NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

REPORT No. 656

THE COLUMN STRENGTH OF TWO EXTRUDED ALUMINUM-ALLOY H-SECTIONS

By WILLIAM R. OSGOOD and MARSHALL HOLT

1939

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AERONAUTIC SYMBOLS

1. FUNDAMENTAL AND DERIVED UNITS

	Symbol	Metric		English	
		Unit	Abbreviation	Unit	Abbreviation
Length	l	meter	m	foot (or mile)	ft. (or mi.)
Time	t	second	s	second (or hour)	sec. (or hr.)
Force	F	weight of 1 kilogram	kg	weight of 1 pound	lb.
Power	P	horsepower (metric)		horsepower	hp.
Speed	V	kilometers per hour	k.p.h.	miles per hour	m.p.h.
		meters per second	m.p.s.	feet per second	f.p.s.

2. GENERAL SYMBOLS

W, Weight$=mg$

g, Standard acceleration of gravity$=9.80665$ m/s^2 or 32.1740 ft./sec.^{2}

m, Mass$=\frac{W}{g}$

I, Moment of inertia$=mk^2$. (Indicate axis of radius of gyration k by proper subscript.)

μ, Coefficient of viscosity

ν, Kinematic viscosity

ρ, Density (mass per unit volume)

Standard density of dry air, 0.12497 kg-m^{-4}-s^2 at 15° C. and 760 mm; or 0.002378 lb.-ft.$^{-4}$ sec.^{2}

Specific weight of "standard" air, 1.2255 kg/m^3 or 0.07651 lb./cu. ft.

3. AERODYNAMIC SYMBOLS

S, Area

S_w, Area of wing

G, Gap

b, Span

c, Chord

$\frac{b^2}{S}$, Aspect ratio

V, True air speed

q, Dynamic pressure$=\frac{1}{2}\rho V^2$

L, Lift, absolute coefficient $C_L=\frac{L}{qS}$

D, Drag, absolute coefficient $C_D=\frac{D}{qS}$

D_0, Profile drag, absolute coefficient $C_{D_0}=\frac{D_0}{qS}$

D_i, Induced drag, absolute coefficient $C_{D_i}=\frac{D_i}{qS}$

D_p, Parasite drag, absolute coefficient $C_{D_p}=\frac{D_p}{qS}$

C, Cross-wind force, absolute coefficient $C_C=\frac{C}{qS}$

R, Resultant force

i_w, Angle of setting of wings (relative to thrust line)

i_t, Angle of stabilizer setting (relative to thrust line)

Q, Resultant moment

Ω, Resultant angular velocity

$\rho\frac{Vl}{\mu}$, Reynolds Number, where l is a linear dimension (e.g., for a model airfoil 3 in. chord, 100 m.p.h. normal pressure at 15° C., the corresponding number is 234,000; or for a model of 10 cm chord, 40 m.p.s., the corresponding number is 274,000)

C_p, Center-of-pressure coefficient (ratio of distance of c.p. from leading edge to chord length)

α, Angle of attack

ϵ, Angle of downwash

α_0, Angle of attack, infinite aspect ratio

α_i, Angle of attack, induced

α_a, Angle of attack, absolute (measured from zero-lift position)

γ, Flight-path angle

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By WILLIAM R. OSGOOD and MARSHALL HOLT

National Bureau of Standards and Aluminum Company of America

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

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By William R. Osgood and Marshall Holt

SUMMARY

Extruded aluminum-alloy members of various cross sections are used in aircraft as compression members either singly or as stiffeners for aluminum-alloy sheet. In order to design such members, it is necessary to know their column strength or, in the case of stiffeners, the value of the double modulus, which is best obtained for practical purposes from column tests.

Column tests made on two extruded H-sections are described, and column formulas and formulas for the ratio of the double modulus to Young's modulus, based on the tests, are given.

INTRODUCTION

Extruded aluminum-alloy members of various cross sections are used in aircraft as compression members either singly or as stiffeners for aluminum-alloy sheet. In order to design such members, it is necessary to know their column strength, or in the case of stiffeners, the value of the double modulus (references 1 and 2), which is best obtained for practical purposes from column tests.

The interest of the National Advisory Committee for Aeronautics in stiffened-sheet construction as applied to monocoque design led to the allotment of funds to the National Bureau of Standards for research in this field, and a part of these funds was used to investigate the column strength of an extruded aluminum-alloy shape comparable with those used in stiffened-sheet construction. The data obtained in the tests made at the National Bureau of Standards are presented and discussed in part I of this report. The material for this investigation was supplied by the Aluminum Company of America.

Column tests were conducted at the Aluminum Research Laboratories on pieces of extruded aluminum alloy taken from the same lot of material supplied to the National Bureau of Standards. Column tests were also made at the Aluminum Research Laboratories on another extruded aluminum-alloy shape, the data on which had been requested by the National Advisory Committee for Aeronautics. The results of these tests are presented and discussed in part II of this report.

A correlation of the test data from the National Bureau of Standards and those from the Aluminum Research Laboratories is made in part III of this report.

MATERIAL

The material used in these investigations of column strength is designated Alcoa 24S–T by the Aluminum Company of America and complies with Navy Department Specifications 46A9a, June 1, 1938: Aluminum-alloy (aluminum-copper-magnesium (1.5 percent)-manganese): Bars, Rods, Shapes, and Wire. The material was furnished in the form of extruded H-beams. The nominal dimensions of the cross sections are shown in

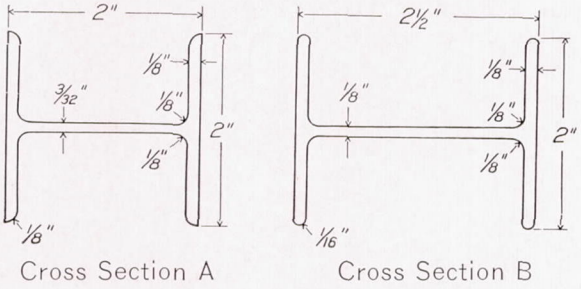


FIGURE 1.—Dimensions of extruded 24S–T H-sections.

figure 1. The National Bureau of Standards tests were made only on cross section A and the Aluminum Company tests included both cross sections.

The mechanical tests to determine properties and the results of these tests are discussed in the following three parts of this report.

I. TESTS MADE AT THE NATIONAL BUREAU OF STANDARDS

PREPARATION OF SPECIMENS

The tensile specimens were three standard type-5 tensile-test specimens, as defined in Navy Department General Specifications for Inspection of Material, Appendix II (Metals). They were cut from the same length of extruded shape, one specimen from the middle of the web and the other two from diagonally opposite positions in the two flanges. The cross-sectional areas of the reduced portions of these specimens were determined by calipering them.

The lengths of all the compressive and the column specimens were measured. In order to determine the required cross-sectional properties of the compressive and the column specimens, more than half of them were weighed and, for each of these specimens, measurements were made of the thickness and the width of each flange and the depth of the section at the middle and of the thickness of the web at each end. The density of a sample of the material was determined by the Division of Weights and Measures of the National Bureau of Standards. The cross-sectional areas were computed from the weights, the lengths, and the densities; and the least radii of gyration were computed from the measured cross-sectional dimensions and the nominal radii of the roundings and fillets.

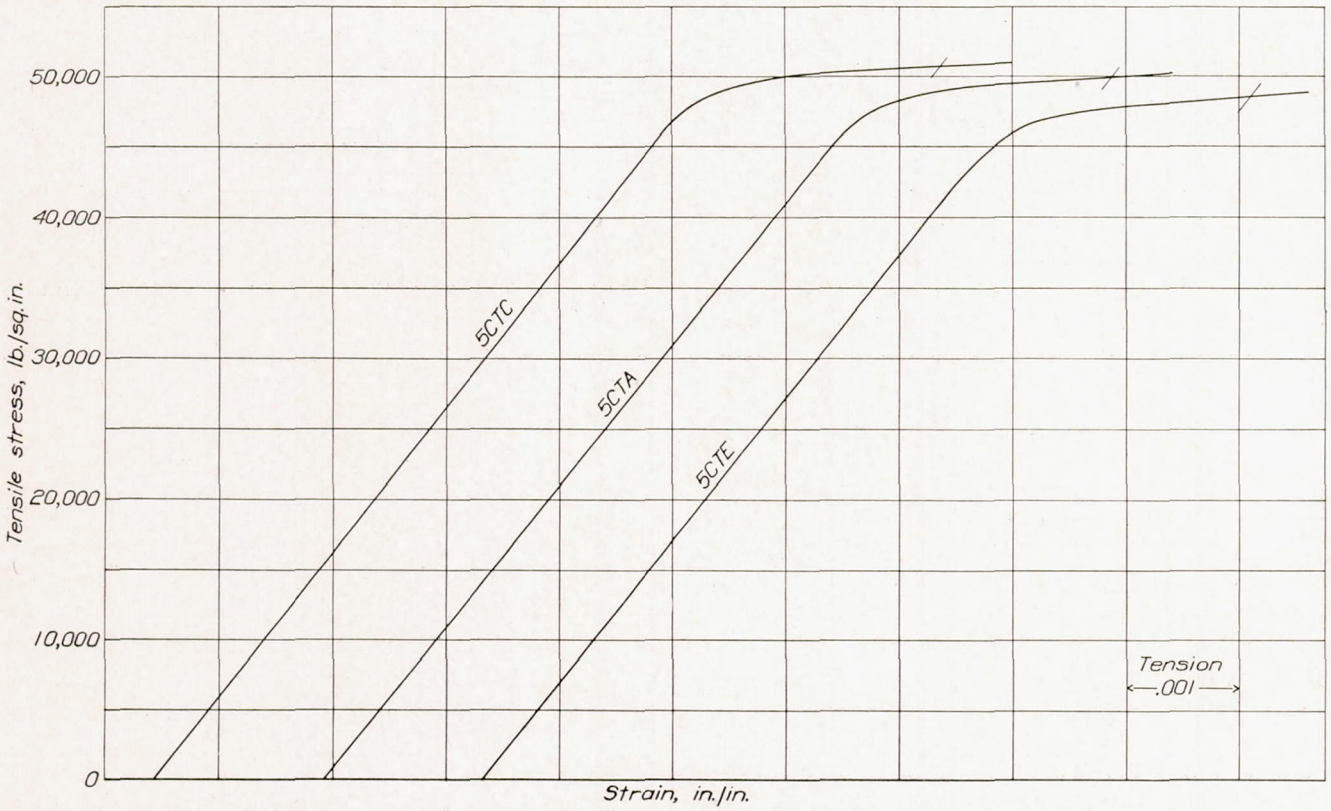


FIGURE 2.—Typical tensile stress-strain diagrams for 24S-T of cross section A. Strains measured on 2-inch gage length with Ewing extensometer. National Bureau of Standards.

TENSILE AND COMPRESSIVE TESTS

Tensile tests were made in screw-driven, beam-and-poise testing machines. The specimens were held in Templin grips supported by spherical bearings. Strains were measured in 2-inch gage lengths by means of Ewing extensometers. Three typical tensile stress-strain diagrams are shown in figure 2. Specimens 5CTC and 5CTA were taken from the flanges and specimen 5CTE from the web. The tensile yield strength was obtained from the stress-strain diagram as the stress at a strain 0.002 in excess of the elastic strain corresponding to this stress.

The most suitable machine available for making the compressive tests was considered to be a fluid-support, Bourdon-tube, hydraulic machine. Auxiliary nuts on the screws of this machine were tightened against the lower surface of the adjustable head to bring it into contact with the lower surface of the threads on the screws, so that rotation of the head relative to the platen of the machine due to clearance between the nuts of the head and the screws was obviated. The unsymmetrical position of the motor, the handwheel, and the other mechanism for raising and lowering the adjustable head causes it to exert on the portion of the two screws below it a constant moment of roughly 1,000 pound-inches in a plane normal to that of the screws. Consequently the screws are slightly bent elastically and, as they tend to straighten under load, produce rotation of the head. This condition causes a slight eccentricity of loading, which is especially undesirable in compression testing; but, with the short specimens and comparatively low loads (maximum, one-third the capacity of the machine) of the present investigation, the effect was not considered serious. Another possible source of error in making compressive tests in this type of machine arises from the possibility of rotation of the platen about a horizontal axis. The platen is rigidly connected to the piston of the hydraulic jack, which is packed, and the clearance between the cylinder and the piston permits rotation of the platen under eccentric

load. This effect can be minimized by keeping as much of the piston in the cylinder as possible.

The compressive specimens were 8-inch lengths of the extruded shape, with ends machined plane and normal to the axis. A specimen to be tested was placed centrally on a ground hardened-steel bearing block located centrally in the testing machine, and a similar, smaller block was placed centrally on the upper end of the specimen. In order to secure as nearly uniform bearing as possible, the upper bearing block was capped with plaster of paris. The capping was done by placing a stiff mix of plaster between two sheets of relatively nonabsorbent oiled tracing paper and transferring it to the bearing block. A load of 1,000 pounds was then applied immediately and held for about 15 minutes to allow the plaster to set.

Strains were measured on 2-inch gage lengths along the middle of each flange. Tuckerman optical strain gages were used to measure strains up to about 0.002 to determine the modulus of elasticity, and Huggenberger Tensometers were used to measure the larger strains. Compressive stress-strain diagrams were obtained for two specimens and are shown in figure 3. The compressive specimens ultimately failed by local buckling, as shown by 1B–1 in figure 4. The compressive yield strength was obtained as the stress corresponding to the intersection with the stress-strain curve of a line drawn through the origin with a slope 2/3 E, where E is the modulus of elasticity (reference 3).

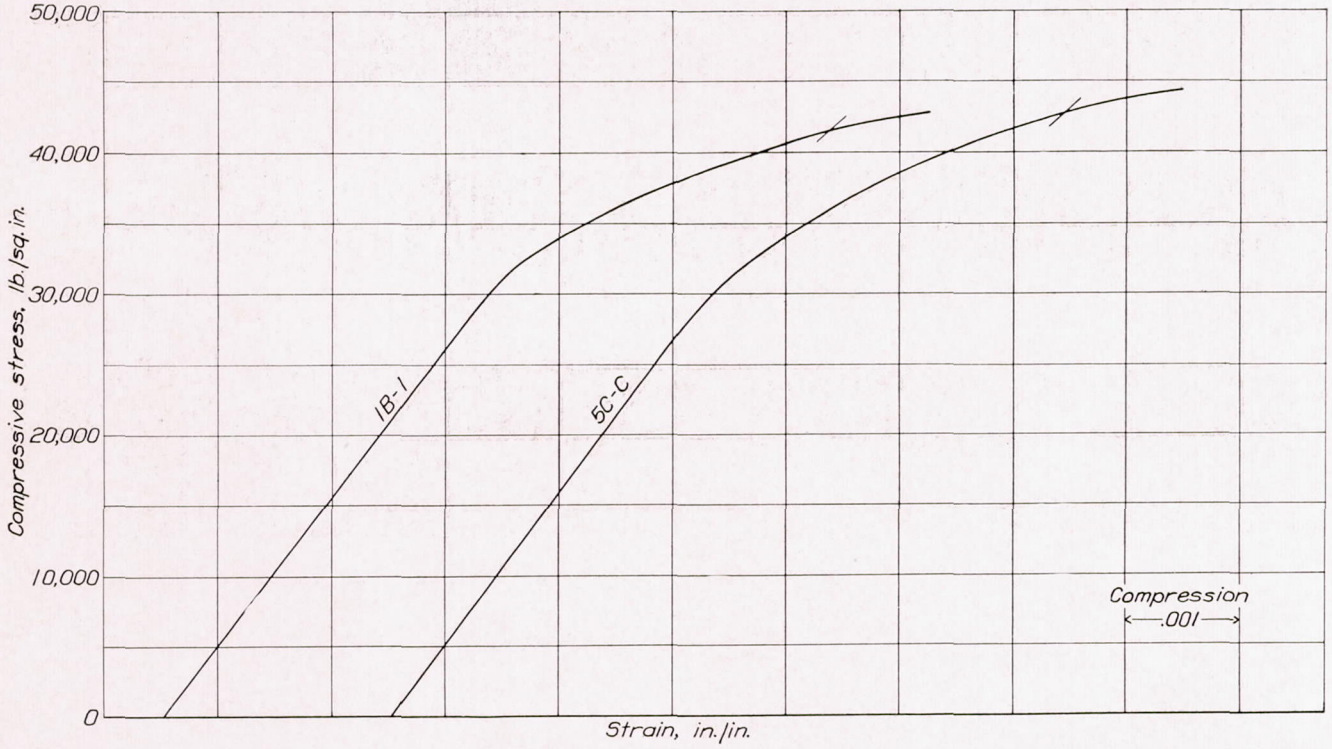


FIGURE 3.—Compressive stress-strain diagrams for 24S–T of cross section A. Strains measured with Tuckerman optical strain gages (up to 0.002) and with Huggenberger strain gages on 2-inch gage length. Length of specimen, 8 inches. National Bureau of Standards.

