https://ntrs.nasa.gov/search.jsp?R=19930092281 2018-03-18T11:32:58+00:00Z

REPORT 1284

THEORY OF WING-BODY DRAG AT SUPERSONIC SPEEDS [1]

By Robert T. Jones

SUMMARY

The relation of Whitcomb's "area rule" to the linear formulas for wave drag at slightly supersonic speeds is discussed. By adopting an approximate relation between the source strength and the geometry of a wing-body combination, the wave-drag theory is expressed in terms involving the areas intercepted by oblique planes or Mach planes. The resulting formulas are checked by comparison with the drag measurements obtained in wind-tunnel experiments and in experiments with falling models in free air. Finally, a theory for determining wing-body shapes of minimum drag at supersonic Mach numbers is discussed and some preliminary experiments are reported.

DISCUSSION

At subsonic speeds the pressure drag arising from the thickness of the body or wings is negligible so long as the shapes are sufficiently well streamlined to avoid flow separation. In that range there exists no possibility of either favorable or adverse interference on the pressure distributions themselves. If one body is so placed as to receive a drag from the pressure field of another then the second body is sure to receive a corresponding increment of thrust from the first.

At supersonic speeds this tolerance which was permitted the designer disappears, and the drag becomes sensitive to the shape and arrangement of the bodies. To be sure, the primary factor here is the thickness ratio, but nevertheless there exist arrangements in which a large cancellation of drag occurs. Examples of the latter are the sweptback wing and the Busemann biplane.

Recently R. T. Whitcomb (ref. 1) has shown how the drag at transonic speeds may be reduced to a surprising extent by simply cutting out a portion of the fuselage to compensate for the area blocked by the wing. The purpose of the present paper is to discuss some of the theoretical aspects of this method of drag reduction and to show how the basic idea may be extended to higher speeds in the supersonic range.

Whitcomb's deduction of the "area rule" was based on considerations of stream-tube area and the phenomenon of "choking"—which follow from one-dimensional-flow theory. Each individual stream tube of a three-dimensional-flow field must obey the law of one-dimensional flow. While we cannot actually determine the three-dimensional field on this basis alone, nevertheless it provides a good starting point for our thinking. The results demonstrate again the effectiveness of basic and simple considerations.

While one-dimensional-flow theory thus provides a clue to the area rule, the necessary principle appears more specifically in the three-dimensional-flow theory. Thus, the formulas for wave drag given by linear theory, if followed toward the limit as M approaches 1.0 (from above), show that the wave drag of a system of wings and bodies depends solely on the longitudinal area distribution of the system as a whole. This was first noted by W. D. Hayes in his 1946 thesis (ref. 2). However, because of the limitations of the theory at transonic speeds, this result was not thought to be of practical significance. Later G. N. Ward (ref. 3), E. W. Graham (ref. 4), and others, restricting themselves to very narrow shapes, expressed the wave drag in terms of the longitudinal area distribution for Mach numbers above 1.0, where the linear theory has a better justification.

It should be noted, however, that both of the problems cited are limiting cases of the more general problem of supersonic drag and it should be borne in mind that only in certain cases has it been possible to reduce the general theoretical formulas to the form of an area rule. It can be shown that the flow field about any system of bodies may be created by a certain distribution of sources and sinks over the surfaces of the bodies. Hayes' formula and the formulas given in reference 5 relate the drag of such a system to the distribution of these singularities. To obtain a formula for the wave drag in terms of area distributions we have to adopt a simplified relation between the source strength and the geometry of the bodies, namely, that the source strength is proportional to the normal component of the stream velocity at the body surface. There are examples (e. g., Busemann biplanes and ducted bodies) for which this assumption is not valid. If, on the other hand, we limit ourselves to thin symmetrical wings mounted on vertically symmetrical fuselages, there are indications that a good estimate of the wave drag at supersonic speeds can be obtained on the basis of the simplified relation assumed.

Following Hayes' method of calculation, we find that at $M=1.0$ the expression for the wave drag of a system of wings and bodies reduces to Kármán's well-known formula (ref. 6) for the wave drag of a slender body of revolution, that is,

$$D_{M\to 1}=-\frac{\rho V^2}{4\pi}\int_{-x_o}^{+x_o}\int_{-x_o}^{+x_o} S''(x)S''(x_1)\log|x-x_1|dx\,dx_1$$

Here $S(X)$ represents the total cross-sectional area intercepted by a plane perpendicular to the stream at the station

[1] Supersedes NACA RM A53H18a by Robert T. Jones, 1953.

