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THE PROBLEM OF LONGITUDINAL STABILITY AND CONTROL AT HIGH SPEEDS

By Manley J. Hood and H. Julian Allen

SUMMARY

The difficulty in pulling out of high-speed dives, experienced with some airplanes, is shown to be due, primarily, to the effect of compressibility on the lift of the wing. As the Mach number is increased above the critical for the wing, there is, for unswept wings, a marked decrease in the lift-curve slope and in the downwash at the tail plane, and, for cambered wings, also a shift in the angle of attack for zero lift. These changes bring about an increase in the static longitudinal stability and alter the trim. Frequently the change in trim is such as to promote a diving tendency which, because of the increased stability, cannot be overcome by use of the normal elevator control.

Means indicated for providing sufficiently powerful longitudinal control to overcome the trim and stability changes include auxiliary flaps and controllable stabilizer. It is also shown that, at least in some instances, it is possible to modify the aerodynamic characteristics of the airplane so that recovery from high-speed dives can be effected with the normal controls.

INTRODUCTION

In high-speed dives many airplanes exhibited a dangerous tendency to continue diving in spite of the application of large control forces. Wind-tunnel tests have confirmed that these difficulties are not peculiar to any particular configuration, so that the problem is of interest to all designers of high-speed airplanes.

The purpose of this report, which was issued in confidential form in 1943, is to acquaint designers with the cause of the difficulties and with the means now known for their alleviation.

CAUSES

STABILITY

For an airplane to be statically stable in pitch, it is necessary that the pitching-moment coefficient decrease as the lift coefficient increases. Neglecting the contribution of the tail plane to the total airplane lift, all drag and thrust components, and all lift and moment components except those due to the wing and horizontal tail plane, the static stability can be expressed in the familiar simplified form by

\[-\frac{dC_m}{dC_L} = \frac{dC_{m_w}}{dC_L} \left( \frac{S_t}{S_w} \right) \left( \frac{c_t}{c_w} \right) \frac{dC_{m_t}}{dC_L} + \left( \frac{S_t}{S_w} \right) \left( \frac{c_t}{c_w} \right) \left( 1 - \frac{d\alpha}{d\alpha} \right) \left( \frac{l_t}{l_w} \right) \]

where the variables are:

- \(C_m\) airplane pitching-moment coefficient about the center of gravity
- \(C_L\) airplane lift coefficient
- \(C_{m_w}\) wing pitching-moment coefficient about the quarter-chord point of its mean aerodynamic chord
- \(C_{m_t}\) tail-plane, pitching-moment coefficient about the quarter-chord point of its mean aerodynamic chord
- \(a_t\) lift-curve slope of the tail plane
- \(a_w\) lift-curve slope of the wing
- \(e\) downwash angle at the tail plane
- \(\alpha\) airplane angle of attack

and the constants are:

- \(S_t\) horizontal tail-plane area
- \(S_w\) wing area
- \(c_t\) horizontal tail-plane mean chord
- \(c_w\) wing mean chord
- \(l_t\) distance from the center of gravity to the quarter-chord point of the horizontal tail plane (considered positive when the center of gravity is forward of the tail quarter-chord point)
- \(l_w\) distance from the center of gravity to the wing quarter-chord point (considered positive when the wing quarter-chord point is forward of the center of gravity)

These constants will usually all be positive. Hence, it is seen that the first, second, and fourth terms of the equation represent destabilizing components, while the third term, expressing the contribution of the horizontal tail-plane lift, is the only stabilizing component.

The effects of compressibility on the values of the right-hand terms of this equation will be discussed in order.

In references 1, 2, and 3, the effects of compressibility on the aerodynamic characteristics of a number of airfoils are given. From these data, it may be deduced that the effect of compressibility on the stability of unswept airfoils in pitch \(-C_{m_w}/dC_L\) may be either to increase or decrease the stability as the Mach number is increased, depending upon the airfoil and the lift coefficient. Wind-tunnel tests of the airplane model designated as A in the figures (reference 4) with the horizontal tail plane removed showed that the stability (expressed by the term \(-dC_{m_w}/dC_L\) in accordance with the simplifying assumptions) does not change appreciably as the Mach number is increased up to the critical of the wing, and that above the critical the effect of compressibility is destabilizing, but is small relative to the effect
on the complete airplane. Tests of a model of the airplane designated as B in the figures in the Ames 16-foot high-speed wind tunnel indicated that with the horizontal tail plane removed there was even less change than with airplane A. When the four engine nacelles were also removed, no appreciable variation of the stability with Mach number was found. As a third instance, tests of a model of the airplane designated as C in the figures with the tail plane removed showed a negligible decrease in stability with increasing Mach number up to 0.775, the limit of the test.