COLUMN TESTS

Fifteen column specimens were tested with freely supported ends and, upon recommendation of the Committee on Aircraft Structures of the National Advisory Committee for Aeronautics, six specimens were tested with elastically restrained ends. The apparatus and the procedure used for making the tests were identical with those described in reference 3. It will be sufficient to explain here that the specimens were supported on knife-edge carriers and centered under load. That is, a load was applied which would not produce anywhere in the specimen stresses greater than the expected maximum average column stress, the deflection of the middle of the specimen and the rotations of the ends were noted, the load was reduced to a low value (150 pounds), one or both ends of the specimen were shifted on the carriers to reduce the deflection under load and equalize the rotations, and the process was repeated with increasing loads until at 90 percent or more of the expected maximum column load the deflection was only a few ten-thousandths of an inch (not over 0.0005 inch).[1] When this condition was reached, the load was reduced to 150 pounds and then gradually increased to the maximum value that could be supported by the specimen. Readings of deflection were taken while applying the load.

Curves of load divided by maximum load plotted against deflection within the free length divided by the

[1] Lest anyone be alarmed by such high centering loads, let him read the appendix.

free length serve as a check on the centering operation. Figure 5 shows typical curves of this kind. If the knee of any such curve is blunt relative to the knees of the curves for the other specimens, as occasionally happens, it indicates that the specimen represented by that curve was not so well centered as the others. In the present investigation all specimens tested with freely supported ends appeared to be well centered.

RESULTS

The results of the column tests are given in table I and in figures 6 and 7. The free lengths l_0, of the test columns were determined from the equation (reference 3)

$$\cot\frac{\phi}{2}+\nu\phi=0 \tag{1}$$

where

$$\phi=\frac{\pi l}{l_0}, \qquad \nu=\frac{m}{Pl}-\frac{s}{l} \tag{2}$$

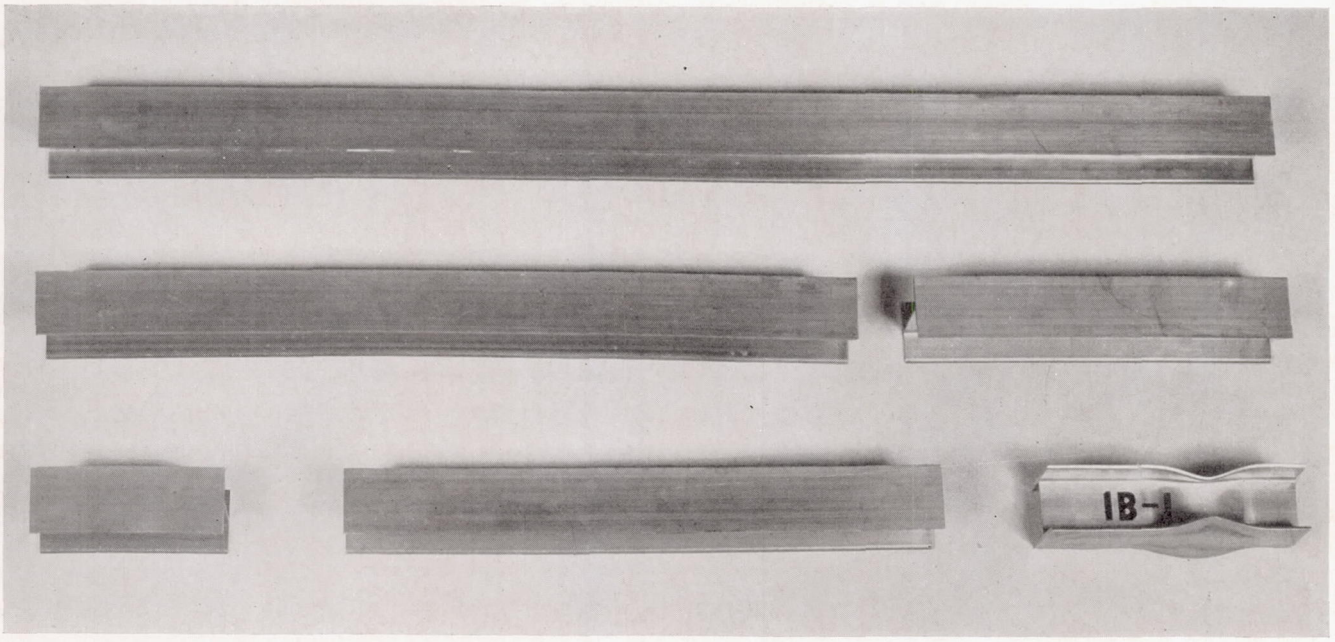


FIGURE 4.—Some column specimens and one compressive specimen (1B–1) of cross section A after test.

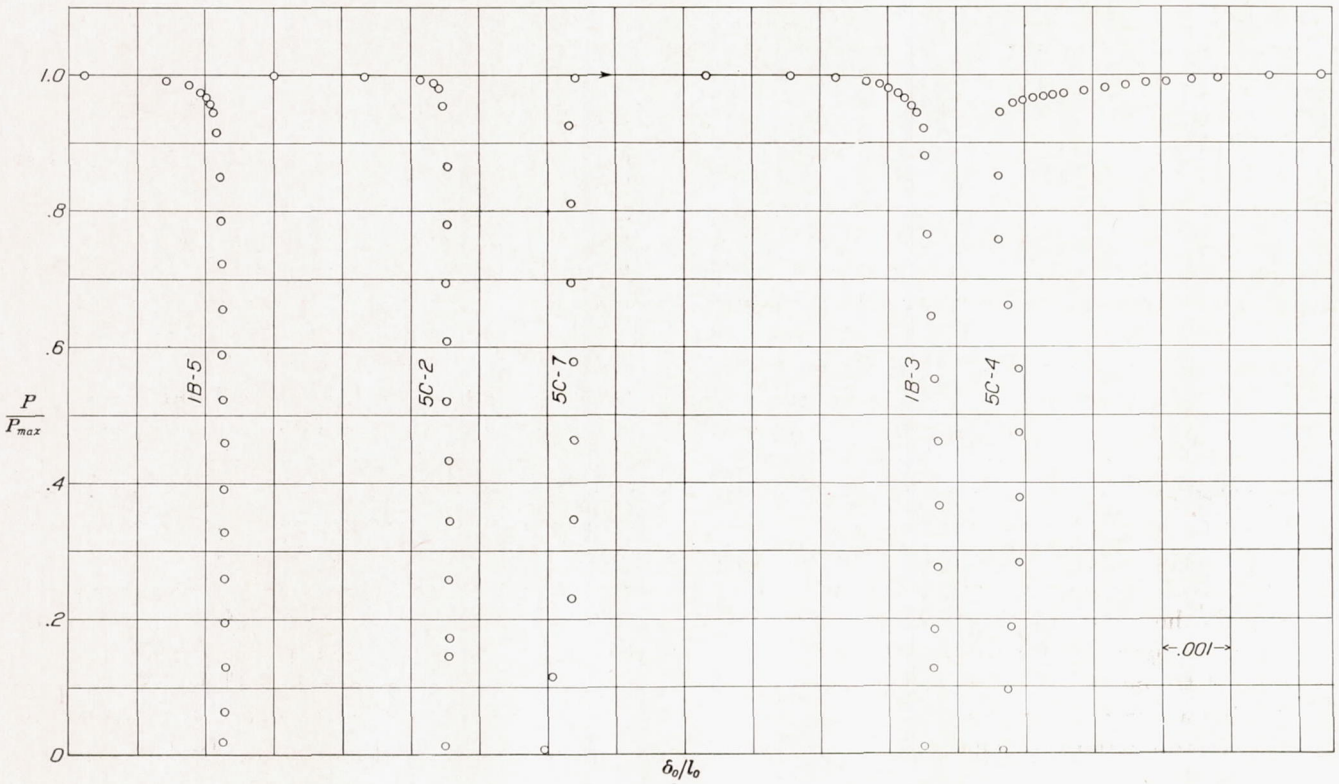


FIGURE 5.—Typical P/P_{max}, δ_0/l_0-diagrams for 24S–T of cross section A.

l is the length of the test specimen, s is the length of the carrier (distance from a supporting knife edge to the adjacent end of the specimen), m is the elastic restraint at each end of the column ($m=0$ for freely supported ends), and P is the maximum load carried by the column. The least radius of inertia, or radius of gyration, of the cross section has been denoted by i, the cross-sectional area by A, and the compressive yield strength by S. The significance of plotting

$$\sigma=\frac{P}{AS} \text{ against } \lambda_0=\frac{1}{\pi}\frac{l_0}{i}\sqrt{\frac{S}{E}} \tag{3}$$

has been discussed in reference 3. The use of these non-dimensional variables makes it possible to reduce in a rational way data representing column tests made with material exceeding specified values in yield strength to a representation that would be expected from material just complying with the specified values.

In figure 6 values of σ have been plotted against λ_0. The three specimens having the lowest values of λ_0 showed evidence of local buckling due to bending before their maximum loads were reached, so that the corresponding values of σ may be slightly low (on the safe side). It is much more difficult to center a short specimen under load than a long one, because the deflection at the middle becomes so small for a short specimen. The results obtained from the tests with elastically restrained ends agree with those obtained from the other tests as well as could be expected.

If the load-deflection curve (fig. 5) for any specimen has a blunt knee relative to the knees of the other curves, and if also the value of σ for this specimen in the σ, λ_0-plot is low, justification exists for throwing it out. No such cases arose in this investigation, however, and low points in figure 6 are to be explained largely by unavoidable variations in the material.

Figure 6 shows the reduced Euler curve and a straight line fitted to the observed σ, λ_0-values for the condition of freely supported ends. The straight line has been cut off at the top at the average value of σ found for the two compressive specimens, which failed by local buckling (specimen 1B–1 in fig. 4). The column strength of the aluminum-alloy shape tested can be given in non-dimensional form by

$$\sigma_{max}=1.153 \tag{4}$$

$$\left.\begin{aligned}\sigma&=0.55+\frac{0.5}{\sqrt{0.55}}-0.5\lambda_0\\ \sigma&=1.224-0.5\lambda_0\end{aligned}\right\}1.153>\sigma>0.55 \tag{5}$$

$$\sigma=\frac{1}{\lambda_0^2},\ \sigma<0.55 \tag{6}$$

It is to be expected that equations (5) and (6) would hold closely for any heat-treated bar, rod, or shape complying with Navy Department Specification 46A9a so long as failure occurred by primary buckling. Equation (4) expresses the condition of failure by local buckling, or wrinkling, or crinkling, and this equation would have to be modified depending on the shape of cross section.

In figure 7 the observed values of the maximum average stress, P/A, have been plotted against the ratio of slenderness, l_0/i. By introducing in equations (4), (5), and (6) the values of λ_0 and σ from equations (3), relations can be obtained between P/A and l_0/i in terms of S and E. For use in design, these relations should contain numerical values of S and E related as far as possible to specified minimum properties of the material. The specified property most closely related to the compressive yield strength is the tensile yield strength. The average ratio of the compressive yield strength to the tensile yield strength[2] of the material of this investigation was 0.85. Navy Department Specification 46A9a for aluminum-alloy shapes such as those tested requires a minimum tensile yield strength of 42,000 pounds per square inch. Material just complying with this specification may therefore be expected to have a compressive yield strength $S=0.85\times42{,}000=35{,}700$ pounds per square inch. If then, this value is taken for S and for E the average value found, 10,660,000 pounds per square inch, there is obtained for design, P/A in pounds per square inch.

$$\left(\frac{P}{A}\right)_{max}=41{,}200 \tag{7}$$

$$\frac{P}{A}=43{,}700\left(1-0.00752\frac{l_0}{i}\right),\quad 41{,}200>\frac{P}{A}>19600 \tag{8}$$

$$\frac{P}{A}=\frac{105\,200\,000}{\left(\frac{l_0}{i}\right)^2},\quad \frac{P}{A}<19{,}600 \tag{9}$$

The curves represented by these equations are shown in figure 7.

In analyzing stiffened-sheet construction it may be necessary to know the quantity $\tau=\frac{\overline{E}}{E}$ as a function of the average stress P/A in the stiffener (reference 1), where $\overline{E}$ is the double modulus (reference 2). The desired relation between τ and P/A may be obtained by eliminating λ_0 between the "universal" column formula (reference 3)

$$\sigma=\frac{\tau}{\lambda_0^2} \tag{10}$$

and each of equations (4), (5), and (6), and then substituting σ from equation (3). Elimination of λ_0 between equation (10) and equations (4), (5), and (6) gives for τ in terms of σ

$$\tau_{min}=0.0231 \tag{11}$$

$$\left.\begin{aligned}\tau&=4\sigma\left(0.55+\frac{0.5}{\sqrt{0.55}}-\sigma\right)^2\\ \tau&=4\sigma(1.224-\sigma)^2\end{aligned}\right\}1.153>\sigma>0.55 \tag{12}$$

$$\tau=1 \qquad \sigma<0.55 \tag{13}$$

[2] The values of tensile yield strength were weighted averages from tests of two specimens of flange material and one specimen of web material.

