REPORT No. 797

APPLICATION OF SPRING TABS TO ELEVATOR CONTROLS

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SUMMARY

Equations are presented for calculating the stick-force characteristics obtained with a spring-tab type of elevator control. The main problems encountered in the design of a satisfactory elevator spring tab are to provide stick forces in the desired range, to maintain the force per g sufficiently constant throughout the speed range, to avoid undesirable "feel" of the control in ground handling or in flight at low airspeeds, and to prevent flutter. Examples are presented to show the design features of spring tabs required to solve these problems for airplanes of various sizes. It appears possible to provide satisfactory elevator-force characteristics over a large center-of-gravity range on airplanes weighing from about 16,000 to 300,000 pounds. On airplanes weighing less than 16,000 pounds, some difficulty may be encountered in obtaining sufficiently heavy stick forces for rapid movements of the control stick. On large airplanes, the control on the ground or at low airspeeds may be unsatisfactory if an ordinary spring tab with a spring flexible enough to avoid a large variation of force per g with speed is used.

Some special tab designs, including geared and preloaded spring tabs, are discussed. The geared spring tab is shown to offer a means of obtaining satisfactory ground control without introducing excessive variation of force per g with speed. Theoretically, if the geared spring tab is used in conjunction with an elevator that has zero variation of hinge moment with angle of attack, the force per g may be made independent of speed at any center-of-gravity location regardless of the value of the spring stiffness.

By the use of spring tabs on elevators, the control forces may be made more closely predictable and the variation of stick-force characteristics among different airplanes of the same type may be greatly reduced. One of the principal objections to the use of spring tabs is the amount of weight required for mass balance to prevent flutter.

INTRODUCTION

Difficulties have been encountered in obtaining desirable control-force characteristics on large or high-speed airplanes, because the hinge moments on the control surfaces must be very closely balanced and because slight changes in the hinge-moment parameters result in large changes in control forces. The advantages of spring tabs in overcoming these difficulties have been pointed out in reference 1 and other reports. It has been recognized, however, that the use of a spring tab on an elevator results in a decreasing value of the stick force per g normal acceleration with increasing speed that might be considered undesirable. An analysis is presented herein of the effects of spring tabs on elevator forces for airplanes of various sizes. The results indicate that an elevator equipped with an ordinary spring tab of suitable design may avoid any serious disadvantage from this effect and may still obtain the advantage of having the control forces predictable and relatively insensitive to variations in the elevator hinge-moment characteristics.

The ordinary, or ungeared, spring tab (fig. 1) may present certain difficulties in obtaining satisfactory control on the ground or at low flight speeds. The geared spring tab (fig. 2) differs from the ordinary spring tab in that, when the elevator is moved with the stick free at zero airspeed, the tab moves with respect to the elevator in the same manner as a conventional geared tab or balancing tab. The geared spring tab presents the theoretical possibility of obtaining a value of force per g in maneuvers that does not vary with speed even though a stiff spring is used to provide adequate ground control. The present report briefly outlines the theory of the geared spring tab, gives formulas for use in design, and indicates the practical possibilities and limitations of the device.

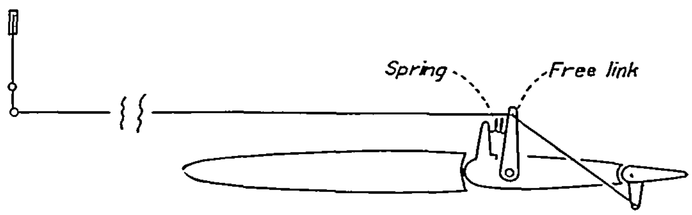


FIGURE 1.—Mechanism for ordinary, or ungeared, spring tab.

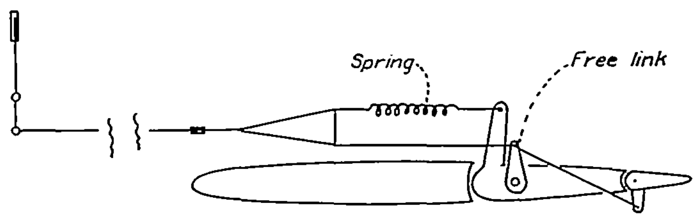


FIGURE 2.—Mechanism for geared spring tab.

SYMBOLS

W	weight
b	span
S	wing area
c	chord
l	tail length
S_T	tail area
$\left(\frac{dC_L}{d\alpha}\right)_w$	slope of lift curve of wing
ϵ	downwash angle
q	dynamic pressure
q_T	dynamic pressure at tail
τ	elevator effectiveness factor $\left(\frac{\partial C_{L_T}/\partial\delta_e}{\partial C_{L_T}/\partial\alpha_T}\right)$
C_L	lift coefficient
V_s	stalling speed
I	elevator moment of inertia
K_1	ratio of stick movement to elevator deflection, tab fixed; normally positive
K_2	ratio of stick movement to tab deflection, elevator fixed; normally negative
K_3	ratio of stick force to tab angle at zero airspeed, elevator fixed; normally positive
K_4	ratio of stick force to elevator angle at zero airspeed; elevator held in deflected position by external means, tab deflection held at zero by application of required force at control stick; positive for balancing tab
H	hinge moment
C_h	hinge-moment coefficient $\left(\frac{H}{qbc^2}\right)$
δ_e	elevator deflection (positive down)
δ_t	tab deflection with respect to elevator (positive down)
x_s	stick movement (positive forward)
F	stick force (pull force positive)
α	angle of attack of wing
α_T	angle of attack of tail
ρ	mass density of air
n	normal acceleration in g units
g	acceleration of gravity (32.2 ft/sec²)
x	distance between center of gravity and stick-fixed neutral point in straight flight (positive when center of gravity is rearward)
$\left(\frac{dC_{h_e}}{d\alpha_T}\right)_{tf}$	variation of elevator hinge-moment coefficient with angle of attack of tail, measured with tab free
$\left(\frac{dC_{h_e}}{d\delta_e}\right)_{tf}$	variation of elevator hinge-moment coefficient with elevator angle, measured with tab free
d	distance between tab mass-balance weight and tab hinge line
j	distance between elevator hinge line and tab hinge line

$$A=\frac{W\left(1-\frac{d\epsilon}{d\alpha}\right)}{\left(\frac{dC_L}{d\alpha}\right)_w S}+g\frac{\rho}{2}l$$

$$B=\frac{Wx}{\frac{\partial C_{L_T}}{\partial\delta_e}\frac{q_T}{q}S_Tl}-\frac{1}{\tau}g\frac{\rho}{2}l$$

Subscripts

T tail

t tab

e elevator

b value for equivalent balancing tab

EQUATIONS FOR ELEVATOR FORCES

The method of deriving the equations for the elevator control force in maneuvers with an ordinary spring tab will be briefly outlined. These equations are similar to equations given in reference 2 but have been arranged to give a clearer physical significance to the various terms.

The change in elevator hinge moment caused by any change in angle of attack, elevator angle, or tab angle is given by the following formula:

$$\Delta H_e=\left(\Delta\alpha_T\frac{\partial C_{h_e}}{\partial\alpha_T}+\Delta\delta_e\frac{\partial C_{h_e}}{\partial\delta_e}+\Delta\delta_t\frac{\partial C_{h_e}}{\partial\delta_t}\right)q_Tb_ec_e^2 \tag{1}$$

The corresponding change in tab hinge moment is given by the expression

$$\Delta H_t=\left(\Delta\alpha_T\frac{\partial C_{h_t}}{\partial\alpha_T}+\Delta\delta_e\frac{\partial C_{h_t}}{\partial\delta_e}+\Delta\delta_t\frac{\partial C_{h_t}}{\partial\delta_t}\right)q_Tb_tc_t^2 \tag{2}$$

The change in elevator angle and the corresponding change in angle of attack at the tail—both of which enter into the calculation of the change in elevator hinge moment—may be derived for any type of maneuver. The change in tab angle required to insert in equation (1) depends on the particular linkage arrangement under consideration. The present discussion will consider the spring-tab arrangement shown in figure 1. For this arrangement, the relation between the stick force, the elevator hinge moment, and the tab hinge moment, when the system is in equilibrium, is given by the formula

$$\left.\begin{aligned}\Delta F&=\frac{\Delta H_e}{K_1}\\&=\frac{\Delta H_t+\Delta\delta_tK_2K_3}{K_2}\end{aligned}\right\} \tag{3}$$

in which the constants K_1 and K_2 are the gear ratios between the stick and elevator and between the stick and tab, respectively, defined by the formula

$$x_s=K_1\delta_e+K_2\delta_t \tag{4}$$

and the constant K_3 depends on the stiffness of the spring. This spring constant for an unpreloaded spring tab is defined in terms of the stick force required at zero airspeed to deflect the tab with the elevator fixed; thus,

$$F=K_3\delta_t \tag{5}$$

By simultaneous solutions of equations (1), (2), and (3), the stick force required in any maneuver for an elevator equipped with an unpreloaded ordinary spring tab may be derived. The elevator force required to produce a given change in acceleration in gradual pull-ups is used as a criterion of the elevator control characteristics. In a pull-up, the change in angle of attack at the tail is given by the formula

$$\Delta\alpha_T=\left[\frac{W\left(1-\frac{d\epsilon}{d\alpha}\right)}{\left(\frac{dC_L}{d\alpha}\right)_w qS}+\frac{g\frac{\rho}{2}l}{q}\right](n-1) \tag{6}$$

and, if the tab is assumed to have a negligible effect on the lift of the tail, the change in elevator angle required is given by the formula

$$\Delta\delta_e=\left[\frac{Wx}{\frac{\partial C_{L_T}}{\partial\delta_e}q_TS_Tl}-\frac{1}{\tau}\frac{g\frac{\rho}{2}l}{q}\right](n-1) \tag{7}$$

