^2 - 1} \right) + \frac{1}{E_2} \left( 1 - 3\sigma_2 \right) \right] \]  

(4)

in which

\[ \delta, \text{ deformation} \]
\[ \sigma_1, \text{ Poisson's ratio} \]
\[ E_1, \text{ modulus of elasticity} \]
\[ r_1, \text{ radius of plunger or inner radius of sleeve} \]
\[ r_2, \text{ outer radius of sleeve} \]
\[ r, \text{ radius of that point at which deformation is being considered} \]
\[ p, \text{ mean pressure between plunger and sleeve} \]
\[ p, \text{ mean pressure on outer surface of sleeve} \]

The radial deformation of the plunger caused by the pressure on the end of the plunger is

\[ \delta = \frac{D p \sigma}{2E} \]  

Therefore, determining the expansion of the sleeve at its inner surface and the contraction or expansion of the plunger at its outer surface, and expressing the ratio \( r_2/r_1 \) by \( n \), the mean increase in clearance produced by a pressure \( p \) at the high-pressure end becomes

\[ \delta = \frac{D p}{4} \left[ \frac{1}{E_1} \left( \frac{1 + n^2}{n^2 - 1} \right) + \frac{1}{E_2} \left( 1 - 3\sigma_2 \right) \right] \]  

(5)

in which the subscript 1 denotes the material from which the sleeve is made and the subscript 2 the material from which the plunger is made.

With the mountings similar to those used for plungers and sleeves 2 and 3 the difference between the contraction of the sleeve and the contraction of the plunger is

\[ \delta = \frac{D p}{4} \left[ \frac{1}{E_1} \left( \frac{1 - 3n^2}{n^2 - 1} \right) + \frac{1}{E_2} \left( 1 - 3\sigma_2 \right) \right] \]  

(6)

In the derivation of equations (5) and (6) it has been assumed that the mean pressure acting between the sleeve and the plunger is one half the pressure acting on the outside of the sleeve, which is the total hydraulic pressure. This assumption is made because there is a pressure gradient from \( p \) to 0 between the plunger and sleeve in the direction of the axis, whereas the full pressure \( p \) is exerted on the outer surface of the sleeve from the high-pressure end to the surface of seal. In equation (5) the deformation \( \delta \) is positive, indicating an increase in mean clearance, whereas in equation
computations it was assumed that the effective lap fit ended at the point d for plunger and sleeve 3. This assumption was made for two reasons: As the tabulated dimensions show, the clearance increases continually beyond this point; since the pressure drop varies as the square of the clearance, most of the resistance to flow occurs in those positions at which the clearance is small. The particular point d was chosen because it is the extreme point at which the pressure is applied to the outside surface of the sleeve. Therefore, before this point the deformation caused by the hydraulic pressure decreases the clearance, the deformation beyond this point increases the clearance. Both curves in figure 9 (a) show that the clearance is decreased proportionally to the increase in the hydraulic pressure. The difference between the two curves is caused by the assumptions on which the calculations are based, and by the fact that due to irregularities in the clearance, the pressure gradient in the direction of the plunger axis is not constant. In addition, for plunger and sleeve 3 the deformation of the sleeve caused by the assembly of the sleeve in the block does result in a smaller initial clearance according to equation (6).

With both plungers 4 and 5 the clearance, whether computed by equation (2) or (5), shows a steadily increasing value as the pressure is increased. The agreement between the two methods of computation was better with these plungers and sleeves than was the case with plunger 3. The closer agreement is to be expected because with plungers and sleeves 4 and 5 the whole length of the lapped fit could be considered. The curves in figures 8 and 9 show that a slight increase or decrease in the mean clearance between a lapped plunger and sleeve materially changes the rate of leakage.

The mean clearances between the two plungers and the sleeve tested by Alden (reference 4) were 0.000038 inch and 0.000040 inch, respectively. These values are approximately one half of those given for plungers and sleeves 4 and 5. Alden suggests as an acceptance test for plungers and sleeves that the rate of air leakage be not greater than four well-defined air bubbles a minute with the low-pressure end of the sleeve submerged in water and an air pressure of 85 pounds per square inch applied to the high-pressure end. Plunger and sleeve 4 when submitted to this test gave a steady stream of bubbles with a pressure of 5 or 10 pounds per square inch. Plunger and sleeve 5 gave a stream of six bubbles in 10 minutes at a pressure of 50 pounds per square inch and a steady stream of bubbles when the pressure was increased to 85 pounds per square inch.

