NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

REPORT No. 493

THE PHYSICAL EFFECTS OF DETONATION IN A CLOSED CYLINDRICAL CHAMBER

By C. S. DRAPER

1934

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AERONAUTIC SYMBOLS

1. FUNDAMENTAL AND DERIVED UNITS

	Symbol	Metric		English	
		Unit	Abbreviation	Unit	Abbreviation
Length	l	meter	m	foot (or mile)	ft. (or mi.)
Time	t	second	s	second (or hour)	sec. (or hr.)
Force	F	weight of 1 kilogram	kg	weight of 1 pound	lb.
Power	P	horsepower (metric)		horsepower	hp.
Speed	V	kilometers per hour	k.p.h.	miles per hour	m.p.h.
		meters per second	m.p.s.	feet per second	f.p.s.

2. GENERAL SYMBOLS

W, Weight $= mg$

g, Standard acceleration of gravity $= 9.80665$ m/s^2 or 32.1740 ft./sec.^{2}

m, Mass $= \frac{W}{g}$

I, Moment of inertia $= mk^2$. (Indicate axis of radius of gyration k by proper subscript.)

μ, Coefficient of viscosity

ν, Kinematic viscosity

ρ, Density (mass per unit volume)

Standard density of dry air, 0.12497 kg-m^{-4}-s^2 at 15° C. and 760 mm; or 0.002378 lb.-ft.$^{-4}$ sec.^{2}

Specific weight of "standard" air, 1.2255 kg/m^3 or 0.07651 lb./cu.ft.

3. AERODYNAMIC SYMBOLS

S, Area

S_w, Area of wing

G, Gap

b, Span

c, Chord

$\frac{b^2}{S}$, Aspect ratio

V, True air speed

q, Dynamic pressure $= \frac{1}{2}\rho V$

L, Lift, absolute coefficient $C_L \frac{L}{qS}$

D, Drag, absolute coefficient $C_D = \frac{D}{qS}$

D_o, Profile drag, absolute coefficient $C_{D_o} = \frac{D_o}{qS}$

D_i, Induced drag, absolute coefficient $C_{D_i} = \frac{D_i}{qS}$

D_p, Parasite drag, absolute coefficient $C_{D_p} = \frac{D_p}{qS}$

C, Cross-wind force, absolute coefficient $C_C = \frac{C}{qS}$

R, Resultant force

i_w, Angle of setting of wings (relative to thrust line)

i_t, Angle of stabilizer setting (relative to thrust line)

Q, Resultant moment

Ω, Resultant angular velocity

$\rho\frac{Vl}{\mu}$, Reynolds Number, where l is a linear dimension (e.g., for a model airfoil 3 in. chord, 100 m.p.h. normal pressure at 15° C., the corresponding number is 234,000; or for a model of 10 cm chord, 40 m.p.s. the corresponding number is 274,000)

C_p, Center-of-pressure coefficient (ratio of distance of $c.p.$ from leading edge to chord length)

α, Angle of attack

ϵ, Angle of downwash

α_o, Angle of attack, infinite aspect ratio

α_i, Angle of attack, induced

α_a, Angle of attack, absolute (measured from zero-lift position)

γ, Flight-path angle

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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

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By C. S. Draper

SUMMARY

Detonation in the internal-combustion engine is studied as a physical process. It is shown that detonation is accompanied by pressure waves within the cylinder charge.

Sound theory is applied to the calculation of resonant pressure-wave frequencies.

Apparatus is described for direct measurement of pressure-wave frequencies.

Frequencies determined from two engines of different cylinder sizes are shown to agree with the values calculated from sound theory.

An outline of the theoretically possible modes of vibration in a right circular cylinder with flat ends is included.

An appendix by John P. Elting gives a method of calculating pressure in the sound wave following detonation.

INTRODUCTION

Detonation in the internal-combustion engine has received a vast amount of attention because of its importance in engine design and fuel selection. Discussion of the general subject has often been hampered by lack of a common definite understanding of the component parts of a complex process. Measurements carried out on a detonating engine must be of a physical nature, whatever the ultimate conclusions reached. For this reason, a study of the exact nature of the physical processes involved should be of direct assistance in detonation research.

If a firecracker is exploded inside a closed drum containing air, two effects naturally follow: A series of sound waves is set up by the sudden local increase in pressure at the explosion and the general pressure within the drum rises because of the energy liberated. The frequency of the resulting sound waves will depend on the dimensions of the drum, the air pressure, and the position of the firecracker within the drum. It is reasonable to suppose that the process known as detonation in internal-combustion engines is similar to that taking place in the case outlined above. A sudden pressure rise in some part of the cylinder charge would correspond to the firecracker explosion and the resulting pressure waves would be analogous to the sound waves in the drum. This assumption is the basis of the physical picture used in the following discussion.

Work on a particular type of physical process is coordinated and simplified if the natural laws controlling the process can be discovered. Specializing this principle to the present case, the problem of detonation in its physical aspects will be reduced to relatively simple terms if it can be shown that the disturbances within the cylinder charge follow the simple laws of sound. The primary object of the work reported in this paper was to check the applicability of simple sound theory to the physical processes characteristic of detonation.

Simple sound theory (reference 1) is concerned with the propagation of small disturbances through an elastic medium. The theory holds only for those cases in which pressure and density changes due to the passage of the wave are small compared to the pressure and density of the medium in its undisturbed state. If experiment indicates that pressure waves occurring during detonation follow the simple sound law, the necessary conclusion is that these changes are small compared to the pressure and density of the undisturbed medium.

Detonation considered from the physical point of view may be analyzed into two stages: (1) A sudden pressure rise in some part of the cylinder charge, which excites; (2) intense pressure waves within the charge.

The first stage of the phenomenon presents two aspects: (*a*) a chemical process characterized by the very rapid reaction of a portion of the burning mixture; (*b*) a sudden local rise in pressure which is the physical counterpart of the chemical process. The chemical phase of the process is treated at length in reference 2.

Measurements of detonation intensity made with an instrument such as the Midgley Bouncing Pin (see reference 3) depend upon the sudden initial pressure rise. The existence of this sharp pressure rise is unquestioned. However, it is much more difficult, from the standpoint of both theory and experiment, to treat sudden discontinuous processes than to handle the case of a series of waves having a simple wave form. For this reason, the first step in considering the physical aspects of detonation was to study the second phase of the process. The material

presented here will deal with the pressure waves following the initial pressure rise rather than with the impulse itself.

The work to be described was carried out at the Massachusetts Institute of Technology under the supervision of Prof. C. F. Taylor. All experimental work was completed in the Internal Combustion Engine laboratory with the cooperation of Prof. E. S. Taylor and Mr. G. L. Williams. Work on the problem was started some years ago under the Sloan and Crane Automotive Fellowship Grants.

PRELIMINARY EXPERIMENTS

In the years 1928 to 1930 experiments were made with indicators in which the sensitive elements were carbon-granule microphones. The microphone units were coupled mechanically to stiff steel diaphragms. In all cases the diaphragms were one-half inch in diameter. Several units were constructed, each with a different diaphragm thickness (thickness varied from 0.015 to 0.060 inch). These units were so designed that they could be screwed into an 18 mm spark-plug hole, the diaphragm being freely exposed to cylinder gas pressures. All tests were made in engines with heavy cast-iron cylinder walls, so that the effects of cylinder-wall vibrations were minimized. Sound records were also taken from a condenser microphone placed in the free air near the engine. Oscillograph records were made with the aid of a specially constructed instrument (reference 4). This oscillograph was adapted to record photographically the events of a succession of engine cycles on motion-picture film. Check runs made throughout the course of the work showed the recorded frequencies to be independent of the natural constants of the detecting and recording apparatus. These experiments led to the conclusions noted below:

(1) In a thick-walled cylinder the indicators were sensitive to vibrations transmitted through the gas of the cylinder charge, but not to cylinder-wall vibrations.

(2) Sound frequencies recorded by an external microphone are the same as those shown by the indicator in contact with the cylinder gases.

(3) Pressure waves exist in the cylinder charge before knocking becomes audible. These pressure waves greatly increase in intensity when the characteristic sound of detonation appears.

(4) For a moderate detonation intensity in a thick-walled engine cylinder using one spark plug, the recorded vibrations are of simple wave form.

(5) The vibrations continue over a considerable portion of the power stroke.

(6) The recorded frequency decreases, as the piston descends, by some 10 to 20 percent of its initial value.

PRELIMINARY CONCLUSIONS

Pressure waves will be set up in an elastic medium by any sudden local pressure change occurring in that medium. In detonation, the excitation of pressure waves is certainly due to an impulse, so the resulting wave system will depend upon free vibration of some elastic system.

Two possibilities present themselves: (1) The wave frequency may be determined by mechanical parts of the cylinder and head structure, or (2) resonant frequency vibrations in the cylinder charge may be responsible for the observed results.

In a thick-walled cylinder of cast iron the first source of vibration will be less prominent than for the case of a cylinder having thin steel walls. It seems improbable that changes occurring in an engine during the power stroke would greatly affect the natural frequency of the mechanical structure. The theory of vibrating systems (references 5, 6, 7, and 8) shows that for a system of unvarying mass, elasticity, and damping the frequency will be constant during a series of free vibrations. These considerations indicate that, in the engine, frequencies due to mechanical vibrations will probably be substantially constant.

Resonant frequency pressure waves within the cylinder charge exist in a medium under changing conditions of pressure, density, and boundaries. These changes will alter the frequency of any system of pressure waves that may exist in the medium.

Experiment showed that for the thick-walled cylinders the records were of simple wave form with frequency decreasing during the power stroke. This result indicated that in the engine the pressure-wave frequency accompanying detonation was probably due to resonant frequency vibrations in the cylinder charge. The subsequent work was directed toward checking this hypothesis by use of simple sound theory.

THEORETICAL CONSIDERATIONS

WAVE EQUATION

The general theory of elasticity leads to an equation for the motion of a small volume within the body of a continuous medium. If the general result is specialized to the case of a fluid (i.e., a medium which will not support shear), the fundamental equation of hydro-dynamics appears. (See reference 9.) Solution of this equation for an elastic fluid under the restricted case of small changes in pressure and density leads to the wave equation in three dimensions. (See references 1, 7, 9, and 10.) In rectangular coordinates the wave equation has the form

$$\frac{\partial^2\Phi}{\partial x^2}+\frac{\partial^2\Phi}{\partial y^2}+\frac{\partial^2\Phi}{\partial z^2}=\frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2} \tag{1}$$

where Φ is the velocity potential of ordinary hydrodynamical theory, i.e., if u, v, w are the velocity components along xyz,

$$u=-\frac{\partial\Phi}{\partial x},\ v=-\frac{\partial\Phi}{\partial y},\ w=-\frac{\partial\Phi}{\partial z},$$

and c is a constant which has the significance of phase velocity in the pressure wave.

In general for any medium

$$c^2=\left(\frac{dp}{d\rho}\right)_0 \tag{2}$$

p is pressure

ρ is mass density

and the zero subscript indicates that the derivative is to be taken under equilibrium conditions.