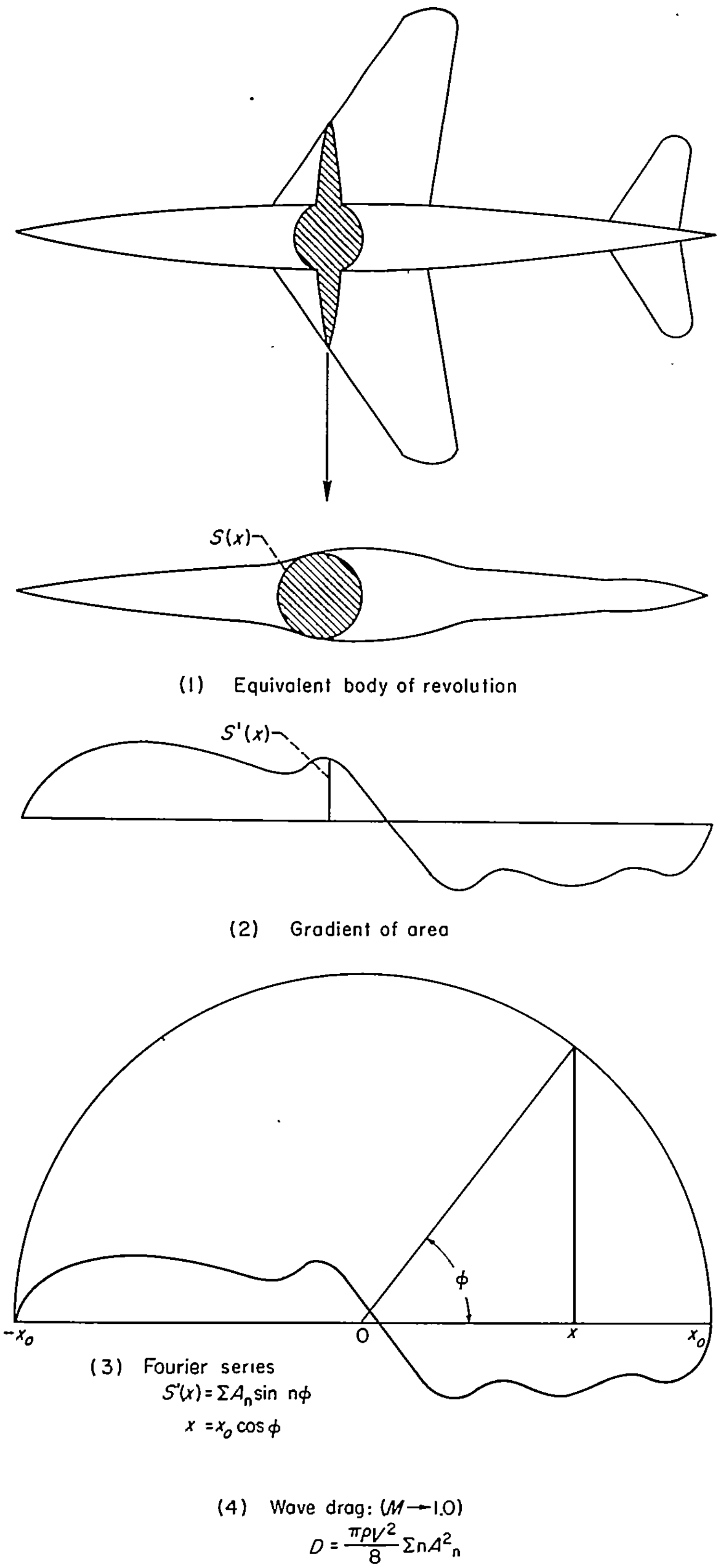


FIGURE 1.—Steps in the calculation of wave drag for $M \rightarrow 1.0$.

x (see fig. 1) and $S''(x)$ is the second derivative of S with respect to x. Following Sears (ref. 7) we may expand $S'(x)$ in a Fourier series and obtain in this way a formula for the drag which is completely analogous to the well-known formula for the induced drag of a wing in terms of its spanwise load distribution. Thus, if we write

$$x = x_0 \cos \phi$$

and

$$S'(x) = \Sigma A_n \sin n\phi$$

we obtain for the wave resistance

$$D = \frac{\pi \rho V^2}{8} \Sigma n A_n^2$$

Of all the terms of the series, each contributes to the drag but only A_1 and A_2 contribute to the volume or the base area of the system. Thus, to achieve a small drag with a given base area, or with a given over-all volume within the given length, the higher harmonics in the curve $S'(x)$ should be suppressed. This formula enables us to characterize the smoothness of a given shape in a quantitative fashion.

To extend these considerations to supersonic speeds we have to consider a series of cross sections of the system made not by planes perpendicular to the stream, but by planes inclined at the Mach angle, or "Mach planes." By means of

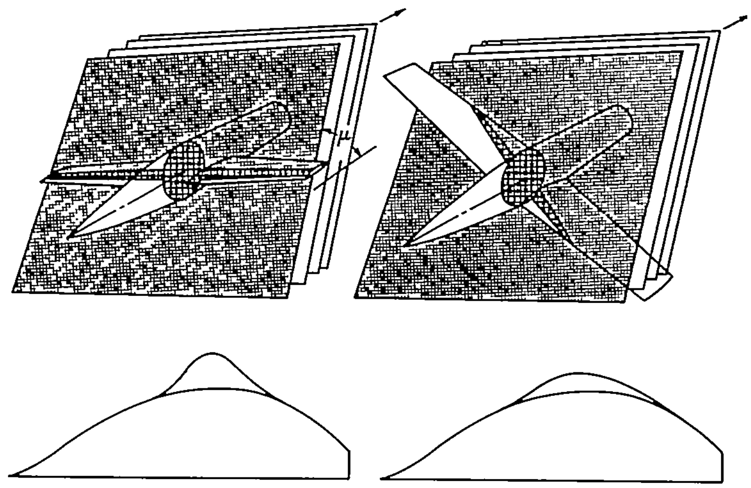


FIGURE 2.—Area distribution given by intersections of Mach planes.

a set of parallel Mach planes (see fig. 2) we construct an "equivalent body of revolution," using the intercepted areas, and compute the drag by von Kármán's formula. The theoretical basis of this step is the fact that the complete three-dimensional disturbance field may be constructed by the superposition of elementary one-dimensional disturbances in the form of plane waves (ref. 8). It is evident that the set of parallel Mach planes may be placed at various angles around the x axis. In constructing the flow field it is necessary to superimpose disturbances at all of these angles and, in computing the drag, to consider the drags of all the equivalent bodies of revolution. The final value of the drag is simply the average of the values obtained through a complete rotation of the Mach planes.