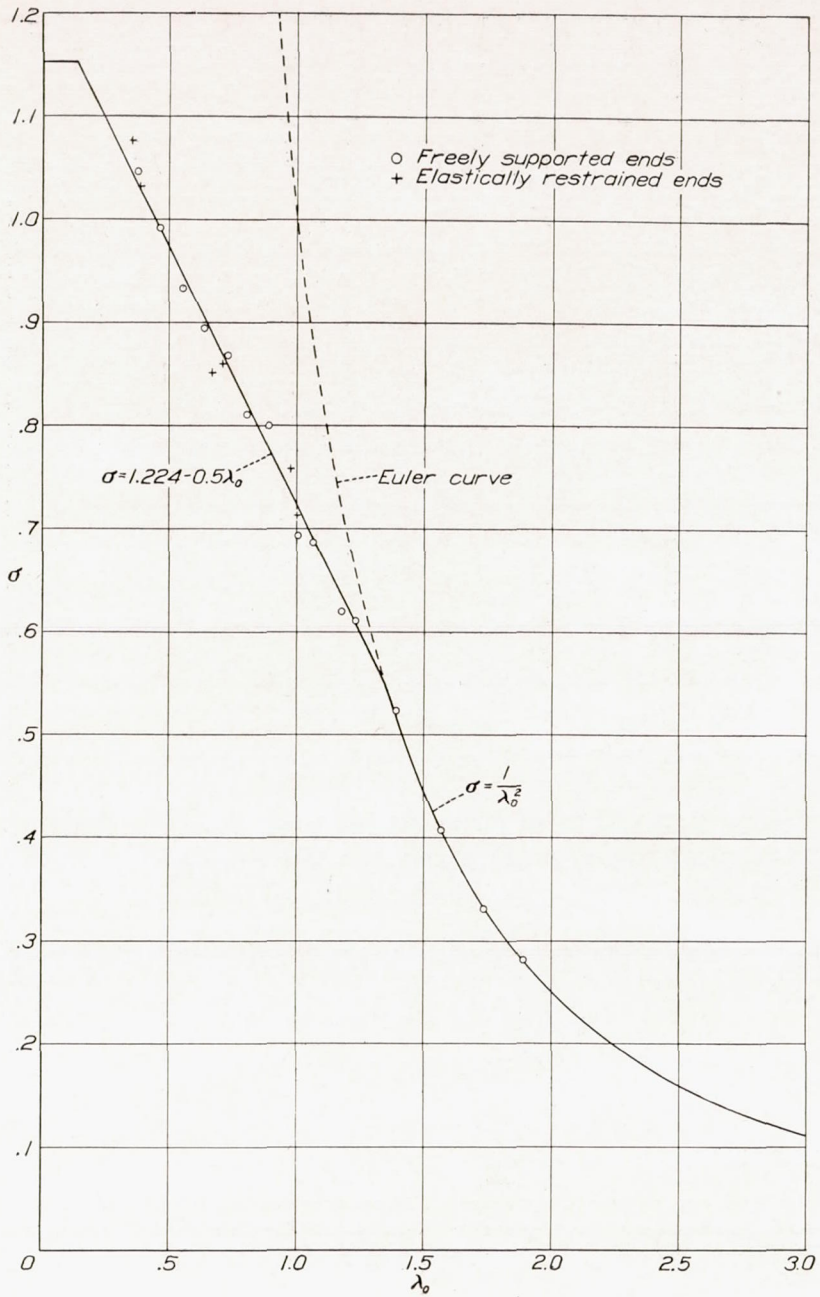


FIGURE 6.—The σ, λ_0-diagram for 24S-T of cross section A.

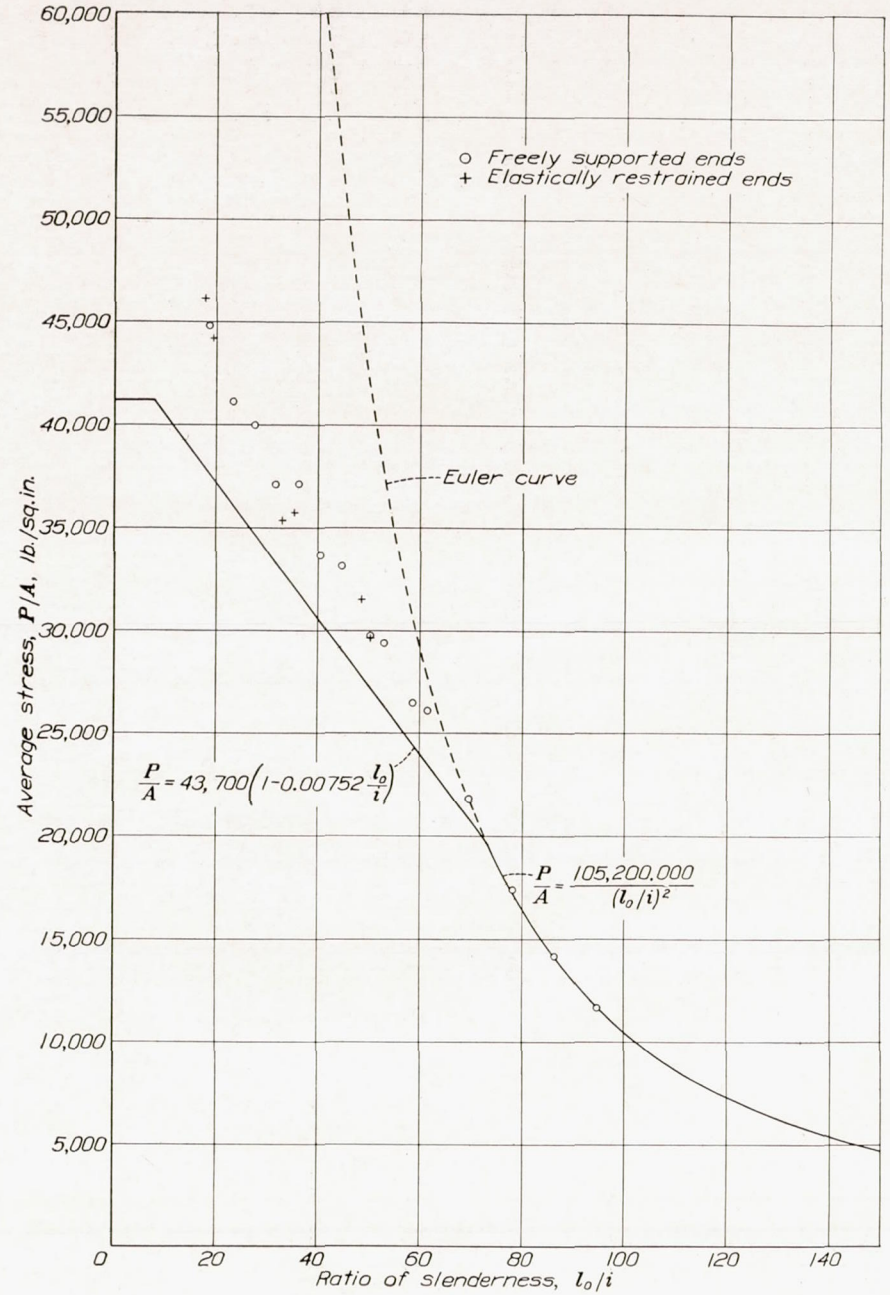


FIGURE 7.—The P/A, l_0/i-diagram for 24S-T of cross section A.

Figure 8 shows the curves represented by these equations. Substitution of σ from equation (3), with S taken as 35,700 pounds per square inch gives, P/A in pounds per square inch,

$$\tau_{min}=0.0231 \tag{14}$$

$$\tau=\frac{P}{8925A}\left(1.224-\frac{P}{35700A}\right)^2, \quad 41{,}200>\frac{P}{A}>19{,}600 \tag{15}$$

$$\tau=1, \quad \frac{P}{A}<19{,}600 \tag{16}$$

Figure 9 shows the curves represented by these equations.

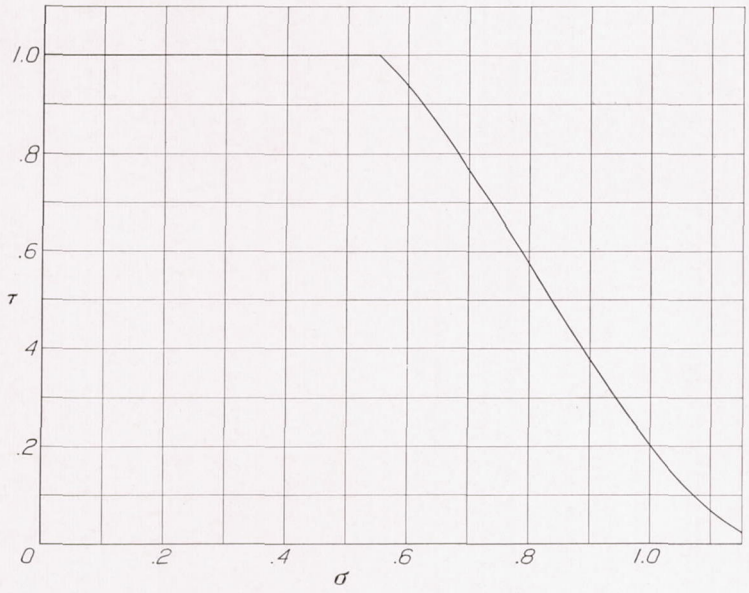


FIGURE 8.—The τ, σ-curve for 24S–T of cross section A.

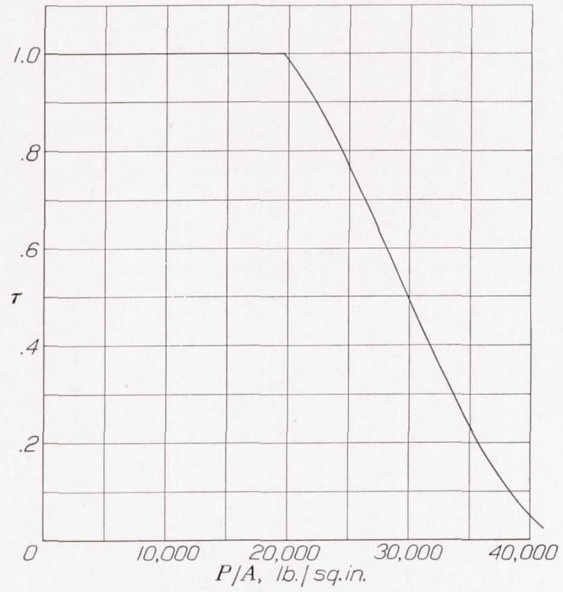


FIGURE 9.—The τ, P/A-curve for 24S–T of cross section A.

II. TESTS MADE AT THE ALUMINUM RESEARCH LABORATORIES

MECHANICAL PROPERTIES TESTS

The test specimens for the mechanical properties tests were identified by the same numbers assigned the extruded pieces: 1 to 4, inclusive, for cross section A and 5 to 8, inclusive, for cross section B. In the case of cross section A, pieces 1 and 2 were from one 46-foot extruded piece, likewise pieces 3 and 4.

The tensile properties of the material were obtained on standard ½-inch wide flat tensile specimens and the results of the tests are summarized in table II. The average tensile properties are as follows:

		Cross section A	*Cross section* B
Tensile strength	lb. per sq. in.	63, 440	62, 200
Yield strength,[3]	lb. per sq. in.	48, 200	49, 900
Elongation in 2 inches	percent	18. 4	19. 2

These tensile properties compare favorably with typical values for 24S–T extruded shapes. Figures 10 and 11 show typical tensile stress-strain curves. The stress-strain relations were determined with the use of Huggenberger tensometers using a gage length of 0.5 inch. Specimens were taken from both the flange and the web.

The compressive properties of the material were obtained on short lengths of the full cross section ($l/i=10$). The average values of the compressive yield strength of the several pieces of material are given in table II as:

	Lb. per sq. in.
Cross section A	44, 700
Cross section B	40, 000

The stress-strain relations shown in figures 12 and 13 were obtained by use of Huggenberger tensometers using a gage length of 1 inch.

Additional compressive tests were made on specimens consisting of a pack of three or five pieces, each five-eighths inch wide, cut from either the web or the flange of the section. Specimens were taken both longitudinally and transversely from both the flange and the web. The jig for holding the pack specimen during testing is shown in figure 14. Typical stress-strain curves determined with the pack specimens are shown in figures 15 and 16. The values of yield strength are summarized in table II.

All these values of mechanical properties indicate that each lot of extruded material was uniform.

COLUMN SPECIMENS AND METHODS OF TEST

The specimens used in the column tests are described in table III. The specimen number, which is a combination of two numbers, designates the piece of material from which the specimen was cut and the approximate length in inches. The actual average area of each specimen was computed from the length and the

[3] Yield strength is the stress that produces a permanent set of 0.2 percent of the initial gage length (Navy Department Specification 46A9a, also American Society for Testing Materials Standard Definitions of terms Relating to Methods of Testing, E6–36).

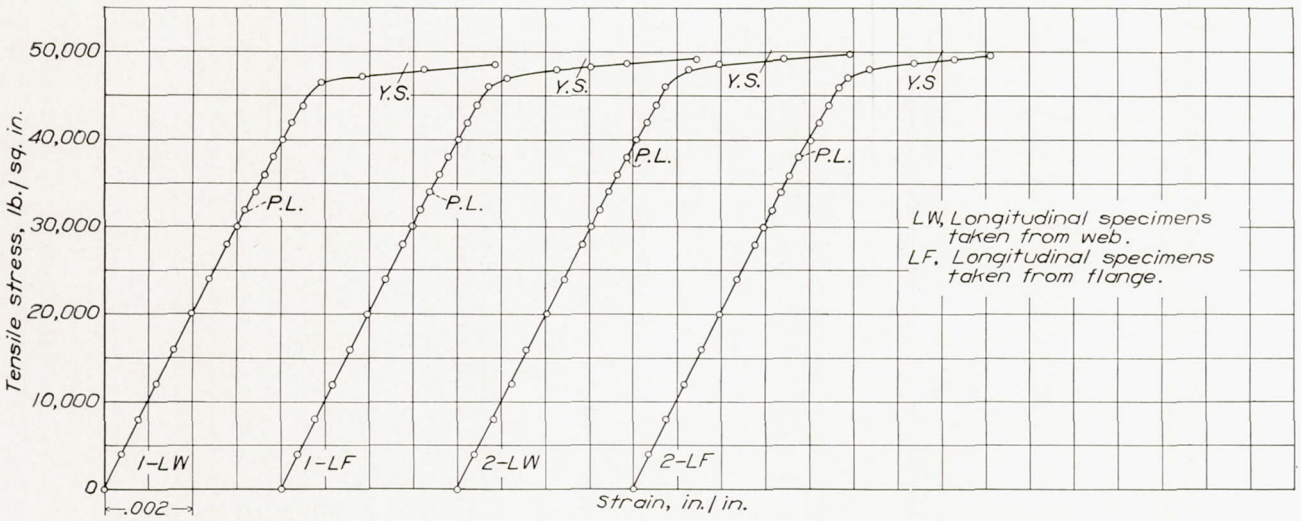


FIGURE 10.—Tensile stress-strain diagrams for 24S-T of cross section A. Strains measured on 0.5-inch gage length with Huggenberger Tensometers.

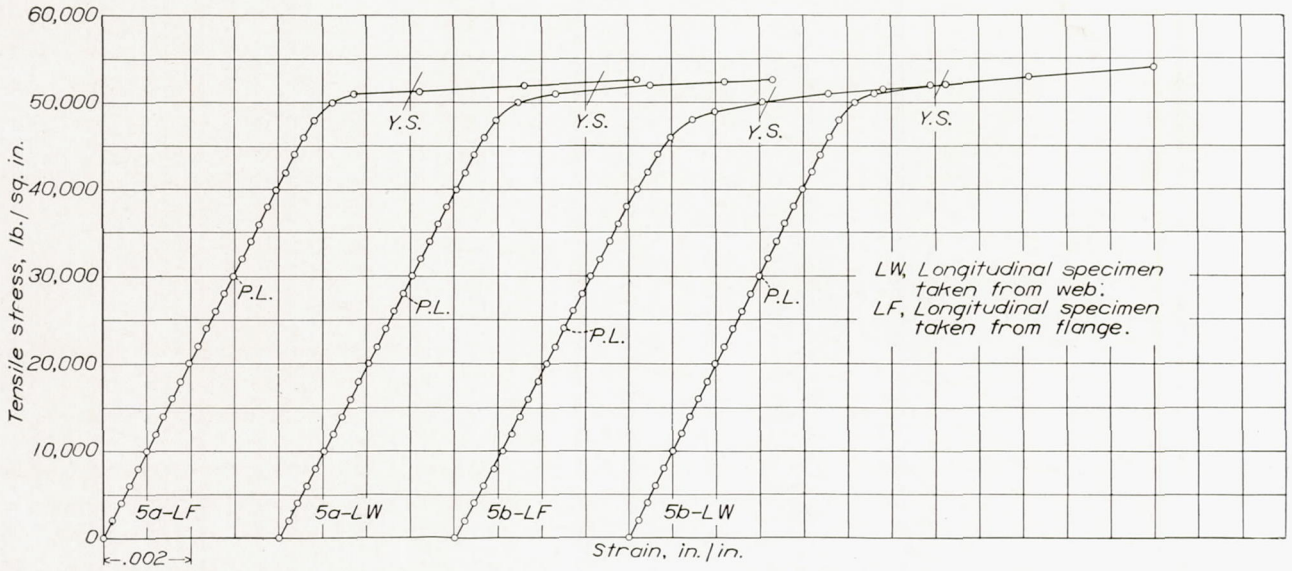


FIGURE 11.—Tensile stress-strain diagrams for 24S-T of cross section B. Strains measured on 0.5-inch gage length with Huggenberger Tensometers.

