RELATIVE LOADING ON BIPLANE WINGS OF UNEQUAL CHORDS

By WALTER S. DIEHL

SUMMARY

It is shown that the lift distribution for a biplane with unequal chords may be calculated by the method developed in N.A.C.A. Technical Report No. 468 if corrections are made for the inequality in chord lengths. The method is applied to four cases in which the upper chord was greater than the lower and good agreement is obtained between observed and calculated lift coefficients.

INTRODUCTION

In reference 1 it was shown that for conventional biplane arrangements the lift coefficient for the upper wing is given by

$$C_L = C_L + \Delta C_{LU}$$ (1)

and the lift coefficient for the lower wing by

$$C_L = C_L + \Delta C_{LL}$$ (2)

where $C_L$ is the biplane lift coefficient and $\Delta C_{LU}$ and $\Delta C_{LL}$ are lift coefficient increments for the upper and lower wings, respectively. It was also shown that $\Delta C_{LU}$ and $\Delta C_{LL}$ are connected by the relation

$$\Delta C_{LU} = -\Delta C_{LL} S_U S_L$$ (3)

where $S_U$ and $S_L$ are the areas of the upper and lower wings, respectively.

$\Delta C_{LU}$ is given by an equation of the form

$$\Delta C_{LU} = K_1 + K_2 C_L$$ (4)

where the constant $K_1$ is a function of gap, chord, wing thickness, stagger, decalage, and overhang and the constant $K_2$ is a function of stagger, gap, chord, span, decalage, and overhang. Equations and charts in reference 1 enable the determination of $K_1$ and $K_2$ for any biplane with equal chords. Application of this method to biplanes with extreme differences in chords and spans has indicated considerable discrepancies between the calculated and observed values. A further study of the problem in the light of some rather limited test data indicates that a simple correction for the ratio of the wing chords will bring the calculated and experimental values into excellent agreement and that a chord correction should therefore be incorporated as an integral part of the general method.

In the discussion that follows the symbols used will be the same as in reference 1.

THE EFFECT OF WING CHORD ON $K_1$

When there is no stagger, decalage, or overhang the value of $K_1$ in equation (4) is a function of the ratio of wing thickness to gap. This basic value of $K_1$ may be designated $K_{10}$. It is due principally to the restriction in area which increases the velocity and decreases the static pressure between the wings. The curve of $K_{10}$ against the ratio of wing thickness to gap given in figure 1 is the same as figure 9 of reference 1. This curve was based on biplanes of equal chords but it will apply to any biplane if the thickness of the lower wing is used in determining the ratio $t/g$ and if the necessary correction is made to transfer the coefficient to the upper wing.

The first condition is met by using the gap-chord ratio referred to the chord of the lower wing, so that

$$\frac{t}{g} = \left(\frac{t}{c_L}\right) + \left(\frac{g}{c_L}\right).$$

The transfer on the coefficient basis requires division by the ratio of areas, lower to upper ($S_L/S_U$) since by definition the $C_L$ for the cellule is so adjusted between the individual values. This means, however, that the correction must be made on the basis of the relative...
chords since the value of $K_{10}$ assumes no overhang. Consequently, to find the value of $K_{10}$ for a biplane having upper and lower chords of $c_u$ and $c_l$, read the

value of $\Delta K_1$ from figure 1 and correct according to the ratio of the chords, or

$$K_{10} = \Delta K_1 \times \left( \frac{c_u}{c_l} \right)$$

(5)

The effect of stagger on $K_1$ may be designated $K_{11}$ and it is given in figure 2 by the curve of $\Delta K_1/s$ as a function of $t/G$ or the ratio of thickness to gap. The curve of figure 2 is the same as that of figure 10 in reference 1 and is based on biplanes of equal chord. It may be applied to any biplane if the value of $t/G$ is based on lower wing thickness and a chord correction is made as in the calculation of $K_{10}$. The stagger should be measured between the $1/4$ chord points at zero lift and referred to the chord of the lower wing.

The value of $K_{11}$ is then given by

$$K_{11} = \Delta K_1 \times s \times \left( \frac{c_u}{c_l} \right)$$

(6)

where $s$ is the stagger in terms of the chord of the lower wing.

The effect of decalage on $K_1$ varies with gap-chord as shown by figure 3, which is the same as figure 17 of reference 1. This curve is based on biplanes with equal chords but it may be applied to any biplane if the chord correction is used. As before, a chord correction is equivalent to an area correction since the

$K_{12}$ = $\frac{\Delta K_1}{\delta} \times \delta \times \left( \frac{c_u}{c_l} \right)$

(7)

where $\Delta K_1/\delta$ is read from figure 3.

