REPORT No. 749

PROPELLER CHARTS FOR THE DETERMINATION OF THE
ROTATIONAL SPEED FOR THE MAXIMUM RATIO
OF THE PROPULSIVE EFFICIENCY TO THE
SPECIFIC FUEL CONSUMPTION

By DAVID BIERMANN and ROBERT N. CONWAY

SUMMARY

A set of propeller operating efficiency charts, based on
a coefficient from which the propeller rotational speed has
been eliminated, is presented. These charts were pre-
pared with data obtained from tests of full-size metal pro-
pellers in the NACA propeller-research tunnel. Working
charts for nine propeller-body combinations are presented,
including results from tests of dual-rotating propellers.

These charts are to be used in the calculation of the
range and the endurance of airplanes equipped with
constant-speed propellers in which, for given flight condi-
tions, it is desired to determine the propeller revolution
speed that gives the maximum ratio of the propulsive
efficiency to the specific fuel consumption. The coefficient
on which the charts are based may be written in the form
of a thrust coefficient or a thrust-power coefficient.

A method of using the charts is outlined and sample
computations for a typical airplane are included.

INTRODUCTION

In the calculation of the range and the endurance of
airplanes equipped with constant-speed propellers, the
thrust horsepower required for flight at a given gross
weight, airspeed, and altitude is known. It is desired
to determine, from the propeller-engine operating char-
acteristics, the propeller rotational speed that gives the
maximum ratio of the propulsive efficiency to the specific
fuel consumption. The usual propeller coefficients are
insufficient for use in such calculations because knowl-
edge of both the engine power and the rotational speed
is required and these quantities are unknown.

In order to facilitate the calculations, the Bureau
of Aeronautics, Navy Department, requested the NACA
to prepare working plots of a thrust coefficient that is
independent of propeller revolution speed. The present
report contains such working charts for nine propeller-
body combinations and includes results obtained from
tests of dual-rotating propellers. A method that
can be used for the determination of the maximum value
of the ratio of the propulsive efficiency to the specific
fuel consumption is also included.

SYMBOLS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P$</td>
<td>power of engine, foot-pounds per second</td>
</tr>
<tr>
<td>$T_e$</td>
<td>effective thrust $(T - \Delta D)$, pounds</td>
</tr>
<tr>
<td>$T$</td>
<td>total propeller thrust</td>
</tr>
<tr>
<td>$\Delta D$</td>
<td>increased drag of body due to propeller slipstream</td>
</tr>
<tr>
<td>$W$</td>
<td>weight, pounds</td>
</tr>
<tr>
<td>$n$</td>
<td>propeller rotational speed, rps</td>
</tr>
<tr>
<td>$L/D$</td>
<td>lift-drag ratio</td>
</tr>
<tr>
<td>$V$</td>
<td>airspeed, feet per second</td>
</tr>
<tr>
<td>$D$</td>
<td>propeller diameter, feet</td>
</tr>
<tr>
<td>$\rho$</td>
<td>mass density of air, slugs per cubic foot</td>
</tr>
<tr>
<td>$\rho_0$</td>
<td>mass density of air at sea level (0.002378)</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>relative density of air $(\rho/\rho_0)$</td>
</tr>
<tr>
<td>$q$</td>
<td>dynamic pressure $(\tfrac{1}{2} \rho V^2)$</td>
</tr>
<tr>
<td>$C$</td>
<td>engine specific fuel consumption, pounds per brake horsepower-hour</td>
</tr>
<tr>
<td>$Q$</td>
<td>engine torque, foot-pounds</td>
</tr>
<tr>
<td>$C_t$</td>
<td>speed-power coefficient $(\tfrac{4}{5} \pi V^2 / P_e)$</td>
</tr>
<tr>
<td>$V/nD$</td>
<td>advance-diameter ratio</td>
</tr>
<tr>
<td>$T_r$</td>
<td>thrust coefficient $(T_e / \pi D^2 V^2)$</td>
</tr>
<tr>
<td>$C_p'$</td>
<td>power coefficient $(P / \pi D^2 V^2 = \rho V^2)$</td>
</tr>
<tr>
<td>$P_e$</td>
<td>power disk-loading coefficient $(8P / \pi D^2 V^2)$</td>
</tr>
</tbody>
</table>

DISCUSSION

In the calculation of the range and the endurance of
airplanes equipped with constant-speed propellers it is
desired to determine, from the engine-propeller operat-
ing characteristics, the propeller speed that gives the
maximum ratio of the propulsive efficiency to the specific
fuel consumption. The thrust horsepower re-
quired for a given flight at a given gross weight, air-
speed, and altitude is already known.