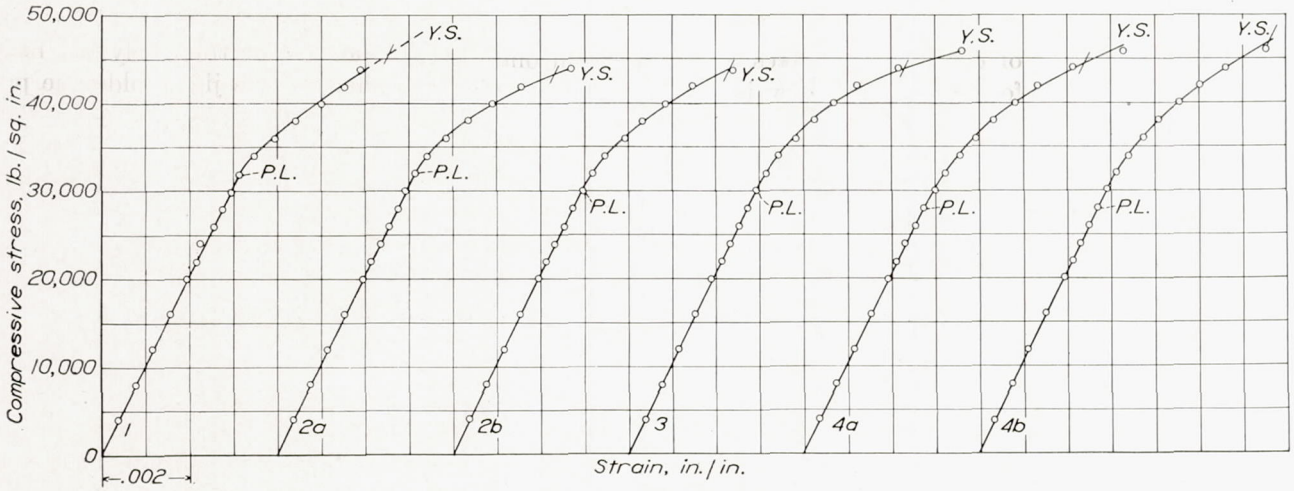


FIGURE 12.—Compressive stress-strain diagrams for 24S-T of cross section A. Strains measured with two Huggenberger Tensometers mounted at the middle of the flanges on 1-inch gage length. Slenderness ratio of specimens, 10; length of specimens, 4.9 inches; ultimate strength, about 50,900 pounds per square inch.

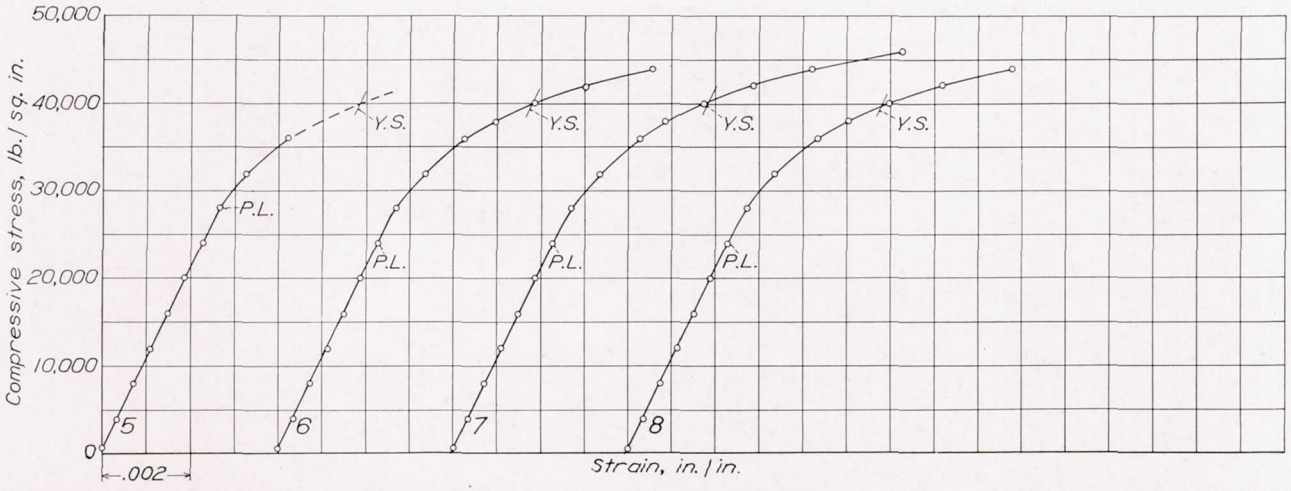


FIGURE 13.—Compressive stress-strain diagrams for 24S–T of cross section B. Strains measured with two Huggenberger Tensometers mounted at the middle of the flanges on 1-inch gage length. Slenderness ratio of specimen, 10; length of specimen, 4.6 inches; ultimate strength, about 50,000 pounds per square inch.

weight of the specimen and the nominal density of the material (0.100 lb. per cu. in.). The crookedness was measured by placing thickness gages between the specimen and a plane surface on which it rested. The maximum crookedness of 1 part in 1,500 was found in specimen 4–19 with a length of 19.44 inches and a measured crookedness of 0.013 inch. The ends of the specimens were carefully finished flat, mutually parallel, and perpendicular to the axis.

Column tests were made using the conditions of flat ends and round ends.

The condition of flat ends was obtained by centering the specimens on the fixed heads of the testing machine as shown in figure 17. The ends of the specimens were restrained to the extent that the bearings did not tip. Under a large sidewise deflection, usually greater than that corresponding to the maximum column load, the ends of the specimen could lift free of the bearing plate on one side.

The condition of round ends was produced by two methods. In the tests of cross section A the bearing plates permitted the specimen to deflect in any direction and twist with practically no restraint. The bearing plates were provided with a spherical seat resting in a nest of 25 hardened-steel balls. The center of rotation of the plates coincided with the ends of the specimen. A specimen was centered on the plates by shifting it on the bearing surfaces until comparable dial readings, representing shortening of the specimen, were obtained at the four corners of the bearing plates for several increments of load. The test set-up is shown in figure 18.

In the tests of cross section B the condition of round ends was obtained by centering the specimens on bearing plates equipped with roller-bearing supports that allow free tipping about only one axis. The specimens were placed on the bearing plates with the axis of least stiffness parallel to the axis of tipping of the plates. The center of rotation of the plates coincided with the end of the specimen. Figure 19 shows a specimen in the testing machine. Because of the relatively low capacity of these bearings (10,000 pounds) only relatively long specimens ($l/i>90$) could be tested. In each test the specimen was placed as nearly centrally on the heads as possible and loaded until a maximum load was reached, after which the loading was quickly stopped to prevent permanent set. The specimen was then moved on the bearing plates a very small distance in the direction opposite to that in which the specimen bent in the first loading. The loading was then repeated. This procedure was continued until failure occurred by bending in the direction opposite to the first failure. From the several loads thus obtained the greatest was taken as the column strength of the specimen. It should be recognized that this method of centering could be used only in cases where the failure was entirely elastic, as in these tests.

Both sets of the column tests of cross section A and the flat-end tests of cross section B were made in an Amsler hydraulic testing machine having a maximum capacity of 300,000 pounds. The round-end tests of cross section B were made in a similar machine with a maximum capacity of 40,000 pounds. In all cases, an intermediate load range was used.

RESULTS AND DISCUSSION

The results of the column tests are given in table III and are shown with column-strength curves in figures 20 and 21. The results from cross sections A and B are shown in different figures, not because the column action of the two sections is different but because of the difference in the values of compressive yield strength, 44,700 pounds per square inch and 40,000 pounds per square inch. It will be noted that the results from the round-end tests and flat-end tests are both plotted in the same figure, using values of effective slenderness ratio, l_0/i, for the abscissas in which $l_0/l=1.0$ for the round-end tests and 0.5 for the flat-end tests.

This method of plotting is used because it yields a more direct comparison of the two types of test than

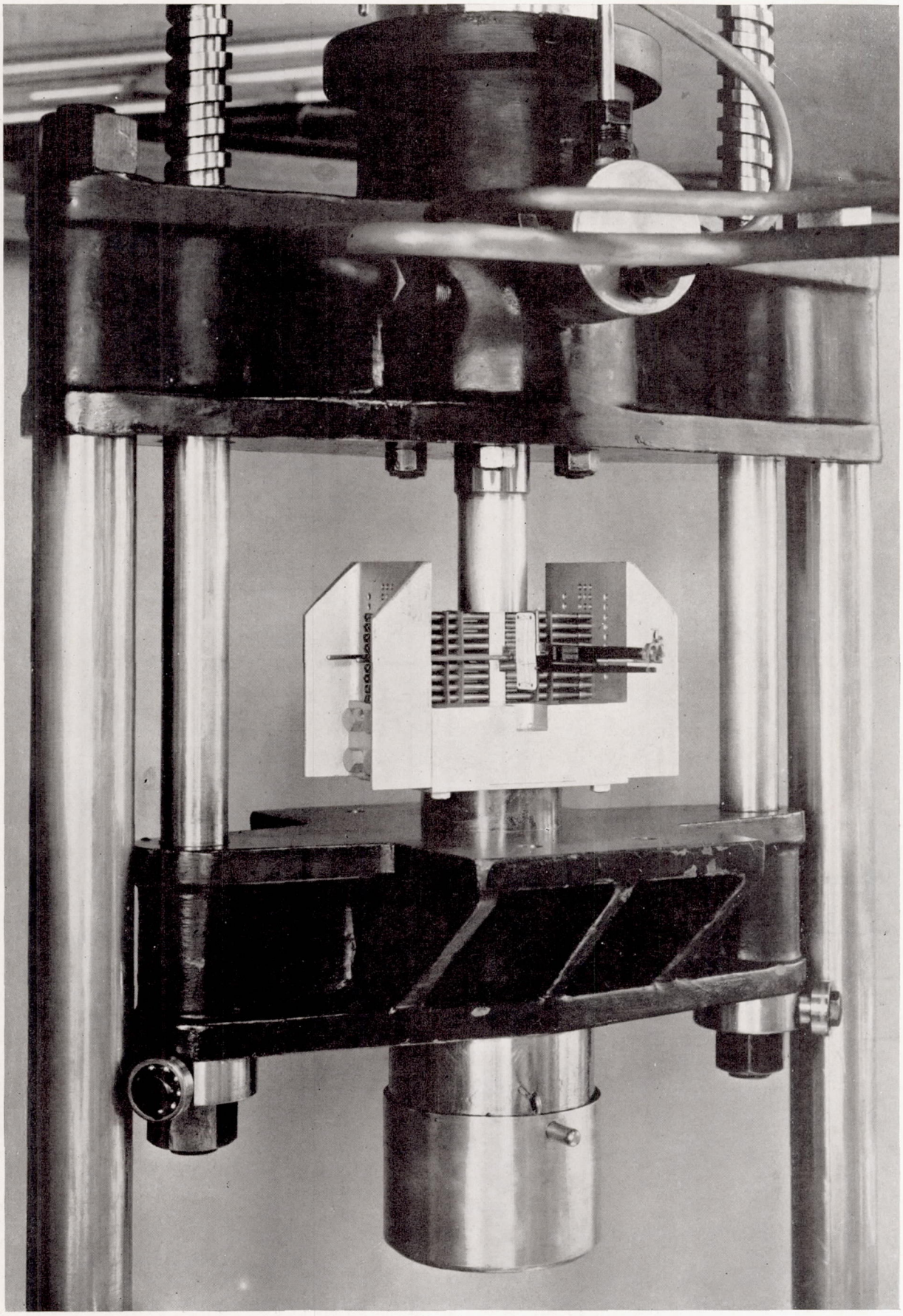


FIGURE 14.—Test of thin sheet-metal specimen in compression.

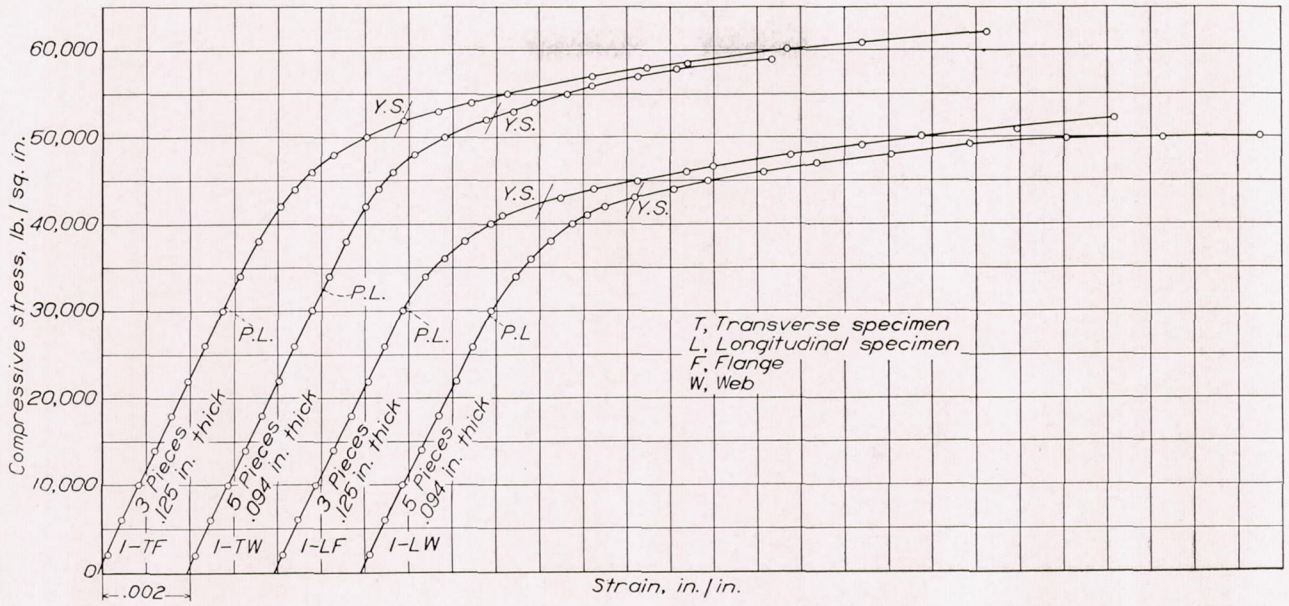


FIGURE 15.—Compressive stress-strain diagrams for 24S-T of cross section A. Strains measured with Huggenberger Tensometers. Pack compression specimen used.