If the medium behaves as a perfect gas, it is found experimentally that changes occurring in a pressure wave of small amplitude (pressure change small compared to total pressure of the medium) are so rapid that heat flow can be neglected and the process follows the adiabatic law. Under these restrictions

$$c=\sqrt{\frac{\gamma p_0}{\rho_0}} \tag{3}$$

where

$$\gamma=\frac{C_p}{C_v}=\frac{\text{specific heat at constant pressure}}{\text{specific heat at constant volume}}$$

WAVE EQUATION IN CYLINDRICAL COORDINATES

For the study of pressure waves in an elastic medium enclosed by a vessel with the shape of a right circular cylinder, it is convenient to use the wave equation in a modified form. The change consists in replacing the rectangular coordinates $x\ y\ z$ by the cylindrical coordinates $r\ \theta\ z$. With this alteration the wave equation becomes (see references 7, 9, and 11):

$$\frac{\partial^2\Phi}{\partial r^2}+\frac{1}{r}\frac{\partial\Phi}{\partial r}+\frac{1}{r^2}\frac{\partial^2\Phi}{\partial\theta^2}+\frac{\partial^2\Phi}{\partial z^2}=\frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2} \tag{4}$$

This equation may be solved by separation of the variables in the ordinary manner. (See references 7 and 11.) The solution may be written:

$$\Phi=\Sigma[A_sJ_s(\beta r)+$$

$$B_sK_s(\beta r)]\begin{bmatrix}\cos\ (s\theta)\\ \sin\ (s\theta)\end{bmatrix}\begin{bmatrix}\cos\ (pz)\\ \sin\ (pz)\end{bmatrix}\begin{bmatrix}\cos\ (2\pi nt+\epsilon)\\ \sin\ (2\pi nt+\epsilon)\end{bmatrix} \tag{5}$$

Where $J_s(\beta r)$ is Bessel's function of the first kind and sth order

$K_s(\beta r)$ is Bessel's function of the second kind and sth order

$$\beta^2=\frac{4\pi^2n^2}{c^2}-p^2$$

c is the velocity of sound in the medium

A_s, B_s, s, p, n and ϵ are constants to be determined.

SOLUTION OF THE WAVE EQUATION FOR CASE OF CIRCULAR CYLINDER WITH FLAT ENDS

In general, for any fixed closed vessel completely filled by an elastic medium, only certain systems of standing waves may exist after an initial disturbance has subsided. The characteristic wave systems are determined by the fact that at rigid boundaries, the velocity of the elastic medium must always be zero in a direction at right angles to the boundary.

The wave equation solution in cylindrical coordinates may be fitted to the case of a right circular cylinder of radius a with plane ends at $z=0$ and $z=h$. The table below presents a summary of the conditions to be satisfied and the results on the wave equation.

Condition	Result
$\frac{\partial\Phi}{\partial r}$ finite everywhere	$B_s=0$
$\frac{\partial\Phi}{\partial r}=0$ when $r=a$	$a=$ a root of $\frac{dJ_s(\beta_a)}{dr}=0$
Φ at $\theta=\Phi$ at $(\theta+2\pi)$	$s=0, 1, 2\ \dots$
$\frac{\partial\Phi}{\partial z}=0$ when $z=0$, $z=h$	$p=\frac{g\pi}{h}$, $g=0, 1, 2\ \dots$ (the cosine term must be used)

If these requirements are fulfilled, the solution becomes (see references 1 and 7):

$$\Phi=\Sigma_{s,\,g,\,\beta}A_{s,\,g,\,\beta}J_s(\beta r)\,\frac{\sin}{\cos}\,(s\theta)\cos\left(\frac{g\pi}{h}z\right)\cos\,(2\pi nt) \tag{6}$$

for the case in which the phase angle ϵ, is made zero by a proper selection of the initial instant. In Equation (6)

$$n=c\sqrt{\frac{\beta^2}{4\pi^2}+\frac{g^2}{4h^2}} \tag{7}$$

(from reference 9).

A few of the allowed values of βa are: (9)

$s=0$ $\beta a=$ 3.832	$s=1$ $\beta a=$1.841	$s=2$ $\beta a=$ 3.00
$\beta a=$ 7.016	$\beta a=$5.332	$\beta a=$ 6.75
$\beta a=$10.173	$\beta a=$8.536	$\beta a=$10.15

$A_{s,\,g,\,\beta}$, are constants depending upon the amplitude of the vibration and the particular modes involved

h is height of cylinder in feet

n is frequency in cycles per second

a is radius of cylinder in feet

c is velocity of sound in feet per second

LONGITUDINAL VIBRATIONS

Equation (6) will give a solution of the wave equation for a case in which s, g, and β have any arbitrary combination of the allowed values. It will be useful to study briefly certain special cases.

If βa and s are each taken as zero, (6) reduces to

$$\Phi=\Sigma_gA_g\cos\left(\frac{g\pi}{h}z\right)\cos\,(2\pi nt) \tag{8}$$

and (7) becomes

$$n=g\frac{c}{2h} \tag{9}$$

Fluid velocity parallel to the longitudinal axis of the cylinder is obtained by differentiating (8) with respect to z.

$$V_z=-\frac{\partial\Phi}{\partial z}=-\Sigma_g B_g \sin\left(\frac{g\pi}{h}z\right)\cos(2\pi nt) \tag{10}$$

This expression applies to the case of longitudinal vibrations of a gas column in a tube closed at both ends. The sine term depending upon z acts merely as a coefficient for the cosine term depending upon time. Physically this means that the amplitude of the sinusoidal velocity variation with time depends upon distance along the axis of the tube.

For $z=0$ and $z=h$, equation (10) fulfills the condition that velocity perpendicular to the end walls must always be zero. If g is taken as unity, a region of

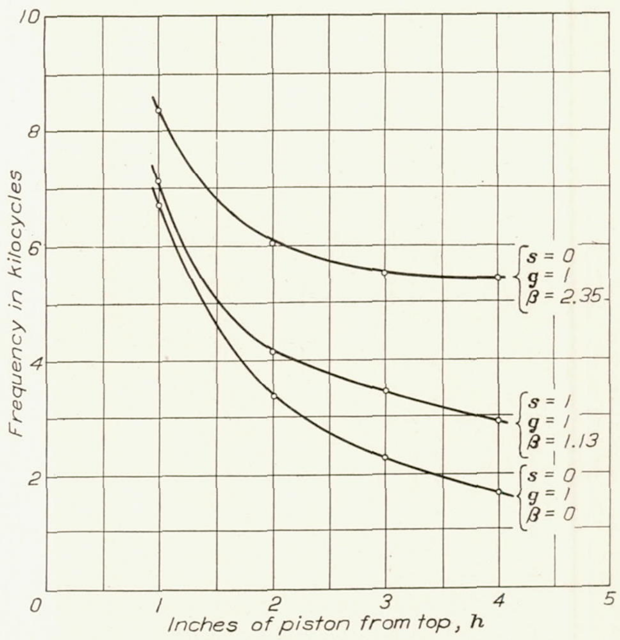


FIGURE 1.—Sound-wave frequency in cylindrical chamber. Longitudinal waves. Air at 760 mm and 70° F. cylinder radius=1.625 inches.

maximum velocity variation will occur at the midpoint of the tube. For larger values of g there will be positions of zero velocity along the length of the tube separated by positions at which the amplitude of the velocity variation passes through a maximum value. The positions of zero velocity are called "nodes" and the positions of maximum velocity variation are called "loops." In general, the loops will be in regions of constant pressure and the nodes will correspond to regions of maximum pressure variation. (See reference 1.)

At a fixed position along the length of the tube, one cycle of the pressure wave will be completed in a time interval $1/n$. The distance traveled by a point on the wave in the time $1/n$ is called the wave length and is denoted by λ. The relation between wave length, sound velocity, and frequency is

$$c=n\lambda \tag{11}$$

For the case of longitudinal vibrations in a closed tube, the wave length is

$$\lambda=2h/g \tag{12}$$

Figure 1 is a plot showing calculated frequency as a function of piston position in a Waukesha C.F.R.

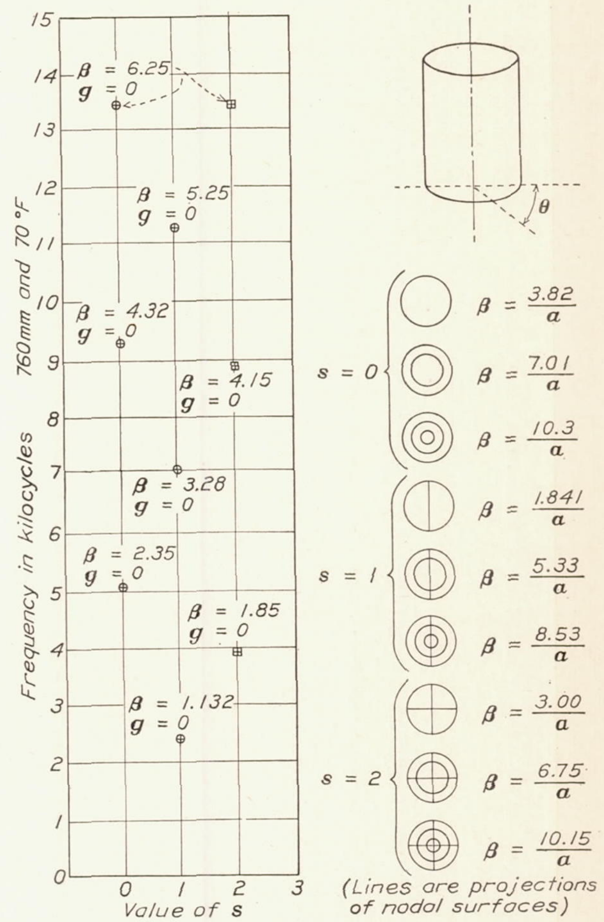


FIGURE 2.—Sound-wave frequency in cylindrical chamber. Transverse waves.

$$\Phi=A_{sg\beta}\,J_s(\beta r)\cos(s\theta)\cos\frac{g\pi}{h}z\cos 2\pi nt$$

$s=0, 1, 2 \ldots\ldots$
$g=0, 1, 2 \ldots\ldots$

$$n=c\sqrt{\frac{\beta^2}{4\pi^2}+\frac{g^2}{4h^2}}$$

$r=a=1.625$ inches
h=height of chamber
n=frequency
c=velocity of sound
a=radius of cylinder

engine, the cylinder containing air under atmospheric pressure. The lowest of the three curves is plotted for the simple case discussed above. Since β is made zero for this case, the frequency must be independent of the cylinder diameter. If any others of the allowed values of β are selected, the vibrations become more complicated and the corresponding value of frequency will depend upon cylinder diameter. The two upper curves are plotted for values of β as indicated.

TRANSVERSE VIBRATIONS

In equation (6), if g is made zero the resulting values of ϕ will be independent of the height of the cylinder. The wave systems may be represented in two dimensions; in fact, the vibrations are strictly analogous to the waves that occur on the surface of water contained in a circular vessel.

If both g and s are made zero, the vibrations will be independent of cylinder height and of angular position, i.e., the pressure waves will be symmetrical. For the lowest allowed value of β the only nodal surface will be at the cylinder wall, which must always be a node. For the next highest allowed value of β there will be one cylindrical nodal surface in addition to the wall itself; the third allowed value of β will correspond to the case of two cylindrical nodes, etc.

For the case of $s=1$ there will be one nodal surface in the form of a plane passing through the axis of the cylinder. In general, there will be a number of equally spaced nodal planes equal to the value of s. The various combinations of values of s and β will give corresponding combinations of nodal planes and nodal circles. Since all the cases in which s is not zero show a variation in conditions with angle, these types of wave systems are called unsymmetrical modes.