In order to show the relation between the elevator forces required with a spring tab and the forces obtained with a conventional elevator, the equations for the force per g in a pull-up are derived first for an elevator without a tab, then for an elevator with a servotab, and finally for an elevator with an ordinary spring tab. In the case of a conventional elevator, the change in elevator hinge moment may be derived from equation (1). By use of the values for $\Delta\alpha_T$ and $\Delta\delta_e$ obtained from equations (6) and (7) and by setting $\Delta\delta_t=0$, the force per g normal acceleration is found to be

$$\frac{\partial F}{\partial n}=\frac{1}{K_1}\left[A\frac{\partial C_{h_e}}{\partial\alpha_T}+B\frac{\partial C_{h_e}}{\partial\delta_e}\right]\frac{q_T}{q}b_ec_e^2 \tag{8}$$

where

$$\left.\begin{aligned}A&=\frac{W\left(1-\frac{d\epsilon}{d\alpha}\right)}{\left(\frac{dC_L}{d\alpha}\right)_w S}+g\frac{\rho}{2}l\\B&=\frac{Wx}{\frac{\partial C_{L_T}}{\partial\delta_e}\frac{q_T}{q}S_Tl}-\frac{1}{\tau}g\frac{\rho}{2}l\end{aligned}\right\} \tag{9}$$

The second case considered is that of a servotab, which is defined as the system shown in figure 1 with the spring omitted. In this case, the stick force in a pull-up may be obtained from equations (1), (2), and (3) by setting the spring constant K_3 equal to zero. The relation obtained for the force per g is

$$\frac{\partial F}{\partial n}=\frac{\frac{1}{K_1}\left[A\left(\frac{dC_{h_e}}{d\alpha_T}\right)_{tf}+B\left(\frac{dC_{h_e}}{d\delta_e}\right)_{tf}\right]\frac{q_T}{q}b_ec_e^2}{1-\frac{K_2\frac{\partial C_{h_e}}{\partial\delta_t}b_ec_e^2}{K_1\frac{\partial C_{h_t}}{\partial\delta_t}b_tc_t^2}} \tag{10}$$

This equation differs from that for the force without a tab in two ways. The first difference is that the terms $\partial C_{h_e}/\partial\alpha_T$ are replaced by the corresponding values which would be measured on the elevator with the tab free. The values for the tab-free condition are given by the expressions

$$\left.\begin{aligned}\left(\frac{dC_{h_e}}{d\alpha_T}\right)_{tf}&=\frac{\partial C_{h_e}}{\partial\alpha_T}-\frac{\frac{\partial C_{h_t}}{\partial\alpha_T}\frac{\partial C_{h_e}}{\partial\delta_t}}{\frac{\partial C_{h_t}}{\partial\delta_t}}\\\left(\frac{dC_{h_e}}{d\delta_e}\right)_{tf}&=\frac{\partial C_{h_e}}{\partial\delta_e}-\frac{\frac{\partial C_{h_t}}{\partial\delta_e}\frac{\partial C_{h_e}}{\partial\delta_t}}{\frac{\partial C_{h_t}}{\partial\delta_t}}\end{aligned}\right\} \tag{11}$$

If the tab does not have any floating tendencies, the values obtained with equations (11) are the same as those obtained for the elevator with the tab fixed. The second difference is that in the denominator a term is added which depends upon the ratio of the elevator dimensions to the tab dimensions, the ratio of the effectiveness of the tab to its aerodynamic hinge moment, and the ratio between the tab and elevator gearing constants. This added term, which in practical designs may range in value from five to several hundred, effectively divides the elevator stick force that would be obtained without a tab by a large factor. The force per g for a servotab, like that for the elevator without a tab, is essentially independent of speed.

The force per g for an elevator equipped with an unpreloaded ordinary spring tab is found to be

$$\frac{\partial F}{\partial n}=\frac{\frac{1}{K_1}\left\{A\left[\left(\frac{dC_{h_e}}{d\alpha_T}\right)_{tf}+\frac{K_2K_3\frac{\partial C_{h_e}}{\partial\alpha_T}}{\frac{\partial C_{h_t}}{\partial\delta_t}q_Tb_tc_t^2}\right]+B\left[\left(\frac{dC_{h_e}}{d\delta_e}\right)_{tf}+\frac{K_2K_3\frac{\partial C_{h_e}}{\partial\delta_e}}{\frac{\partial C_{h_t}}{\partial\delta_t}q_Tb_tc_t^2}\right]\right\}\frac{q_T}{q}b_ec_e^2}{1-\frac{K_2\frac{\partial C_{h_e}}{\partial\delta_t}b_ec_e^2}{K_1\frac{\partial C_{h_t}}{\partial\delta_t}b_tc_t^2}+\frac{K_2K_3}{\frac{\partial C_{h_t}}{\partial\delta_t}q_Tb_tc_t^2}} \tag{12}$$

Three terms are added when the tab-spring constant is taken into account. All three terms are seen to be of the same form and contain the dynamic pressure q_T in the denominator. At very low speeds, therefore, these three terms will be very large compared with any other terms in equation (12) and, in this case, the equation reduces to the form of equation (8), the force per g of the elevator without a spring tab. At very high speeds, the three added terms in equation (12) approach zero and the equation for force per g reduces to that derived for a servotab (equation (10)). The actual variation of force per g with speed for various values of the spring constant K_3 is shown for a typical spring-tab installation in figure 3.

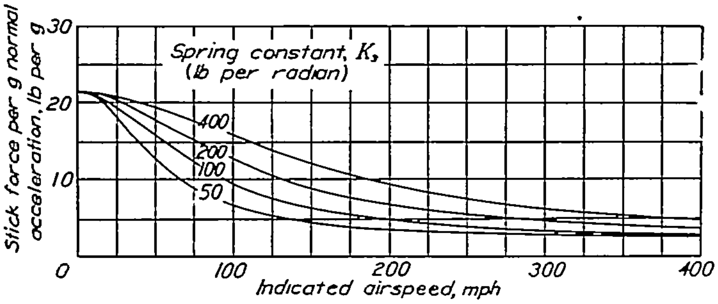


FIGURE 3.—Variation of force per g normal acceleration with speed for typical spring-tab installation with various amounts of spring stiffness. (Values of force per g below stalling speed have no physical significance.)

DESIGN PROBLEMS

The main problems that arise in connection with the design of a spring tab for an elevator are as follows:

(a) To provide stick forces in the desired range

(b) To maintain force per g sufficiently constant through the speed range

(c) To avoid undesirable "feel" of control for ground handling

(d) To prevent flutter

These four conditions will be shown to restrict the design characteristics of a satisfactory ordinary elevator spring tab to a rather narrow range for any particular type of airplane.

Some additional discussion may be necessary to clarify points (b) and (c). The force per g obtained with a servotab has been shown not to vary with speed. A servotab has been found to be undesirable, however, because the elevator does not follow movements of the stick smoothly when the airplane is on the ground, taxying, or making landings and take-offs. A banging action of the control has been experienced because the elevator does not move until the tab hits its stops. The use of a spring tab provides a mechanical connection between the stick and the elevator and relieves this difficulty. One of the main problems in the design of a spring tab is to avoid an undesirably large variation of force per g with speed in flight and still to provide a sufficiently rigid connection between the stick and the elevator to give control while the airplane is taxying. With an ordinary spring tab, the variation of force per g with speed in flight may be reduced to a small value by using a spring sufficiently weak that, in the normal-flight speed range, the control behaves essentially as a servotab. It is necessary to decide upon some criterion for the minimum value of spring stiffness required for control while the airplane is taxying.

The response of the elevator to a sudden stick movement depends upon the elevator hinge moment that results from a unit stick deflection. If the elevator is held fixed, the variation of elevator hinge moment with stick deflection for an elevator equipped with a spring tab is given by the following equation:

$$\frac{\partial H_e}{\partial x_s}=\frac{-K_1K_3}{K_2}+\frac{\frac{\partial C_{h_e}}{\partial \delta_t}q_Tb_ec_e^2}{K_2}-\frac{K_1\frac{\partial C_{h_t}}{\partial \delta_t}q_Tb_tc_t^2}{K_2^2} \tag{13}$$

At zero speed the elevator hinge moment comes entirely from the spring but, as the speed increases, the aerodynamic hinge moment due to tab deflection is added. The initial angular acceleration of the elevator, which occurs after a sudden stick movement, depends on the ratio of elevator hinge moment to stick deflection divided by the moment of inertia of the elevator about its hinge line. In flight tests of a small fighter airplane, the minimum value of spring stiffness required for satisfactory feel of the controls on the ground corresponded to the value (at zero airspeed)

$$\frac{1}{I}\frac{\partial H_e}{\partial x_s}=\frac{-K_1K_3}{K_2I}=200 \text{ foot-pounds per foot per slug-foot}^2$$

This value is, of course, many times smaller than the degree of rigidity present in a conventional control system but has nevertheless been shown to be satisfactory for the case of the small fighter airplane. For a large airplane, particularly one equipped with a tricycle landing gear, elevator control should not be required until speeds approaching the take-off speed are reached. In such a case, then, a lower value of the ratio might be acceptable at zero airspeed. The value of $\frac{1}{I}\frac{\partial H_e}{\partial x_s}$ should, however, be reasonably large at speeds approaching the take-off speed.