The effect of overhang on $K_1$ may be denoted by $K_{13}$. In figure 21 of reference 1, contour curves were given of $K_1$ against overhang. These curves were to be used by entering at zero overhang with the value of $K_1$ obtained by adding $K_{10} + K_{11} + K_{12}$ and passing along the appropriate contour to the desired overhang. In this manner the value of $K_{13}$ was not determined directly. Since $K_{13}$ is subjected to the same chord correction as the preceding factors, it is desirable to replot the data as in figure 4, giving the value of $K_{13}$ directly. For any biplane the value of $K_{13}$ is then obtained from

$$K_{13} = \Delta K_1 \times \left[ \frac{c_u}{c_l} \right]$$

(8)
The final value of $K_1$ is now obtained by addition of the four factors

$$K_1 = K_{10} + K_{11} + K_{12} + K_{13}$$ \hspace{1cm} (9)

**THE EFFECT OF WING CHORD ON $K_1$**

The basic value of $K_1$ in equation (4) is determined by stagger. For biplanes with individual wings of aspect ratio 6 and equal chords, zero decalage and no overhang it was shown in reference 1 that

$$K_{20} = 0.050 + 0.17 \left( \frac{\delta}{c} \right)$$ \hspace{1cm} (10)

![Graphical representation of $K_1$ vs aspect ratio and gap-chord ratio.](image)

**Figure 5.-Effect of gap and aspect ratio on $K_1$ for equal chords.**

To apply this equation to any biplane the stagger should be measured between the $\frac{1}{4}$ chord points at zero lift and referred to the chord of the lower wing $c_L$. The basic value of $K_{20}$ should then be multiplied by the chord ratio or

$$K_{20} = \left[ 0.050 + 0.17 \left( \frac{\delta}{c_L} \right) \right] \times \frac{c_L}{c_0}$$ \hspace{1cm} (11)

The influence of aspect ratio and gap-chord ratio is combined in a factor $F_2$ which may be read from figure 5. Figure 5 is the same as figure 12 in reference 1. In finding $F_2$ the gap-chord ratio should be based on the lower wing.

The effect of decalage on $K_2$ may be denoted by $K_{21}$. In reference 1 it was shown that for the equal chord biplane

$$K_{21} = +0.0186 \delta^\circ$$ \hspace{1cm} (12)

where $\delta$ is the angle between the zero lift lines of the wings, considered positive when these intersect for-ward of the leading edge. When the chord lengths differ $K_{21}$ should be corrected accordingly, to give

$$K_{21} = 0.0186 \delta^\circ \times \left[ \frac{c_L}{c_0} \right]$$ \hspace{1cm} (13)

In reference 1 the effect of overhang on $K_3$ was given for the equal-chord biplane by figure 21 which consisted of a series of contour lines of $K_3$ plotted against overhang. To use these curves it was necessary to find $K_3 = (F_3 \times K_{20}) + K_{21}$ for zero overhang and
passing along this contour to the desired overhang. The actual value of the effect of overhang which may be denoted by \( K_2 \) was not directly determined. Since \( K_2 \) is subjected to the same chord correction as the previous factors, it is desirable to replot figure 21 of reference 1 so that \( K_2 \) can be read directly, as in figure 6. For any biplane the value of \( K_2 \) is obtained by

\[
K_2 = \Delta K_2 \times \left[ \frac{\epsilon}{\epsilon_0} \right]
\]

where \( \Delta K_2 \) is the overhang correction for equal chords as read from figure 6.

The value of \( K_3 \) in equation (4) is then obtained by addition of the three corrected terms

\[
K_3 = [F_2 \times K_2] + K_2 + K_2
\]

COMPARISON OF CALCULATED AND OBSERVED DATA

Available load distribution tests on biplanes with unequal chords are limited to four cases. Reference 2 reports tests on a biplane having the following characteristics:

Upper wing: span \( b_u = 36 \) inches, chord \( c_u = 6 \) inches

Lower wing: span \( b_l = 24 \) inches, chord \( c_l = 4 \) inches

Gap: \( G = 4\frac{3}{4} \) inches, section R.A.F. 15

Stagger 20° on leading edge at \( \alpha = \) 0°, or 1.12 inches between \( \frac{1}{4} \) chord point at zero lift.

\[
\text{Overhang} = \frac{36 - 24}{36} = 0.33
\]