The usual propeller coefficients of power, thrust, and
propulsive efficiency that are defined as
are insufficient for this purpose because neither the engine power nor the engine speed is known.

In order to solve these special problems with the least work it is necessary to use plots of propeller coefficients that do not involve the rotational speed. Suitable forms of the coefficients may be obtained from the usual thrust or power coefficients as follows:

\[
T_c = \frac{C_T}{(V/nD)^2} = \frac{T_2}{\rho D^2 V^2}
\]

\[
\eta C_p' = \frac{P}{\eta D^2 V^2}
\]

Because the brake horsepower is unknown, the equation for \( C_p' \) should be multiplied by \( \eta \) to obtain a thrust-power coefficient:

\[
\eta C_p' = \frac{\eta P}{(V/nD)^2} = \frac{\eta P}{\rho D^2 V^2}
\]

### PROPELLER DATA

The propeller charts were prepared from data obtained in the NACA 20-foot propeller-research tunnel. The tests that provided the data were made with tractor propellers of approximately 10-foot diameter operating in conjunction with various representative body types. A list of the charts and the basic propeller data are given in the following table; principal model dimensions are given directly on the propeller charts.

<table>
<thead>
<tr>
<th>Figure</th>
<th>Propeller drawing number</th>
<th>Total number of blades</th>
<th>Body condition</th>
<th>Approximate propeller diameter (ft)</th>
<th>Rotation</th>
<th>Blade angle range at 0.75 ( n ) (deg)</th>
<th>Source of propeller data, reference number</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, 2</td>
<td>3166-4</td>
<td>3</td>
<td>Streamline body, with spinner, on symmetrical wing</td>
<td>10</td>
<td>Single</td>
<td>20 to 65</td>
<td>1</td>
</tr>
<tr>
<td>3, 4</td>
<td>0168-A-3</td>
<td>3</td>
<td>Streamline body, with spinner</td>
<td>11.20</td>
<td>Single</td>
<td>20 to 65</td>
<td>1</td>
</tr>
<tr>
<td>5, 6</td>
<td>3166-6</td>
<td>4</td>
<td>Streamline body, with spinner, on symmetrical wing</td>
<td>12</td>
<td>Single</td>
<td>20 to 65</td>
<td>2</td>
</tr>
<tr>
<td>7, 8</td>
<td>3166-6, L. E.</td>
<td>4</td>
<td>……</td>
<td>12</td>
<td>Single</td>
<td>20 to 65</td>
<td>2</td>
</tr>
<tr>
<td>9, 10</td>
<td>3166-6, r. h.</td>
<td>4</td>
<td>……</td>
<td>12</td>
<td>Single</td>
<td>20 to 65</td>
<td>2</td>
</tr>
<tr>
<td>11, 12</td>
<td>3166-6, L. E.</td>
<td>6</td>
<td>……</td>
<td>12</td>
<td>Single</td>
<td>20 to 65</td>
<td>2</td>
</tr>
<tr>
<td>13, 14</td>
<td>5066-5</td>
<td>3</td>
<td>Liquid-cooled engine nacelle, with spinner</td>
<td>10</td>
<td>Single</td>
<td>15 to 60</td>
<td>3</td>
</tr>
<tr>
<td>15, 17</td>
<td>5066-6</td>
<td>3</td>
<td>Radial engine nacelle</td>
<td>10</td>
<td>Single</td>
<td>15 to 60</td>
<td>4</td>
</tr>
<tr>
<td>16, 18</td>
<td>5066-8</td>
<td>4</td>
<td>Liquid-cooled engine nacelle, with spinner</td>
<td>10</td>
<td>Single</td>
<td>15 to 60</td>
<td>6</td>
</tr>
</tbody>
</table>