EXAMPLES

Design considerations.—In order to illustrate the application of ordinary spring tabs to elevator controls of airplanes of various sizes, the stick-force characteristics in maneuvers have been calculated for four airplanes ranging in size from a scout bomber to an airplane weighing 300,000 pounds, which represents about the largest type of airplane now being contemplated by aircraft designers. In each case, a practical spring-tab design has been arrived at that provides stick-force characteristics which satisfy the requirements of reference 3. These examples show what design features of a spring tab are required to obtain stick forces for maneuvering within the range required for each class of airplane and indicate also special problems that may arise in the design of spring tabs for aircraft of particular sizes. The characteristics of the airplanes chosen as examples are given in table I. Certain factors that were considered in designing the spring tabs are as follows:

(a) The spring stiffness has been selected on the basis of providing satisfactory ground control by making the value of $\frac{1}{I}\frac{\partial H_e}{\partial x_s}$ at zero airspeed equal to or greater than 200 foot-pounds per foot per slug-foot^2 except where otherwise noted.

(b) A reasonable degree of aerodynamic balance of the elevator, corresponding to a value of $\frac{\partial C_{h_e}}{\partial \delta_e}=-0.002$ or -0.003, has been assumed so that large elevator deflections may be obtained without having the tab size or deflection exceed practical limits. The value of $\frac{\partial C_{h_e}}{\partial \alpha_T}=0$, which has been used in all calculations, may be attained in practice by suitable choice of the elevator contours. Variation in the value of $\partial C_{h_e}/\partial \alpha_T$ will not, however, alter the effects of the spring tab but will simply shift the stick-free neutral points in straight and turning flight by the same amount for a spring tab as for a conventional type of balance.

(c) The tab hinge-moment characteristics were assigned the representative values $\frac{\partial C_{h_t}}{\partial \delta_t}=-0.003$ or -0.005, $\frac{\partial C_{h_t}}{\partial \delta_e}=0$, and $\frac{\partial C_{h_t}}{\partial \alpha_T}=0$. By suitable modification of the tab design, considerable variation in these values may be obtained. The effects of such changes on the stick forces may be determined from formulas (11) and (12).

Scout bomber (weight, 16,000 lb).—The variation of force per g with speed and with center-of-gravity position for a scout bomber weighing 16,000 pounds is shown in figure 4.

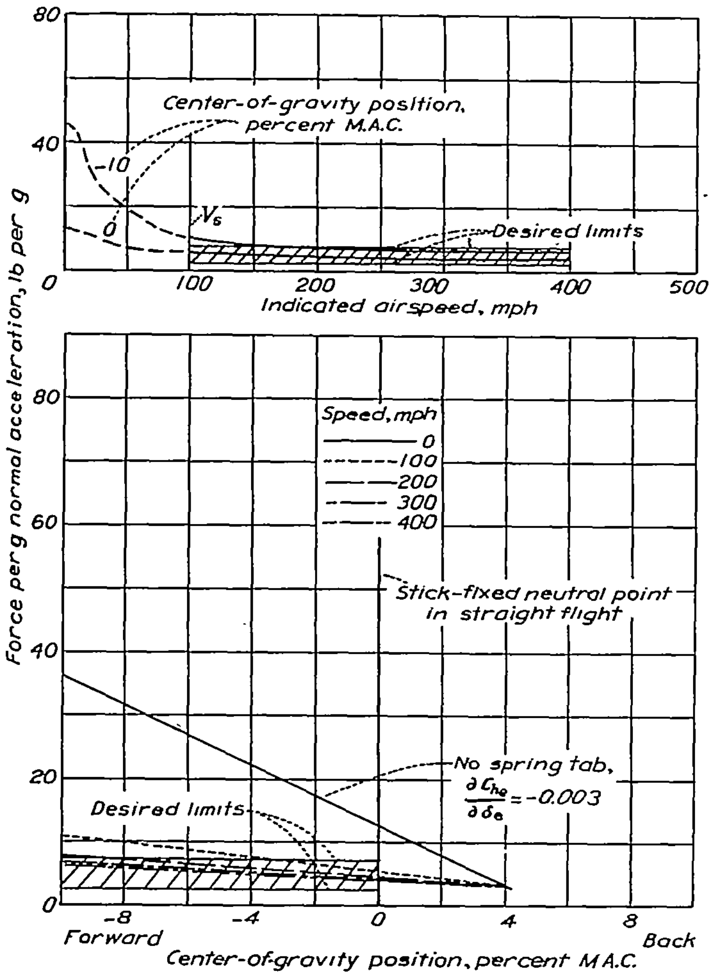


FIGURE 4.—Variation of force per g with speed and with center-of-gravity position for scout bomber (weight, 16,000 lb).

The desirable range of stick forces (shown by cross hatching in figures) is indicated in accordance with the requirement of reference 3. A center-of-gravity range of 10 percent of the mean aerodynamic chord has been assumed.

The hypothetical curve of force per g at zero speed, which also represents the force per g throughout the speed range when a spring tab is not used, shows that a conventional elevator with the degree of balance used would give heavy stick forces and an excessive variation of force per g with the center-of-gravity position. The assumed spring tab reduces the variation of force per g with center-of-gravity position to an acceptable amount. The variation of force per g with speed, for the spring stiffness chosen to give satisfactory ground control, also appears to be desirably small. Somewhat larger values of force per g are obtained near the minimum speed, but this fact is thought to be unimportant because the airplane stalls at low values of normal acceleration in this speed range. The stick forces were generally too low with a spring tab alone but have been raised to an acceptable value by the use of a small bobweight that requires a pull force of about 3 pounds on the stick.

Although the combination of spring tab and bobweight gives stick forces that satisfy the requirements, recent flight tests have shown that such an arrangement might be considered unsatisfactory to the pilots because of the lightness of the stick force required to make large rapid movements of the stick. This lightness, of course, results from the small effective value of the variation of hinge-moment coefficient with elevator deflection which is necessary in order to obtain a small variation of force per g with center-of-gravity position. The requirement for light stick forces over such a large center-of-gravity range on an airplane of this type seems, in fact, to be incompatible with the pilot's desire for forces large enough to prevent inadvertent movements of the control stick.

The problem of providing sufficient heaviness of the control stick for quick movements (with the resultant undesirable variation of force per g with center-of-gravity position) when a spring tab is used may present some difficulties on an airplane as small as a scout bomber. The following possibilities are available for making the forces heavier:

(a) To decrease K_2, the mechanical advantage of the stick over the tab
(b) To increase the tab chord
(c) To increase $\partial C_{h_t}/\partial \delta_t$ by use of strips on the tab trailing edge
(d) To reduce the amount of aerodynamic balance on the elevator

Of these possibilities, (a) and (b) may excessively increase the amount of mass balance required to prevent flutter, a subject that will be discussed in a later section of the paper. Only a limited advantage is gained by method (c). Method (d) will require the use of a larger tab to obtain large elevator deflections. By a combination of these methods, however, it appears practicable to obtain a sufficiently large centering tendency of the stick on an airplane of the scout-bomber class. For a given value of $\frac{1}{I}\frac{\partial H_e}{\partial x_s}$ at zero airspeed, changes (a), (b), and (c) give a favorable reduction in the variation of force per g with speed.

Satisfactory control feel might possibly be provided, even on an airplane that has no variation of force per g with center-of-gravity position, by suitable inertia weights or damping devices in the control system. Several systems for accomplishing this result have been proposed, but none have yet been tested in flight.

Medium bomber (weight, 50,000 lb).—The stick-force characteristics of a medium bomber weighing 50,000 pounds with the assumed spring-tab design are shown in figure 5. The spring stiffness, chosen on the basis of ground control, provides a sufficiently small variation of force per g with speed. The stick forces lie within the desired limits. It is believed that the centering tendency of the control stick associated with these forces would be considered sufficiently large, although no tests have been made of an airplane of this size to verify this belief.

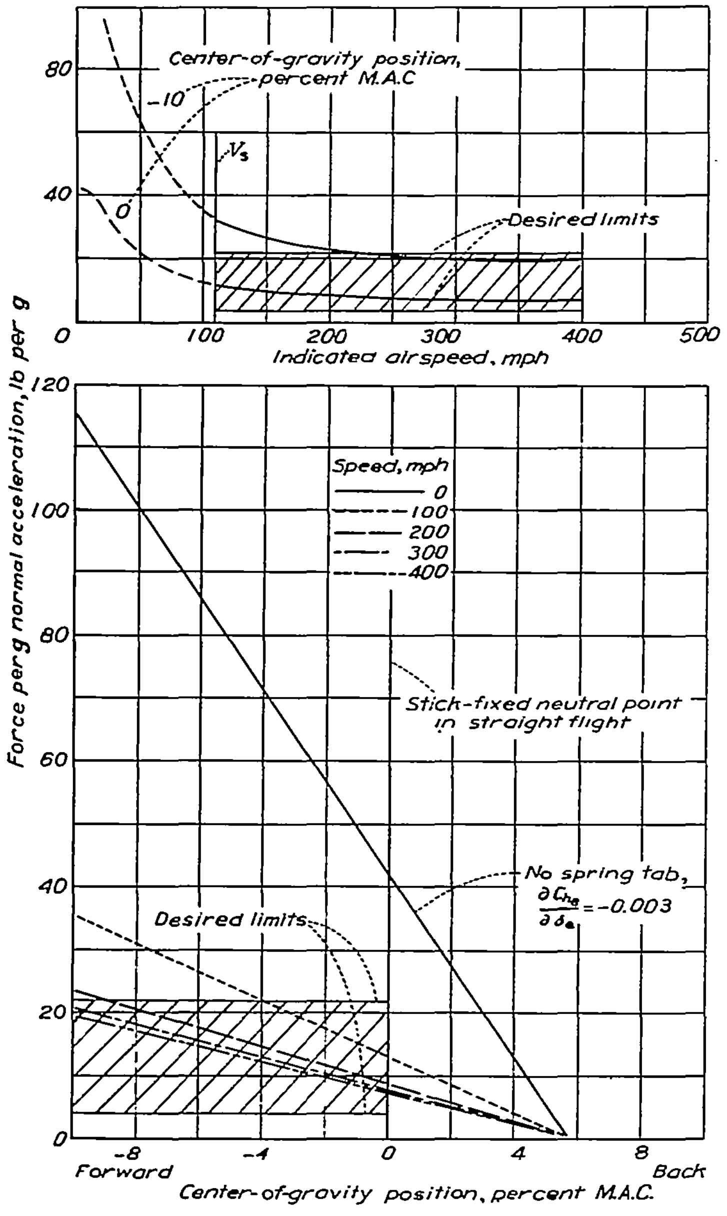


FIGURE 5.—Variation of force per g with speed and with center-of-gravity position for medium bomber (weight, 50,000 lb).