From the above:

\[
\begin{align*}
\frac{s}{c_l} = \frac{1.12}{4.0} &= 0.28 \\
\frac{G}{c_u} &= \frac{4.5}{4.0} = 1.125 \\
\frac{t}{c} &= 0.070 \\
\frac{c_l}{c_u} &= 0.28 \\
\frac{c}{c_u} &= 0.3
\end{align*}
\]

From figure 1 and equation (5)

\[
K_{10} = -0.005 \times \frac{2}{3} = -0.0033
\]

From figure 2 and equation (6)

\[
\frac{\Delta K_1}{(\sigma/c_l)} = 0.010 \quad K_{11} = 0.010 \times 0.28 \times \frac{2}{3} = +0.0010
\]

From figure 4 and equation (8)

\[
\Delta K_3 = -0.025 \quad K_{13} = (-0.025) \times \frac{2}{3} = -0.0167
\]

Hence \( K_i = -0.0033 + 0.0019 - 0.0167 = -0.018 \)

From equation (11)

\[
K_{20} = [0.050 + (0.17 \times 0.28)] \times \frac{2}{3} = 0.065
\]

From figure 5, \( F_2 = 0.90 \)

\[
F_2 \times K_{20} = 0.90 \times 0.065 = 0.058 \quad K_{21} = 0
\]

From figure 6, \( \Delta K_4 = 0.096 \)

From equation (14), \( K_{22} = 0.096 \times \frac{2}{3} = 0.062 \)

Hence, \( K_4 = 0.058 + 0.062 = 0.120 \)

and

\[
\Delta C_{L_0} = -0.018 + 0.120 C_L
\]

The test data are as follows:

<table>
<thead>
<tr>
<th>Angle of attack</th>
<th>(-4.25°)</th>
<th>(-0.25°)</th>
<th>(3.75°)</th>
<th>(7.75°)</th>
<th>(11.75°)</th>
<th>(15.75°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upper wing</td>
<td>(C_{N_{U}})</td>
<td>(-122)</td>
<td>(+178)</td>
<td>(+488)</td>
<td>(+756)</td>
<td>(+1,016)</td>
</tr>
<tr>
<td>Lower wing</td>
<td>(C_{N_{L}})</td>
<td>(-076)</td>
<td>(+140)</td>
<td>(+374)</td>
<td>(+550)</td>
<td>(+704)</td>
</tr>
<tr>
<td>Biplane</td>
<td>(C_N)</td>
<td>(-108)</td>
<td>(+166)</td>
<td>(+463)</td>
<td>(+692)</td>
<td>(+920)</td>
</tr>
<tr>
<td>(\Delta C_{N_{U}})</td>
<td>(-014)</td>
<td>(+012)</td>
<td>(+038)</td>
<td>(+064)</td>
<td>(+090)</td>
<td>(+1,050)</td>
</tr>
</tbody>
</table>

These values of \( \Delta C_{N_{U}} \) are plotted against \( C_N \) on figure 7. Two points calculated from equation (10) are given on figure 7 and it will be noted that the agreement is satisfactory. The equation of \( \Delta C_{N_{U}} \) from the experimental data is \( \Delta C_{N_{U}} = -0.007 + 0.106 C_N \), which may be compared with equation (16).

Reference 3 reports tests on a biplane differing from the one preceding only in the overhang, the spans being
equal in this case. The aspect ratio for the upper wing was 6, for the lower wing 9, average 7.5.
\[K_{10} = -0.0033 \text{ as for first arrangement.} \]
\[K_{11} = +0.0019 \text{ as for first arrangement.} \]
\[K_{12} = 0 \text{ (no decalage).} \]
\[K_{13} = 0 \text{ (no overhang).} \]
\[\therefore K_1 = -0.0014. \]
\[K_{20} = 0.065 \text{ as for first arrangement.} \]
From figure 5, \(F_2 = 0.73\).
\[F_2 \times K_{20} = 0.73 \times 0.065 = 0.0475. \]
\[K_{21} = 0 \text{ (no decalage).} \]
\[K_{22} = 0 \text{ (no overhang).} \]
\[\therefore K_2 = 0.048. \]

The test data obtained are as follows:

<table>
<thead>
<tr>
<th>Angle of attack</th>
<th>(-4^\circ)</th>
<th>(0^\circ)</th>
<th>(4^\circ)</th>
<th>(8^\circ)</th>
<th>(12^\circ)</th>
<th>(16^\circ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upper wing (\Delta C_{LU})</td>
<td>-0.098</td>
<td>+ 0.174</td>
<td>0.438</td>
<td>0.696</td>
<td>0.968</td>
<td>1.050</td>
</tr>
<tr>
<td>Lower wing (\Delta C_{LU})</td>
<td>-0.074</td>
<td>+ 0.170</td>
<td>0.312</td>
<td>0.618</td>
<td>0.818</td>
<td>0.945</td>
</tr>
<tr>
<td>Biplane (\Delta C_{LU})</td>
<td>-0.088</td>
<td>+ 0.173</td>
<td>0.428</td>
<td>0.665</td>
<td>0.908</td>
<td>1.012</td>
</tr>
</tbody>
</table>