### USE OF CHARTS

It is possible that propeller charts of the type given in this report may have a number of uses that are not anticipated at the present time. Also, it is quite likely that different methods can be devised for using the charts in solving any specific problem. The chief object of these charts is, however, to furnish a means of determining the optimum propeller speed for given flight conditions to obtain the maximum ratio of the propulsive efficiency to the engine fuel consumption. A method of obtaining the optimum propeller speed is presented in the following outline.
Figure 1.—Chart of $\frac{V}{nD}$ against $T_0$ for three-blade propeller S155-6. Streamline body, with spinner, on symmetrical wing; single rotation. Propeller data taken from reference 1.
Figure 2.—Chart of $\eta$ against $V/nD$ for three-blade propeller 618A-8. Streamline body, with spinner, on symmetrical wing; single rotation. Propeller data taken from reference 1.

Figure 4.—Chart of $\eta$ against $V/nD$ for three-blade propeller 618A-8. Streamline body, with spinner; single rotation. Propeller data taken from reference 1.
FIGURE 8.—Chart of $V/nD$ against $T_1$ for three-blade propeller 6103A-3. Streamline body, with spinner; single rotation. Propeller data taken from reference 1.
Figure 5—Chart of $\frac{V}{nD}$ against $T_3$ for four-blade propeller N19-6. Streamline body, with spinner, on symmetrical wing; single rotation. Propeller data taken from reference 2.
Figure 6.—Chart of $\eta$ against $V/wD$ for four-blade propeller 316A. Streamline body, with spinner, on symmetrical wing; single rotation. Propeller data taken from reference 2.

Figure 7.—Chart of $\eta$ against $V/wD$ for six-blade propeller 316A. Streamline body, with spinner, on symmetrical wing; single rotation. (The 50° efficiency curve was interpolated.) Propeller data taken from reference 2.
Figure 7.—Chart of $V/nD$ against $\eta$ for six-blade propeller $170^\circ$. Streamline body, with spinner, on symmetrical wing; single rotation. Propeller data taken from reference 5.
Figure 9.—Chart of $\frac{V_n D}{n D}$ against $T_n$ for two two-blade propellers 8150-5 and 8150-4. Streamline body, with spinner, on symmetrical wing; dual rotation. Propeller data taken from reference 2.
Figure 10.—Chart of $\eta$ against $V/nD$ for two two-blade propellers 3156-8 and 3156-6. Streamlined body, with spinner, on symmetrical wing; dual rotation.
Propeller data taken from reference 7.

Figure 12.—Chart of $\eta$ against $V/nD$ for two three-blade propellers 3156-8 and 3156-6. Streamlined body, with spinner, on symmetrical wing; dual rotation.
Propeller data taken from reference 2.
Figure 11.—Chart of $V/nD$ against $T_t$ for two three-blade propellers 2155-8 and 2166-6. Streamline body, with spinner, on symmetrical wings dual rotation. Propeller data taken from reference 2.
Figure 12.—Chart of $V/nD$ against $T_e$ for three-blade propeller 8000-h. Liquid-cooled engine model, with spinner; single rotation. Propeller data taken from reference 8.
Figure 1A—Chart of $\alpha$ against $V/\pi D$ for three-blade propeller 5565-D. Liquid-cooled engine nozzle, with spinner; single rotation. Propeller data taken from reference 2.
Figure 15.—Chart of $V/nD$ against $T_e$ for three-blade propeller 3005-9. Radial engine mount; single rotation. Propeller data taken from reference 4.
Figure 16.—Chart of $V/nD$ against $T_r$ for four-blade propeller S686-9. Liquid-cooled engine nacelle, with spinner; single rotation. Data taken from reference 8.
Given: Aerodynamic characteristics of airplane and flight conditions:

1. $L/D$ for speed of flight and altitude (or thrust power for speed of flight and altitude)
2. Weight, $W$
3. Airspeed, $V$
4. Diameter of propeller, $D$
5. Density at altitude, $\rho$
6. Engine gear ratio

Characteristics of engine, including:

1. Brake horsepower for altitude of flight as a function of engine speed and manifold pressure
2. Specific fuel consumption as a function of engine speed and manifold pressure or torque

To obtain: Propeller speed to effect the maximum ratio of propulsive efficiency to specific fuel consumption

Solution based on thrust coefficient, $T_c$:

1. Determine effective thrust $T_e$ from $L/D$ and $W$
2. Solve for thrust coefficient $T_c = \frac{T_e}{\rho D^2 V^2} = \frac{195.4}{\sigma D^3 (mph)^3}$
3. For several values of propeller speed $n$, solve for $V/nD$

(4) For several values of $V/nD$ computed under step (3), read from propeller chart values of propulsive efficiency $\eta$

(5) For several values of $n$, compute propeller torque $Q = \frac{T_c V}{2\pi\eta}$

(6) For several propeller values of $n$, compute $n/r$ and $Q/Q_{\text{rated}}$ ($n_{\text{rated}}$ and $Q_{\text{rated}}$ correspond to rated engine speed and torque values with gear ratio taken into account)

(7) From fuel-consumption data, determine values of engine specific fuel consumption $C$ corresponding to engine speeds and torque values computed under step (6)

(8) For several values of $n$, compute and plot values of $\eta/C$

(9) From curve obtained in step (8), determine $\eta$ for maximum value of $\eta/C$

Solution based on thrust coefficient, $\eta C_f$:

1. Determine thrust power $\eta P$ or thrust horsepower from curves of power against speed of given data
2. Solve for $\eta C_f' = \frac{\eta P}{\rho D^2 V^2} = \frac{73300 \, \text{thp}}{\sigma D^3 (mph)^3}$
(3) The rest of the solution corresponds to steps in the solution based on thrust coefficient, except that $Q = \frac{\eta P}{2\pi n T}$

**SAMPLE CALCULATION**

A calculation for a sample airplane is herein performed to demonstrate the use of the charts. Values of $\eta$ corresponding to the maximum value of $\eta/C$ were determined for three flight speeds; the computations for only the condition of flight at maximum $L/D$ of the airplane are included. Range computations have also been made for the three flight speeds based on a specific fuel load.

In addition, range computations were made for the same flight speeds but with the engine torque and speed adjusted according to the relation $Q \propto n^3$ to demonstrate the value of adjusting the engine speed to obtain the maximum engine-propeller operating efficiency. (The relation $Q \propto n^3$ corresponds to the operation with a fixed-pitch propeller.)

The two charts necessary for the calculations are included: One chart is a plot of $L/D$ against airspeed for the sample airplane (fig. 19); the other chart is a fuel-consumption chart (fig. 20). The fuel-consumption chart was prepared from data taken during NACA tests of a 1340-H cylinder from a Pratt & Whitney 1340-SIH1-G engine.

Calculations were made for three conditions of flight, namely: maximum $L/D$, 50-percent power, and 75-percent power.

Given: Weight: gross = 17,500 pounds; gas load = 5830 pounds

Power plant: two 700-horsepower engines; engine speed = 2380 rpm, geared 16:9

Propellers: two three-blade Hamilton Standard 3155-6 propellers; diameter = 11 feet; propeller speed = 1340 rpm

Maximum velocity: 210 miles per hour at 8100 feet

Flight altitude: 10,000 feet

**Solution for flight at maximum $L/D$ based on thrust coefficient $T_e$ (calculations are shown for one point on fig. 21; other values are given in table 1):**

1. Maximum $L/D = 15.1$ (from fig. 19)
   
   $V$ for maximum $L/D = 116$ miles per hour = 170 feet per second
   
   $T_e = \frac{V}{L/D} = \frac{17500}{15.1} = 1160$ pounds or 580 pounds per engine

2. $T_e = \frac{195.4 \times 580}{0.738 \times 121 \times (1.346 \times 10^5)} = 0.0945$

3. If it is assumed that $n = 20$ rps,

   $V/nD = \frac{170}{20 \times 11} = 0.773$

4. For $V/nD = 0.773$ and $T_e = 0.0945$,

   $\eta = 0.823$ (fig. 1)

5. $Q = \frac{580 \times 170}{2\pi \times 20 \times 0.823} = 953$ foot-pounds

6. $n_{r,\text{rel}} = 22.33$ rps,

   $\frac{550 \text{ hp}_{\text{rel}}}{2\pi n_{r,\text{rel}}} = \frac{550 \times 700}{2 \times 22.33} = 2750$ foot-pounds