Heavy bomber (weight 125,000 lb).—The calculated stick-force characteristics of a heavy bomber (weight, 125,000 lb) are shown in figure 6. In order to obtain stick forces within the desired range, a tab of rather narrow chord and an increased value of K_2 (the mechanical advantage of the stick over the tab) have to be used. When these measures are adopted, it is no longer possible to meet the criterion for ground control $\left(\frac{1}{I}\frac{\partial H_t}{\partial x_t}\right.$ at zero speed=200 foot-pounds per foot per slug-foot$\left.^2\right)$ and still maintain a sufficiently small variation of force per g with speed. Although the spring stiffness required to obtain the characteristics shown in figure 6 is greater than the stiffness used on the smaller airplanes, the value of $\frac{1}{I}\frac{\partial H_t}{\partial x_t}$ at zero speed is considerably reduced but reaches a value of 200 at a speed of 80 miles per hour. This condition would probably be acceptable, however, on a large airplane with a tricycle landing gear.

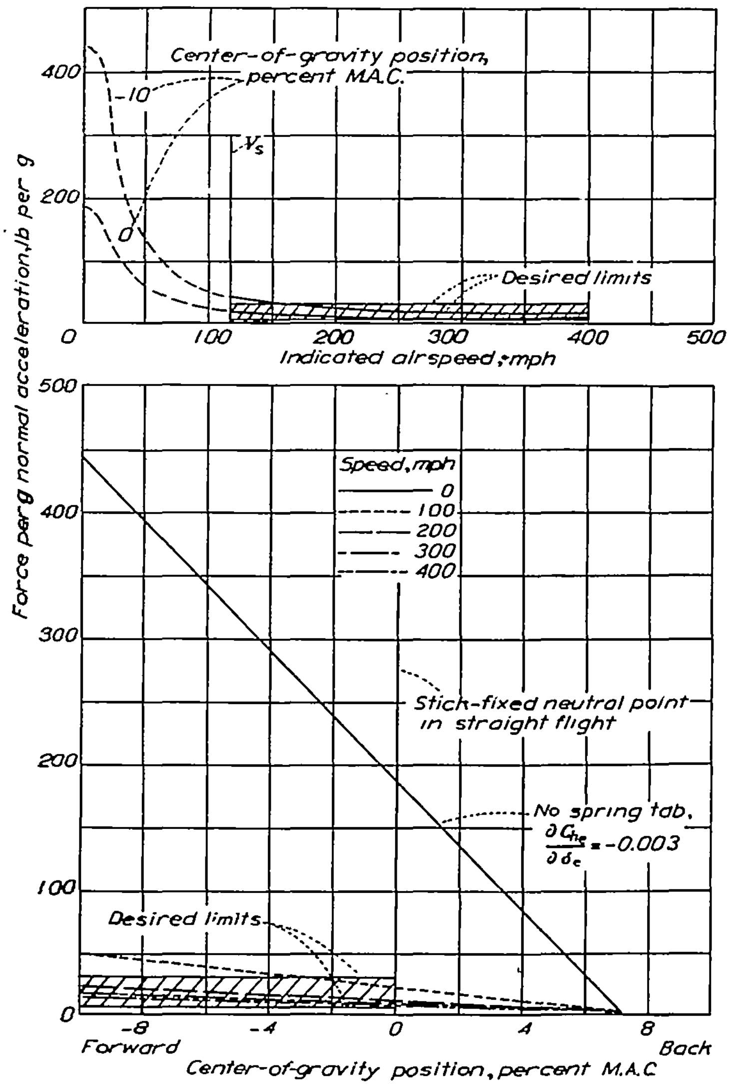


FIGURE 6.—Variation of force per g with speed and with center-of-gravity position for heavy bomber (weight, 125,000 lb).

Airplane of 300,000 pounds weight.—The calculated stick-force characteristics of an airplane weighing approximately 300,000 pounds are shown in figure 7. On an airplane of this size, considerable care must be taken to balance aerodynamically both the elevator and the tab in order to obtain sufficiently light stick forces. A very small value of $\frac{1}{I}\frac{\partial H_t}{\partial x_t}$ at zero speed must also be accepted in order to avoid excessive variation of force per g with speed. The value of

$\frac{1}{I}\frac{\partial H_e}{\partial x_s}$ for this tab arrangement exceeds 200 at speeds above 102 miles per hour.

The stick forces on an airplane of this size depend rather critically on the elevator and tab hinge-moment characteristics. In view of the rather limited data available at present on the hinge-moment characteristics of tabs, special tests would undoubtedly be required to develop a design that provides the desired hinge-moment parameters. The degree of balance required is not so high that small variations in contours among different airplanes would cause excessive variations in the stick forces. It therefore appears that a spring tab may be used to provide satisfactory elevator control on an airplane of at least 300,000 pounds gross weight. The limiting size of airplane that could be adequately controlled by this means is difficult to estimate, inasmuch as factors such as the response of the elevator to stick movements, rather than the magnitude of the stick forces, would

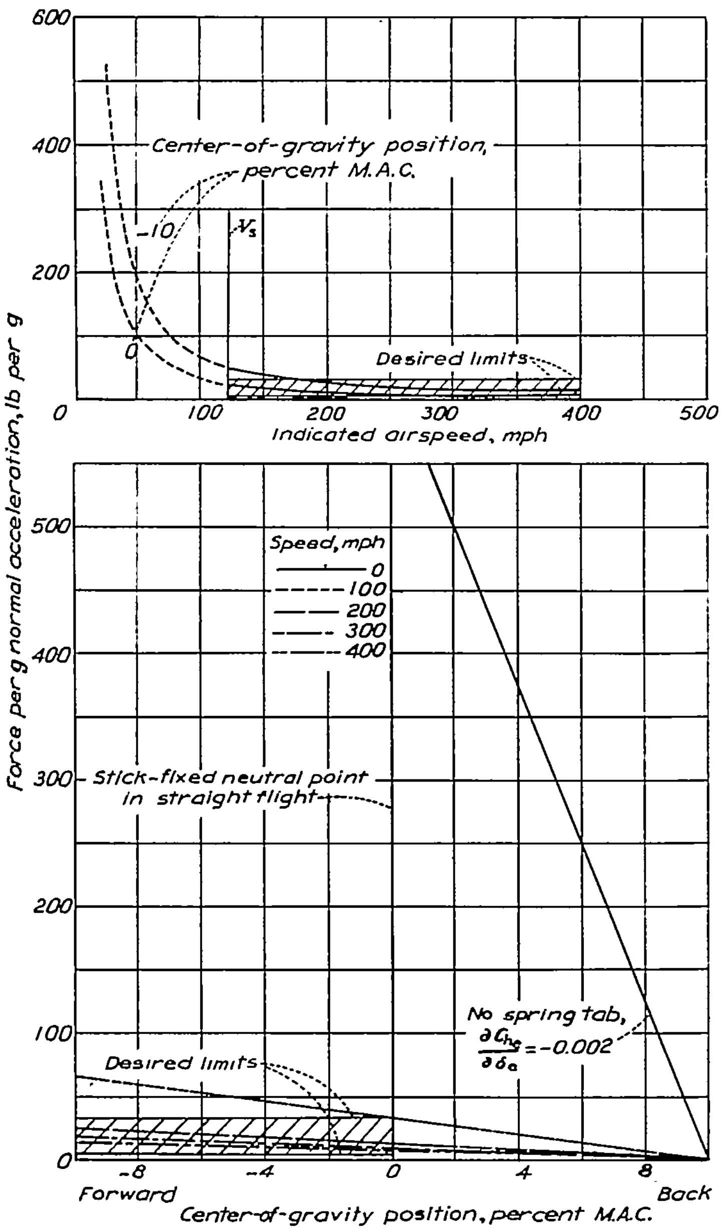


FIGURE 7.—Variation of force per g with speed and with center-of-gravity position for 300,000-pound airplane.

probably set the upper limit on the size of airplane that could be controlled. The increasing importance of the elevator inertia on large airplanes is caused by the fact that the moment of inertia of the elevator tends to increase as approximately the fourth power of the linear dimension, whereas the aerodynamic hinge moments due to the tab vary as the cube of the linear dimension.