It would be expected that, as for the wing, the effect of compressibility on the pitching-moment characteristics of the horizontal tail plane \( \frac{dC_m}{dC_L} \) would be slight. Moreover, \( \left( \frac{S_i}{S_e} \frac{C_i}{C_e} \right) \) is a small number, since the tail area is much less than the wing area. In consequence, the variation of the value of the second term of the equation with Mach number is negligible.

The predominant effect of compressibility on the static longitudinal stability is its effect on the stabilizing contribution of the tail plane, expressed by the third term of the equation. At speeds below the critical of the wing and tail plane, the ratio of the lift-curve slope of the tail plane to that of the wing \( \alpha, d\alpha \) remains nearly constant as the Mach number is increased. However, as shown by the approximate analysis of reference 5, the rate of change of downwash with respect to the angle of attack \( d\alpha \) is slightly increased. This increase causes a decrease in stability. Nevertheless, the effect is small and is important only in cases for which the low-speed stability is marginal.

As the Mach number is increased above the critical of the wing, but not above that of the horizontal tail plane, the value of \( \alpha, d\alpha \) is materially decreased. This decrease, in addition to the change in span load distribution (due to the relatively lower critical speed of the midspan sections of the wing) causes a marked decrease in the value of \( d\alpha \). The effects of compressibility on \( \alpha, d\alpha \) and \( d\alpha \) combine to produce a serious increase in the static longitudinal stability.

Figure 1 shows that, for four representative airplanes, the variations of static longitudinal stability with Mach number follow the trends indicated by the analysis of the stability equation. The results for airplane A (reference 4), airplane B, and airplane C were obtained from model tests in the Ames 16-foot high-speed wind tunnel. The curve for airplane D was obtained from results of flight tests conducted at Langley Memorial Aeronautical Laboratory (reference 6). The curves for the first three airplanes are based on the moment-lift characteristics with fixed elevator. The curve for airplane D is based on the variation of normal acceleration with stick force and so may not be strictly comparable with the other curves, since the elevator hinge-moment coefficient may vary with Mach number. This variation may explain the apparent decrease in stability of this airplane as the Mach number increased above 0.72.

It should be noted that the relative stability shown in figure 1 is dependent upon the value of \( \frac{dC_m}{dC_L} \) at low Mach number, which in turn is dependent upon the center-of-gravity position. With center-of-gravity positions farther back than those used, the low-speed stability would be reduced so that the relative stability at high Mach numbers would be greater than shown.

When the critical Mach number of the tail plane is exceeded sufficiently to materially alter the tail-plane lift-curve slope \( \alpha, d\alpha \), a decrease in static longitudinal stability would be expected.

**Controllability**

The effect of compressibility on the longitudinal controllability depends upon (1) its effect on the ability of the elevators to change the airplane pitching moment, and (2) its effect on the trim and the static longitudinal stability.

The ability of the elevators to change the airplane pitching moment has been investigated for airplanes A (reference 7), B, and C up to Mach numbers of 0.725, 0.775 and 0.775, respectively. The results show that up to the maximum test Mach numbers, the pitching-moment coefficient resulting from a given elevator deflection was essentially constant. However, in tests of the model of airplane B with an abnormally thick horizontal tail plane, the critical Mach number of the tail plane was exceeded to such an extent as to reduce the effectiveness of the elevators in changing the pitching moment.

![Figure 1: Variation of static longitudinal stability with Mach number for four airplanes at zero lift coefficient. Relative stability = \( \frac{dC_m}{dC_L} \) at Mach number.](image)
The effect of compressibility on the lift coefficient for trim, in addition to its effect on the static longitudinal stability, is shown in figure 2 for airplanes A (reference 4), B, and C. For each model, the results shown are for the fixed elevator angle which provides trim for level flight at 340 miles per hour and 30,000 feet altitude \((M=0.50)\). The trim lift coefficients are for the normal wing loadings of 45, 61, and 36 pounds per square foot for airplanes A, B, and C, respectively. Within the range in which the horizontal tail plane is not stalled, the curves for other trim conditions can be closely approximated by translating the scale of pitching-moment coefficients. For each model, the curves of figure 2 show that, except in the regions where the curves tend to cross, there is a large change in trim as the Mach number is increased. The fact that the curves tend to cross at negative values of the lift coefficient, coupled with the increase of stability, produces large changes in trim at the positive lift coefficients required for pull-outs from dives.