Based on equation (2), the relative leakage of these four plungers and sleeves in order of their clearances would be 1, 1.3, 4.0, and 11.0. According to Alden's specifications, only the first plunger and sleeve would be acceptable. It seems that the specifications are too strict, because even with plunger and sleeve 4 the leakage, as will be shown later, is only a fraction of 1 percent of the fuel quantity injected when the plunger and sleeve are operating in an injection pump. If a leakage test is made with water, for which the viscosity does not vary over the pressure range used in hydraulic injection, the mean clearance can be determined from equation (3a). The mean clearance being known, the leakage under any pressure can then be estimated from equations (2a) and (5) or (6).

Figure 10 shows the effect of the viscosity on the leakage. As would be expected from equation (2), the rate of leakage varies inversely as the viscosity of the liquid. Because the present range of fuels used in
EFFECT OF VISCOSITY ON FUEL LEAKAGE BETWEEN LAPPED PLUNGERS AND SLEEVES

Injection systems vary considerably in viscosity, it is to be expected that the leakage of any particular injection system must vary considerably with the type of fuel used.

The data shown in figure 11 were obtained by changing the position of the plunger relative to the sleeve. The data obtained with plunger and sleeve 3 are particularly interesting. As long as the high-pressure end of the plunger was to the left of the point d of the sleeve, the leakage increased to a maximum and then decreased as the pressure was increased. When the point of minimum clearance was passed, the leakage increased considerably. When the plunger was extended so that no part of it was in that portion of the sleeve in which contraction took place, the leakage not only did not reach a maximum but also showed marked increase over the values obtained previously. The marked increase is caused by the increased clearance in the lapped fit and the increased clearance due to deformation. With plunger and sleeve 5, all the curves are of the same general form, but there is a marked increase in the leakage after the plunger was withdrawn beyond the point of minimum clearance (point f, fig. 5). These curves show that the length/diameter ratio of the lapped fit is not as important in its control of leakage as is the clearance. It should be emphasized that, whereas the leakage varies as the first power of the plunger diameter and inversely as the first power of the length of the lapped fit, it varies as the third power of the clearance.

For the three plungers and sleeves for which the measurements were made the clearance reaches a definite minimum. With 4 and 5, this minimum occurs about the midpoint of the sleeve. In each case the leakage became much greater as soon as the plunger was placed in a position in which this minimum was not included. Consequently, although the curves in figure 11 indicate that the length/diameter ratio should be between 3 and 5, it is probable that the lower value will be satisfactory with respect to leakage.

The leakages obtained with the two injection valves are shown in figure 12. Because the walls of the valve-stem sleeves were much thicker in proportion than the walls of the plunger sleeves, and because the diameters of the stems were less than those of the plungers, the increase of mean clearance with pressure was considerably less than with the plungers and sleeves. As a result, the rate-of-leakage curves are more nearly straight lines than those obtained with most of the plungers and sleeves. The rates of leakage are comparable to those obtained with plunger and sleeve 4, indicating a clearance of 0.00009 inch. These injection valves had been used in engine tests with both gasoline and hydrogenated safety fuel. Injection valve no. 13-C was used by Schey and Young in the majority of their tests with hydrogenated safety fuel (reference 7). The injection valve had been used for approximately 200 hours of engine operation at maximum power. Injection valve no. 18 had been operated for a much shorter time. The leakages obtained from the injection valves when they were operated from the port-control injection pump are shown in tables I and II. Because of the small discharge area of the injection nozzle (equivalent to a 0.022-inch orifice) the injection pressures were considerably higher than the injection valve opening pressure of 3,000 pounds per square inch (reference 2).

**Table I — Leakage from No. 13-C Injection Valve**

<table>
<thead>
<tr>
<th>Injection pressure, pounds per square inch</th>
<th>Pump speed, revolutions per minute</th>
<th>Rate of discharge, grams per hour</th>
<th>Rate of leakage, grams per hour</th>
<th>Duration of run, hours</th>
<th>Temperature, °F.</th>
<th>Leakage, percentage of fuel injected</th>
</tr>
</thead>
<tbody>
<tr>
<td>2,400</td>
<td>501</td>
<td>6,240</td>
<td>5.8</td>
<td>61</td>
<td>61</td>
<td>0.69</td>
</tr>
<tr>
<td>2,400</td>
<td>500</td>
<td>6,200</td>
<td>5.8</td>
<td>61</td>
<td>58</td>
<td>0.66</td>
</tr>
<tr>
<td>5,600</td>
<td>1,002</td>
<td>12,700</td>
<td>5.8</td>
<td>20</td>
<td>44</td>
<td>0.14</td>
</tr>
<tr>
<td>5,600</td>
<td>847</td>
<td>12,700</td>
<td>5.2</td>
<td>20</td>
<td>44</td>
<td>0.14</td>
</tr>
<tr>
<td>7,200</td>
<td>806</td>
<td>9,400</td>
<td>6.5</td>
<td>61</td>
<td>96</td>
<td>1.11</td>
</tr>
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</table>