Figure 2 shows a summary of the lowest frequency transverse modes of vibration for the C.F.R. engine cylinder containing air at atmospheric pressure. The corresponding nodal surfaces are indicated diagrammatically in the same figure.

The diagrams of figure 3 (obtained from reference 9) show the forms of the lines of equal pressure, to which the motions of the particles are orthogonal, for two transverse modes of vibration. Diagram (a) shows the equal pressure lines for the case of a single nodal plane. Diagram (b) shows the case in which one nodal cylinder exists in addition to the nodal plane. If the full lines represent instantaneous pressures above the equilibrium pressure, the dotted lines will represent pressures below the equilibrium pressure.

Since the circle is a symmetrical shape, there can be no reason based on the form of the circular cylinder for the existence of unsymmetrical modes of vibration. In the unsymmetrical cases, the position of the nodal plane must be determined by the manner in which the vibrations are excited. For an engine cylinder, unsymmetrical vibration modes could be produced by local disturbances occurring near the cylinder wall. The angular positions of these disturbances would determine orientation of the resulting nodal planes. Thus, it would be expected that a single local disturbance occurring near the cylinder wall would set up the first unsymmetrical mode of vibration with the nodal plane at right angles to the diameter upon which the disturbance occurs.

GENERAL MODES OF VIBRATION

In general, any combination of longitudinal and transverse vibration may occur that corresponds to allowed values of g, s, and β. The corresponding nodal surfaces may become very complicated but they always consist of combinations of the simple types considered above.

RELATION BETWEEN FREQUENCY AND CYLINDER CHARGE CONDITIONS

Equation (11) gives the relation between sound velocity, wave length, and frequency in an elastic medium. For the case of pressure waves within an engine cylinder, wave length—for a given mode—will depend upon the size and shape of the cylinder and the combustion chamber. If the engine is of the valve-in-head type with both the head and piston flat, the theory outlined above may be applied. By use of equation (11) with equation (7) the various wave lengths corresponding to simple modes of vibration

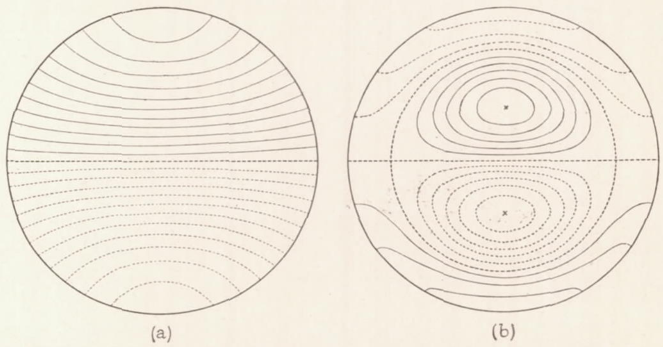


FIGURE 3.—Lines of constant pressure for two transverse modes of vibration in circular cylinder: (a) Single nodal plane; (b) one nodal cylinder in addition to nodal plane.

may be estimated. The expression for wave length becomes

$$\lambda=\frac{1}{\sqrt{\frac{\beta^2}{4\pi^2}+\frac{g^2}{4h^2}}} \tag{13}$$

For the special case of $g=0$, (13) becomes

$$\lambda=\frac{2\pi}{\beta}=\frac{2\pi}{(\beta a)}a \tag{14}$$

where βa may take any of the values allowed by (6).

If the first unsymmetrical mode of vibration is considered

$$\lambda=\frac{2\pi}{1.84}a=3.41a \tag{15}$$

In the engine cycle, piston position changes in a known manner with crank angle, so that size and shape of the enclosed chamber are completely determined at any instant. On the other hand, sound velocity depends on conditions in the cylinder charge. Through proper measurements, it is possible to esti-

mate instantaneous values of sound velocity with a reasonable degree of accuracy. A continuous record of pressure-wave frequency may be easily obtained. By combination of the frequency data with calculated sound velocities in equation (11) the corresponding value of wave length may be obtained. Comparison of the wave length determined in this manner with values computed from cylinder dimensions for different modes, will serve to indicate which mode is excited in a given case. For complicated modes difficulty may be encountered in carrying out this process, however, for the simpler cases, wave-length differences are so great that definite results can be attained.

The relation between the variables concerned is found by combining equation (3) with equation (11) to obtain

$$n=\frac{1}{\lambda}\sqrt{\frac{\gamma p_0}{\rho_0}} \tag{16}$$

Equations (13) and (16) summarize the predictions of simple sound theory with regard to the vibration

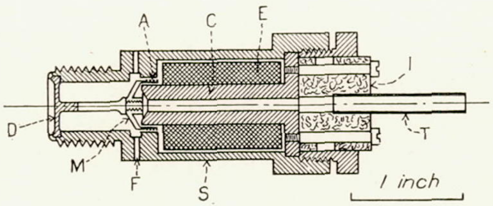


FIGURE 4.—M.I.T. rate of change of pressure indicator.

frequency of resonant wave systems in a right circular cylinder with flat ends.

It will be shown that experiment produces results consistent with calculations based on sound theory. This agreement indicates that simple sound theory may be applied to the pressure-wave phenomena which accompany detonation.

The following section will outline the experimental methods and apparatus used in obtaining the necessary data.

APPARATUS

GENERAL

The experimental work requires, in addition to the ordinary laboratory equipment for engine research, apparatus for precise measurement of equilibrium pressures and an instrument to determine pressure-wave frequencies in the engine cylinder.

PRESSURE MEASUREMENTS

The M.I.T. engine indicator gave satisfactory records for pressure determinations. The instrument is described in a recent article (reference 12). In operation, the M.I.T. engine indicator supplies a spark record made on a rotating drum similar to that of the Farnboro indicator. (See reference 2.) The sensitive unit is of the balanced pressure type using a diaphragm in the manner of the Bureau of Standards indicator. An electrical circuit incorporating a General Electric Thyratron tube produces a spark of sufficient intensity for recording purposes. The circuit is controlled by the make or break of contact between a thin steel diaphragm and an insulated contact. Since only the grid current of the Thyratron flows between the contact and the diaphragm, satisfactory operation is possible over relatively long periods of time. An extended series of tests has shown that the M.I.T. indicator is both accurate and reliable.

FREQUENCY MEASUREMENTS

An instrument [1] for recording pressure-wave frequencies was developed especially for use in detonation work. This device has three component parts:

(*a*) A unit sensitive to pressure changes.

(*b*) A vacuum-tube amplifier.

(*c*) An oscillograph.

Sensitive unit.—Figure 4 shows a cross section of the indicator unit. An exciting winding E is carried by a core C of cold-rolled steel. The projecting end of C, with a cylindrical opening in the shell S, forms an annular air gap A. A diaphragm D of heat-treated alloy steel is rolled into the steel shell S and carries the moving coil M at the end of a small pillar. The coil is wound on a very light spider of aluminum alloy. The complete assembly of the diaphragm and coil weighs less than 2.5 grams. A copper tube T is mounted in a bakelite insulater I so that its opening lines up with a central hole in the core C, through which cooling air is supplied to the moving coil assembly. The cooling air is discharged through a series of openings F in the shell S. The central air passage also serves as a means of bringing the insulated lead from M out to T, which acts as one electrical terminal.

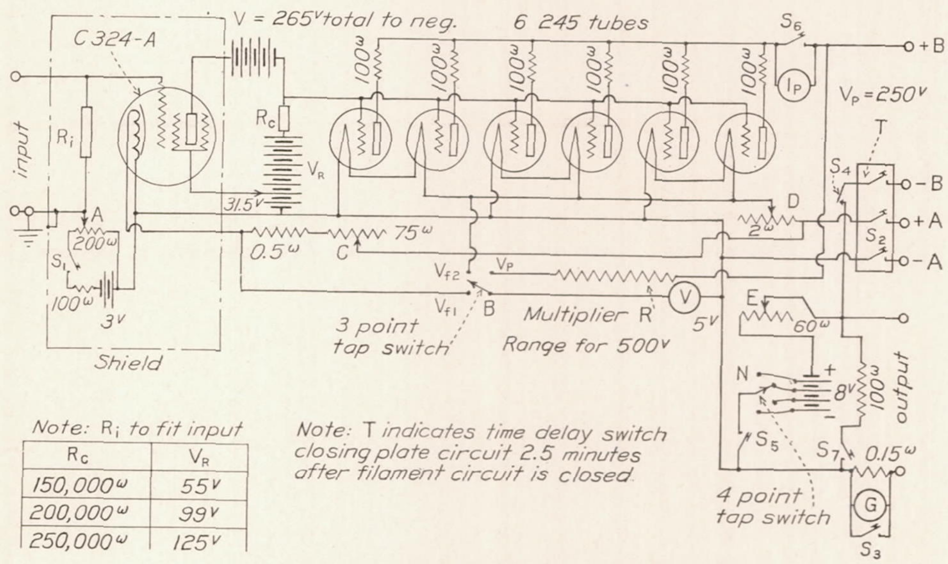
The moving coil indicator depends upon the voltage generated in a wire when that wire is made to cut lines of magnetic flux. This voltage is directly proportional to the rate at which flux lines are cut by the wire. In the present case, the velocity of the coil will depend upon the rate of change of pressure in the engine cylinder. The induced voltage will be a function of the rate of change of pressure, rather than of the magnitude of the pressure. A certain reaction on the constant magnetic field of the air gap will result from current flowing either in the pick-up coil itself or in the metallic cylinder carrying the coil. Since the coil currents are small no serious error is to be expected from this source. A series of tests were made using coil-carrying cylinders with slots parallel to the axis of the shell and extending almost to the base of the coil showed no detectible effect on the

[1] The indicator described in the present paper was developed without knowledge of the work of Prof. Augustus Trowbridge of Princeton University. (Trans. of Society of Automotive Engineers, vol. 17, t. I, 1922.) The writer wishes to call attention to the priority of Professor Trowbridge in the construction of a moving coil indicator for engine research.

records as compared with coils wound on solid shells. This result justifies the conclusion that no considerable error is present from currents induced in the metal of the coil structure.

Amplifier.—Experiment showed that satisfactory equipment should be able to produce records over a frequency range of 3,000 to 10,000 cycles per second. In practice, the sensitive units do not produce sufficient current to operate a suitable oscillograph directly. This difficulty was overcome by the construction of a two-stage resistance coupled amplifier having a reasonably linear output-current range of about 400 milliamperes. The amplifier was adapted for use with the General Motors indicator (see reference 13) as well as the moving coil indicators.

Figure 5 is a diagram of the amplifier unit. A single screen grid tube is coupled by the resistance R_c with an output stage of 6 tubes in parallel. Plate voltage for the first stage is supplied by a battery of dry cells, while a bank of storage batteries furnishes current for the output stage. The grid biasing potentiometer A makes it possible to obtain any desired direct-current component in the output tubes as measured by the meter I_p. Since the amplifier is ordinarily coupled directly to an oscillograph vibrator, a balancing arrangement consisting of the rheostat E and a storage battery connected to the tap switch N is used to reduce the direct-current component in the output circuit to zero. The correct adjustment is indicated by the sensitive galvanometer G. A meter V connected to a three-point tap switch B is useful in checking filament and plate voltages during operation.