**SUMMARY OF EFFECTS ON STABILITY AND CONTROLLABILITY**

The previous discussion has shown that as the Mach number increases above the critical for the wing, a marked reduction of the lift-curve slope occurs. As a result, the angle of attack required for a given lift coefficient is increased and the downwash at the tail is decreased. These effects cause a large increase in static longitudinal stability and a decrease in the lift coefficient for trim. The combined effects can make pull-outs from high-speed dives dangerously difficult.

**CURES**

The means which are known for correcting or alleviating the difficulties fall into the following five classes:

1. Limitation of diving speed,
2. Elevation of critical Mach number,
3. Provision of special control,
4. Reduction of trim change,
5. Reduction of stability change.

These means are considered in order.

**LIMITATION OF DIVING SPEED**

It would be possible with the use of dive brakes to limit the diving speed to values below those at which control and stability difficulties are encountered. For fighter airplanes this means is generally not acceptable since the high-speed dive may be tactically useful.

**ELEVATION OF CRITICAL MACH NUMBER**

Obviously the difficulties would be eliminated if, by astute choice of wing and body contours, the critical Mach number were raised above the value attainable in dives. In fact, for most of the airplanes which have been investigated, longitudinal control has been adequate for Mach numbers exceeding the critical by about 0.1. Avoiding the dive difficulties by raising the critical Mach number is possible for low-altitude airplanes, especially those with light wing loadings, because of the relatively lower Mach numbers attainable in dives with these types of airplane.

Assume, for example, an airplane having a wing loading of 30 pounds per square foot and a critical Mach number of 0.70 (corresponding to an NACA 66, 2-215 wing section). Also, assume that the longitudinal control remains adequate at Mach numbers up to 0.78 and that the drag coefficient has a constant value of 0.018 up to 0.70 Mach number and then increases linearly to 0.045 at a Mach number of 0.78. For this airplane the terminal dive Mach number is less than 0.78 at altitudes below 8,500 feet, so that the control difficulties herein discussed would not be experienced below this altitude. As an additional illustration, assume that the airplane starts a dive vertically at 400 miles per hour. A step-by-step integration indicates that in order to exceed a Mach number of 0.78, the dive (as assumed) must be started at an altitude greater than 20,000 feet. It is clear that some additional altitude would be necessary to reach the assumed starting conditions.

For high-altitude airplanes, especially those with high wing-loadings, it is probably impracticable to make the critical Mach number high enough to avoid stability and control difficulties by this means alone because of the re-
quired thinness of the wing sections, but in any event it is desirable to raise the critical speed to the highest practicable value. There have been several reports of airplanes which could not be controlled in high-speed dives at high altitude but became controllable as lower altitudes were reached. As lower altitudes are reached, the Mach number decreases for two reasons: (1) the dive speed decreases because of the increased drag accompanying the increased air density, and (2) the speed of sound increases in accordance with the increasing air temperature.

PROVISION OF SPECIAL CONTROL

One obvious means for improving the longitudinal control is the provision of tabs, or other servo or boost devices, to enable the pilot to deflect the elevator through a greater range at high speeds. Wind-tunnel tests have indicated this expedient to be effective in some cases—airplane B and reference 8—but, as previously noted, there are airplanes which cannot be controlled even by large elevator deflections because of the extreme stability and trim changes involved.

A means for obtaining control when the elevator is inadequate, is the controllable stabilizer. In reference 7 it is shown that, for airplane A, the controllable stabilizer would provide longitudinal control at Mach numbers up to 0.74, the limit of the tests. It is reasonable to expect that this control would remain satisfactory for Mach numbers at least up to the critical Mach number for the tail plane.

Another powerful method for providing additional longitudinal control at Mach numbers above the critical of the wings is the use of dive-recovery flaps mounted on the under-surface of the wing. Reference 7 shows the effectiveness of dive-recovery flaps on a model of airplane A. Results of other tests indicate that the flaps would be effective also on airplane B. The powerful effect of these flaps is illustrated by an example in reference 7. On airplane A it is shown that, when deflected 45°, a flap of 9-inch chord (full scale) located one-third of the wing chord from the leading edge and extending from the fuselage to the booms, will increase the lift coefficient for trim by 0.55 at a Mach number of 0.725.