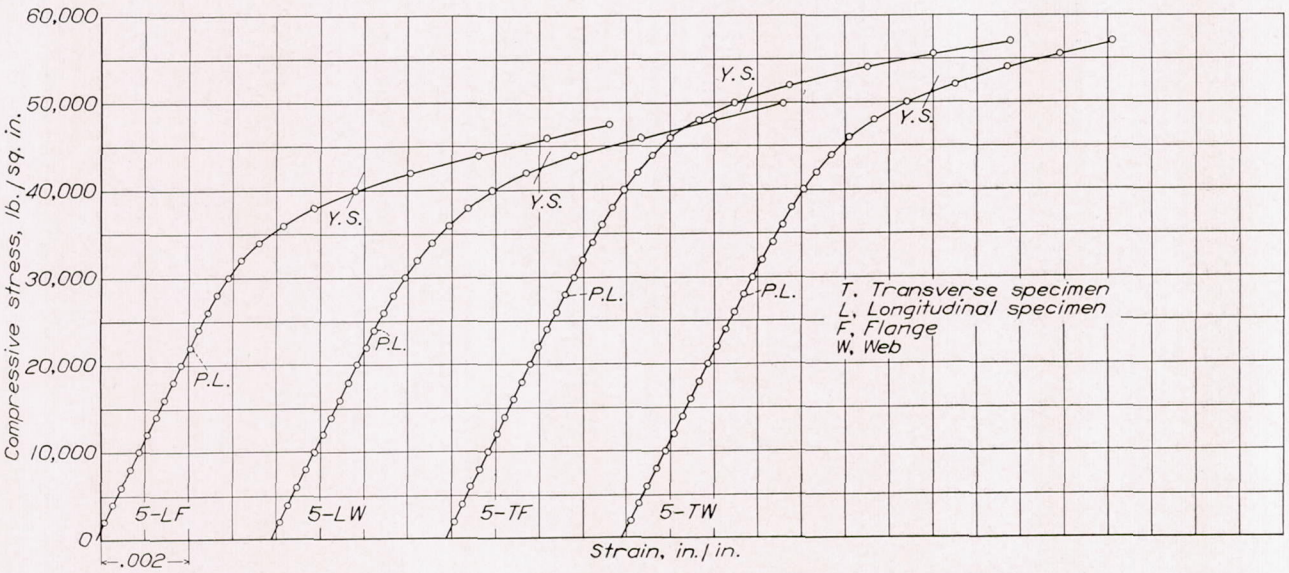
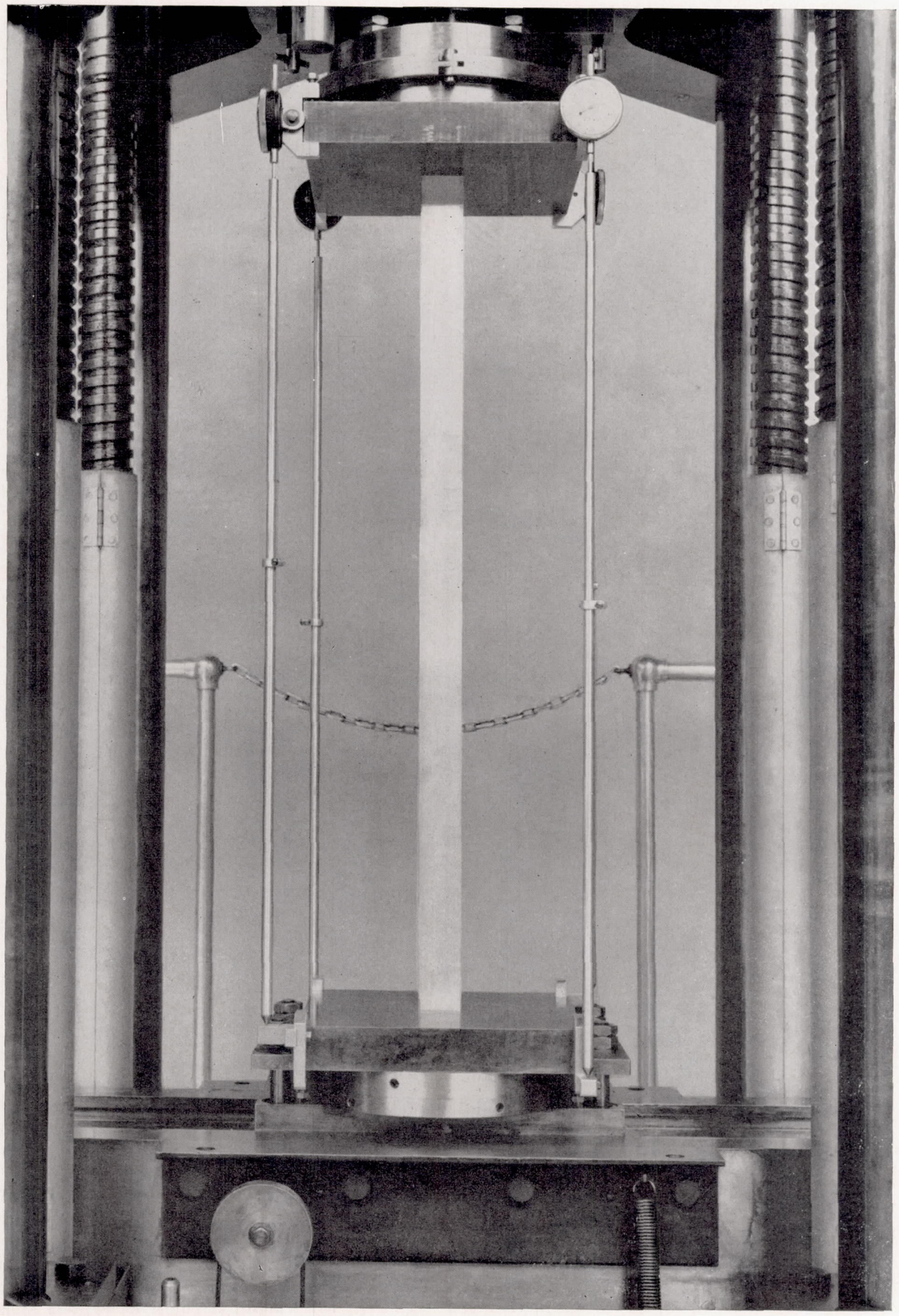


FIGURE 16.—Compressive stress-strain diagrams for 24S-T of cross section B. Strains measured with Huggenberger Tensometers. Pack compression specimen used, three pieces in pack.

would be possible if the data were plotted separately. The value of 0.5 for l_0/l in the flat-end tests has been found to be justified by the results of previous investigations, and the good agreement between these results from the two methods of test gives additional evidence that this value of l_0/l is satisfactory.

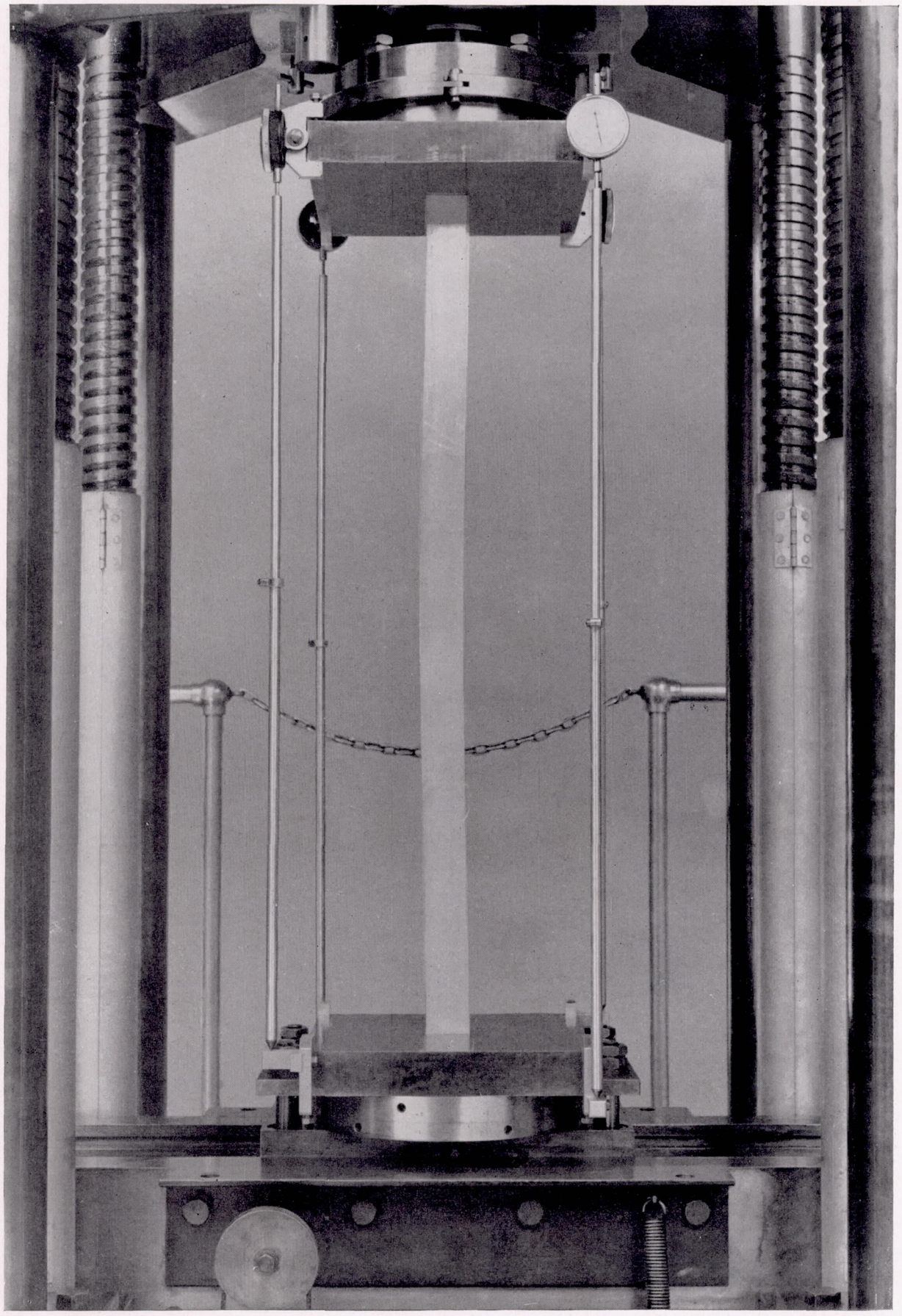
In addition to the test results, figures 20 and 21 show four curves of column strength. One of these is the ordinary Euler column curve. The equations of two of the other curves are of the same form as the Euler curve. These two equations take into account the inelastic behavior of the material at stresses greater than the proportional limit by using reduced values of the modulus instead of the initial modulus in the range of plastic action. In one, the initial modulus has been replaced by the tangent modulus and in the other by the effective modulus based on the double-modulus theory. (See reference 2.) These curves and the Euler curve are exactly the same, of course, for stresses below the proportional limit. In these curves the values of tangent modulus and the values of effective modulus based on the double-modulus theory were obtained from the compressive stress-strain data plotted in figures 12 and 13. The stress-modulus relations are shown in figures 22 and 23.

The fourth curve in figures 20 and 21 is simply a straight line drawn tangent to the Euler curve. The equations of these lines are the ones that would be predicted for this material on a basis of previous investigations of the column strength of various aluminum



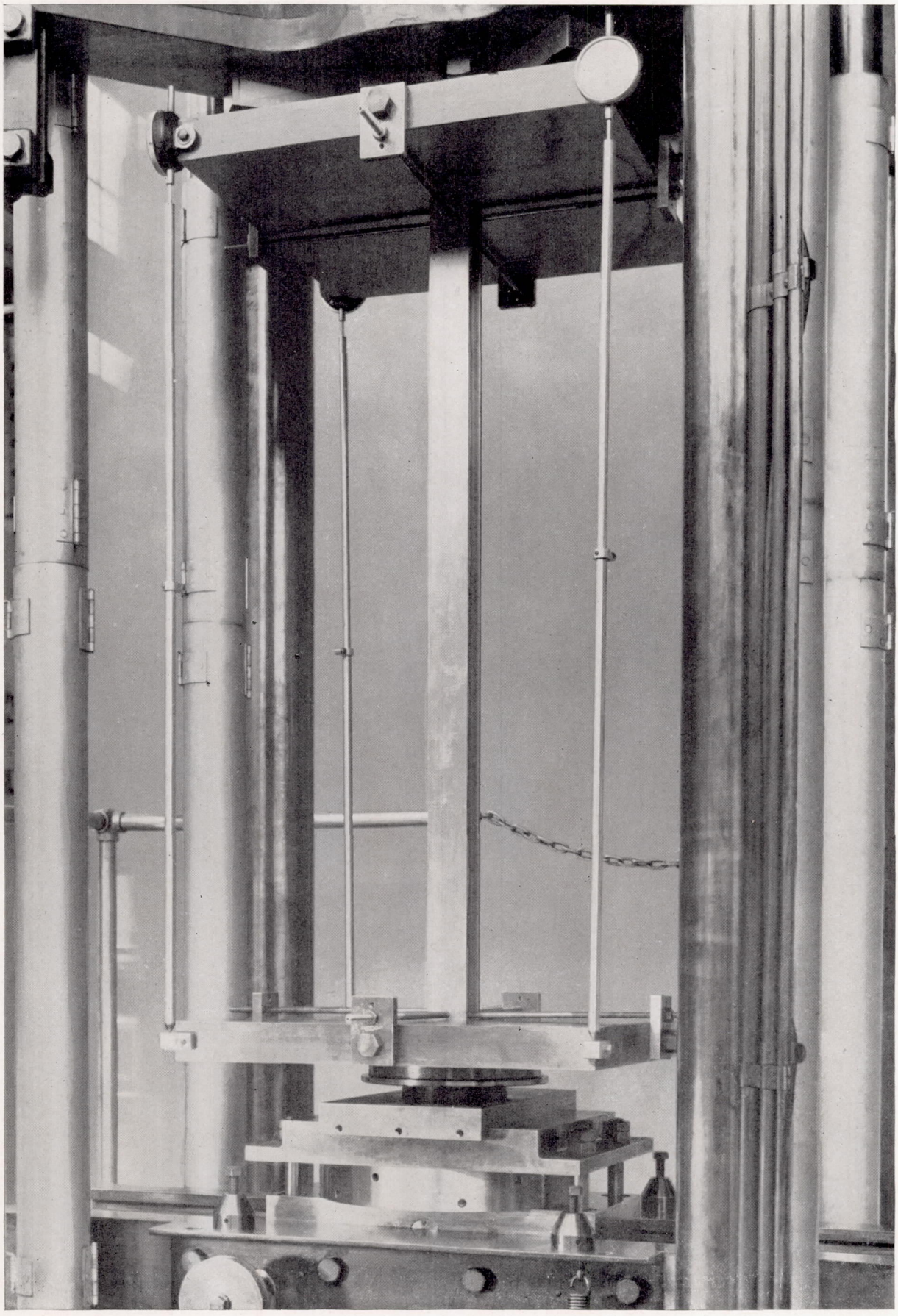
(a) Set-up for test.

FIGURE 17a.—Test of a column with flat ends.