In order to make these statements more specific, we may write the equation of one such Mach plane as follows:

$$X = x - y' \cos \theta - z' \sin \theta$$

where $y' = \sqrt{M^2-1}\ y$, $z' = \sqrt{M^2-1}\ z$, and θ is the angle of rotation of the Mach plane. By assigning different values to X while keeping θ constant, we obtain a series of parallel planes at the same angle θ around the x axis. By assigning different values to θ while keeping X a constant, we obtain a set of planes enveloping that Mach cone whose apex lies at the point $x=X$.

Selecting a value of θ, we cut through the wing-body system with a series of planes corresponding to different values of X. The total intercepted area in each plane is then equated to the area intercepted by this plane passing through the equivalent body of revolution. If we denote the area intercepted obliquely by $s(X,\theta)$, then the area $S(X,\theta)$ is defined by

$$S=s \sin \mu$$

where μ is the Mach angle (i. e., $\sin \mu=1/M$). Thus, S is the area intercepted by normal planes passing through the equivalent body of revolution on the assumption that this body is slender. Again, we write

$$S'(X,\theta)=\frac{\partial}{\partial X} S(X,\theta)=\Sigma A_n \sin n\phi$$

with

$$\cos \phi=\frac{X}{X_o}$$

Here, however, both the length $2X_o$ and the shape of the equivalent body vary with the angle θ. The drag of each equivalent body of revolution, which we may denote by D' (θ) is then determined by applying Sears' formula:

$$D'(\theta)=\frac{\pi\rho V^2}{8} \Sigma n A_n^2$$

The total drag of the wing-body system is the average of all these values between $\theta=0$ and $\theta=2\pi$, that is,

$$D=\frac{1}{2\pi}\int_0^{2\pi} D'(\theta)d\theta$$

In general, the coefficients A_n will be functions of the angle of projection θ. However, calculation shows that the first two coefficients A_1 and A_2 are again related in a simple way to the base area and the volume v. Thus,

$$A_1=\frac{2}{\pi}\frac{S(X_o)}{X_o}$$

$$A_2=2A_1-\frac{4}{\pi}\frac{v}{X_o^2}$$

None of the higher coefficients contribute to the base area or volume, but they invariably contribute to the drag.

The rules for obtaining a low wave drag now reduce to the rule that each of the equivalent bodies obtained by the oblique projections should be as smooth and slender as possible, the "smoothness" again being related to an absence of higher harmonics in the series expression for $S'(X)$. Thus in the case of given length and volume the series should contain only the term $A_2 \sin 2\phi$ (see fig. 3). It should be noted that in this theory, the equivalent bodies of revolution do not have a physical significance. The concept is simply an aid in visualizing the magnitude of the drag of the complete system.

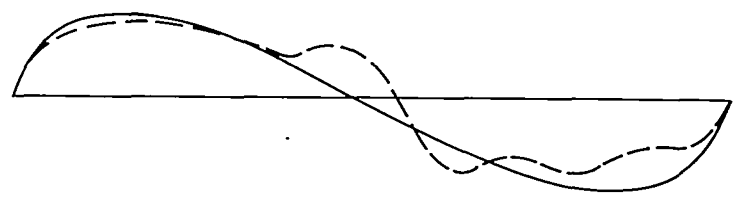


FIGURE 3.—Optimum area distribution for given length and volume.

To check the agreement between these theoretical formulas for the wave drag and experimental values, we have compared our calculations with the results of tests made by dropping models from a high altitude. This comparison was made by George H. Holdaway of Ames Laboratory, who supplied the accompanying illustration (fig. 4). In some of these cases it was found necessary to retain more than 20 terms of the Fourier series in order to obtain a convergent expression for the drag.

Considering the variety of the shapes represented here, the agreement is certainly as good as we ought to expect from our linear simplifications. The agreement is naturally better in those interesting cases in which the drag is small.

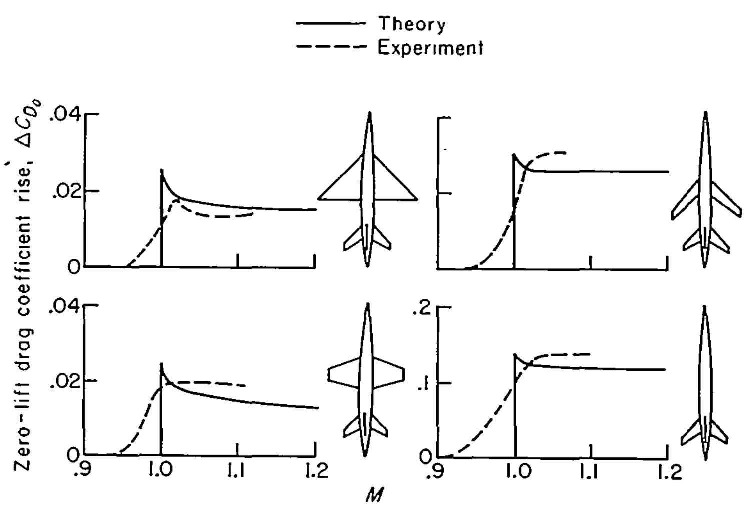


FIGURE 4.—Comparison of theory with results of Ames Laboratory drop tests.