   At $n = 20$ rps,

   $\frac{n}{n_{r,\text{rel}}} = \frac{20}{22.33} = 0.897$

   and $Q/Q_{\text{rel}} = 0.346$

**Figure 19.—Variation of $L/D$ with airspeed for sample airplane.**

**Figure 20.—Effect of engine speed on minimum fuel consumption of several torque values. The 1340-H cylinder.**

7. From figure 20, by extrapolation,

   at $n/n_{r,\text{rel}} = 0.897$ and $Q/Q_{\text{rel}} = 0.346$,

   $C = 0.600$ pound per brake horsepower per hour

8. $\eta/C = 0.823 \div 0.600 = 1.37$
(9) \( \eta/C \) is plotted against \( n \) in figure 21, together with the corresponding values of \( Q, C, \) and \( \eta \), which have been added as a matter of interest. From this figure it is seen that the maximum value of \( \eta/C \) occurs at \( n = 13.7 \).

The results of similar computations for flight at 50-per cent and 75-per cent power are given in Table I.

The range for the three flight speeds was calculated from the Breguet range formula:

\[
\text{Range, miles} = 863 \frac{L}{D} \log \frac{W_0}{W_g}
\]

where \( W_0 \) is the gross weight and \( W_g \) is the gross weight less the gas load.

In the Breguet formula, \( L/D \) is assumed constant and the values of \( \eta/C \) are average during flight; whereas, the values of \( \eta/C \) as used in the problem are for the start of flight.

For maximum \( \eta/C \) at maximum \( L/D \),

\[
\text{Range, miles} = 863 \times 15.1 \times 1.62 \times 0.181 = 3820
\]

With the data from Table I, the range at 50-per cent power was computed as 3860 miles and at 75-per cent power the range was 2940 miles. The variation of the range with the airspeed is given in figure 22. For comparison, figure 22 contains a plot of the range for the same airplane equipped with both controllable and fixed-pitch propellers, the fixed-pitch propeller being set for the condition of high-speed flight. Because the gain in range by the use of controllable-pitch propellers set for maximum engine-propeller efficiency was relatively small for the conditions assumed in the sample computations, it would appear that the optimum blade-angle setting is fairly close to the high-speed setting. The stating of such a generality is hazardous, however, chiefly because the fuel-consumption data herein presented may not apply to engines in general.

### Table I

**Calculation of \( \eta/C \)**

<table>
<thead>
<tr>
<th>Flight conditions at start of flight</th>
<th>( \eta/C )</th>
<th>( L/D )</th>
<th>( V )</th>
<th>( T )</th>
<th>( T_\text{p} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \text{r.m.s.} )</td>
<td>( \text{m} )</td>
<td>( \text{m} )</td>
<td>( \text{m} )</td>
<td>( \text{m} )</td>
</tr>
<tr>
<td>1.0</td>
<td>0.80</td>
<td>0.94</td>
<td>1.05</td>
<td>1.35</td>
<td>0.456</td>
</tr>
<tr>
<td>1.2</td>
<td>0.90</td>
<td>0.94</td>
<td>1.20</td>
<td>1.30</td>
<td>0.455</td>
</tr>
<tr>
<td>1.4</td>
<td>0.90</td>
<td>0.94</td>
<td>1.25</td>
<td>1.30</td>
<td>0.454</td>
</tr>
<tr>
<td>1.6</td>
<td>0.90</td>
<td>0.94</td>
<td>1.30</td>
<td>1.30</td>
<td>0.453</td>
</tr>
<tr>
<td>1.8</td>
<td>0.90</td>
<td>0.94</td>
<td>1.35</td>
<td>1.30</td>
<td>0.452</td>
</tr>
<tr>
<td>2.0</td>
<td>0.90</td>
<td>0.94</td>
<td>1.40</td>
<td>1.30</td>
<td>0.451</td>
</tr>
</tbody>
</table>

**Figure 21.** Variation of engine-propeller operating characteristics for flight at maximum \( L/D \) of sample airplanes.

**Figure 22.** Comparison of flight ranges for fixed- and controllable-pitch propellers on sample airplanes at 10,000 feet. The foregoing comparisons were based on values of \( \eta/C \) for the start of flight, instead of on average values as stipulated in the Breguet formula.

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**REFERENCES**