R_c	V_R
150,000ω	55ᵛ
200,000ω	99ᵛ
250,000ω	125ᵛ

FIGURE 5.—Amplifier used in making detonation records.

In practice, usable records may be obtained by connecting the moving coil directly to the amplifier input and the oscillograph directly to the amplifier output. The records made with direct coupling were supplemented with records made using specially wound input and output transformers. In general, a transformer so constructed that the same flux links each turn of the primary and each turn of the secondary, will have the same voltage induced in every turn of both windings by changes in the linking flux. If the primary circuit has a small resistance, while its inductance is high, the resistance drop may be neglected in comparison with the self-induced voltage of the winding for sufficiently rapid changes in primary current. Under these conditions the induced primary voltage may be taken as equal in magnitude to the externally applied primary voltage and consequently the induced secondary voltage will be equal in magnitude to the applied primary voltage multiplied by the ratio of the number of turns on the secondary to the number of turns on the primary. The argument outlined above will apply in any case for which the voltage applied to the primary is never constant over a time interval comparable to a reasonably small fraction of the time constant of the primary circuit. (See reference 14.)

A series of tests were made with a "square wave" voltage impressed on the primary of the input transformer. These tests showed that the amplifier and coupling transformer system will transmit satisfactorily an arbitrary voltage variation if the disturbance is completed in a sufficiently short time. Using the input transformer and amplifier without the output transformer, the time requirement is such that a good reproduction of the rate of change of pressure card, as distinct from the pressure waves of detonation, can be obtained at engine speeds over 800 revolutions per minute. Throughout the work all data were taken from 2 sets of records, 1 set made with the amplifier alone and a second set with the input transformer and amplifier but without the output transformer. There are no observable differences between the two sets of records so far as detonation frequencies are concerned, but the increased sensitivity resulting from use of the input transformer produces records more suited to direct inspection without enlargement.

Oscillograph.—The problem of making a continuous record of pressure-wave frequencies accompanying detonation led to the construction of a special oscillograph capable of resolving frequencies over 10,000 cycles per second and including a number of successive engine cycles on the same record. This oscillograph is described in an article by the author and Mr. D. G. C. Luck (reference 4).

The instrument takes advantage of a short distance between the oscillograph mirror and the recording

film to obtain good photographic records at high effective film speeds. It is possible to use lengths of motion picture film up to 40 feet.

Combined amplifier and oscillograph system.—Figure 6 is an experimentally determined sensitivity curve for the amplifier-oscillograph system with coupling transformers.

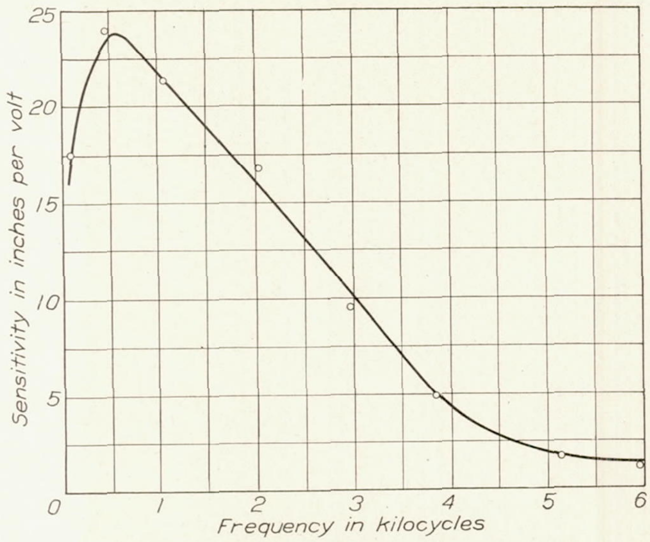


FIGURE 6.—Sensitivity of amplifier-oscillograph system. Amplifier with output and input transformers.

The nonlinear nature of the response is undesirable, but if frequency data only are to be considered, no difficulty is introduced so long as consideration is limited to simple wave forms. An extended series of tests showed the apparatus to be free from any tendency toward self-oscillation.

It was found in practice that sufficient coupling exists between the ignition circuit of the engine and the indicator system to give a reliable reference point on the oscillograph record for each engine cycle. The magnitude of the spark pickup effect can be varied over a considerable range by the proper application of electrical shielding or by alteration of the input circuit of the amplifier.

EXPERIMENTAL WORK

General.—Records of pressures and frequencies were taken from two engines, one a Waukesha C.F.R. engine and the other an N.A.C.A. universal test engine equipped with a special cylinder head. In both cases the cylinder head and piston were flat and the combustion chamber had the same bore as the cylinder proper. The bore of the C.F.R. engine was 3¼ inches while the N.A.C.A. engine had a bore of 5 inches. Figure 7 indicates diagrammatically the experimental arrangement with the C.F.R. engine. All runs were made with the indicator units located directly across the cylinder from a single spark plug. The records showing detonation were taken under conditions of moderate intensity as judged by ear.

Figures 8 and 9 show two sets of specimen records, one made without detonation and the other with detonation present. Figure 8 shows comparable records taken with the M.I.T. indicator, the General Motors indicator and the rate of change of pressure indicator during normal operation of the C.F.R. engine. Figure 9 is a set of records like that of figure 8 except that the C.F.R. engine was operating with detonation induced by the addition of 1 percent ethyl nitrite to the fuel. In this case, the M.I.T. indicator record has a greatly extended line of points near maximum pressure corresponding to the sudden pressure rise characteristic of detonation. It will be noticed that the expansion line starts from about the same pressure as in normal operation but has its points somewhat more widely dispersed. The high peak pressures without a corresponding change in the expansion line are proof that the high pressure is a local phenomenon rather than a general cylinder pressure. Figure 10 is a composite

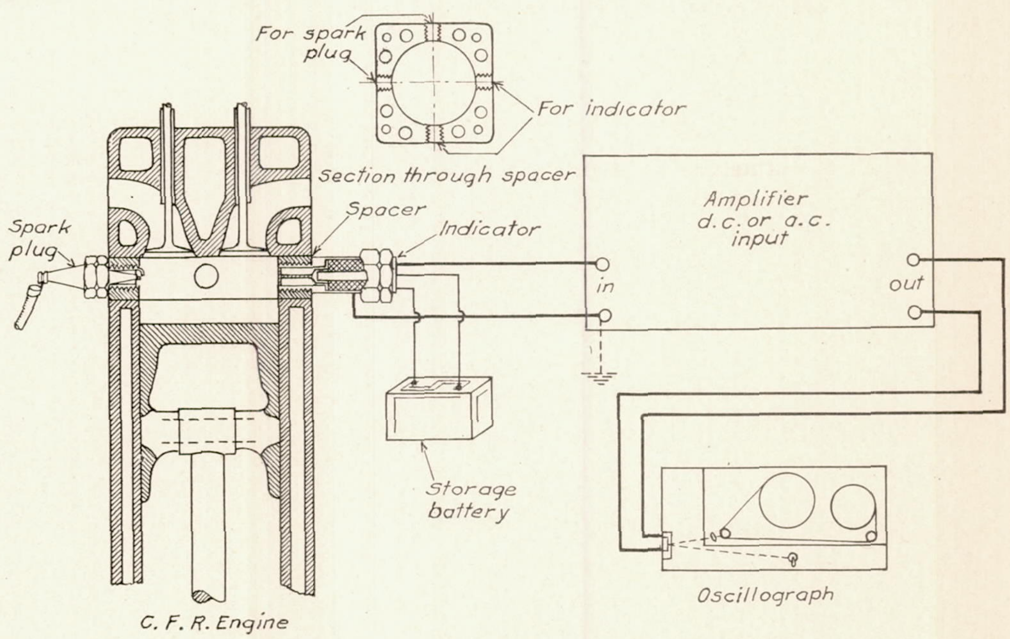


FIGURE 7.—Diagram of connections for M.I.T. rate of change pressure of indicator.

enlargement to the same crank angle scale of the General Motors indicator record and the rate of change of pressure indicator record taken from figure 9. These records show that these two instruments which are widely different in construction indicate the same frequency during detonation.

Detonation frequency measurements.—Oscillograms were made for each engine using the moving-coil indicator. Check runs were made in both cases using a "blind plug" adapted to shield the sensitive unit from gas pressures. Records made under this latter condition showed no trace of the cylinder resonance frequency. Frequency data to be used for comparison seems certain that variations from cycle to cycle in engine operation are responsible for a major portion of the observed dispersion of the points in the indicator record.

In order to obtain good values, only pressure data taken for crank angles well after top center were used in the calculations.

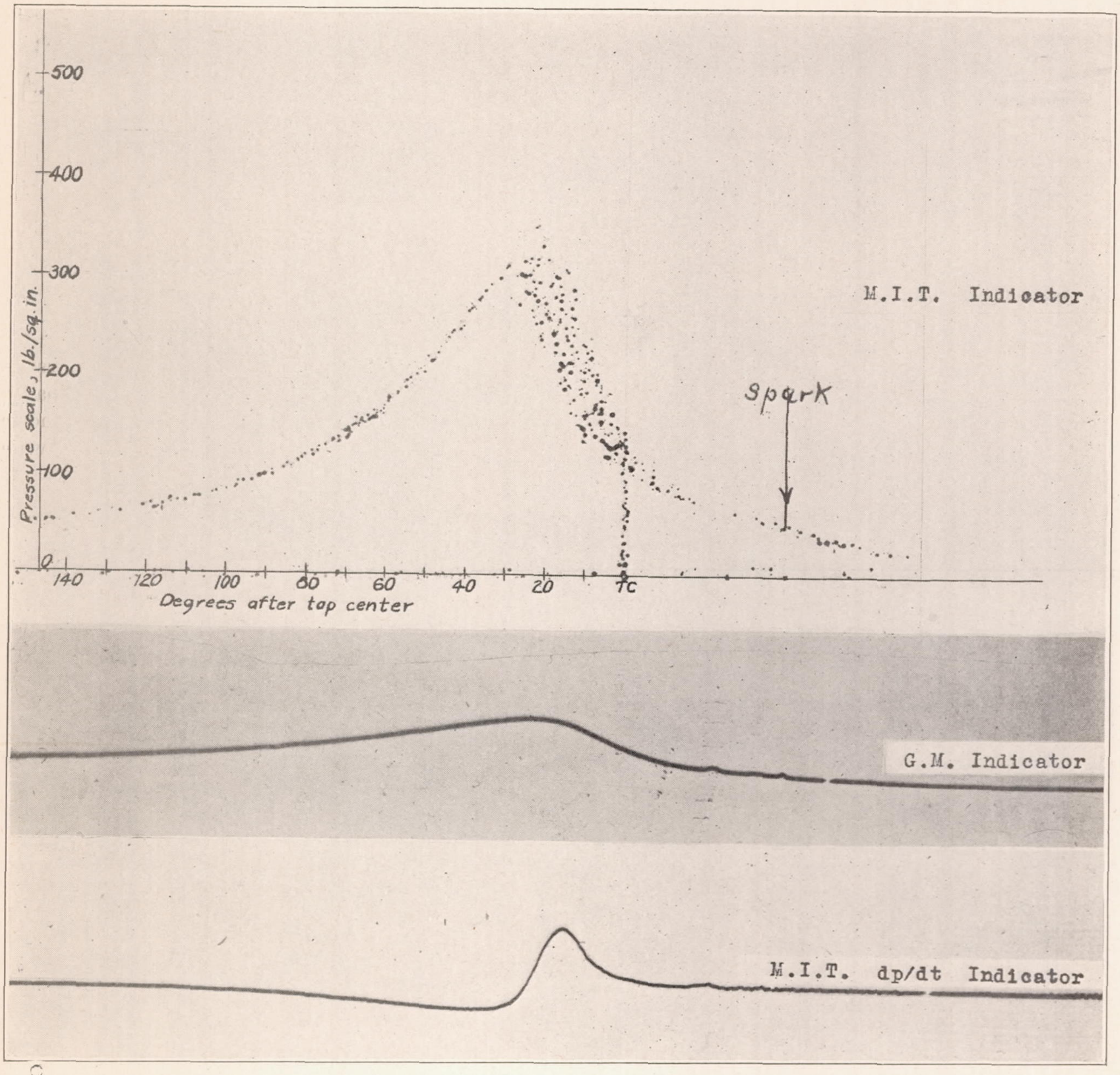


FIGURE 8.—Indicator records taken during normal engine operation. C.F.R. engine speed, 1,200 r.p.m.

with computed values were taken from the records of figure 11.