With any of the means suggested, caution must be exercised to avoid catastrophically high accelerations in pulling out from dives. With the setting of the longitudinal control required for recovery, as the Mach number decreases during the pull-out, the lift coefficient at which the airplane trims increases rapidly as may be seen in figure 2. This rapid increase in the lift coefficient for trim may cause excessive accelerations unless the control is promptly shifted. In this respect, the dive-recovery flaps (reference 7) have the advantage that their effectiveness decreases as the Mach number decreases. In any event the operation of the control should be such as to permit rapid return to the normal setting. A number of instances have been reported with different airplanes wherein the pilots have been unable to effect recovery at high altitudes but, as the Mach number decreased with decreasing altitude, the airplanes suddenly recovered with accelerations so extreme as to cause severe blacking-out of the pilot and, in some cases, structural failure of the airplane.

REDUCTION OF TRIM CHANGE

For most airplanes that have been investigated in the wind tunnel there is a value of the lift coefficient for which there is but a small change in trim with Mach number, up to the limits of the tests. In general, this lift coefficient is negative (fig. 2) so that, for positive lift coefficients, the increase of stability with Mach number causes the pitching-moment coefficient to decrease as the Mach number is increased, producing a diving tendency. If the lift coefficient for which the trim change is small were shifted to a suitable positive value, two advantages would be gained: (1) In a vertical dive (zero lift), as the Mach number increases, the pitching-moment coefficient would increase, so producing an automatic tendency to recover from the dive; and (2) in the range of lift coefficients required for recovery, there would be less change in trim with Mach number so that less powerful control would be necessary to effect recovery.

Means for bringing about this desirable positive shift of the lift coefficient at which the trim change with Mach number is small have been investigated with the model of airplane A. For this particular model the effective means included a change of contour of the wing center section (reference 7), and a partial extension of the outboard flaps coupled with aileron droop (reference 4). The flap deflection and aileron droop were effective only in the absence of the powerful adverse effect of the standard fuselage on this model and caused objectionably large drag increases. A change of fuselage shape delayed the divergence to higher Mach numbers (reference 4). Apparently the benefits resulting from the fuselage and wing-contour change are due to the decreased effects of compressibility on the lift of the wing center section. On the other hand, the benefit derived from the use of the outboard flaps and the aileron droop appears to be due to the increased lift that they provide. Present knowledge is insufficient for promulgation of general design recommendations for making the small trim change occur at positive values of the lift coefficient. A better understanding of this problem is dependent on increased knowledge of the effects of compressibility on the aerodynamic characteristics of airfoils.

REDUCTION OF STABILITY CHANGE

Inasmuch as the change of longitudinal trim and stability is due to changes of the angle of attack of the tail plane with Mach number, it has been suggested that the changes could be eliminated by the use of a floating horizontal tail plane. In this case, longitudinal stability could be provided by spring-loading the tail plane, together with a proper choice of the pivot location. With this arrangement, the stability and trim would be independent of the downwash angle at the tail plane. This device has not yet been thoroughly investigated. It may involve difficult structural and flutter problems as well as longitudinal instability owing to the increase with Mach number of the instability of the wings. A tail plane having a larger-than-usual ratio of elevator chord to stabilizer chord might provide a desirable compro-
mize. With this arrangement, a spring or other special device would be necessary to provide stability at low speeds.

As may be seen from the stability equation previously discussed, a reduction of the lift-curve slope of the tail plane would tend to decrease the stability. If the tail-plane lift-curve slope decreased with Mach number in the proper relation with the wing lift-curve slope, the variation of longitudinal stability with Mach number would be reduced or eliminated. This method of improving the diving characteristics was attempted in the case of the model of airplane B by increasing the thickness of the tail plane so that its critical Mach number corresponded approximately to that of the wing. The results showed that the modification did decrease the change of stability with Mach number. Unfortunately, the elevator effectiveness was simultaneously diminished so that no over-all improvement in controllability was realized.

CONCLUDING REMARKS

It has been shown that the difficulty of pulling out of high-speed dives, experienced with some airplanes, is due to large changes of static longitudinal stability and trim which result chiefly from the decrease of wing lift-curve slope, and downwash at the tail plane, as the Mach number increases above the critical for the wing.

Wind-tunnel tests have indicated that adequate control in high-speed dives can be provided by controllable stabilizers or by dive-recovery flaps. The upper limit of the Mach number range in which these devices are effective is not known.

AMES AERONAUTICAL LABORATORY,
NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS,
MOFFETT FIELD, CALIF.

REFERENCES