DISCUSSION OF EXAMPLES

The ability of the spring tab to provide desirable stick-force characteristics over a large center-of-gravity range on airplanes weighing between about 16,000 and 300,000 pounds has been shown by the preceding examples. The lower limit on the size of airplane that can be controlled is determined by the requirement for a definite centering tendency of the control stick. The upper limit is not clearly defined but probably is set by the ability of the elevator to follow rapid stick movements.

One advantage of the spring-tab control is that any variation in the stick-force characteristics between airplanes of the same type, caused by slight differences in the contours of the elevators, would be much less for a spring-tab elevator than for an elevator equipped with a conventional type of balance such as a balancing tab or an inset hinge. This difference may be explained as follows: In order to obtain desirable stick forces with a conventional type of balance, the elevator hinge-moment parameters $\partial C_{h_e}/\partial \delta_e$ and $\partial C_{h_e}/\partial \delta_T$ must be reduced to very small values. Variations of these parameters caused by slight differences in the elevator contours are likely to be of the same order of magnitude as the desired values. Such variations would cause changes in the stick-force characteristics of 100 percent or more. In the case of the spring tab, however, a high degree of balance of the elevator is not required. The stick forces are reduced to desirable values by the action of the tab. A properly designed spring tab has been shown to act essentially as a servotab at normal flight speeds. The formula for the force per g with a servotab (equation (10)) shows that the force per g is reduced by a large factor in the denominator that depends on the tab and linkage characteristics. The effects of any variations in the values of $\partial C_{h_e}/\partial \alpha_T$ and $\partial C_{h_e}/\partial \delta_e$ will be reduced by the same ratio. Inasmuch as this ratio varies from about 1:10 in the case of the scout bomber to 1:100 in the case of the 300,000-pound airplane, the spring tab should effectively eliminate any difficulties caused by variations in elevator hinge-moment parameters.

Errors in the predicted stick-force characteristics for a proposed spring-tab design, caused by failure to obtain the desired elevator hinge-moment characteristics, are likewise reduced by this ratio. As a result, the control characteristics of a spring-tab elevator should be more closely predictable than those of a conventional elevator, especially on a large airplane. This advantage is somewhat offset by the fact that the stick forces obtained with a spring tab depend on the hinge-moment parameters of the tab as well as of the elevator. At present, information on the hinge-moment characteristics of tabs is not very complete.

The spring tab should provide an effective means of control in high-speed flight, especially as regards recovery from high Mach number dives, where the control forces on a conventional elevator may become excessive. It is known that trim tabs may be used to recover from dives, at least at the Mach numbers reached by present-day airplanes; but this procedure is known to be extremely dangerous because, when the airplane reaches lower altitudes and Mach numbers, excessive accelerations may be experienced before the trim tabs can be returned to neutral. The spring tab directly controlled by the stick should eliminate this difficulty. Furthermore, the stick forces with a spring tab would not be likely to become excessive in the pull-out. The effects of compressibility may in many cases be considered as a large rearward shift of the neutral point (of the order of 20 to 30 percent of the mean aerodynamic chord). Figures 4 to 7 show that such a shift would lead to excessive stick forces for recovery with a conventional elevator but to reasonable forces for a spring-tab control. In order to effect recovery, the elevator and tail would have to be built sufficiently strong to withstand the large loads imposed.

PREVENTION OF FLUTTER

A theoretical investigation of the flutter of spring tabs is presented in reference 4 and the practical results are given in reference 5. These reports show that both the elevator and tab must be mass-balanced about their hinge lines and that the tab mass-balance weight must be placed closer to the tab hinge line than a certain distance defined by the relation

$$d=\frac{f}{1-\frac{K_1}{K_2}} \tag{14}$$

In order to be most effective, the tab mass-balance weight should be placed about half this distance ahead of the tab hinge line. Equation (14) shows that, if the mechanical advantage of the stick over the tab K_2 is reduced to a small value, the tab mass-balance weight must be placed so close to the tab hinge line that a prohibitively large weight may be required. Equation (4) indicates that K_1 and K_2 cannot be reduced simultaneously without unduly decreasing the stick travel.

A small value of the mechanical advantage of the stick over the tab has been shown to be advantageous on small airplanes in order to provide sufficiently large stick-force gradients and small variation of force per g with speed. An experimental investigation to determine the validity of equation (14) is, therefore, urgently required. Because of effects of flexibility in the control linkages, the applicability of equation (14) is open to some question in cases in which K_2 is small. In some instances spring tabs without mass balance have been used without the occurrence of flutter. Special devices with a smaller penalty due to weight have also been proposed to prevent flutter.

STICK-FORCE CHARACTERISTICS IN STRAIGHT FLIGHT

In figures 4 to 7, the rear limit of the assumed center-of-gravity range was taken as the stick-fixed (actually, elevator- and tab-fixed) neutral point in straight flight. Because $\partial C_{h_e}/\partial \alpha_T$ was taken equal to zero, this point also represents the stick-free neutral point. For all center-of-gravity positions ahead of this point, the stick-force variation with speed will be stable and the gradient will be reduced by the spring tab in the same proportion as the maneuvering forces. The effects of changes in the hinge-moment parameters $\partial C_{h_e}/\partial \delta_e$ and $\partial C_{h_e}/\partial \alpha_T$ and the effects of altitude on the neutral point and maneuver point may be shown to follow the same rules with a spring tab as with a conventional elevator.

SPECIAL SPRING-TAB ARRANGEMENTS

The formulas set up for the stick forces obtained in maneuvers with a spring tab may be used to determine the characteristics of several special arrangements.

Tab controlled independently of elevator.—The mechanism for a tab controlled independently of elevator is shown diagrammatically in figure 8. This arrangement is a special case of the previously used system in which the elevator gearing constant K_1 equals zero. The stick-force characteristics may be found from equations (12) and (13) by setting K_1 equal to zero.

If K_1 equals zero, the value of K_2 must be large enough to require full stick travel for full tab deflection. For airplanes weighing about 50,000 pounds or less, a small value of K_2 was required to provide sufficiently heavy stick forces. The tab controlled independently of elevator would therefore be considered satisfactory only on large airplanes. Equation (13) furthermore indicates that, when $K_1=0$, the elevator will not be constrained to follow stick movements at zero airspeed no matter how stiff a spring is used. The system of figure 8 will thus have no advantages over a servotab from the standpoint of ground control. The spring should therefore be omitted in order to avoid a force per g that varies with speed. This system is more likely than an ordinary spring tab to result in instability of the short-period oscillation of the airplane with stick fixed, because the stability of the elevator itself with stick fixed is essentially the same as with stick free. As a result, the dynamic stability of the airplane with stick fixed is no greater than with stick free. With a conventional spring tab such as that shown in figure 1, on the other hand, the effective restoring moment on the elevator with stick fixed is greatly increased by the leading action of the tab, so that the stick-fixed dynamic stability of the airplane is close to the elevator-fixed value. The only benefit that appears to result from the use of the system of figure 8 is a possible reduction of stick forces on a very large airplane because of the increased allowable mechanical advantage of the stick over the tab. Use of this alternative does not appear to be necessary, however, for the largest airplane considered (300,000 pounds weight).

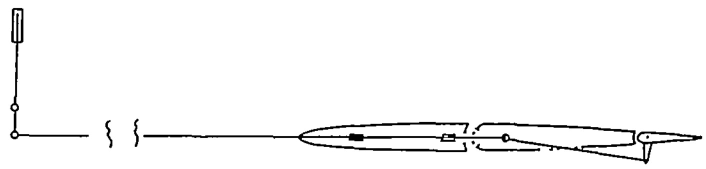


FIGURE 8.—Tab controlled independently of elevator.

Preloaded spring tab.—If the tab spring is preloaded to prevent deflection of the tab until the stick force exceeds a certain amount, the stick force per g will equal that of the elevator without a spring tab up to the point where the stick force reaches the preload. Beyond this point, the force per g will equal the force calculated for an unpreloaded spring tab. The force variation with acceleration will therefore be nonlinear, a characteristic usually considered to be undesirable.