Figure 5 shows an analysis of one of Whitcomb's experiments. The linear theory, of course, shows the transonic drag rise simply as a step at $M=1.0$. We may expect such a variation to be approached more closely as the thickness vanishes. To represent actual values here a nonlinear theory would be needed. For many purposes it will be sufficient to estimate roughly the width of the transonic zone by considerations such as those given in reference 9. In the present case it will be noted that agreement with the linear theory is reached at Mach numbers above about 1.08, and the linear theory clearly shows the effect of the modification.

For further theoretical studies of wing-body drag, shapes have been selected which are especially simple analytically, namely, the Sears-Haack body and biconvex wings of elliptic plan form, having aspect ratios of 2.54 and 0.635. Figure 6 shows the effect of wing proportions on the variation of wave drag with Mach number, both with and without the Whitcomb modification. In each case the modification has the effect of reducing the wave drag to that of the body alone at $M=1.0$. In the case of the low-aspect-ratio wing this drag reduction remains effective over a considerable range of higher Mach numbers. With the higher aspect ratio, however, the drag increases sharply at higher speeds, so that at $M=1.6$ the modification nearly doubles the wave drag.

The rapid increase of drag in the case of the high-aspect-ratio wing is, of course, the result of the relatively abrupt curvatures introduced into the fuselage lines by the cutout. Such abrupt cutouts are necessarily associated with wings having small fore and aft dimensions, that is, unswept wings of high aspect ratio.

These considerations led to the problem of determining a fuselage shape for such wings that is better adapted to the higher Mach numbers. The first step in this direction is, obviously, simply to lengthen the region of the cutout—thus avoiding the rapid increase of drag with Mach number. The problem of actually determining the best shape for the fuselage cutout at any specified Mach number has been under-

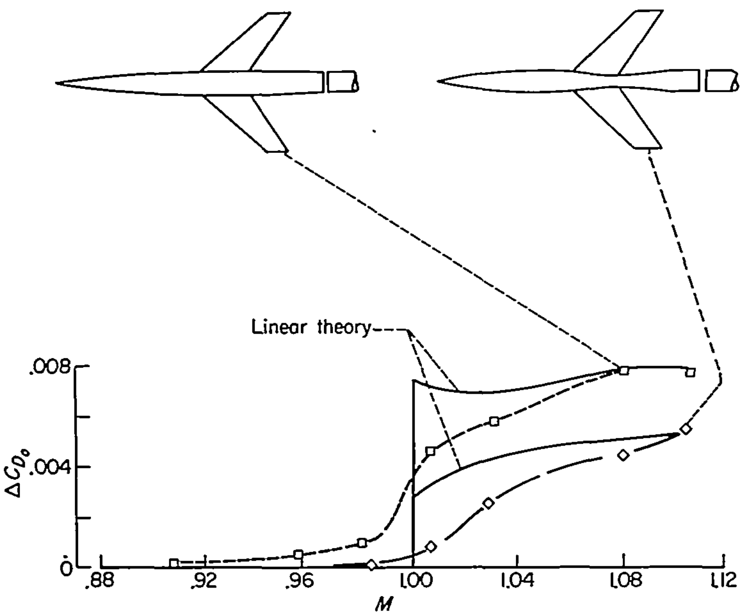


FIGURE 5.—Comparison of Whitcomb's experiments with theory.

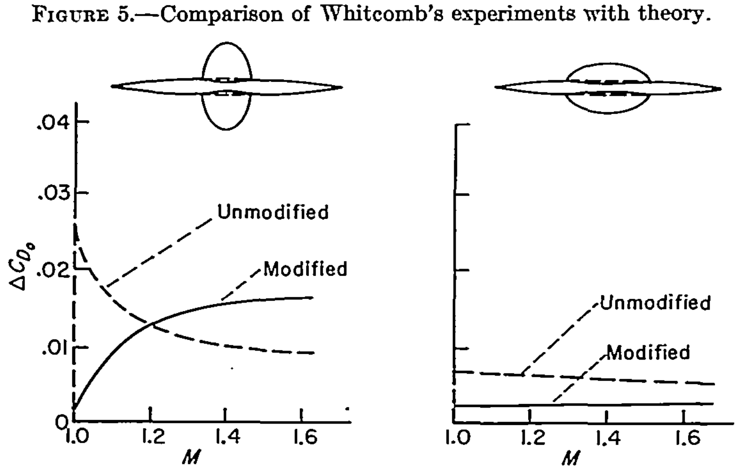


FIGURE 6.—Effect of Whitcomb modification on calculated wave drag.