Pressure measurements.—Figures 8 and 9 show the type of records obtained with the M.I.T. indicator. The dispersion of points on the combustion line may be varied by mixture ratio adjustment and control of the engine intake. Tests on certain engines have shown records with well-defined combustion lines so it

Measurement of indicator natural frequency.—No means for determination of a complete frequency-response curve of the moving coil unit was available. However, it was found that the natural frequency of the diaphragm could be excited by the impact of a hardened-steel ball driven by compressed air.

Records made with the usual equipment showed the natural frequency of the diaphragm and coil system

to be between 10,000 and 11,000 cycles per second. In all cases the disturbance due to the impact of the ball was entirely damped out in about one-thousandth of a second. In an engine running at 1,800 revolutions per minute one-thousandth of a second corresponds to about 10° of crank travel. To make sure that the effect of any possible transient in the indicator diaphragm was negligible, data were taken from the records only after a minimum of 30 crank degrees had passed since the initial impulse.

The possible effect of temperature on the elastic properties of the indicator diaphragm was considered. It was found that the outer surface of a polished indicator diaphragm showed at worst a straw color after an extended period of operation in a detonating engine. The temperature corresponding to a straw "temper color" is less than 500° F., so it is valid to conclude that under operating conditions the diaphragm never reached this temperature. Experiments carried out at M.I.T. have shown that the variation in the elastic modulus of steel is less than 10 percent up to a temperature of 500° F. (reference 15). Since the elastic modulus enters as a square root into the expression for the natural frequency of a diaphragm (reference 7) the natural frequency of the moving system in the indicator could not have been affected by more than 5 percent at operating temperatures. On the basis of these results it is certain

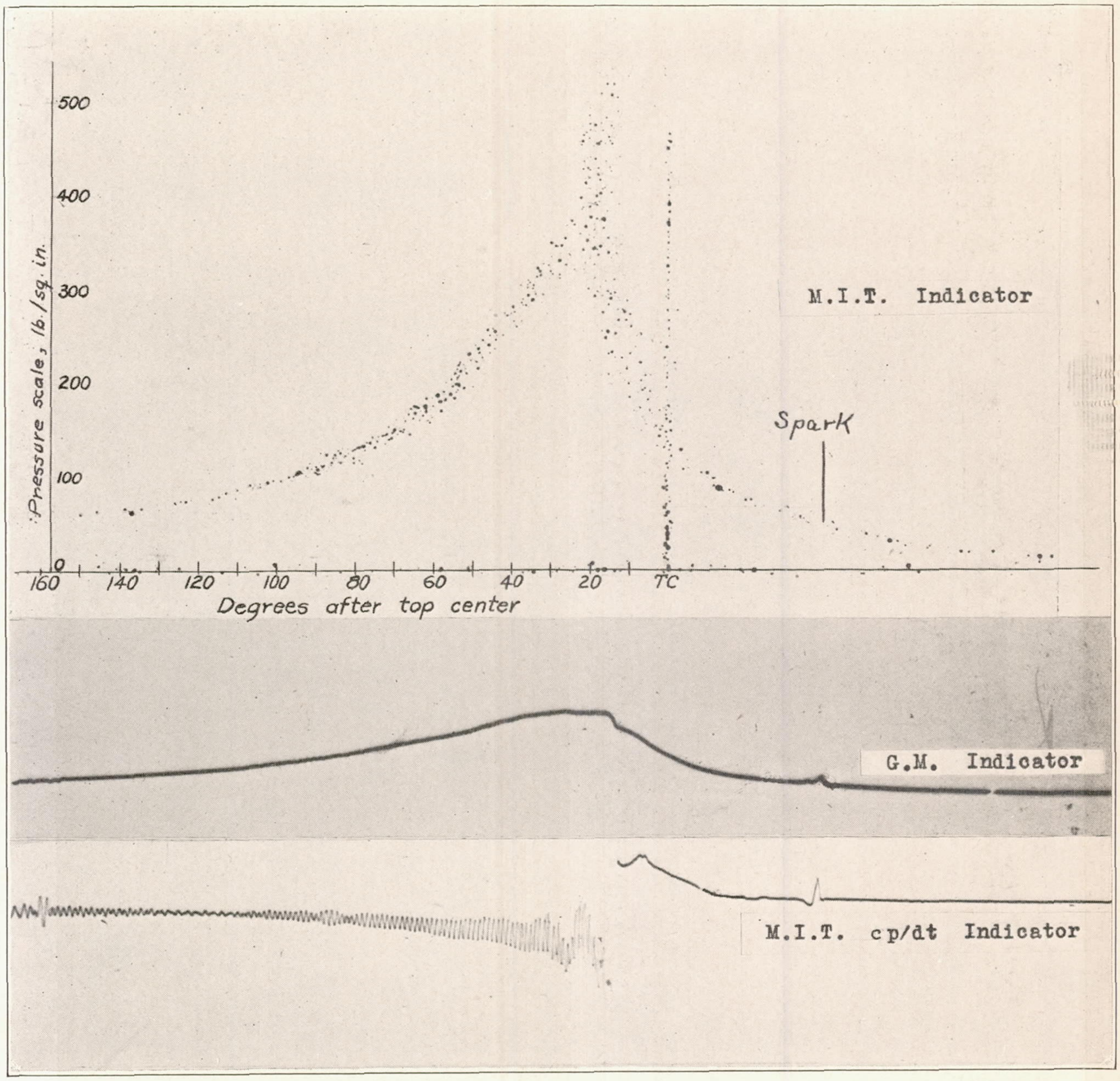


FIGURE 9.—Indicator records taken during detonation induced by addition of ethyl nitrite to the fuel. C.F.R. engine, 1,200 r.p.m.

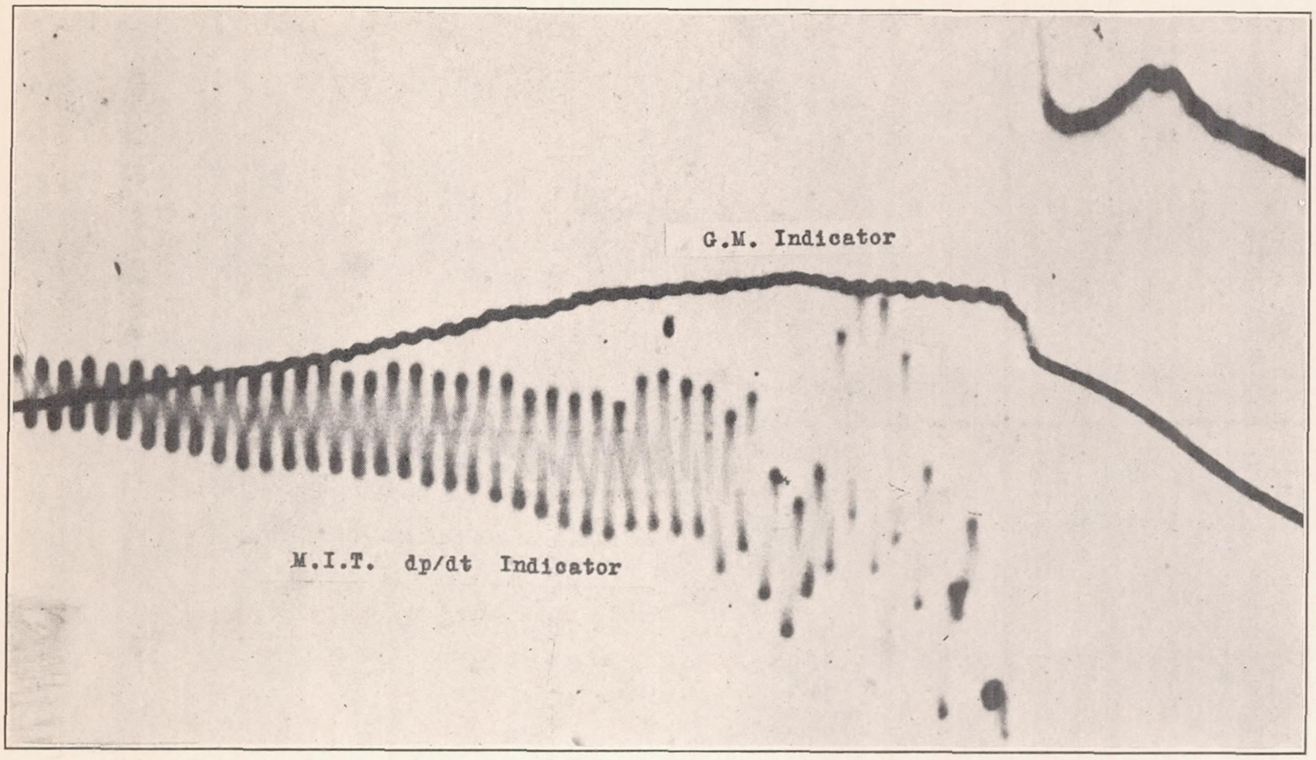


FIGURE 10.—Composite enlargement of records showing detonation frequency.