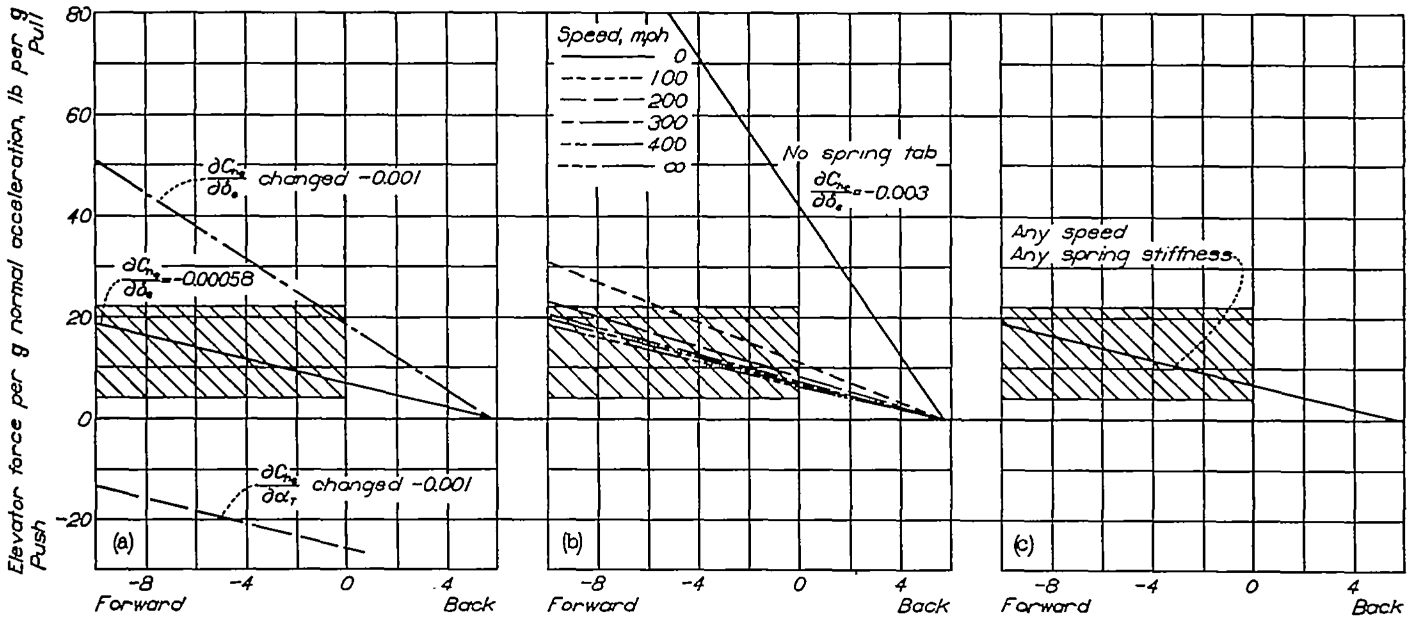
If friction is present in the tab system, an unpreloaded spring tab may not return to a definite equilibrium position and, as a result, the pilot may experience difficulty in maintaining a specified trim speed. A small amount of preload may be used to center the tab definitely in trimmed flight and thereby to overcome this difficulty. In view of the mechanical complications involved in the use of a preloaded spring, as well as the nonlinear force characteristics mentioned previously, it appears desirable to avoid the necessity for preload by reducing friction in the tab system to a minimum.

Geared spring tab.—The mechanism for a geared spring tab is shown diagrammatically in figure 2. As noted previously, this device differs from an ordinary spring tab in that, when the elevator is moved with the stick free at zero airspeed, the tab deflects with respect to the elevator in the same manner as a conventional geared tab or balancing tab.

It has been shown that, when an ordinary spring tab is used, the variation of force per g with speed may be reduced to an acceptable amount by using a tab spring sufficiently flexible to make the control behave essentially as a servotab at normal flight speeds. The ground control provided by this flexible spring might be considered acceptable but a stiffer spring would be very desirable, especially on large airplanes that have elevators with high moments of inertia. If a geared spring tab is used, it will be shown that a stiffer spring may be employed without increasing the variation of force per g with speed.

In the appendix of the present paper, the theoretical derivation of the stick forces with an ordinary spring tab is extended to allow calculation of the stick forces with a geared spring tab. The force per g obtained with an ordinary spring tab has been shown to vary with speed. As the speed approaches zero the force per g approaches that obtained with the tab fixed and, at very high speeds, approaches the value for a servotab. With a geared spring tab, as the speed approaches zero the force per g is shown to approach that of an equivalent balancing tab and, at very high speeds, is shown to approach the value for a servotab. The geared spring tab therefore provides a means of reducing the force per g at low speeds while leaving the force per g at high speeds unchanged. The force per g may theoretically be made to remain constant throughout the speed range, no matter what spring stiffness is used. This arrangement therefore embodies the advantage provided by either the conventional balance or the servotab, namely, that the stick-force gradient does not vary with speed. The undesirable sensitivity of the conventional balance to small changes in hinge-moment characteristics and the poor ground control of the servotab are avoided by the geared spring tab.

In order to compare the merits of conventional types of balance, ungeared spring tabs, and geared spring tabs, the stick-force characteristics have been computed for the medium bomber (weight, 50,000 lb) with the various types of elevator control. The results of these calculations are shown in figure 9. The airplane characteristics are assumed, as before, to be those given in table I. The control-system characteristics are given in table II. The stick forces of a closely balanced elevator with conventional balance (as, for example, a balancing tab) are shown in figure 9(a).



(a) Conventional balance. (b) Ungeared spring tab. (c) Geared spring tab.

FIGURE 9.—Stick-force characteristics of various types of elevator control. Desirable range of stick forces indicated by shaded area.

The critical nature of the balance is also shown by the large changes in stick-force gradients caused by changes in $\partial C_{h_e}/\partial \delta_e$ and $\partial C_{h_e}/\partial \alpha_T$ of -0.001 per degree. Variations of this order of magnitude may result from slight differences in contours of the elevator, within production tolerances, on different airplanes of the same type. The characteristics of an ordinary, or ungeared, spring tab are illustrated in figure 9(b). These values are the same as those presented previously in figure 5.

The characteristics of a geared spring tab that was designed to provide the same control-force characteristics as the conventional balance are shown in figure 9(c). The method of calculating the values of the hinge-moment parameters and gear ratio that were used to obtain stick-force gradients independent of speed is given in the appendix. The same characteristics will be obtained with any spring stiffness.

The exact values of hinge-moment parameters required to give the characteristics shown in figure 9(c) will not be attained in practice. It is therefore desirable to investigate the effects of changing the hinge-moment parameters slightly. If the spring in the geared spring tab had infinite stiffness, the system would be identical with the balancing tab (fig. 9(a)) and the stick forces would be equally sensitive to small changes in hinge-moment parameters. The spring stiffness must therefore be limited to a point at which normal changes in $\partial C_{h_e}/\partial \delta_e$ and $\partial C_{h_e}/\partial \alpha_T$ do not cause large changes in the stick-force characteristics.

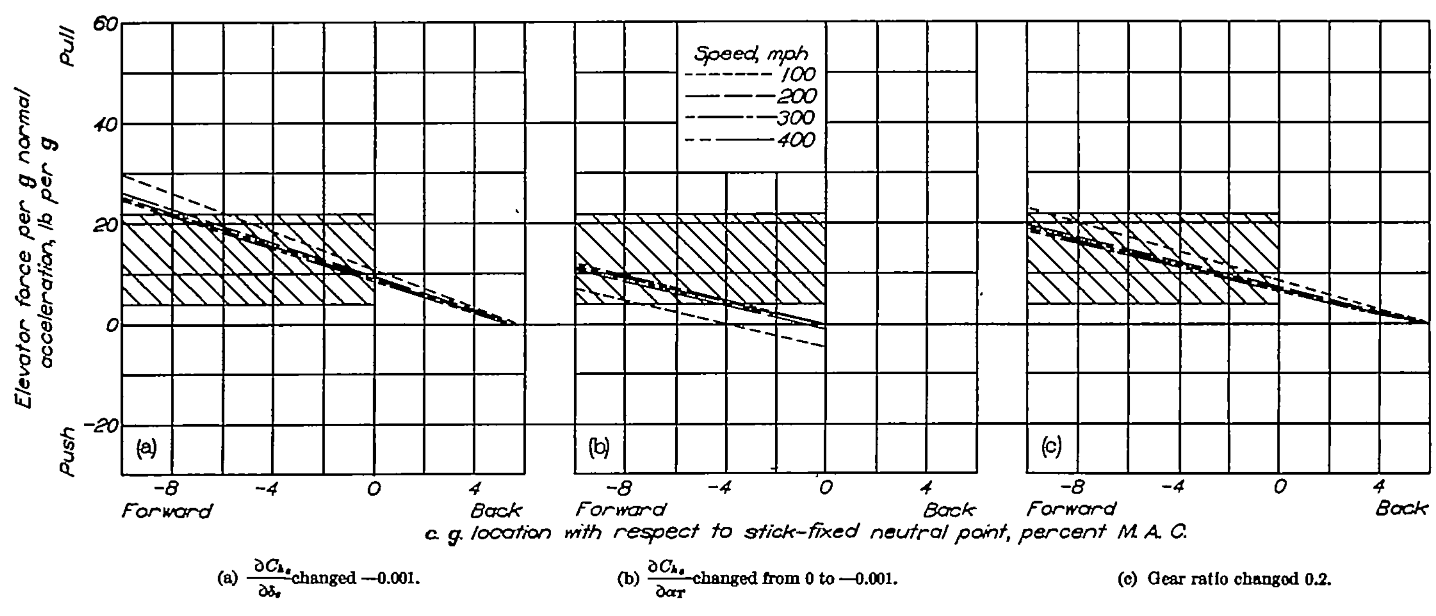
In order to determine the effects of errors in the values of $\partial C_{h_e}/\partial \delta_e$ and $\partial C_{h_e}/\partial \alpha_T$ when a finite value of spring stiffness is used, the stick forces have been computed for a geared spring tab that has the same spring stiffness as the ungeared spring tab of figure 9(b). The effects of changing $\partial C_{h_e}/\partial \delta_e$ and $\partial C_{h_e}/\partial \alpha_T$ by -0.001 for the geared spring tab are shown in figures 10(a) and 10(b), respectively. Some variation of force per g with speed is introduced but the variation is considerably smaller than that normally encountered with the ungeared spring tab (fig. 9(b)). Inasmuch as a greater variation of force per g with speed probably can be tolerated, an increase in spring stiffness to improve the ground control appears desirable.

The changes in $\partial C_{h_e}/\partial \delta_e$ and $\partial C_{h_e}/\partial \alpha_T$ cause changes in the order of magnitude of the stick forces as well as some variation in force per g with speed. These changes are, however, much smaller than those that occur with the conventional balance (fig. 9(a)). At high speeds, in fact, they approach the changes that would occur if a servotab were used.