These values of \(\Delta C_{LU}\) are plotted against \(C_N\) on figure 8. The equation of the line through the test points is \(\Delta C_{LU} = -0.006 + 0.054 C_N\) which should be compared with equation (17). The agreement is again satisfactory.

Two special biplane tests have been made at Wright Field by the Army Air Corps and reported in reference 4. The biplane used in the first test had the following characteristics:

Upper wing: span \(b_U = 36\) inches, chord \(c_U = 6\) inches.

Lower wing: span \(b_L = 18\) inches, chord \(c_L = 3\) inches.

Gap: \(G = 4\frac{1}{2}\) inches, wing section Clark Y.

Stagger 3.63 inches measured on L.E. at \(\alpha = 0^\circ\) or 3.06 inches measured between \(\%\) chord points at zero lift.

Overhang \(= \frac{36 - 18}{36} = 0.50\).
The biplane used in the second test reported in reference 4 had the following characteristics:
- Upper wing: span $b_u = 36$ inches, chord $c_u = 6$ inches
- Lower wing: span $b_l = 18$ inches, chord $c_l = 3$ inches
- Gap: $G = 4\frac{1}{4}$ inches, wing section Clark Y
- Stagger 0 measured on L.E. at $a = 0^\circ$, -0.47 inch measured between $\frac{1}{4}$ chord points at zero lift.

Overhang $= \frac{36 - 18}{36} = 0.50$

From the above data:

\[ \frac{s}{c_l} = -0.47 \times 3.0 = 1.18 \quad \frac{G}{c_l} = 4.5 \times 3.0 = 1.5 \]

\[ \frac{t}{c} = 0.117 \quad \frac{t}{G} = \frac{0.117}{1.5} = 0.078 \quad \frac{c_l}{c_u} = \frac{1}{2} \]

From figure 1 and equation (5)

\[ K_{10} = -0.009 \times \frac{G}{c_l} = -0.0045 \]

From figure 2 and equation (6)

\[ \Delta K = 0.017 \quad K_{11} = 0.017 \times (-0.16) \times \frac{G}{c_l} = -0.0014 \]

\[ K_{12} = 0 \text{ (no decalage)} \]

From figure 4 and equation (8)

\[ \Delta K = -0.012 \quad K_{13} = -0.012 \times \frac{G}{c_l} = -0.006 \]

Hence, $K_1 = -0.0045 - 0.0014 - 0.006 = -0.012$

From equation (11)

\[ K_{20} = [0.050 + 0.173(-0.16)] \times \frac{G}{c_l} = +0.0114 \]

From figure 5, $F_1 = 0.67$

\[ F_1 \times K_{20} = 0.67 \times 0.0114 = 0.008 \]

\[ K_{21} = 0 \text{ (no decalage)} \]

From figure 6 and equation (14)

\[ \Delta K = 0.100 \quad K_{22} = 0.100 \times \frac{1}{2} = 0.050 \]

Hence,

\[ K_2 = 0.008 + 0.050 = 0.058 \]

and

\[ \Delta C_{L_{uv}} = -0.012 + 0.058 \quad C_L \quad (20) \]

From equation (3)

\[ \Delta C_{L_{uv}} = +0.048 - 0.232 \quad C_L \quad (21) \]

The report tabulates the lift coefficients at two points only. These are compared with the calculated values below:

<table>
<thead>
<tr>
<th></th>
<th>From test</th>
<th>Calculated</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upper wing $C_{L_{u}}$</td>
<td>1.025</td>
<td>1.031</td>
</tr>
<tr>
<td>Lower wing $C_{L_{l}}$</td>
<td>0.776</td>
<td>0.805</td>
</tr>
<tr>
<td>Biplane $C_L$</td>
<td>0.985</td>
<td>0.985</td>
</tr>
<tr>
<td>$\Delta C_{L_{uv}}$</td>
<td>0.040</td>
<td>0.046</td>
</tr>
<tr>
<td>Ratio $\frac{C_{L_{u}}}{C_{L_{l}}}$</td>
<td>1.32</td>
<td>0.87</td>
</tr>
</tbody>
</table>