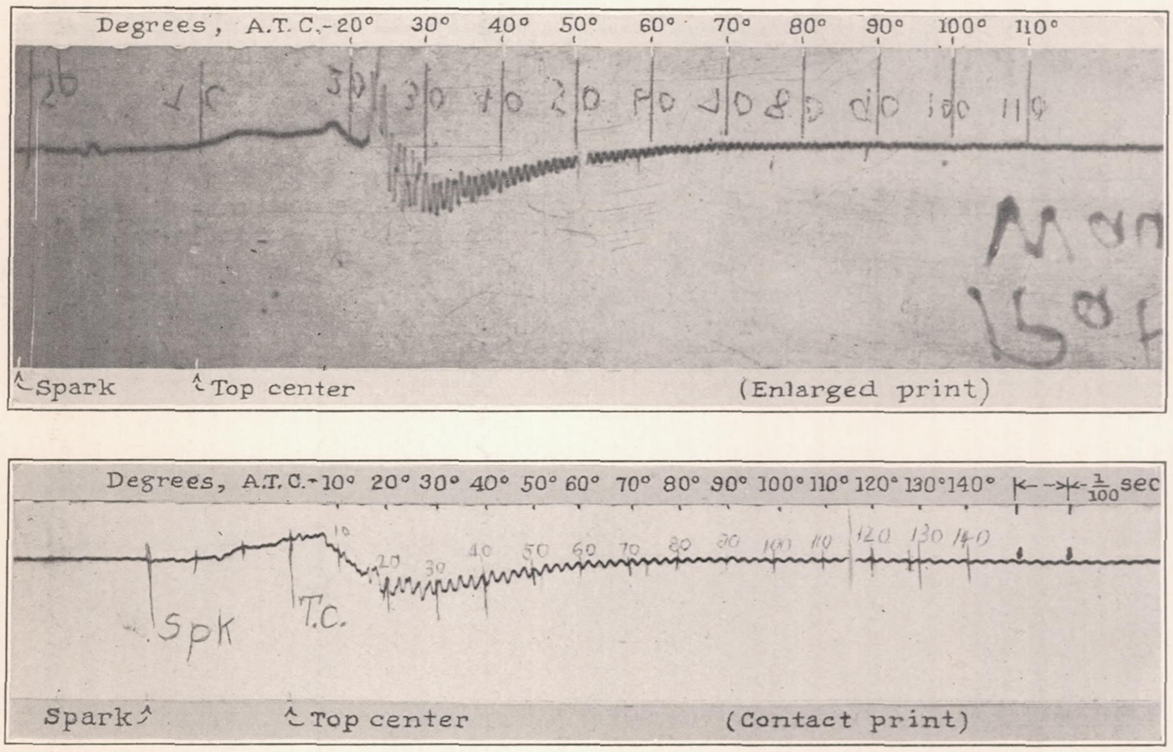
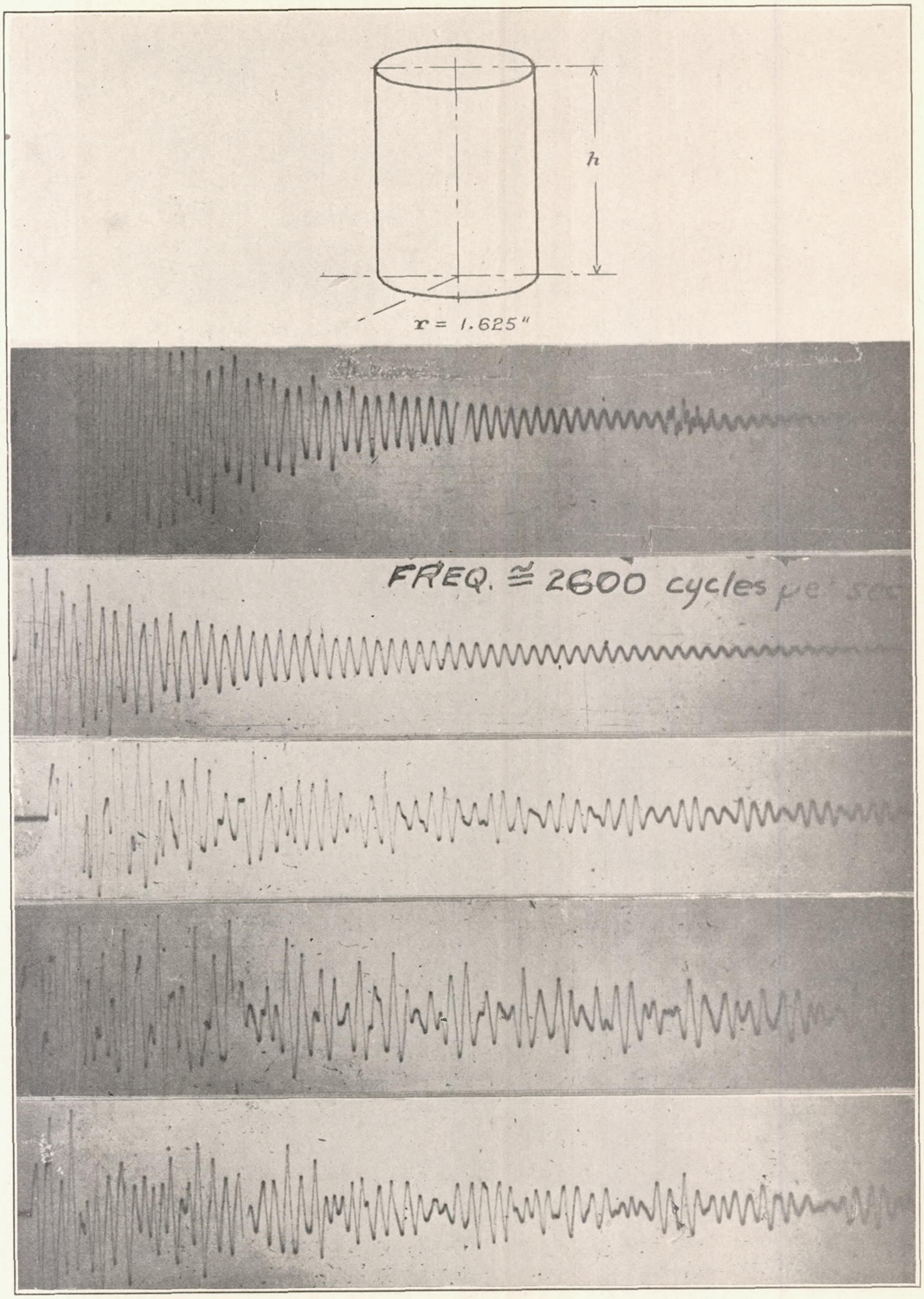


FIGURE 11.—C.F.R. engine. N.A.C.A. universal test engine. Detonation frequency records.



h=0.95″; crank angle=0°.

h=0.98″; crank angle=30°.

h=2.18″; crank angle=60°.

h=3.45″; crank angle=90°

h=4.45″; crank angle=120°.

FIGURE 12.—Recorded sound waves in cylinder due to explosion of primer.

NOTE.—The film speed used for these records was 225 inches per second.

that the natural frequency of the indicator diaphragm was well above any frequency found in the engine detonation records.

Measurement of cylinder-resonance frequency.—Determination of the vibration mode excited in the gas enclosed by a cylinder involves measurement of frequency under known conditions of pressure and density. This problem was attacked for the present case by explosion of the primer from a pistol shell within the engine cylinder. The primer was exploded near the cylinder wall with the moving-coil indicator located diametrically opposite. Records were made with air at atmospheric pressure for six different crank positions of the C.F.R. engine.

The oscillograms resulting from this procedure are shown in figure 12.

DISCUSSION OF DATA

Calculation of cylinder-resonance frequency on the basis of simple sound theory involves the use of equation (16). The necessary data are simultaneous values of equilibrium pressure p_0, equilibrium density ρ_0, ratio of specific heats γ, and wave length λ.

DETERMINATION OF WAVE LENGTH

Theory indicates that for a given mode, the wave length of standing waves in a vessel with rigid walls is a function only of the size and shape of the enclosure. Theory also shows the relation between wave length and frequency for the present case. It seems reasonable to assume that for a given chamber, the same mode of vibration will be excited by a certain type of disturbance, no matter what the constants of the enclosed elastic medium. On this basis it seemed probable that a pistol primer exploded near the cylinder wall would set up the same type of wave system as that excited by the rapid burning of a similarly located portion of the cylinder charge in detonation. For the operating engine, the excitation of pressure waves must occur with the piston near its upper position. To have an analogous case for the pistol-primer explosion the experiment should be carried out with the piston near its top-center position.

The two upper records of figure 12 show the frequency resulting from explosion of a pistol primer near the cylinder wall for crank angles less than 30°. The measured frequency is about 2,600 cycles per second. Figure 1 indicates that for corresponding conditions, the lowest longitudinal frequency is about 6,500 cycles per second. It follows that the mode actually excited is not a longitudinal mode. On the other hand, figure 2 shows that the frequency of one transverse mode is about 2,400 cycles per second, the next lowest frequency being about 4,000 cycles per second. Evidently the mode excited by the pistol primer is the first unsymmetrical mode with one nodal plane, as indicated in figure 3 (a).

The small discrepancy between the measured and calculated frequencies may be attributed to the increase in air temperature resulting from energy liberated by the pistol-primer explosion.

The theory indicates that the longitudinal frequencies are not necessarily multiples of the transverse frequencies, so the wave form for a case in which both transverse and longitudinal frequencies are excited will not repeat itself on the record. The nature of the phenomenon thus makes exact frequency estimates difficult; however, the oscillograms of figure 12 for chamber heights of 2.2 and 3.5 inches show beat frequencies of about 550 cycles per second and 650 cycles per second, respectively. If it is assumed in each case that the beats are due to combination of two frequencies, one of which is 2,600 cycles per second, the second frequency would be $2{,}600 \pm 550 = 3{,}150$, or 2,050 cycles per second for $h=2.2$ inches and $2{,}600 \pm 650 = 3{,}250$, or 1,950 cycles per second for $h=3.5$ inches. Figure 1 shows the simple longitudinal mode corresponding to a frequency of 3,200 cycles per second for the first case (corresponding to the higher frequency) and of 1,950 cycles per second in the second case (corresponding to the lower frequency). The experimental results thus show that, for a chamber of height comparable with the cylinder diameter, both longitudinal and transverse modes are actually excited.

If the first unsymmetrical mode of vibration is excited with the piston near its top-center position, it is reasonable to expect that this mode would persist as the piston travels downward. For the case in which no further excitation occurs, the simple wave form of the pressure-time record should be preserved during the piston descent.

On the basis of the considerations outlined above, detonation pressure-wave frequency was assumed to be due to the first unsymmetrical transverse mode. The corresponding wave length as computed by equation (15) was used in the frequency calculations. This wave length depends simply on the cylinder diameter and will introduce no appreciable uncertainty in the calculated frequency.

DETERMINATION OF CHARGE DENSITY

For any given cycle in the 4-stroke-cycle engine, the cylinder charge is composed of exhaust gas left in the clearance space from the preceding cycle, air from the atmosphere, and fuel vapor. Once the intake valve is closed, the weight of gas within the cylinder is constant throughout the compression and power strokes except for leakage. No experimental work on the problem of leakage was carried out in the present case. In the absence of better information, it was assumed that leakage did not affect the results more than 2 percent.

Computation of the weight of exhaust gas left in the clearance space involves a knowledge of the molecular weight of this gas and its temperature. Following the data given by Pye in reference 2, 900° C. seems to be a reasonable value for the temperature. Study of a number of exhaust-gas analyses leads to the selection of 28 as a good average value for the molecular weight. (See reference 16.) The residual gases furnish somewhat less than one-tenth of the total cylinder charge so that an assumed 10-percent error in their quantity would introduce about 1-percent error in the final density.