**Table II — Leakage from No. 18 Injection Valve**

<table>
<thead>
<tr>
<th>Injection pressure, pounds per square inch</th>
<th>Pump speed, revolutions per minute</th>
<th>Rate of discharge, grams per hour</th>
<th>Rate of leakage, grams per hour</th>
<th>Duration of run, hours</th>
<th>Temperature, °F.</th>
<th>Leakage, percentage of fuel injected</th>
</tr>
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<tr>
<td>2,100</td>
<td>553</td>
<td>8,080</td>
<td>2.0</td>
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<td>0.14</td>
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<td>2,600</td>
<td>508</td>
<td>8,080</td>
<td>2.0</td>
<td>3.5</td>
<td>136</td>
<td>0.14</td>
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<tr>
<td>4,200</td>
<td>508</td>
<td>16,300</td>
<td>36.5</td>
<td>8</td>
<td>20</td>
<td>0.10</td>
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<td>4,500</td>
<td>968</td>
<td>17,000</td>
<td>19.7</td>
<td>7</td>
<td>136</td>
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<td>14,700</td>
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<tr>
<td>8,800</td>
<td>1,042</td>
<td>21,100</td>
<td>36.5</td>
<td>6</td>
<td>95</td>
<td>0.16</td>
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<tr>
<td>6,799</td>
<td>1,042</td>
<td>21,100</td>
<td>36.5</td>
<td>6</td>
<td>95</td>
<td>0.16</td>
</tr>
</tbody>
</table>

1 Low fuel quantity discharged in one cycle was 0.1 percent of the quantity injected.

With the injection pump used, the check valve maintained a residual pressure in the injection tube after cut-off at the fuel-injection pump. Therefore, the time during which the injection valve was under pressure was in some cases greater than the injection period. In general the leakage quantity was small, between 0.1 and 0.2 percent of the quantity injected. In two of the tests in which the discharge nozzle became partly clogged, increasing the maximum injection pressure and increasing the period of discharge (see reference 2), the leakage reached a value of 1 percent of the quantity injected. The value of 0.2 percent of the quantity injected can be expressed as approximately 0.1 pound or 3 cubic inches per 100 horsepower per hour. When gasoline is used this quantity is sufficient to necessitate a return tube for this type of injection valve to prevent the leaking fuel from wetting the engine or to provide adequate ventilation of the engine compartment.
Figure 10.—Effect of viscosity on the leakage of lapped plungers and sleeves.
Figure 11.—Effect of length of lapped fit on the leakage of lapped plungers and sleeves with Auto Diesel fuel.
The leakage rates obtained with a continuous hydraulic pressure give approximately the same rates as those obtained with the injection valve operated from the pump. For instance, referring to table 1 at 1,000 r.p.m., the maximum injection pressure was 4,600 pounds per square inch for hydrogenated safety fuel. The injection period (estimated from reference 2, fig. 10) was 27.5 pump degrees. The rate of leakage was therefore 3.8 grams per minute, as compared to 4.1 grams per minute shown in figure 12 for the same pressure. Similar results were obtained with the Auto Diesel fuel. Static tests showed a leakage of 0.32 gram per minute and leakage of 0.30 gram per minute with pump operation. The measurements of the leakage with and without a check valve in the pump indicated a slightly increased leakage with the check valve.

Although the different liquids do show a decided variation in the rate of leakage, the effect of this leakage on the fuel quantity discharged by the injection pump and on the rate of discharge is small. Figure 13 shows the rates of discharge at three pump speeds with three of the liquids from the port-control injection pump. With this pump the length of the lap sealing the injection pressure from the cam case was approximately 1.47 inches. In each case the rates of discharge are close together and the most viscous liquid gave rates between the other two. At 500 r.p.m. the mean injection pressure was approximately 3,000 pounds per square inch. The injection period was 0.01 second. Assuming that the lap clearance was approximately the same as that for plunger 5, the leakage was at the rate of 0.03 gram per minute for Auto Diesel fuel (fig. 8(e)), or 0.000005 gram for the...
injection period. The quantity injected was 0.3 gram. Therefore, the leakage is estimated at 0.0017 percent of the quantity injected with the Auto Diesel fuel, or 0.016 percent with gasoline. At 1,000 r.p.m. these rates amount to approximately 0.02 cubic inch per pump cylinder per hour with the Auto Diesel and 0.2 cubic inch per hour with the gasoline. The leakage of the less viscous fuel that is fed into the camshaft case may, in some cases, cause pump failure because of dilution of the injection-pump lubricant. However, the injection pump of which plungers and sleeves 4, 5, and 6 were a part has had about 250 hours operation with gasoline and with hydrogenated safety fuel, and there has been no sign of wear on either the pump cam or the rollers. It can be concluded that within the range of viscosities used in these tests, the viscosity of the fuel does not affect the metering ability of the pump nor is the leakage sufficient to present an appreciable loss of fuel, but the leakage may in some cases, particularly with the lighter fuels, be detrimental to the lubrication system of the injection pump.