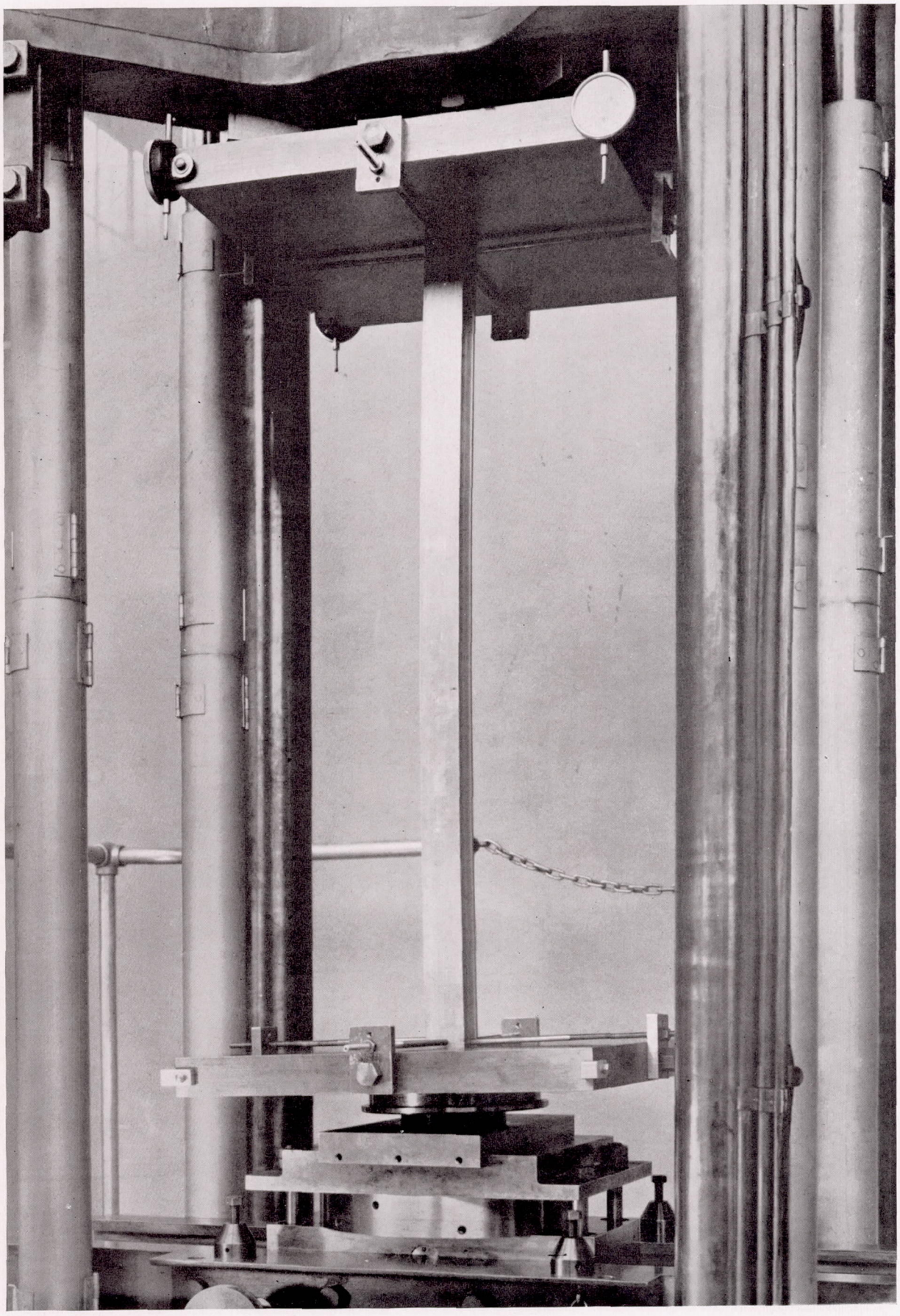
(b) Specimen after failure.

FIGURE 17b.—Test of column with flat ends.



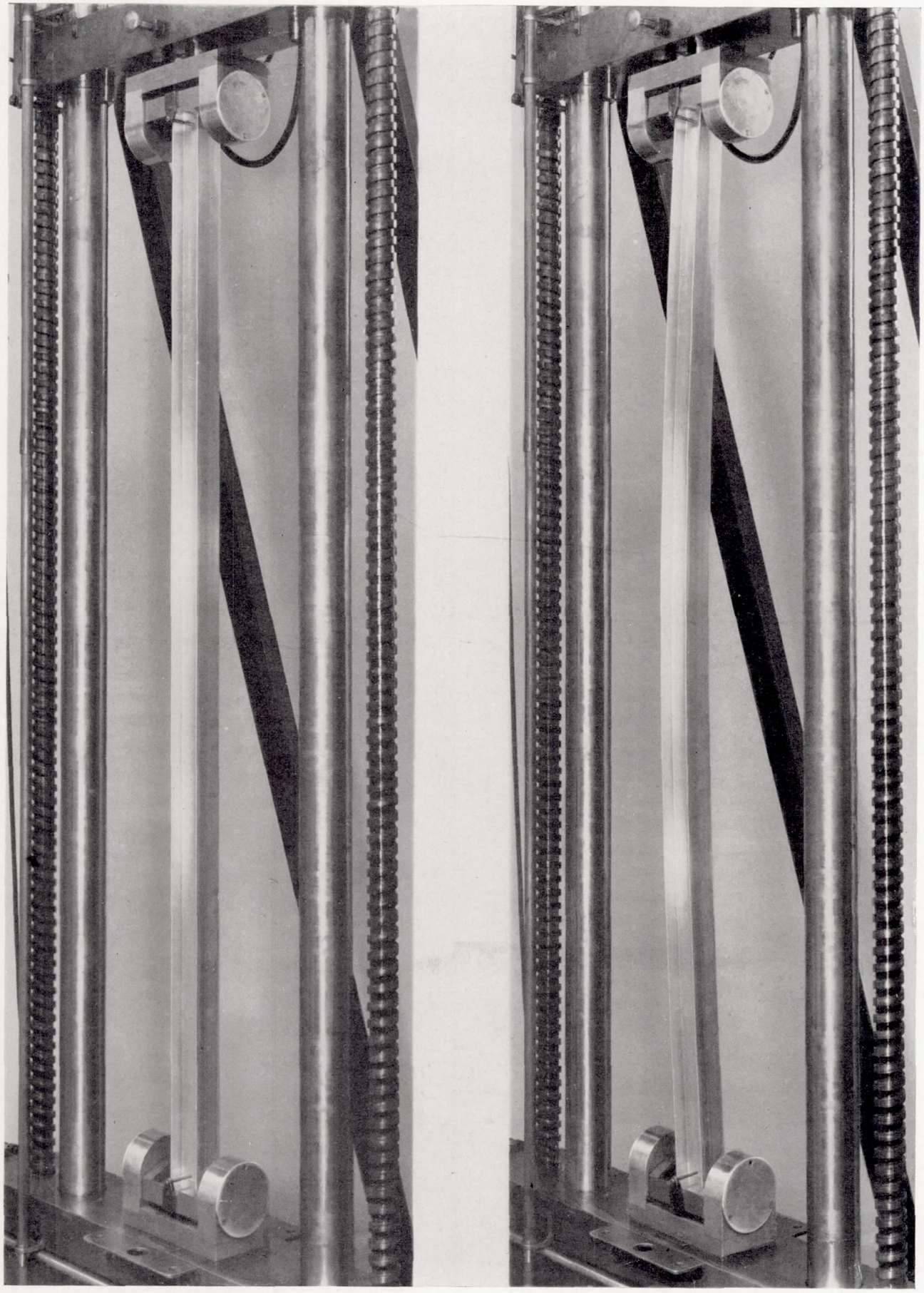
(a) Set-up for test.

FIGURE 18a.—Test of a column with round ends. Cross section A.



(b) Specimen after failure.

FIGURE 18b.—Test of a column with round ends. Cross section A.



(a) Set-up for test. (b) Specimen after failure.

FIGURE 19.—Test of a column with round ends. Cross section B.

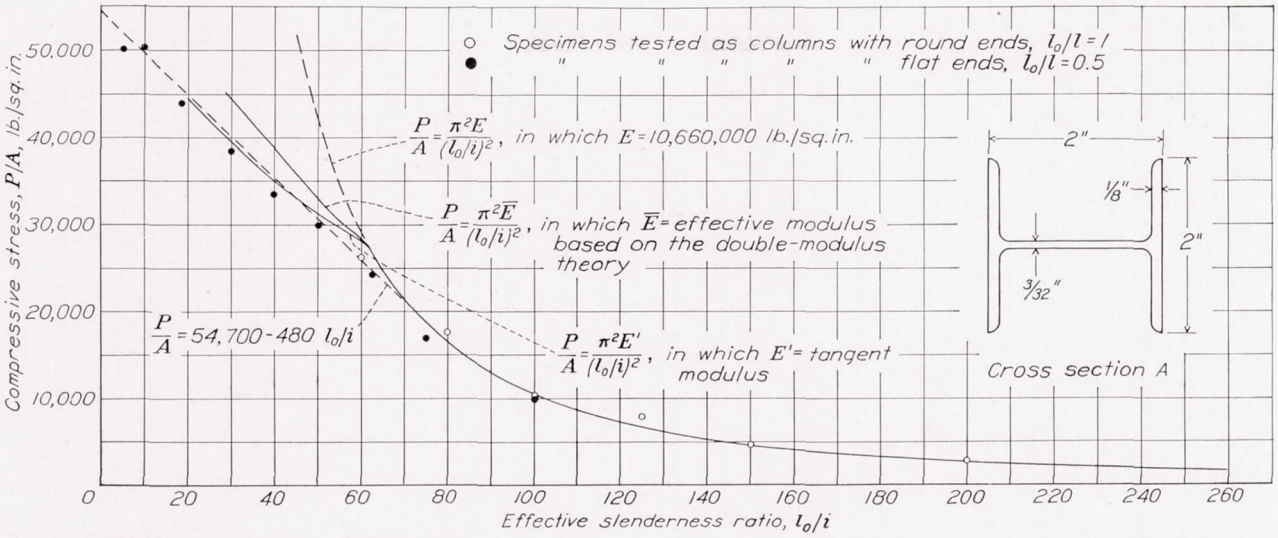


FIGURE 20.—Column strength of 24S-T of cross section A. Average compressive yield strength (offset=0.2 percent), 44,700 pounds per square inch.

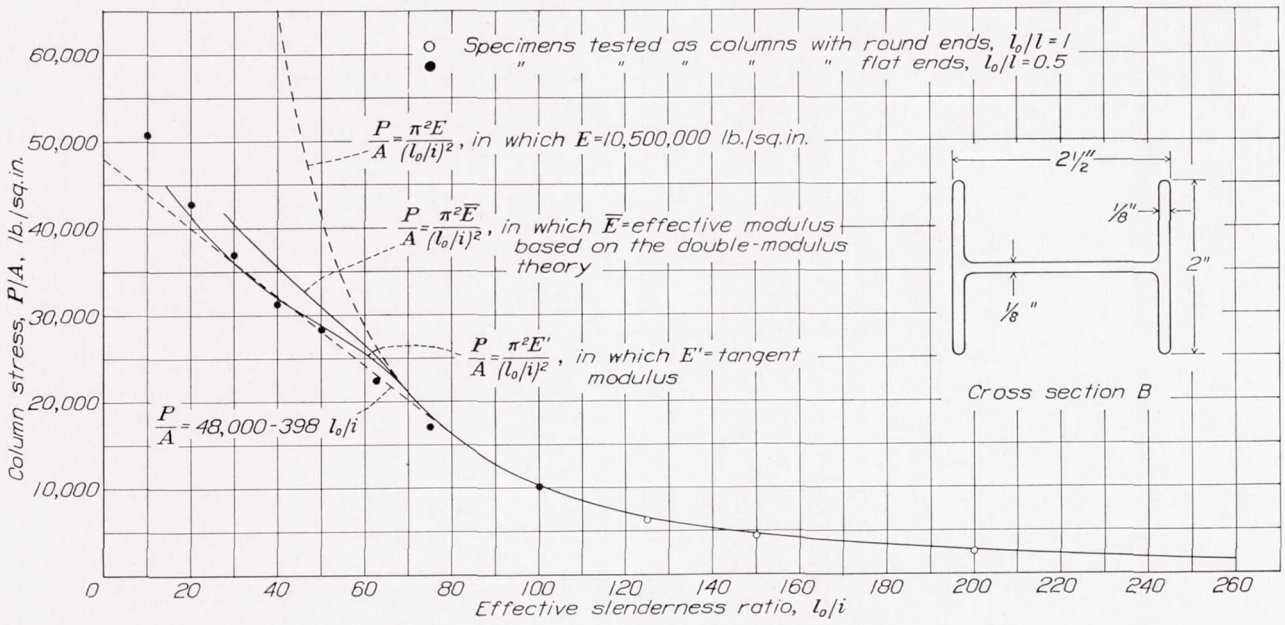


FIGURE 21.—Column strength of 24S-T of cross section B. Average compressive yield strength (offset=0.2 percent), 40,000 pounds per square inch.

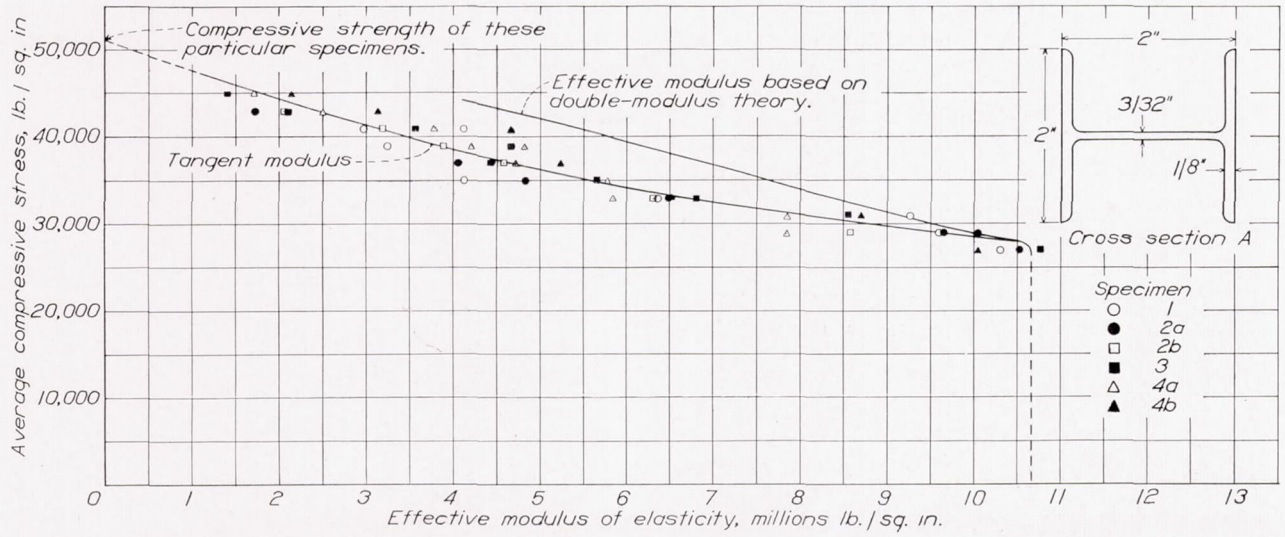


FIGURE 22.—Stress-tangent modulus curve for 24S-T of cross section A. Values of tangent modulus taken from compressive stress-strain relations obtained with Huggenberger Tensometers mounted at the middle of the flanges. Slenderness ratio of specimen, 10.

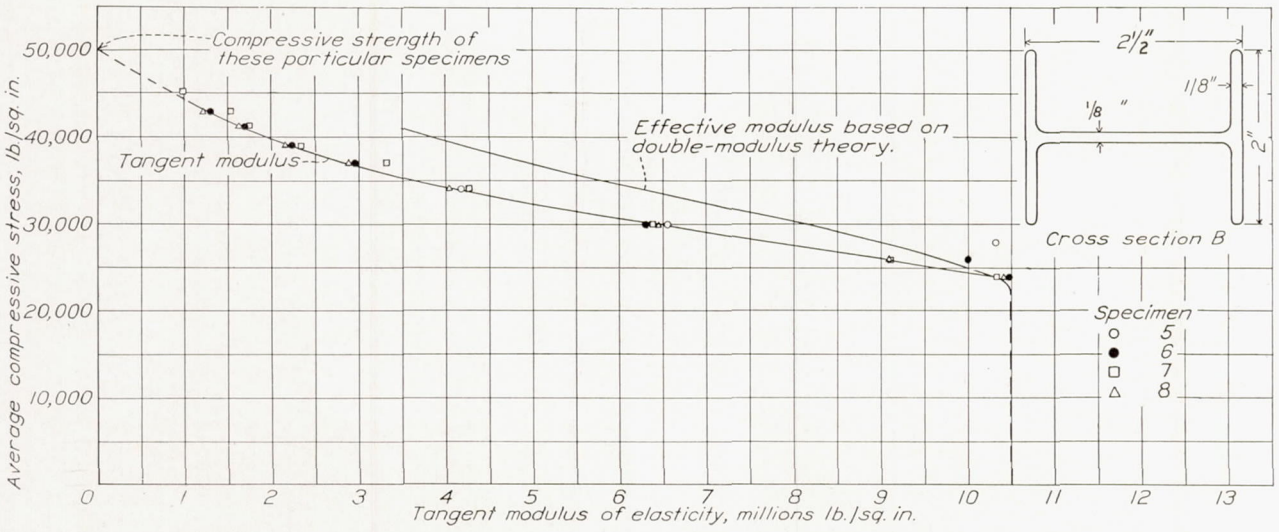


FIGURE 23.—Stress-tangent modulus curve for 24S-T of cross section B. Values of tangent modulus taken from compressive stress-strain relations obtained with Huggenberger Tensometers mounted at the middle of the flanges. Slenderness ratio of specimen, 10.

alloys (reference 4). The equation of the straight lines is of the form

$$\frac{P}{A}=B-D\left(\frac{l_0}{i}\right)$$

in which

P is the ultimate column load, pound.