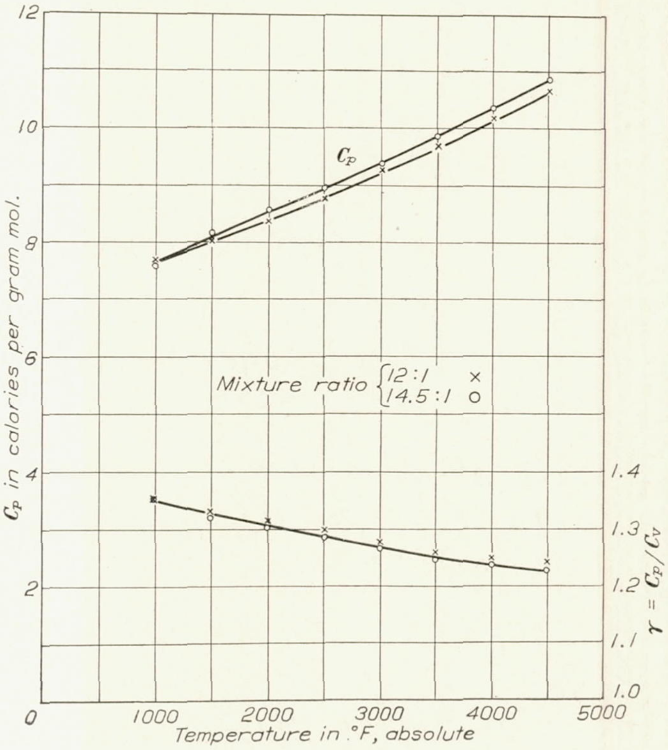


FIGURE 13.—Variation of C_p and γ with change of temperature and mixture ratio. Data taken from reference 15. Gas composition:

Mixture	12:1	14.5:1
CO_2	8.2	11.7
O_2	.5	1.0
CO	6.1	1.0
CH_4	.6	.1
H_2	2.8	.2
N_2	68.8	73.4
H_2O	13.0	12.6

For the C.F.R. engine experiments, an orifice was used in air-flow measurements. The resulting data are certainly accurate to within 10 percent. A Roots blower type of air meter was used in the N.A.C.A. engine work. The results should be valid to within 2 percent.

Fuel-flow measurements were made in the usual manner and should introduce less than 1 percent error in the final result.

Assuming that each uncertainty is as likely to be in one direction as the other, the resultant uncertainty in density will be equal to the square root of the sum of the squares of the component uncertainties. (See reference 17.)

For the C.F.R. engine test the uncertainty in density will be about 11 percent, while for the N.A.C.A. engine the corresponding uncertainty will be about 3 percent.

DETERMINATION OF PRESSURES

Pressures may be read from the M.I.T. indicator records to within 5 pounds over the portion of the cycle adopted for study. Pressure magnitudes are above 100 pounds per square inch absolute. It follows that the uncertainty in pressure measurement is less than 5 percent.

DETERMINATION OF RATIO OF SPECIFIC HEATS

The ratio of specific heats may be written

$$\gamma=\frac{C_p}{C_v}=\frac{C_p}{C_p-1.99} \tag{17}$$

where C_p and C_v are expressed in gram calories per gram mol. (See reference 2.)

Data are available on C_p as a function of temperature for various gases (reference 18). For temperatures in excess of 1,800° C., the uncertainty in C_p is rather large. However, for temperatures less than this value, Eastman (reference 18) estimates that the uncertainties in the values of C_p for the gases CO_2, H_2O, N_2, CO, etc., are all less than 3 percent. Calculations in the present report were limited to a range of the expansion stroke for which temperatures estimated from the gas law were between 1,600° and 2,100° C. Under these circumstances, it seems reasonable to assume that the C_p values for the cylinder charge are known to within 6 percent.

An estimate of the uncertainty in the final result, made in the usual manner, shows that C_p/C_v should be within about 2 percent of its true value. (See reference 17.)

Figure 13 is a plot, based on Eastman's data, of C_p and γ against temperature.

DIRECT MEASUREMENT OF FREQUENCIES

Data were taken from the records of detonation pressure-wave frequencies only for crank angles well after the initial exciting impulse. This procedure eliminated the effects of possible transients in the indicating apparatus.

In making frequency estimates, the record was measured over a period from 5° before to 5° after the crank angle considered. Since the variation in frequency over a 10° crank interval was of the order of 2 percent, the procedure used could not introduce any very serious error in the final result.

A microscope comparator was used to measure frequencies from the records. The N.A.C.A. engine records were taken with a film speed of about 225 inches per second. At this speed it is possible to obtain check frequency readings to about 2 percent.

The C.F.R. engine records were made with somewhat lower film speeds and the uncertainty in the measured frequencies is about 5 percent.

PRECISION ESTIMATE

Equation (16) shows that ρ_0, p_0, and γ each occur under a square-root sign in the calculation of pressure-wave frequency. It follows (reference 17) that a 1-percent uncertainty in any one of the three quantities will introduce but one-half-percent uncertainty in the computed value of frequency. Using the assumption of equally likely positive and negative errors in each case, the resultant uncertainty in frequency will be the square root of the sum of the squares of the individual components. Table I is a summary of precision data for both the experimental cases.

The resultant uncertainty of frequency calculated from the C.F.R. engine data is about 7 percent. The corresponding uncertainty in calculated frequency for the N.A.C.A. engine is about 4 percent.

DISCUSSION OF RESULTS

Table II is a summary of results from the N.A.C.A. engine data. The second, third, and fourth columns give the estimated values of ρ_o, p_o, and C_p/C_v while the last two columns contain the calculated and observed frequencies for the same crank angles.

Figure 14 contains plots of measured and calculated frequencies against crank angle for both the experimental cases. Certain points on the C.F.R. engine frequency curve were checked 2 or 3 times in separate runs with alterations in the indicating system. The results of these individual runs are shown as distinct points on the plot.

Calculated values agree with the experimental values within an average of about 6 percent for the C.F.R. engine curves and within 3 percent for the N.A.C.A. engine curves. The results of experiment thus check with the predictions of simple sound theory within the experimental uncertainty for two engines with a relatively large difference in cylinder size.

CONCLUSIONS

The work outlined leads to the following conclusions with regard to detonation in the internal combustion engine:

(1) Detonation is accompanied by pressure waves within the cylinder charge.

(2) These pressure waves follow the laws of simple sound theory.

(3) Resonance phenomena within the cylinder charge determine the pressure-wave frequencies.

GENERAL DISCUSSION

At first glance it seems remarkable that the experimental procedure should produce results agreeing so closely with the predictions of an approximate mathematical treatment. However, the assumptions used in development of simple sound theory are similar to the approximations made when Hooke's law is used in the theory of elasticity. In both cases the essence of the matter is that attention is limited to relatively small disturbances from the equilibrium state. Applied to the present study, this means that for not too severe conditions, the pressure changes involved in the cylinder pressure waves are small compared to the absolute pressure existing in the cylinder. The oscillograph records used for frequency determinations are of little value for direct study of pressure mag-

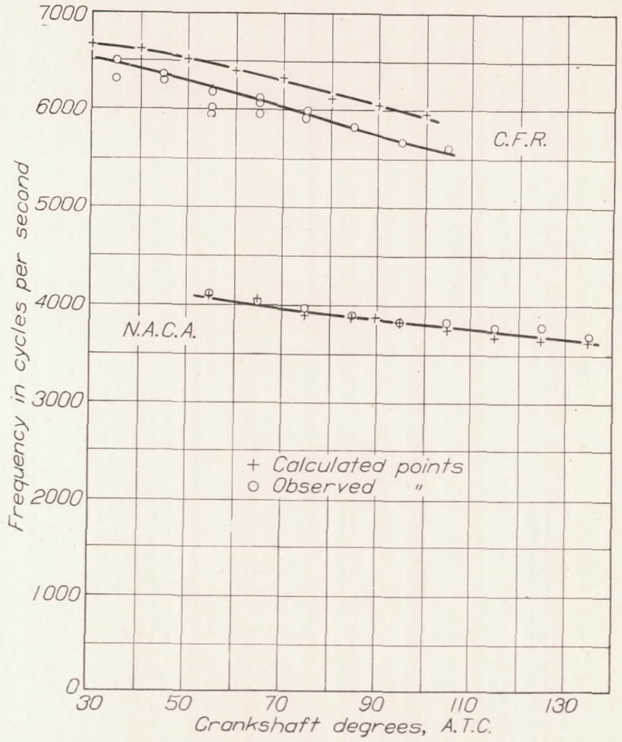


FIGURE 14.—Comparison of calculated and observed detonation sound frequencies. Waukesha C.F.R. engine: Bore=3.25 in.; stroke=4.5 in. N.A.C.A. engine: $\frac{\text{con. rod length}}{\text{crank throw}}$=3.32 in.; compression ratio=5.5; bore=5 in.; stroke=7 in.; speed=1,540 r.p.m. Air consumption=4.89 lb./min. Fuel consumption=0.313 lb./min. Assumed molecular weight of exhaust gas=28; temperature of exhaust gas=900° C.

nitudes in the waves. However, a reasonably good estimate of the pressures existing in the wave front has been made by Mr. John P. Elting (see appendix), who bases his result on the amplitude of particle motion in the wave as shown in flame photographs. Mr. Elting's work indicates that the maximum pressure above equilibrium in the wave is about 25 pounds per square inch. Comparing this with an equilibrium pressure of 415 pounds per square inch absolute, shows the wave involves changes which are of the order of 5 percent of the total pressure. This evidence from an independent experimental method confirms the predictions of sound theory.

An examination of the initial pressure rise accompanying detonation requires a knowledge of the indicator-system response to sudden nonrepeating pressures.

It is probable that theoretical treatment of the initial disturbance and of the problems entering under conditions of intense detonation will necessitate methods taking into account relatively high wave-front pressures.

It is obvious from the theoretical discussion that any change in the size or shape of the combustion chamber will alter the resulting pressure-wave record. Under the limitations of present mathematical knowledge, the range of forms for which a rigorous theoretical treatment may be developed is rather small. Studies of free liquid surfaces in models might yield interesting results for combustion chambers of irregular shapes. If the matter proves to be of sufficient importance, it is probable that graphical methods similar to those of optics might be developed.

Theory predicts that the pressure-wave system excited in a particular case will depend upon the location of the initial impulse. It is thus to be expected that the shape of the pressure-wave records for a given cylinder will depend upon the location of the detonating portion of the charge; hence upon the number and relative positions of spark plugs. Since the wave system exists in the cylinder charge for a comparatively long time, any general motion of the charge will alter the relation between a necessarily stationary indicator and the nodal surfaces of the waves. The resulting record will be complicated by the variation in wave amplitude due to orientation changes, in addition to the normal decrease between successive cycles.

A paper published early in 1933 by Professor Wawrziniok (reference 19) describes certain experiments carried out on the noises accompanying explosions of fuels. This reference was received after the work reported above was completed.

The experiments of Professor Wawrziniok were carried out in closed chambers, the sound frequencies being measured with suitable apparatus placed in the free air outside the chamber. These chambers had lengths somewhat longer than their diameters and the ignition spark occurred near the middle of one end wall. Additional tests were made on sound frequencies resulting from a single fuel explosion within the cylinder of an engine.

Sound frequencies observed outside the closed chamber corresponded to resonant longitudinal waves within the chamber. The engine experiments led to the conclusion, "* * * the maximum length of the combustion chamber seems to be the determining factor in the development of noise."

No quantitative data are available at present on the rates of change of pressure responsible for pressure-wave excitation in a detonating engine, so no comparison can be made with the pressure changes measured in the closed chamber experiments. However, once the exciting impulse has subsided the resulting system of pressure waves in the engine should be determined by the same laws as in the closed chamber. On this basis, the results of the writer may be compared with those of Professor Wawrziniok.

The records of figure 12 that were taken from pistol-primer explosions in the C.F.R. engine cylinder for various piston positions were made under "closed-chamber" conditions. For the cases in which cylinder diameter was over three times the chamber height, the record is due to transverse vibrations. As the cylinder height becomes approximately equal to the diameter, longitudinal frequencies appeared in addition to the transverse vibrations. These observations check nicely with the conclusions of Professor Wawrziniok and supply the amendment that chamber shape as well as maximum chamber length will affect the sound record.