With the injection pump used in these tests, there was a short length of lapped fit that sealed the fuel under the injection pressure from the fuel in the intake tube. One of the objections raised to port control is that this short length of lapped fit will result in sufficient leakage to the intake line to affect the fuel quantity discharged. Therefore, rates-of-discharge data were obtained for part loads, because this condition becomes worse as the fuel quantity injected is decreased. The results are shown in figure 14. The curves show that over the range of viscosities tested, the fuel injection was satisfactory down to one fourth load, the smallest quantity tested—approximately 0.075 gram per injection.

The leakage past the ports is shown in figure 15. As the figure shows, the lapped area is small compared to the area presented by the remainder of the sleeve. Although the rate of leakage is considerably higher than that between the remainder of the plunger and the sleeve, the rate is not excessive, amounting to approximately 2 grams per minute, or 0.0003 gram for the injection period, approximately 0.11 percent of the fuel quantity injected. With a less viscous fuel, such as gasoline, the leakage is estimated to be between 2 and 3 percent of the fuel quantity injected. Of course, this leakage does not represent a loss of fuel because the leakage is returned to the intake tube. However, if the leakage is excessive and varies considerably between the different plungers of a multicylinder pump, unequal distribution to the different engine cylinders may result. The leakage shown in figure 15 is for the setting at which the lapped area is a maximum. It can be concluded that with fuels with viscosities in the range of diesel fuels, the leakage past the port of a port-controlled fuel-injection pump is not objectionable, but that with less viscous fuels, such as gasoline, unless particular care is taken to insure a good lapped fit, the leakage may be sufficient to cause unequal distribution of fuel to the various engine cylinders, particularly at light loads.

Figure 16 shows the effect of injection-tube diameter on the rates of fuel discharge with the three fuels. With the exception of the tube having a 1/4-inch internal diameter, the curves are in the same order as in the preceding figures. With the smallest tube, the S.A.E. 30 lubricating oil shows the lowest rates of discharge during the first part of the injection, probably because of the increased resistance. In the other curves in which the S.A.E. 30 oil, in general, shows rates between the Auto Diesel fuel and the water, the relationship is caused by the combined effects of the density and compressibility of the liquids. As in the previous figures, the curves show that the injection pump operates satisfactorily with fuels covering a wide range of viscosities.

The explanation of the order of the rates of discharge can be obtained from an analysis of the factors controlling the rates. As has been shown in reference 1, the hydraulic pressure produced by the moving plunger is equivalent to \( \rho S \beta \), in which \( S \) is the velocity of the pressure waves in the medium, and \( \beta \), the velocity of the pump plunger times the ratio of the plunger area to the injection-tube area. The value of \( S \) is \( \sqrt{\frac{E}{\rho}} \), in which \( E \) is the reciprocal of the modulus of compressibility of the liquid. The mass rate of fuel discharges from the orifice is proportional to \( \sqrt{\rho_0} \), in which \( p \) is the pressure at the discharge orifice. The mass rate of discharge is therefore proportional to \( \sqrt{\rho_0} \sqrt{\frac{E}{\rho}} \) or \( \rho E \). Since the expression is a proportionality and not an equality, specific gravity may be substituted for the density. The values of \( E \) for the Auto Diesel fuel and the S.A.E. 30 oil are not known. They are estimated, however (from reference 1 and the data given in reference 7), to be 284,000 and 392,000 pounds per square inch, respectively. The value of \( E \) for water at a pressure of 3,000 pounds per square inch is 332,000 pounds per square inch. From these data, the proportionality factors are computed to be 24.0, 23.7, and 20.0 for the water, S.A.E. 30 oil, and the Auto Diesel fuel, respectively, which is the same order as the results in figures 13 and 14. Since the proportionality factor varies as the three quarters power of the density and as the one quarter power of the compressibility, it is probably safe to conclude that the rate of discharge from a fuel-injection system will vary in the order of the density of the fuel used, although the variation between the different fuels is small. Of course, when the diameter of the injection tube is such that the resistance to flow is appreciable, this order of variation is destroyed, as is shown in figure 16.
Figure 11.—Effect of viscosity and pump speed on the rate of discharge.