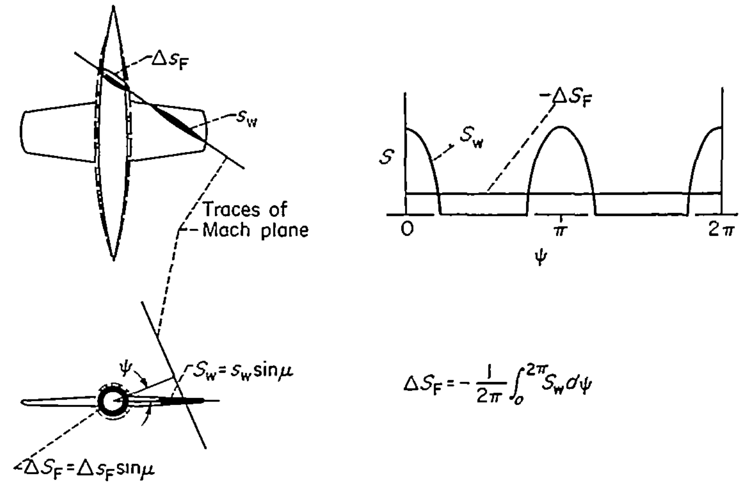


FIGURE 7.—Design of fuselage modification for specified Mach number.

taken by Harvard Lomax and Max. A. Heaslet at Ames Laboratory (ref. 10). Their solution of this problem provides a definite method for determining the distribution of sources and sinks along the fuselage axis that will achieve a minimum value of the drag for a given *wing shape* at any specified Mach number. Furthermore, by admitting singularities of higher order—quadrupoles, etc., which would distort the rotational symmetry of the fuselage—they have been able to show that the wave drag of a wing-body system can be reduced, in principle at least, to a minimum value associated with the given overall length and volume of the system, that is, to the value for a simple Sears-Haack body containing the whole volume of the system.[2]

By adopting our simplified relation between the source strength and the body shape, we may describe the result of this theory by a relatively simple concept, which is illustrated in figure 7. For modifications of the first type, the problem is to determine the area ΔS_F to be removed from the fuselage to best compensate for a given wing. (See fig. 7.) Selecting a station along the fuselage axis and a Mach plane passing through this station, we revolve this plane around the axis, measuring at each angle ψ the normal projection, or frontal projection, of the area intercepted where the plane cuts through the wing. After plotting these areas against ψ and integrating between 0 and 2π, we obtain ΔS_F as the average of the values of S_w. At any Mach number the total volume to be subtracted from the fuselage is equal to the wing volume. At higher Mach numbers, since the modification extends over a greater length, the area subtracted at individual cross sections becomes less.

Figure 8 shows the calculated result of designing the fuselage cutout for a specific Mach number, 1.2 in this case. The lower curve is an envelope showing the minimum values that can be achieved by such a radially symmetric cutout.

Figure 9 shows the magnitude of the gain that is theoretically possible by higher order modifications of the fuselage shape. There are three lower bounds here, and the symbols a_0, a_2, etc., attached to them refer to a representation of the fuselage shape by singularities of increasingly higher order.

[2] This value is, of course, not an absolute minimum for a given volume since, as shown by Ferrari, the wave drag of a body can be reduced to zero by special volume distributions (see ref. 11).

The curve labeled a_0 is that given on the previous figure and shows the maximum effect of radially symmetric modifications. While the fuselage shapes for the other curves have not actually been determined, the curve labeled a_0+a_2 may be thought of as referring to a cutout with an additional elliptic modification.

In order to test this theory of determining optimum body shapes we have started a program using models similar to those investigated theoretically. Several of these models have already been tested in the Ames 2- by 2-foot wind tunnel, with results that agree quite well with calculations made on the assumptions given earlier. Shown in figure 10 are the experimental and theoretical curves. It is evident that the calculated differences are all reproduced approximately in the experimental values.

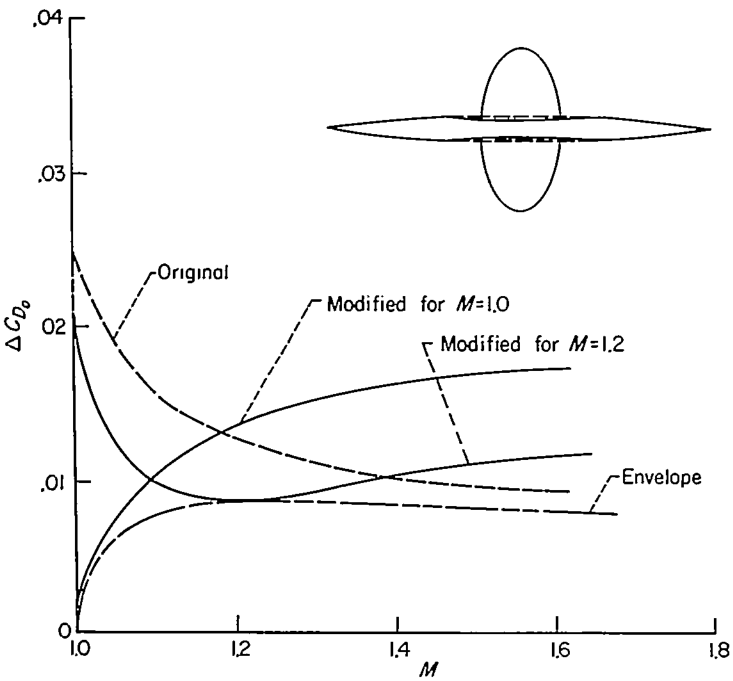


FIGURE 8.—Effect of modification designed for a specified Mach number.