A systematic investigation of the effect of spark-plug location on the pressure-wave systems both in fixed bombs and in engine cylinders should furnish valuable information in connection with the general problem.

The various problems suggested above are now being studied at the Massachusetts Institute of Technology.

Massachusetts Institute of Technology,
Cambridge, Mass., *October 1933.*

APPENDIX

CALCULATION OF PRESSURES IN THE SOUND WAVE FOLLOWING DETONATION

By John P. Elting

The general theory of sound provides a means of calculating the pressure existing in a sound wave if the displacement of the particles can be determined. A flame record that shows the sound trace after detonation permits such a measurement. The frequency and the velocity of the sound wave may also be determined by a knowledge of the film speed and the length of the combustion chamber.

A flame record is presented (fig. 15) showing detonation for which data were taken. An L-head engine was used having a shallow combustion chamber with rectangular boundaries. For a plane wave, the difference in pressure between the average and that in the condensation of the wave is given by:

$$P-P_0=\frac{2\pi\gamma n}{c}P_0x$$

(see reference 1, and reference 20),

where n is the frequency
c is the velocity of sound
$\gamma=\frac{C_p}{C_v}$, ratio of specific heats
P_0 is the average pressure
P is the pressure in the condensation
x is the displacement of the particles

In order to measure these quantities from the flame record, we see that:

$$c=2L\left(\frac{d}{TD}\right)$$

where
L is the length of the combustion chamber
d is the distance between timing marks
T is the time interval represented by d
D is the separation of successive wave traces
and so

$$n=\frac{d}{TD}$$

Hence

$$P-P_0=\pi\frac{\gamma}{L}P_0x$$

where we remember that x is the actual displacement of the particles from equilibrium in the combustion chamber.

The displacement x is considered to be equal to one-half the apparent motion of the flame. A mean value for several of the flames was obtained by direct measurement on the film with a comparator. Likewise, the magnification ratio was also determined.

The following quantities were obtained:

Actual displacement of particles=0.12 inch
Length of combustion chamber=7.00 inches
Pressure from M.I.T. indicator (approx.)=400 lb./sq.in.
The ratio of specific heats taken from figure 13=1.27

The computation:

(1.27) (3.14) (400) (0.12)/(7)=27.4 pounds per square inch. This result shows that the order of magnitude of the pressures existing in the condensation of the sound wave is about 25 to 30 pounds per square inch above the average pressure existing in the combustion chamber after detonation.

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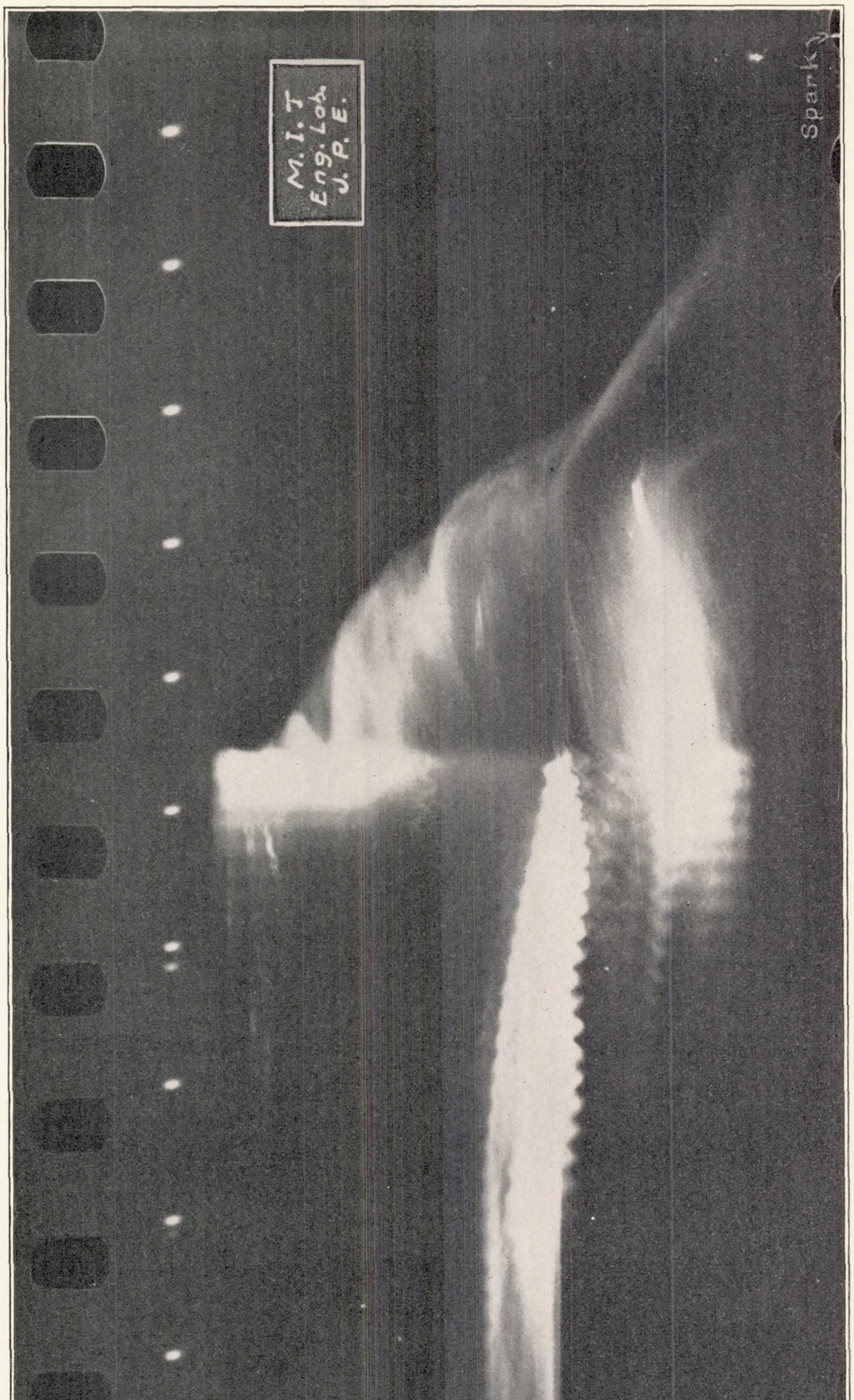

FIGURE 15.—Flame record showing detonation. Timing flashes every 0.002 second. Frequency of wave about 2,600 per second.

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TABLE I.—PRECISION OF EXPERIMENTAL DATA

C.F.R. engine			N.A.C.A. engine	
Quantity	Uncertainty in quantity	Uncertainty introduced in frequency	Uncertainty in quantity	Uncertainty introduced in frequency
C_p/C_v	0.02	0.01	0.02	0.01
p_0	.05	.03	.05	.03
ρ_0	.11	.06	.03	.02
λ	0	0	0	0

TABLE II.—N.A.C.A. ENGINE—SUMMARY OF DATA AND RESULTS

Crank angle	Charge density	Charge pressure	$\frac{C_p}{C_v}$	Sound velocity	Sound frequency [1] (cycles/second)	
					Calculated	Observed
Degrees	*Lb./cu. ft.*	*Lb./sq. in.*		*Ft./sec.*		
55	0.183	270	1.24	2,900	4,090	4,120
65	.156	225	1.25	2,870	4,050	4,040
75	.136	180	1.25	2,760	3,890	3,960
85	.119	155	1.26	2,740	3,860	3,880
90	.112	145	1.26	2,740	3,860	--------
95	.106	135	1.26	2,720	3,830	3,840
105	.097	120	1.26	2,660	3,750	3,820
115	.090	105	1.27	2,610	3,680	3,780
125	.085	98	1.27	2,590	3,650	3,760
135	.080	90	1.27	2,560	3,610	3,690

[1] The calculated frequency is based on the first transverse mode ($s=1$, $g=0$, $\beta=1.13$).

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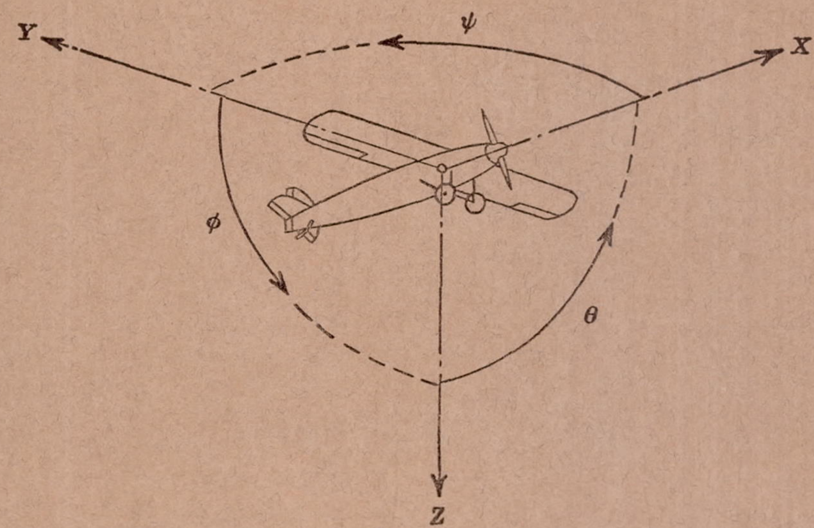

Positive directions of axes and angles (forces and moments) are shown by arrows

Axis		Force (parallel to axis) symbol	Moment about axis			Angle		Velocities	
Designation	Symbol		Designation	Symbol	Positive direction	Designation	Symbol	Linear (component along axis)	Angular
Longitudinal	X	X	Rolling	L	$Y \longrightarrow Z$	Roll	ϕ	u	p
Lateral	Y	Y	Pitching	M	$Z \longrightarrow X$	Pitch	θ	v	q
Normal	Z	Z	Yawing	N	$X \longrightarrow Y$	Yaw	ψ	w	r

Absolute coefficients of moment

$C_l = \frac{L}{qbS}$ (rolling) $C_m = \frac{M}{qcS}$ (pitching) $C_n = \frac{N}{qbS}$ (yawing)

Angle of set of control surface (relative to neutral position), δ. (Indicate surface by proper subscript.)

4. PROPELLER SYMBOLS

D, Diameter

p, Geometric pitch

p/D, Pitch ratio

V', Inflow velocity

V_s, Slipstream velocity

T, Thrust, absolute coefficient $C_T = \frac{T}{\rho n^2 D^4}$

Q, Torque, absolute coefficient $C_Q = \frac{Q}{\rho n^2 D^5}$

P, Power, absolute coefficient $C_P = \frac{P}{\rho n^3 D^5}$

C_s, Speed-power coefficient $= \sqrt[5]{\frac{\rho V^5}{P n^2}}$

η, Efficiency

n, Revolutions per second, r.p.s.

Φ, Effective helix angle $= \tan^{-1}\left(\frac{V}{2\pi rn}\right)$

5. NUMERICAL RELATIONS

1 hp. = 76.04 kg-m/s = 550 ft-lb./sec.

1 metric horsepower = 1.0132 hp.

1 m.p.h. = 0.4470 m.p.s.

1 m.p.s. = 2.2369 m.p.h.

1 lb. = 0.4536 kg.

1 kg = 2.2046 lb.

1 mi. = 1,609.35 m = 5,280 ft.

1 m = 3.2808 ft.