A, cross-sectional area of member, square inch.

$\frac{l_0}{i}$, effective slenderness ratio.

B, constant depending on the compressive yield strength of the material.

D, constant depending on the compressive yield strength and on the modulus of elasticity of the material.

The intercept on the axis of zero slenderness, B, is arrived at by the following simple calculation involving the compressive yield strength of the material:

$$\text{Intercept } B=\text{yield strength}\left(1+\frac{\text{yield strength}}{200{,}000}\right).$$

None of the curves shown agree with the data exactly but both the straight line and the curve based on the tangent modulus of elasticity show good agreement with the data. The curve based on the double-modulus theory lies somewhat above the test results in the region of plastic action of the material.

The specimens after failure are shown in figures 24 and 25. Although the shorter specimens show considerable local distortion, it should be pointed out that this action was not apparent until the average stress exceeded the yield strength or until the maximum column strength had been attained.

III. CORRELATION OF TEST RESULTS

The average mechanical properties of the material of cross section A may be summarized as follows:

	Lbs. per sq. in.
Tensile yield strength (offset=0.2 percent):	
National Bureau of Standards	49, 800
Aluminum Research Laboratories	48, 200
Compressive yield strength:	
National Bureau of Standards (2/3 E method)	42, 050
National Bureau of Standards (offset=0.2 percent)	42, 200
Aluminum Research Laboratories (offset=0.2 percent)	44, 700

The difference in the values of the tensile yield strength is 1,600 pounds per square inch or about 3 percent, and the difference in the values of the compressive yield strength is 2,500 pounds per square inch or about 6 percent. The National Bureau of Standards tests indicate the higher tensile yield strength and the lower compressive yield strength. The ratios of the average compressive yield strength to the average tensile yield strength are:

National Bureau of Standards,

$$\frac{\text{compressive yield strength}}{\text{tensile yield strength}}=0.85$$

Aluminum Research Laboratories,

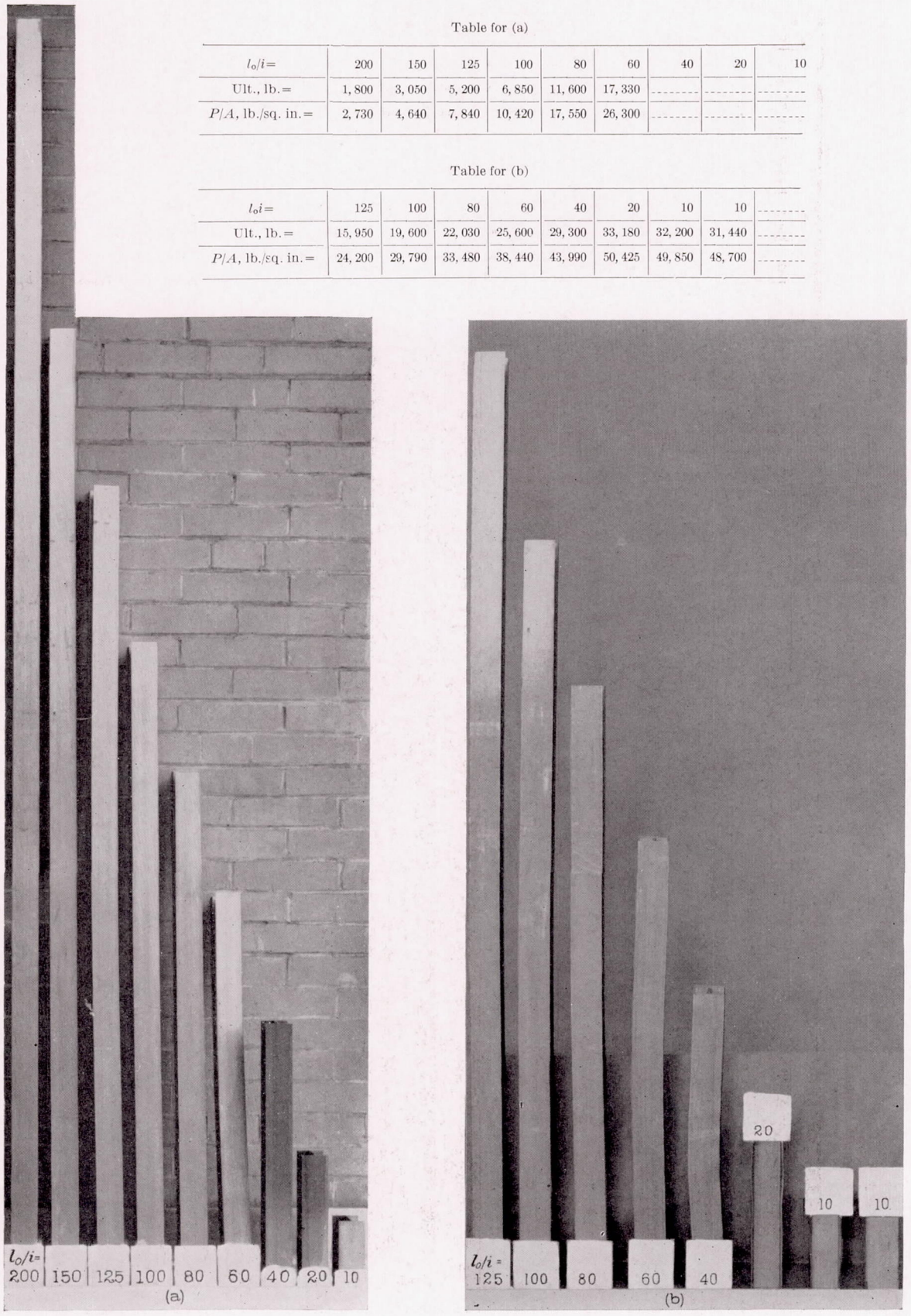
$$\frac{\text{compressive yield strength}}{\text{tensile yield strength}}=0.93$$

Table for (a)

$l_o/i=$	200	150	125	100	80	60	40	20	10
Ult., lb.=	1, 800	3, 050	5, 200	6, 850	11, 600	17, 330	--------	--------	--------
P/A, lb./sq. in.=	2, 730	4, 640	7, 840	10, 420	17, 550	26, 300	--------	--------	--------

Table for (b)

$l_o i=$	125	100	80	60	40	20	10	10	--------
Ult., lb.=	15, 950	19, 600	22, 030	25, 600	29, 300	33, 180	32, 200	31, 440	--------
P/A, lb./sq. in.=	24, 200	29, 790	33, 480	38, 440	43, 990	50, 425	49, 850	48, 700	--------



(a) Tested with round ends.

(b) Tested with flat ends.

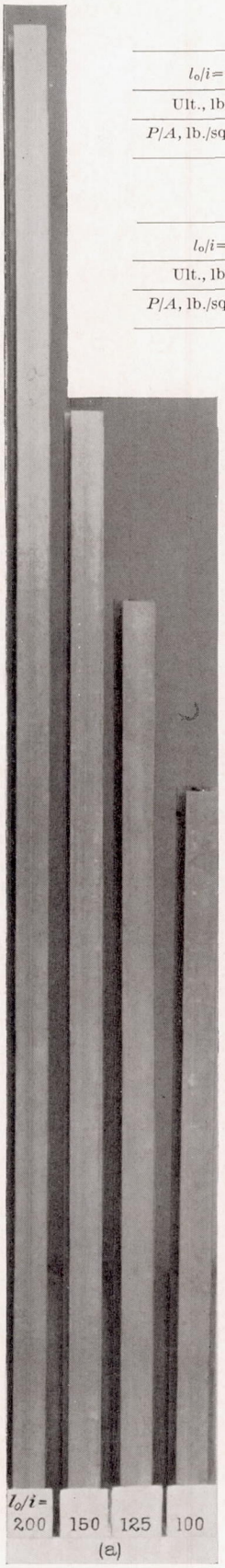
FIGURE 24.—Specimens of 24S-T of cross section A after testing.

Table for (a)

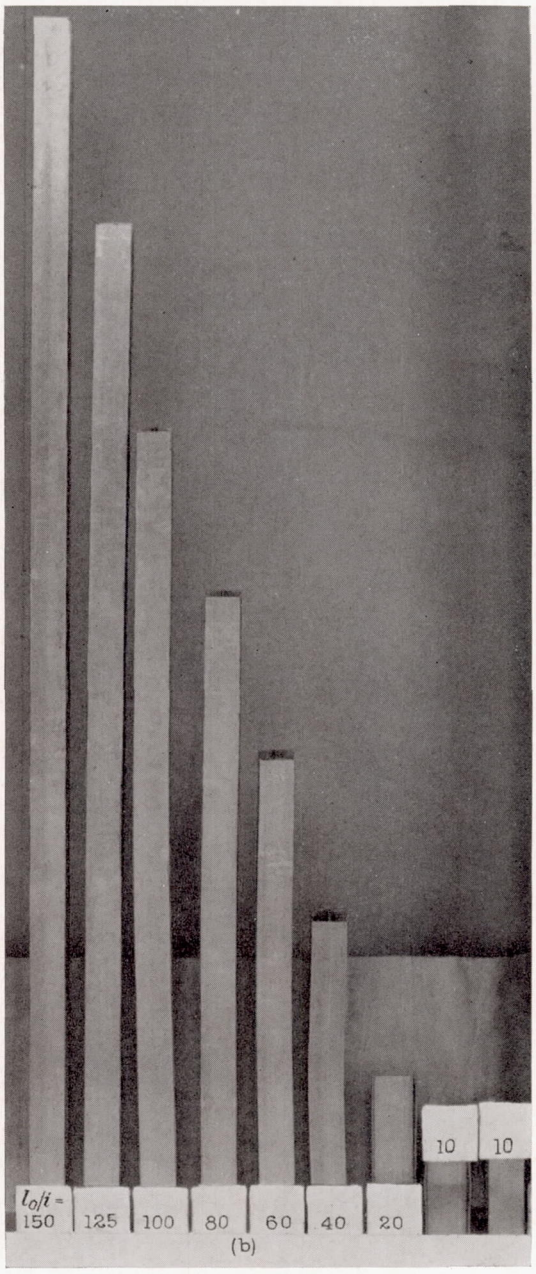
$l_o/i=$	200	150	125	100	--------	--------	--------	--------	--------
Ult., lb.=	1, 995	3, 510	5, 000	8, 000	--------	--------	--------	--------	--------
P/A, lb./sq. in.=	2, 550	4, 480	6, 390	10, 190	--------	--------	--------	--------	--------

Table for (b)

$l_o/i=$	150	125	100	80	60	40	20	10	10
Ult., lb.=	13, 375	17, 600	22, 260	24, 500	28, 930	33, 700	39, 900	39, 100	38, 500
P/A, lb./sq. in.=	17, 038	22, 478	28, 357	31, 370	36, 995	42, 875	50, 763	49, 810	49, 110



(a) Tested with round ends.



(b) Tested with flat ends.

FIGURE 25.—Specimens of 24S-T of cross section B after testing.

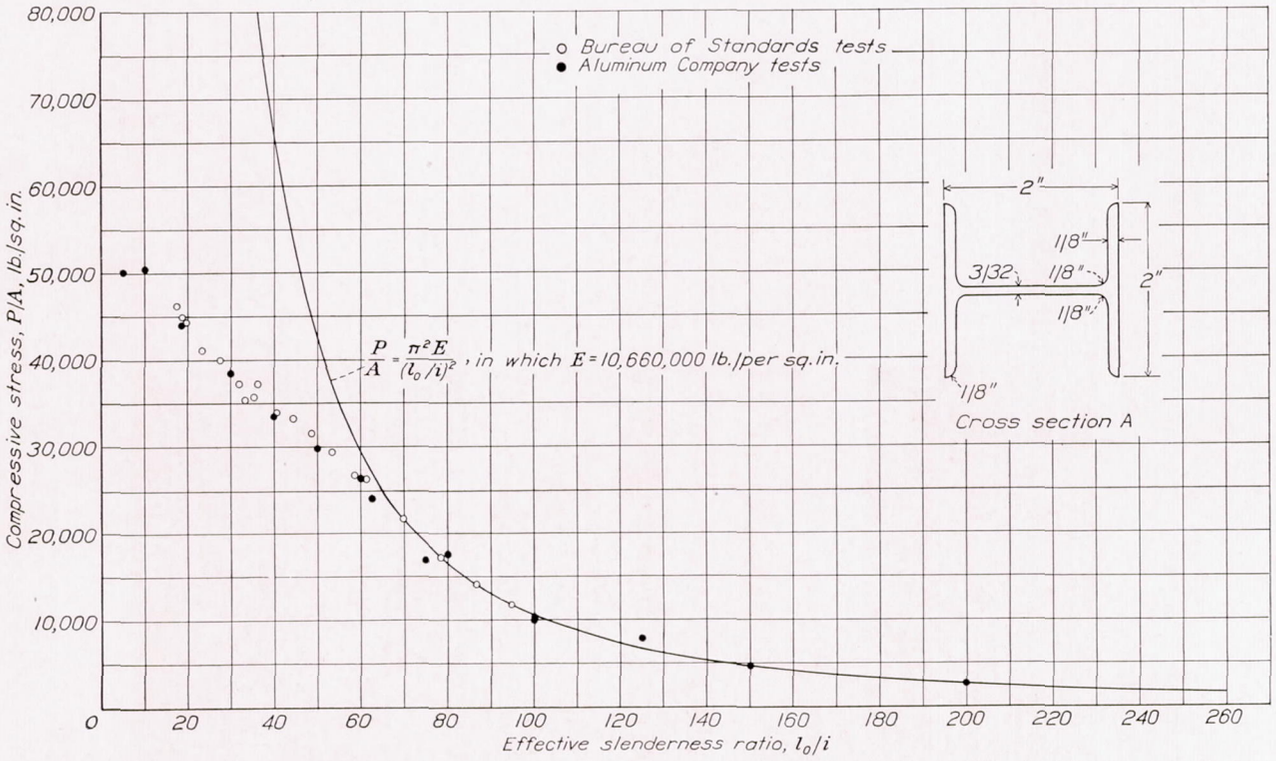


FIGURE 26.—Comparison of results from National Bureau of Standards and Aluminum Company tests for 24S-T of cross section A. $l_0/l=0.5$, assumed for flat ends.