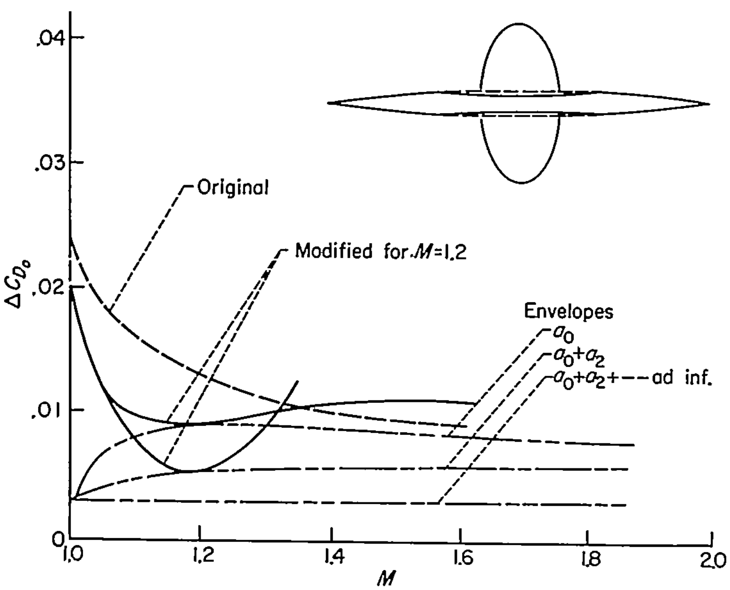
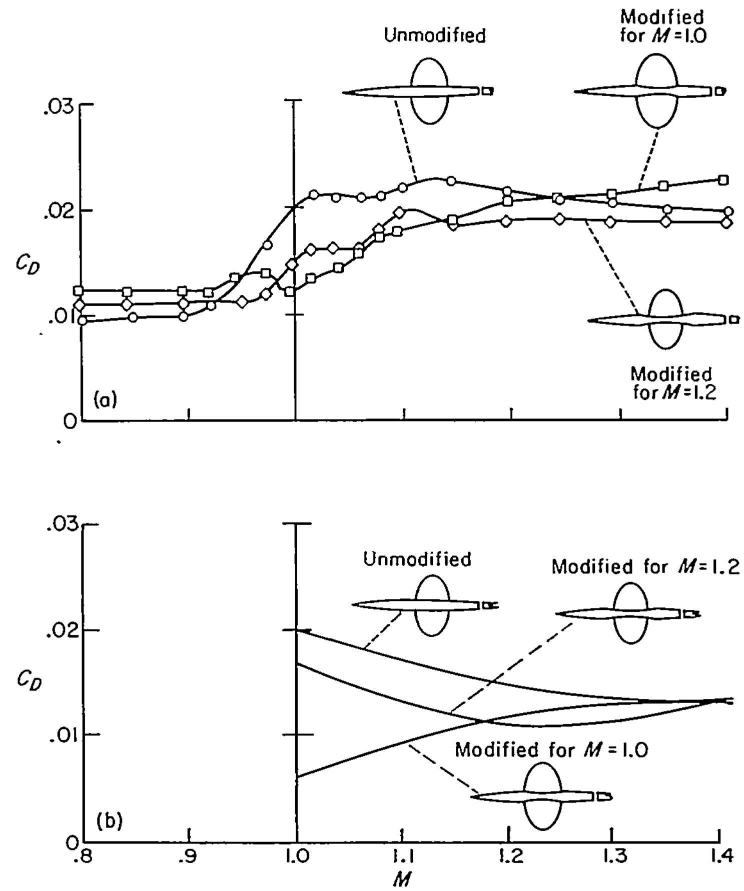


FIGURE 9.—Envelopes for drag at design Mach number.



(a) Experimental values.

(b) Calculated values.

FIGURE 10.—Drag of bodies with elliptic wings.

There are, of course, examples of wing-body systems which would hardly benefit by any change in shape of the fuselage. It is easy to decide whether a gain is possible, or worthwhile, by comparing the actual wave drag of the system with that of a Sears-Haack body containing the over-all volume of the system. In the case of 63° the wing-body combination, which has been described in several previous reports, this comparison yields 0.0045 as a lower bound for the wave-drag coefficient and 0.005 for the actual value. In such cases, for which the wave drag is initially very low, further reduction by reshaping the fuselage is not worthwhile.

It is clear from the foregoing, however, that appreciable savings in drag can be made in many cases by a suitable shaping of the fuselage. Unswept wings of high aspect ratio are benefited most and require the most careful consideration of the fuselage shape.

These new developments illustrate, again, the fact that the disturbance fields at transonic and supersonic speeds are essentially three-dimensional phenomena. It was not long ago that our ideas concerning the wing section—which had their origin in the older incompressible flow theory—had to be relinquished because of the predominating effects of the wing plan form. Now we must learn how to design the wing and the fuselage together.

AMES AERONAUTICAL LABORATORY
NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS
MOFFETT FIELD, CALIF., *July 8, 1953*

APPENDIX A

SIMPLIFIED CALCULATION OF DRAG IN SPECIAL CASES

If special shapes such as the Sears-Haack body (ref. 7) and the elliptic wing (ref. 8) are selected for exploratory studies, then the calculation of drag can be greatly simplified.