The effect of changing the gear ratio of the geared spring tab from its ideal value is shown in figure 10(c). The effect of changing the gear ratio is nearly equivalent to changing the value of $\partial C_{h_e}/\partial \delta_e$. An error in providing the ideal value of $\partial C_{h_e}/\partial \delta_e$ on an actual airplane may therefore be corrected by suitable adjustment of the gear ratio.

The geared spring tab used to obtain the characteristics shown in figure 9(c) had values of the hinge-moment parameters $\partial C_{h_e}/\partial \alpha_T$ and $\partial C_{h_t}/\partial \alpha_T$ equal to zero. The equations given in the appendix show that this condition must be satisfied if the stick-force gradient is to be independent of speed at any center-of-gravity location. The value of $\partial C_{h_e}/\partial \alpha_T$, in practice, may be made equal to zero by use of elevators with a beveled trailing edge or with horn balances. The value of $\partial C_{h_t}/\partial \alpha_T$ is normally very small and may likewise be adjusted by varying the trailing-edge angle. If the values of $\partial C_{h_e}/\partial \alpha_T$ and $\partial C_{h_t}/\partial \alpha_T$ are not equal to zero, the force per g may still be made independent of speed by use of a geared spring tab for one particular center-of-gravity location, but the force per g will vary somewhat with speed at other center-of-gravity locations.

The effect of an increase in altitude on the stick-force gradients obtained with a geared spring tab is to shift forward the center-of-gravity location for zero force per g (the maneuver point) and to leave the slopes of the curves of force per g against center-of-gravity location unchanged. In this respect, the geared spring tab may be shown to follow the same rules as a conventional elevator. The stick-force variation with speed in straight flight is related to the force per g in maneuvers in the same way for a geared spring-tab elevator as for a conventional elevator.



(a) $\frac{\partial C_{h_e}}{\partial \delta_e}$ changed -0.001. (b) $\frac{\partial C_{h_e}}{\partial \alpha_T}$ changed from 0 to -0.001. (c) Gear ratio changed 0.2.

FIGURE 10.—Effects of design variations on stick-force characteristics of geared spring tab of figure 9(c).

The application of ordinary spring tabs to airplanes of various sizes was considered previously. The results of this analysis, in general, may be applied to the geared spring tab. In order to avoid excessive stick-force variation with speed with an ordinary spring tab, the spring must be sufficiently flexible to make the control behave essentially as a servotab in the normal-flight speed range. The stick-force gradient obtained with a geared spring tab must also equal that of a servotab if force variation with speed is to be avoided. Because the stick forces obtained with a servotab result from the aerodynamic hinge moments on the tab, some difficulty may be encountered in providing sufficiently heavy stick-force gradients with normal tab designs on airplanes much smaller than the 50,000-pound airplane considered in the present report.

CONCLUSIONS

An analysis of the effects of spring tabs on elevator forces for airplanes of various sizes has indicated the following conclusions:

1. By the use of spring tabs, satisfactory elevator control-force characteristics may be obtained over a large center-of-gravity range on airplanes varying in weight from about 16,000 to at least 300,000 pounds.

2. The spring tab offers the possibility of greatly reducing the changes in stick forces that result from small variations in contours of the elevators on different airplanes of the same type.

3. The elevator control-force characteristics resulting from the use of a spring tab should be more closely predictable than those with other types of aerodynamic balance such as a balancing tab or inset-hinge balance; in order to take advantage of this effect, however, more complete information on the hinge-moment characteristics of tabs is required.

4. One of the chief objections to the use of spring tabs is the amount of weight required for mass balance to prevent flutter. Experimental work is recommended in order to find means of reducing the amount of balance weight required.

5. By means of a geared spring tab, it is theoretically possible to provide a value of stick-force gradient in maneuvers that does not vary with speed, no matter what spring stiffness is used. If the geared spring tab is used in conjunction with an elevator that has zero variation of hinge moment with angle of attack, the force per g may be made independent of speed at any center-of-gravity location.

6. A geared spring tab may be designed to provide adequate ground control and small sensitivity of the control forces to slight changes in the hinge-moment parameters. The poor ground control associated with a servotab and the sensitivity of a conventional balance of small changes in hinge-moment parameters may therefore be avoided.

7. The geared spring tab appears to be most suitable for application to large airplanes.

Langley Memorial Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., *November 24, 1944.*

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APPENDIX

EQUATIONS FOR ELEVATOR FORCES WITH GEARED SPRING TAB

The tab system considered is shown in figure 2. The mechanical characteristics of the linkage are completely determined when four constants are specified. These constants are defined by the following equations:

$$x_s=K_1\delta_e+K_2\delta_t \tag{A1}$$

$$F=K_3\delta_t+K_4\delta_e \tag{A2}$$

Equation (A2) applies when the airspeed is zero. The ratio between the tab deflection and the elevator deflection, stick fixed, equals $-K_1/K_2$ and the ratio between the tab deflection and the elevator deflection at zero airspeed, stick free, equals $-K_4/K_3$. The ratio K_4/K_3 is defined as the linkage ratio of an equivalent balancing tab. When the system is in equilibrium, the relations between stick force, elevator hinge moments, and tab hinge moments are given in terms of these constants by the expressions

$$\left.\begin{aligned}\Delta F&=\frac{\Delta H_e-\Delta H_t\dfrac{K_4}{K_3}}{K_1-\dfrac{K_4}{K_3}K_2}\\[2ex]\Delta F&=\frac{\Delta H_t}{K_2}+K_3\left(\Delta\delta_t+\frac{K_4}{K_3}\Delta\delta_e\right)\end{aligned}\right\} \tag{A3}$$

These equations may be solved simultaneously with equations (1) and (2) to obtain an expression for the force per g for a geared spring tab. The resulting equation is the same as was derived for an ordinary spring tab, provided that certain substitutions are made for some of the parameters. These substituted values may be interpreted physically as the characteristics of the equivalent balancing tab previously defined. The complete equation is

$$\frac{\partial F}{\partial n}=\frac{\dfrac{1}{(K_1)_b}\left\{A\left[\left(\dfrac{dC_{h_e}}{d\alpha_T}\right)_{tf}+\dfrac{K_2K_3\left(\dfrac{\partial C_{h_e}}{\partial\alpha_T}\right)_b}{\dfrac{\partial C_{h_t}}{\partial\delta_t}q_Tb_tc_t^2}\right]+B\left[\left(\dfrac{dC_{h_e}}{d\delta_e}\right)_{tf}+\dfrac{K_2K_3\left(\dfrac{\partial C_{h_e}}{\partial\delta_e}\right)_b}{\dfrac{\partial C_{h_t}}{\partial\delta_t}q_Tb_tc_t^2}\right]\right\}\dfrac{q_T}{q}b_ec_e^2}{1-\dfrac{K_2\left(\dfrac{\partial C_{h_e}}{\partial\delta_t}\right)_b b_ec_e^2}{(K_1)_b\dfrac{\partial C_{h_t}}{\partial\delta_t}b_tc_t^2}+\dfrac{K_2K_3}{\dfrac{\partial C_{h_t}}{\partial\delta_t}q_Tb_tc_t^2}} \tag{A4}$$

where the quantities with the subscript b are defined in the following table:

Quantity	Definition	Physical significance
$(K_1)_b$	$K_1\left(1-\frac{K_4K_2}{K_3K_1}\right)$	Ratio between stick travel and elevator deflection for equivalent balancing tab
$\left(\frac{\partial C_{h_e}}{\partial\delta_e}\right)_b$	$\frac{\partial C_{h_e}}{\partial\delta_e}-\frac{K_4}{K_3}\frac{\partial C_{h_e}}{\partial\delta_t}-\frac{K_4}{K_3}\frac{\partial C_{h_t}}{\partial\delta_e}\frac{b_tc_t^2}{b_ec_e^2}+\left(\frac{K_4}{K_3}\right)^2\frac{\partial C_{h_t}}{\partial\delta_t}\frac{b_tc_t^2}{b_ec_e^2}$	Value of $\partial C_{h_e}/\partial\delta_e$ for equivalent balancing tab
$\left(\frac{\partial C_{h_e}}{\partial\alpha_T}\right)_b$	$\frac{\partial C_{h_e}}{\partial\alpha_T}-\frac{K_4}{K_3}\frac{\partial C_{h_t}}{\partial\alpha_T}\frac{b_tc_t^2}{b_ec_e^2}$	Value of $\partial C_{h_e}/\partial\alpha_T$ for equivalent balancing tab
$\left(\frac{\partial C_{h_e}}{\partial\delta_t}\right)_b$	$\frac{\partial C_{h_e}}{\partial\delta_t}-\frac{K_4}{K_3}\frac{\partial C_{h_t}}{\partial\delta_t}\frac{b_tc_t^2}{b_ec_e^2}$	Value of $\partial C_{h_e}/\partial\delta_t$ for equivalent balancing tab; measured with tab link connected. Physical significance may be visualized as effect of deflecting tab as a trim tab by changing length of tab link

The stick-force characteristics of an ordinary spring tab were discussed in the main text. At very high speeds the stick force per g normal acceleration was shown to approach the value obtained with a servotab, and at low speeds the force per g was shown to approach the value obtained with the tab fixed. By similar reasoning, the stick-force gradient with a geared spring tab may be shown to approach that of a servotab at high speeds and to approach that obtained with the equivalent balancing tab at low speeds. By varying the gear ratio, the force per g at low speeds may be adjusted to any desired value without affecting the force per g at high speeds. In particular, the force per g at low speeds may be adjusted to the value obtained at high speeds. The stick-force gradient, in this case, is found to be independent of the speed.