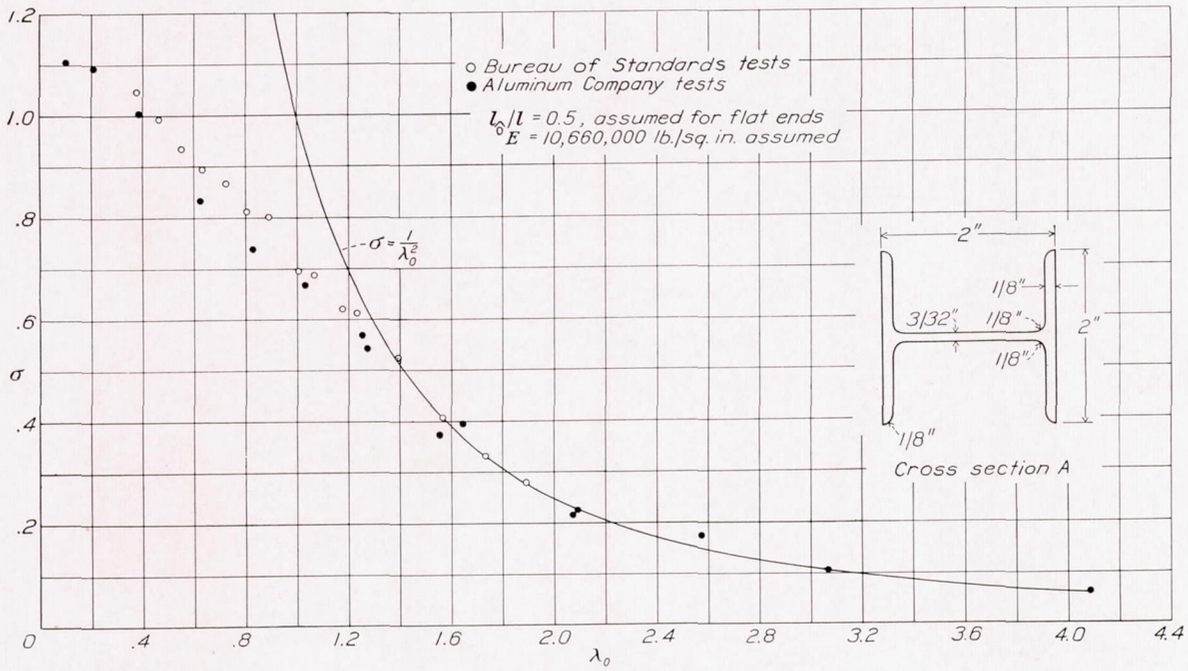


FIGURE 27.—Comparison of results from National Bureau of Standards and Aluminum Company tests for 24S-T of cross section A. The nondimensional method of part I in which σ is plotted against λ_0.

Figure 26 shows the column test results of cross section A and the Euler column curve, plotted as P/A against l_0/i ($l_0/l=0.5$, assumed for flat-end specimens, and $E=10{,}660{,}000$ pounds per square inch assumed for Euler curve).

Figure 27 shows the column test results of cross section A and the nondimensional Euler column curve, plotted as $\sigma=P/(AS)$ against $\lambda_0=\frac{1}{\pi}\frac{l_0}{i}\sqrt{\frac{S}{E}}$ (the free length of the Aluminum Company's flat-end specimens has been assumed one-half of the length of the specimen, and the modulus of elasticity for their specimens has been assumed as 10,660,000 pounds per square inch).

National Bureau of Standards,
Washington, D. C.,
and
Aluminum Research Laboratories,
Aluminum Company of America,
New Kensington, Pa.,
September 30, 1938.

APPENDIX

CENTERING LOADS

When centering is done under load in the plastic range, it is necessary to limit the maximum deflections to values which insure that the maximum stress at no point of the middle cross section exceeds the expected maximum average column stress. An estimate of the maximum allowable deflection may be obtained by considering that the center line of the specimen goes into a sine curve. Then for the middle cross section

$$\frac{\epsilon}{c}=\frac{1}{\rho}=\delta_0\frac{\pi^2}{l_0^2}$$

where ϵ is the bending strain at the extreme fiber distance c, ρ is the radius of curvature, δ_0 is the maximum deflection, and l_0 is the free length of the specimen. Solved for ϵ, this equation gives

$$\epsilon=\frac{\pi^2 c\delta_0}{l_0^2}$$

In the present investigation, corresponding to a maximum value of $\delta_0=0.0005$ inch,

$$\epsilon=\frac{0.005}{l_0^2}$$

when l_0 is measured in inches. If $l_0=10$ inches, a low value, $\epsilon=0.00005$; and if $l_0=25$ inches, a medium value, $\epsilon=0.000008$.

The column stresses corresponding to these values of l_0 $\left(\frac{l_0}{i}=21 \text{ and } 52, \text{ respectively}\right)$ may be obtained roughly from figure 7 as 42,000 and 30,000 pounds per square inch. The stress-strain diagrams (fig. 3) show that at a stress of 42,000 pounds per square inch an increase of strain of 0.00005 results in an increase of stress of only about 100 pounds per square inch, or 0.24 percent; and at 30,000 pounds per square inch an increase of strain of 0.000008 results in an increase of stress of about 80 pounds per square inch, or 0.27 percent. These small increases of stress due to a deflection of 0.0005 inch at the maximum column load indicate that the final centering load may be close to the column load without danger of overstressing any part of the cross section.

REFERENCES

1. Lundquist, Eugene E., and Fligg, Claude M.: A Theory for Primary Failure of Straight Centrally Loaded Columns. T. R. No. 582, N. A. C. A., 1937.
2. Osgood, William R.: The Double-Modulus Theory of Column Action. Civil Engineering, vol. 5, no. 3, March 1935, pp. 173–175.
3. Osgood, William R.: Column Strength of Tubes Elastically Restrained against Rotation at the Ends. T. R. No. 615, N. A. C. A.. 1938.
4. Templin, R. L., Sturm, R. G., Hartmann, E. C., and Holt, M.: Column Strength of Various Aluminum Alloys. Tech. Paper No. 1, Aluminum Res. Lab., Aluminum Co. of America, 1938.

TABLE I.—RESULTS OF COLUMN TESTS AND COMPRESSIVE TESTS MADE AT THE NATIONAL BUREAU OF STANDARDS ON 24S–T EXTRUDED H-BEAM SPECIMENS OF CROSS SECTION A

Specimen	Slenderness ratio l_0/i	Average stress P/A (lb./sq. in.)	$\lambda_0=\frac{1}{\pi}\frac{l_0}{i}\sqrt{\frac{S}{E}}$	$\sigma=\frac{P}{AS}$	Specimen	Slenderness ratio l_0/i	Average stress P/A (lb./sq. in.)	$\lambda_0=\frac{1}{\pi}\frac{l_0}{i}\sqrt{\frac{S}{E}}$	$\sigma=\frac{P}{AS}$
Freely supported ends, $m=0$									
5C–5	18.82	44,820	0.378	1.047	5C–3	53.01	29,400	1.064	0.687
1B–5c	23.24	41,140	.464	.991	5D–1	58.62	26,520	1.177	.620
5C–4	27.62	39,990	.554	.934	5C–7	61.40	26,140	1.233	.611
1B–5b	31.92	37,130	.637	.895	1B–4	69.81	21,750	1.392	.524
5C–6	36.12	37,160	.725	.868	5C–2	78.08	17,410	1.567	.407
1B–5a	40.42	33,650	.806	.811	5C–8	86.44	14,170	1.735	.331
1B–3	44.60	33,180	.890	.800	1B–5	94.79	11,680	1.891	.281
5D–2	50.17	29,720	1.007	.694					
Elastically restrained ends, $m=192{,}000$ lb.-in. per radian									
5C–9	17.79	46,110	0.357	1.077	1A–2	48.87	31,460	0.975	0.758
1A–4	33.22	35,310	.663	.851					
Elastically restrained ends, $m=385{,}000$ lb.-in. per radian									
5C–1	19.32	44,180	0.388	1.032	1A–1	50.34	29,610	1.004	0.713
1A–3	35.28	35,670	.704	.860					
Flat ends (compressive specimens)									
1B–1	----------	48 390	----------	1.166	5C–C	----------	48,850	----------	1.141

TABLE II.—SUMMARY OF MECHANICAL PROPERTIES OF MATERIAL

[24S-T Extruded H-Beam]

	Specimen	Cross section A			Cross section B		
		Minimum	Maximum	Average	Minimum	Maximum	Average
Tensile strength lb. per sq. in.	Web	62,770	62,960	62,890	60,450	63,670	62,070
	Flange	62,930	64,980	63,990	60,560	64,150	62,370
Tensile yield strength (offset=0.2 percent) lb. per sq. in	Web	47,700	49,100	48,470	48,100	51,900	50,075
	Flange	48,100	49,000	47,940	49,000	51,300	49,750
Elongation in 2 in percent	Web	16.5	18.5	17.3	16.5	22.5	18.8
	Flange	17.0	21.5	19.5	15.0	22.0	19.6
Compressive yield strength (offset=0.2 percent) lb. per sq. in.	Full section	43,400	46,700	44,700	39,700	40,300	40,000
	Longitudinal pack, web	42,600	43,800	43,100	39,600	43,000	41,925
	Longitudinal pack, flange	42,600	43,800	43,000	38,400	40,000	39,150
	Transverse pack, web	52,200	52,600	52,400	47,600	54,700	51,725
	Transverse pack, flange	51,800	53,400	52,800	48,000	51,200	49,975

TABLE III.—DESCRIPTION OF SPECIMENS AND RESULTS OF COLUMN TESTS MADE AT ALUMINUM RESEARCH LABORATORIES

24S-T Extruded H-Beam

Specimen	Length (in.)	Weight (lb.)	Slenderness ratio	Measured crookedness [1] (in.)	Actual average area [2] (sq. in.)	Maximum column load (lb.)	Column strength (lb./sq. in.)
Cross section A, specimens tested as columns with flat ends							
1-96	96.46	6.357	200	0.025	0.659	6,530	9,910
1-72	72.42	4.765	150	.013	.658	11,150	16,950
3-60a	60.34	3.976	125	.030	.659	15,950	24,200
3-48	48.27	3.176	100	.006	.658	19,600	29,790
1-39	38.96	2.564	80	----------	.658	22,030	33,480
4-29a	28.96	1.929	60	.011	.666	25,600	38,440
2-19	18.04	1.201	37	----------	.666	29,300	43,990
4-10	9.73	.640	20	----------	.658	33,180	50,430
1-5	4.94	.319	10	----------	.646	32,330	50,050
Cross section A, specimens tested as columns with round ends.[3]							
2-96	96.48	6.358	200	0.040	0.659	1,800	2,730
2-72	72.44	4.770	150	.014	.658	3,050	4,640
3-60b	60.35	4,000	125	.024	.663	5,200	7,840
4-48	48.30	3.173	100	.021	.657	6,850	10,420
3-39	38.68	2.557	80	.014	.661	11,600	17,550
4-29b	29.01	1.909	60	.015	.658	17,330	26,300
4-19	19.44	1.276	40	.013	.656	(4)	(4)

[1] Crookedness measured by placing thickness gages between the specimen and a plane surface on which it rested.

TABLE III.—DESCRIPTION OF SPECIMENS AND RESULTS OF COLUMN TESTS MADE AT ALUMINUM RESEARCH LABORATORIES—Continued

24S-T Extruded H-Beam

Specimen	Length (in.)	Weight (lb.)	Slenderness ratio	Measured crookedness (in.)	Actual average area (sq. in.)	Maximum column load (lb.)	Column strength (lb./sq. in.)
Cross section B, specimens tested as columns with flat ends							
5-90a	90.28	7.096	200	0.008	0.786	8,000	10,180
6-68a	67.80	5.320	150	.011	.785	13,375	17,040
7-56a	56.60	4.430	125	.008	.783	17,600	22,480
8-45a	45.25	3.550	100	.006	.785	22,260	28,360
7-36	36.10	2.820	80	.005	.781	24,500	31,370
7-27	27.15	2.122	60	----------	.782	28,930	37,000
8-18	18.10	1.422	40	----------	.786	33,700	42,880
5-9	8.97	.705	20	----------	.786	39,900	50,760
Cross section B, specimens tested as columns with round ends [5]							
5-90b	90.15	7.053	200	0.014	0.782	1,995	2,550
6-68b	67.81	5.310	150	.010	.783	5,310	4,480
7-56b	56.60	4.429	125	.011	.783	5,000	6,390
4-45b	45.25	3.551	100	.007	.785	8,000	10,190

[2] Area computed from the weight and length of the specimen and the nominal specific gravity of the material.
[3] Ball-bearing spherical heads used, specimen free to deflect in any direction and twist.
[4] Strength greater than the capacity of the ball-bearing spherical seats.
[5] Roller-bearing heads used, specimen free to deflect in only one direction.

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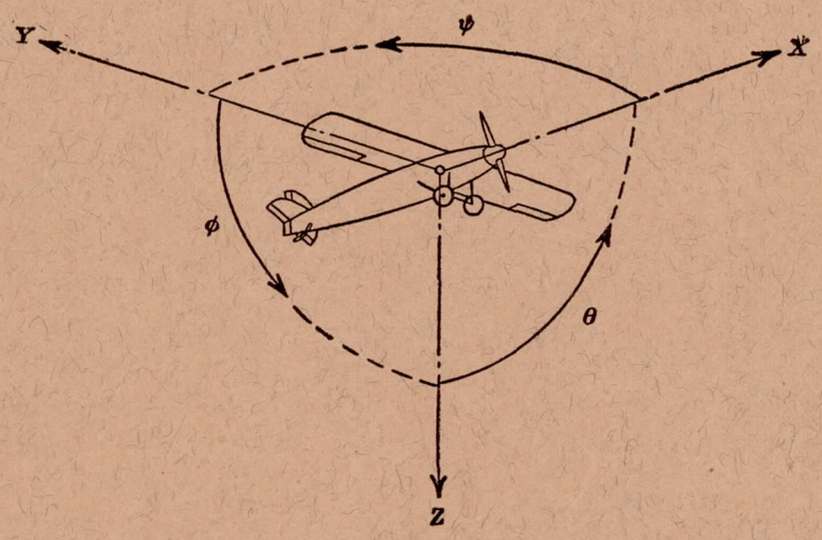

Positive directions of axes and angles (forces and moments) are shown by arrows

Axis		Force (parallel to axis) symbol	Moment about axis			Angle		Velocities	
Designation	Symbol		Designation	Symbol	Positive direction	Designation	Symbol	Linear (component along axis)	Angular
Longitudinal	X	X	Rolling	L	$Y \longrightarrow Z$	Roll	ϕ	u	p
Lateral	Y	Y	Pitching	M	$Z \longrightarrow X$	Pitch	θ	v	q
Normal	Z	Z	Yawing	N	$X \longrightarrow Y$	Yaw	ψ	w	r

Absolute coefficients of moment

$C_l=\frac{L}{qbS}$ (rolling) $\quad C_m=\frac{M}{qcS}$ (pitching) $\quad C_n=\frac{N}{qbS}$ (yawing)

Angle of set of control surface (relative to neutral position), δ. (Indicate surface by proper subscript.)

4. PROPELLER SYMBOLS

D, Diameter

p, Geometric pitch

p/D, Pitch ratio

V', Inflow velocity

V_s, Slipstream velocity

T, Thrust, absolute coefficient $C_T=\frac{T}{\rho n^2 D^4}$

Q, Torque, absolute coefficient $C_Q=\frac{Q}{\rho n^2 D^5}$

P, Power, absolute coefficient $C_P=\frac{P}{\rho n^3 D^5}$

C_s, Speed-power coefficient$=\sqrt[5]{\frac{\rho V^5}{P n^2}}$

η, Efficiency

n, Revolutions per second, r.p.s.

Φ, Effective helix angle$=\tan^{-1}\left(\frac{V}{2\pi r n}\right)$

5. NUMERICAL RELATIONS

1 hp.=76.04 kg-m/s=550 ft-lb./sec.
1 metric horsepower=1.0132 hp.
1 m.p.h.=0.4470 m.p.s.
1 m.p.s.=2.2369 m.p.h.

1 lb.=0.4536 kg.
1 kg=2.2046 lb.
1 mi.=1,609.35 m=5,280 ft.
1 m=3.2808 ft.