The radius r of the Sears-Haack body at any station X is given by

$$\frac{r}{r_{max}}=\left[1-\left(\frac{x}{x_0}\right)^2\right]^{3/4} \tag{A1}$$

For this shape

$$S'(X)=A_2 \sin 2\phi \tag{A2}$$

and the drag has a minimum value for the given volume and length. The value of the drag is given by

$$C_D=\frac{D}{\frac{\rho}{2}V^2 S_{max}}=\frac{9}{8}\pi^2\frac{(r_{max})^2}{x_0^2} \tag{A3}$$

The elliptic wing has symmetrical biconvex sections, with ordinates z given by

$$\frac{z}{z_{max}}=1-\frac{x^2}{a^2}-\frac{y^2}{b^2} \tag{A4}$$

where a and b are the semiaxes. The area distribution for every angle of projection is similar to that of the Sears-Haack body, but the projected length varies with the angle. The wing thus yields a minimum value of the wave drag consistent with a given volume and the elliptic plan form. The value of this drag is:

$$C_D=\frac{D}{\frac{\rho}{2}V^2 S}=4\frac{(z_{max})^2}{a^2}\frac{1}{\sqrt{M^2-1+\frac{a^2}{b^2}}}\left(2-\frac{M^2-1}{M^2-1+\frac{a^2}{b^2}}\right) \tag{A5}$$

where S is the plan area of the wing.

By making use of the reversal theorem for drag we may compute the wave drag of any body from the fictitious pressure field obtained by superimposing the perturbation velocities for forward and reversed motion (refs. 12 and 13). This process leads to some interesting relations for the shapes selected. Thus in the case of the Sears-Haack body it may be shown that the combined pressure distribution $\overline{p}$ consists of a uniform gradient of pressure over the whole interior R of its "characteristic envelope" defined by the Mach cone from the nose together with the reversed Mach cone from the tail. (See fig. 11.)

By thinking of the characteristic region R as a region of uniform horizontal buoyancy, and of the body b in terms of a certain volume, ν_b, we see that the drag is simply the product

$$D_{bb}=\nu_b\frac{d\overline{p}_b}{dx} \tag{A6}$$

The existence of a constant pressure gradient makes the computation of interference drag particularly simple for such shapes, provided the interfering body lies entirely within the characteristic region R. Thus the additional drag of an airfoil a placed within the double cone of the fuselage will be given by

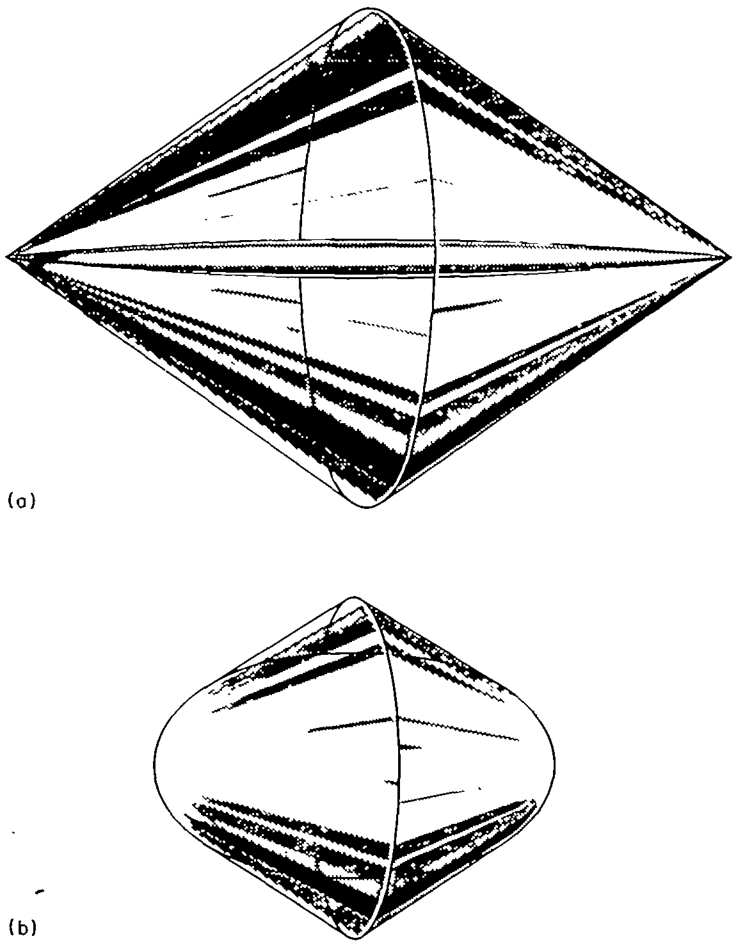
$$D_{ab}=\nu_a\frac{D_{bb}}{\nu_b} \tag{A7}$$

Now, by the mutual drag theorem (ref. 13) we have

$$D_{ab}=D_{ba} \tag{A8}$$

or, "the drag of the fuselage caused by the presence of the wing is equal to the drag of the wing caused by the presence of the fuselage." In this way we obtain the general formula

$$D(a+b)=D_{bb}+2D_{ab}+D_{aa} \tag{A9}$$



(a) Body of revolution.

(b) Elliptic wing.

FIGURE 11.—Characteristic envelopes.

and for the special shapes selected:

$$D(a+b)=D_{bb}\left(1+2\frac{\nu_a}{\nu_b}\right)+D_{aa} \qquad \text{(A10)}$$

The effect of an indentation or cutout in the fuselage may be calculated by introducing a second "body," c, shorter than the fuselage, and having a negative volume equal to the volume subtracted by the indentation. In order to simplify the situation as much as possible it will be assumed that the wing lies entirely within the characteristic region of the indentation, and furthermore that the latter may be represented by a "negative" Sears-Haack body with volume equal to that of the wing.