The conditions that must be satisfied in order to provide a force gradient independent of speed may be found from equation (A4). The assumption is made that the ratio q_T/q is independent of speed—a condition approximately true at maneuvering speeds. The force per g will be independent of speed if the ratio of the terms in the numerator that contain q_T to the terms in the denominator that contain q_T is the same as the ratio of the remaining terms in the numerator to the remaining terms in the denominator. For one particular center-of-gravity location, this condition may always be satisfied by suitable choice of the gear ratio. If it is desired to provide a force gradient independent of speed at any center-of-gravity location, the following relations must be satisfied:

$$\frac{\left(\dfrac{dC_{h_e}}{d\alpha_T}\right)_{tf}}{1-\dfrac{K_2\left(\dfrac{\partial C_{h_e}}{\partial\delta_t}\right)_b b_ec_e^2}{(K_1)_b\dfrac{\partial C_{h_t}}{\partial\delta_t}b_tc_t^2}}=\left(\frac{\partial C_{h_e}}{\partial\alpha_T}\right)_b \tag{A5}$$

$$\frac{\left(\dfrac{dC_{h_e}}{d\delta_e}\right)_{tf}}{1-\dfrac{K_2\left(\dfrac{\partial C_{h_e}}{\partial\delta_t}\right)_b b_ec_e^2}{(K_1)_b\dfrac{\partial C_{h_t}}{\partial\delta_t}b_tc_t^2}}=\left(\frac{\partial C_{h_e}}{\partial\delta_e}\right)_b \tag{A6}$$

In practice, equation (A5) can be satisfied only by making $\partial C_{h_e}/\partial \alpha_T$ and $\partial C_{h_t}/\partial \alpha_T$ very close to zero. Equation (A6) may then be used to determine the gear ratio K_4/K_3 that must be employed to provide a value of force per g which does not vary with speed.

After substituting in equation (A6) the value of $\left(\frac{dC_{h_e}}{d\delta_e}\right)_{tf}$ obtained from equation (11) and the values of $(K_1)_b$, $\left(\frac{\partial C_{h_e}}{\partial \delta_t}\right)_b$, and $\left(\frac{\partial C_{h_e}}{\partial \delta_e}\right)_b$ given in the preceding table, the equation may be solved explicitly for the gear ratio K_4/K_3.* The gear ratio is obtained by solving a quadratic equation which yields the two values

$$\left.\begin{aligned}\left(\frac{K_4}{K_3}\right)_1&=\frac{\frac{K_2}{K_1}\frac{\partial C_{h_e}}{\partial \delta_e}-\frac{\partial C_{h_t}}{\partial \delta_e}\frac{b_t c_t^2}{b_e c_e^2}}{\frac{K_2}{K_1}\frac{\partial C_{h_e}}{\partial \delta_t}-\frac{\partial C_{h_t}}{\partial \delta_t}\frac{b_t c_t^2}{b_e c_e^2}}\\ \left(\frac{K_4}{K_3}\right)_2&=\frac{\dfrac{K_2\frac{\partial C_{h_e}}{\partial \delta_t}b_e c_e^2}{K_1\frac{\partial C_{h_t}}{\partial \delta_t}b_t c_t^2}-1}{\dfrac{K_2}{K_1}-\dfrac{\frac{\partial C_{h_t}}{\partial \delta_t}b_t c_t^2}{\frac{\partial C_{h_e}}{\partial \delta_t}b_e c_e^2}}\end{aligned}\right\} \tag{A7}$$

The significance of the two solutions may be seen by substituting values for the airplane and control-system characteristics given in tables I and II. The following numerical values are obtained for the gear ratios:

$$\left(\frac{K_4}{K_3}\right)_1=0.84$$

$$\left(\frac{K_4}{K_3}\right)_2=21$$

Of these two solutions, only the smaller value is of practical interest. The larger value would result in excessive tab deflections that would very likely cause the lift increment due to the tab, which has been neglected in the present analysis, to reverse the direction of lift on the surface. For practical use, therefore, only the formula for $(K_4/K_3)_1$ need be considered.

*This solution was pointed out to the author by Mr. H. Gumbel of the Republic Aviation Corporation.

When the gear ratio and elevator hinge-moment characteristics are selected by this procedure to give a force gradient independent of speed at any center-of-gravity location, the force gradient may be computed from the equation

$$\frac{\partial F}{\partial n}=\frac{B}{(K_1)_b}\left(\frac{\partial C_{h_e}}{\partial \delta_e}\right)_b\frac{q_t}{q}b_e c_e^2 \tag{A8}$$

The criterion given in the main text for the spring stiffness required for satisfactory ground control may be used for a geared spring tab as well as for an ordinary spring tab. For a geared spring tab, the variation of elevator hinge moment with stick deflection when the elevator is held fixed is given by the following equation, which is very similar to equation (13) of the main text.

$$\frac{\partial H_e}{\partial x_s}=\frac{-(K_1)_b K_3}{K_2}+\frac{\left(\frac{\partial C_{h_e}}{\partial \delta_t}\right)_b q_T b_e c_e^2}{K_2}-\frac{(K_1)_b\frac{\partial C_{h_t}}{\partial \delta_t}q_T b_t c_t^2}{K_2^2} \tag{A9}$$

If it is desired to satisfy the criterion at zero airspeed, the terms containing q_T may be neglected and the following relation is obtained:

$$\begin{aligned}\frac{1}{I}\frac{\partial H_e}{\partial x_s}&=200\\ &=\frac{-(K_1)_b K_3}{K_2 I}\end{aligned}$$

This expression may be used to solve for K_3, which determines the spring stiffness. For the example under consideration,

$$\begin{aligned}K_3&=\frac{K_2(200)I}{-(K_1)_b}\\ &=\frac{(-0.45)(200)(1.5)}{-1.80\left[1-\frac{(0.84)(-0.45)}{1.80}\right]}\\ &=95.0 \text{ pounds per radian}\end{aligned}$$

A value of K_3 of 100 pounds per radian has been used in the examples of this paper. From the value of K_4/K_3 determined previously, the value of K_4 may be readily obtained.

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TABLE I.—CHARACTERISTICS OF VARIOUS AIRPLANES

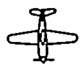
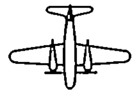
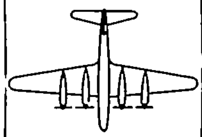
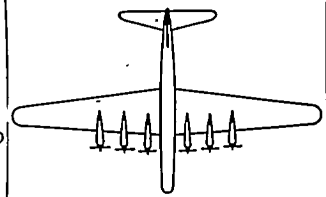
	Scout bomber	Medium bomber	Heavy bomber	300,000-pound airplane
				
Scale, ft	0 50	0 100	0 100	0 100 200
W, lb	16,000	50,000	125,000	300,000
b, ft	49	89.3	143	223.5
S, sq ft	400	1,000	2,275	5,000
c, ft	8.16	11.18	15.90	22.35
l, ft	20	35	50	75
S_T, sq ft	100	200	455	1,000
$\left(\frac{dC_L}{d\alpha}\right)_w$, per radian	4.2	4.5	4.6	4.7
$1-\frac{d\epsilon}{d\alpha}$	0.5	0.55	0.57	0.60
q_T/q	1.0	1.0	1.0	1.0
b_e, ft	20	34	50	75
c_e, ft	1.8	2.2	3.2	4.8
b_t, ft	5.0	7.35	15.0	26.2
c_t, ft	0.50	0.80	0.60	0.666
τ	0.5	0.5	0.5	0.5
$\frac{\partial C_{L_T}}{\partial \delta_e}$, per radian	1.7	1.7	1.7	1.7
I, slug-ft^2	0.5	1.5	7.0	35
K_1, ft per radian	1.80	1.80	1.80	1.80
K_2, ft per radian	−0.60	−0.45	−1.20	−1.20
K_3, lb per radian	33.3	100	124	200
$\frac{\partial C_{h_e}}{\partial \alpha_T}$, per deg	0	0	0	0
$\frac{\partial C_{h_e}}{\partial \delta_e}$, per deg	−0.003	−0.003	−0.003	−0.002
$\frac{\partial C_{h_e}}{\partial \delta_t}$, per deg	−0.003	−0.003	−0.003	−0.003
$\frac{\partial C_{h_t}}{\partial \alpha_T}$, per deg	0	0	0	0
$\frac{\partial C_{h_t}}{\partial \delta_e}$, per deg	0	0	0	0
$\frac{\partial C_{h_t}}{\partial \delta_t}$, per deg	−0.005	−0.005	−0.005	−0.003

TABLE II.—CONTROL-SYSTEM CHARACTERISTICS

	Conventional balance (fig. 9(a))	Ungeared spring tab (fig. 9(b))	Geared spring tab (fig. 9(c))
K_1, ft per radian	2.18	1.80	1.80
K_2, ft per radian	----------	−0.45	−0.45
K_3, lb per radian	----------	100	100
K_4, lb per radian	----------	----------	84
$\frac{\partial C_{h_e}}{\partial \alpha_T}$, per deg	0	0	0
$\frac{\partial C_{h_e}}{\partial \delta_e}$, per deg	−0.00058	−0.003	−0.003
$\frac{\partial C_{h_e}}{\partial \delta_t}$, per deg	----------	−0.003	−0.003
$\frac{\partial C_{h_t}}{\partial \alpha_T}$, per deg	----------	0	0
$\frac{\partial C_{h_t}}{\partial \delta_e}$, per deg	----------	0	0
$\frac{\partial C_{h_t}}{\partial \delta_t}$, per deg	----------	−0.005	−0.005