Figure 14.—Effect of viscosity and throttle setting on the rate of discharge.

Figure 15.—Leakage past ports of port-controlled pump used in tests.
CONCLUSIONS

The following conclusions are presented:

1. The rate of leakage between a lapped plunger and sleeve varies directly as the density of the fuel, the pressure producing the leakage, the diameter of the plunger, and as the cube of the mean clearance between the plunger and sleeve. The rate of leakage varies inversely as the length of the lapped fit, and inversely as the viscosity of the liquid in absolute units. Because of these relationships, the length of the lapped fit is of minor importance compared to the clearance.

2. With a pump-injection system the leakage past the plunger is approximately 0.2 to 0.01 percent, depending on the viscosity, of the fuel injected provided that the clearance is not greater than 0.0001 inch.

3. In general, the length of lapped fit necessary for a lapped plunger and sleeve is from three to five times the plunger diameter.

4. The rate of leakage is increased considerably under pressure unless the walls of the sleeve are sufficiently thick and the diameter of the plunger sufficiently small to make the expansion of the former and the contraction of the latter negligible in comparison with the clearance.

5. The rate of leakage can be made to decrease with increased pressure either by making the plunger hollow so that the pressure increases the plunger diameter, or by maintaining the pressure on the outside of the sleeve so that under pressure the actual clearance is decreased, or both.

6. The quantity of fuel that leaks past the injection valve stem when an automatic injection valve is employed is insufficient to affect the fuel consumption of the engine, but is sufficient to wet the engine if non-volatiles such as fuel oil are used.

7. Present-day commercial fuel injection pumps will operate satisfactorily on fuels ranging in viscosity from 0.0042 to 3.00 poises provided that the fuel is supplied to the pump under sufficient pressure to fill the receiving volume.

8. The effects of the above range of viscosities on the mass rates of fuel discharge of the injection system are negligible.

9. There is a slight variation in the rates of fuel discharge for fuels of different densities, the denser fuels showing the higher rates.

FIGURE 16.—Effect of viscosity and injection-tube diameter on the rate of discharge.

LANGLEY MEMORIAL AERONAUTICAL LABORATORY, NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS, LANGLEY FIELD, VA., DECEMBER 13, 1933.

REFERENCES

Positive directions of axes and angles (forces and moments) are shown by arrows.

<table>
<thead>
<tr>
<th>Axis</th>
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<th>Symbol</th>
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<td>Z</td>
<td>Z</td>
<td>Yawing</td>
<td>N</td>
<td>Yaw</td>
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</tbody>
</table>

Absolute coefficients of moment
\[ C_l = \frac{L}{\overline{q}bS} \] (rolling)
\[ C_m = \frac{M}{\overline{q}cS} \] (pitching)
\[ C_n = \frac{N}{\overline{q}bS} \] (yawing)

Angle of set of control surface (relative to neutral position), \( \delta \). (Indicate surface by proper subscript.)

4. PROPELLER SYMBOLS

- \( D \), Diameter
- \( \rho \), Geometric pitch
- \( \rho/D \), Pitch ratio
- \( V' \), Inflow velocity
- \( V_n \), Slipstream velocity
- \( T \), Thrust, absolute coefficient \( C_T = \frac{T}{\rho n^2 D^3} \)
- \( Q \), Torque, absolute coefficient \( C_Q = \frac{Q}{\rho n^2 D^3} \)

\[ P, \text{ Power, absolute coefficient } C_P = \frac{P}{\rho n^2 D^3} \]
\[ C_n, \text{ Speed-power coefficient } = \frac{V}{P n^2} \]
\[ \eta, \text{ Efficiency } \]
\[ n, \text{ Revolutions per second, r.p.s. } \]
\[ \phi, \text{ Effective helix angle } = \tan^{-1} \left( \frac{V}{2\pi n} \right) \]

5. NUMERICAL RELATIONS

1 hp. = 76.04 kgm/s = 550 ft-lb./sec.
1 metric horsepower = 1.0132 hp.
1 m.p.h. = 0.4470 m.p.s.
1 m.p.s. = 2.2369 m.p.h.

1 lb. = 0.4536 kg.
1 kg = 2.2046 lb.
1 mi. = 1,609.35 m = 5,280 ft.
1 m = 3.2808 ft.