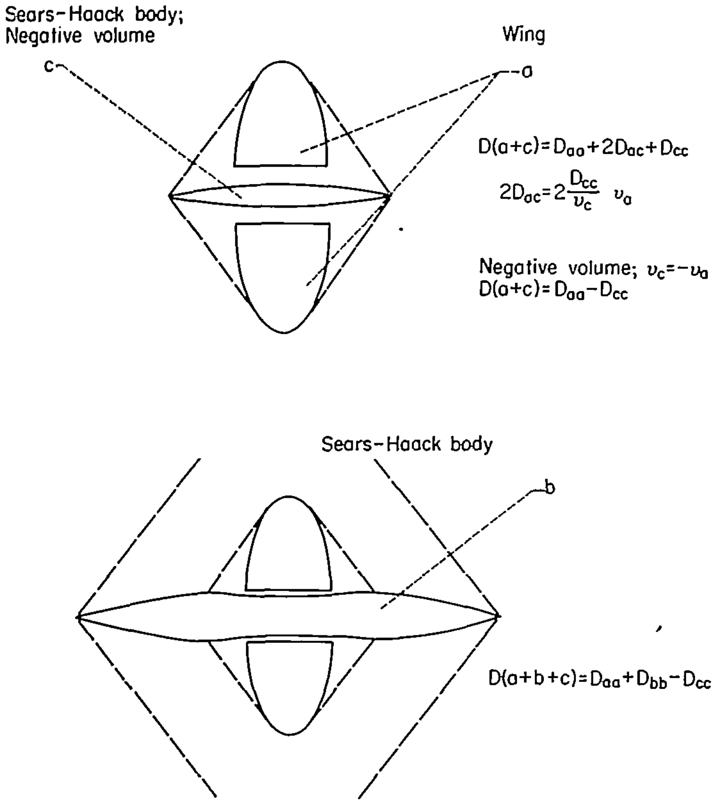


FIGURE 12.—Simplified calculation of interference drag.

The calculation of drag in this case is illustrated in figure 12. For the airfoil and cutout we have

$$\left.\begin{aligned} &D(a+c)=D_{aa}+2D_{ac}+D_{cc} \\ \text{but, since,}\quad &2D_{ac}=-2D_{cc} \\ &D(a+c)=D_{aa}-D_{cc} \end{aligned}\right\} \qquad \text{(A11)}$$

Now, the combination $(a+c)$ may be placed inside the characteristic region of the body b without interference, since $\nu_a+\nu_c=0$. Hence,

$$D(a+b+c)=D_{aa}+D_{bb}-D_{cc} \qquad \text{(A12)}$$

This formula yields the minimum drag for the shapes selected under the assumption that ν_{b+c} is fixed. In this case the drag saving is equal to the drag of the indentation alone.

The negative Sears-Haack body is not the optimum shape of the indentation c for the elliptic wing, as shown by the result of Heaslet and Lomax quoted earlier (ref. 10). Again, however, in the case of the optimum shape for c, our previous equation holds. However, the calculation of D_{cc} is more complex in this case and its value is somewhat greater.

REFERENCES

1. Whitcomb, Richard T.: A study of the Zero-Lift Drag Rise Characteristics of Wing-Body Combinations Near the Speed of Sound. NACA RM L52H08, 1952.
2. Hayes, W. D.: Linearized Supersonic Flow. North American Aviation, Inc., Rep. No. AL-222, June 1947, pp. 94–95.
3. Ward, G. N.: Supersonic Flow Past Slender Pointed Bodies. Quart. Jour. Mech. and Appl. Math., vol. II, pt. 1, 1949.
4. Graham, E. W.: Pressure and Drag on Smooth Slender Bodies in Linerarized Flow. Douglas Aircraft Co., Rep. SM-13417, 1949.
5. Heaslet, Max. A., Lomax, Harvard, and Spreiter, John R.: Linearized Compressible Flow Theory for Sonic Flight Speeds. NACA Rep. 956, 1950.
6. de Kármán, Th.: The Problem of Resistance in Compressible Fluids. Estratto dagli Atti del V Convegno della Fondazione Alessandro Volta, 1935, Rome, Reale Accademia d'Italia, 1936.
7. Sears, W. R.: On Projectiles of Minimum Wave Drag. Quart. Appl. Math., vol. IV, no. 4, Jan. 1947.
8. Jones, Robert T.: Theoretical Determination of the Minimum Drag of Airfoils at Supersonic Speeds. Jour. Aero. Sci., vol. 19, no. 12, Dec. 1952.
9. Busemann, A.: Application of Transonic Similarity. NACA TN 2687, 1952.
10. Lomax, Harvard, and Heaslet, Max. A.: A Special Method for Finding Body Distortions That Reduce the Wave Drag of Wing and Body Combinations at Supersonic Speeds. NACA Rep. 1282, 1956.
11. Ferri, Antonio: Application of the Method of Characteristics to Supersonic Rotational Flow. NACA Rep. 841, 1946.
12. Munk, Max M.: The Reversal Theorem of Linearized Supersonic Airfoil Theory. Jour. Appl. Phy., vol. 21, no. 2, Feb. 1950, pp. 159–161.
13. Jones, Robert T.: The Minimum Drag of Thin Wings in Frictionless Flow. Jour. Aero. Sci., vol. 18, no. 2, Feb. 1951, pp. 75–81.